Differential Calibration for A/B-steps on CoreXY Printers

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Abstract

Contrary to established practice, the rotation distance for the A- and B-motors on Core XY printers is not an exactly calculated value, it's a function of belt tension and manufacturing tolerances[1]; As such it is subject to calibration, which will be discussed in this paper. Current calibration processes either rely on directly measuring toolhead motion using a dial gauge indicator, or by printing a calibration model. Measuring the toolhead motion directly works poorly for CoreXY printers as the each motor drives the toolhead diagonally instead of parallell to the X and Y axes and it's difficult to correctly position the dial gauge indicator colinearly to the motors' driven axis. On the other hand, current methods relying on calibration prints do not in general account for both the material shrinkage, thermal expansion of the print bed and the fact that the A/B-steps of the motors affect the toolhead motion along the diagonals instead of directly along the X and Y axes. This paper proposes a method which is specifically designed to cancel out errors from material shrinkage and extrusion width, as well as minimize the impact from thermal expansion of the print bed. The method can also be applied to cartesian printers with slight modification. *If you're looking for the step-by-step tutorial, please navigate to: github.com/cmdremily/BoronTrident/, this document aims to prove the correctness and accuracy of the method rather than being a calibration guide.*

1 Introduction

The proposed method calibrates the physical XY movement of the toolhead and is by design not intended to calibrate the actually printed parts that the printer produces. In other words, the purpose of the calibration is to make sure that the toolhead performs the movements in the XY plane as commanded in G-Code. The A/B steps is a fundamental property of the printer, if they are incorrect other calibrations such as skew correction and part scaling in the slicer can be used to attempt to correct for them. However, since the toolhead isn't performing the movements as intended by the slicer, these calibrations can only *indirectly* adjust for the error in the A/B-steps, often with side effects such as perimeters being too close or too far away from eachother, or local over-/under-extrusion.

It's always best to correct any error at its source, which is the goal of this method.

2 Sources of Error

In order to produce an accurate calibration for the A/B steps we must devise a method of accurately measuring how far the toolhead has actually moved when commanded to move a specific distance. As measuring the toolhead movement directly on a CoreXY is difficult we are left with the option to indirectly calibrate the toolhead movenemt through printing a calibration model. The sources of error that we have to contend with are:

- Thermal expansion/contraction of the print bed
- Extrusion width error, both filament width inconsistency and extrusion feed rate error
- Material shrinkage
- Measurement error, both from the measuring device and the measuring technique

[1] Please see the appendix for more details.

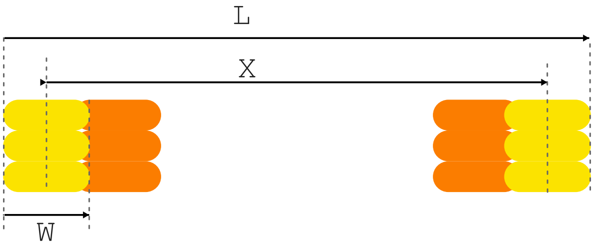

Figure 1: Two opposing walls without infill inbetween.

We will keep these in mind as we derive the method.

3 Method Overview

Let's consider the case with two opposing walls printed without any infill, as in figure 1. The distance between the outer walls is L, the distance from centre to centre of the exterior perimeter is X, and the extrusion width is W. These quantities trivially relate to each other as:

$$L = X + 2\frac{W}{2} = X + W. \tag{1}$$

However, if we have two pairs of such walls with different dimensions, call them: $L_1 = X_1 + W$ and $L_2 = X_2 + W$, then by taking the difference as:

$$\Delta L_{1,2} = L_1 - L_2 = X_1 - X_2 \tag{2}$$

we see that the effect of the extrusion width W is cancelled out. We will call this a "differential measurement" going forward. In fact, we can see that any constant that is added to L will be cancelled out in (2). This means that any error which is independent of X, and affects both pairs of walls equally, will be cancelled out in (2). This leads us to our first lemma:

Lemma 1. *Any error affecting the measured length L between the outer edges of two exterior perimeters, which is independent of the distance between the centres of the two opposing exterior extrusions (i.e. independent of X in (1)), and which equally affects two such measured lengths: L_1 and L_2 will be cancelled out when taking the difference between those: $\Delta L_{1,2} = L_1 - L_2$. In other words, any constant errors added to (1) will be cancelled out in (2).*

This is a key result that will allow us to cancel out the error from extrusion width and material shrinkage.

In the ideal case, X in (1) is equal to the distance that the toolhead moves between these two perimeters, and W is the extrusion width set in the slicer. However, in reality the toolhead doesn't move the exact intended distance, nor is the extrusion width exactly W. We call these non-ideal values as: $x = X\lambda$ where λ is a scale factor, and $w = W\delta + \epsilon$ where δ is a scale error on the extrusion width and ϵ is any error which can be described by lemma 1. We then get measured distance l in the non-ideal case:

$$l = x + 2\frac{w}{2} = x + w \tag{3}$$

and of course:

$$\Delta l_{1,2} = l_1 - l_2 = x_1 - x_2. \tag{4}$$

If we then take the ratio of the differential non-ideal measurement and the ideal differential measurement:

$$\frac{\Delta l_{1,2}}{\Delta L_{1,2}} = \frac{x_1 - x_2}{X_1 - X_2} = \frac{X_1\lambda - X_2\lambda}{X_1 - X_2} = \lambda \tag{5}$$

we get the scale error on the toolhead movement. We formulate this into our second lemma:

Lemma 2. *By taking two differential measurements along the same axis, the ratio between them will produce the scale error of the toolhead movement along that axis.*

We now have everything we need to design a calibration model which is immune to error in extrusion width (but not inconsistency) and material shrinkage as long as lemma 1 applies.

4 Designing a Calibration Model

Consider a calibration model that consists of three parallell, single width, single layer extrusions as in figure 2. It's obvious that the requirements for lemma 1 are met when taking the differential measurement between the two line pairs as indicated; Material shrinkage will affect all lines homogeneously and the distance between the lines isn't

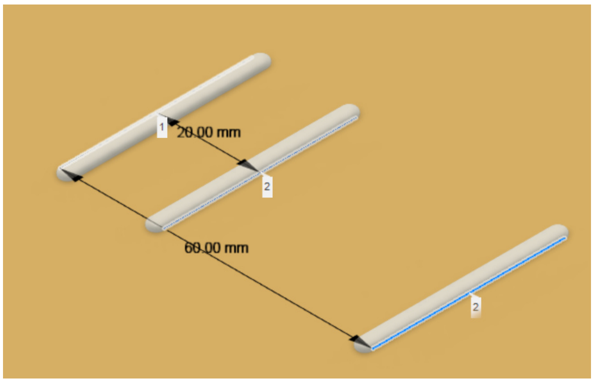

Figure 2: One extrusion wide lines to measure L_i from.

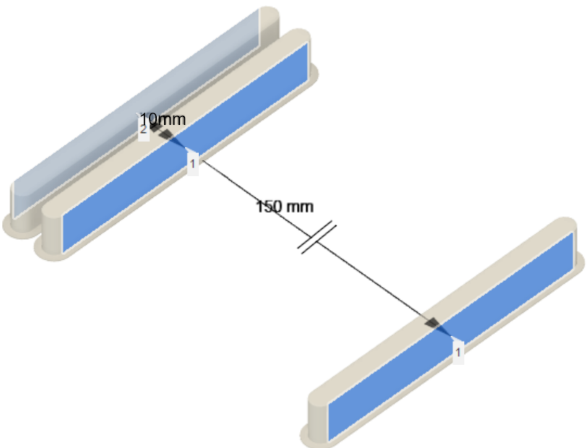

Figure 3: Improved calibration model for a single axis. Distances not to scale for image size reasons.

affected by material shrinkage, as they are trivial lines perpendicular to the measuring direction.

However, this simple calibration model while theoretically useful, is unpractical. Measuring single extrusions accurately is difficult as there is very little for the jaws of the calipers to align on, and making the extrusions taller just means that they will delaminate from the bed when measuring or deflect too easily, again making measuring difficult. A better calibration model is needed.

Inspired by the simple three parallell lines model above, we create a more rigid model that will be more practical for measurements as shown in figure 3.

Note that built-in brims are added to keep the pylons stable on the build plate during measuring. The brims are chosen so that they never touch another pylon's brim, therefore making sure that each pylon's shrinkage will be independent of each other. Each pylon should be printed with external perimeters first and have the seams positioned away from the measurement surfaces (marked blue in 3) to guarantee surface consistency. The pylons must also be printed with enough perimeters to produce a solid part that will shrink symmetrically. Finally, the part must be printed without any over-extrusion to prevent material build-up in the middle of the pylon during printing which may affect the measuring surface on the higher layers.

From the above we see that as the brims don't touch, no shrinking depends on X_1 or X_2. We also see that each pair of measurement surfaces will shrink towards the pylon centre identically so that means that the requirements for lemma 1 hold.

Although somewhat counter intuitive, the amount that the pylons shrink by doesn't matter, as long as they shrink in the same way, the requirements for lemma 1 are still met and the shrinkage is cancelled out in equation 2.

4.1 Impact from Filament Cross-Sectional Inconsistency

Let's assume that an extrusion width of w has been requested at a layer height of h and a linear XY speed of v, then we expect that a total flow rate of whv will be achieved. For a nominal filament diameter of d, linear extruder feed rate then becomes $f = \frac{whv}{\frac{d^2}{4}\pi}$. How much does the extrusion width w change if the filament diameter changes by ϵ?

Rewriting the above, gives w as:

$$\frac{fd^2\pi}{4hv} = w$$

introducing an error of ϵ to d gives:

$$\frac{f\left(d+\epsilon\right)^2\pi}{4hv} = w_\epsilon$$

$$w + \frac{f\left(2d\epsilon + \epsilon^2\right)\pi}{4hv} \overset{2d\epsilon \gg \epsilon^2}{\approx} w + \frac{f2d\epsilon\pi}{4hv} \approx w_\epsilon$$

plugging in $f = \frac{whv}{\frac{d^2}{4}\pi}$ again:

$$\frac{2\epsilon}{d}w \approx w_\epsilon - w$$

which gives the extrusion width error. The average of the reported deviation of the diameter on my spools of Prusament seems to be around $\epsilon = 0.02$ mm, taking that as a representative value of filament with low variance, this results in an extrusion width error of: $0.04mm/1.75mm \cdot 0.6mm \approx 13.7$µm for a 0.6 mm extrusion width. However, only half of this will be on the measuring surface, i.e. 6.9 µm (nice) surface inconsistency can be expected. This error is sufficiently small to not have a significant impact on the accuracy of the calibration.

It should be noted, that the calibration model proposed above uses a relatively low amount of filament so it's likely that the local consistency of the diameter of the filament will be better than the above. We can safely ignore this error, assuming that the calibration is done using filament with a low variance in the cross-sectional area.

4.2 Dimensioning the Calibration Model

Recall equation (5) for the scale factor λ. As the denominator consists of exact values from the calibration model, there are no sources of errors here. However, the numerator contains the measurement error twice, once for each reference dimension. In the best case, these errors can cancel, in the worst case they can stack. In order to minimize this error, the denominator should be made as large as possible, this is achieved by making the minor reference dimension as small as possible and the major dimension as large as possible. For the model provided, we chose 150mm for the major dimension as it's a common size for calipers (which can typically measure a few mm extra), and likewise we chose 10 mm for the minor dimensions as this was the smallest dimension that allowed the pylons to be sturdy enough to not deflect during measurement and the brim be large enough to securely keep the pylons on the print bed.

Using these dimensions, a stacked measurement error of 0.05 mm works out to an error in the steps value of $\frac{0.05}{150-10} \approx 0.0357\%$. For reference on a 20 tooth 2GT pulley, this is approximately 14µm per 40mm.

It's worth noting that if the measurement error is a bias error (i.e. consistently measuring slightly over or slightly under, or the instrument has an offset error) then that error will also cancel in equation 2 and doesn't affect the accuracy of λ. Therefore it's important to measure multiple times and practice measuring until consistent measurements are obtained.

For reference, a good pair of digital calipers can have an accuracy of 0.02 or 0.01 mm over the full 150 mm range. Assuming that the measurement technique used produces consistent measurements within 0.01mm (i.e. the error which isn't cancelled by equation 2 is 0.01mm), and the error from the instrument is 0.015 mm, and there's 0.005 mm error from the filamen inconsistency, then the total error is: $2\,(0.015 + 0.005) + 0.01 = 0.05$ mm. So the above accuracy is a representative upper bound on accuracy which is achievable with good instruments, good filament and good technique.

4.3 Aligning the Calibration Model

The calibration model as described above should be aligned so that the scale factor λ is measured in the primary direction of the motors. This means that for a cartesian printer, the measurement directions should be axis aligned. For a Core XY printer the calibration model should be on a 45° diagonal as each of the motor A-/B-motors drive a diagonal, see figure (4). A final note on Core XY printers, it's easy to get confused about which λ is for which motor, if on repeated calibrations the steps values doesn't converge, then it's highly likely that the motors' step values have been swapped.

Failure to do these steps will cause the calibration process to not converge on the correct steps value for the motors.

Do not rotate the calibration model in the slicer, rebuild it in the correct orientation in your CAD program to avoid errors from the floating point math in the matrix multiplication of the rotation in the slicer.

5 About Thermal Expansion of the Print Bed

For reference, if we take $14 \cdot 10^{-6}$ as a typical thermal expansion coefficient for a stainless steel

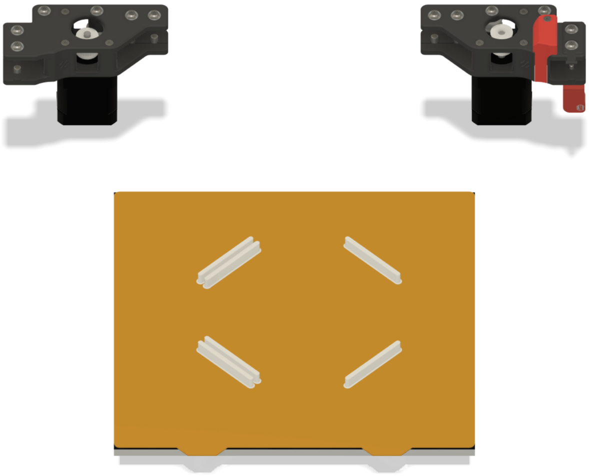

Figure 4: Calibration model correctly aligned for Core XY.

print sheet, then a 80° C bed when cooling down to 20°C will cause a 150mm distance to shrink by $14 \cdot 10^{-6} \cdot (80-20) \cdot 150 = 0.126$ mm which is large in comparison to the errors we're trying to calibrate for.

We will mitigate this source of error by reducing the temperature of the print-bed as far as possible and by measuring the calibration model on the heated print bed, thus not letting the bed contract by cooling.

6 Process Overview

The process of calibration is described in brief here, a step-by-step tutorial together with calibration models is available on github.com/cmdremily/BoronTrident/. This section should be considered as informative.

1. On the printer
   - Make sure belts are equally and properly tensioned, changing belt tensions means that the calibration has to be redone.
   - Clear any previous skewcorrections. On Core XY printers, the skew can often be corrected for solely by the steps calibration in this document. Re-do skew correction after steps have been calculated.
   - Reset motor steps to the default value calculated from pulley geometry.
   - Keep the nozzle clean from oozing, so it doesn't affect the measurements.
2. For your filament
   - Make sure that pressure advance has a reasonably good value for your filament of choice.
   - Make sure that retractions are tuned decently.
   - Reduce extrusion multiplier slightly.
   - Disable cooling fan and use the lowest temperatures possible to make shrinkage ocurr uniformly and minimally.
3. Slice the calibration model as noted in section 4
   - Slow down the print, around 60mm/s and 800mm/s2 is good. It's important that there's no ringing artefacts affecting the measuring surfaces.
4. Print the model
5. Measure the reference directions in both directions
6. Use equation (5) to compute the scale factor to correct the steps values by and update printer configuration
7. Repeat from step 4 until the change to steps values between iterations is small. The repetition may be needed if a significant skew was present on a Core XY printer to start, the skew would mean that the pylons form a rhombus instead of a rectangle, introducing a scale error on the measurement. Every iteration corrects the skew until eventually the method converges.

7 Closing Words

We have proven via lemmas (1) and (2) that if a few conditions are met, it's possible to design a calibration model that allows for the accurate calibration of A-/B-steps even in the presence of extrusion error and material shrinkage. We have also show an example of one such a calibration model and argued for its correctness and estimated that errors as low as 14µm or lower per 40mm can be achieved under good conditions.

8 Appendix: Why is the calculated A/B-steps value incorrect?

The A/B rotation distance is commonly considered a "calculated" quantity, where the value is given from the pulley geometry and considered an exact value. To see why this is wrong, consider that we can also calculate the exact value for the rotation distance for the extruder, and this is often done, yet the norm is to calibrate this value even after calculating it. Why would the extruder be special in this way? Another way to see that the calculated A/B rotation should be calibrated is to consider the following scenario: Two 20 tooth pulleys are attached to a shaft, and two idlers are attached to another shaft 1 meter away. We cut a belt that has the ideal length and pull it over the bottom pulley and idler, let's say that this belt has 1020 notches. Next we cut another belt, this time with 980 notches, and we stretch it over the top pulley and idler. Next we turn the pulley 51 full revolutions. The bottom belt with 1020 notches will have rotated one whole revolution ($\frac{51 \cdot 20}{1020} = 1$), but the top belt with 980 notches will have rotated $\frac{51 \cdot 20}{980} \approx 1.04$ revolutions. This means that as both belts span the same length, if you would draw a dot on both belts, the dot on the top belt will move faster and further than the bottom dot. In other words, the belt tension affects how far the belt (or the X carriage) moves with each revolution, therefore the rotation distance must be calibrated for A/B motors.

In a way this can be understood as, the rotation distance is the actual belt pitch multiplied by the tooth count on the pulley, but the belt pitch is a function of the belt tension and manufacturing tolerance, it's not an universal constant.