CHAPTER 2

Discrete-Time Signals and Systems

Basic Problems

21. See book companion toolbox for the function.

22. (a) $x[n]$ versus n as shown in Figure 2.1 on page 2.

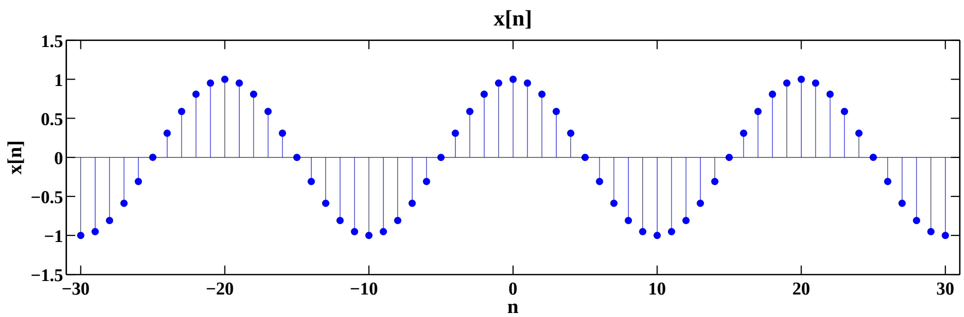

FIGURE 2.1: $x[n]$ versus n.

(b) A down sampled signal $y[n]$ for $M = 5$.

(c) A down sampled signal $y[n]$ for $M = 20$.

(d) Comments: The downsampled signal is compressed.

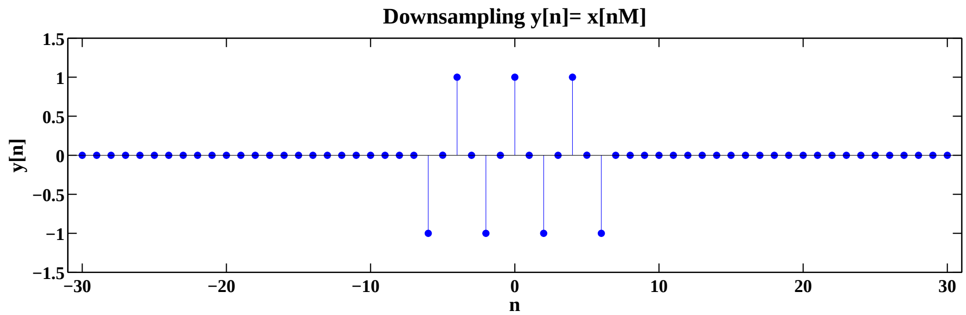

FIGURE 2.2: A down sampled signal $y[n]$ for $M = 5$.

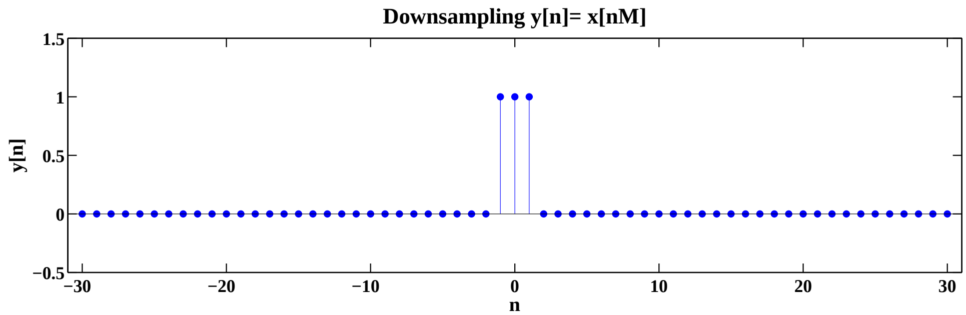

FIGURE 2.3: A down sampled signal $y[n]$ for $M = 20$.

23. (a) $y[n] = x[-n]$ (Time-flip)
linear, time-variant, noncausal, and stable

(b) $y[n] = \log(|x[n]|)$ (Log-magnitude)
nonlinear, time-invariant, causal, and unstable

(c) $y[n] = x[n] - x[n-1]$ (First-difference)
linear, time-invariant, causal, and stable

(d) $y[n] = \text{round}\{x[n]\}$ (Quantizer)
nonlinear, time-invariant, causal, and stable

24. Comments: The filtered data are smoother and $y_1[n]$ is 25 samples delayed than $y_2[n]$.

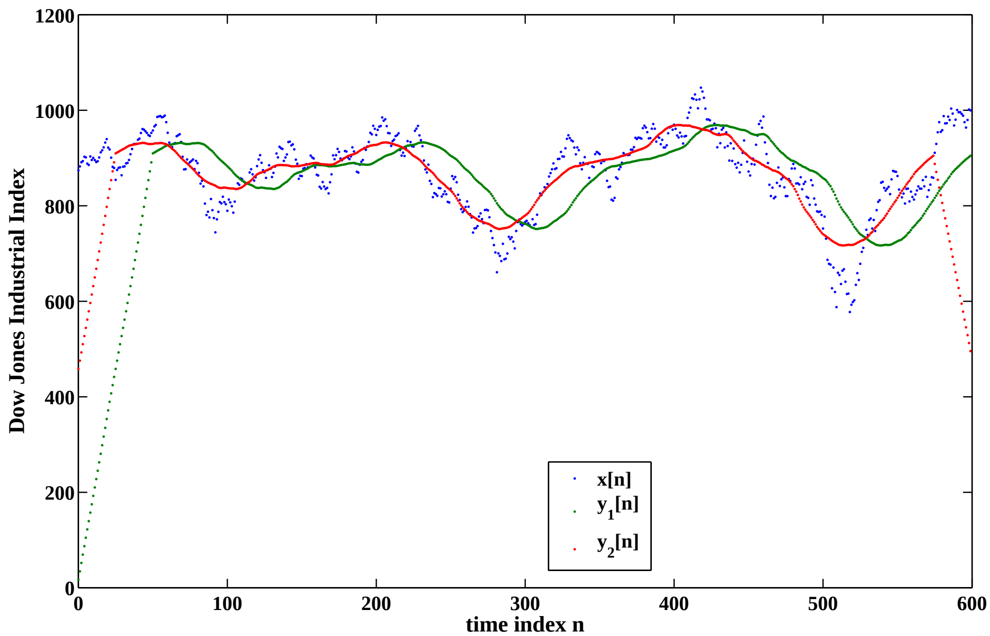

FIGURE 2.4: Dow Jones Industrial Average weekly opening value $x[n]$ and its moving averages.

25. (a) Solution:

$$\begin{aligned}
&\text{if } n \in [0, M-1] \\
&\quad y[n] = \frac{n(n+1)}{2} \\
&\text{if } n \in [M-1, N-1] \\
&\quad y[n] = \frac{M(M-1)}{2} \\
&\text{if } n \in [N-1, M+N-3] \\
&\quad y[n] = \frac{M(M-1)}{2} - \frac{(n-N+1)(n-N)}{2}
\end{aligned}$$

(b) Comments: The analytical solution can be verified.

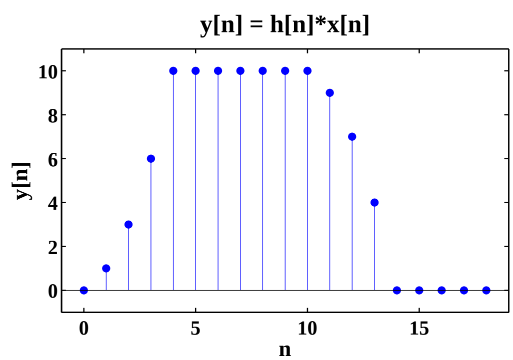

FIGURE 2.5: MATLAB verification of analytical expression for the sequence $y[n] = h[n] * x[n]$.

26. Solution:

$$
\begin{aligned}
&\text{if } n \in [0, N-1] \\
&\quad y[n] = \frac{1-a^{n+1}}{1-a} \\
&\text{if } n \in [N-1, M-1] \\
&\quad y[n] = \frac{a^{n+1}(a^{-N}-1)}{1-a}
\end{aligned}
$$

$$
\begin{aligned}
&\text{if } n \in [M-1, M+N-2] \\
&\quad y[n] = \frac{a^{n-N+1}-a^{M}}{1-a} \\
&\quad y[n] = 0, \quad \text{otherwise}
\end{aligned}
$$

27. Solution:

$$
y[n] = \frac{b^{n+1}-a^{n+1}}{b-a} u[n]
$$

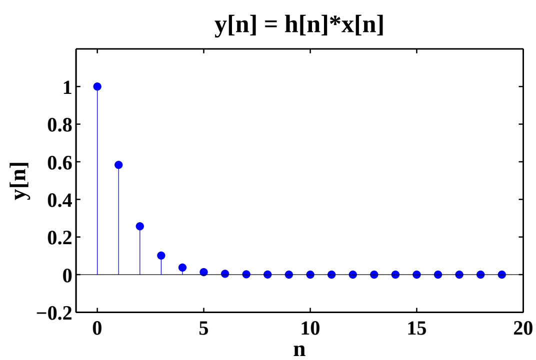

FIGURE 2.6: MATLAB verification of analytical expression for the sequence $y[n] = h[n] * x[n]$.

28. (a) Solution:

$$y[n] = (n+1)(0.9)^n u[n]$$

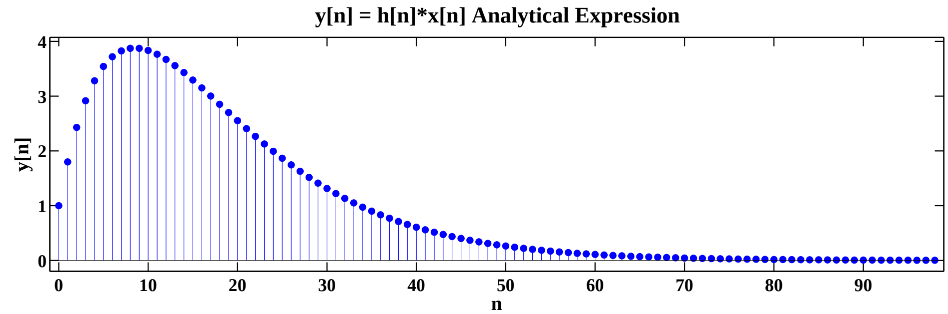

FIGURE 2.7: $y[n]$ plot determined analytically.

(b) $y[n]$ computed by `conv` function.

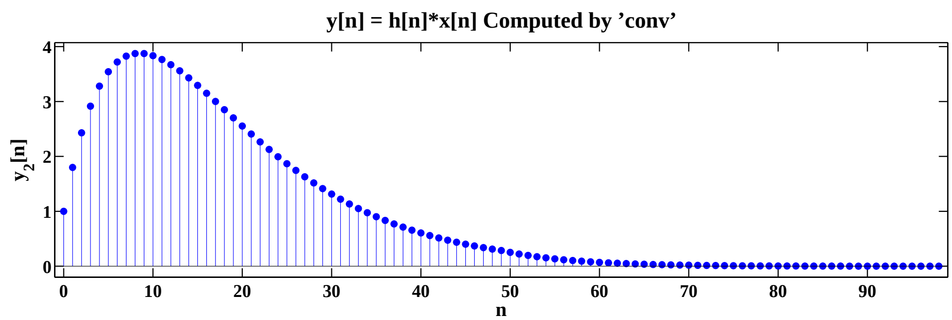

FIGURE 2.8: $y[n]$ plot determined by `conv` function.

(c) $y[n]$ computed by `filter` function.

(d) Comments: (c) comes closer to (a). Because in (b) the tail parts (samples from $n = 50$) of both $x[n]$ and $h[n]$ are curtailed, the second part samples (samples from $n = 50$) of (b) differ from the ones in (a).

29. See plots below.

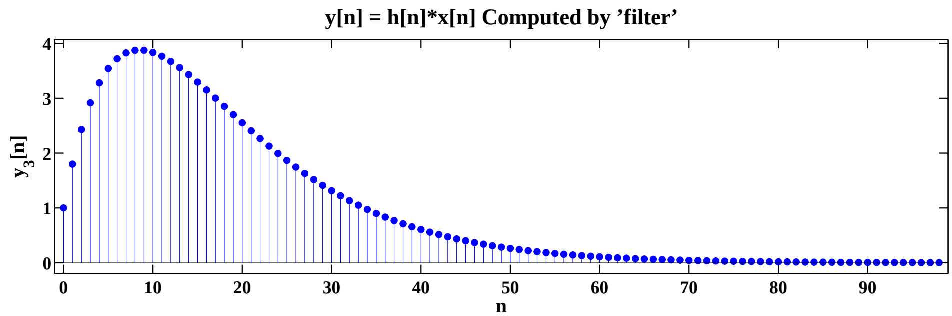

FIGURE 2.9: $y[n]$ plot determined by `filter` function.

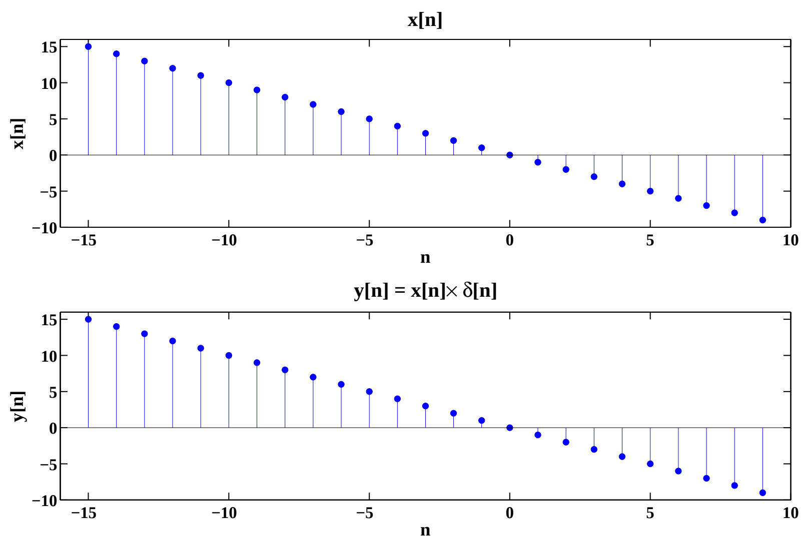

FIGURE 2.10: Verify identity property.

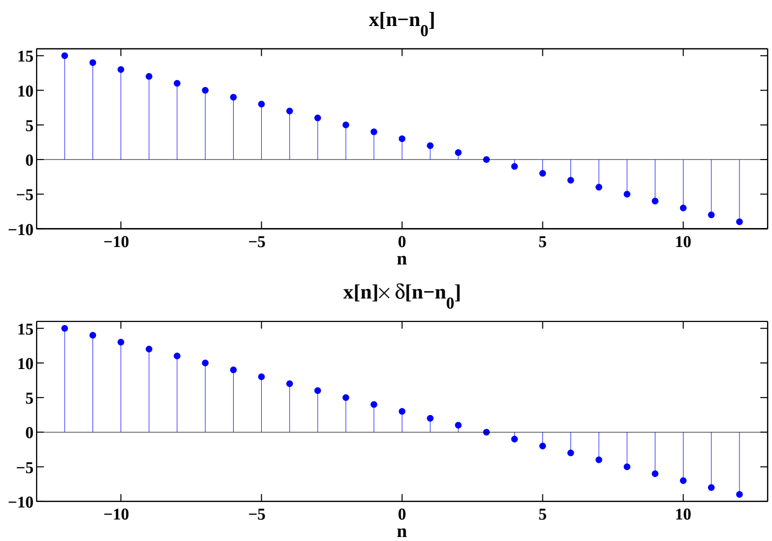

FIGURE 2.11: Verify delay property.

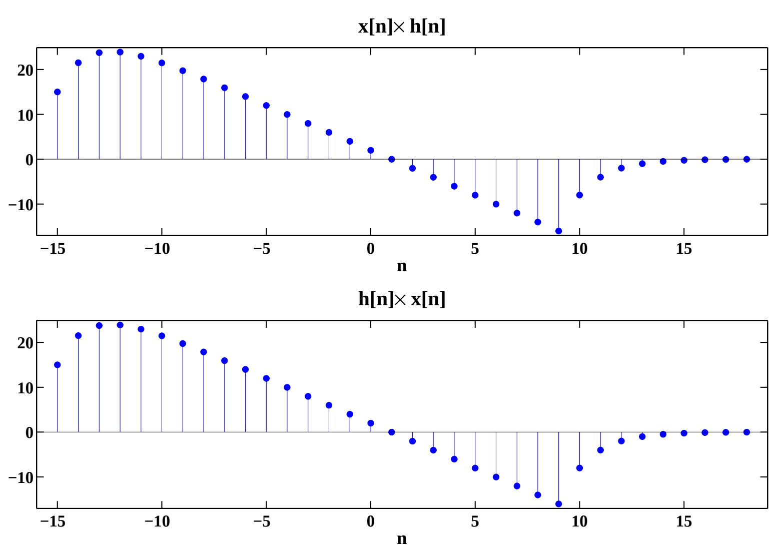

FIGURE 2.12: Verify commutative property.

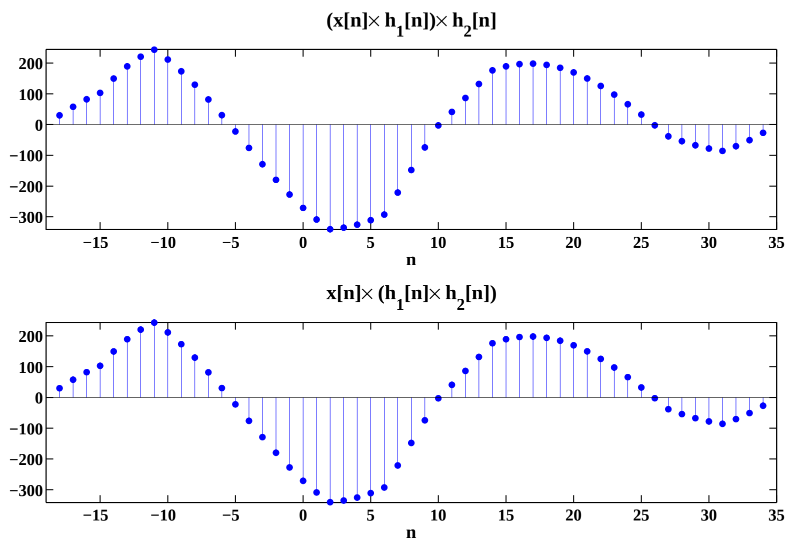

FIGURE 2.13: Verify associative property.

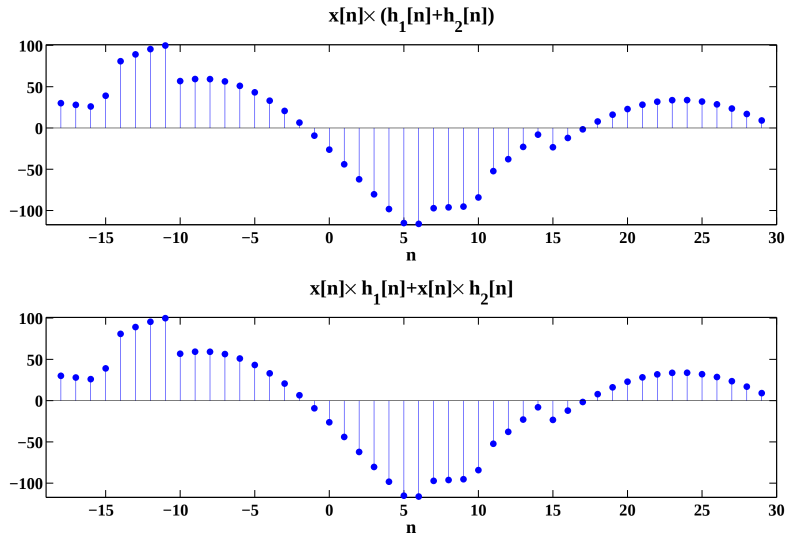

FIGURE 2.14: Verify distributive property.

30.

31. (a) See plot below.

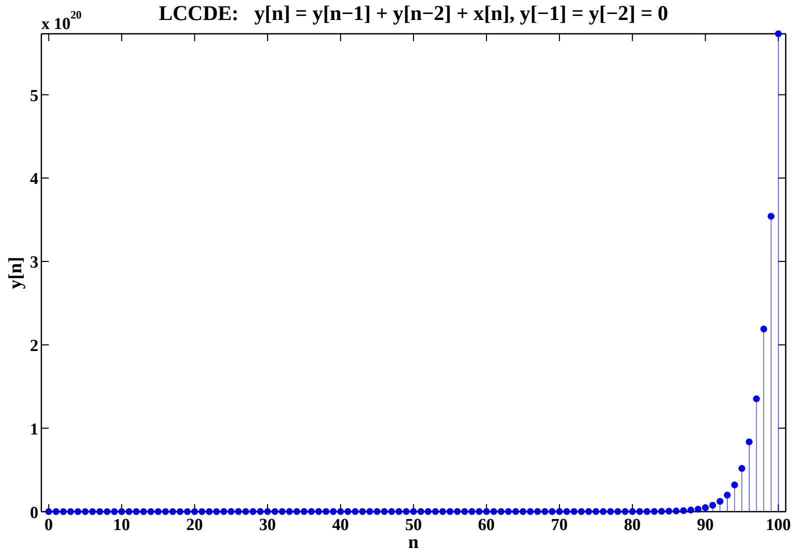

FIGURE 2.15: System impulse response for $0 \leq n \leq 100$, using function `filter`.

(b) Comments: The system is unstable.

(c) Comments: $h[n]$ is 1 sample left moved Fibonacci sequence.

32. See plot below.

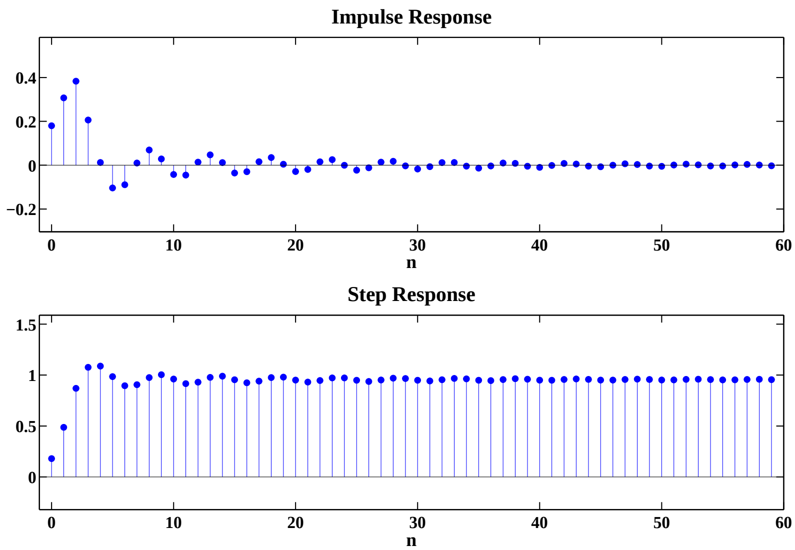

FIGURE 2.16: System impulse response and step response for first 60 samples using function `filter`.

33. See plots below.

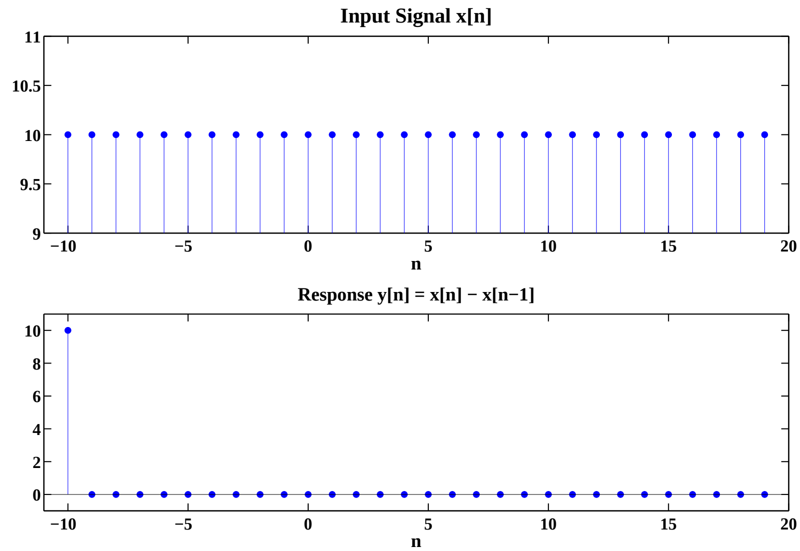

FIGURE 2.17: Differentiator output if input is $x[n] = 10\{u[n+10] - u[n-20]\}$.

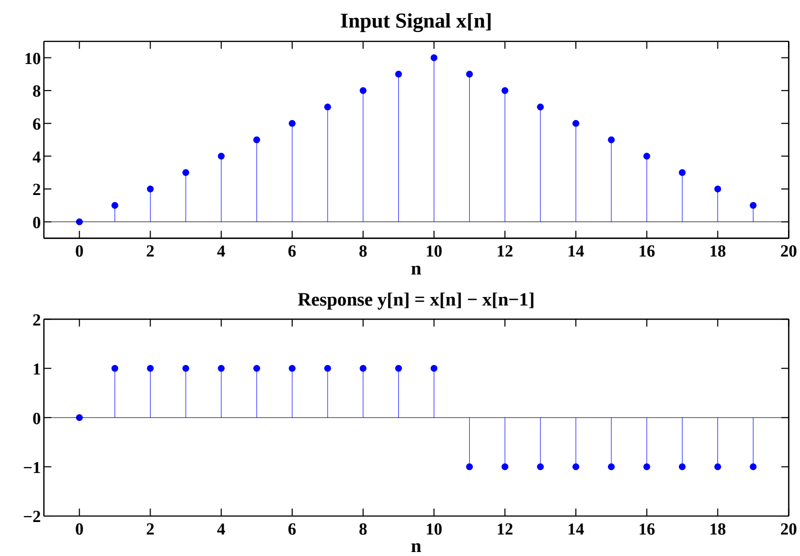

FIGURE 2.18: Differentiator output if input is $x[n] = n\{u[n] - u[n-10]\} + (20 - n)\{u[n-10] - u[n-20]\}$.

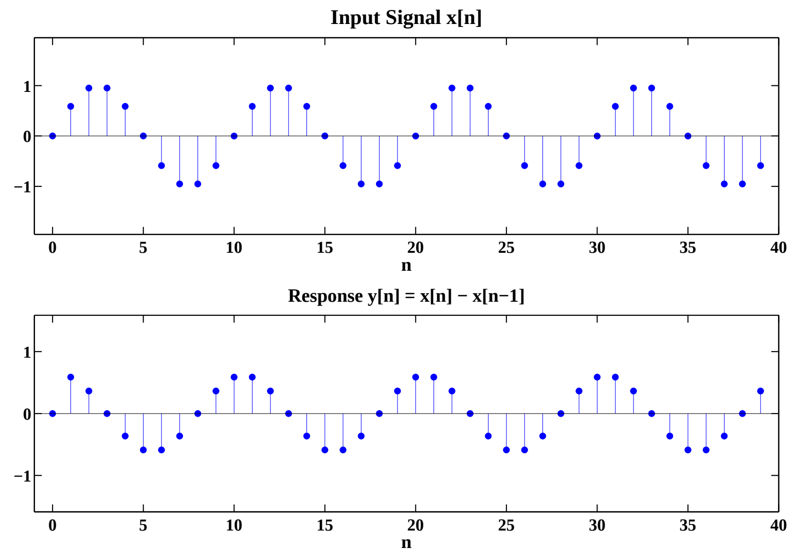

FIGURE 2.19: Differentiator output if input is $x[n] = \cos(0.2\pi n - \pi/2)\{u[n] - u[n-40]\}$.

34. See plots below.

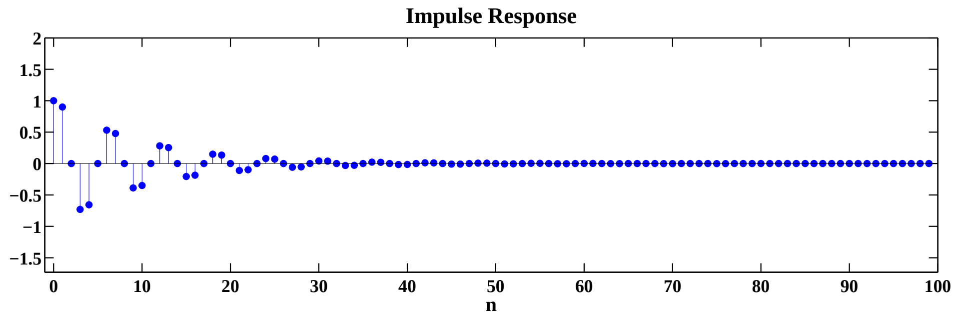

FIGURE 2.20: System impulse response.

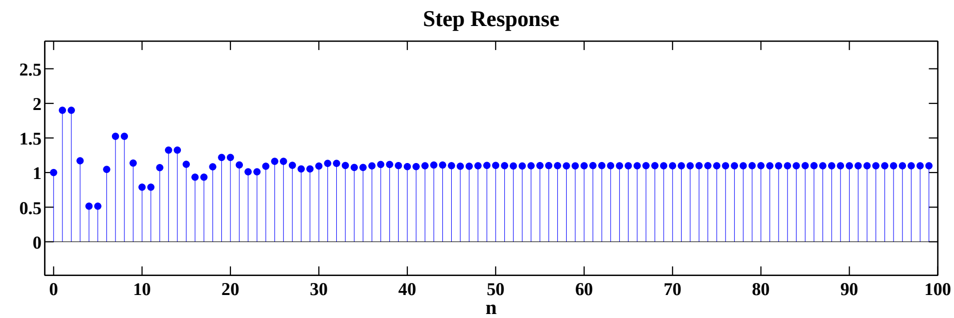

FIGURE 2.21: System step response.

35. (a) $y(t) = x(t-1) + x(2-t)$
linear, time-variant, noncausal, and stable

(b) $y(t) = dx(t)/dt$
linear, time-invariant, causal, and unstable

(c) $y(t) = \int_{-\infty}^{3t} x(\tau)\mathrm{d}\tau$
linear, time-variant, noncausal, and unstable

(d) $y(t) = 2x(t) + 5$
nonlinear, time-invariant, causal, and stable

36. (a) Solution:

$$\begin{aligned} \text{if } t \in [-1, 1],\ y(t) &= \frac{(1+t)^2}{6} \\ \text{if } t \in [1, 2],\ y(t) &= \frac{2t}{3} \\ \text{if } t \in [2, 4],\ y(t) &= \frac{-t^2+2t+8}{6} \\ y(t) &= 0 \quad \text{otherwise} \end{aligned}$$

(b)

(c) Comments: When $T = 0.01$, the error becomes negligible.

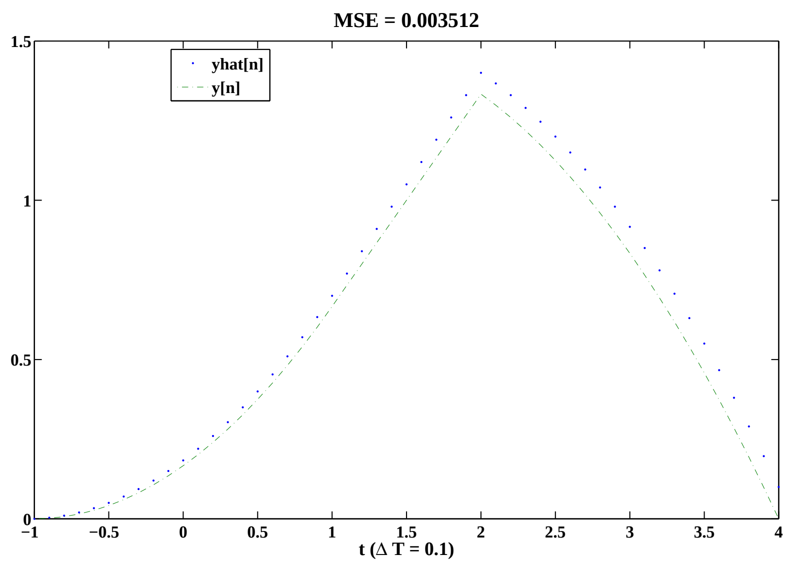

FIGURE 2.22: Plot of sequences $\hat{y}(nT)$ and $y(nT)$ for $T = 0.1$.

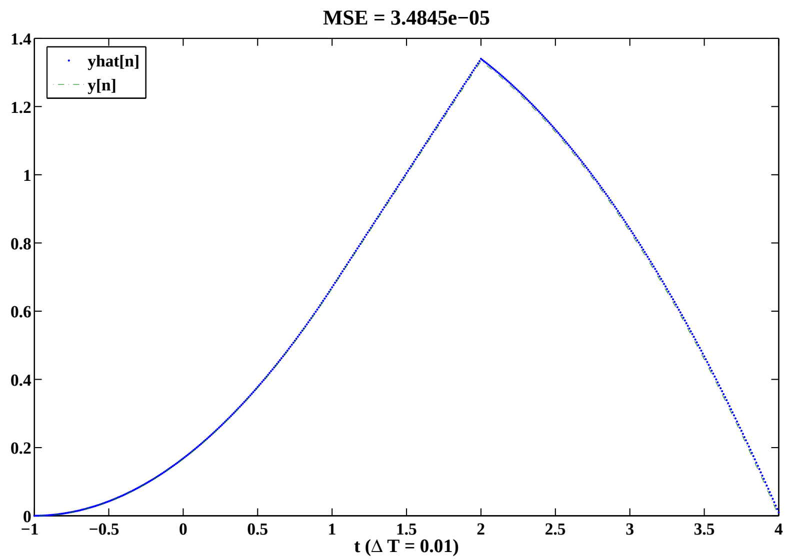

FIGURE 2.23: Plot of sequences $\hat{y}(nT)$ and $y(nT)$ for $T = 0.01$.