CHAPTER 4

Fourier Representation of Signals

Tutorial Problems

1. Solution:
   If there exists a fundamental period T, we have

$$
\begin{aligned}
x(t+T) &= x_1(t+T) + x_2(t+T) = x_1(t+mT_1) + x_2(t+nT_2) \\
&= x_1(t) + x_2(t) = x(t), \quad m, n = 1, 2, 3, \ldots
\end{aligned}
$$

   The condition is a finite T exists that

$$T = mT_1 = nT_2, \quad m, n = 1, 2, 3, \ldots$$

2. (a) Solution:
   $x_1(t)$ is periodic and its fundamental period is $T = 24$.

   (b) Solution:
   $x_2(t)$ is aperiodic.

   (c) Solution:
   $x_3[n]$ is aperiodic.

   (d) Solution:
   $x_4[n]$ is periodic and its fundamental period is $N = 24$.

   (e) Solution:
   $x_5(t)$ is periodic and its fundamental period is $T = 6$.

3. (a) $x_1(t) = 2\cos(10\pi t) \times 3\cos(20\pi t)$, $-0.2 \le t \le 0.2$.

   (b) $x_2(t) = 3\sin(0.2\pi t) \times 5\cos(2\pi t)$, $0 \le t \le 20$.

   (c) $x_3(t) = 5\cos(5\pi t) \times 4\sin(10\pi t)$, $0 \le t \le 2$.

   (d) $x_4(t) = 4\sin(100\pi t) \times 2\cos(400\pi t)$, $0 \le t \le 0.01$.

   MATLAB script:

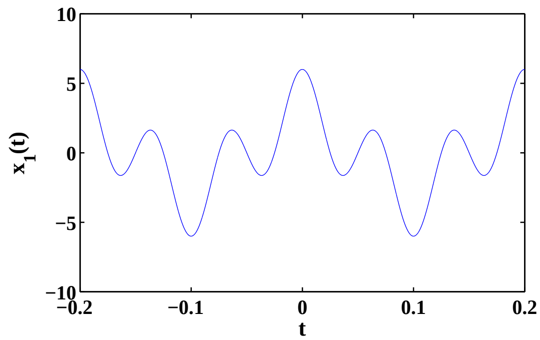


FIGURE 4.1: $x_1(t) = 2\cos(10\pi t) \times 3\cos(20\pi t)$, $-0.2 \le t \le 0.2$.

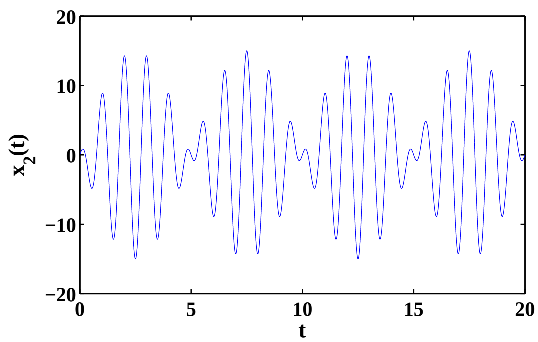


FIGURE 4.2: $x_2(t) = 3\sin(0.2\pi t) \times 5\cos(2\pi t)$, $0 \le t \le 20$.

```
% P0403: Verify the area under the function is zero
close all; clc
N = 100000;
%% Part (a):
% t = linspace(-0.2,0.2,N);
% x1 = 2*cos(10*pi*t).*3.*cos(20*pi*t);
% hf = figconfg('P0403','small');
% % hf = figconfg('P0403');
% plot(t,x1)
% xlabel('t','fontsize',LFS)
% ylabel('x_1(t)','fontsize',LFS)
% sum(x1.*0.4/N)
```

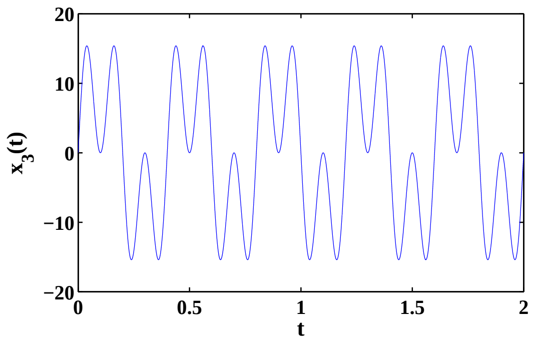


FIGURE 4.3: $x_3(t) = 5\cos(5\pi t) \times 4\sin(10\pi t)$, $0 \leq t \leq 2$.

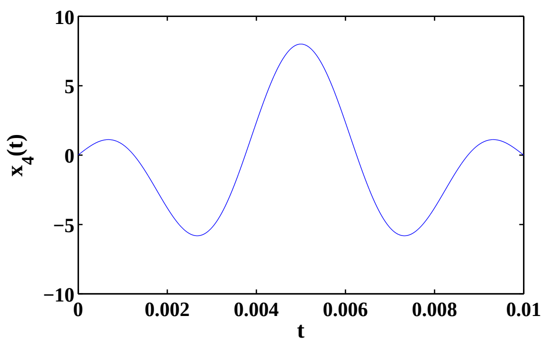


FIGURE 4.4: $x_4(t) = 4\sin(100\pi t) \times 2\cos(400\pi t)$, $0 \leq t \leq 0.01$.

```
%% Part (b):
% t = linspace(0,20,N);
% x2 = 3*sin(0.2*pi*t).*5.*cos(2*pi*t);
% hf = figconfg('P0403','small');
% plot(t,x2)
% xlabel('t','fontsize',LFS)
% ylabel('x_2(t)','fontsize',LFS)
% sum(x2.*20/N)

%% Part (c):
% t = linspace(0,2,N);
% x3 = 5*cos(5*pi*t).*4.*sin(10*pi*t);
% hf = figconfg('P0403','small');
```

```
% plot(t,x3)
% xlabel('t','fontsize',LFS)
% ylabel('x_3(t)','fontsize',LFS)
% sum(x3.*2/N)

%% Part (d):
t = linspace(0,0.01,N);
x4 = 4*sin(100*pi*t).*2.*cos(400*pi*t);
hf = figconfg('P0403','small');
plot(t,x4)
xlabel('t','fontsize',LFS)
ylabel('x_4(t)','fontsize',LFS)
sum(x4.*0.01/N)
```

4. (a) Solution:
The fundamental period of $x(t)$ is $T = 2$.

$$\int_0^2 \sin(3\pi t)dt = \int_0^2 \cos(8\pi t+\pi/3)dt = \int_0^2 \sin(3\pi t)\cos(8\pi t+\pi/3)dt = 0$$

$$\begin{aligned} P_{\text{av}} &= \frac{1}{T}\int_0^2 |x(t)|^2 dt \\ &= \frac{1}{2}\int_0^2 4dt + \frac{1}{2}\int_0^2 16\cos^2(3\pi t - \pi/2)dt + \frac{1}{2}\int_0^2 36\cos^2(8\pi t + \pi/3)dt \\ &= 4 + 8\int_0^2 \frac{1-\cos(6\pi t - \pi)}{2}dt + 18\int_0^2 \frac{1-\cos(16\pi t + 2\pi/3)}{2}dt \\ &= 30 \end{aligned}$$

(b) Solution:

$$\Omega_0 = 2\pi \cdot \frac{1}{T} = \pi$$

(c) Solution:

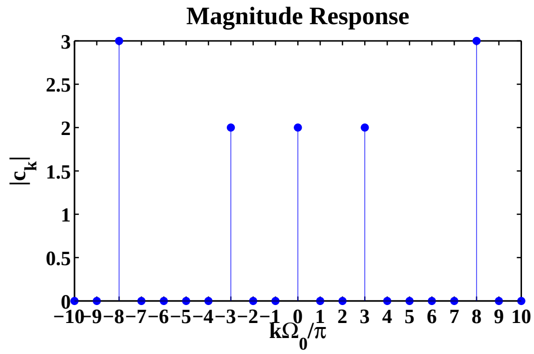
$$x(t) = 2\mathrm{e}^{\mathrm{j}0\pi t} + 2\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}\mathrm{e}^{j3\pi t} + 2\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}\mathrm{e}^{-j3\pi t} + 3\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{j8\pi t} + 3\mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{-j8\pi t}$$

$$c_0 = 2, c_3 = 2\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}, c_{-3} = 2\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}, c_8 = 3\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}, c_{-8} = 3\mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}.$$

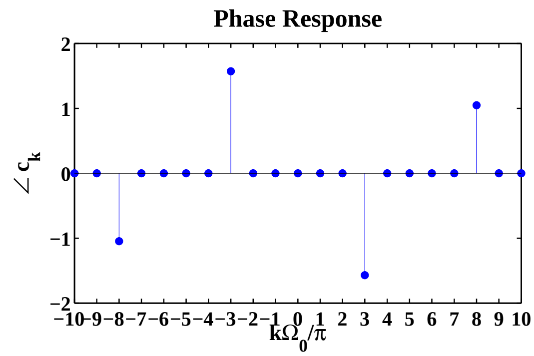
(d) Solution:

$$P_{\text{av}} = \sum_{k=-\infty}^{\infty} |c_k|^2 = 30$$

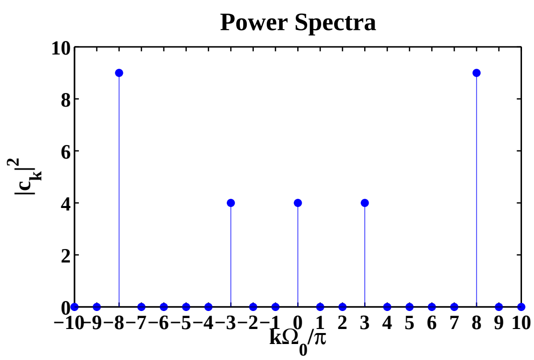
which verifies our computation in part (a).



(a)



(b)



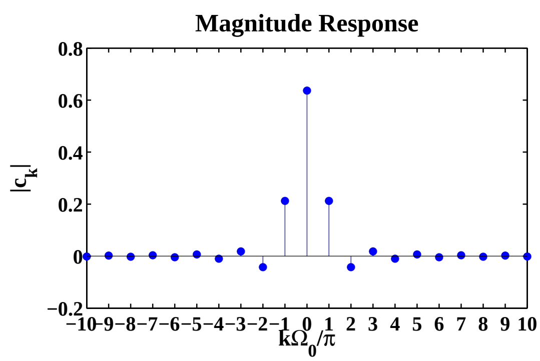
(c)

FIGURE 4.5: (a) Magnitude response of $x(t)$. (b) Phase response of $x(t)$. (c) Power spectra of $x(t)$.

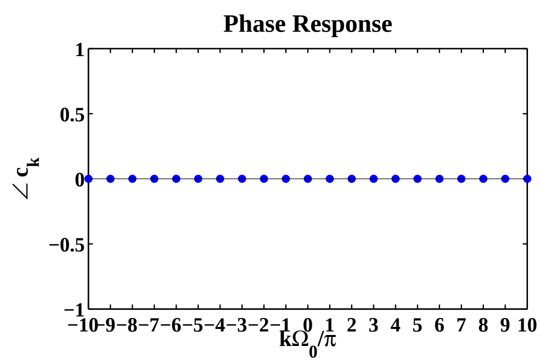
5. Solution:

$$T = \frac{2\pi}{10\pi} \cdot \frac{1}{2} = \frac{1}{10}, \quad \Omega_0 = 20\pi.$$

$$\begin{aligned}
c_k &= \frac{1}{T}\int_T \cos(10\pi t)\mathrm{e}^{\mathrm{j}k\Omega_0 t}dt = 5\int_{-\frac{1}{20}}^{\frac{1}{20}} \left(\mathrm{e}^{\mathrm{j}10\pi t} + \mathrm{e}^{-\mathrm{j}10\pi t}\right)\mathrm{e}^{\mathrm{j}k\cdot 2\pi t}dt \\
&= 5\int_{-\frac{1}{20}}^{\frac{1}{20}} \left(\mathrm{e}^{\mathrm{j}(2k+1)10\pi t} + \mathrm{e}^{-\mathrm{j}(2k-1)10\pi t}\right)dt \\
&= \frac{5}{\mathrm{j}(2k+1)10\pi} \cdot \mathrm{e}^{\mathrm{j}(2k+1)10\pi t}\Big|_{-\frac{1}{20}}^{\frac{1}{20}} + \frac{5}{\mathrm{j}(2k-1)10\pi} \cdot \mathrm{e}^{\mathrm{j}(2k-1)10\pi t}\Big|_{-\frac{1}{20}}^{\frac{1}{20}} \\
&= \frac{\sin\left[\frac{(2k+1)\pi}{2}\right]}{(2k+1)\pi} + \frac{\sin\left[\frac{(2k-1)\pi}{2}\right]}{(2k-1)\pi}
\end{aligned}$$



(a)



(b)

FIGURE 4.6: (a) Magnitude response of $x(t)$. (b) Phase response of $x(t)$.

6. Proof:

$$P_{\text{av}} = \frac{1}{T_0}\int_{T_0} x(t)x^*(t)dt = \frac{1}{T_0}\int_{T_0}\left(\sum_k c_k \mathrm{e}^{\mathrm{j}k\Omega_0 t}\right)\left(\sum_m c_m \mathrm{e}^{\mathrm{j}m\Omega_0 t}\right)^* dt$$
$$= \frac{1}{T_0}\sum_k\sum_m c_k c_m^* \int_{T_0} \mathrm{e}^{\mathrm{j}k\Omega_0 t}\mathrm{e}^{-\mathrm{j}m\Omega_0 t}$$

$$\int_{T_0} \mathrm{e}^{\mathrm{j}k\Omega_0 t}\mathrm{e}^{-\mathrm{j}m\Omega_0 t} = \begin{cases} 0, & k \neq m \\ T_0, & k = m \end{cases}$$

$$P_{\text{av}} = \frac{1}{T_0}\sum_k T_0 \cdot c_k \cdot c_k^* = \sum_{k=-\infty}^{\infty} |c_k|^2$$

7. Solution:

$$c_k = \frac{1}{T_0}\int_{T_0} y(t)\mathrm{e}^{-\mathrm{j}k\Omega_0 t}dt = \frac{1}{T_0}\int_{T_0} h(t)x(t)\mathrm{e}^{-\mathrm{j}k\Omega_0 t}dt$$
$$= \frac{1}{T_0}\int_{T_0}\left(\sum_m a_m \mathrm{e}^{\mathrm{j}m\Omega_0 t}\right)\left(\sum_n b_n \mathrm{e}^{\mathrm{j}n\Omega_0 t}\right)\mathrm{e}^{-\mathrm{j}k\Omega_0 t}dt$$
$$= \sum_m\sum_n a_m b_n \cdot \frac{1}{T_0}\int_{T_0} \mathrm{e}^{\mathrm{j}(m+n)\Omega_0 t}\cdot \mathrm{e}^{-\mathrm{j}k\Omega_0 t}dt$$
$$= \sum_{m+n=k} a_m b_n = \sum_{\ell=-\infty}^{\infty} a_\ell b_{k-\ell}$$

8. (a) Solution:

$$X(\mathrm{j}2\pi F) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-\mathrm{j}2\pi F t}dt = \int_{-1}^{1} \mathrm{e}^{-t}\cdot \mathrm{e}^{-\mathrm{j}2\pi F t}dt$$
$$= -\frac{\mathrm{e}^{-(\mathrm{j}2\pi F+1)t}}{\mathrm{j}2\pi F + 1}\Bigg|_{-1}^{1} = \frac{\mathrm{e}^{\mathrm{j}2\pi F+1} - \mathrm{e}^{-\mathrm{j}2\pi F-1}}{\mathrm{j}2\pi F + 1}$$

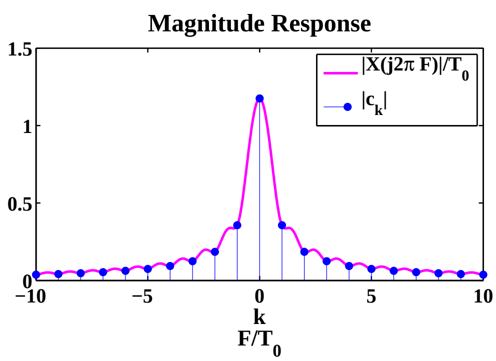
$$T = 2, \quad F_0 = \frac{1}{T} = \frac{1}{2}, \quad \Omega_0 = 2\pi F_0 = \pi$$

$$c_k = \frac{1}{T}\int_T \tilde{x}(t)\mathrm{e}^{-\mathrm{j}2\pi k F_0 t}dt = \frac{1}{2}\int_{-1}^{1} \mathrm{e}^{-t}\cdot \mathrm{e}^{-\mathrm{j}\pi k t}dt$$
$$= \frac{1}{2}\cdot \frac{\mathrm{e}^{-(\mathrm{j}k\pi+1)t}}{-(\mathrm{j}k\pi + 1)}\Bigg|_{-1}^{1} = \frac{\mathrm{e}^{\mathrm{j}k\pi+1} - \mathrm{e}^{-\mathrm{j}k\pi-1}}{2(\mathrm{j}k\pi + 1)}$$

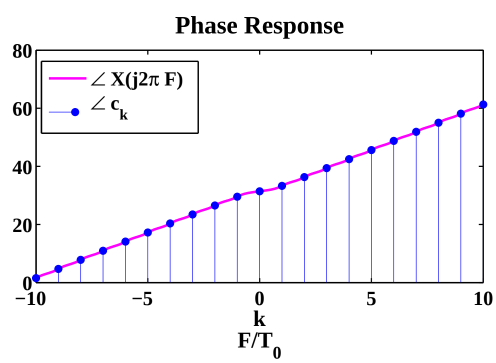
(b) Solution:

$$
\begin{aligned}
X(\mathrm{j}2\pi k/T_0)\,/T_0 &= \frac{\mathrm{e}^{\mathrm{j}2\pi\frac{k}{2}+1} - \mathrm{e}^{-\mathrm{j}2\pi\frac{k}{2}-1}}{\mathrm{j}2\pi\frac{k}{2}+1} \cdot \frac{1}{2} \\
&= \frac{\mathrm{e}^{\mathrm{j}k\pi+1} - \mathrm{e}^{-\mathrm{j}k\pi-1}}{2(\mathrm{j}k\pi+1)} = c_k
\end{aligned}
$$

(c)



(a)



(b)

FIGURE 4.7: (a) $|X(\mathrm{j}2\pi F)|$ and $|c_k|$. (b) $\angle X(\mathrm{j}2\pi F)$ and $\angle c_k$.

9. (a) Solution:

$$X(\mathrm{j}2\pi F) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-\mathrm{j}2\pi Ft}dt = \int_{-\infty}^{\infty} \frac{2\sin 2\pi t}{2\pi t}\mathrm{e}^{-\mathrm{j}2\pi Ft}dt$$
$$= \begin{cases} 1, & -1 < F < 1 \\ 0, & \text{otherwise} \end{cases}$$

(b) Solution:

$$c_k = 4\int_{-\frac{1}{80}}^{\frac{1}{80}} 1 \cdot \mathrm{e}^{-\mathrm{j}2\pi kF_0 t}dt = 4 \cdot \left.\frac{\mathrm{e}^{-\mathrm{j}2\pi kF_0 t}}{-\mathrm{j}2\pi kF_0}\right|_{-\frac{1}{80}}^{\frac{1}{80}} = \frac{\sin\frac{\pi}{10}k}{\pi k}$$

(c) Solution:

$$X_s(\mathrm{j}2\pi F) = \sum_{k=-\infty}^{\infty} \frac{\sin\frac{\pi}{10}k}{\pi k} \cdot X[\mathrm{j}2\pi(F-4k)]$$

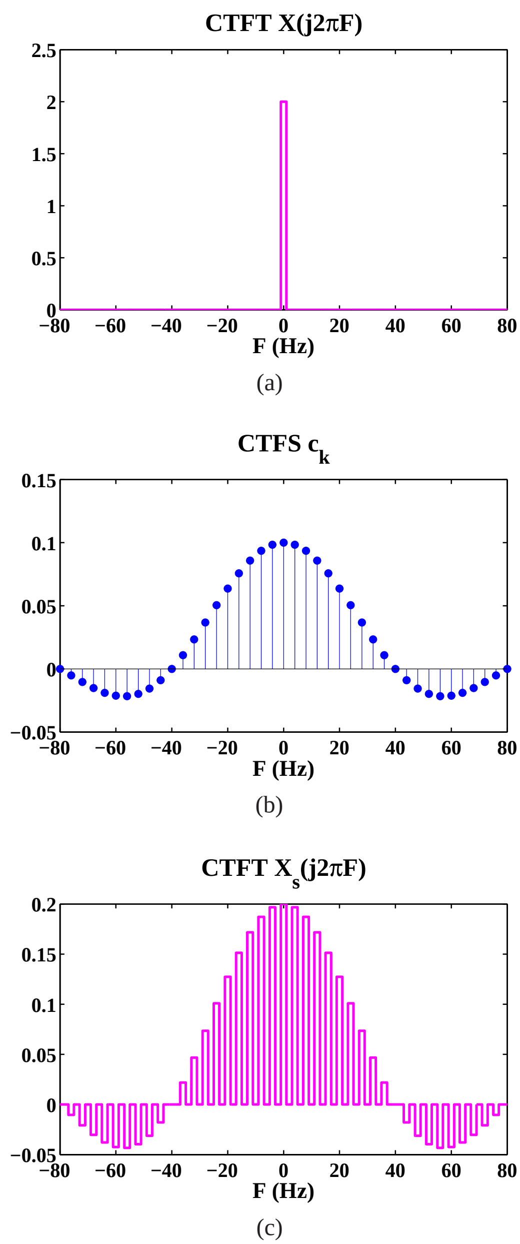


FIGURE 4.8: (a) Plot of CTFT $X(\mathrm{j}2\pi F)$. (b) Plot of CTFS coefficients c_k. (c) Plot of CTFT $X_s(\mathrm{j}2\pi F)$.

10. (a)
```
function c = dtfs0(x)
% P0410(a): Write a function c=dtfs0(x) which compute
% the DTFS coefficients (4.67) of a periodic signal
N = length(x);
x = x(:)';
k = 0:N-1;
n = 0:N-1;
nk = n'*k;
matexp = exp(-j*2*pi/N*nk);
c = x*matexp/N;
```

(b)
```
function x = idtfs0(c)
% P0410(b): Write a function x=idtfs0(c) which compute
% the inverse DTFS (4.63)
N = length(c);
c = c(:)';
k = 0:N-1;
n = 0:N-1;
kn = k'*n;
matexp = exp(j*2*pi/N*kn);
x = c*matexp;
```

(c)
```
% P0410c: Verify functions c=dtfs0(x) and x=idtfs0(c)
%         using specification in Example4.9
clc; close all;
x = [1 1 1 0 0 0 0 0 1 1]; N = length(x);
c1 = fft(x)/N;
c2 = real(dtfs0(x));
x1 = ifft(c1)*N;
x2 = real(idtfs0(c2));
```

11. (a) Solution:

$$x_1[n] = \sin[2\pi(3/10)n] = \frac{1}{2\mathrm{j}}\left[\mathrm{e}^{\mathrm{j}\frac{2\pi}{10}3\pi} - \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}3\pi}\right] = \frac{1}{2\mathrm{j}}\left[\mathrm{e}^{\mathrm{j}\frac{2\pi}{10}3\pi} - \mathrm{e}^{\mathrm{j}\frac{2\pi}{10}7\pi}\right]$$

$$c_3 = \frac{1}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}, \quad c_7 = \frac{1}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}$$

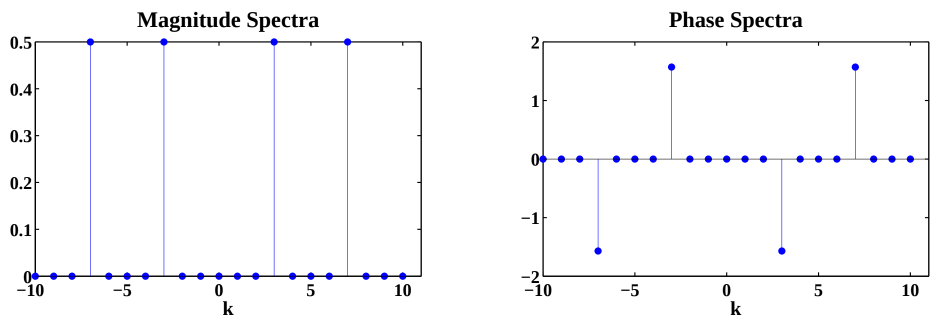


FIGURE 4.9: Magnitude and phase spectra of periodic sequence $x_1[n] = \sin[2\pi(3/10)n]$.

(b) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{6}\sum_{n=0}^{5} x_2[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{6}kn} \\
&= \frac{1}{6}\cdot\left[\mathrm{e}^{-\mathrm{j}\frac{2\pi}{6}k0} + 2\mathrm{e}^{-\mathrm{j}\frac{2\pi}{6}k1} - \mathrm{e}^{-\mathrm{j}\frac{2\pi}{6}k2} + 0 - \mathrm{e}^{-\mathrm{j}\frac{2\pi}{6}k4} + 2\mathrm{e}^{-\mathrm{j}\frac{2\pi}{6}k5}\right] \\
&= \frac{1}{6}\left[1 + 4\cos\left(\frac{2\pi}{6}k\right) - 2\cos\left(\frac{4\pi}{6}k\right)\right]
\end{aligned}
$$

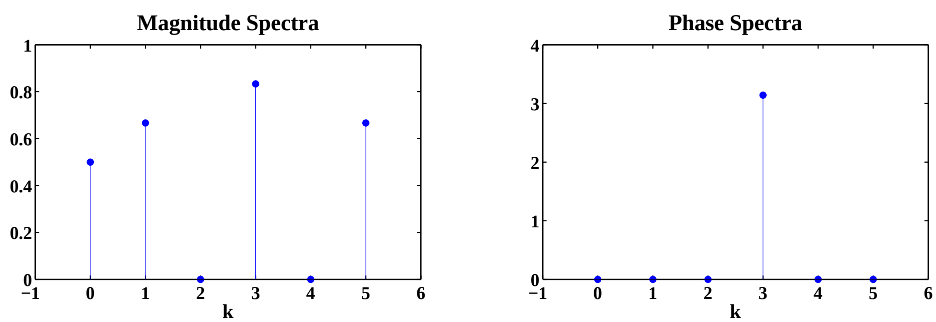


FIGURE 4.10: Magnitude and phase spectra of periodic sequence $x_2[n] = \{1, 2, -1, 0, -1, 2\}, 0 \leq n \leq 5$ (one period).

(c) Solution:

$$\begin{aligned} c_k &= \frac{1}{4}\sum_{n=0}^{3}\left[1-\sin\left(\frac{\pi n}{4}\right)\right] \mathrm{e}^{-\mathrm{j}\frac{2\pi}{4}kn} \\ &= \frac{1}{4}\left[1+(1-\sin(\frac{\pi}{4})\mathrm{e}^{-\mathrm{j}\frac{2\pi}{4}k})+0+(1-\sin(\frac{\pi}{4})\mathrm{e}^{-\mathrm{j}\frac{2\pi}{4}k3})\right] \\ &= \frac{1}{4}\left[1+(1-\sin(\frac{\pi}{4})\mathrm{e}^{-\mathrm{j}\frac{2\pi}{4}k})+0+(1-\sin(\frac{\pi}{4})\mathrm{e}^{\mathrm{j}\frac{2\pi}{4}k})\right] \\ & \frac{1}{4}\left[1+2(1-\sin(\frac{\pi}{4}))\cos(\frac{k\pi}{2})\right] \end{aligned}$$

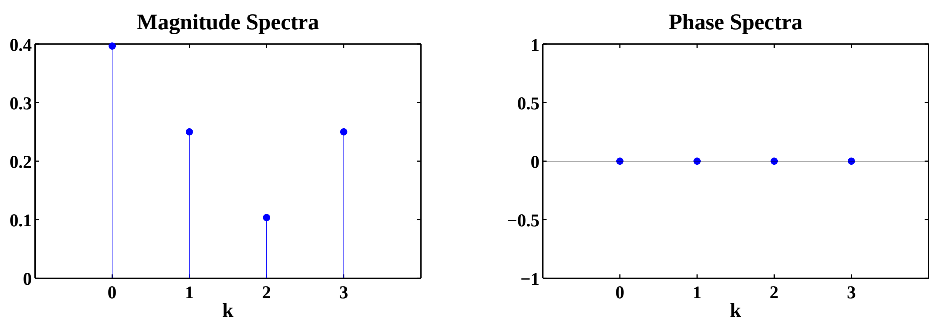


FIGURE 4.11: Magnitude and phase spectra of periodic sequence $x_3[n] = 1 - \sin(\pi n/4), 0 \leq n \leq 3$ (one period).

(d) Solution:

$$\begin{aligned} c_k &= \frac{1}{12}\sum_{n=0}^{11}\left[1-\sin\left(\frac{\pi n}{12}\right)\right] \mathrm{e}^{-\mathrm{j}\frac{2\pi}{4}kn} \\ &= \frac{1}{12}\left[1+(1-\sin(\frac{\pi}{4}))2\cos(\frac{k\pi}{6})+(1-\sin(\frac{3\pi}{4}))2\cos(\frac{k\pi}{2})\right. \\ &\left. +2\cos(\frac{2k\pi}{3})+(1-\sin(\frac{5\pi}{4}))2\cos(\frac{5k\pi}{6})+2\cos(k\pi)\right] \end{aligned}$$

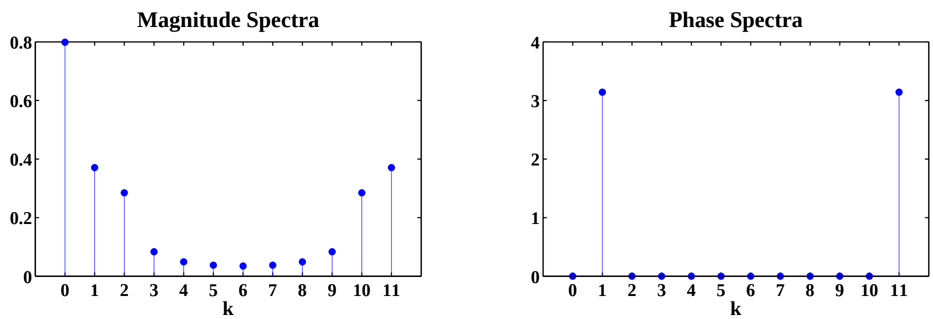


FIGURE 4.12: Magnitude and phase spectra of periodic sequence $x_4[n] = 1 - \sin(\pi n/4), 0 \leq n \leq 11$ (one period).

(e) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{8}\sum_{n=0}^{7} x_5[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}kn} \\
&= \frac{1}{8}\left[1 + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k3} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k4} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k5} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k7}\right] \\
&= \frac{1}{8}\left[1 + 2\cos(\frac{k\pi}{4}) + 2\cos(\frac{3k\pi}{4}) + \cos(k\pi)\right]
\end{aligned}
$$

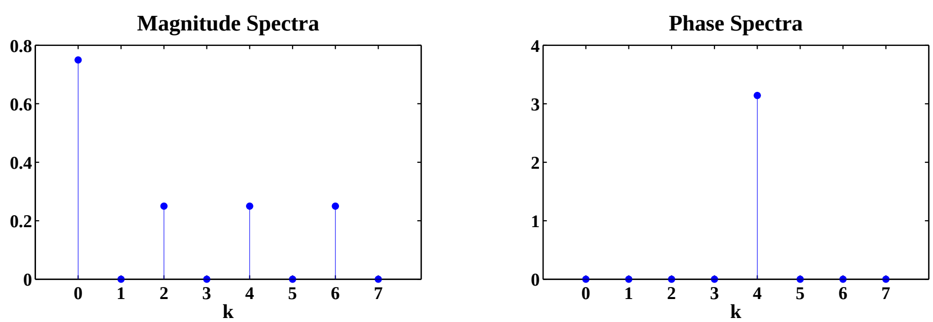


FIGURE 4.13: Magnitude and phase spectra of periodic sequence $x_5[n] = \{1, 1, 0, 1, 1, 1, 0, 1\}, 0 \leq n \leq 7$ (one period).

(f) Solution:

$$c_k = \frac{1}{N_0}\sum_{n=0}^{N_0-1} 1 \cdot \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N_0}kn} = \delta[k]$$

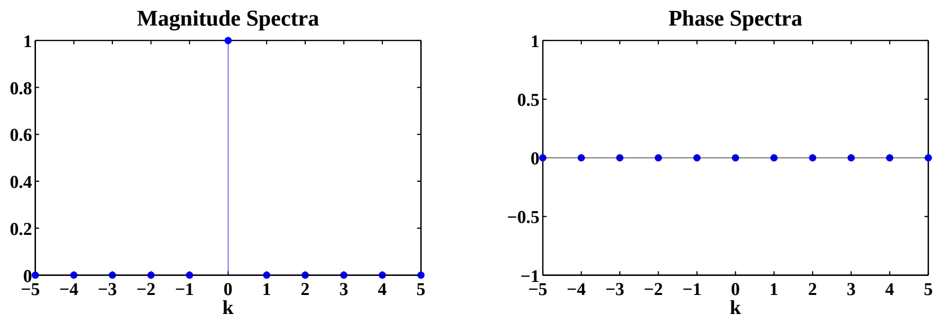


FIGURE 4.14: Magnitude and phase spectra of periodic sequence $x_6[n] = 1$ for all n.

12. Solution:
    (a)

$$X_1(\omega) = \frac{1}{1 - \mathrm{e}^{-\mathrm{j}\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2k\pi)$$

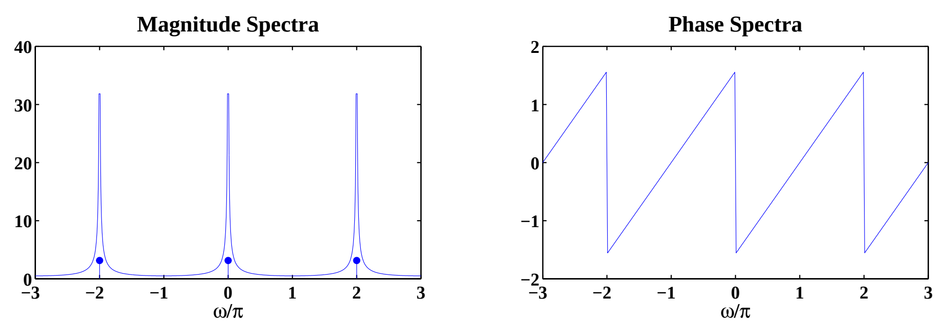


FIGURE 4.15: Magnitude and phase response for sequence $x_1[n] = u[n]$.

(b)

$$x_2[n] = \frac{1}{2}\left(e^{j\omega_0 n} + e^{-j\omega_0 n}\right) u[n]$$
$$= \frac{1/2}{1 - e^{-j(\omega - \frac{\pi}{3})}} + \frac{1}{2}\sum_{k=-\infty}^{\infty} \pi\delta(\omega - \frac{\pi}{3} - 2k\pi)$$
$$\frac{1/2}{1 - e^{-j(\omega + \frac{\pi}{3})}} + \frac{1}{2}\sum_{k=-\infty}^{\infty} \pi\delta(\omega + \frac{\pi}{3} - 2k\pi)$$

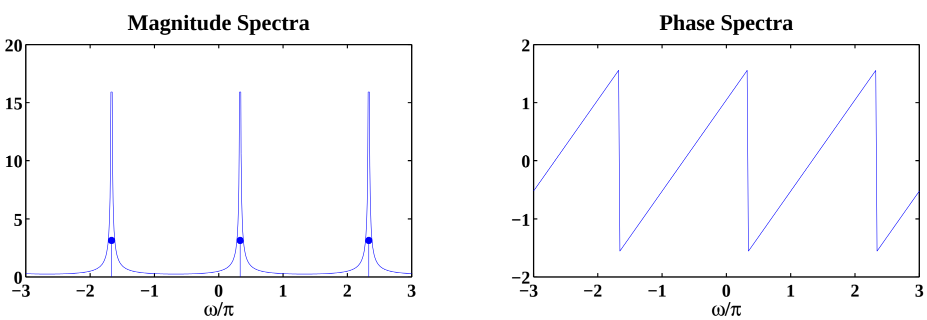


FIGURE 4.16: Magnitude and phase response for sequence $x_2[n] = \cos(\omega_0 n)u[n]$, $\omega_0 = \pi/3$.

13. (a) Solution:

$$x_1[n] = (1/2)^{|n|}\left(\frac{1}{2}e^{j\pi(n-1)/8} + \frac{1}{2}e^{-j\pi(n-1)/8}\right)$$
$$\text{DTFT}\left\{(1/2)^{|n|}\right\} = \sum_{n=-\infty}^{\infty}(1/2)^{|n|}e^{-j\omega n}$$
$$= \sum_{n=-\infty}^{-1}(1/2)^{-n}e^{-j\omega n} + 1 + \sum_{n=1}^{\infty}(1/2)^{n}e^{-j\omega n}$$
$$= \frac{3/2}{5/4 - \cos\omega}$$
$$X_1(\omega) = \frac{1}{2}e^{j\pi/8}\frac{3/2}{5/4 - \cos(\omega - \pi/8)} + \frac{1}{2}e^{-j\pi/8}\frac{3/2}{5/4 - \cos(\omega + \pi/8)}$$

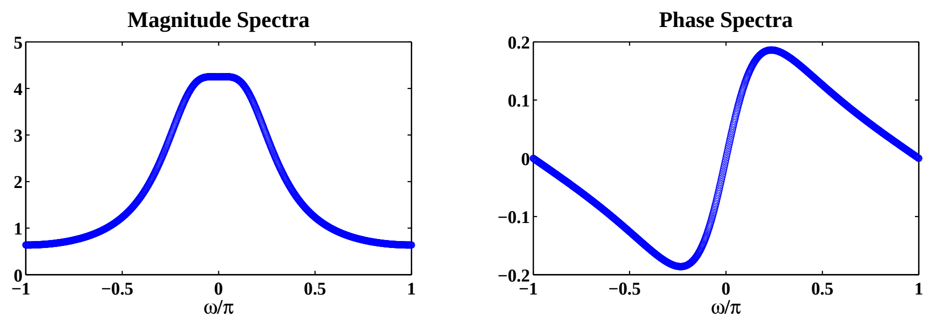


FIGURE 4.17: Magnitude and phase spectra of signal $x_1[n] = (1/2)^{|n|}\cos(\pi(n-1)/8)$.

(b) Solution:

$$X_2(\omega) = \sum_{n=-3}^{3} n\mathrm{e}^{-\mathrm{j}\omega n} = -2\mathrm{j}\sin(\omega) - 4\mathrm{j}\sin(2\omega) - 6\mathrm{j}\sin(3\omega)$$

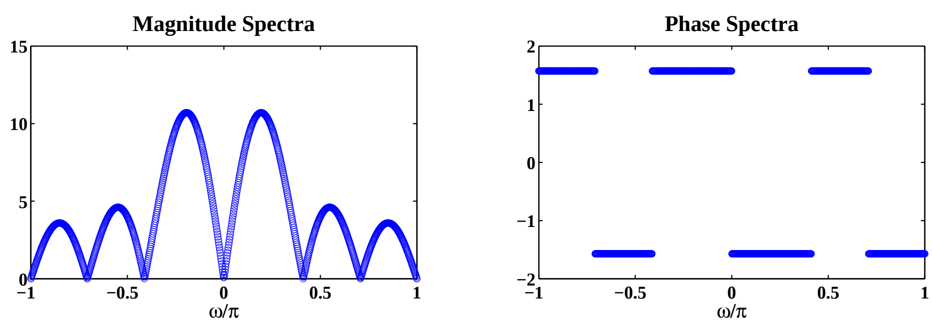


FIGURE 4.18: Magnitude and phase spectra of signal $x_2[n] = n(u[n+3] - u[n-4])$.

(c) Solution:

$$\begin{aligned} X_3(\omega) &= \sum_{n=-4}^{4} (2 - n/2)\mathrm{e}^{-\mathrm{j}\omega n} \\ &= 4\mathrm{e}^{4\mathrm{j}\omega} + \frac{7}{2}\mathrm{e}^{3\mathrm{j}\omega} + 3\mathrm{e}^{2\mathrm{j}\omega} + \frac{5}{2}\mathrm{e}^{\mathrm{j}\omega} + 2 + \frac{3}{2}\mathrm{e}^{-\mathrm{j}\omega} + \mathrm{e}^{-2\mathrm{j}\omega} + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\omega} \end{aligned}$$

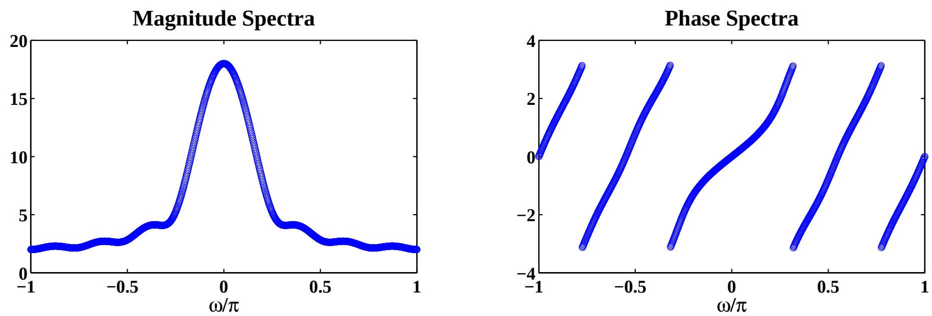


FIGURE 4.19: Magnitude and phase spectra of signal $x_3[n] = (2 - n/2)(u[n+4] - u[n-5])$.

14. (a) Solution:

$$\begin{aligned} X_1(\mathrm{e}^{\mathrm{j}\omega}) &= \cos^2(\omega) + \sin^2(3\omega) \\ &= 1 + \frac{1}{4}\left(\mathrm{e}^{2\mathrm{j}\omega} + \mathrm{e}^{-2\mathrm{j}\omega}\right) - \frac{1}{4}\left(\mathrm{e}^{6\mathrm{j}\omega} + \mathrm{e}^{-6\mathrm{j}\omega}\right) \\ x_1[n] &= \{-\frac{1}{4}, 0, 0, 0, \frac{1}{4}, 0, \underset{\uparrow}{1}, 0, \frac{1}{4}, 0, 0, 0, -\frac{1}{4}\} \end{aligned}$$

(b) Solution:

$$\begin{aligned} x_2[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_2(\mathrm{e}^{\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}\omega n} d\omega \\ &= \frac{1}{2\pi}\left(\int_{-\pi}^{-\omega_c} \mathrm{e}^{\mathrm{j}\omega n} d\omega + \int_{\omega_c}^{\pi} \mathrm{e}^{\mathrm{j}\omega n} d\omega\right) = \frac{-\sin\omega_c n}{\pi n} \end{aligned}$$

(c) Solution:

$$\begin{aligned} x_3[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_3(\mathrm{e}^{\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}\omega n} d\omega \\ &= \frac{1}{2\pi}\left[\int_{-\pi/2}^{0}(1 + 2\omega/\pi)\mathrm{e}^{\mathrm{j}\omega n} d\omega + \int_{0}^{\pi/2}(1 - 2\omega/\pi)\mathrm{e}^{\mathrm{j}\omega n} d\omega\right] \\ &= \frac{-2\sin(\frac{\pi}{2}n)}{(\pi n)^2} \end{aligned}$$

(d) Solution:

$$\begin{aligned} x_4[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_4\big(\mathrm{e}^{\mathrm{j}\omega}\big)\,\mathrm{e}^{\mathrm{j}\omega n} d\omega \\ &= \frac{1}{2\pi}\left[\int_{-\omega_c-\frac{\Delta}{2}}^{-\omega_c+\frac{\Delta}{2}} \mathrm{e}^{\mathrm{j}\omega n} d\omega + \int_{\omega_c-\frac{\Delta}{2}}^{\omega_c+\frac{\Delta}{2}} \mathrm{e}^{\mathrm{j}\omega n} d\omega\right] \\ &= \frac{2\sin(\frac{\Delta}{2}n)\cos(\omega_c n)}{\pi n} \end{aligned}$$

15. (a) Solution:

Time-shifting, Folding, and Linearity

$$X_1(\omega) = \mathrm{e}^{\mathrm{j}\omega} X(\omega) + \mathrm{e}^{\mathrm{j}\omega} X(-\omega)$$

(b) Solution:

Conjugation and Linearity

$$X_2(\omega) = \left(X(\omega) + X^*(-\omega)\right)/2$$

(c) Solution:

Differentiation and Linearity

$$X_3(\omega) = X(\omega) + 2\mathrm{j}\frac{dX(\omega)}{d\omega} + \frac{d^2X(\omega)}{d\omega^2}$$

16. Solution:

(a)

$$X\big(\mathrm{e}^{\mathrm{j}0}\big) = \sum_n x[n] = -1$$

(b)

$x[n]$ real and even $\implies X\big(\mathrm{e}^{\mathrm{j}\omega}\big)$ real and even $\implies \angle X\big(\mathrm{e}^{\mathrm{j}\omega}\big) = 0$

(c)

$$\int_{-\pi}^{\pi} X\big(\mathrm{e}^{\mathrm{j}\omega}\big)\,\mathrm{d}\omega = 2\pi x[0] = -6\pi$$

(d)

$$X(e^{j\pi}) = \sum_n x[n]e^{-j\pi n} = \sum_n x[n]\cos(\pi n) = -1-2-3-4-1 = -9$$

(e)

$$\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 2\pi \sum_n |x[n]|^2 = 38\pi, \quad \text{Parseval's Theorem}$$

17. (a) Solution:

$$r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]y[n-\ell]$$
$$x[n] = [1, 2, \underset{\uparrow}{3}, 2, 1]$$
$$y[n] = [2, 1, \underset{\uparrow}{0}, -1, -2]$$
$$\ell = 1, y[n-1] = [2, \underset{\uparrow}{1}, 0, -1, -2], \quad r_{xy}[1] = 6$$

Compute $r_{xy}[\ell]$ for $\ell \in [-4, 4]$, we have

$$r_{xy}[\ell] = [-2, -5, -8, -6, \underset{\uparrow}{0}, 6, 8, 5, 2]$$

(b) Solution:

$$\rho_{xy}[\ell] = \frac{r_{xy}[\ell]}{\sqrt{E_x}\sqrt{E_y}}$$
$$E_x = \sum_n |x[n]|^2 = 19, \quad E_y = \sum_n |y[n]|^2 = 10$$
$$\rho_{xy}[\ell] = \frac{1}{\sqrt{190}}[-2, -5, -8, -6, \underset{\uparrow}{0}, 6, 8, 5, 2]$$

(c) Comments:
The two signal has exactly the same shape and only differs by a scale factor.

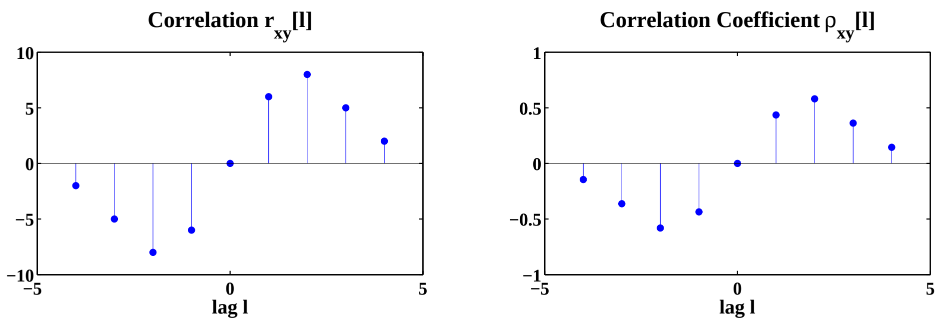


FIGURE 4.20: Plot of the correlation $r_{xy}[\ell]$ and correlation coefficient $\rho_{xy}[\ell]$ between the two signals.

18. (a) Solution:

$$\begin{aligned}
r_{xy}[\ell] &= \sum_{n=-\infty}^{\infty} (0.9)^n u[n](0.9)^{n-\ell} u[n-\ell] \\
&= u[-\ell-1]\sum_{n=0}^{\infty}(0.9)^{2n-\ell} + u[\ell]\sum_{n=0}^{\ell}(0.9)^{\ell} \\
&= \frac{1}{1-0.9^2}\left(0.9^{-\ell}u[-\ell-1] + 0.9^{\ell}u[\ell]\right) \\
E_x &= \sum_{n=0}^{\infty} |(0.9)^n|^2 = \frac{1}{1-0.9^2}, \quad E_y = E_x \\
\rho_{xy}[\ell] &= \frac{r_{xy}[\ell]}{\sqrt{E_x}\sqrt{E_y}} = 0.9^{-\ell}u[-\ell-1] + 0.9^{\ell}u[\ell]
\end{aligned}$$

(b) Solution:

$$\begin{aligned}
r_{xy}[\ell] &= \sum_{n=-\infty}^{\infty} (0.9)^n u[n](0.9)^{-n+\ell} u[-n+\ell] \\
&= u[\ell]\sum_{n=\ell}^{\infty}(0.9)^{2n-\ell} = (\ell+1)(0.9)^{\ell}u[\ell] \\
E_x &= \sum_{n=0}^{\infty} |(0.9)^n|^2 = \frac{1}{1-0.9^2}, \quad E_y = E_x \\
\rho_{xy}[\ell] &= \frac{r_{xy}[\ell]}{\sqrt{E_x}\sqrt{E_y}} = (1-0.9)^2(\ell+1)(0.9)^{\ell}u[\ell]
\end{aligned}$$

(c) Solution:

$$
\begin{aligned}
r_{xy}[\ell] &= \sum_{n=-\infty}^{\infty} (0.9)^n u[n] (0.9)^{n+5-\ell} u[n+5-\ell] \\
&= u[-\ell+4] \sum_{n=0}^{\infty} (0.9)^{2n+5-\ell} + u[\ell-5] \sum_{n=\ell-5}^{\infty} (0.9)^{2n+5-\ell} \\
&= \frac{1}{1-0.9^2} \left(0.9^{5-\ell} u[-\ell+4] + 0.9^{\ell-5} u[\ell-5] \right) \\
E_x &= \sum_{n=0}^{\infty} |(0.9)^n|^2 = \frac{1}{1-0.9^2}, \quad E_y = E_x \\
\rho_{xy}[\ell] &= \left(0.9^{5-\ell} u[-\ell+4] + 0.9^{\ell-5} u[\ell-5] \right)
\end{aligned}
$$

19.
```
function [rxy,l] = ccrs(x,nx,y,ny)
% P0419: Define function computing correlation rxy
%        between two finite length signals
% % Verification:
% nx = -2:2;
% ny = -2:2;
% x = [1 2 3 2 1];
% y = [2 1 0 -1 -2];
[rxy l] = conv0(x(:),nx(:),flipud(y(:)),sort(-ny));
```

Basic Problems

20. Solution:

$$x_1[n] = x_1[n + mN_1], \quad x_2[n] = x_1[n + mN_2], \quad m = 0, \pm 1, \pm 2, \ldots$$

$$x[n + N_1N_2] = x_1[n + N_1N_2] + x_2[n + N_1N_2] = x_1[n] + x_2[n] = x[n]$$

$x[n]$ is always periodic, and the fundamental period N is the least common multiple of N_1, N_2.

21. (a) Solution:

$$T_1 = \frac{2\pi}{3\pi} = \frac{2}{3}, \quad T_2 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$x_1(t)$ is aperiodic.

(b) Solution:

$$N = \frac{2\pi}{0.1\pi} = 20$$

$x_2[n]$ is periodic with fundamental period $N = 20$.

(c) Solution:

$$T_1 = \frac{2\pi}{3000\pi} = \frac{1}{1500}, \quad T_2 = \frac{2\pi}{2000\pi} = \frac{1}{1000}$$

$x_3(t)$ is periodic with fundamental period $T = \frac{1}{500}$.

(d) Solution:

$$N_1 = \frac{2\pi}{1/11} = 22\pi$$

$x_4[n]$ is aperiodic.

(e) Solution:

$$N_1 = \frac{2\pi}{\pi/5} = 10, \quad N_2 = \frac{2\pi}{\pi/6} = 12, \quad N = \frac{2\pi}{\pi/2} = 4$$

$x_5[n]$ is periodic with fundamental period $N = 60$.

22. (a) Solution:

$$x(t) = \cos(15\pi t) \implies x[n] = x(nT) = \cos(15\pi nT)$$

$$N = \frac{2\pi}{15\pi T} = \frac{2}{15T}$$

T is a rational number so that $x[n]$ is periodic.

(b) Solution:

$$T = 0.1 \implies N = \frac{2}{15T} = \frac{4}{3}$$

Hence, the fundamental period of the sequence $x[n]$ is $N = 4$.

23. Solution:

$$c_k = \frac{1}{T_0}\int_{T_0} x(t)\mathrm{e}^{-\mathrm{j}2\pi kF_0 t}dt, \quad F_0 = \frac{1}{T_0}$$

$$c_k = \frac{1}{T_0}\left(A\int_{-\frac{T_0}{2}}^{0}(1+2t/T_0)\mathrm{e}^{-\mathrm{j}2\pi k\frac{1}{T_0}t}dt + A\int_0^{\frac{T_0}{2}}(1-2t/T_0)\mathrm{e}^{-\mathrm{j}2\pi k\frac{1}{T_0}t}dt\right)$$

$$= \frac{2A}{T_0}\left(\int_0^{\frac{T_0}{2}}(1-2t/T_0)\cos 2\pi k\frac{1}{T_0}t\,dt\right) = \frac{A(1-\cos\pi k)}{(\pi k)^2}$$

(a)

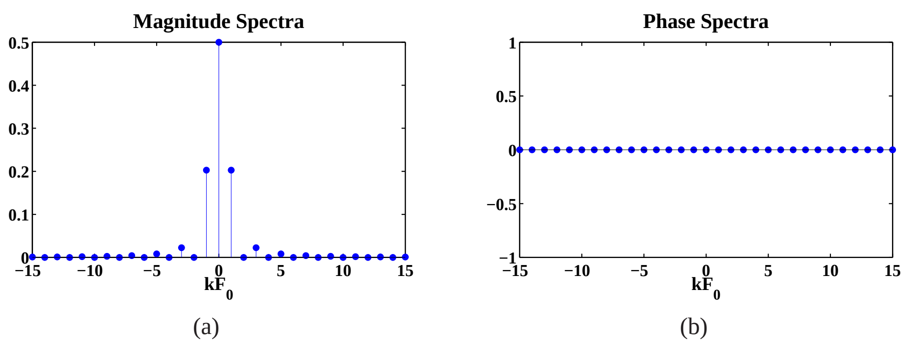


FIGURE 4.21: (a) Magnitude spectra of $x(t)$ for $A = 1$ and $T_0 = 1$. (b) Phase spectra of $x(t)$ for $A = 1$ and $T_0 = 1$.

(b)

MATLAB script:

```
% Determine the Fourier series coefficients
% and plot its magnitude and phase spectra
close all; clc
%% Plot spectra
T0 = 1; F = 1/T0;
A = 1;
m = 15;
```

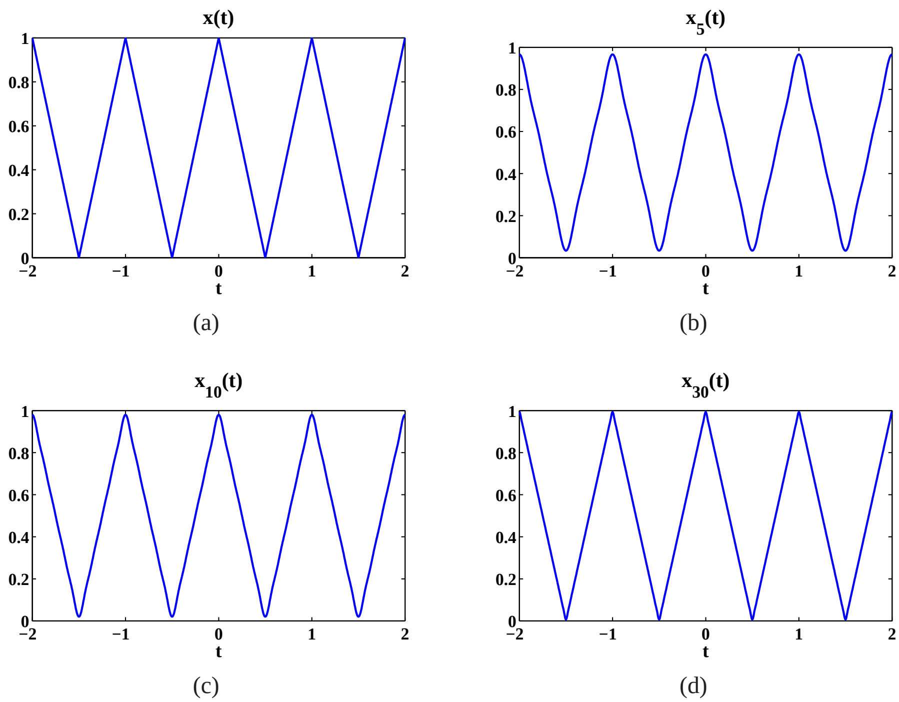


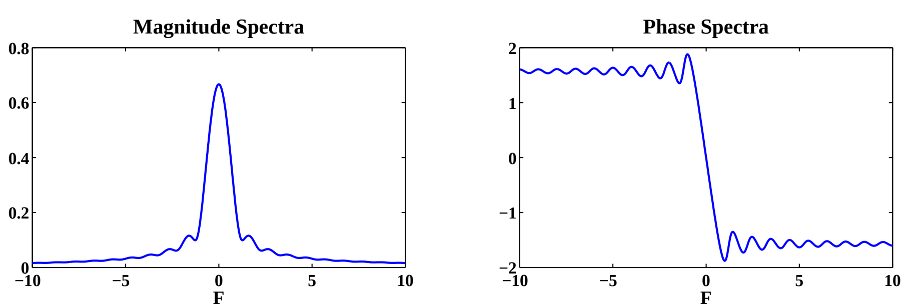
FIGURE 4.22: (a) $x(t)$ for $A = 1$ and $T_0 = 1$. (b) $x_5(t)$. (c) $x_{10}(t)$. (b) $x_{30}(t)$.

```
% m = 10;
% m = 30;
k = -m:m;
ck = A*(1-cos(pi*k))./(pi*k).^2;
ck(k==0) = A/2;
ck_mag = abs(ck);
ck_phase = angle(ck);
hfa = figconfg('P0423a','small');
stem(k,ck_mag,'filled')
xlabel('kF_0','fontsize',LFS)
title('Magnitude Spectra','fontsize',TFS)
hfb = figconfg('P0423b','small');
stem(k,ck_phase,'filled')
xlabel('kF_0','fontsize',LFS)
title('Phase Spectra','fontsize',TFS)
%% Part (b):
```

```
t = linspace(-2*T0,2*T0,1000)';
tt = t;
while any(tt<-T0/2)
    tt(tt<-T0/2) = tt(tt<-T0/2)+T0;
end
while any(tt>T0/2)
    tt(tt>T0/2) = tt(tt>T0/2)-T0;
end
xt = A*(1-2*abs(tt)/T0);
xmt = real(exp(j*2*pi*F*t*k)*ck(:));
hfc = figconfg('P0423c','small');
plot(t,xt,'linewidth',2)
xlabel('t','fontsize',LFS)
title('x(t)','fontsize',TFS)
hfd = figconfg('P0423d','small');
plot(t,xmt,'linewidth',2)
xlabel('t','fontsize',LFS)
title(['x_{',num2str(m),'}(t)'],'fontsize',TFS)
```

24. (a) Solution:

$$X(F) = \int_{-\infty}^{\infty} x_1(t) \mathrm{e}^{-\mathrm{j}2\pi F t} dt = \int_{-\infty}^{\infty} (1-t^2)[u(t)-u(t-1)] \mathrm{e}^{-\mathrm{j}2\pi F t} dt$$
$$= \frac{1}{\mathrm{j}2\pi F} + \frac{2\mathrm{e}^{-\mathrm{j}2\pi F}}{(\mathrm{j}2\pi F)^2} + \frac{2(\mathrm{e}^{-\mathrm{j}2\pi F}-1)}{(\mathrm{j}2\pi F)^3}$$



(a) (b)

FIGURE 4.23: (a) Magnitude and phase spectra of signal $x_1(t) = (1-t^2)[u(t) - u(t-1)]$.

(b) Solution:

$$X(F) = \int_{-\infty}^{\infty} x_2(t)\mathrm{e}^{-\mathrm{j}2\pi Ft}dt = \int_{-\infty}^{\infty} \mathrm{e}^{-3|t|}\sin 2\pi t\mathrm{e}^{-\mathrm{j}2\pi Ft}dt$$
$$= \frac{-48\mathrm{j}\pi^2 F}{(4\pi^2F^2 + 12\mathrm{j}\pi F - 9 - 4\pi^2)(4\pi^2F^2 - 12\mathrm{j}\pi F - 9 - 4\pi^2)}$$

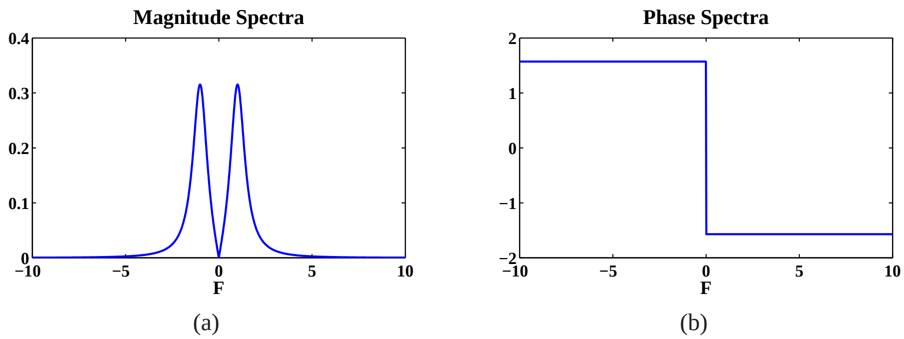


FIGURE 4.24: (a) Magnitude and phase spectra of signal $x_2(t) = \mathrm{e}^{-3|t|}\sin 2\pi t$.

(c) Solution:

$$x_3(t) = 2\frac{\sin \pi t}{\pi t}\frac{\sin 2\pi t}{2\pi t} = 2\mathrm{sinc}(t)\mathrm{sinc}(2t)$$

$$\mathrm{CTFT}(\mathrm{sinc}(t)) = \begin{cases} 1, & -\frac{1}{2} \le F \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$
$$\mathrm{CTFT}(\mathrm{sinc}(2t)) = \begin{cases} \frac{1}{2}, & -1 \le F \le 1 \\ 0, & \text{otherwise} \end{cases}, \quad \text{Time Scaling Property}$$
$$X_3(F) = 2\cdot \mathrm{CTFT}(\mathrm{sinc}(t)) * \mathrm{CTFT}(\mathrm{sinc}(2t))$$
$$= \begin{cases} 1, & -\frac{1}{2} \le F \le \frac{1}{2} \\ F + \frac{3}{2}, & -\frac{3}{2} \le F \le -\frac{1}{2} \\ -F + \frac{3}{2}, & \frac{1}{2} \le F \le \frac{3}{2} \\ 0, & \text{otherwise} \end{cases}$$

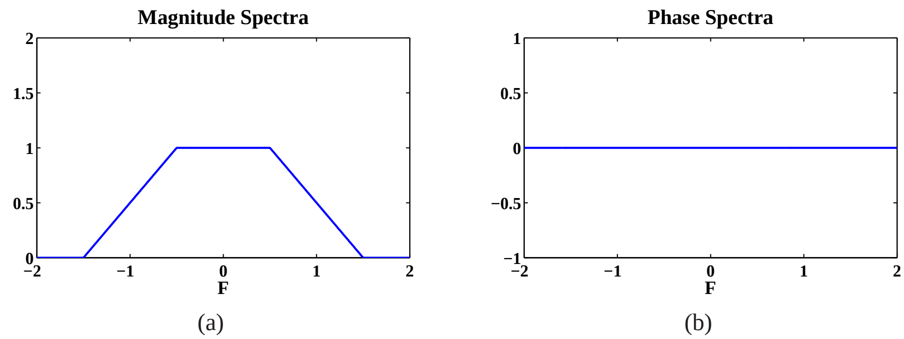


FIGURE 4.25: (a) Magnitude and phase spectra of signal $x_3(t) = \frac{\sin \pi t}{\pi t}\frac{\sin 2\pi t}{\pi t}$.

25. (a) Solution:

```
% Compute partial sum defined in P0425
close all; clc
%% Part (a):
L = 2; N = 9;
M = [1,2,3,4];
ind = 4;
k = -M(ind):M(ind);
ak = (2*L+1)/N*ones(1,length(k));
indk = mod(k,N)~=0;
ak(indk) = sin(k(indk)*(L+1/2)*2*pi/N)./sin(k(indk)*pi/N)/N;
n = -20:20;
xhatM = ak*exp(j*2*pi*k(:)*n/N);
isreal(xhatM)
% Plot
hfa = figconfg('P0425','small');
stem(n,xhatM,'filled')
xlabel('n','fontsize',LFS)
ylabel('x_M[n]','fontsize',LFS)
title(['M = ',num2str(M(ind))],'fontsize',TFS)
```

(b) tba

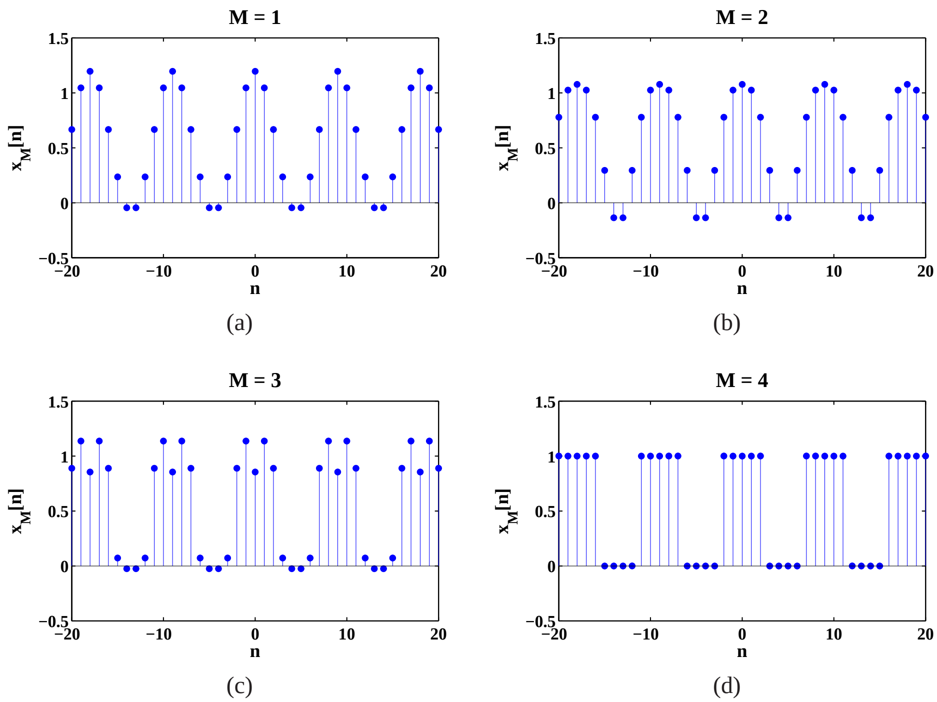


FIGURE 4.26: Sequences $\hat{x}_M[n]$. (a) $M = 1$. (b) $M = 2$. (c) $M = 3$. (d) $M = 4$.

26. (a) Solution:

$$\begin{aligned} x_1[n] &= 4\cos(1.2\pi n + 60^\circ) + 6\sin(0.4\pi n - 30^\circ) \\ &= 4\left(\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{\mathrm{j}\frac{2\pi}{5}3n} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{\mathrm{j}\frac{2\pi}{5}(-3)n}\right) + 6\left(\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{\mathrm{j}\frac{2\pi}{5}n} - \mathrm{e}^{\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{\mathrm{j}\frac{2\pi}{5}(-1)n}\right) \end{aligned}$$

$$c_k = \begin{cases} 0, & k = 5m \\ 6\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}, & k = 5m+1 \\ 4\mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}, & k = 5m+2 \qquad m = 0, \pm 1, \pm 2 \\ 4\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}, & k = 5m+3 \\ -6\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}, & k = 5m+4 \end{cases}$$

(b) Solution:

$$\begin{aligned} c_k &= \frac{1}{4}\sum_{n=0}^{3} x_2[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{4}kn} \\ &= \frac{1}{4}\left(1 + \cos\frac{\pi}{4}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}k} + 0 - \cos\frac{3\pi}{4}\mathrm{e}^{-\mathrm{j}\frac{3\pi}{2}k}\right) = \frac{1}{4}\left(1 + 2\cos\frac{\pi}{4}\cos\frac{\pi}{2}k\right) \end{aligned}$$

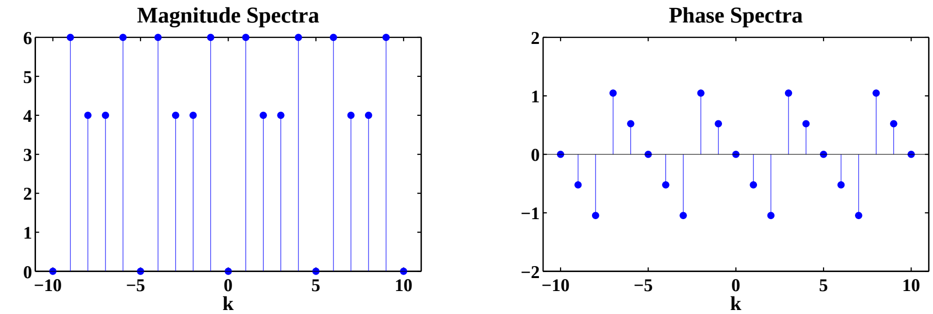


FIGURE 4.27: Magnitude and phase spectra of periodic sequence $x_1[n] = 4\cos(1.2\pi n + 60^\circ) + 6\sin(0.4\pi n - 30^\circ)$.

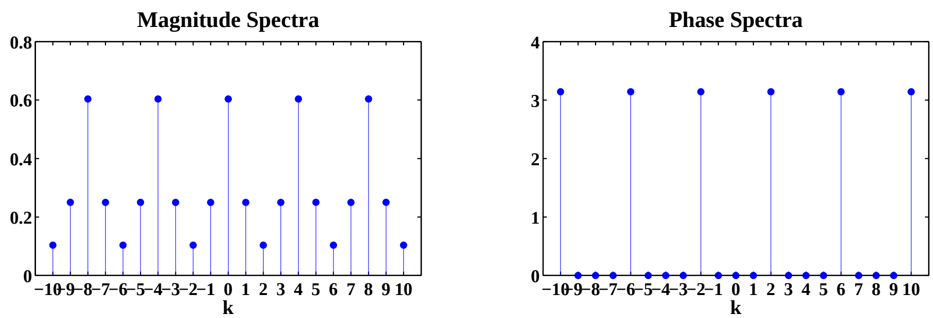


FIGURE 4.28: Magnitude and phase spectra of periodic sequence $x_2[n] = |\cos(0.25\pi n)|, 0 \leq n \leq 3$.

(c) Solution:

$$\begin{aligned} c_k &= \frac{1}{8}\sum_{n=0}^{7} x_5[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}kn} \\ &= \frac{1}{8}\left[1 + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k3} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k4} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k5} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}k7}\right] \\ &= \frac{1}{8}\left[1 + 2\cos(\frac{k\pi}{4}) + 2\cos(\frac{3k\pi}{4}) + \cos(k\pi)\right] \end{aligned}$$

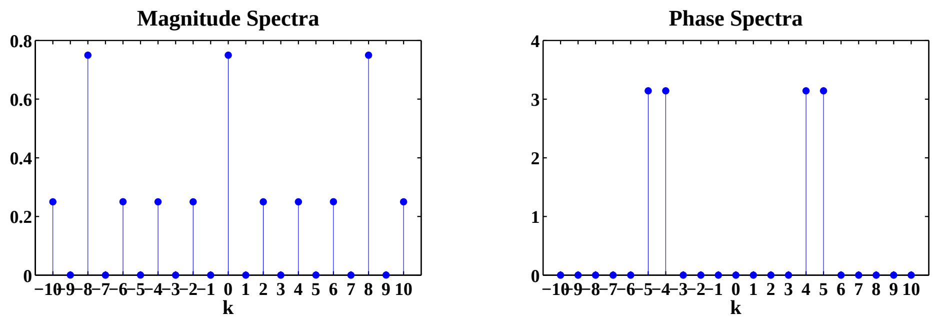


FIGURE 4.29: Magnitude and phase spectra of periodic sequence $x_3[n]$.

(d) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{12}\sum_{n=0}^{11}\left[1-\sin\left(\frac{\pi n}{12}\right)\right]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{4}kn} \\
&= \frac{1}{12}\Bigg[1+(1-\sin(\frac{\pi}{4}))2\cos(\frac{k\pi}{6})+(1-\sin(\frac{3\pi}{4}))2\cos(\frac{k\pi}{2}) \\
&+2\cos(\frac{2k\pi}{3})+(1-\sin(\frac{5\pi}{4}))2\cos(\frac{5k\pi}{6})+2\cos(k\pi)\Bigg]
\end{aligned}
$$

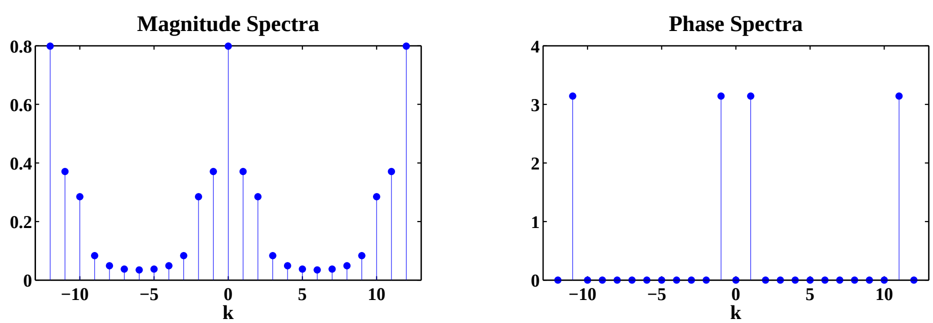


FIGURE 4.30: Magnitude and phase spectra of periodic sequence $x_4[n] = 1 - \sin(\pi n/4), 0 \le n \le 11$ (one period).

(e) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{7}\sum_{n=0}^{6} x_5[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}kn} \\
&= \frac{1}{7}\left(1-2\mathrm{e}^{-j\frac{2\pi}{7}k}+\mathrm{e}^{-j\frac{2\pi}{7}k\cdot 2}-\mathrm{e}^{-j\frac{2\pi}{7}k\cdot 4}+2\mathrm{e}^{-j\frac{2\pi}{7}k\cdot 5}-\mathrm{e}^{-j\frac{2\pi}{7}k\cdot 6}\right)
\end{aligned}
$$

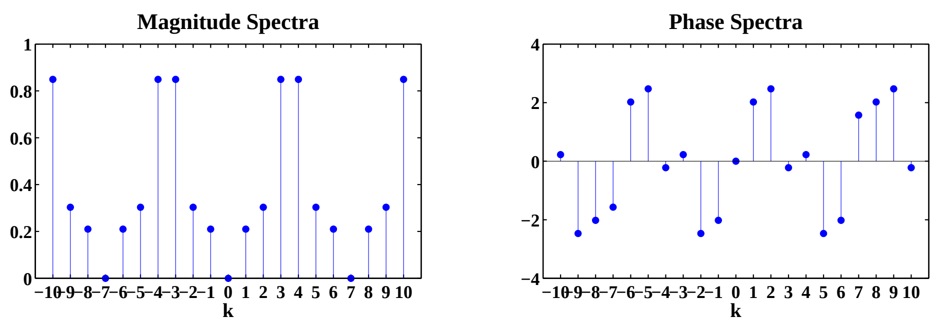


FIGURE 4.31: Magnitude and phase spectra of periodic sequence $x_5[n]$.

27. (a) Solution:

$$\frac{1}{N}\sum_{n=0}^{N-1} x[n-n_0]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = \frac{1}{N}\sum_{n=n_0}^{N-1+n_0} x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn}\cdot\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}kn_0} = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn_0}a_k$$

(b) Solution:

$$\frac{1}{N}\sum_{n=0}^{N-1}(x[n]-x[n-1])\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = a_k - \mathrm{e}^{\mathrm{j}\frac{2\pi}{N}k}a_k$$

(c) Solution:

$$\begin{aligned}\frac{1}{N}\sum_{n=0}^{N-1}(-1)^n x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} &= \frac{1}{N}\sum_{n=0}^{N-1}\mathrm{e}^{\mathrm{j}n\pi}x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} \\ &= \frac{1}{N}\sum_{n=0}^{N-1}x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}\left(k-\frac{N}{2}\right)n} = a_{k-\frac{N}{2}}\end{aligned}$$

(d) tba

28. (a) Solution:

$$y[n] = |x[n]|^2 = x[n]\cdot x^*[n]$$

$$
\begin{aligned}
b_k &= \frac{1}{N}\sum_{n=0}^{N-1} y[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\cdot x^*[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\left(\sum_{m=0}^{N-1} a_m \mathrm{e}^{\mathrm{j}\frac{2\pi}{N}mn}\right)\cdot x^*[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} \\
&= \sum_{m=0}^{N-1} a_m\left(\frac{1}{N}\sum_{n=0}^{N-1} x^*[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}(k-m)n}\right) \\
&= \sum_{m=0}^{N-1} a_m \cdot a^*_{m-k}
\end{aligned}
$$

(b) Solution:
If a_k are real, we can claim that b_k are real as well.

29. (a) Proof:

$$
\begin{aligned}
c_k &= \frac{1}{N}\sum_{n=0}^{N-1} y[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = \frac{1}{N}\sum_{n=0}^{N-1} h[n]x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\left(\sum_{\ell=0}^{N-1} a_\ell \mathrm{e}^{\mathrm{j}\frac{2\pi}{N}\ell n}\right) x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} \\
&= \sum_{\ell=0}^{N-1} a_\ell\left(\frac{1}{N}\sum_{n=0}^{N-1} x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}(k-\ell)n}\right) \\
&= \sum_{\ell=0}^{N-1} a_\ell b_{k-\ell}
\end{aligned}
$$

(b) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{8}\sum_{n=0}^{7} h[n]x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}kn} = \frac{1}{8}\sum_{n=0}^{3}\sin(3\pi n/4)\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}kn}\\
&= \frac{1}{8}\left(\sin\left(\frac{3\pi}{4}\right)\cdot \mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k} + \sin\left(\frac{3\pi}{2}\right)\cdot \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}k} + \sin\left(\frac{9\pi}{4}\right)\cdot \mathrm{e}^{-\mathrm{j}\frac{3\pi}{4}k}\right)\\
a_k &= \left\{0,0,0,\frac{1}{2\mathrm{j}},0,-\frac{1}{2\mathrm{j}},0,0\right\}, \quad 0 \le k \le 7\\
b_k &= \frac{1}{8}\left(1+\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}k} + +\mathrm{e}^{-\mathrm{j}\frac{3\pi}{4}k}\right)\\
\sum_{\ell=0}^{7} a_\ell b_{k-\ell} &= \frac{1}{2\mathrm{j}}\frac{1}{8}\left(1+\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}(k-3)} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}(k-3)} + +\mathrm{e}^{-\mathrm{j}\frac{3\pi}{4}(k-3)}\right)\\
&- \frac{1}{2\mathrm{j}}\frac{1}{8}\left(1+\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}(k+3)} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}(k+3)} + +\mathrm{e}^{-\mathrm{j}\frac{3\pi}{4}(k+3)}\right)\\
&= \frac{1}{8}\left(\sin\left(\frac{3\pi}{4}\right)\cdot \mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k} + \sin\left(\frac{3\pi}{2}\right)\cdot \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}k} + \sin\left(\frac{9\pi}{4}\right)\cdot \mathrm{e}^{-\mathrm{j}\frac{3\pi}{4}k}\right)\\
&= c_k
\end{aligned}
$$

30. (a) Solution:

$$x_1[n] = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} u[n-1]$$

$$X_1(\mathrm{e}^{\mathrm{j}\omega}) = \frac{\frac{1}{3}\mathrm{e}^{-\mathrm{j}\omega}}{1-\frac{1}{3}\mathrm{e}^{-\mathrm{j}\omega}}$$

(b) Solution:

$$x_2[n] = \left(\frac{1}{2}\right)^5 \left(\frac{1}{4}\right)^n u[n]\left(\mathrm{e}^{\mathrm{j}\pi n/4} + \mathrm{e}^{-\mathrm{j}\pi n/4}\right)$$

$$X_2(\mathrm{e}^{\mathrm{j}\omega}) = \left(\frac{1}{2}\right)^5\left[\frac{\mathrm{e}^{-2\mathrm{j}(\omega-\pi/4)}}{1-\frac{1}{4}\mathrm{e}^{-\mathrm{j}(\omega-\pi/4)}} + \frac{\mathrm{e}^{-2\mathrm{j}(\omega+\pi/4)}}{1-\frac{1}{4}\mathrm{e}^{-\mathrm{j}(\omega+\pi/4)}}\right]$$

(c) Solution:

$$X_3(\mathrm{e}^{\mathrm{j}\omega}) = \begin{cases} \frac{16}{\pi^2}\mathrm{e}^{-\mathrm{j}4\omega}, & 0 \le |\omega| \le \frac{\pi^2}{4}\\ 0, & \frac{\pi^2}{4} < |\omega| < \pi \end{cases}$$

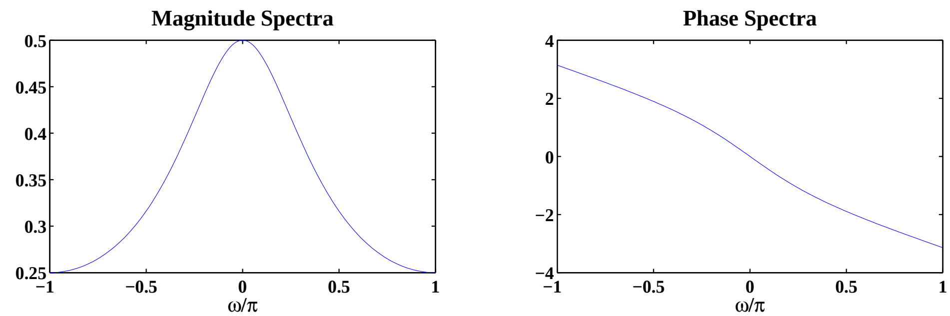


FIGURE 4.32: Magnitude and phase response for sequence $x_1[n] = (1/3)^n u[n-1]$.

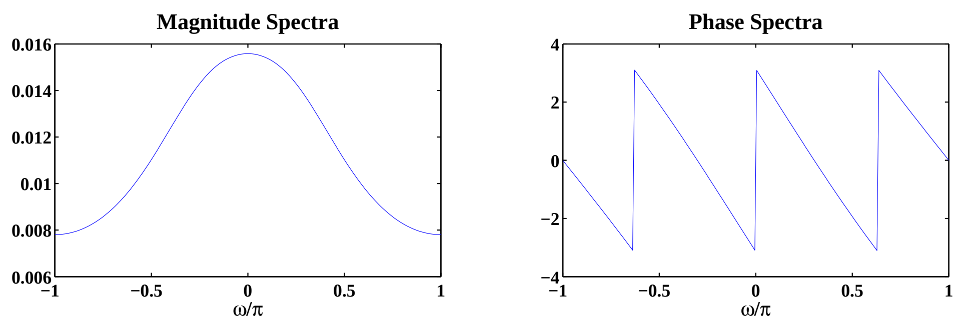


FIGURE 4.33: Magnitude and phase response for sequence $x_2[n] = (1/4)^n \cos(\pi n/4) u[n-2]$.

(d) Solution:

$$\begin{aligned} X_4(\mathrm{e}^{\mathrm{j}\omega}) &= \sum_{n=0}^{9} \frac{1}{2\mathrm{j}} \left(\mathrm{e}^{\mathrm{j}0.1\pi n} - \mathrm{e}^{-\mathrm{j}0.1\pi n}\right) \mathrm{e}^{-\mathrm{j}n\omega} \\ &= \frac{1}{2\mathrm{j}} \left[\frac{1-\mathrm{e}^{-10\mathrm{j}(\omega-0.1\pi)}}{1-\mathrm{e}^{-\mathrm{j}(\omega-0.1\pi)}} - \frac{1-\mathrm{e}^{-10\mathrm{j}(\omega+0.1\pi)}}{1-\mathrm{e}^{-\mathrm{j}(\omega+0.1\pi)}} \right] \end{aligned}$$

(e) Solution:

$$X_5(\mathrm{e}^{\mathrm{j}\omega}) = \begin{cases} \frac{8}{\pi^3}\left(\frac{\pi^2}{2} - |\omega|\right), & 0 \le |\omega| \le |2\pi - \frac{\pi^2}{2}| \\ \frac{8(\pi-2)}{\pi^2}, & |2\pi - \frac{\pi^2}{2}| < |\omega| < \pi \end{cases}$$

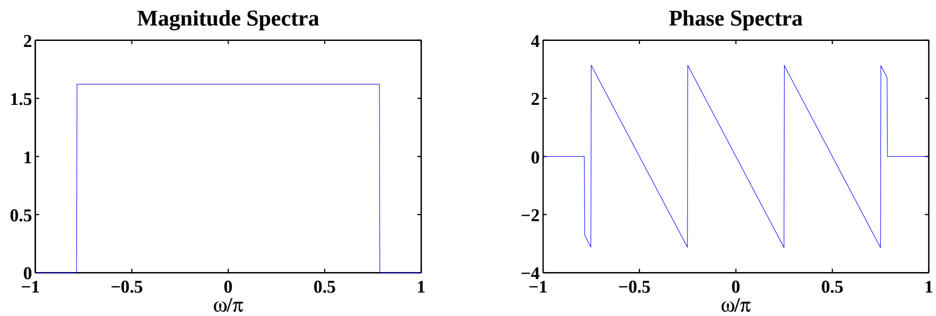


FIGURE 4.34: Magnitude and phase response for sequence $x_3[n] = \text{sinc}(2\pi n/8) * \text{sinc}\{2\pi(n-4)/8\}$.

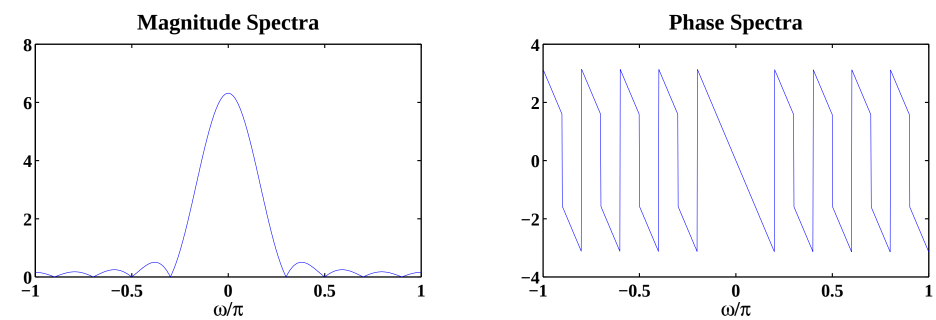


FIGURE 4.35: Magnitude and phase response for sequence $x_4[n] = \sin(0.1\pi n)(u[n] - u[n-10])$.

31. (a) Solution:

$$x_1[n] = \frac{1}{2\pi}\left(1 - e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}\right) = \frac{1}{2\pi}\left(1 - 2\cos\frac{\pi}{2}n\right)$$

(b) Solution:

$$x_2[n] = \frac{1}{5}\text{sinc}\left(\frac{n}{5}\right)$$

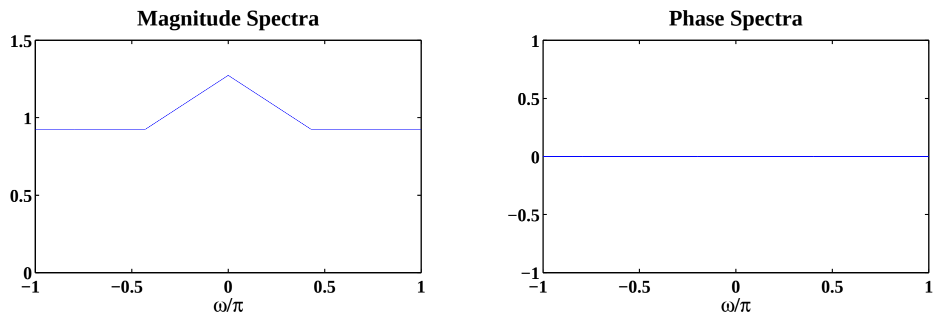


FIGURE 4.36: Magnitude and phase response for sequence $x_5[n] = \text{sinc}^2(\pi n/4)$.

(c) Solution:

$$\begin{aligned} x_3[n] &= \frac{1}{2\pi}\left(\int_{-\frac{\pi}{2}}^{0} \frac{-2\omega}{\pi}\mathrm{e}^{\mathrm{j}n\omega}d\omega + \int_{0}^{\frac{\pi}{2}} \frac{2\omega}{\pi}\mathrm{e}^{\mathrm{j}n\omega}d\omega\right) \\ &= \frac{1}{2\pi}\int_{0}^{\frac{\pi}{2}} \frac{2\omega}{\pi} 2\mathrm{j}\sin(n\omega)d\omega \\ &= \frac{-\mathrm{j}}{n\pi}\cos(\frac{\pi}{2}n) + \frac{2\mathrm{j}}{n^2\pi^2}\sin(\frac{\pi}{2}n) \end{aligned}$$

(d) Solution:

$$\begin{aligned} x_4[n] = \frac{1}{2\pi}&\left(\int_{-\pi}^{-\omega_c-\frac{\Delta\omega}{2}} \mathrm{e}^{\mathrm{j}n\omega}d\omega + \int_{-\omega_c+\frac{\Delta\omega}{2}}^{0} \mathrm{e}^{\mathrm{j}n\omega}d\omega + \int_{0}^{\omega_c-\frac{\Delta\omega}{2}} \mathrm{e}^{\mathrm{j}n\omega}d\omega \right.\\ &\left. + \int_{\omega_c+\frac{\Delta\omega}{2}}^{\pi} \mathrm{e}^{\mathrm{j}n\omega}d\omega\right) \\ &= \frac{1}{2\pi}\left(\int_{\omega_c+\frac{\Delta\omega}{2}}^{\pi} 2\mathrm{j}\sin(n\omega)d\omega + \int_{0}^{\omega_c-\frac{\Delta\omega}{2}} 2\mathrm{j}\sin(n\omega)d\omega\right) \\ &= \frac{\mathrm{j}}{\pi n}\left\{1-\cos(\pi n)+\cos[(\omega_c+\frac{\Delta\omega}{2})n]-\cos[(\omega_c-\frac{\Delta\omega}{2})n]\right\} \end{aligned}$$

32. (a) Solution:

$$X_1(\mathrm{e}^{\mathrm{j}\omega}) = 2\mathrm{e}^{2\mathrm{j}\omega}X(\mathrm{e}^{\mathrm{j}\omega}) + 3\mathrm{e}^{-3\mathrm{j}\omega}X(\mathrm{e}^{\mathrm{j}\omega})$$

(b) Solution:

$$x_2[n] = \frac{1}{2}\left(\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{\mathrm{j}0.2\pi n} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{-\mathrm{j}0.2\pi n}\right) + \frac{1}{2}\left(\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{\mathrm{j}0.2\pi n} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{-\mathrm{j}0.2\pi n}\right)x[n]$$

$$X_2(e^{j\omega}) = \frac{1}{2}e^{j\frac{\pi}{6}}\delta(\omega-\frac{\pi}{5})+\frac{1}{2}e^{-j\frac{\pi}{6}}\delta(\omega+\frac{\pi}{5})+\frac{1}{2}e^{j\frac{\pi}{6}}X(e^{j(\omega-\frac{\pi}{5})})+\frac{1}{2}e^{-j\frac{\pi}{6}}X(e^{j(\omega+\frac{\pi}{5})})$$

(c) Solution:

$$x_3[n] = (2e^{-j\pi})e^{j0.5\pi n}x[n+2]$$
$$X_3(e^{j\omega}) = -2e^{j2(\omega-0.5\pi)}X(e^{j(\omega-0.5\pi)})$$

(d) Solution:

$$X_4(e^{j\omega}) = \frac{1}{2}X(e^{j\omega}) - \frac{1}{2}X(e^{j\omega})^*$$

(e) Solution:

$$x_5[n] = e^{j\frac{\pi}{2}n}x[n+1] + e^{-j\frac{\pi}{2}n}x[n-1]$$
$$X_5(e^{j\omega}) = X(e^{j(\omega-\frac{\pi}{2})})e^{j(\omega-\frac{\pi}{2})} + X(e^{j(\omega+\frac{\pi}{2})})e^{-j(\omega+\frac{\pi}{2})}$$

33. (a) Solution:

$$X_1(e^{j\omega}) = X(e^{j(\omega-\frac{\pi}{2})})e^{2j(\omega-\frac{\pi}{2})} = \frac{e^{2j(\omega-\frac{\pi}{2})}}{1+0.8e^{-j(\omega-\frac{\pi}{2})}}$$

(b) Solution:

$$x_2[n] = x[n]\left(\frac{1}{2}e^{j0.4\pi n} + \frac{1}{2}e^{-j0.4\pi n}\right)$$

$$\begin{aligned} X_2(e^{j\omega}) &= \frac{1}{2}X(e^{j(\omega-0.4\pi)}) + \frac{1}{2}X(e^{j(\omega-0.4\pi)}) \\ &= \frac{\frac{1}{2}}{1+0.8e^{-j(\omega-0.4\pi)}} + \frac{\frac{1}{2}}{1+0.8e^{-j(\omega+0.4\pi)}} \end{aligned}$$

(c) Solution:

$$\begin{aligned} X_3(e^{j\omega}) &= X(e^{j\omega})X\left(e^{-j\omega}\right) = \frac{1}{1+0.8e^{-j\omega}} \cdot \frac{1}{1+0.8e^{j\omega}} \\ &= \frac{1}{1+1.6\cos(\omega)+0.64} \end{aligned}$$

(d) Solution:

$$\begin{aligned} X_4(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[2n]e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\frac{\omega}{2}n} = X(e^{j\frac{\omega}{2}}) \\ &= \frac{1}{1+0.8e^{-j\frac{\omega}{2}}} \end{aligned}$$

(e) Solution:

$$x[n] = (-0.8)^n u[n]$$

$$X_5\left(e^{j\omega}\right) = \sum_{m=0}^{\infty} (-0.8)^{2m} e^{-j\omega 2m} = \frac{1}{1 - 0.8^2 e^{-j2\omega}}$$

34. (a) Solution:

$$X_R\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x_I[n] \sin(\omega n), \quad \text{odd symmetric}$$

$$X_I\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x_I[n] \cos(\omega n), \quad \text{even symmetric}$$

$$x_R[n] = 0,$$

$$x_I[n] = \frac{1}{2\pi} \int_{2\pi} [X_R\left(e^{j\omega}\right) \sin(\omega n) + X_I\left(e^{j\omega}\right) \cos(\omega n)] d\omega, \quad \text{nonsymmetric}$$

(b) Solution:

$$X_R\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x_I[n] \sin(\omega n), \quad \text{odd symmetric}$$

$$X_I\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x_I[n] \cos(\omega n), \quad \text{even symmetric}$$

$$x_R[n] = 0,$$

$$x_I[n] = \frac{1}{2\pi} \int_{2\pi} [X_R\left(e^{j\omega}\right) \sin(\omega n) + X_I\left(e^{j\omega}\right) \cos(\omega n)] d\omega, \quad \text{even symmetric}$$

(c) Solution:

$$X_R\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x_I[n] \sin(\omega n), \quad \text{odd symmetric}$$

$$X_I\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x_I[n] \cos(\omega n), \quad \text{even symmetric}$$

$$x_R[n] = 0,$$

$$x_I[n] = \frac{1}{2\pi} \int_{2\pi} [X_R\left(e^{j\omega}\right) \sin(\omega n) + X_I\left(e^{j\omega}\right) \cos(\omega n)] d\omega, \quad \text{odd symmetric}$$

35. (a) Proof:

$$r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]y[n-\ell], \quad -\infty \leq \ell \leq \infty$$

$$\begin{aligned}
R_{xy}(\omega) &= \sum_{\ell=-\infty}^{\infty} r_{xy}[\ell]\mathrm{e}^{-\mathrm{j}\omega\ell} = \sum_{\ell=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} x[n]y[n-\ell]\mathrm{e}^{-\mathrm{j}\omega\ell} \\
&= \sum_{n=-\infty}^{\infty} x[n]\left(\sum_{\ell=-\infty}^{\infty} y[n-\ell]\mathrm{e}^{-\mathrm{j}\omega\ell}\right) \\
&= \sum_{n=-\infty}^{\infty} x[n]\mathrm{e}^{-\mathrm{j}\omega n}\left(\sum_{\ell=-\infty}^{\infty} y[n-\ell]\mathrm{e}^{-\mathrm{j}\omega(\ell-n)}\right) \\
&= \left(\sum_{n=-\infty}^{\infty} x[n]\mathrm{e}^{-\mathrm{j}\omega n}\right)\left(\sum_{m=-\infty}^{\infty} y[m]\mathrm{e}^{\mathrm{j}\omega m}\right) \\
&= X\!\left(\mathrm{e}^{\mathrm{j}\omega}\right)Y\left(\mathrm{e}^{-\mathrm{j}\omega}\right)
\end{aligned}$$

(b) Proof:

$$R_x(\omega) = X\!\left(\mathrm{e}^{\mathrm{j}\omega}\right)X\left(\mathrm{e}^{-\mathrm{j}\omega}\right)$$

Since $x[n]$ is real, $X\left(\mathrm{e}^{-\mathrm{j}\omega}\right) = X\!\left(\mathrm{e}^{\mathrm{j}\omega}\right)^*$, hence

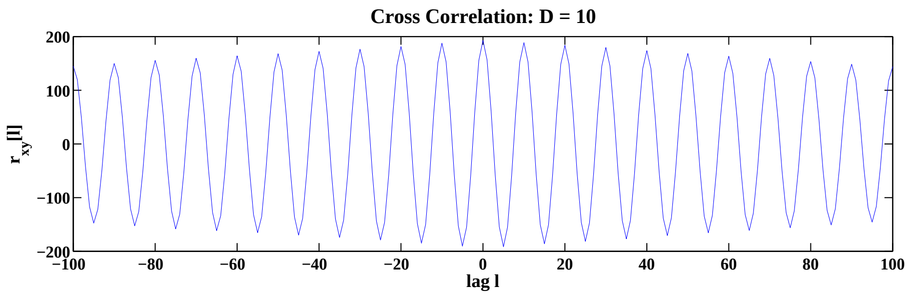
$$R_x(\omega) = |X\!\left(\mathrm{e}^{\mathrm{j}\omega}\right)|^2$$

36. (a) See plot below.

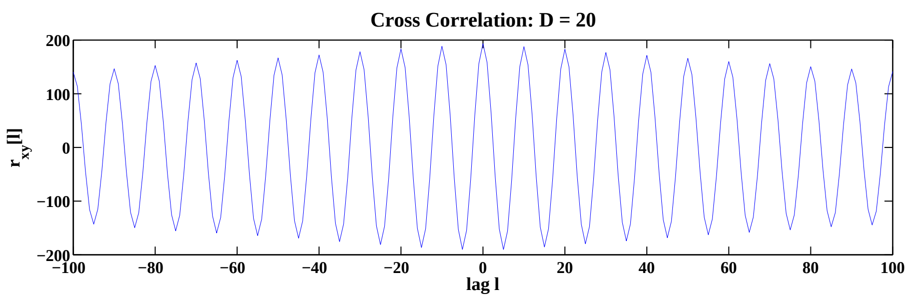
(b) Comments:
The larger the delay D is, the smaller the correspondent $r_{xy}[\ell]$ will be. Hence, we can distinguish the delay D from the observation of $r_{xy}[\ell]$.

```
% P0436: Compute and plot correlation between x[n] and y[n]
% close all; clc
nx = -200:200;
xn = sin(0.2*pi*nx);
wn = randn(1,length(xn));
wn = sqrt(0.1)*wn;

D = 10;
% D = 20;
% D = 50;
```

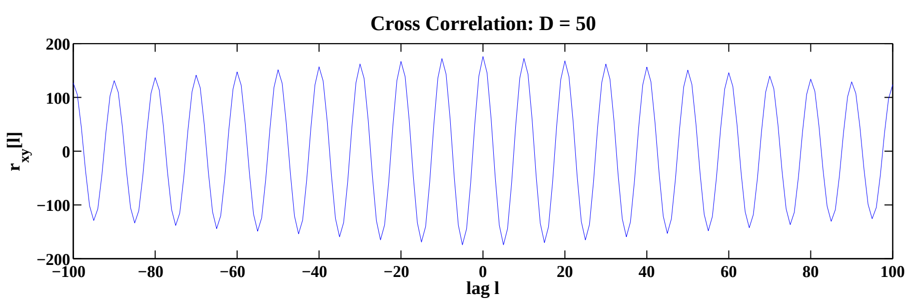
```
ny = nx+D;
yn = xn + wn;

[c lagc] = xcorr(xn(1+D:end),yn(1:end-D),100);
% Plot:
hf = figconfg('P0436','long');
plot(lagc,c)
xlabel('lag l','fontsize',LFS)
ylabel('r_{xy}[l]','fontsize',LFS)
title(['Cross Correlation: D = ',num2str(D)],'fontsize',TFS)
```



(a)



(b)



(c)

FIGURE 4.37: Cross correlation $r_{xy}[\ell]$ plot of (a) $D = 10$. (b) $D = 20$. (c) $D = 50$.

Assessment Problems

37.

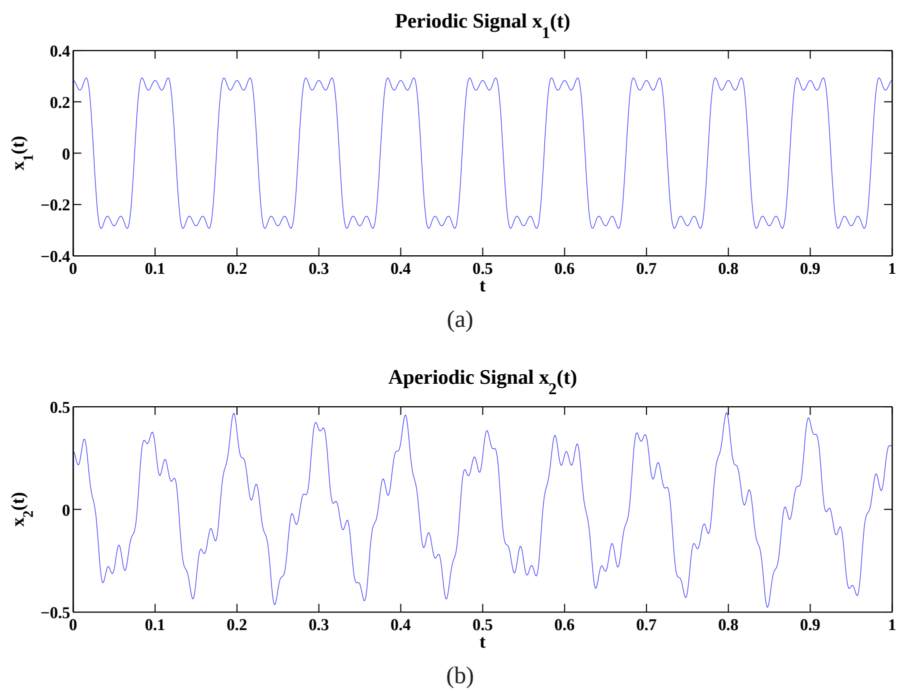


FIGURE 4.38: Examples of (a) a periodic signal $x_1(t)$ and (b) an "almost"-periodic signal $x_2(t)$.

```
% P0437: Generate and plot signals given in Figure4.3
close all; clc
t = linspace(0,1,1000);
F0 = 10;
x1 = cos(2*pi*F0*t)/3 - cos(2*pi*3*F0*t)/10 + ...
    cos(2*pi*5*F0*t)/20;
x2 = cos(2*pi*F0*t)/3 - cos(2*pi*sqrt(8)*F0*t)/10 + ...
    cos(2*pi*sqrt(51)*F0*t)/20;

% Plot:
hfa = figconfg('P0437a','long');
plot(t,x1)
xlabel('t','fontsize',LFS)
```

```
ylabel('x_1(t)','fontsize',LFS)
title('Periodic Signal x_1(t)','fontsize',TFS)
hfb = figconfg('P0437b','long');
plot(t,x2)
xlabel('t','fontsize',LFS)
ylabel('x_2(t)','fontsize',LFS)
title('Aperiodic Signal x_2(t)','fontsize',TFS)
```

38. (a) Solution:

$$T_1 = \frac{2\pi}{7\pi} = \frac{2}{7}, \quad T_2 = \frac{2\pi}{11\pi} = \frac{2}{11}, \quad T = 2$$

$x_1(t)$ is periodic with fundamental period $T = 2$.

(b) Solution:

$$T_1 = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi, \quad T_2 = \frac{2\pi}{2\sqrt{2}} = \frac{\sqrt{2}}{2}\pi, \quad T = \sqrt{2}\pi$$

$x_2(t)$ is periodic with fundamental period $T = \sqrt{2}\pi$.

(c) Solution:

$$T_1 = 22\pi, \quad T_2 = 158\pi, \quad T_3 = 68\pi, \quad T_1 = 22\pi, \quad T = 2\times 11\times 79\times 31\pi$$

$x_3(t)$ is periodic with fundamental period $T = 2 \times 11 \times 79 \times 31\pi = 53,878\pi$.

(d) Solution:

$$N_1 = \frac{2\pi}{\pi/7} = 14, \quad N_2 = \frac{2\pi}{\pi/11} = 22, \quad N = 154$$

$x_4[n]$ is periodic with fundamental period $N = 154$.

(e) Solution:

$$N_1 = \frac{2\pi}{0.1\pi} \times \frac{1}{2} = 10, \quad N_2 = \frac{2\pi}{2\pi/11} = 11, \quad N = 110$$

$x_5[n]$ is periodic with fundamental period $N = 110$.

39. Proof:

$$\sum_{n=<N>} s_k[n]s_m^*[n] = \sum_{n=<N>} \mathrm{e}^{\mathrm{j}\frac{2\pi}{N}kn}\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}mn} = \sum_{n=0}^{N-1} \mathrm{e}^{\mathrm{j}\frac{2\pi}{N}(k-m)n}$$

if $k - m \neq 0$,

$$\sum_{n=<N>} s_k[n]s_m^*[n] = \frac{1-\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}(k-m)N}}{1-\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}(k-m)}} = \frac{1-1}{1-\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}(k-m)}} = 0$$

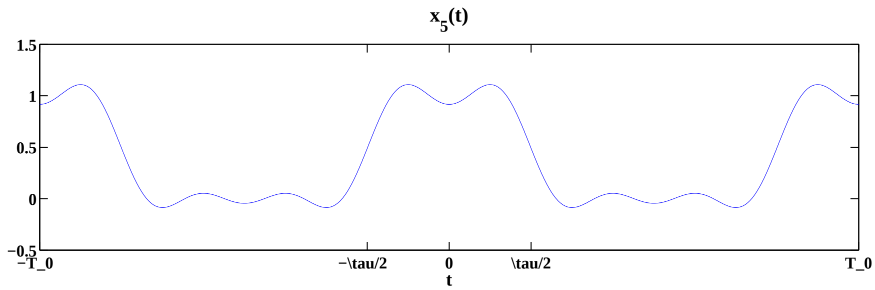
if $k - m = 0$,

$$\sum_{n=<N>} s_k[n]s_m^*[n] = \sum_{n=0}^{N-1} 1 = N$$

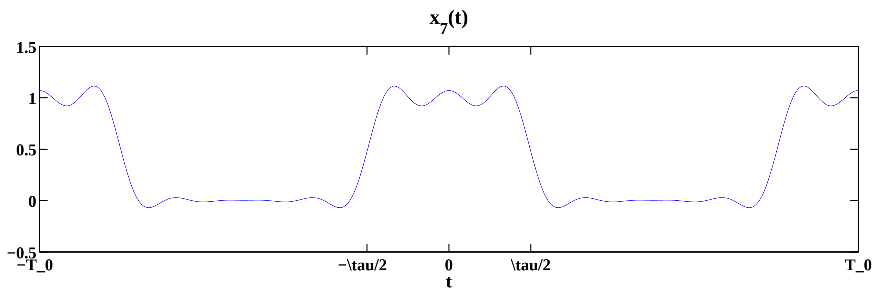
40.

MATLAB script:

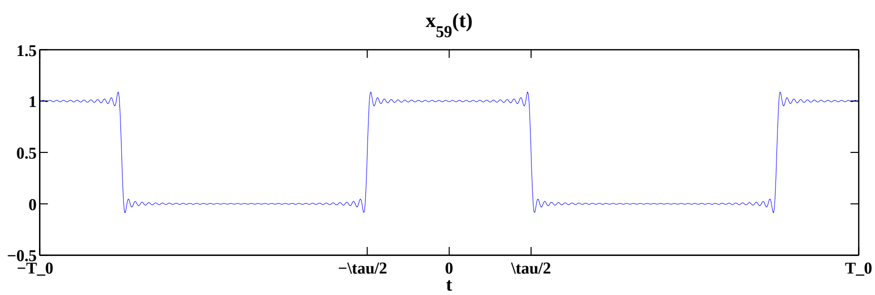
```
% P0440: Generate and plot signals given in Figure4.12
close all; clc
tau = 0.4;
F0 = 1;
T0 = 1/F0;
t = linspace(-T0,T0,1000);
A = 1;
% m = [5,7,59];
m = 59;
k = -100:100;
c = A*tau*F0*sinc(k*F0*tau);
temp = repmat(c,length(t),1).*exp(j*2*pi*F0*t'*k);
ind = (k>=-m)&(k<=m);
xm = sum(temp(:,ind),2);
% Plot:
hf = figconfg('P0440','long');
plot(t,xm)
set(gca,'Xtick',[-T0,-tau/2,0,tau/2,T0])
set(gca,'XtickLabel',{'-T_0','-\tau/2','0','\tau/2','T_0'})
xlabel('t','fontsize',LFS)
title(['x_{',num2str(m),'}(t)'],'fontsize',TFS)
```



(a)



(b)



(c)

FIGURE 4.39: Fourier series approximation of a rectangular pulse train.

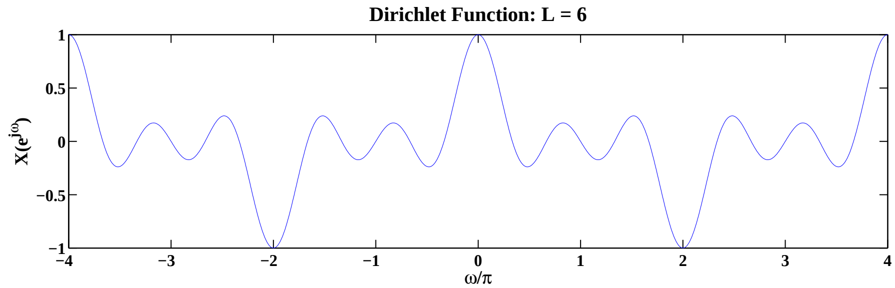
41. Proof:

$$
\begin{aligned}
P_{av} &= \frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2 = \frac{1}{N}\sum_{n=0}^{N-1}x[n]\cdot x^*[n] \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\left(\sum_{k=0}^{N-1}c_k\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}kn}\right)\left(\sum_{m=0}^{N-1}c_m^*\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}mn}\right) \\
&= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}c_kc_m^*\left(\sum_{n=0}^{N-1}\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}kn}\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}mn}\right) \\
&= \frac{1}{N}\sum_{k=0}^{N-1}c_kc_k^*N \\
&= \sum_{k=0}^{N-1}|c_k|^2
\end{aligned}
$$

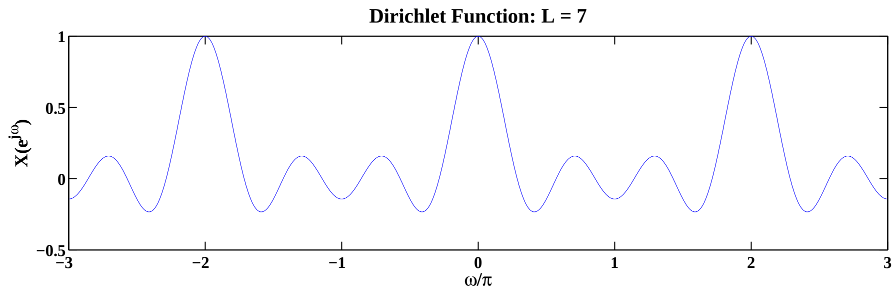
42. Comments:

The fundamental period for $L = 6$ is 4π and the fundamental period for $L = 7$ is 2π.

```
% P0442: Compute and plot the Dirichlet function
%        defined in (4.80)
close all; clc
L = 6;
% L = 7;
% w = linspace(-3*pi,3*pi,1000);
w = linspace(-4*pi,4*pi,1000);
D = diric(w,L);
% Plot:
hf = figconfg('P0442','long');
plot(w/pi,D)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('X(e^{j\omega})','fontsize',LFS)
title(['Dirichlet Function: L = ',num2str(L)],'fontsize',TFS)
```



(a)



(b)

FIGURE 4.40: The Dirichlet function for (a) $L = 6$ and (b) $L = 7$.

43. Solution:

$$X\left(e^{j\omega}\right) = \sum_{n=0}^{N-1} x[n] e^{-j\omega n}$$

The linear system is

$$\begin{bmatrix} e^{-j\omega_0 \cdot 0} & e^{-j\omega_0 \cdot 1} & \cdots & e^{-j\omega_0 \cdot (N-1)} \\ e^{-j\omega_1 \cdot 0} & e^{-j\omega_1 \cdot 1} & \cdots & e^{-j\omega_1 \cdot (N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\omega_{N-1} \cdot 0} & e^{-j\omega_{N-1} \cdot 1} & \cdots & e^{-j\omega_{N-1} \cdot (N-1)} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} = \begin{bmatrix} X\left(e^{j\omega_0}\right) \\ X\left(e^{j\omega_1}\right) \\ \vdots \\ X\left(e^{j\omega_{N-1}}\right) \end{bmatrix}$$

The N samples of $x[n]$ can be solved by

$$\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} = \begin{bmatrix} e^{-j\omega_0 \cdot 0} & e^{-j\omega_0 \cdot 1} & \cdots & e^{-j\omega_0 \cdot (N-1)} \\ e^{-j\omega_1 \cdot 0} & e^{-j\omega_1 \cdot 1} & \cdots & e^{-j\omega_1 \cdot (N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\omega_{N-1} \cdot 0} & e^{-j\omega_{N-1} \cdot 1} & \cdots & e^{-j\omega_{N-1} \cdot (N-1)} \end{bmatrix}^{-1} \begin{bmatrix} X\left(e^{j\omega_0}\right) \\ X\left(e^{j\omega_1}\right) \\ \vdots \\ X\left(e^{j\omega_{N-1}}\right) \end{bmatrix}$$

44. (a) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{7}\sum_{n=-1}^{5} x_1[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}kn} \\
&= \frac{1}{7}\left(\mathrm{e}^{\mathrm{j}\frac{2\pi}{7}k} + 2 + 3\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k} + 3\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k\cdot 2} + 3\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k\cdot 3} + 2\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k\cdot 4} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k\cdot 5}\right)
\end{aligned}
$$

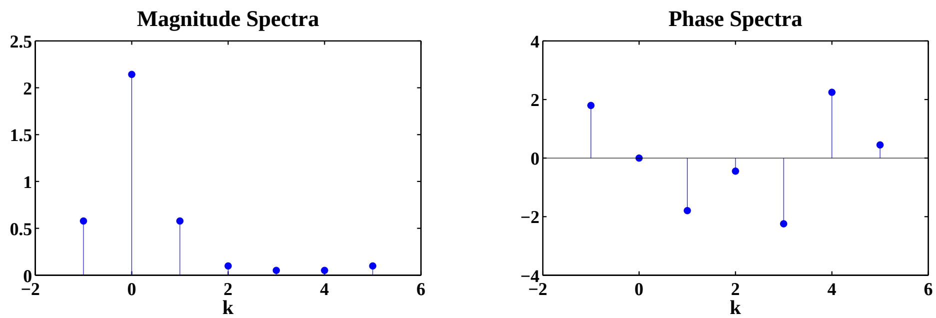


FIGURE 4.41: Magnitude and phase spectra of periodic sequence $x_1[n]$.

(b) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{10}\sum_{n=-5}^{4} x_2[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}kn} \\
&= (-\frac{\mathrm{j}}{5})\sum_{n=1}^{4}\sin(0.2\pi n)\sin(0.2\pi kn)
\end{aligned}
$$

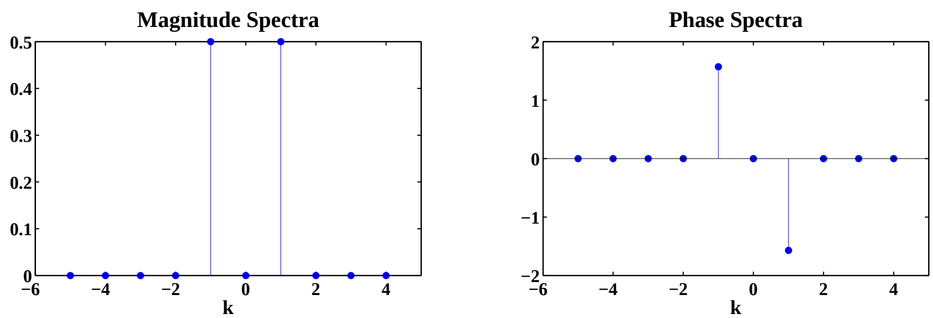


FIGURE 4.42: Magnitude and phase spectra of periodic sequence $x_2[n]$.

(c) Solution:

$$x_3[n] = \mathrm{e}^{\mathrm{j}2\pi n/7} + \mathrm{e}^{\mathrm{j}\pi n/3} + \mathrm{e}^{\mathrm{j}\pi n/7}$$
$$= \mathrm{e}^{\mathrm{j}\frac{2\pi}{42}n\cdot 6} + \mathrm{e}^{\mathrm{j}\frac{2\pi}{42}n\cdot 7} + \mathrm{e}^{\mathrm{j}\frac{2\pi}{42}n\cdot 3}$$

$$c_k = 1, \quad k = 3, 6, 7$$

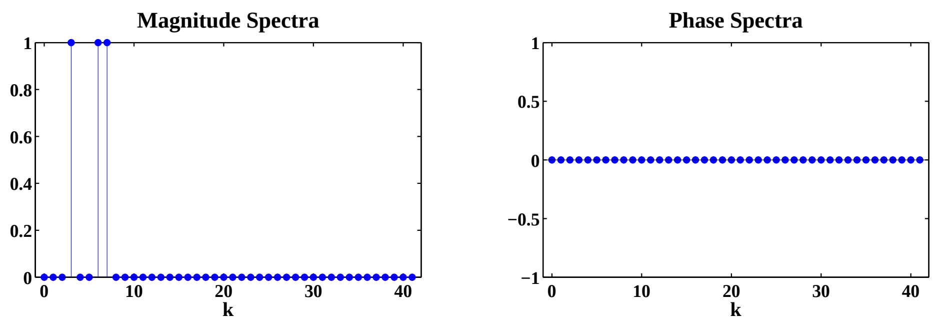


FIGURE 4.43: Magnitude and phase spectra of periodic sequence $x_3[n]$.

(d) Solution:

$$c_k = \frac{1}{8}\sum_{n=0}^{7} x_4[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}kn}$$
$$= \frac{1}{8}\left(1 + 2\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k} + 3\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 2} + 4\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 3} + 5\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 4} + 6\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 5} + 7\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 6} + 8\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 7}\right)$$

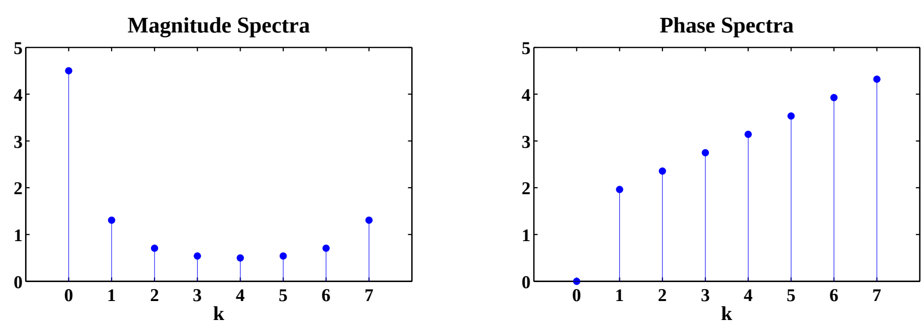


FIGURE 4.44: Magnitude and phase spectra of periodic sequence $x_4[n]$.

(e) Solution:

$$\begin{aligned} c_k &= \frac{1}{2}\sum_{n=0}^{1} x_5[n] \mathrm{e}^{-\mathrm{j}\frac{2\pi}{2}kn} \\ &= \frac{1}{2}(1\cdot 1 - 1\cdot \mathrm{e}^{-\mathrm{j}\pi k}) = \frac{1}{2}(1-\cos \pi k) \end{aligned}$$

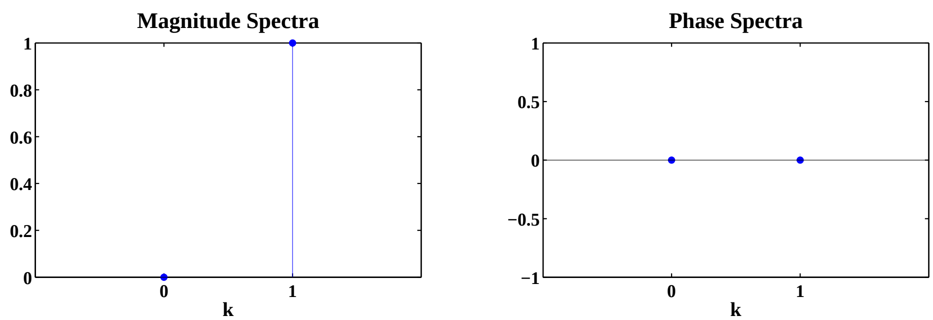


FIGURE 4.45: Magnitude and phase spectra of periodic sequence $x_5[n]$.

45. (a) Solution:

$$b_k = a_k(\mathrm{e}^{\mathrm{j}k\frac{2\pi}{N}} + 2 + \mathrm{e}^{-\mathrm{j}k\frac{2\pi}{N}}) = 2a_k(1+\cos(\frac{2\pi}{N}k))$$

(b) Solution:

$$\mathrm{e}^{-\mathrm{j}6\pi n/N} x[n-2] = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}n3} x[n-2]$$

$$b_k = a_{k+3}\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}(k+3)2}$$

(c) Solution:

$$3\cos(2\pi 5n/N)x[-n] = \frac{3}{2}\left(\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}n5} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}n5}\right) x[-n]$$

$$b_k = \frac{3}{2}a_{-(k-5)} + \frac{3}{2}a_{-(k+5)}$$

(d) Solution:

$$b_k = a_k + a_k^*$$

46. Proof:

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x_1[n]x_2^*[n] &= \sum_{n=-\infty}^{\infty}\left(\frac{1}{2\pi}\int_{2\pi} X_1(\mathrm{e}^{\mathrm{j}\omega})\,\mathrm{e}^{\mathrm{j}\omega n} d\omega\right) x_2^*[n] \\
&= \frac{1}{2\pi}\int_{2\pi} X_1(\mathrm{e}^{\mathrm{j}\omega})\left(\sum_{n=-\infty}^{\infty} x_2^*[n]\mathrm{e}^{\mathrm{j}\omega n}\right) d\omega \\
&= \frac{1}{2\pi}\int_{2\pi} X_1(\mathrm{e}^{\mathrm{j}\omega})\left(\sum_{n=-\infty}^{\infty} x_2[n]\mathrm{e}^{-\mathrm{j}\omega n}\right)^* d\omega \\
&= \frac{1}{2\pi}\int_{2\pi} X_1(\mathrm{e}^{\mathrm{j}\omega})X_2(\mathrm{e}^{\mathrm{j}\omega})^* d\omega
\end{aligned}$$

47. tba

48. (a) Solution:

$$X_1(\mathrm{e}^{\mathrm{j}\omega}) = \frac{3}{1-0.9\mathrm{e}^{-\mathrm{j}\omega}}$$

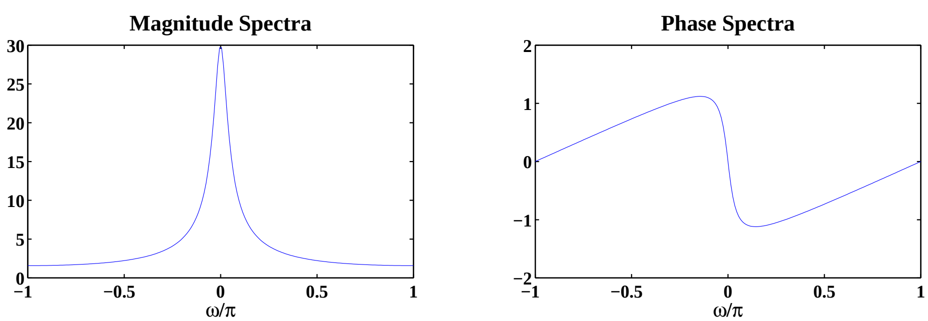


FIGURE 4.46: Magnitude and phase response for sequence $x_1[n] = 3(0.9)^n u[n]$.

(b) Solution:

$$x_2[n] = 2\cdot 0.8^4(-0.8)^{n-2}u[n-2]$$

$$X_2(\mathrm{e}^{\mathrm{j}\omega}) = \frac{2\cdot 0.8^4\mathrm{e}^{-\mathrm{j}2\omega}}{1+0.8\mathrm{e}^{-\mathrm{j}\omega}}$$

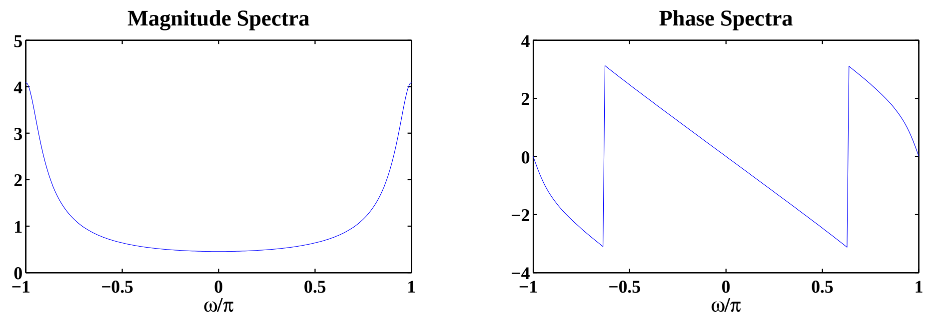


FIGURE 4.47: Magnitude and phase response for sequence $x_2[n] = 2(-0.8)^{n+2}u[n-2]$.

(c) Solution:

$$x_3[n] = (-0.7)(n-2)(-0.7)^{n-2}u[n-2] + 4\cdot(-0.7)(-0.7)^{n-2}u[n-2]$$

$$X_3(\mathrm{e}^{\mathrm{j}\omega}) = \frac{0.7^2\,\mathrm{e}^{-\mathrm{j}3\omega}}{(1+0.7\mathrm{e}^{-\mathrm{j}\omega})^2} - \frac{4\cdot 0.7\cdot \mathrm{e}^{-\mathrm{j}2\omega}}{1+0.7\mathrm{e}^{-\mathrm{j}\omega}}$$

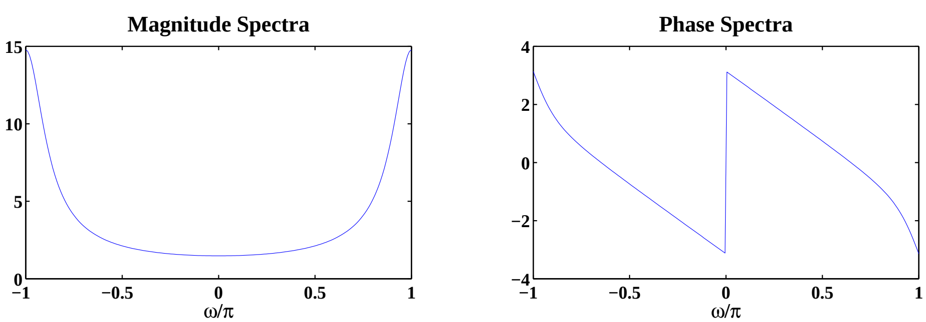


FIGURE 4.48: Magnitude and phase response for sequence $x_3[n] = (n+2)(-0.7)^{n-1}u[n-2]$.

(d) Solution:

$$x_r[n] = \frac{5}{2}\cdot(-0.8)^n(\mathrm{e}^{\mathrm{j}0.1\pi n} + \mathrm{e}^{-\mathrm{j}0.1\pi n})u[n]$$

$$X_4(\mathrm{e}^{\mathrm{j}\omega}) = \frac{5}{2}\left[\frac{1}{1+0.8\mathrm{e}^{-\mathrm{j}(\omega-0.1\pi)}} + \frac{1}{1+0.8\mathrm{e}^{-\mathrm{j}(\omega+0.1\pi)}}\right]$$

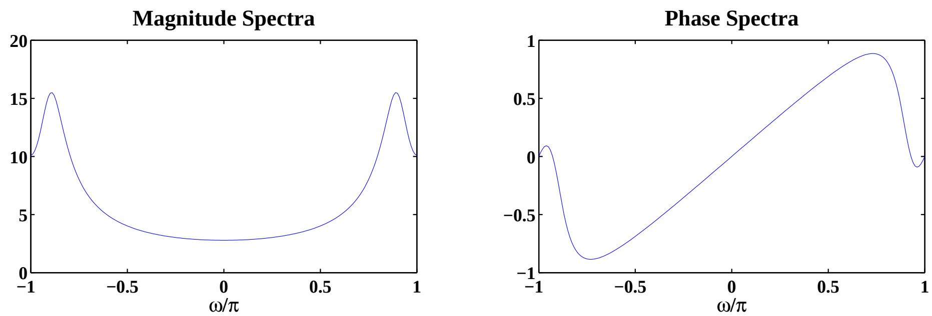


FIGURE 4.49: Magnitude and phase response for sequence $x_4[n] = 5(-0.8)^n\cos(0.1\pi n)u[n]$.

(e) Solution:

$$X_5(\mathrm{e}^{\mathrm{j}\omega}) = \sum_{n=-10}^{10}(0.7).^{|n|}\mathrm{e}^{-\mathrm{j}n\omega} = \sum_{n=1}^{10}(0.7).^{n}\mathrm{e}^{\mathrm{j}n\omega} + 1 + \sum_{n=1}^{10}(0.7).^{n}\mathrm{e}^{-\mathrm{j}n\omega}$$
$$= \frac{0.7\mathrm{e}^{\mathrm{j}\omega}(1-0.7^{10}\mathrm{e}^{\mathrm{j}10\omega})}{1-0.7\mathrm{e}^{\mathrm{j}\omega}} + 1 + \frac{0.7\mathrm{e}^{-\mathrm{j}\omega}(1-0.7^{10}\mathrm{e}^{-\mathrm{j}10\omega})}{1-0.7\mathrm{e}^{-\mathrm{j}\omega}}$$

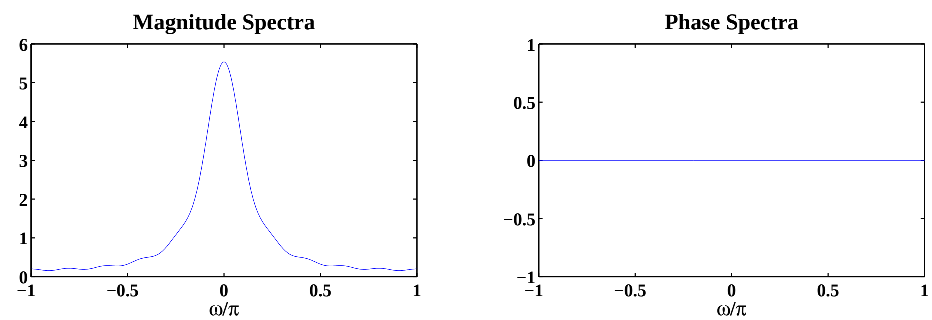


FIGURE 4.50: Magnitude and phase response for sequence $x_4[n] = 5(-0.8)^n\cos(0.1\pi n)u[n]$.

49. (a) Solution:

$$X_1(\mathrm{e}^{\mathrm{j}\omega}) = 2 + \frac{3}{2}(\mathrm{e}^{\mathrm{j}\omega} + \mathrm{e}^{-\mathrm{j}\omega}) + 2(\mathrm{e}^{\mathrm{j}3\omega} + \mathrm{e}^{-\mathrm{j}3\omega})$$

$$x_1[n] = 2\delta[n] + \frac{3}{2}(\delta[n+1] + \delta[n-1]) + 2(\delta[n+3] + \delta[n-3])$$

(b) Solution:

$$X_2(\mathrm{e}^{\mathrm{j}\omega}) = \left[1 + \frac{5}{2}(\mathrm{e}^{\mathrm{j}2\omega} + \mathrm{e}^{-\mathrm{j}2\omega}) + 4(\mathrm{e}^{\mathrm{j}4\omega} + \mathrm{e}^{-\mathrm{j}4\omega})\right] \mathrm{e}^{-\mathrm{j}3\omega}$$

$$x_2[n] = \delta[n-3] + \frac{5}{2}(\delta[n-1] + \delta[n-5]) + 4(\delta[n+1] + \delta[n-7])$$

(c) Solution:

$$X_3(\mathrm{e}^{\mathrm{j}\omega}) = \mathrm{j}\mathrm{e}^{-\mathrm{j}4\omega}\left[2 + \frac{3}{2}(\mathrm{e}^{\mathrm{j}\omega} + \mathrm{e}^{-\mathrm{j}\omega}) + \frac{1}{2}(\mathrm{e}^{\mathrm{j}2\omega} + \mathrm{e}^{-\mathrm{j}2\omega})\right]$$

$$x_3[n] = 2\mathrm{j}\delta[n-4] + \frac{3}{2}\mathrm{j}(\delta[n-3] + \delta[n-5]) + \frac{1}{2}\mathrm{j}(\delta[n-2] + \delta[n-6])$$

(d) Solution:

$$X_4(\mathrm{e}^{\mathrm{j}\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \pi/8 \\ 0, & \pi/8 \le |\omega| \le \pi \end{cases} + \begin{cases} 1, & 0 \le |\omega| \le 3\pi/4 \\ 0, & 3\pi/4 \le |\omega| \le \pi \end{cases}$$

$$x_4[n] = \frac{1}{8}\mathrm{sinc}(\frac{n}{8}) + \frac{3}{4}\mathrm{sinc}(\frac{3n}{4})$$

(e) Solution:

$$X_5(\mathrm{e}^{\mathrm{j}\omega}) = \omega \mathrm{e}^{\mathrm{j}(\pi/2)}\mathrm{e}^{-\mathrm{j}5\omega} = \mathrm{j}\omega\mathrm{e}^{-\mathrm{j}5\omega}$$

$$x[n] = \frac{1}{2\pi}\int_0^{2\pi} \omega \mathrm{e}^{\mathrm{j}n\omega} d\omega = -\frac{\mathrm{j}}{n}$$

$$x_5[n] = \frac{1}{n-5}$$

50. (a) Proof:

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\omega} = \sum_{n=n_0+1}^{n_0+N} \tilde{x}[n]e^{-jn\omega} = \sum_{<N>} \tilde{x}[n]e^{-jn\omega}$$

$$a_k = \frac{1}{N}\sum_{<N>} \tilde{x}[n]e^{-jn\frac{2\pi}{N}k} = \frac{1}{N}X\left(e^{j\frac{2\pi}{N}k}\right)$$

(b) Solution:

$$a_k = \frac{1}{5}\sum_{n=0}^{4} e^{-j\frac{2\pi}{5}kn} = \frac{1}{5}\left(1 + e^{-j\frac{2\pi}{5}k} + e^{-j\frac{2\pi}{5}k\cdot 2} + e^{-j\frac{2\pi}{5}k\cdot 3} + e^{-j\frac{2\pi}{5}k\cdot 4}\right)$$

$$X\left(e^{j\omega}\right) = \sum_{n=0}^{4} e^{-jn\omega} = \left(1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega}\right)$$

$$\frac{1}{5}X\left(e^{j\frac{2\pi}{5}k}\right) = \frac{1}{5}\left(1 + e^{-j\frac{2\pi}{5}k} + e^{-j2\frac{2\pi}{5}k} + e^{-j3\frac{2\pi}{5}k} + e^{-j4\frac{2\pi}{5}k}\right)$$

51. (a) Solution:

$$x_1[n] = \frac{1}{3}x[n+2] + \frac{1}{6}x[n+1] + \frac{1}{6}x[n-1] + \frac{1}{3}x[n-2]$$

$$X_1\left(e^{j\omega}\right) = \frac{1}{3}X\left(e^{j\omega}\right)e^{j2\omega} + \frac{1}{6}X\left(e^{j\omega}\right)e^{j\omega} + \frac{1}{6}X\left(e^{j\omega}\right)e^{-j\omega} + \frac{1}{3}X\left(e^{j\omega}\right)e^{-j2\omega}$$

(b) Solution:

$$x_2[n] = \left[(0.9)^n u[n]\frac{1}{2}(e^{j0.1\pi n} + e^{-j0.1\pi n})\right] * x[n-2]$$

$$X_2\left(e^{j\omega}\right) = \frac{1}{2}\left(\frac{1}{1-0.9e^{-j(\omega-0.1\pi)}} + \frac{1}{1-0.9e^{-j(\omega+0.1\pi)}}\right)X\left(e^{j\omega}\right)e^{-j2\omega}$$

(c) Solution:

$$x_3[n] = j\cdot\frac{n}{j}x[n-1] - \left(\frac{n}{j}\right)^2 x[n-2]$$

$$X_3\left(e^{j\omega}\right) = j\frac{dX\left(e^{j\omega}\right)e^{-j\omega}}{d\omega} - \frac{d^2X\left(e^{j\omega}\right)e^{-j2\omega}}{d\omega^2}$$

(d) Solution:

$$X_4\left(e^{j\omega}\right) = \frac{X\left(e^{j\omega}\right) - jX^*\left(e^{j\omega}\right)}{2}$$

(e) Solution:

$$x_5[n] = (-0.7)^n u[n] \cdot \frac{1}{2\mathrm{j}}(\mathrm{e}^{\mathrm{j}0.4\pi n} - \mathrm{e}^{-\mathrm{j}0.4\pi n}) * x[n+2]$$

$$X_5\big(\mathrm{e}^{\mathrm{j}\omega}\big) = \frac{1}{2\mathrm{j}}\left(\frac{1}{1+0.7\mathrm{e}^{-\mathrm{j}(\omega-0.4\pi)}} - \frac{1}{1+0.7\mathrm{e}^{-\mathrm{j}(\omega+0.4\pi)}}\right) X\big(\mathrm{e}^{\mathrm{j}\omega}\big)\mathrm{e}^{2\mathrm{j}\omega}$$

52. Solution:

Parseval's Theorem: $$\sum_{n=-\infty}^{\infty} x_1[n]x_2^*[n] = \frac{1}{2\pi}\int_{2\pi} X_1\big(\mathrm{e}^{\mathrm{j}\omega}\big)X_2^*\left(\mathrm{e}^{\mathrm{j}\omega}\right) d\omega$$

$$x_1[n] = \frac{\sin(\pi n/4)}{2\pi n} = \frac{1}{8}\text{sinc}(n/4)$$

$$X_1\big(\mathrm{e}^{\mathrm{j}\omega}\big) = \frac{1}{2}\text{rect}(\frac{2\omega}{\pi})$$

$$x_2[n] = \frac{\sin(\pi n/6)}{5\pi n} = \frac{1}{30}\text{sinc}(n/6)$$

$$X_1\big(\mathrm{e}^{\mathrm{j}\omega}\big) = \frac{1}{5}\text{rect}(\frac{3\omega}{\pi})$$

$$S = \frac{1}{2\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2}\cdot\frac{1}{5}d\omega = \frac{1}{60}$$

53. Solution:

$$x[n] = \frac{1}{2}(\mathrm{e}^{\mathrm{j}\omega_0 n} + \mathrm{e}^{-\mathrm{j}\omega_0 n})(u[n] - u[n-M])$$

$$\begin{aligned}\sum_{n=0}^{M-1} \mathrm{e}^{-\mathrm{j}n\omega} &= 1 + \mathrm{e}^{-\mathrm{j}\omega} + \mathrm{e}^{-\mathrm{j}2\omega} + \cdots + \mathrm{e}^{-\mathrm{j}(M-1)\omega} \\ &= \frac{1 - \mathrm{e}^{-\mathrm{j}M\omega}}{1 - \mathrm{e}^{-\mathrm{j}\omega}} = \frac{1 - \cos M\omega + \mathrm{j}\sin M\omega}{1 - \cos\omega + \mathrm{j}\sin\omega}\end{aligned}$$

The real part of $\sum_{n=0}^{M-1} \mathrm{e}^{-\mathrm{j}n\omega}$ is given by

$$\begin{aligned}X_R(\mathrm{e}^{\mathrm{j}\omega}) &= \frac{(1 - \cos M\omega + \mathrm{j}\sin M\omega)(1 - \cos M\omega - \mathrm{j}\sin M\omega)}{(1-\cos\omega)^2 + \sin^2\omega} \\ &= \frac{1 - \cos M\omega - \cos\omega + (\cos M\omega\cos\omega + \sin M\omega\sin\omega)}{2 - 2\cos\omega} \\ &= \frac{2\sin^2(\omega/2) + (\cos(M-1)\omega - \cos(M\omega))}{4\sin^2(\omega/2)} \\ &= \frac{\sin(\omega/2) + \sin(\frac{2M-1}{2}\omega)}{2\sin(\omega/2)} \\ &= \frac{\sin(2M\omega/4)\cos(\frac{2M-2}{4}\omega)}{\sin(\omega/2)} \\ &= \cos\frac{(M+1)\omega}{2}\frac{\sin\frac{M\omega}{2}}{\sin\frac{\omega}{2}}\end{aligned}$$

Simply applying frequency-shifting property, we can prove that:

$$\begin{aligned}X(\mathrm{e}^{\mathrm{j}\omega}) = &\tfrac{1}{2}\cos\left\{\tfrac{(\omega-\omega_0)(M-1)}{2}\right\}\left[\tfrac{\sin\{(\omega-\omega_0)M/2\}}{\sin\{(\omega-\omega_0)/2\}}\right] \\ &+ \tfrac{1}{2}\cos\left\{\tfrac{(\omega+\omega_0)(M-1)}{2}\right\}\left[\tfrac{\sin\{(\omega+\omega_0)M/2\}}{\sin\{(\omega+\omega_0)/2\}}\right]\end{aligned}$$

Comments:

As M increases, the DTFT $X(\mathrm{e}^{\mathrm{j}\omega})$ is closer to the DTFS of $\cos\omega_0 n$.

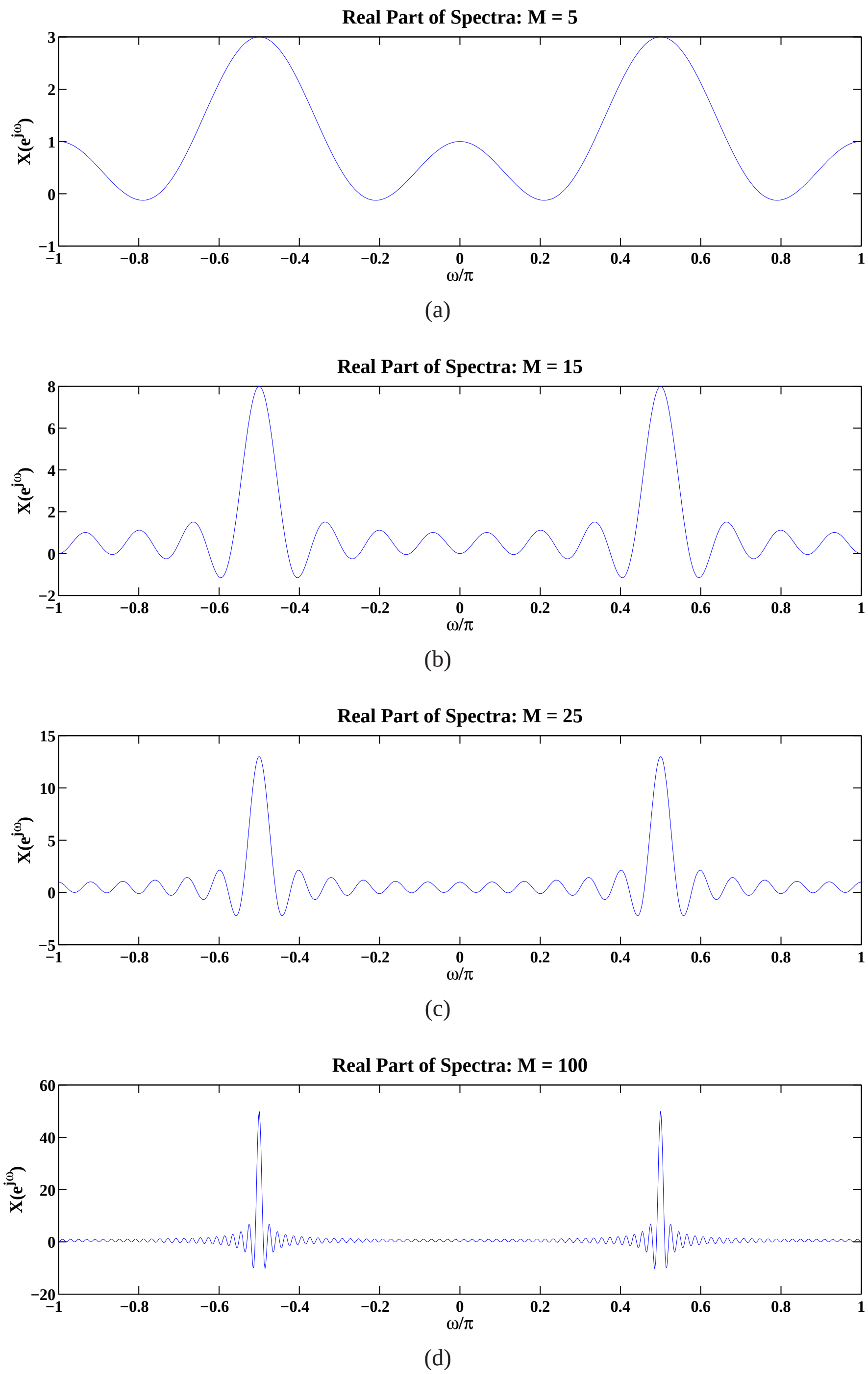


FIGURE 4.51: Plot of $X(e^{j\omega})$ for $\omega_0 = \pi/2$ and $N = 5,\ 15,\ 25,\ 100$.

54. Solution:

(a)

$$X\big(e^{j0}\big) = \sum x[n] = 0$$

(b)

$|X\big(e^{j\omega}\big)| = 0,$ Real and odd in time $\implies$ Imaginary and odd in frequency

(c)

$$\int_{-\pi}^{\pi} X\big(e^{j\omega}\big)\, d\omega = 2\pi x[0] = 0$$

(d)

$$\begin{aligned} X\big(e^{j\pi}\big) &= \sum_n x[n] e^{-j\pi n} = \sum_n x[n] \cos \pi n \\ &= 1 \cdot \cos 4\pi - 2\cos 3\pi + 3\cos 2\pi - 4\cos \pi + 0 \\ &\quad + 4\cos \pi - 3\cos 2\pi + 2\cos 3\pi - \cos 4\pi \\ &= 0 \end{aligned}$$

(e)

$$\int_{-\pi}^{\pi} |X\big(e^{j\omega}\big)|^2\, d\omega = 2\pi \sum_n |x[n]|^2 = 120\pi, \quad \text{Parseval's Theorem}$$

55. (a) Solution:

$$
\begin{aligned}
r_y[\ell] &= \sum_{n=-\infty}^{\infty} y[n]y[n-\ell] = \sum_{n=-\infty}^{\infty} (x[n]+ax[n-D])(x[n-\ell]+ax[n-D-\ell]) \\
&= \sum_{n=-\infty}^{\infty} \big(x[n]x[n-\ell] + x[n]\cdot a \cdot x[n-D-\ell] + a\cdot x[n-D]x[n-\ell] \\
&\quad + a^2 \cdot x[n-D]x[n-D-\ell]\big) \\
&= (1+a^2)r_x[\ell] + a\cdot r_x[\ell+D] + a\cdot r_x[\ell-D]
\end{aligned}
$$

(b) See plot below.

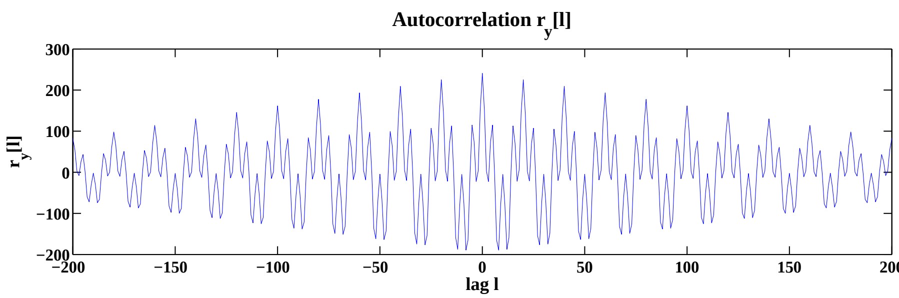


FIGURE 4.52: Plot of autocorrelation $r_y[\ell]$.

(c) tba

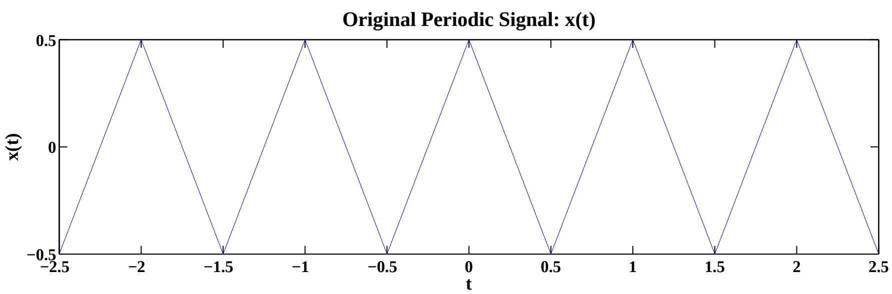
Review Problems

56. See book companion toolbox.

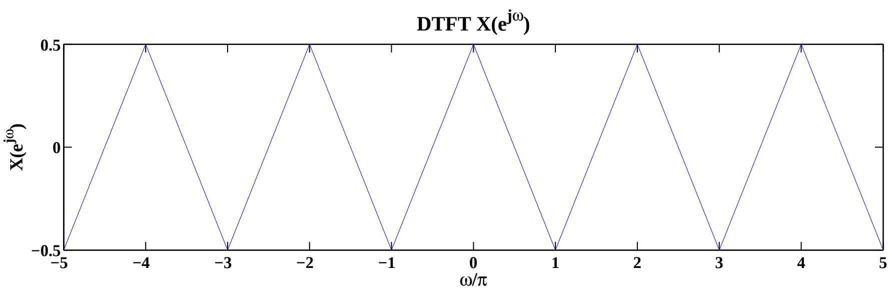
57. (a) Solution:

$$\begin{aligned}
c_k &= \int_{-0.5}^{0.5} \frac{1-4|t|}{2} \mathrm{e}^{-\mathrm{j}2\pi kt} dt \\
&= \int_{-0.5}^{0} \frac{1+4t}{2} \mathrm{e}^{-\mathrm{j}2\pi kt} dt + \int_{0}^{0.5} \frac{1-4t}{2} \mathrm{e}^{-\mathrm{j}2\pi kt} dt \\
&= \int_{0}^{0.5} (1-4t) \cos 2\pi kt dt \\
&= \frac{1-\cos \pi k}{\pi^2 k^2}
\end{aligned}$$

(b) See plot below.



(a)



(b)

FIGURE 4.53: Plot of (a) original periodic signal $x(t)$ and (b) DTFT $C\big(\mathrm{e}^{\mathrm{j}\omega}\big)$.

(c) Solution:

$$x(t) = C\big(\mathrm{e}^{\mathrm{j}2\pi t}\big)$$

(d) tba