CHAPTER 6

Sampling of Continuous-Time Signals

Basic Problems

21.

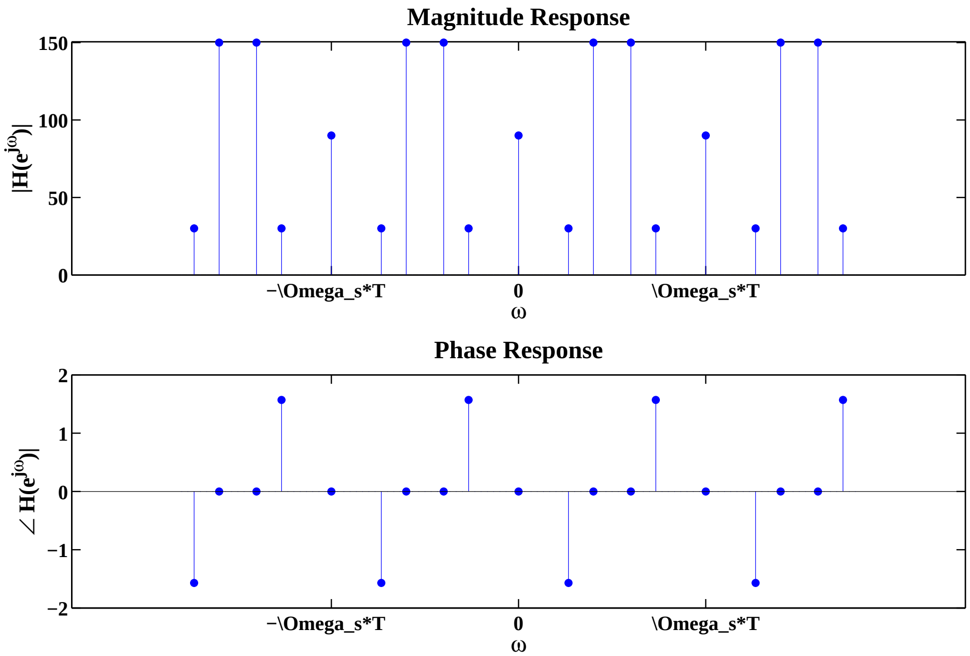
22. Solution:
The spectra of the continuous signal $x_c(t)$ is

$$X_c(j2\pi F) = \begin{cases} 3, & F = 0 \\ j, & F = 8 \\ -j, & F = -8 \\ 5, & F = 12 \\ 5, & F = -12 \\ 0, & \text{otherwise} \end{cases}$$

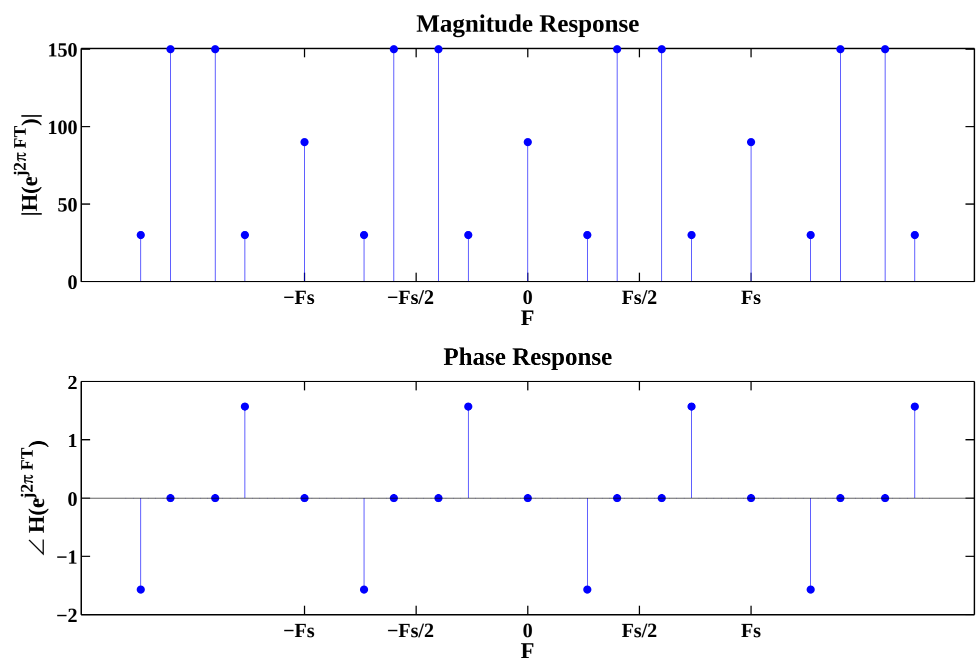
The spectra of sampled sequence $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=2\pi F/F_s} = F_s \sum_{n=-\infty}^{\infty} X_c[j2\pi(F - nF_s)]$$

$x_c(t)$ can be recovered if (a) $F_s = 30$ Hz, and can NOT be recovered if (b) $F_s = 20$ Hz, (c) $F_s = 15$ Hz.

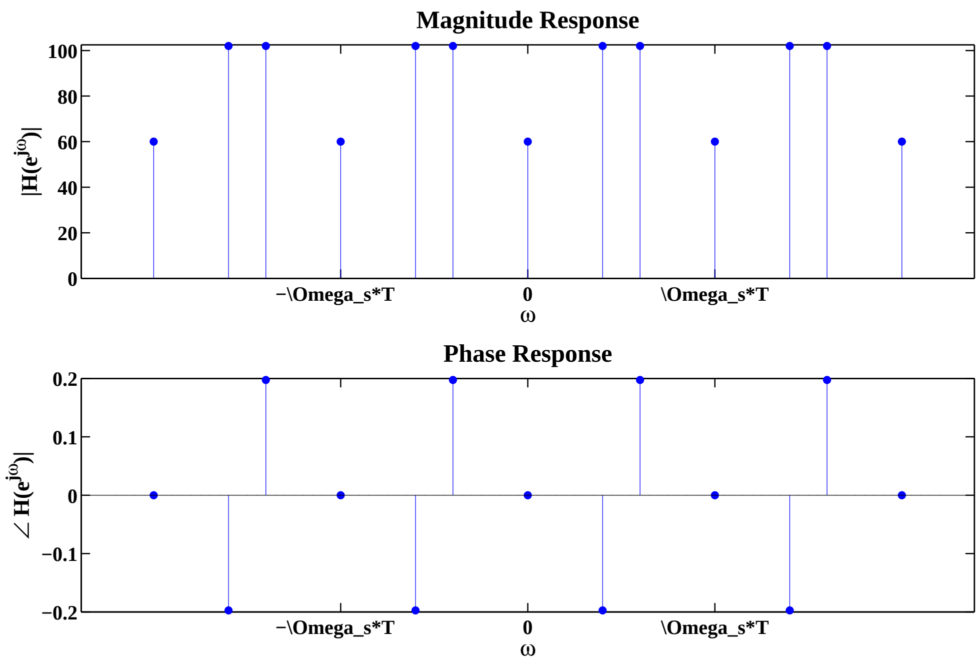


(a)

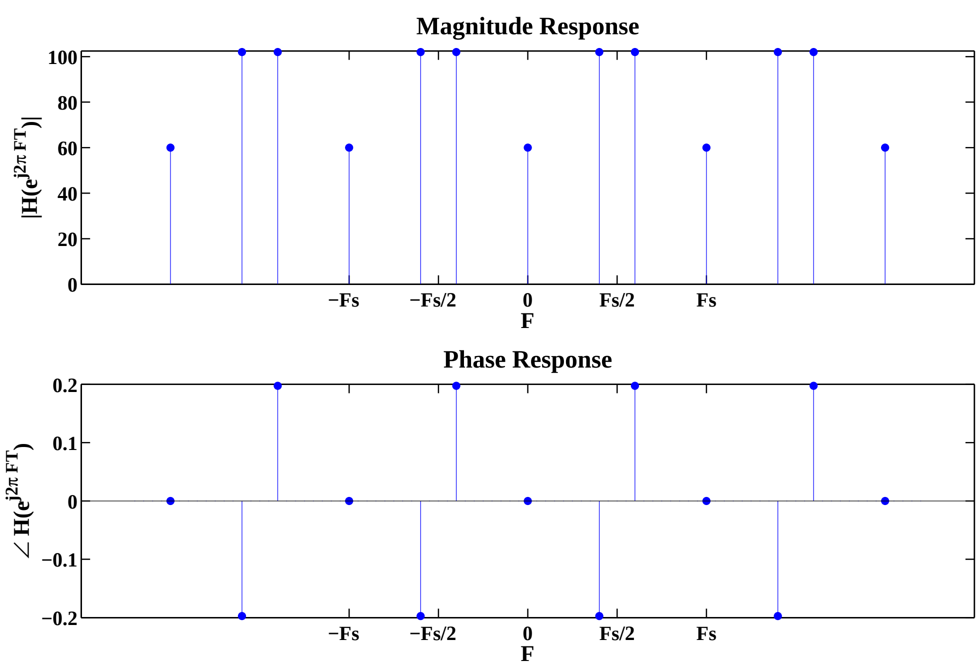


(b)

FIGURE 6.1: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 30$ KHz.

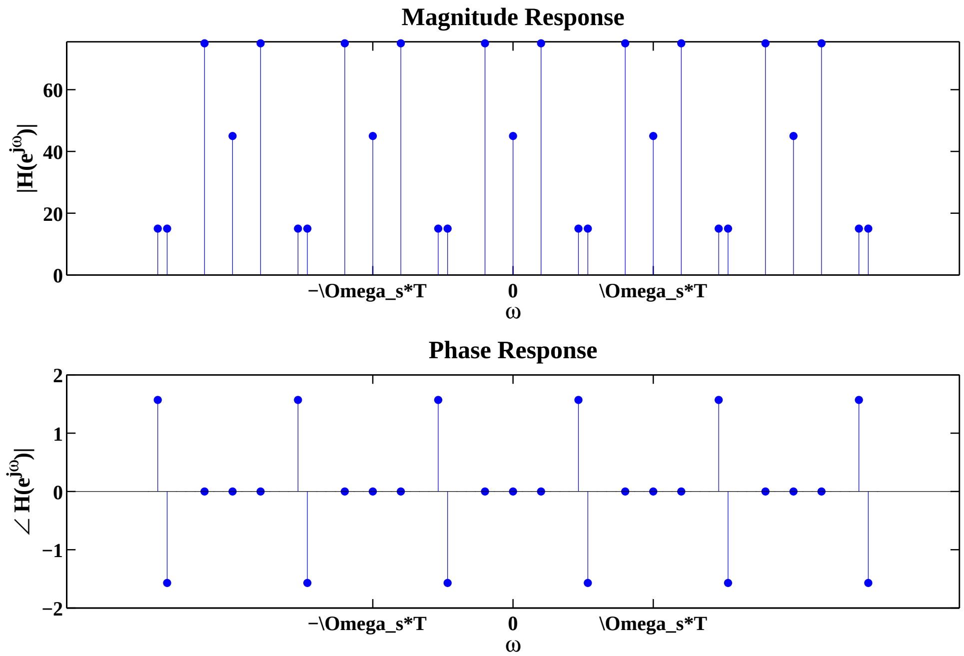


(a)

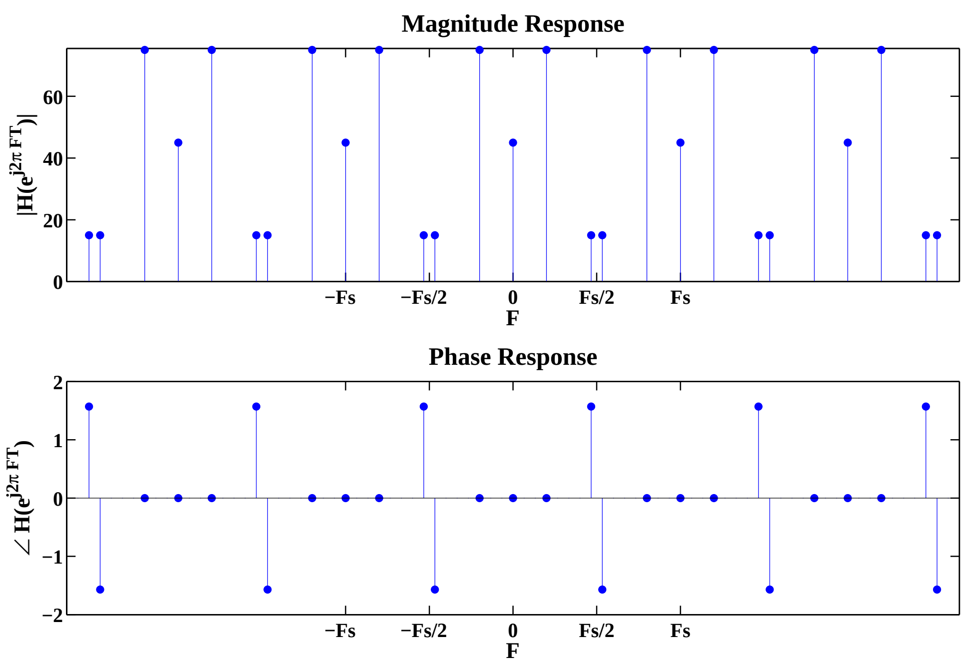


(b)

FIGURE 6.2: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 20$ KHz.



(a)



(b)

FIGURE 6.3: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 15$ KHz.

23. Solution:

The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j\Omega) = \begin{cases} 5, & \Omega = 40 \\ 3, & \Omega = -70 \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=\Omega T} = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_r[j(\Omega - k\Omega_s)]$$

The continuous signal $x_c(t)$ can be recovered if the sampling interval is (a) $T = 0.01$, (b) $T = 0.04$, and can NOT be recovered if the sampling interval is (c) $T = 0.1$.

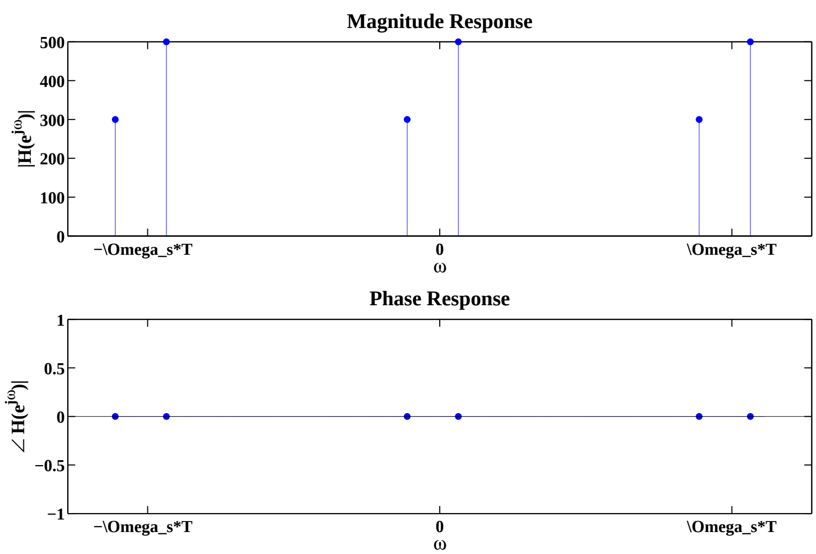


FIGURE 6.4: Magnitude and phase responses of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = 0.01$.

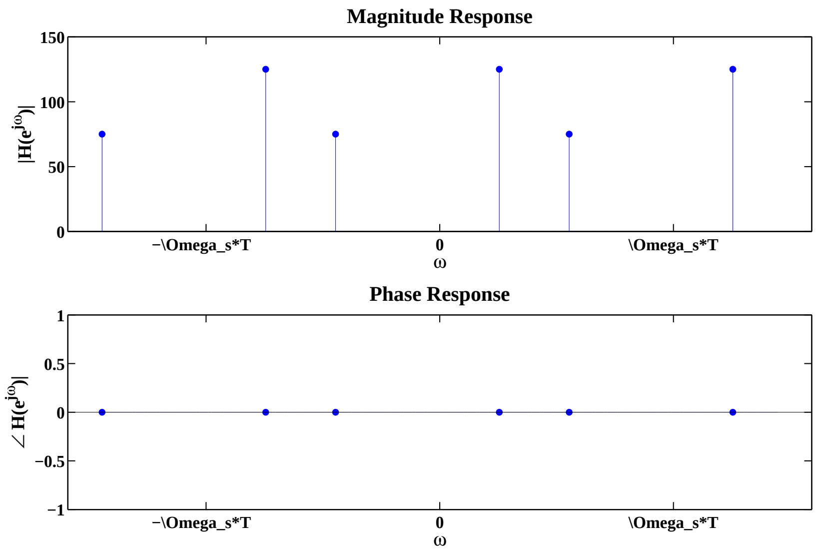


FIGURE 6.5: Magnitude and phase responses of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = 0.04$.

24. Solution:
The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j2\pi F) = \begin{cases} \frac{2}{5}|F| + 2, & |F| \leq 5 \\ -\frac{4}{5}|F| + 8, & 5 < |F| \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=2\pi F/F_s} = F_s \sum_{k=-\infty}^{\infty} X_c[j2\pi(F - kF_s)]$$

The signal $x_c(t)$ can NOT be recovered from $x[n]$ if the sampling rate is (a) $F_s = 10$ Hz and (b) $F_s = 15$ Hz and can be recovered if the sampling rate is (c) $F_s = 30$ Hz.

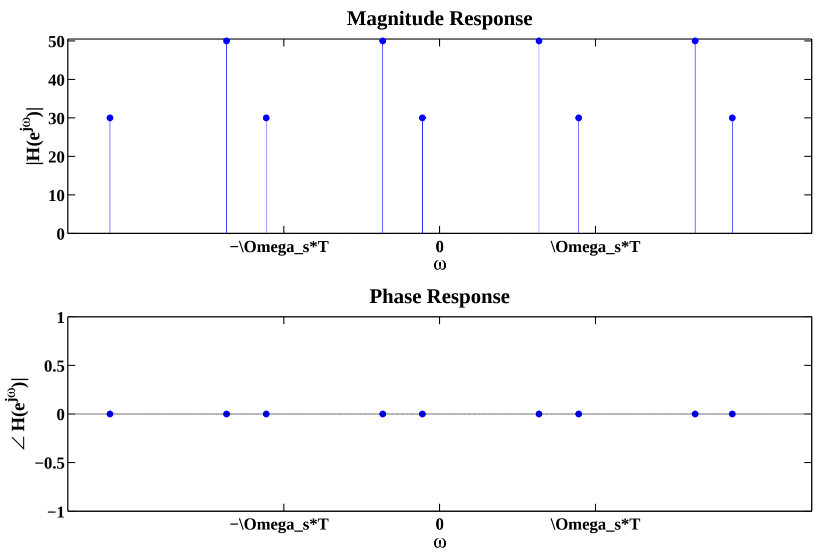


FIGURE 6.6: Magnitude and phase responses of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = 0.1$.

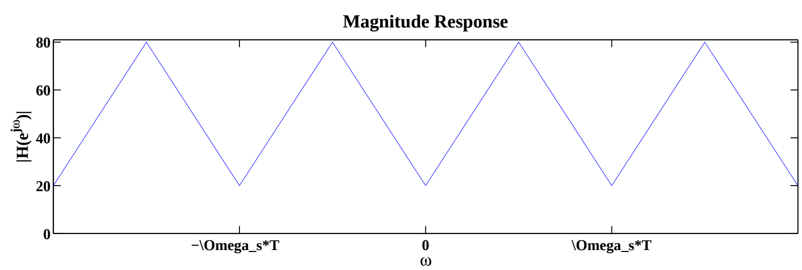


FIGURE 6.7: Magnitude response of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling rate is $F_{\mathrm{s}} = 10$.

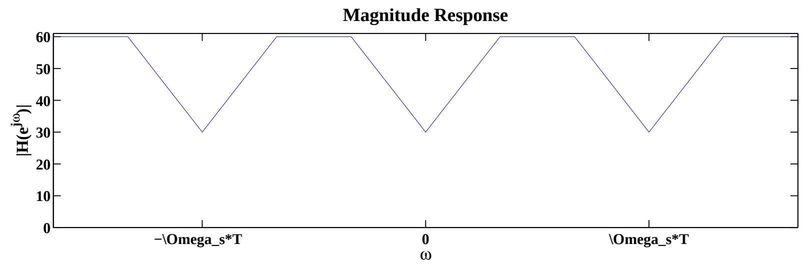


FIGURE 6.8: Magnitude response of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of ω in $\frac{\mathrm{rad}}{\mathrm{sam}}$ when the sampling rate is $F_\mathrm{s} = 15$.

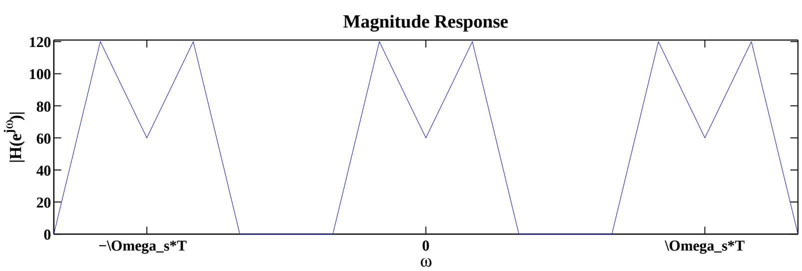


FIGURE 6.9: Magnitude response of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of ω in $\frac{\mathrm{rad}}{\mathrm{sam}}$ when the sampling rate is $F_\mathrm{s} = 30$.

25. Solution:
The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j2\pi F) = \begin{cases} -5|F| + 10, & 0 \le |F| \le 1 \\ 5|F|, & 1 \le |F| \le 2 \\ 10, & 2 \le |F| \le 3 \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X\left(e^{j2\pi F/F_s}\right) = F_s \sum_{k=-\infty}^{\infty} X_c[j2\pi(F - kF_s)]$$

The signal $x_c(t)$ can NOT be recovered from $x[n]$ when the sampling interval is (a) $T = 0.2$, (b) $T = 0.25$, (c) $T = 0.5$.

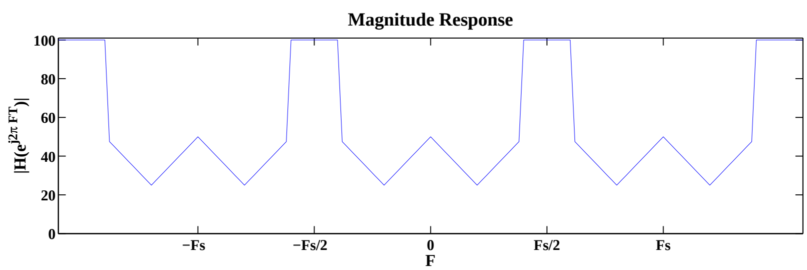


FIGURE 6.10: Magnitude response of $X\left(e^{j\omega}\right)$ as a function of F in Hz when the sampling interval is $T = 0.2$.

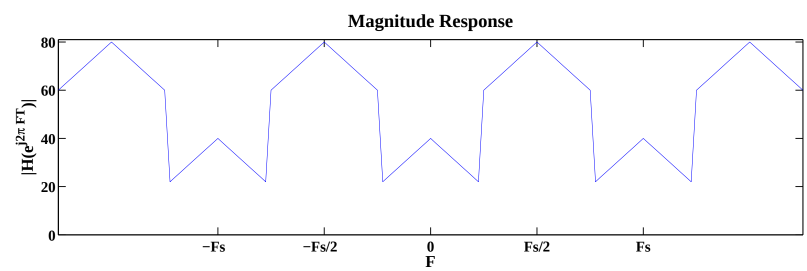


FIGURE 6.11: Magnitude response of $X\left(e^{j\omega}\right)$ as a function of F in Hz when the sampling interval is $T = 0.25$.

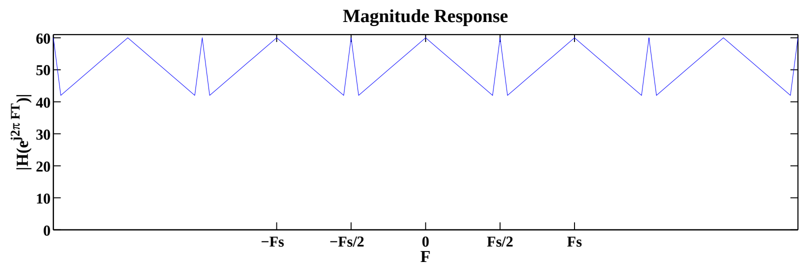


FIGURE 6.12: Magnitude response of $X(e^{j\omega})$ as a function of F in Hz when the sampling interval is $T = 0.5$.

26. (a) Solution:
The sampled sequence $x[n]$ is:

$$x[n] = 3\cos(0.3\pi n + \pi/4) + 3\sin(0.8\pi n)$$

(b) Solution:
The constructed signal $y_{\mathrm{r}}(t)$ is:

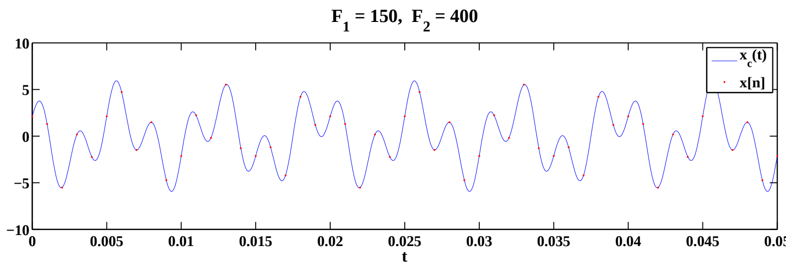
$$y_{\mathrm{r}}(t) = 3\cos(300\pi t + \pi/4) + 3\sin(800\pi t)$$

(c) Solution:
The sampled sequence $x[n]$ is:

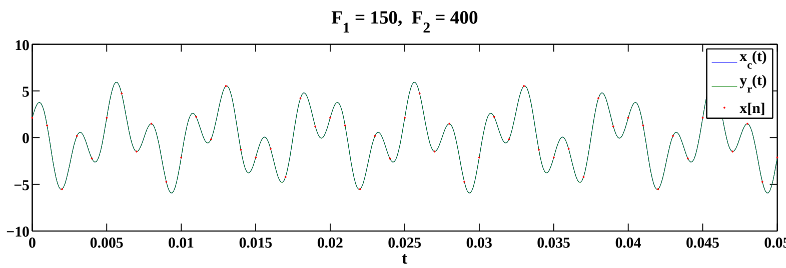
$$x[n] = 3\cos(0.6\pi n + \pi/4) + 3\sin(1.4\pi n)$$

The constructed signal $y_{\mathrm{r}}(t)$ is:

$$y_{\mathrm{r}}(t) = 3\cos(600\pi t + \pi/4) - 3\sin(600\pi t)$$



(a)

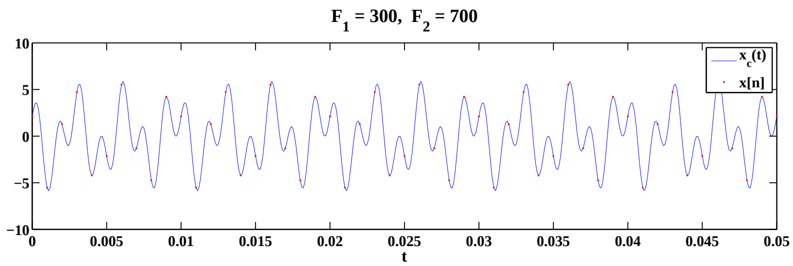


(b)

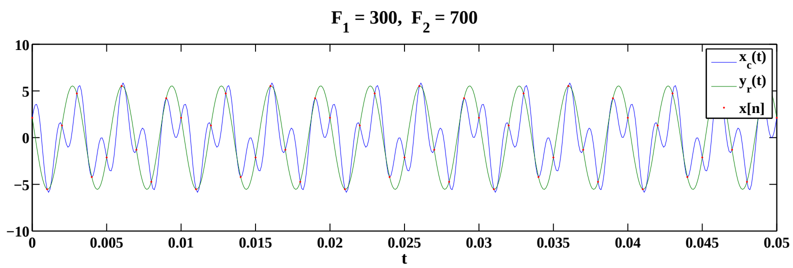
FIGURE 6.13: (a) Plot of $x[n]$ and $x_c(t)$ and (b) plot of $y_r(t)$ when $F_1 = 150$ Hz and $F_2 = 400$ Hz.

27. (a) Solution:
    The quantizer step is:
    $$\frac{10\text{v}}{2^8} = 0.0390625\text{v}$$
    (b) Solution:
    The SQNR is:
    $$\text{SQNR} = 49.92\text{dB}$$
    (c) Solution:
    The folding frequency is $F_s/2 = 4\text{k}$.
    (d) Solution:
    The reconstructed signal $x_r(t)$ is:
    $$x_r(t) = -5\sin[6000\pi t - \pi/2]$$

28. Solution:
    The minimum sampling frequency is:
    $$\min F_s = 4\text{KHz}$$



(a)



(b)

FIGURE 6.14: (a) Plot of $x[n]$ and $x_{\mathrm{c}}(t)$ and (b) plot of $y_{\mathrm{r}}(t)$ when $F_1 = 300$ Hz and $F_2 = 700$ Hz.

29. Solution:
    The minimum sampling rate can be computed by

$$\min F_{\mathrm{s}} = 95.2727273\text{KHz}$$

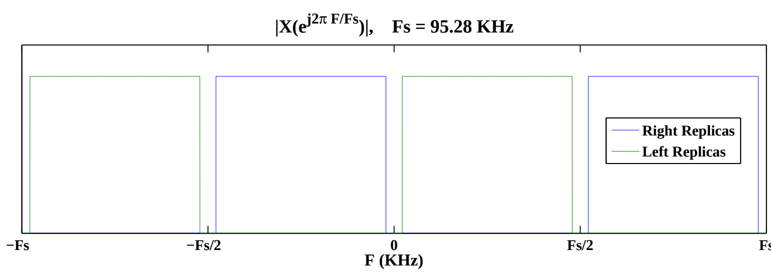


FIGURE 6.15: Baseband signal spectrum after sampling.

30. (a) Solution:

(i) when $\Delta x = \Delta y = 0.5$ meter, the reconstructed signal is:

$$s_{\mathrm{r}}(x, y) = 4\cos(2\pi y)$$

(ii) when $\Delta x = \Delta y = 0.2$ meter, the reconstructed signal is:

$$s_{\mathrm{r}}(x, y) = 4\cos(4\pi x)\cos(4\pi y)$$

(b) tba

31. tba.