CHAPTER 4

Fourier Representation of Signals

Basic Problems

20. Solution:
$x[n]$ is always periodic, and the fundamental period N is the least common multiple of N_1, N_2.

21. (a) Solution:
$x_1(t)$ is aperiodic.

 (b) Solution:
$x_2[n]$ is periodic with fundamental period $N = 20$.

 (c) Solution:
$x_3(t)$ is periodic with fundamental period $T = \frac{1}{500}$.

 (d) Solution:
$x_4[n]$ is aperiodic.

 (e) Solution:
$x_5[n]$ is periodic with fundamental period $N = 60$.

22. (a) Solution:
$x[n]$ is periodic.

 (b) Solution:
The fundamental period of the sequence $x[n]$ is $N = 4$.

23. Solution:

$$c_k = \frac{A(1 - \cos \pi k)}{(\pi k)^2}$$

(a) See plots below.

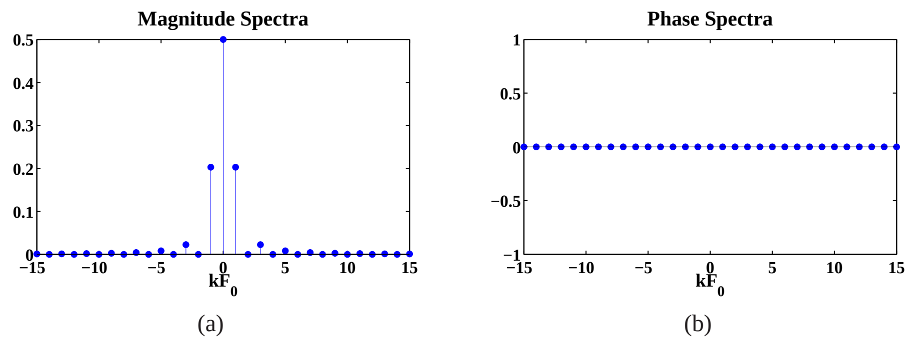


FIGURE 4.1: (a) Magnitude spectra of $x(t)$ for $A = 1$ and $T_0 = 1$. (b) Phase spectra of $x(t)$ for $A = 1$ and $T_0 = 1$.

(b) See plots below.

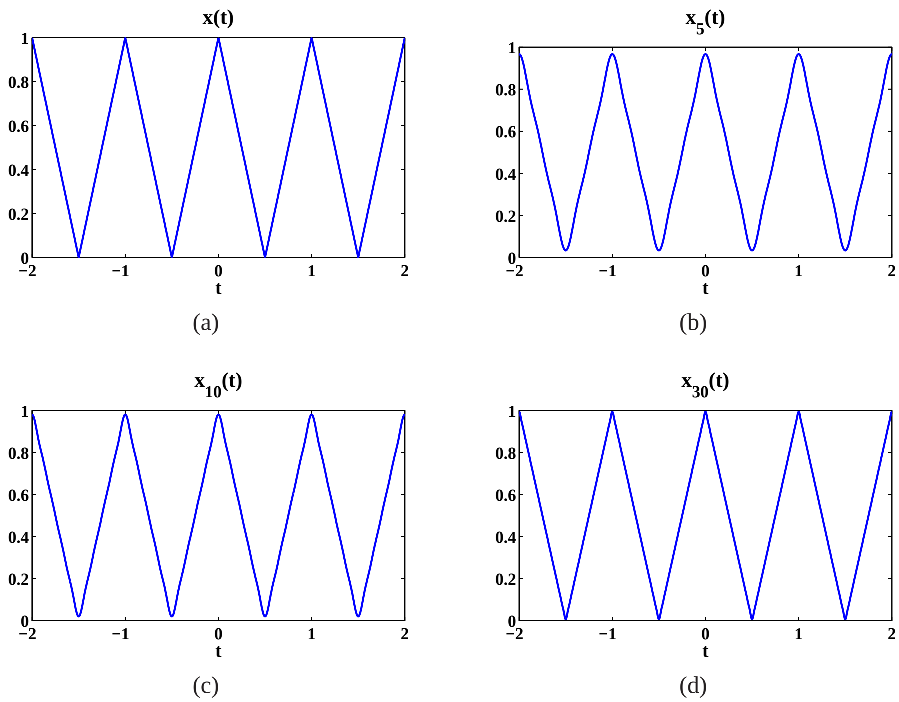


FIGURE 4.2: (a) $x(t)$ for $A = 1$ and $T_0 = 1$. (b) $x_5(t)$. (c) $x_{10}(t)$. (b) $x_{30}(t)$.

24. (a) Solution:

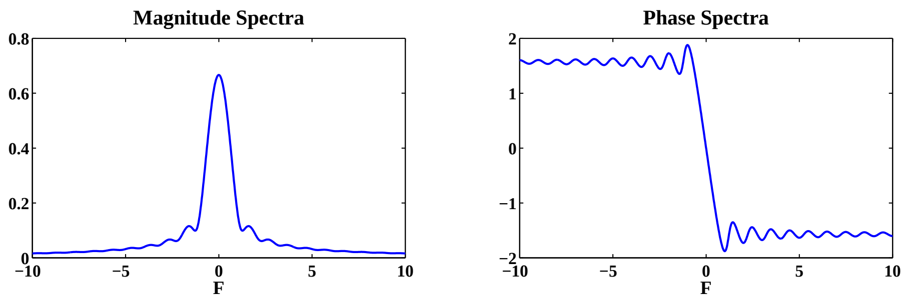
$$X(F) = \frac{1}{\mathrm{j}2\pi F} + \frac{2\mathrm{e}^{-\mathrm{j}2\pi F}}{(\mathrm{j}2\pi F)^2} + \frac{2(\mathrm{e}^{-\mathrm{j}2\pi F} - 1)}{(\mathrm{j}2\pi F)^3}$$

(b) Solution:

$$X(F) = \frac{-48\mathrm{j}\pi^2 F}{(4\pi^2F^2 + 12\mathrm{j}\pi F - 9 - 4\pi^2)(4\pi^2F^2 - 12\mathrm{j}\pi F - 9 - 4\pi^2)}$$

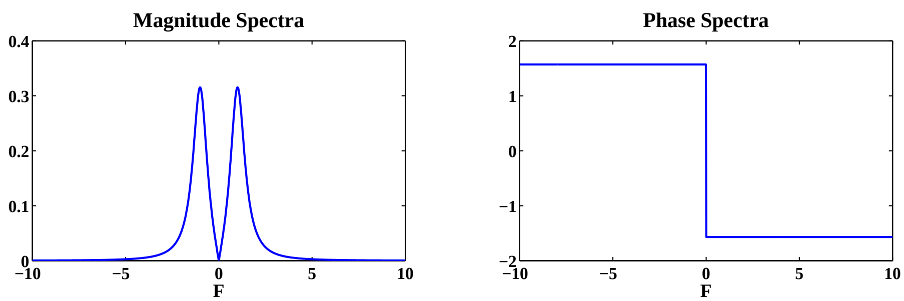
(c) Solution:

$$X_3(F) = \begin{cases} 1, & -\frac{1}{2} \le F \le \frac{1}{2} \\ F + \frac{3}{2}, & -\frac{3}{2} \le F \le -\frac{1}{2} \\ -F + \frac{3}{2}, & \frac{1}{2} \le F \le \frac{3}{2} \\ 0, & \text{otherwise} \end{cases}$$



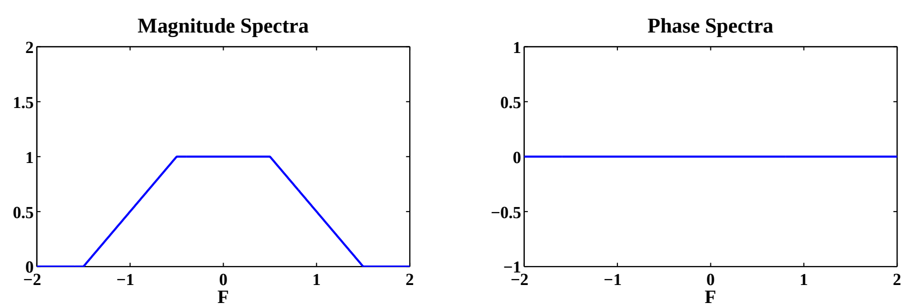
(a) (b)

FIGURE 4.3: (a) Magnitude and phase spectra of signal $x_1(t) = (1 - t^2)[u(t) - u(t-1)]$.



(a) (b)

FIGURE 4.4: (a) Magnitude and phase spectra of signal $x_2(t) = \mathrm{e}^{-3|t|} \sin 2\pi t$.



(a) (b)

FIGURE 4.5: (a) Magnitude and phase spectra of signal $x_3(t) = \frac{\sin \pi t}{\pi t} \frac{\sin 2\pi t}{\pi t}$.

25. (a) Solution:

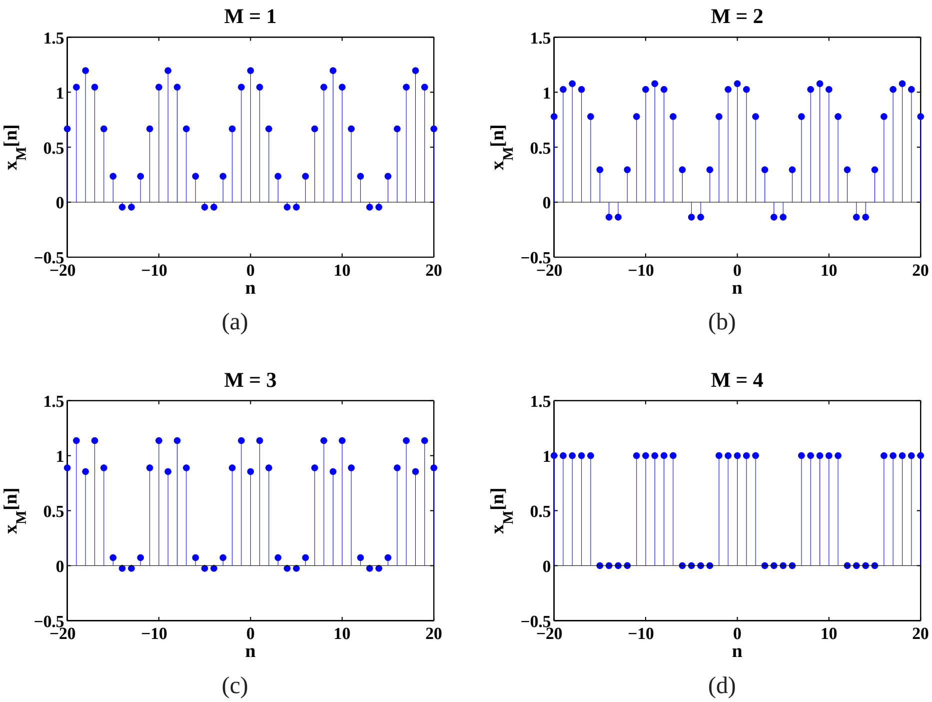


FIGURE 4.6: Sequences $\hat{x}_M[n]$. (a) $M = 1$. (b) $M = 2$. (c) $M = 3$. (d) $M = 4$.

26. (a) Solution:

$$c_k = \begin{cases} 0, & k = 5m \\ 6e^{-j\frac{\pi}{6}}, & k = 5m+1 \\ 4e^{-j\frac{\pi}{3}}, & k = 5m+2 \\ 4e^{j\frac{\pi}{3}}, & k = 5m+3 \\ -6e^{j\frac{\pi}{6}}, & k = 5m+4 \end{cases} \qquad m = 0, \pm 1, \pm 2$$

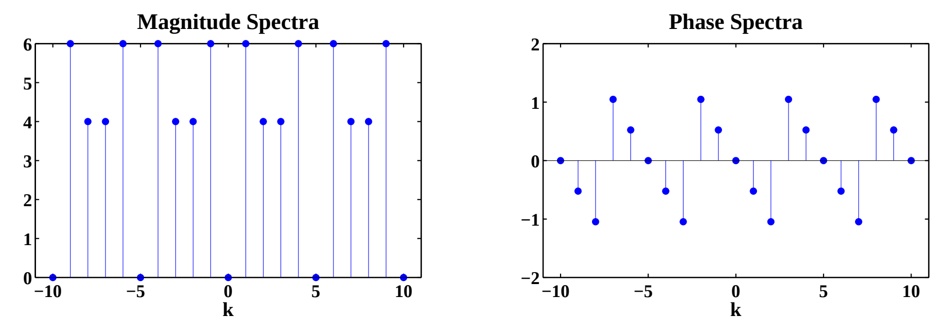


FIGURE 4.7: Magnitude and phase spectra of periodic sequence $x_1[n] = 4\cos(1.2\pi n + 60°) + 6\sin(0.4\pi n - 30°)$.

(b) Solution:

$$c_k = \frac{1}{4}\left(1 + 2\cos\frac{\pi}{4}\cos\frac{\pi}{2}k\right)$$

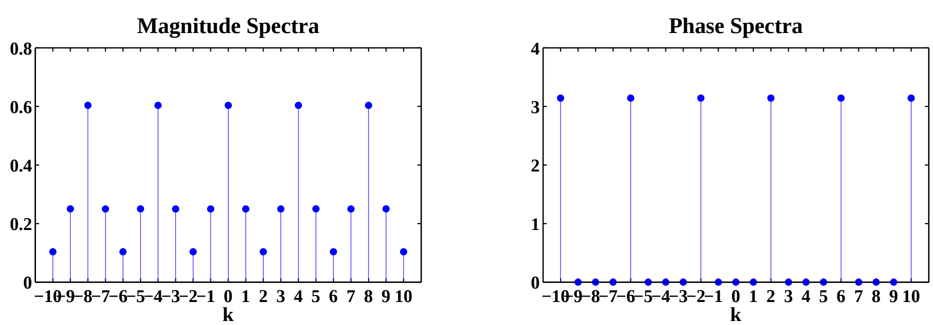


FIGURE 4.8: Magnitude and phase spectra of periodic sequence $x_2[n] = |\cos(0.25\pi n)|, 0 \le n \le 3$.

(c) Solution:

$$c_k = \frac{1}{8}\left[1 + 2\cos(\frac{k\pi}{4}) + 2\cos(\frac{3k\pi}{4}) + \cos(k\pi)\right]$$

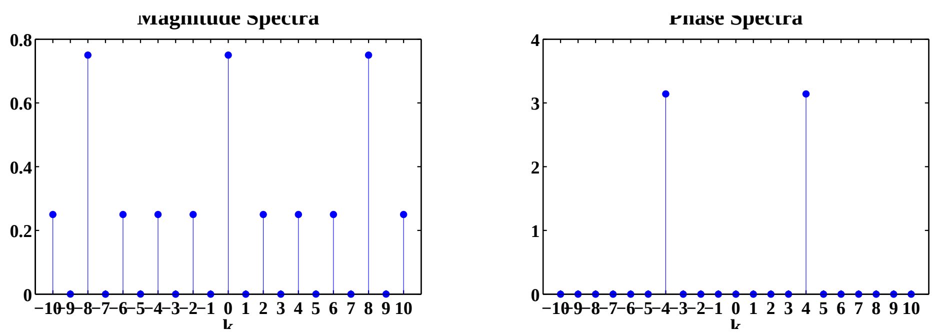


FIGURE 4.9: Magnitude and phase spectra of periodic sequence $x_3[n]$.

(d) Solution:

$$c_k = \frac{1}{12}\Big[1 + (1 - \sin(\frac{\pi}{4}))2\cos(\frac{k\pi}{6}) + (1 - \sin(\frac{3\pi}{4}))2\cos(\frac{k\pi}{2})$$
$$+2\cos(\frac{2k\pi}{3}) + (1 - \sin(\frac{5\pi}{4}))2\cos(\frac{5k\pi}{6}) + 2\cos(k\pi)\Big]$$

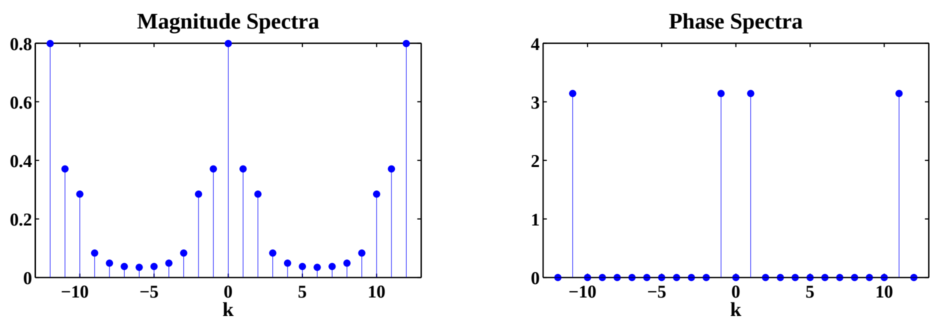


FIGURE 4.10: Magnitude and phase spectra of periodic sequence $x_4[n] = 1 - \sin(\pi n/4), 0 \le n \le 11$ (one period).

(e) Solution:

$$c_k = \frac{1}{7}\left(1 - 2e^{-j\frac{2\pi}{7}k} + e^{-j\frac{2\pi}{7}k\cdot 2} - e^{-j\frac{2\pi}{7}k\cdot 4} + 2e^{-j\frac{2\pi}{7}k\cdot 5} - e^{-j\frac{2\pi}{7}k\cdot 6}\right)$$

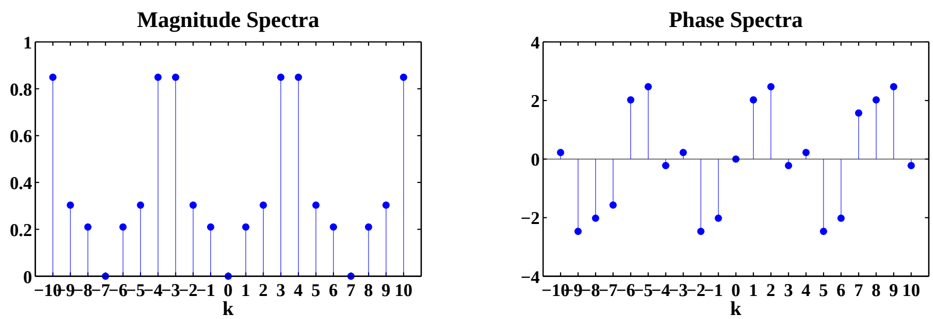


FIGURE 4.11: Magnitude and phase spectra of periodic sequence $x_5[n]$.

27. (a) Solution:

$$\frac{1}{N}\sum_{n=0}^{N-1} x[n-n_0]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn_0}a_k$$

(b) Solution:

$$\frac{1}{N}\sum_{n=0}^{N-1}(x[n]-x[n-1])\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = a_k - \mathrm{e}^{\mathrm{j}\frac{2\pi}{N}k}a_k$$

(c) Solution:

$$\frac{1}{N}\sum_{n=0}^{N-1}(-1)^n x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = a_{k-\frac{N}{2}}$$

(d) tba

28. (a) Solution:

$$b_k = \sum_{m=0}^{N-1} a_m \cdot a^*_{m-k}$$

(b) Solution:
If a_k are real, we can claim that b_k are real as well.

29.

30. (a) Solution:

$$X_1(\mathrm{e}^{\mathrm{j}\omega}) = \frac{\frac{1}{3}\mathrm{e}^{-\mathrm{j}\omega}}{1-\frac{1}{3}\mathrm{e}^{-\mathrm{j}\omega}}$$

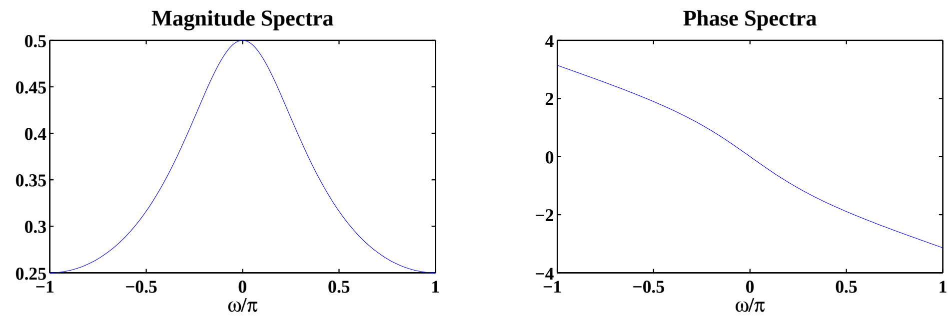


FIGURE 4.12: Magnitude and phase response for sequence $x_1[n] = (1/3)^n u[n-1]$.

(b) Solution:

$$X_2(\mathrm{e}^{\mathrm{j}\omega}) = \left(\frac{1}{2}\right)^5 \left[\frac{\mathrm{e}^{-2\mathrm{j}(\omega-\pi/4)}}{1-\frac{1}{4}\mathrm{e}^{-\mathrm{j}(\omega-\pi/4)}} + \frac{\mathrm{e}^{-2\mathrm{j}(\omega+\pi/4)}}{1-\frac{1}{4}\mathrm{e}^{-\mathrm{j}(\omega+\pi/4)}}\right]$$

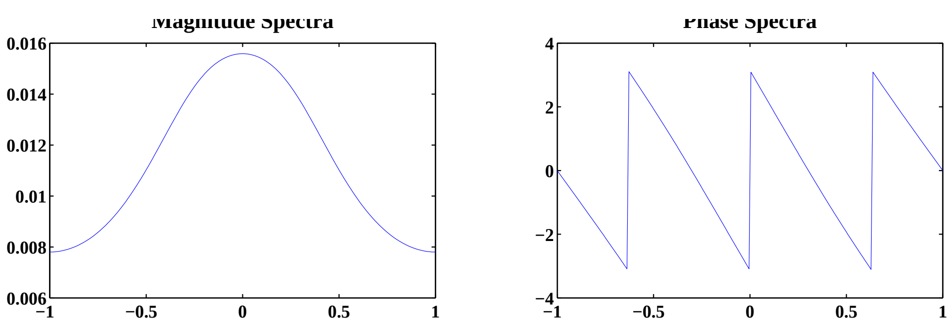


FIGURE 4.13: Magnitude and phase response for sequence $x_2[n] = (1/4)^n \cos(\pi n/4) u[n-2]$.

(c) Solution:

$$X_3(\mathrm{e}^{\mathrm{j}\omega}) = \begin{cases} \frac{16}{\pi^2}\mathrm{e}^{-\mathrm{j}4\omega}, & 0 \le |\omega| \le \frac{\pi^2}{4} \\ 0, & \frac{\pi^2}{4} < |\omega| < \pi \end{cases}$$

(d) Solution:

$$X_4(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1}{2\mathrm{j}}\left[\frac{1-\mathrm{e}^{-10\mathrm{j}(\omega-0.1\pi)}}{1-\mathrm{e}^{-\mathrm{j}(\omega-0.1\pi)}} - \frac{1-\mathrm{e}^{-10\mathrm{j}(\omega+0.1\pi)}}{1-\mathrm{e}^{-\mathrm{j}(\omega+0.1\pi)}}\right]$$

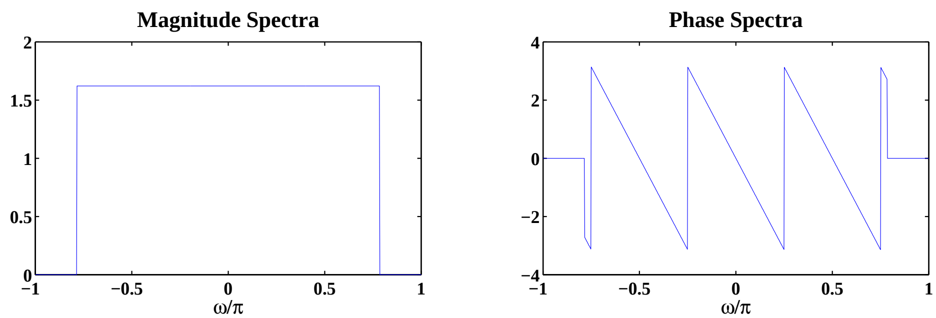


FIGURE 4.14: Magnitude and phase response for sequence $x_3[n] = \text{sinc}(2\pi n/8) * \text{sinc}\{2\pi(n-4)/8\}$.

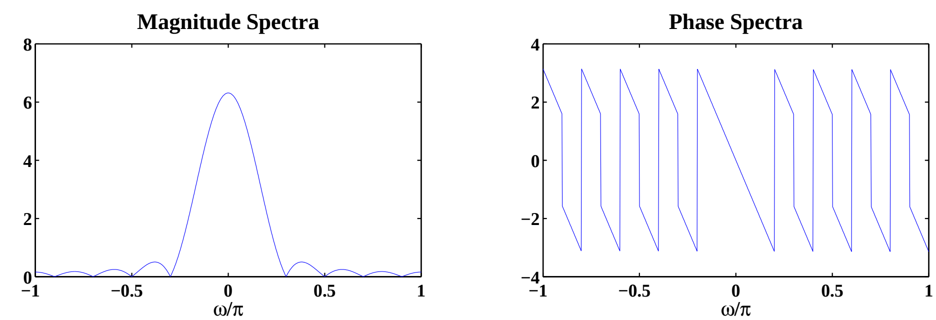


FIGURE 4.15: Magnitude and phase response for sequence $x_4[n] = \sin(0.1\pi n)(u[n] - u[n-10])$.

(e) Solution:

$$X_5(e^{j\omega}) = \begin{cases} \frac{8}{\pi^3}(\frac{\pi^2}{2} - |\omega|), & 0 \le |\omega| \le |2\pi - \frac{\pi^2}{2}| \\ \frac{8(\pi-2)}{\pi^2}, & |2\pi - \frac{\pi^2}{2}| < |\omega| < \pi \end{cases}$$

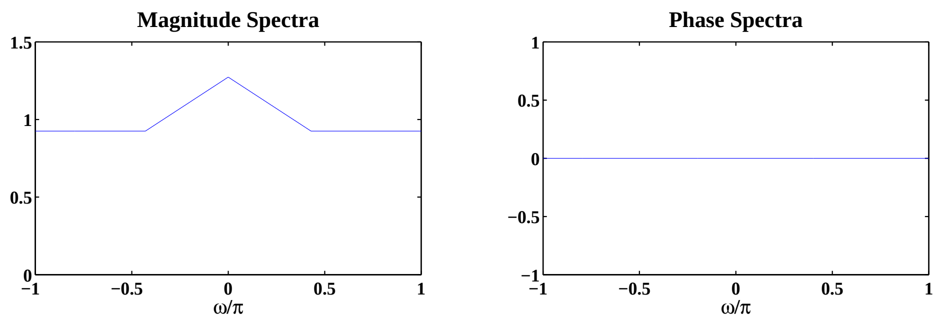


FIGURE 4.16: Magnitude and phase response for sequence $x_5[n] = \text{sinc}^2(\pi n/4)$.

31. (a) Solution:

$$x_1[n] = \frac{1}{2\pi}\left(1 - 2\cos\frac{\pi}{2}n\right)$$

(b) Solution:

$$x_2[n] = \frac{1}{5}\text{sinc}\left(\frac{n}{5}\right)$$

(c) Solution:

$$x_3[n] = \frac{-\text{j}}{n\pi}\cos(\frac{\pi}{2}n) + \frac{2\text{j}}{n^2\pi^2}\sin(\frac{\pi}{2}n)$$

(d) Solution:

$$x_4[n] = \frac{\text{j}}{\pi n}\left\{1 - \cos(\pi n) + \cos[(\omega_c + \frac{\Delta\omega}{2})n] - \cos[(\omega_c - \frac{\Delta\omega}{2})n]\right\}$$

32. (a) Solution:

$$X_1\big(\text{e}^{\text{j}\omega}\big) = 2\text{e}^{2\text{j}\omega}X\big(\text{e}^{\text{j}\omega}\big) + 3\text{e}^{-3\text{j}\omega}X\big(\text{e}^{\text{j}\omega}\big)$$

(b) Solution:

$$X_2\big(\text{e}^{\text{j}\omega}\big) = \frac{1}{2}\text{e}^{\text{j}\frac{\pi}{6}}\delta(\omega-\frac{\pi}{5})+\frac{1}{2}\text{e}^{-\text{j}\frac{\pi}{6}}\delta(\omega+\frac{\pi}{5})+\frac{1}{2}\text{e}^{\text{j}\frac{\pi}{6}}X\big(\text{e}^{\text{j}(\omega-\frac{\pi}{5})}\big)+\frac{1}{2}\text{e}^{-\text{j}\frac{\pi}{6}}X\big(\text{e}^{\text{j}(\omega+\frac{\pi}{5})}\big)$$

(c) Solution:

$$X_3\big(\text{e}^{\text{j}\omega}\big) = -2\text{e}^{\text{j}2(\omega-0.5\pi)}X\big(\text{e}^{\text{j}(\omega-0.5\pi)}\big)$$

(d) Solution:

$$X_4\big(\text{e}^{\text{j}\omega}\big) = \frac{1}{2}X\big(\text{e}^{\text{j}\omega}\big) - \frac{1}{2}X\big(\text{e}^{\text{j}\omega}\big)^*$$

(e) Solution:

$$X_5(e^{j\omega}) = X\left(e^{j(\omega-\frac{\pi}{2})}\right)e^{j(\omega-\frac{\pi}{2})} + X\left(e^{j(\omega+\frac{\pi}{2})}\right)e^{-j(\omega+\frac{\pi}{2})}$$

33. (a) Solution:

$$X_1(e^{j\omega}) = \frac{e^{2j(\omega-\frac{\pi}{2})}}{1+0.8e^{-j(\omega-\frac{\pi}{2})}}$$

(b) Solution:

$$X_2(e^{j\omega}) = \frac{\frac{1}{2}}{1+0.8e^{-j(\omega-0.4\pi)}} + \frac{\frac{1}{2}}{1+0.8e^{-j(\omega+0.4\pi)}}$$

(c) Solution:

$$X_3(e^{j\omega}) = \frac{1}{1+1.6\cos(\omega)+0.64}$$

(d) Solution:

$$X_4(e^{j\omega}) = \frac{1}{1+0.8e^{-j\frac{\omega}{2}}}$$

(e) Solution:

$$X_5(e^{j\omega}) = \frac{1}{1-0.8^2 e^{-j2\omega}}$$

34. (a) Solution:

$$X_R(e^{j\omega}),\quad \text{odd symmetric}$$
$$X_I(e^{j\omega}),\quad \text{even symmetric}$$
$$x_R[n] = 0,$$
$$x_I[n],\quad \text{nonsymmetric}$$

(b) Solution:

$$X_R(e^{j\omega}),\quad \text{odd symmetric}$$
$$X_I(e^{j\omega}),\quad \text{even symmetric}$$
$$x_R[n] = 0,$$
$$x_I[n],\quad \text{even symmetric}$$

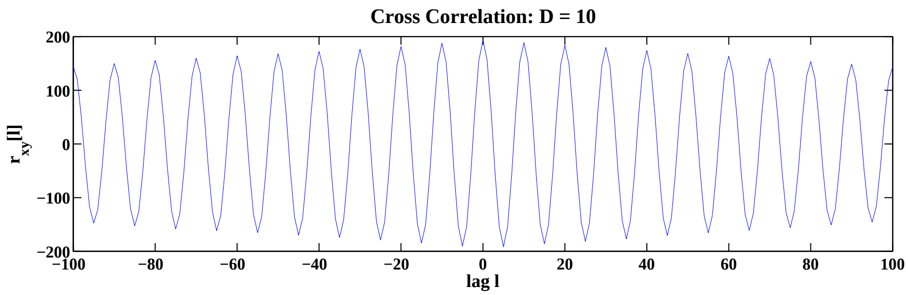
(c) Solution:

$$X_R(\mathrm{e}^{\mathrm{j}\omega}), \quad \text{odd symmetric}$$
$$X_I(\mathrm{e}^{\mathrm{j}\omega}), \quad \text{even symmetric}$$
$$x_R[n] = 0,$$
$$x_I[n], \quad \text{odd symmetric}$$

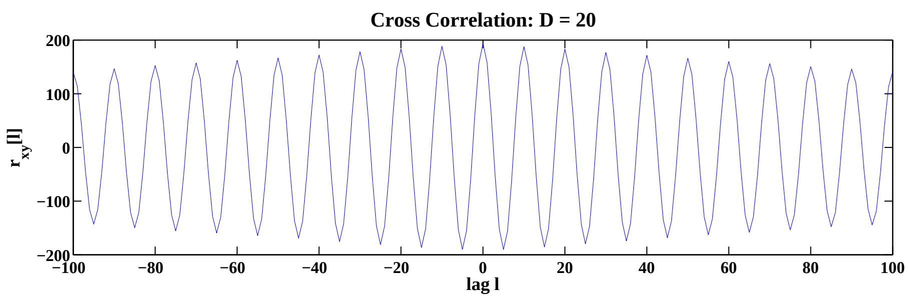
35.

36. (a) See plot below.

(b)



(a)



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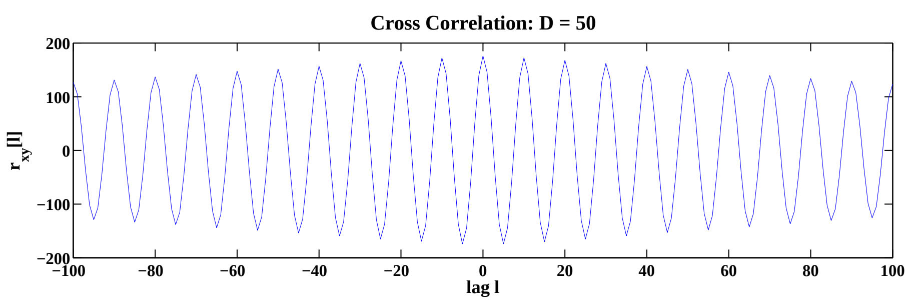

(c)

FIGURE 4.17: Cross correlation $r_{xy}[\ell]$ plot of (a) $D = 10$. (b) $D = 20$. (c) $D = 50$.