CHAPTER 5

Transform Analysis of LTI Systems

Tutorial Problems

1. (a) Solution:

$$y[n] = a\,y[n-1] + b\,x[n], \quad -1 < a < 1$$
$$H\big(e^{j\omega}\big) = \frac{b}{1 - a e^{-j\omega}}$$
$$x[n] = 3\cos(\pi n/2) = \frac{3}{2}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right)$$

$$y[n] = \frac{3}{2}\left(e^{j\frac{\pi}{2}n}H\big(e^{j\frac{\pi}{2}}\big) + e^{-j\frac{\pi}{2}n}H\big(e^{j-\frac{\pi}{2}}\big)\right) = \frac{6}{5}\cos\frac{\pi}{2}n + \frac{3}{5}\sin\frac{\pi}{2}n$$

$$x[n] = 3\sin(\pi n/4) = \frac{3}{2j}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right)$$

$$\begin{aligned} y[n] &= \frac{3}{2j}\left(e^{j\frac{\pi}{4}n}H\big(e^{j\frac{\pi}{4}}\big) - e^{-j\frac{\pi}{4}n}H\big(e^{j-\frac{\pi}{4}}\big)\right) \\ &= \frac{3\left[2\sin\frac{\pi}{4}n - \sin\frac{\pi}{4}(\sin\frac{\pi}{4}n + \cos\frac{\pi}{4}n)\right]}{1.25 - \cos\frac{\pi}{4}} \end{aligned}$$

(b) See plot below.

(c) See plot below.

MATLAB script:

```
% P0501: Plot magnitude and phase response
close all;clc
%% Frequencey Domain:
```

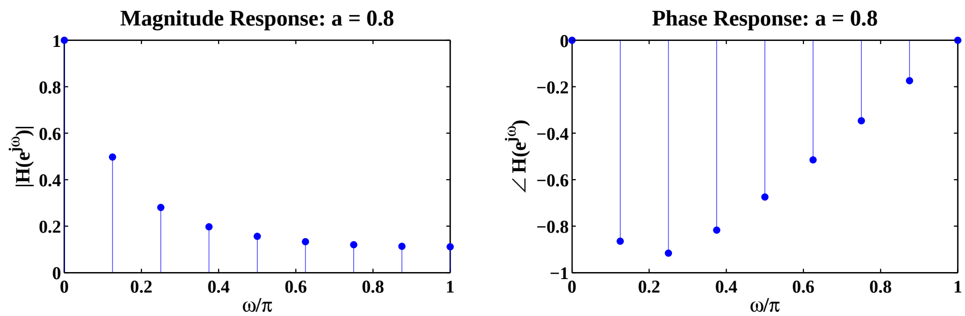


FIGURE 5.1: Magnitude and phase response of the system for $0 \leq \omega \leq \pi$ at increments of $\pi/8$.

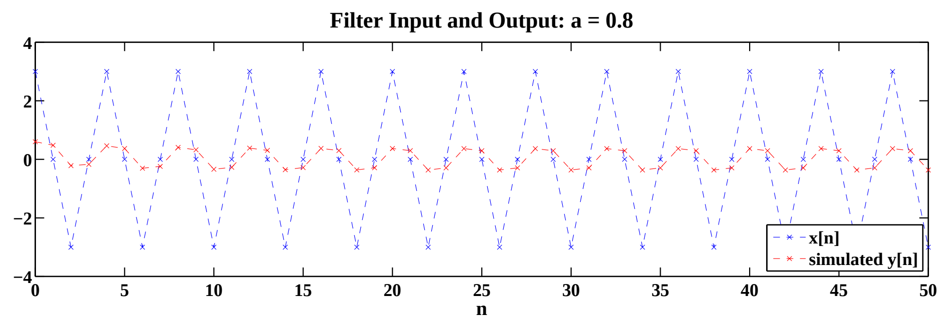


FIGURE 5.2: Input $x[n]$ and the output $y[n]$ where $x[n] = 3\cos(\pi n/2)$.

```
% X = [0 0 0 0 3/2 0 0 0 0]; % (i) x[n]
X = [0 0 3/2/j 0 0 0 0 0 0]; % (ii) x[n]
w = (0:1/8:1)*pi;
% a = 0.5; % part a
a = 0.8; % part b
% a = -0.8; % part c
b = 1-abs(a);
H = freqz(b,[1 -a],w);
H_mag = abs(H);
H_phase = angle(H);
Y = H.*X;
%% Time Domain:
n = 0:50;
xn = 3*cos(pi*n/2);
```

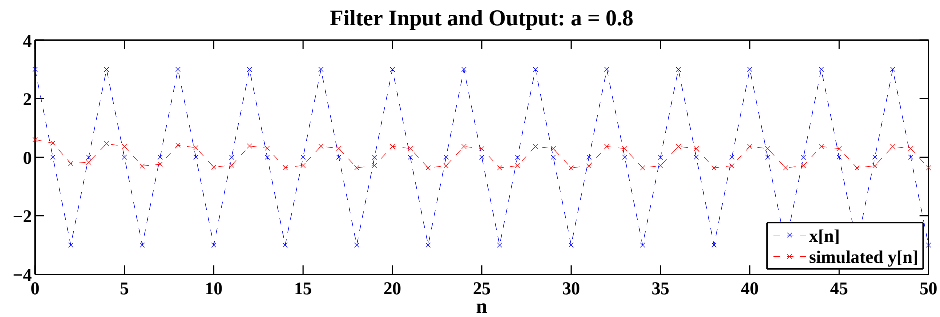


FIGURE 5.3: Input $x[n]$ and the output $y[n]$ where $x[n] = 3\sin(\pi n/4)$.

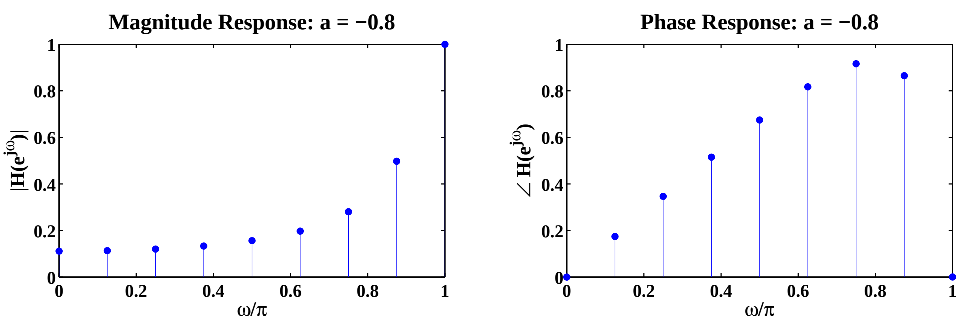


FIGURE 5.4: Magnitude and phase response of the system for $0 \leq \omega \leq \pi$ at increments of $\pi/8$.

```
yn = filter(b,[1 -a],xn);
yn_ref = 6/5*cos(pi*n/2)+3/5*sin(pi*n/2);
%% Plot:
hfa = figconfg('P0501a','long');
subplot(121)
stem(w/pi,H_mag,'filled')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title(['Magnitude Response: a = ',num2str(a)],'fontsize',TFS)
subplot(122)
stem(w/pi,H_phase,'filled')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title(['Phase Response: a = ',num2str(a)],'fontsize',TFS)
```

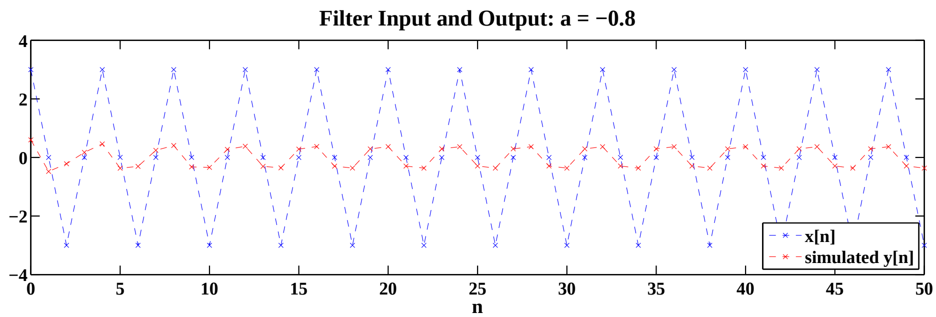


FIGURE 5.5: Input $x[n]$ and the output $y[n]$ where $x[n] = 3\cos(\pi n/2)$.

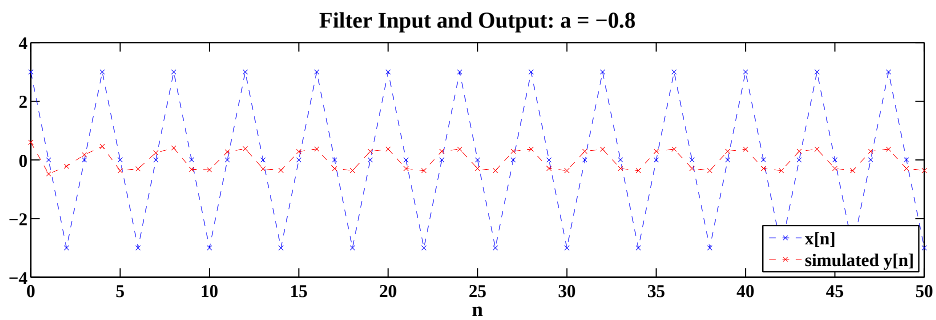


FIGURE 5.6: Input $x[n]$ and the output $y[n]$ where $x[n] = 3\sin(\pi n/4)$.

```
hfb = figconfg('P0501b','long');
plot(n,xn,'--xb',n,yn,'--xr')
legend('x[n]','simulated y[n]','theoretical y[n]'...
    ,'Location','Southeast')
xlabel('n','fontsize',LFS)
title(['Filter Input and Output: a = ',num2str(a)],'fontsize',TFS)
```

2. (a) Solution:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{b}{1 + 0.81\mathrm{e}^{-2\mathrm{j}\omega}}$$

(b) Solution:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{b}{1 + 0.81\cos 2\omega - \mathrm{j}0.81\sin 2\omega}$$

$$\begin{aligned}
|H(\mathrm{e}^{\mathrm{j}\omega})| &= \frac{|b|}{\sqrt{(1 + 0.81\cos 2\omega)^2 + (0.81\sin 2\omega)^2}} \\
&= \frac{|b|}{\sqrt{1 + 0.81^2 + 2 * 0.81\cos 2\omega}}
\end{aligned}$$

$$\begin{aligned}
\max|H(\mathrm{e}^{\mathrm{j}\omega})| &= \frac{|b|}{\sqrt{1 + 0.81^2 - 2 * 0.81}} = 1 \\
|b| &= 0.19
\end{aligned}$$

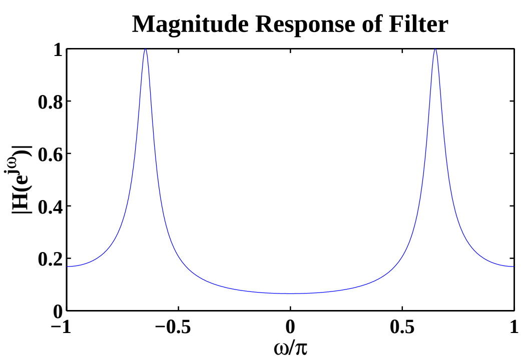


FIGURE 5.7: Magnitude response of the system.

(c) See plot below.

(d) Solution:

$$x[n] = 2\cos(0.5\pi n + 60^\circ) = \mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n}$$

$$\begin{aligned}
y[n] &= \mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n}H(\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}) + \mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n}H(\mathrm{e}^{\mathrm{j}-\frac{\pi}{2}}) \\
&= 2\cos(\frac{\pi}{2}n + \frac{\pi}{3})
\end{aligned}$$

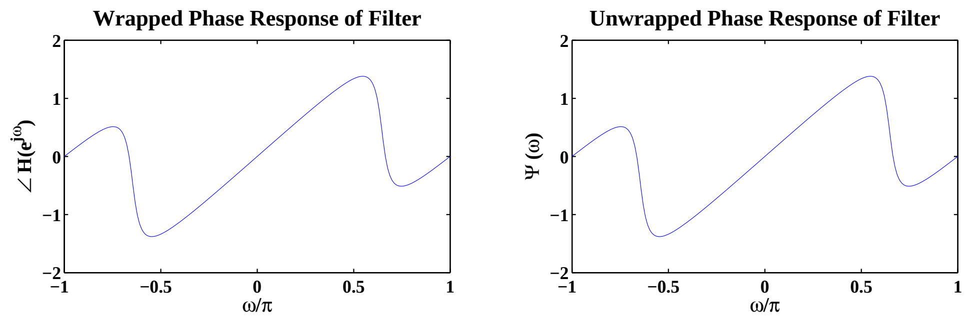


FIGURE 5.8: Wrapped and the unwrapped phase responses of the system.

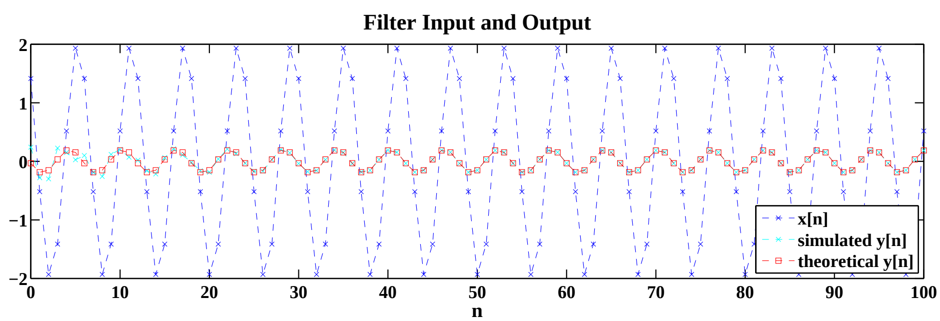


FIGURE 5.9: MATLAB verification of the steady-state response to $x[n]$.

(e) See plot below.
MATLAB script:

```
% P0502: Plot magnitude and phase response
close all;clc
b = sqrt(1+0.8^2+0.81^2-2*0.81-0.8^2*1.81^2/2/2/0.81);
a = [1 0.8 0.81];
w = linspace(-pi,pi,1000);
H = freqz(b,a,w);
H_mag = abs(H);
H_phase = angle(H);
H_phase_unwrap = unwrap(H_phase);
n = 0:100;
xn = 2*cos(pi*n/3+pi/4);
yn = filter(b,a,xn);
yn_ref = 2*0.0577*cos(pi*n/3+pi/4)-2*0.0809*sin(pi*n/3+pi/4);
```

```
%% Plot:
hfa = figconfg('P0502a','small');
plot(w/pi,H_mag)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response of Filter','fontsize',TFS)
hfb = figconfg('P0502b','long');
subplot(121)
plot(w/pi,H_phase)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title('Wrapped Phase Response of Filter','fontsize',TFS)
subplot(122)
plot(w/pi,H_phase_unwrap)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\Psi (\omega)','fontsize',LFS)
title('Unwrapped Phase Response of Filter','fontsize',TFS)
hfc = figconfg('P0502c','long');
plot(n,xn,'--xb',n,yn,'--xc',n,yn_ref,'--sr')
legend('x[n]','simulated y[n]','theoretical y[n]','Location','Southeast')
xlabel('n','fontsize',LFS)
title('Filter Input and Output','fontsize',TFS)
```

3. (a) Solution:

$$\begin{aligned}|H(e^{j2\pi F})| &= \frac{0.2}{\sqrt{(1-0.8\cos\omega)^2+(0.8\sin\omega)^2}}\\ &= \frac{0.2}{\sqrt{1.64-1.6\cos\omega}}, \quad \omega = 2\pi F/F_s\end{aligned}$$

(c) See plot below.
MATLAB script:

```
% P0503: Linear FM signal
close all; clc
%% Specification:
B = 10;
Fs = 100;
tau = 10;
N = tau*Fs;
```

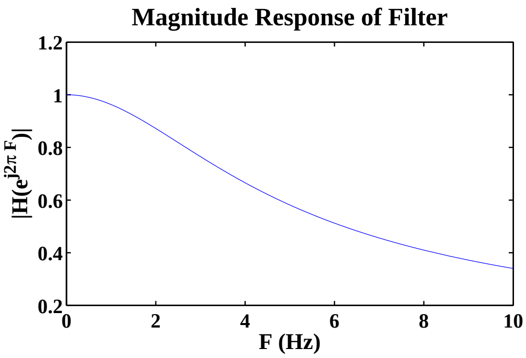


FIGURE 5.10: Plot of $|H(\mathrm{e}^{\mathrm{j}2\pi F})|$ over $0 \le F \le B$ Hz.

(b)

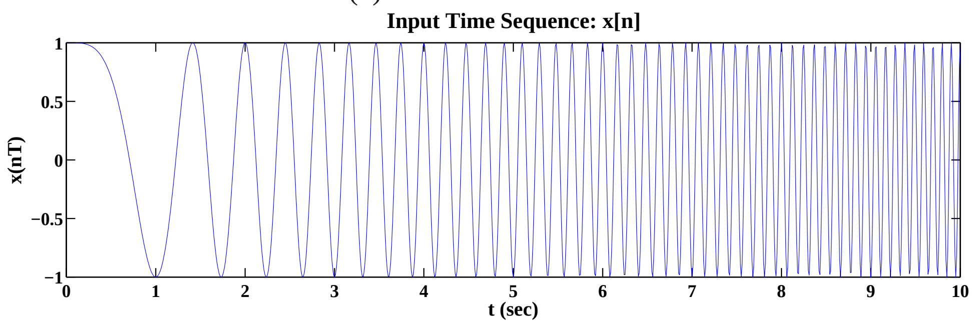


FIGURE 5.11: Plot of $x[n] = x(nT)$ over $0 \le t \le \tau$ sec.

```
n = 0:N;
F = linspace(0,B,1001);
w = 2*pi*F/Fs;
%% Part a:
H = 0.2./(1-0.8*exp(-j*w));
% H_mag = abs(H);
H_mag = 0.2./sqrt(1.64-1.6*cos(w));
%% Part b:
xn = cos(pi*B/Fs/N*n.^2);
%% Part c:
yn = filter(0.2,[1 -0.8],xn);
%% Plot:
hfa = figconfg('P0503a','small');
plot(F,H_mag)
```

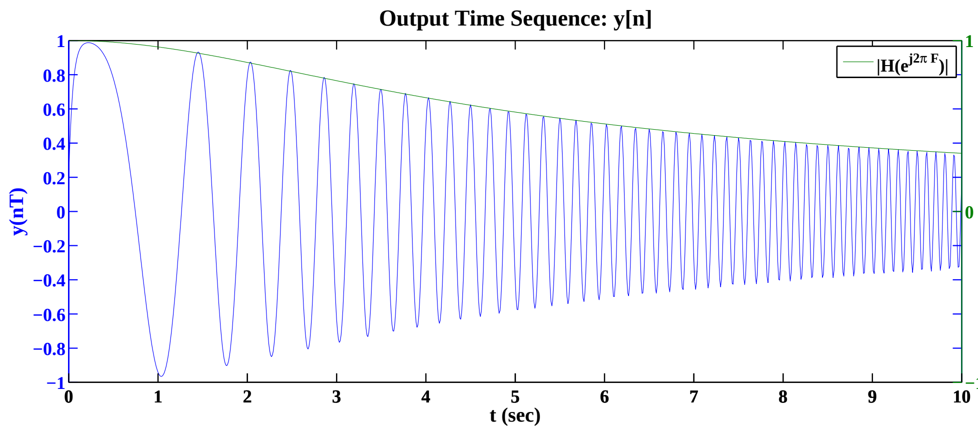


FIGURE 5.12: Plot of $y[n] = y(nT)$ over $0 \le t \le \tau$ sec.

```
xlabel('F (Hz)','fontsize',LFS)
ylabel('|H(e^{j2\pi F})|','fontsize',LFS)
title('Magnitude Response of Filter','fontsize',TFS)
hfb = figconfg('P0503b','long');
plot(n/Fs,xn)
xlabel('t (sec)','fontsize',LFS)
ylabel('x(nT)','fontsize',LFS)
title('Input Time Sequence: x[n]','fontsize',TFS)
hfc = figconfg('P0503c','long');
[AX H1 H2] = plotyy(n/Fs,yn,n/Fs,H_mag);
set(AX(2),'ylim',[-1 1],'YTick',-1:1)
xlabel('t (sec)','fontsize',LFS)
ylabel('y(nT)','fontsize',LFS)
title('Output Time Sequence: y[n]','fontsize',TFS)
legend('|H(e^{j2\pi F})|')
```

4. (a) Solution:

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$$

$$Y(z) = \frac{1}{1-\frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1-\frac{1}{2}e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}$$

which is unique.

(b) Solution:

$$H(e^{j\frac{\pi}{3}}) = 2$$

Hence, the frequency response function exists but not unique.

(c) $x[n] = \frac{\sin \pi n/4}{\pi n} \xmapsto{\mathcal{H}} y[n] = \frac{\sin \pi n/2}{\pi n}$
Solution:

$$y[n] = 2x[2n]$$

which is time-varying.

(d) Solution:

$$y[n] = x[n] - x[n-1]$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = 1 - e^{-j\omega}$$

which is unique.

5. (a) See plot below.

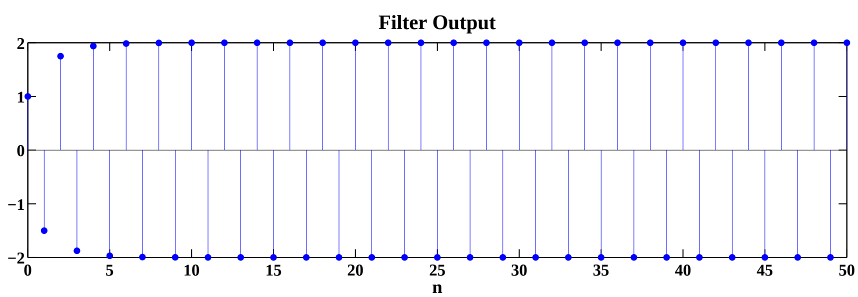


FIGURE 5.13: System response to the input $x[n] = (-1)^n u[n]$.

(b) See plot below.

(c) See plot below.
MATLAB script:

```
% P0505: Checking whether a system is LTI
%        Compute impulse response and frequence response
close all; clc
%% Part a:
n = 0:50;
N = length(n);
xn1 = (-1).^n;
```

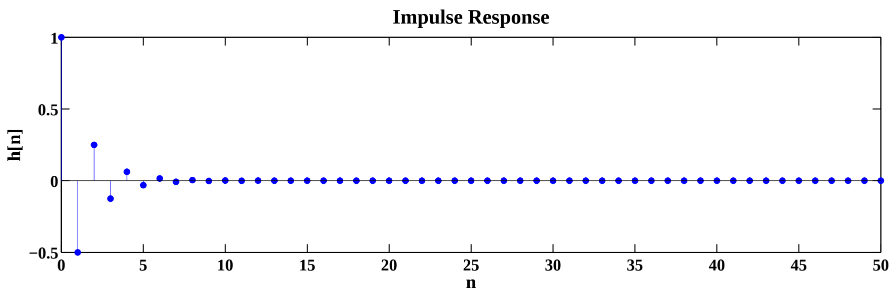


FIGURE 5.14: System frequency response.

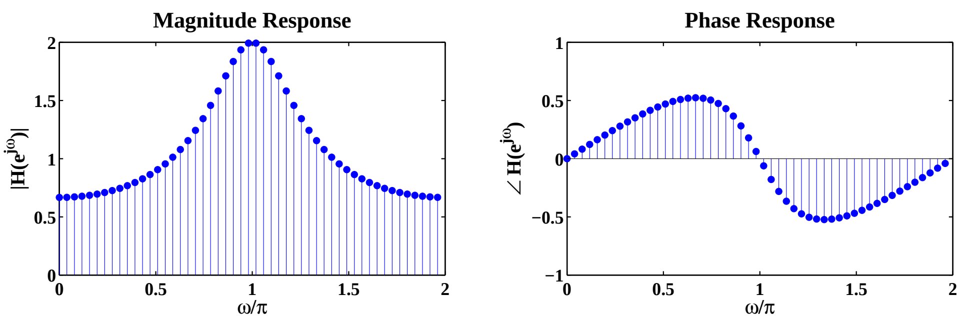


FIGURE 5.15: System frequency response.

```
yn1 = ltiwhich(xn1);
%% Part b:
delta = zeros(size(n));
delta(1) = 1;
hn = ltiwhich(delta);
H = fft(hn);
H_mag = abs(H);
H_phase = angle(H);
%% Part c:
w = (0:10)/10*pi;
H_ref_mag = zeros(size(w));
H_ref_phase = zeros(size(w));
for ii = 1:length(w)
    xn = cos(w(ii)*n);
    yn = ltiwhich(xn);
    [H_ref_mag(ii) ind] = max(yn(10:end));
```

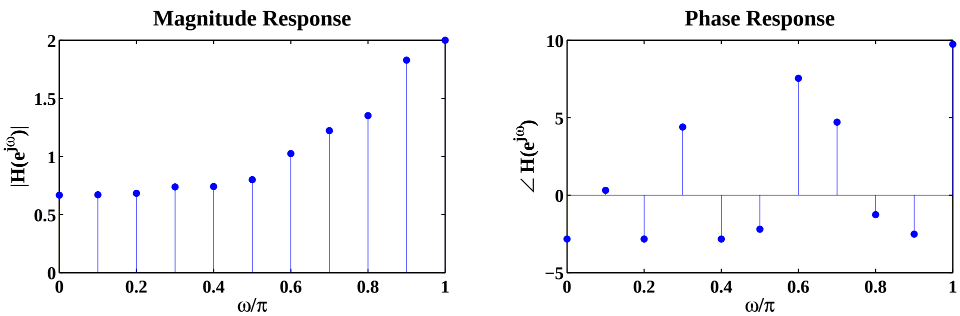


FIGURE 5.16: System frequency response for comparison.

```
    H_ref_phase(ii) = (ind(1)+9-20)/20*2*pi;
%     H_ref_phase(ii) = atan(tan(H_ref_phase(ii)));
end
%% Plot
hfa = figconfg('P0505a','long');
stem(n,yn1,'filled')
xlabel('n','fontsize',LFS)
title('Filter Output','fontsize',TFS)
hfb = figconfg('P0505b','long');
stem(n,hn,'filled')
xlabel('n','fontsize',LFS)
ylabel('h[n]','fontsize',LFS)
title('Impulse Response','fontsize',TFS)
hfc = figconfg('P0505c','long');
subplot(121)
stem(2*(0:N-1)/N,H_mag,'filled')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(122)
stem(2*(0:N-1)/N,H_phase,'filled')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title('Phase Response','fontsize',TFS)
hfd = figconfg('P0505d','long');
subplot(121)
stem((0:10)/10,H_ref_mag,'filled')
```

```
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(122)
stem((0:10)/10,H_ref_phase,'filled')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title('Phase Response','fontsize',TFS)
```

6. (a) Solution:

$$H(e^{j\omega}) = \frac{(1+e^{-j\omega})^6}{2^6} = \frac{(1+\cos\omega - j\sin\omega)^6}{2^6}$$

$$\angle H(e^{j\omega}) = 6\tan^{-1}\frac{-\sin\omega}{1+\cos\omega} = 6\tan^{-1}\frac{-2\sin\frac{\omega}{2}\cos\frac{\omega}{2}}{2\cos^2\frac{\omega}{2}} = -3\omega$$

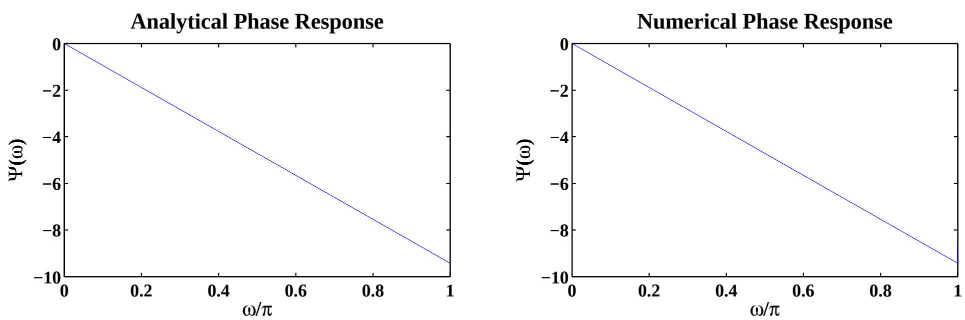


FIGURE 5.17: Phase response comparison between analytical and numerical results.

(b) MATLAB script:

```
  % P0506: Compute and plot phase response
close all; clc
%% Part a: Analytic Solution
w = linspace(0,1,1000)*pi;
H_phase_ana = -3*w;
%% Part b: Numerical Solution
b = poly(-ones(1,6));
H_ref = freqz(b,1,w);
H_phase_ref = unwrap(angle(H_ref));
```

```
%% Plot
hfa = figconfg('P0506','long');
subplot(121)
plot(w/pi,H_phase_ana)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\Psi(\omega)','fontsize',LFS)
title('Analytical Phase Response','fontsize',TFS)
subplot(122)
plot(w/pi,H_phase_ref)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\Psi(\omega)','fontsize',LFS)
title('Numerical Phase Response','fontsize',TFS)
```

7. (a) Solution:

$$\text{System function:}\quad H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2} - \frac{1}{2}z^{-1}$$

$$\text{Frequency response:}\quad H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1}{2} - \frac{1}{2}\mathrm{e}^{-\mathrm{j}\omega} = \frac{1}{2}(1-\cos\omega+\mathrm{j}\sin\omega)$$

$$\text{Magnitude response:}\quad |H(\mathrm{e}^{\mathrm{j}\omega})| = \frac{\sqrt{(1-\cos\omega)^2+\sin^2\omega}}{2} = |\sin\frac{\omega}{2}|$$

$$\text{Phase response:}\quad \angle H(\mathrm{e}^{\mathrm{j}\omega}) = \tan^{-1}\frac{\sin\omega}{1-\cos\omega} = \frac{\pi}{2} - \frac{\omega}{2}$$

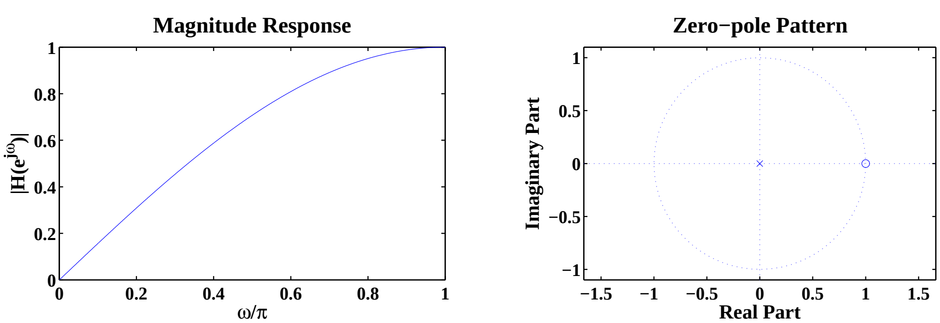


FIGURE 5.18: Magnitude response and pole-zero plot of $y[n] = \frac{1}{2}(x[n]-x[n-1])$.

(b) Solution:

$$\text{System function:}\quad H(z)=\frac{Y(z)}{X(z)}=\frac{1}{2}-\frac{1}{2}z^{-2}$$

$$\text{Frequency response:}\quad H\big(e^{j\omega}\big)=\frac{1}{2}-\frac{1}{2}e^{-2j\omega}=\frac{1}{2}(1-\cos 2\omega+j\sin 2\omega)$$

$$\text{Magnitude response:}\quad |H\big(e^{j\omega}\big)|=\frac{\sqrt{(1-\cos 2\omega)^2+\sin^2 2\omega}}{2}=|\sin\omega|$$

$$\text{Phase response:}\quad \angle H\big(e^{j\omega}\big)=\tan^{-1}\frac{\sin 2\omega}{1-\cos 2\omega}=\frac{\pi}{2}-\omega$$

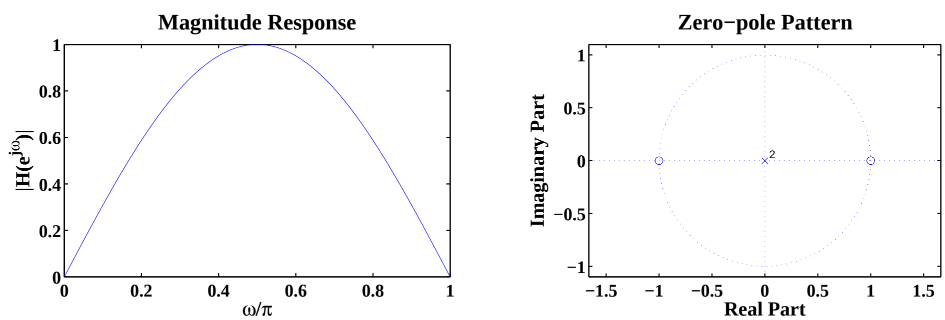


FIGURE 5.19: Magnitude response and pole-zero plot of $y[n]=\frac{1}{2}(x[n]-x[n-2])$.

(c) Solution:

$$\text{System function:}\quad H(z)=\frac{Y(z)}{X(z)}=\frac{1}{4}(1+z^{-1}-z^{-2}-z^{-3})$$

Frequency response:

$$H\big(e^{j\omega}\big)=\frac{1}{4}[(1+\cos\omega-\cos 2\omega-\cos 3\omega)+j(-\sin\omega+\sin 2\omega+\sin 3\omega)]$$

Magnitude response:

$$|H\big(e^{j\omega}\big)|=\frac{1}{4}\sqrt{(1+\cos\omega-\cos 2\omega-\cos 3\omega)^2+(-\sin\omega+\sin 2\omega+\sin 3\omega)^2}$$

$$\text{Phase response:}\quad \angle H\big(e^{j\omega}\big)=\frac{-\sin\omega+\sin 2\omega+\sin 3\omega}{1+\cos\omega-\cos 2\omega-\cos 3\omega}$$

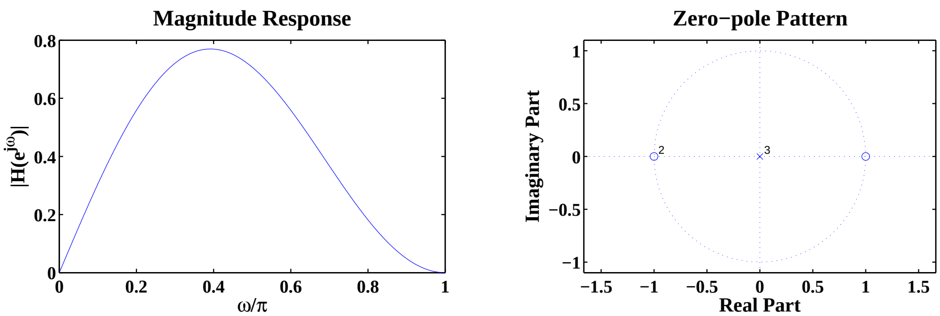


FIGURE 5.20: Magnitude response and pole-zero plot of $y[n] = \frac{1}{4}(x[n] + x[n-1]) - \frac{1}{4}(x[n-2] + x[n-3])$.

(d) Solution:

System function: $$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 + z^{-1} - z^{-3} - z^{-4})$$

Frequency response:

$$H(e^{j\omega}) = \frac{1}{4}[(1+\cos\omega-\cos 3\omega-\cos 4\omega)+j(-\sin\omega+\sin 3\omega+\sin 4\omega)]$$

Magnitude response:

$$|H(e^{j\omega})| = \frac{1}{4}\sqrt{(1+\cos\omega-\cos 3\omega-\cos 4\omega)^2+(-\sin\omega+\sin 3\omega+\sin 4\omega)^2}$$

Phase response: $$\angle H(e^{j\omega}) = \frac{-\sin\omega+\sin 3\omega+\sin 4\omega}{1+\cos\omega-\cos 3\omega-\cos 4\omega}$$

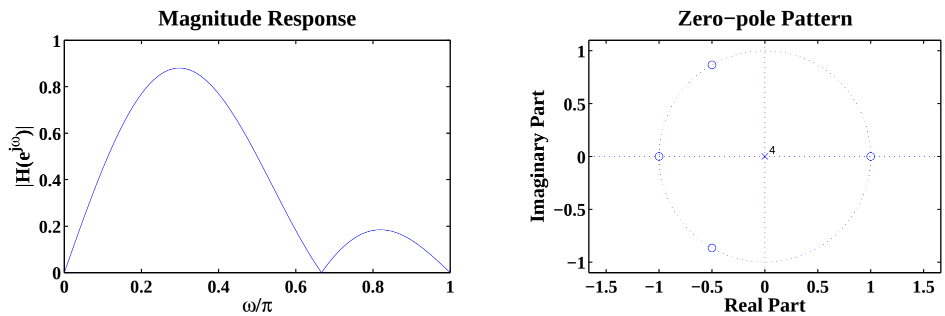


FIGURE 5.21: Magnitude response and pole-zero plot of $y[n] = \frac{1}{4}(x[n] + x[n-1]) - \frac{1}{4}(x[n-3] + x[n-4])$.

8. (a) Solution:

$$h_{bp}[n] = 2\frac{\sin\omega_c(n-n_d)}{\pi(n-n_d)}\cos\omega_0 n \quad (5.72)$$

$$h_{lp}[n] = \frac{\sin\omega_c(n-n_d)}{\pi(n-n_d)} \quad (5.70)$$

Modulation Property:

$$x[n]\cos\omega_0 n = \frac{1}{2}X\big(e^{j(\omega+\omega_0)}\big) + \frac{1}{2}X\big(e^{j(\omega-\omega_0)}\big)$$

$$H_{bp}\big(e^{j\omega}\big) = \begin{cases} e^{-j\omega n_d}, & \omega_\ell \le |\omega| \le \omega_h \\ 0, & \text{otherwise} \end{cases}$$

$$H_{lp}\big(e^{j\omega}\big) = \begin{cases} e^{-j\omega n_d}, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

$$H_{bp}\big(e^{j\omega}\big) = H_{lp}\big(e^{j(\omega-\omega_0)}\big) + H_{lp}\big(e^{j(\omega+\omega_0)}\big)$$

where $\omega_0 = \frac{\omega_\ell+\omega_h}{2}$ and $\omega_c = \frac{\omega_h-\omega_\ell}{2}$.
Hence,

$$h_{bp}[n] = 2h_{lp}\cos\omega_0 n = 2\frac{\sin\omega_c(n-n_d)}{\pi(n-n_d)}\cos\omega_0 n$$

(b) Solution:

$$H_{bp}\big(e^{j\omega}\big) = H_{lp1}\big(e^{j\omega}\big) - H_{lp2}\big(e^{j\omega}\big)$$

where

$$H_{lp1}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & |\omega| \le \omega_h \\ 0, & \omega_h < |\omega| \le \pi \end{cases}$$

$$H_{lp2}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & |\omega| \le \omega_\ell \\ 0, & \omega_\ell < |\omega| \le \pi \end{cases}$$

9. MATLAB function:

```
function [grp,omega] = mygrpdelay(b,a)
% Implement equation (5.89) to compute group delay
p = roots(a);
p = p(:);
N = size(p,1);
r = abs(p);
phi = angle(p);

z = roots(b);
z = z(:);
M = size(z,1);
q = abs(z);
theta = angle(z);

K = 1024;
omega = 2*pi*(0:K-1)/K;

r_epd = repmat(r,1,K);
phi_epd = repmat(phi,1,K);
q_epd = repmat(q,1,K);
theta_epd = repmat(theta,1,K);
temp1 = cos(repmat(omega,N,1)-phi_epd);
temp2 = cos(repmat(omega,M,1)-theta_epd);

grp = sum((r_epd.^2-r_epd.*temp1)./(1+r_epd.^2-2*r_epd.*temp1),1);
grp = -grp + sum((q_epd.^2-q_epd.*temp2)./(1+q_epd.^2-2*q_epd.*temp2),1);
```

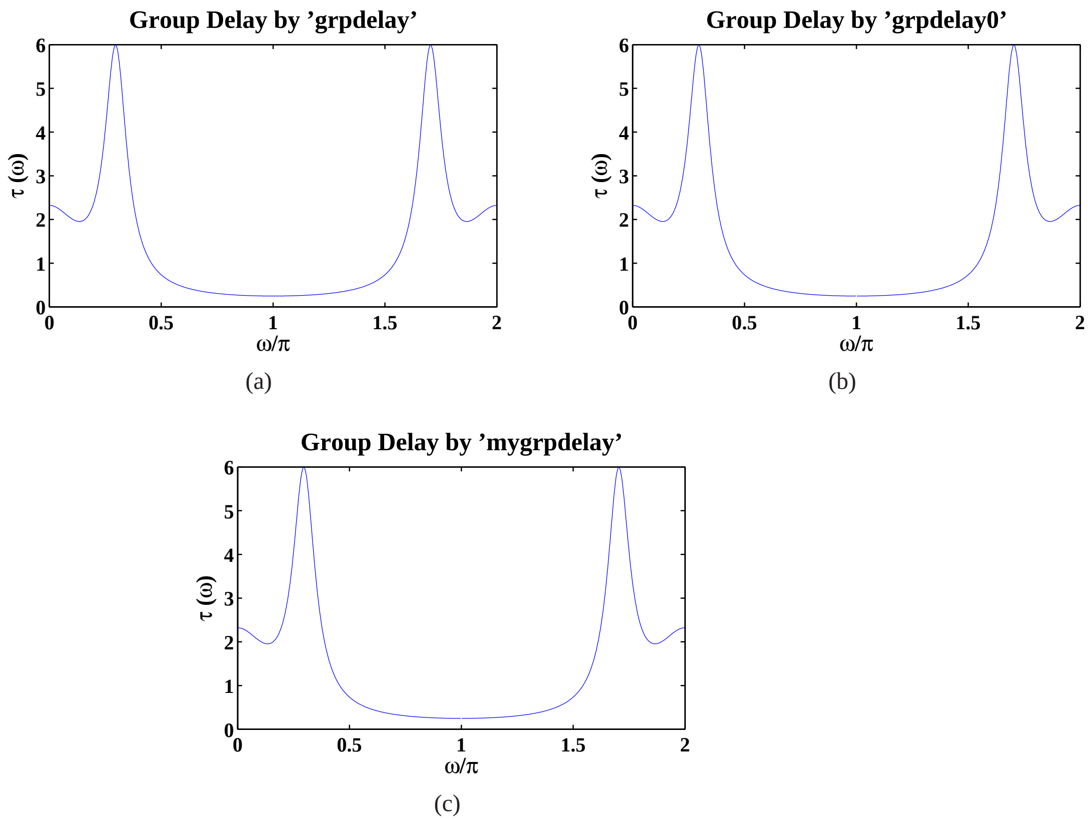


FIGURE 5.22: (a) Group delay computed by MATLAB function `grpdelay`. (b) Group delay computed by MATLAB function `grpdelay0`. (c) Group delay computed by MATLAB function `mygrpdelay`.

MATLAB script:

```
% P0509: Compute group delay using different functions
%        and compare with results for system (5.99)
close all; clc
%% Specification:
b = [1 1.655 1.655 1];
a = [1 -1.57 1.264 -0.4];
w = linspace(0,2,1000)*pi;
gd1 = grpdelay(b,a,w); % method I
[gd2 w2] = grpdelay0(b,a); % method II
```

```
[gd3,w3] = mygrpdelay(b,a); % method III
%% Plot
hfa = figconfg('P0509a','small');
plot(w/pi,gd1)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\tau (\omega)','fontsize',LFS)
title('Group Delay by ''grpdelay''','fontsize',TFS)
hfb = figconfg('P0509b','small');
plot(w2/pi,gd2)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\tau (\omega)','fontsize',LFS)
title('Group Delay by ''grpdelay0''','fontsize',TFS)
hfc = figconfg('P0509c','small');
plot(w3/pi,gd3)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\tau (\omega)','fontsize',LFS)
title('Group Delay by ''mygrpdelay''','fontsize',TFS)
```

10. Proof:

$$\tau_{gd}(\omega) = -\frac{d\Psi(\omega)}{d\omega}$$

$$\Psi(\omega) = \tan^{-1}\frac{H_I(\omega)}{H_R(\omega)} + 2k\pi, \quad \Psi(\omega) \text{ is continuous}$$

$$-\frac{d\Psi(\omega)}{d\omega} = -\frac{d}{d\omega}\left(\tan^{-1}\frac{H_I(\omega)}{H_R(\omega)} + 2k\pi\right) = \frac{\left(\frac{dH_I(\omega)}{d\omega}\right)H_R(\omega) - \left(\frac{dH_R(\omega)}{d\omega}\right)H_I(\omega)}{H_R^2(\omega) + H_I^2(\omega)}$$

$$nh[n] \overset{\text{DTFT}}{\longrightarrow} \mathrm{j}\cdot\frac{d\left[H_R(\omega) + \mathrm{j}H_I(\omega)\right]}{d\omega} = \mathrm{j}\frac{dH_R(\omega)}{d\omega} - \frac{dH_I(\omega)}{d\omega}$$

Hence, we have

$$\frac{dH_R(\omega)}{d\omega} = G_I(\omega); \quad \frac{dH_I(\omega)}{d\omega} = -G_R(\omega)$$

$$H_R^2(\omega) + H_I^2(\omega) = |H\left(\mathrm{e}^{\mathrm{j}\omega}\right)|^2$$

Thus, we proved that

$$\tau_{gd}(\omega) = -\frac{d\Psi(\omega)}{d\omega} = \frac{H_{\mathrm{R}}(\omega)G_{\mathrm{R}}(\omega) + H_{\mathrm{I}}(\omega)G_{\mathrm{I}}(\omega)}{|H(\mathrm{e}^{\mathrm{j}\omega})|^2}$$

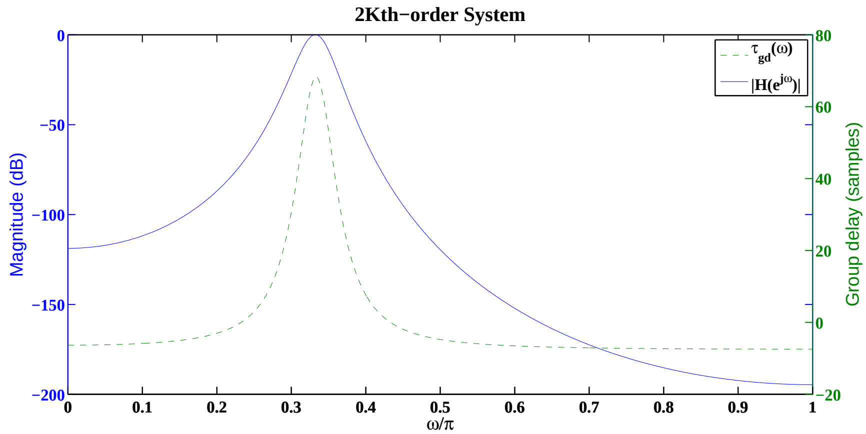


FIGURE 5.23: Frequency response of a single $2K$th-order system.

11. Solution:

    MATLAB script:

```
% P0511: Generate Figures 5.7 and 5.8 in Example5.6
close all; clc
%% Speicification:
r = 0.9;
w0 = pi/3;
K = 8;
b0 = 0.0271^4;
w = linspace(0,1,1000)*pi;

N = 100;
mu = 0;
sigma = 2;
n = linspace(-5,5,N);
sn = normpdf(n,mu,sigma);
n = 0:4*N-1;
w1 = 0.34*pi;
w2 = 0.6*pi;
xn = zeros(1,length(n));
xn(1:N) = sn.*cos(w1*(0:N-1));
xn(N+1:2*N) = sn.*cos(w2*(0:N-1));
```

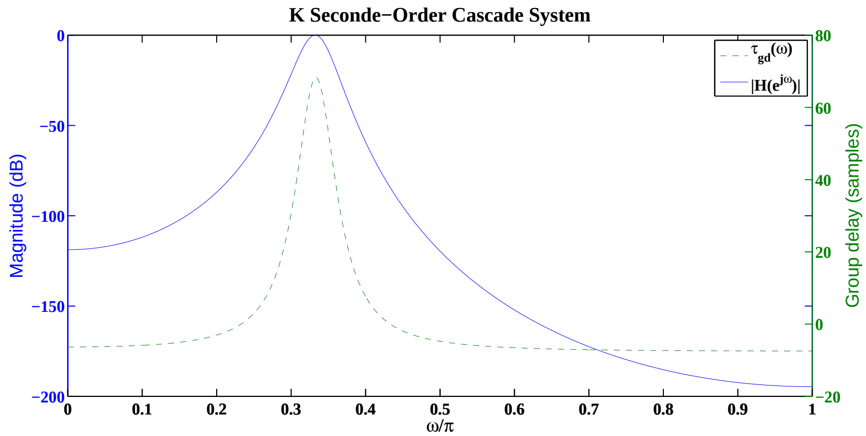


FIGURE 5.24: Frequency response of a cascade connection of K second-order systems.

```
X = dtft12(xn,0,w);

%% Part a:
bcas = b0^(1/K);
acas = [1 -2*r*cos(w0) r^2];

Hcas = freqz(bcas,acas,w);
H_mag_db_cas = 10*log10(abs(Hcas).^2)*K;
gd_cas = K*grpdelay(bcas,acas,w);
```

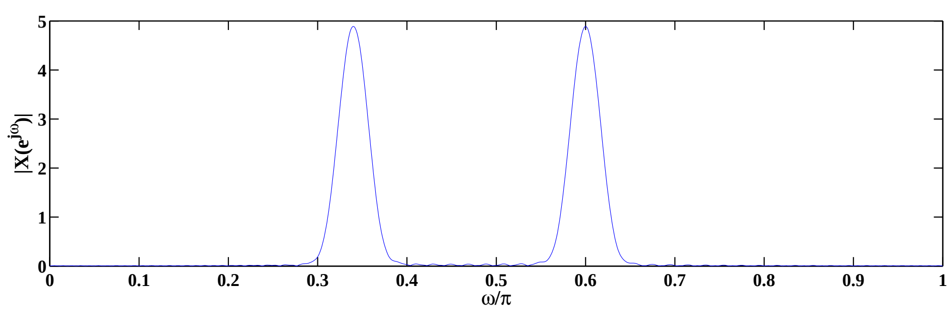


FIGURE 5.25: Frequency response input sequence $x[n]$.

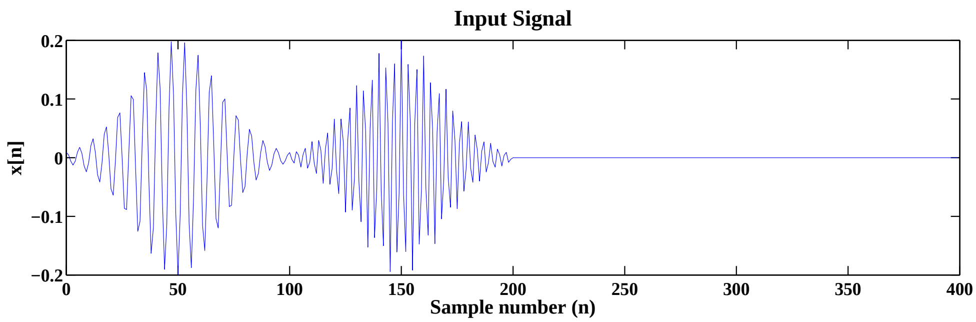


FIGURE 5.26: Input sequence $x[n]$.

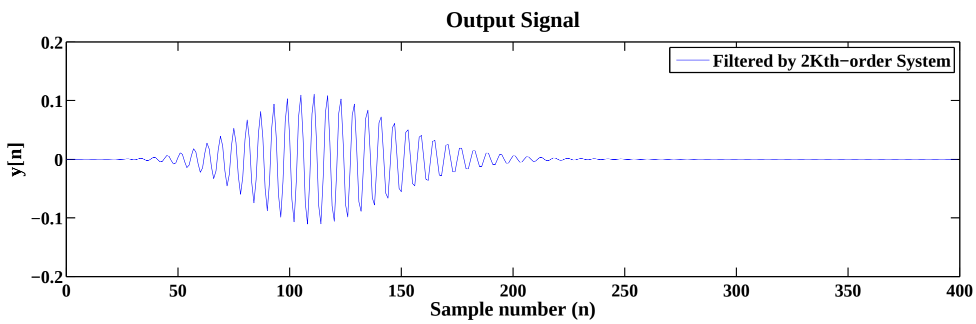


FIGURE 5.27: Output sequence $y[n]$ of a single $2K$th-order system.

```
yn_cas = xn;
for ii=1:K
yn_cas = filter(bcas,acas,yn_cas);
end

%% Part b:
a = acas;
for ii = 1:K-1
    a = conv(a,acas);
end
H = freqz(b0,a,w);
H_mag_db = 10*log10(abs(H).^2);
gd = grpdelay(b0,a,w);
yn = filter(b0,a,xn);
```

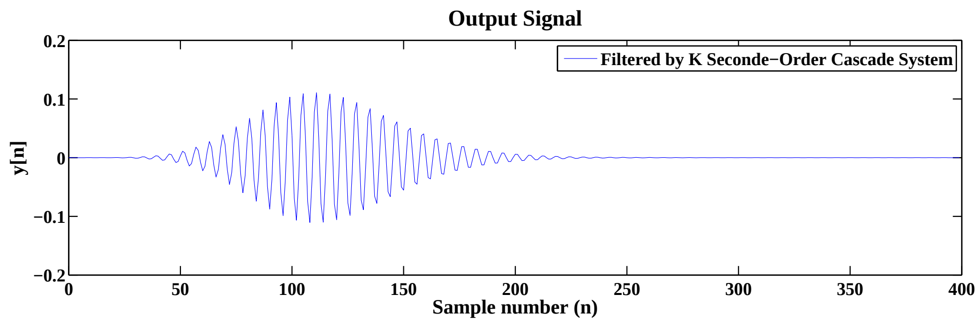


FIGURE 5.28: Output sequence $y[n]$ of a cascade connection of K second-order systems.

```
%% Plot
hfa = figconfg('P0511a','long');
[AX H1 H2] = plotyy(w/pi,H_mag_db,w/pi,gd);
xlabel('\omega/\pi','fontsize',LFS)
set(get(AX(1),'Ylabel'),'String','Magnitude (dB)','fontsize',LFS)
set(get(AX(2),'Ylabel'),'String','Group delay (samples)','fontsize',LFS)
set(AX(1),'ylim',[-200 0])
set(H2,'LineStyle','--')
legend('\tau_{gd}(\omega)','|H(e^{j\omega})|','Location','Northeast')
title('2Kth-order System','fontsize',TFS)

hfb = figconfg('P0511b','long');
[AX H1 H2] = plotyy(w/pi,H_mag_db_cas,w/pi,gd_cas);
xlabel('\omega/\pi','fontsize',LFS)
set(get(AX(1),'Ylabel'),'String','Magnitude (dB)','fontsize',LFS)
set(get(AX(2),'Ylabel'),'String','Group delay (samples)','fontsize',LFS)
set(AX(1),'ylim',[-200 0])
set(H2,'LineStyle','--')
legend('\tau_{gd}(\omega)','|H(e^{j\omega})|','Location','Northeast')
title('K Seconde-Order Cascade System','fontsize',TFS)

hfc = figconfg('P0511c','long');
plot(w/pi,abs(X))
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|X(e^{j\omega})|','fontsize',LFS)
```

```
hfd = figconfg('P0511d','long');
plot(n,xn)
xlabel('Sample number (n)','fontsize',LFS)
ylabel('x[n]','fontsize',LFS)
title('Input Signal','fontsize',TFS)

hfe = figconfg('P0511e','long');
plot(n,yn)
xlabel('Sample number (n)','fontsize',LFS)
ylabel('y[n]','fontsize',LFS)
title('Output Signal','fontsize',TFS)
legend('Filtered by 2Kth-order System','Location','Northeast')

hff = figconfg('P0511f','long');
plot(n,yn_cas)
xlabel('Sample number (n)','fontsize',LFS)
ylabel('y[n]','fontsize',LFS)
title('Output Signal','fontsize',TFS)
legend('Filtered by K Seconde-Order Cascade System','Location','Northeast')
```

12. Proof:

$$x[n] = s[n]\cos\omega_c n \quad (5.62)$$

$$\Psi(\omega) \approx \Psi(\omega_c) + \left.\frac{\Phi(\omega)}{d\omega}\right|_{\omega=\omega_c}(\omega-\omega_c) = -\tau_{pd}\omega_c - \tau_{gd}(\omega_c)(\omega-\omega_c) \quad (5.63)$$

$$y[n] \approx |H(\mathrm{e}^{\mathrm{j}\omega_c})| s[n-\tau_{gd}(\omega_c)]\cos\{\omega_c[n-\tau_{pd}(\omega_c)]\} \quad (5.64)$$

Suppose the system magnitude response is constant near ω_c,

$$\begin{aligned}
y[n] &\approx |H(\mathrm{e}^{\mathrm{j}\omega_c})| \left(\frac{1}{2\pi}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} s(\mathrm{e}^{\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}n\omega} d\omega\right) \cos\omega_c n \mathrm{e}^{-\mathrm{j}\tau_{pd}(\omega_c)\omega_c}\mathrm{e}^{-\mathrm{j}\tau_{gd}(\omega_c)(\omega-\omega_c)} \\
&= |H(\mathrm{e}^{\mathrm{j}\omega_c})| \left(\frac{1}{2\pi}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} s(\mathrm{e}^{\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}n\omega}\mathrm{e}^{-\mathrm{j}\tau_{gd}(\omega_c)\omega} d\omega\right) \cos\omega_c n \mathrm{e}^{-\mathrm{j}\tau_{pd}(\omega_c)\omega_c} \\
&= |H(\mathrm{e}^{\mathrm{j}\omega_c})| \left(\frac{1}{2\pi}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} s(\mathrm{e}^{\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}\omega(n-\tau_{gd}(\omega_c))} d\omega\right) \left(\cos\omega_c n \mathrm{e}^{-\mathrm{j}\tau_{pd}(\omega_c)\omega_c}\right) \\
&= |H(\mathrm{e}^{\mathrm{j}\omega_c})| s[n-\tau_{gd}(\omega_c)]\cos\{\omega_c[n-\tau_{pd}(\omega_c)]\}
\end{aligned}$$

13. (a) Solution:

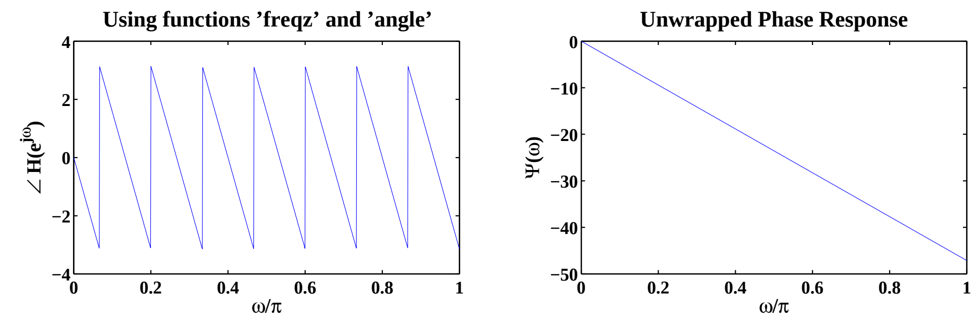


FIGURE 5.29: Phase response of pure delay $y[n] = x[n-15]$ using the functions freqz, angle and unwrap.

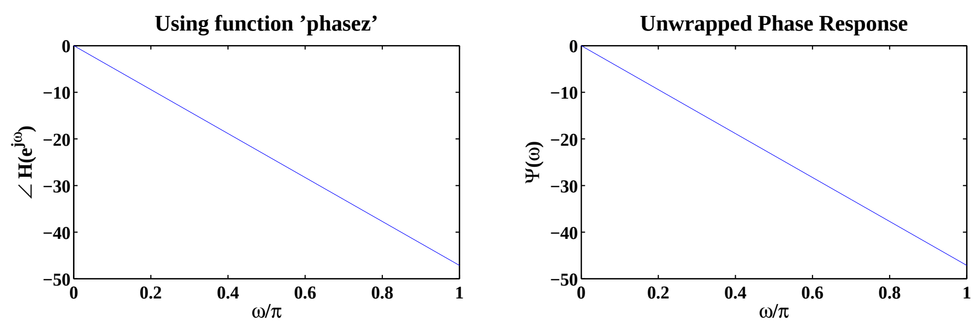


FIGURE 5.30: Phase response of pure delay $y[n] = x[n-15]$ using the function phasez.

(b) Solution:

MATLAB script:

```
% P0513: Plot phase response
close all; clc
%% Specification:
%% Part a:
% b = zeros(1,16);
% b(end) = 1;
% a = 1;

%% Part a:
```

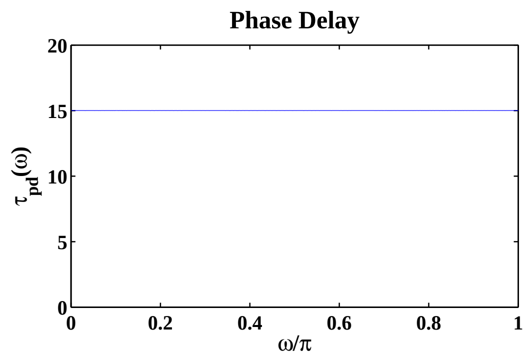


FIGURE 5.31: Phase delay of pure delay $y[n] = x[n-15]$ using the function phasedelay.

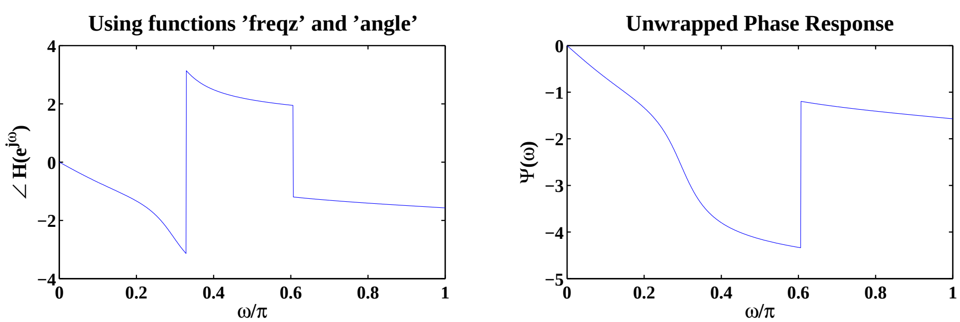


FIGURE 5.32: Phase response of the system defined by (5.99) using the functions freqz, angle and unwrap.

```
b = [1 1.655 1.655 1];
a = [1 -1.57 1.264 -0.4];

%% Computation:
w = linspace(0,1,1000)*pi;
H = freqz(b,a,w);
H_phase = angle(H);
H_phase2 = phasez(b,a,w);
Pd = phasedelay(b,a,w);
%% Plot:
hfa = figconfg('P0513a','long');
subplot(121)
```

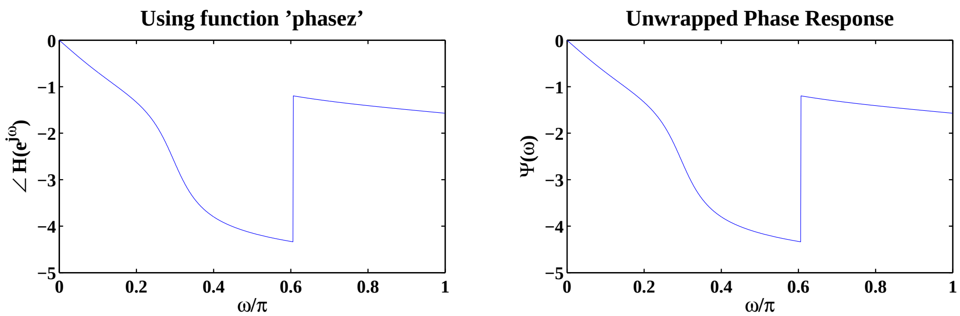


FIGURE 5.33: Phase response of the system defined by (5.99) using the function phasez.

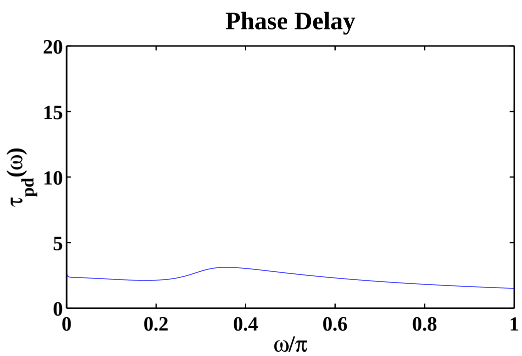


FIGURE 5.34: Phase delay of the system defined by (5.99) using the function phasedelay.

```
plot(w/pi,H_phase)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title('Using functions ''freqz'' and ''angle''','fontsize',TFS)
subplot(122)
plot(w/pi,unwrap(H_phase))
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\Psi(\omega)','fontsize',LFS)
title('Unwrapped Phase Response','fontsize',TFS)

hfb = figconfg('P0513b','long');
subplot(121)
```

```
plot(w/pi,H_phase2)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title('Using function ''phasez''','fontsize',TFS)
subplot(122)
plot(w/pi,unwrap(H_phase2))
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\Psi(\omega)','fontsize',LFS)
title('Unwrapped Phase Response','fontsize',TFS)

hfc = figconfg('P0513c','small');
plot(w/pi,Pd)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\tau_{pd}(\omega)','fontsize',LFS)
ylim([0 20])
title('Phase Delay','fontsize',TFS)
```

14. (a) Solution:

$$H\big(e^{j\omega}\big) = b_0 \frac{1 - e^{-2j\omega}}{1 - (2r\cos(\phi)e^{-j\omega}) + r^2 e^{-2j\omega}}$$

Solve for $|H\big(e^{j\phi}\big)| = 1$, we have

$$b_0 = 0.095$$

(b) Solution:

$$h[n] = b_0 r^n \frac{\sin[(n+1)\phi]}{\sin\phi} u[n] - b_0 r^{n-2} \frac{\sin[(n-1)\phi]}{\sin\phi} u[n-2]$$

(c) Solution:
The approximate 3-dB bandwidth of the resonator is 0.2086.

(d) Solution:
The exact 3-dB bandwidth is 0.2094.

(e) tba

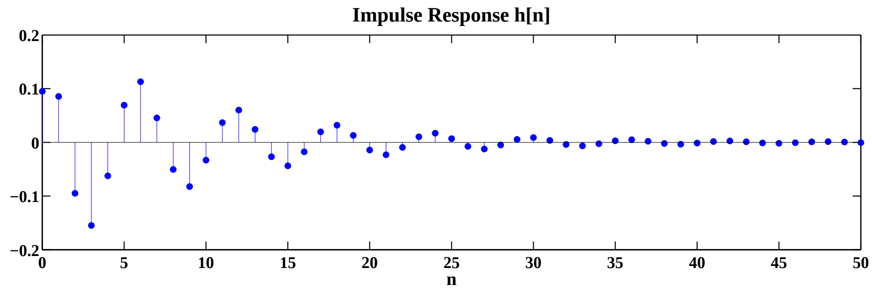


FIGURE 5.35: Impulse response $h[n]$ of the above resonator.

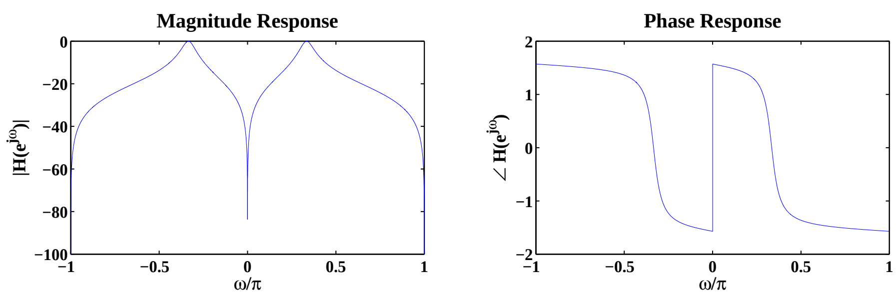


FIGURE 5.36: Magnitude response and phase response of the above resonator.

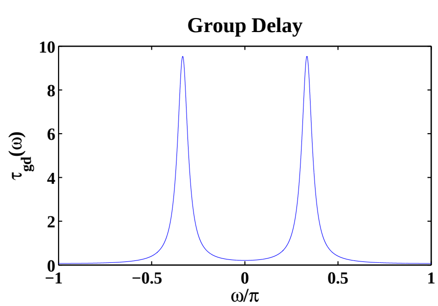


FIGURE 5.37: Group-delay of the above resonator.

15. (a) tba

(b) See plot below.

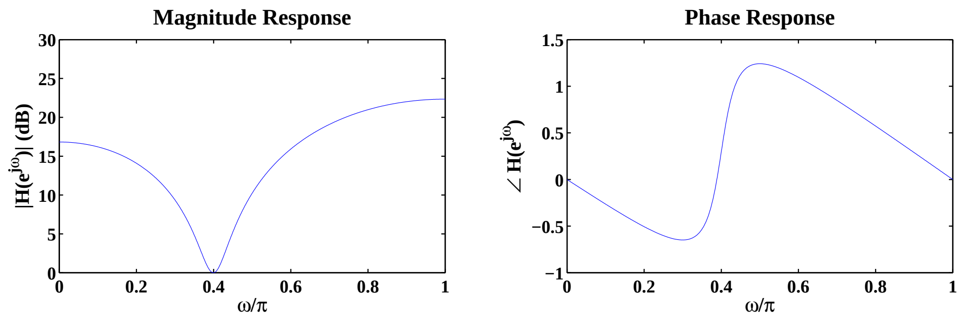


FIGURE 5.38: Magnitude response and phase response of the above resonator.

(c) Solution: The analytic 3-dB bandwidth is 0.2118 while the approximate bandwidth is 0.2107.

16. (a) Solution:

$$H_{lp}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{\sum_{k=0}^{M} b_k \mathrm{e}^{-\mathrm{j}k\omega}}{1 + \sum_{k=0}^{N} a_k \mathrm{e}^{-\mathrm{j}k\omega}}$$

$$H_{hp}(\mathrm{e}^{\mathrm{j}\omega}) = H_{lp}(\mathrm{e}^{\mathrm{j}(\omega-\pi)}) = \frac{\sum_{k=0}^{M} b_k \mathrm{e}^{-\mathrm{j}k(\omega-\pi)}}{1 + \sum_{k=0}^{N} a_k \mathrm{e}^{-\mathrm{j}k(\omega-\pi)}}$$

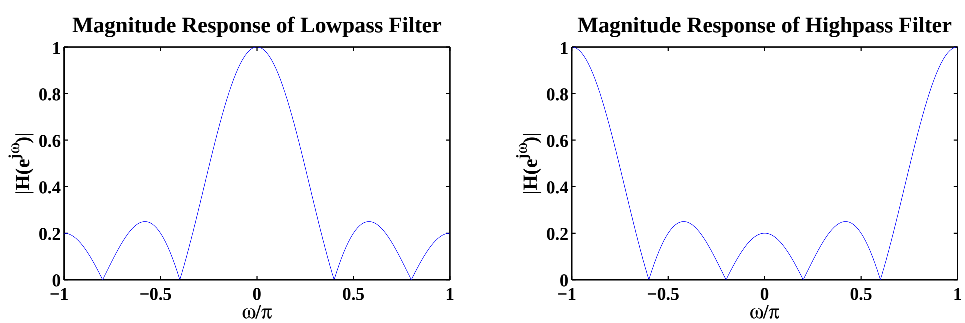


FIGURE 5.39: Magnitude response of lowpass and highpass filter.

(b) Solution:

$$H_{lp}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1 + 1.655\mathrm{e}^{-\mathrm{j}\omega} + 1.655\mathrm{e}^{-2\mathrm{j}\omega} + \mathrm{e}^{-3\mathrm{j}\omega}}{1 - 1.57\mathrm{e}^{-\mathrm{j}\omega} + 1.264\mathrm{e}^{-2\mathrm{j}\omega} - 0.4\mathrm{e}^{-3\mathrm{j}\omega}}$$

$$H_{hp}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1 - 1.655\mathrm{e}^{-\mathrm{j}\omega} + 1.655\mathrm{e}^{-2\mathrm{j}\omega} - \mathrm{e}^{-3\mathrm{j}\omega}}{1 + 1.57\mathrm{e}^{-\mathrm{j}\omega} + 1.264\mathrm{e}^{-2\mathrm{j}\omega} + 0.4\mathrm{e}^{-3\mathrm{j}\omega}}$$

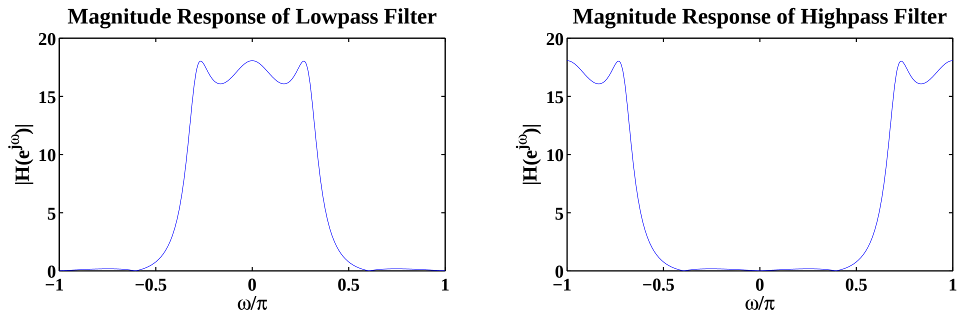


FIGURE 5.40: Magnitude response of lowpass and highpass filter.

17. Proof:

$$H_k(z) = z^{-1}\frac{1 - p_k^* z}{1 - p_k z^{-1}} = \frac{z^{-1} - p_k^*}{1 - p_k z^{-1}} \quad (5.157)$$

$$|H_k(\mathrm{e}^{\mathrm{j}\omega})| = 1, \quad (5.160)$$

$$\angle H_k(\mathrm{e}^{\mathrm{j}\omega}) = -\omega - 2\tan^{-1}\frac{r_k \sin(\omega - \phi_k)}{1 - r_k\cos(\omega - \phi_k)} \quad (5.161)$$

$$\tau_k(\omega) = \frac{1 - r_k^2}{1 + r_k^2 - 2r_k\cos(\omega - \phi_k)} \quad (5.162)$$

The frequency response is

$$H_k(\mathrm{e}^{\mathrm{j}\omega}) = \mathrm{e}^{-\mathrm{j}\omega}\frac{1 - p_k^*\mathrm{e}^{\mathrm{j}\omega}}{1 - p_k\mathrm{e}^{-\mathrm{j}\omega}} = \frac{\mathrm{e}^{-\mathrm{j}\omega} - p_k^*}{1 - p_k\mathrm{e}{-}\mathrm{j}\omega}$$

also we have

$$p_k = r_k\mathrm{e}^{\mathrm{j}\phi_k}$$

First, we prove (5.160)

$$|H_k(\mathrm{e}^{\mathrm{j}\omega})| = |\mathrm{e}^{-\mathrm{j}\omega}|\frac{|1 - p_k^*\mathrm{e}^{\mathrm{j}\omega}|}{|1 - p_k\mathrm{e}^{-\mathrm{j}\omega}|} = 1$$

Then, we prove (5.161)

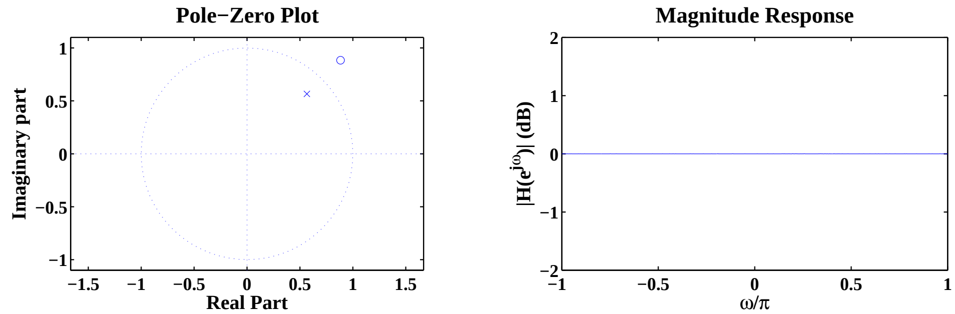
$$
\begin{aligned}
\angle H_k\big(e^{j\omega}\big) &= \angle e^{-j\omega} + \angle(1 - p_k^* e^{j\omega}) - \angle(1 - p_k e^{-j\omega}) \\
&= -\omega + \angle(1 - r_k e^{-j\phi_k} e^{j\omega}) - \angle(1 - r_k e^{j\phi_k} e^{-j\omega}) \\
&= -\omega + \angle(1 - r_k \cos(\omega - \phi_k) - jr_k \sin(\omega - \phi_k)) \\
&\quad - \angle(1 - r_k \cos(\omega - \phi_k) + jr_k \sin(\omega - \phi_k)) \\
&= -\omega - 2\tan^{-1} \frac{r_k \sin(\omega - \phi_k)}{1 - r_k \cos(\omega - \phi_k)}
\end{aligned}
$$

Finally, we prove (5.162)

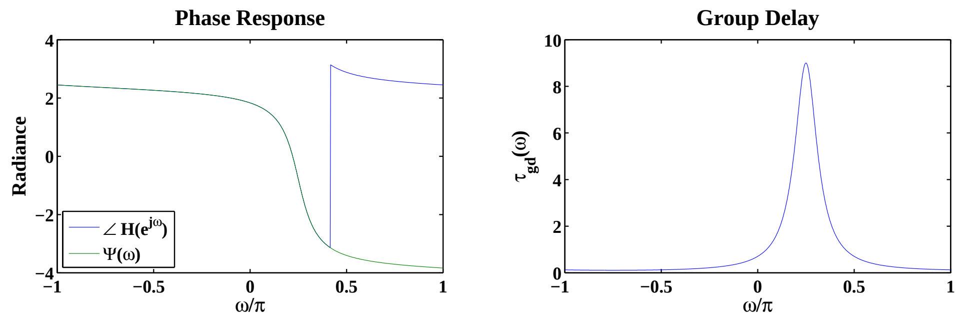
$$
\begin{aligned}
\tau_{gd}(\omega) = -\frac{d\angle H\big(e^{j\omega}\big)}{d\omega} &= 1 + 2 \times \frac{1}{1 + \left[\frac{r_k \sin(\omega-\phi_k)}{1-r_k \cos(\omega-\phi_k)}\right]^2} \\
&\quad \times \frac{r_k \cos(\omega - \phi_k)[1 - r_k \cos(\omega - \phi_k)] - r_k \sin(\omega - \phi_k) r_k \sin(\omega - \phi_k)}{[1 - r_k \cos(\omega - \phi_k)]^2} \\
&= 1 + 2 \times \frac{r_k \cos(\omega - \phi_k) - r_k^2 \cos^2(\omega - \phi_k) - r_k^2 \sin^2(\omega - \phi_k)}{[1 - r_k \cos(\omega - \phi_k)]^2 + [r_k \sin(\omega - \phi_k)]^2} \\
&= \frac{1 - r_k^2}{1 + r_k^2 - 2r_k \cos(\omega - \phi_k)}
\end{aligned}
$$

MATLAB scripts:

```
% P0517: Illustrate a first order allpass filter
close all; clc
%% Specification:
p = 0.8*exp(j*pi/4);
b = [-p',1];
a = [1 -p];
w = linspace(-1,1,1000)*pi;
H = freqz(b,a,w);
H_mag = abs(H);
H_mag_db = 10*log10(H_mag.^2);
H_phase = angle(H);
H_phase_unwrap = unwrap(H_phase);
gd = grpdelay(b,a,w);

%% Plot:
hfa = figconfg('P0517a','long');
```



(a)



(b)

FIGURE 5.41: Figure 5.29 regeneration.

```
subplot(121)
zplane(b,a)
xlabel('Real Part','fontsize',LFS)
ylabel('Imaginary part','fontsize',LFS)
title('Pole-Zero Plot','fontsize',TFS)
subplot(122)
plot(w/pi,H_mag_db)
ylim([-2 2])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})| (dB)','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)

hfb = figconfg('P0517b','long');
subplot(121)
plot(w/pi,H_phase,w/pi,H_phase_unwrap)
xlabel('\omega/\pi','fontsize',LFS)
```

```
ylabel('Radiance','fontsize',LFS)
title('Phase Response','fontsize',TFS)
legend('\angle H(e^{j\omega})','\Psi(\omega)','location','southwest')
subplot(122)
plot(w/pi,gd)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\tau_{gd}(\omega)','fontsize',LFS)
title('Group Delay','fontsize',TFS)
```

18. (a) Solution:
The frequency response is:

$$\begin{aligned}H(\mathrm{e}^{\mathrm{j}\omega}) &= \frac{2+3.125\mathrm{e}^{-2\mathrm{j}\omega}}{1-0.9\mathrm{e}^{-\mathrm{j}\omega}+0.81\mathrm{e}^{-2\mathrm{j}\omega}} \\ &= \frac{2+3.125\cos 2\omega - 3.125\mathrm{j}\sin 2\omega}{(1-0.9\cos\omega+0.81\cos 2\omega)+\mathrm{j}(0.9\sin\omega-0.81\sin 2\omega)}\end{aligned}$$

The magnitude response is:

$$\begin{aligned}|H(\mathrm{e}^{\mathrm{j}\omega})| &= \frac{\sqrt{(2+3.125\cos 2\omega)^2+3.125^2\sin^2 2\omega}}{\sqrt{(1-0.9\cos\omega+0.81\cos 2\omega)^2+(0.9\sin\omega-0.81\sin 2\omega)^2}} \\ &= \frac{\sqrt{4+3.125^2+12.5\cos 2\omega}}{\sqrt{1+0.9^2+0.81^2-2\times 0.9\times 1.81\cos\omega+2\times 0.81\cos 2\omega}}\end{aligned}$$

The phase response is:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) = -\tan^{-1}\frac{3.125\sin 2\omega}{2+3.125\cos 2\omega} - \tan^{-1}\frac{0.9\sin\omega-0.81\sin 2\omega}{1-0.9\cos\omega+0.81\cos 2\omega}$$

(b) Solution:

$$H(z) = \frac{2(1+\frac{25}{16}z^{-2})}{1-0.9z^{-1}+0.81z^{-2}} = \frac{2(1+\frac{5}{4}\mathrm{j}z^{-1})(1-\frac{5}{4}\mathrm{j}z^{-1})}{1-0.9z^{-1}+0.81z^{-2}}$$

$$\begin{aligned}H_{\min}(z) &= \frac{2\times\frac{25}{8}(1-\frac{4}{5}\mathrm{j}z^{-1})(1+\frac{4}{5}\mathrm{j}z^{-1})}{1-0.9z^{-1}+0.81z^{-2}} \\ &= \frac{3.125+2z^{-2}}{1-0.9z^{-1}+0.81z^{-2}}\end{aligned}$$

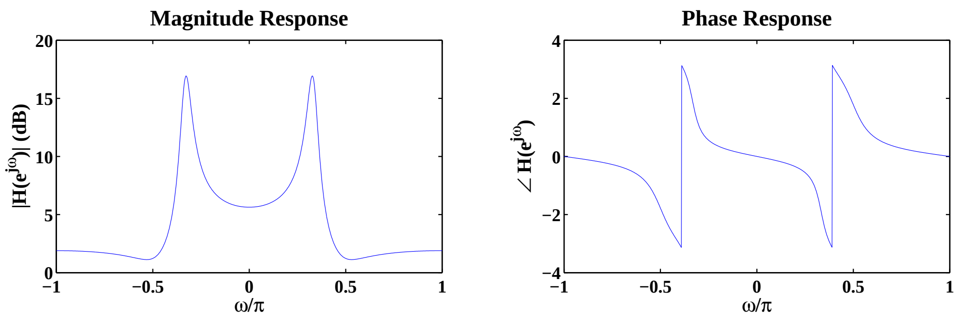


FIGURE 5.42: Magnitude response and phase responses of the system $H(z)$.

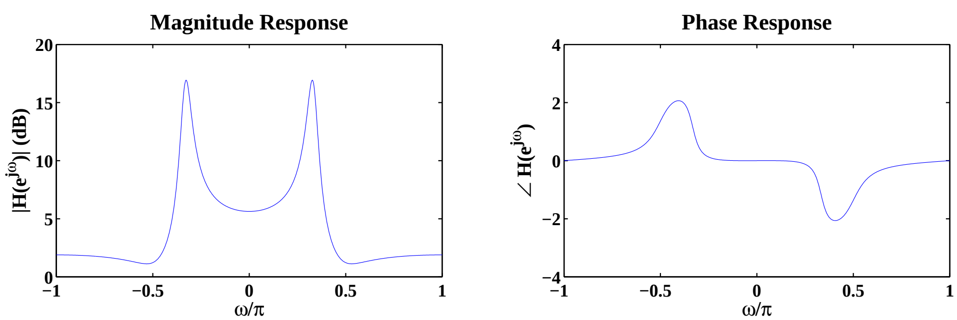


FIGURE 5.43: Magnitude response and phase responses of the minimum-phase system $H_{\min}(z)$.

(c) Solution:

$$H_{\text{eq}}(z) = \frac{Gz^{-n_d}}{H_{\min}(z)}$$

Choose $G = 1$ and $n_d = 0$, we have the equalizer system:

$$H_{\text{eq}}(z) = \frac{1}{H_{\min}(z)} = \frac{1 - 0.9z^{-1} + 0.81z^{-2}}{3.125 + 2z^{-2}}$$

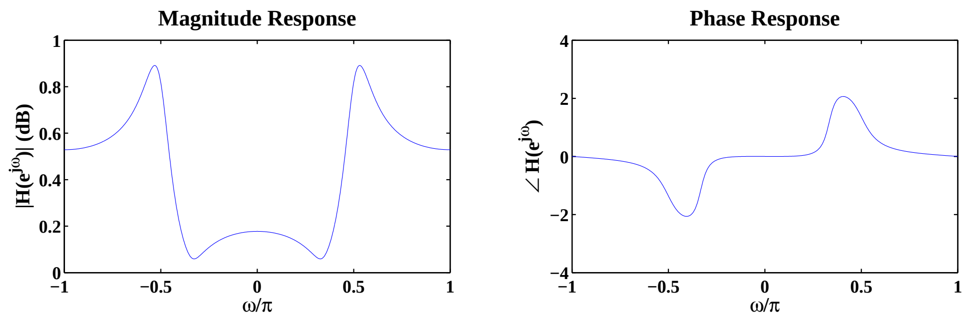


FIGURE 5.44: Magnitude response and phase responses of the equalizer system $H_{\text{eq}}(z)$.

19. (a) Proof:

Substitute $z = r\mathrm{e}^{\mathrm{j}\theta}$ into $H(z)$, we have

$$H(z) = \frac{r^{-1}\mathrm{e}^{-\mathrm{j}\theta} - a}{1 - ar^{-1}\mathrm{e}^{-\mathrm{j}\theta}} = \frac{(-a + r^{-1}\cos\theta) - \mathrm{j}r^{-1}\sin\theta}{(1 - ar^{-1}\cos\theta) + \mathrm{j}ar^{-1}\sin\theta}$$

$$|H(z)| = \sqrt{\frac{(-a + r^{-1}\cos\theta)^2 + r^{-2}\sin^2\theta}{(1 - ar^{-1}\cos\theta)^2 + a^2r^{-2}\sin^2\theta}} = \sqrt{\frac{a^2r^2 + 1 - 2ar\cos\theta}{r^2 + a^2 - 2ar\cos\theta}}$$

where $a^2r^2 + 1 - 2ar\cos\theta > 0$ and $r^2 + a^2 - 2ar\cos\theta > 0$.

$$a^2r^2 + 1 - (r^2 + a^2) = (1 - r^2)(1 - a^2)$$

where $1 - a^2 > 0$. Hence, we proved that

$$|H(z)| \begin{cases} < 1 & \text{for } r > 1, \\ = 1 & \text{for } r = 1, \\ > 1 & \text{for } r < 1. \end{cases}$$

(b) Proof:

The frequency response of the system is

$$H\big(\mathrm{e}^{\mathrm{j}\omega}\big) = \frac{\mathrm{e}^{-\mathrm{j}\omega} - a}{1 - a\mathrm{e}-\mathrm{j}\omega} = \frac{(\cos\omega - a) - \mathrm{j}\sin\omega}{(1 - a\cos\omega) + \mathrm{j}a\sin\omega}$$

The group-delay $\tau(\omega)$ is

$$\tau(\omega) = \frac{1 - a^2}{1 + a^2 - 2a\cos\omega}$$

Substitute $\tau(\omega)$ into the integral, we have

$$\int_0^{\pi} \tau(\omega)\mathrm{d}\omega = \left(\omega + 2\tan^{-1}\frac{a\sin\omega}{1-a\cos\omega}\right)\Bigg|_0^{\pi} = \pi$$

20. (a) Solution:

Using time-expansion property,

$$x_{(k)}[n] = \begin{cases} x[r], & n = rk, \\ 0, & n \neq rk. \end{cases} \xrightarrow{z} X(z^k)$$

The time sequence corresponding to z-transform

$$A(z) = \frac{z^{-1}}{1-az^{-1}}$$

is

$$x[n] = a^{n-1}u[n-1]$$

Hence, the impulse response $h[n]$ can be defined as

$$h[n] = \begin{cases} x[r], & n = r\cdot D \\ 0, & n \neq r\cdot D \end{cases}$$

(b) Solution:

The frequency response can be written as

$$H\big(\mathrm{e}^{\mathrm{j}\omega}\big) = \frac{\mathrm{e}^{-\mathrm{j}D\omega}}{1-a\mathrm{e}^{-\mathrm{j}D\omega}} = \frac{\mathrm{e}^{-\mathrm{j}D\omega}}{1-a\cos D\omega + \mathrm{j}a\sin D\omega}$$

The magnitude response is

$$|H\big(\mathrm{e}^{\mathrm{j}\omega}\big)| = \frac{1}{\sqrt{(1-a\cos D\omega)^2 + a^2\sin^2 D\omega}} = \frac{1}{\sqrt{1+a^2-2a\cos D\omega}}$$

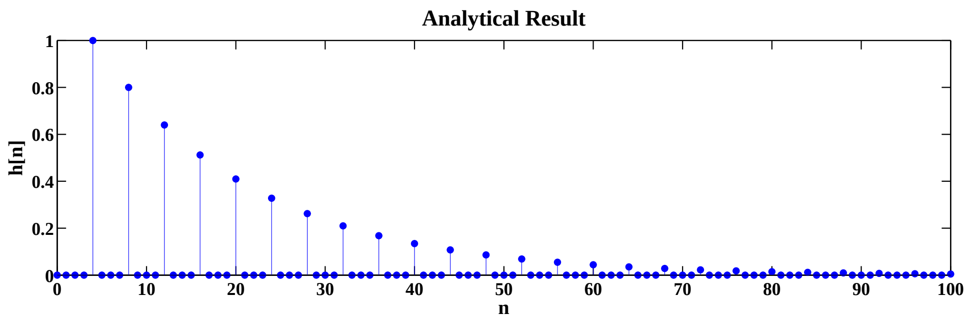
which exhibits D peaks and D dips over $0 \leq \omega \leq 2\pi$.

When $a > 0$,

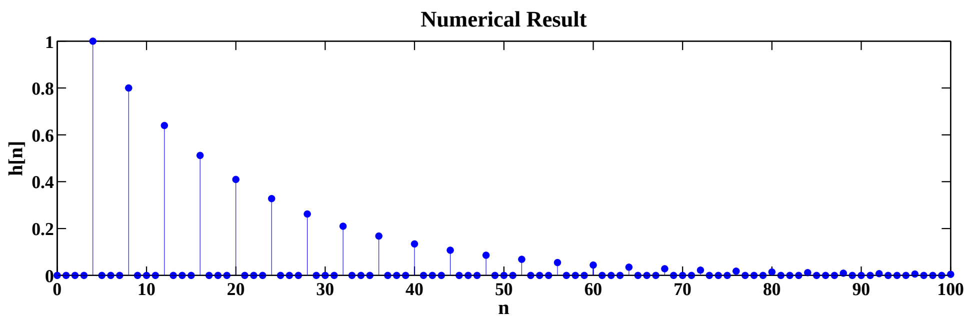
$$\begin{cases} \text{Dip locations:} & \omega = \frac{(2k+1)\pi}{D}, 0 \leq k \leq D-1 \\ \text{Peak locations:} & \omega = \frac{2k\pi}{D}, 0 \leq k \leq D-1 \end{cases}$$

When $a < 0$,

$$\begin{cases} \text{Peak locations:} & \omega = \frac{(2k+1)\pi}{D}, 0 \leq k \leq D-1 \\ \text{Dip locations:} & \omega = \frac{2k\pi}{D}, 0 \leq k \leq D-1 \end{cases}$$



(a)



(b)

FIGURE 5.45: MATLAB verification of analytical expression of the impulse response $h[n]$ for $D = 4$ and $a = 0.8$.

(c) MATLAB script:

```
% P0520: Illustrate Comb filter
close all; clc
%% Specification:
% D = 4;
% a = 0.8;
% D = 5;
% a = 0.9;
D = 8;
a = -0.8;
bh = zeros(1,D+1);
bh(end) = 1;
ah = zeros(1,D+1);
ah(1) = 1; ah(end) = -a;
```

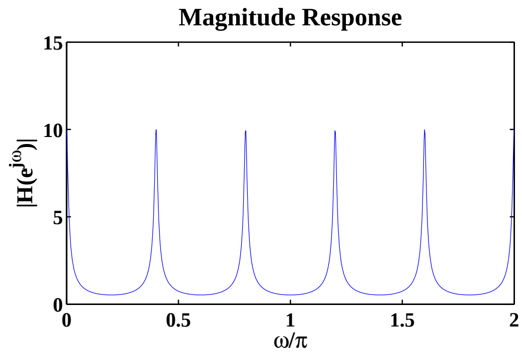


FIGURE 5.46: Magnitude response for $D = 4$ and $a = 0.8$.

```
n = 0:100;
xn = a.^(n-1);
hn_ana = zeros(size(n));
ind = (mod(n,D) == 0);
hn_ana(ind) = xn(1:sum(ind));
hn_ana(1) = 0;

[hn nn] = impz(bh,ah,n(end)+1);

w = linspace(0,2,1000)*pi;
H = freqz(bh,ah,w);
H_mag = abs(H);
%% Plot:
hfa = figconfg('P0520a','long');
stem(n,hn_ana,'filled')
xlabel('n','fontsize',LFS)
ylabel('h[n]','fontsize',LFS)
title('Analytical Result','fontsize',TFS)

hfb = figconfg('P0520b','long');
stem(n,hn,'filled')
xlabel('n','fontsize',LFS)
ylabel('h[n]','fontsize',LFS)
title('Numerical Result','fontsize',TFS)

hfc = figconfg('P0520c','small');
```

```
plot(w/pi,H_mag)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
```

21. (a) Proof:

$$H(s) = \frac{(s-3)(s-2-\mathrm{j})(s-2+\mathrm{j})(s+1)}{(s+5)(s+3-3\mathrm{j})(s+3+3\mathrm{j})(s+2-2\mathrm{j})(s+2+2\mathrm{j})}$$

Hence, there are three zero on the right-hand plane which proved that the system $H(s)$ is NOT minimum phase system.

(b) Solution:

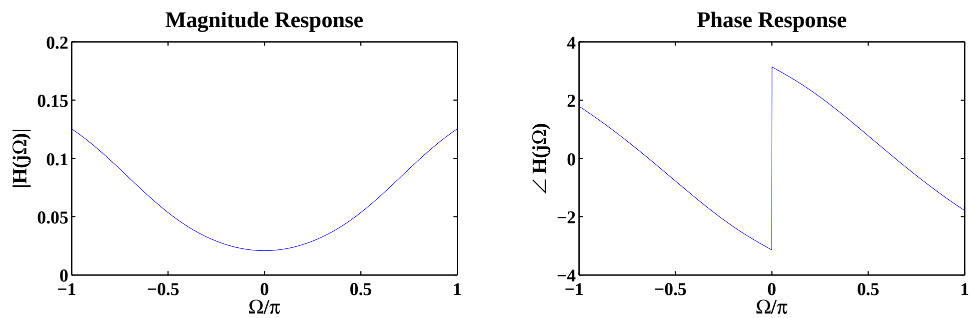
$$H(s) = H_{\text{min}}(s) \cdot H_{\text{ap}}(s)$$

$$H_{\text{min}}(s) = \frac{(s+3)(s+2-\mathrm{j})(s+2+\mathrm{j})(s+1)}{(s+5)(s+3-3\mathrm{j})(s+3+3\mathrm{j})(s+2-2\mathrm{j})(s+2+2\mathrm{j})}$$

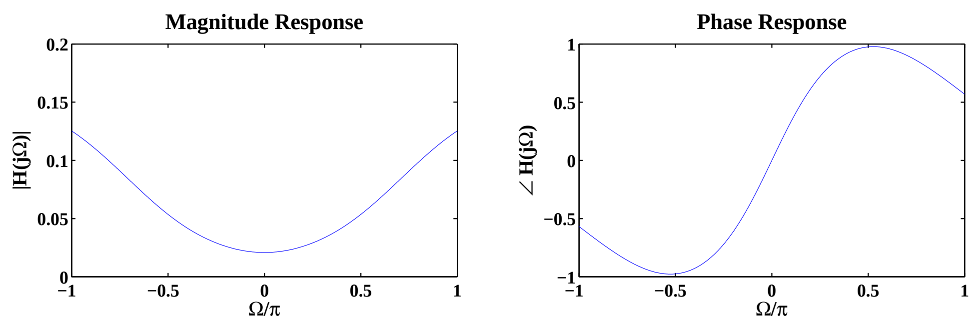
$$H_{\text{ap}}(s) = \frac{(s-3)(s-2-\mathrm{j})(s-2+\mathrm{j})}{(s+3)(s+2-\mathrm{j})(s+2+\mathrm{j})}$$

(c) See plot below.

(d) See plot below.



(a)



(b)

FIGURE 5.47: Magnitude and phase responses of (a) $H(s)$ and (b) $H_{\min}(s)$.

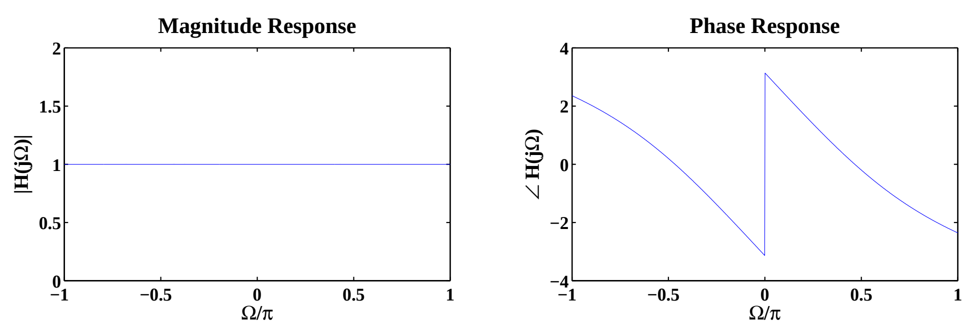


FIGURE 5.48: Magnitude and phase responses of $H_{\text{ap}}(s)$.

22. (a) See plot below.

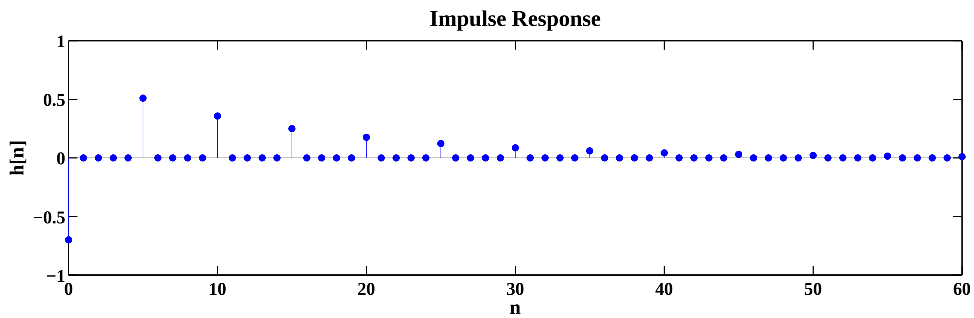


FIGURE 5.49: Impulse response $h[n]$.

(b) See plot below.

(c) See plot below.

(d) tba.

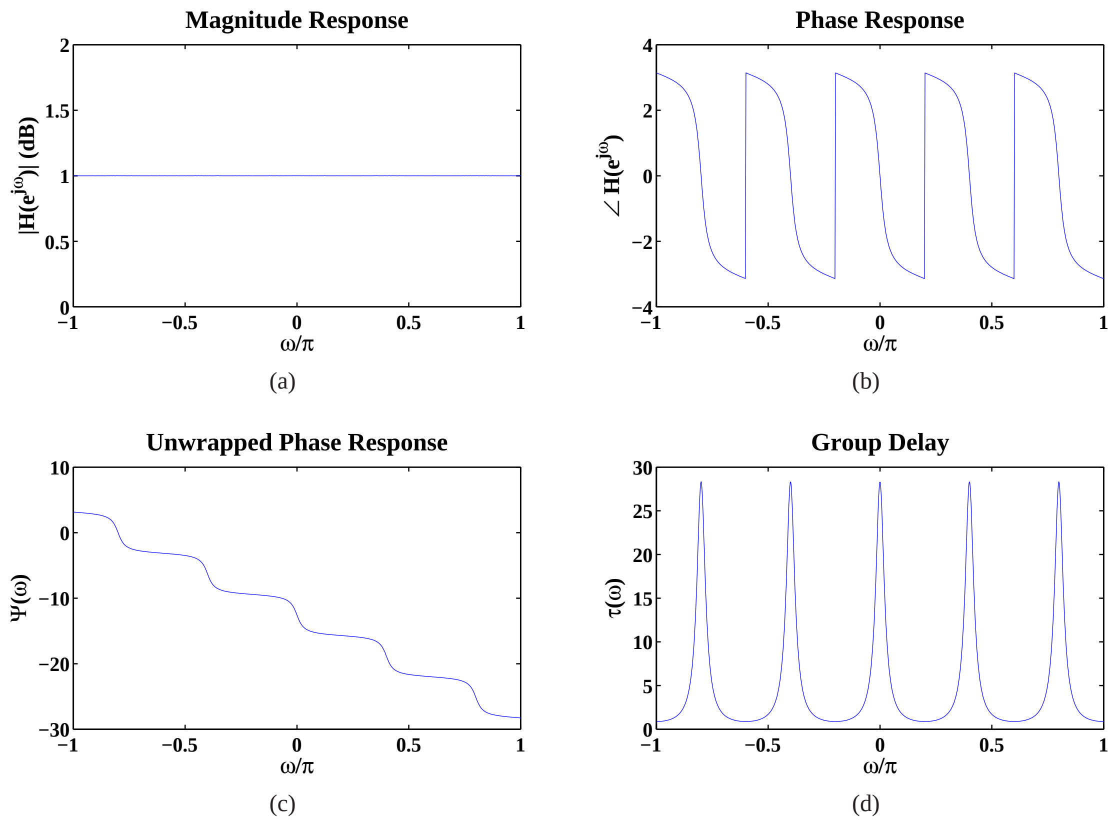


FIGURE 5.50: Magnitude, wrapped-phase, unwrapped-phase, and group-delay responses for $D = 5$ and $a = 0.7$.

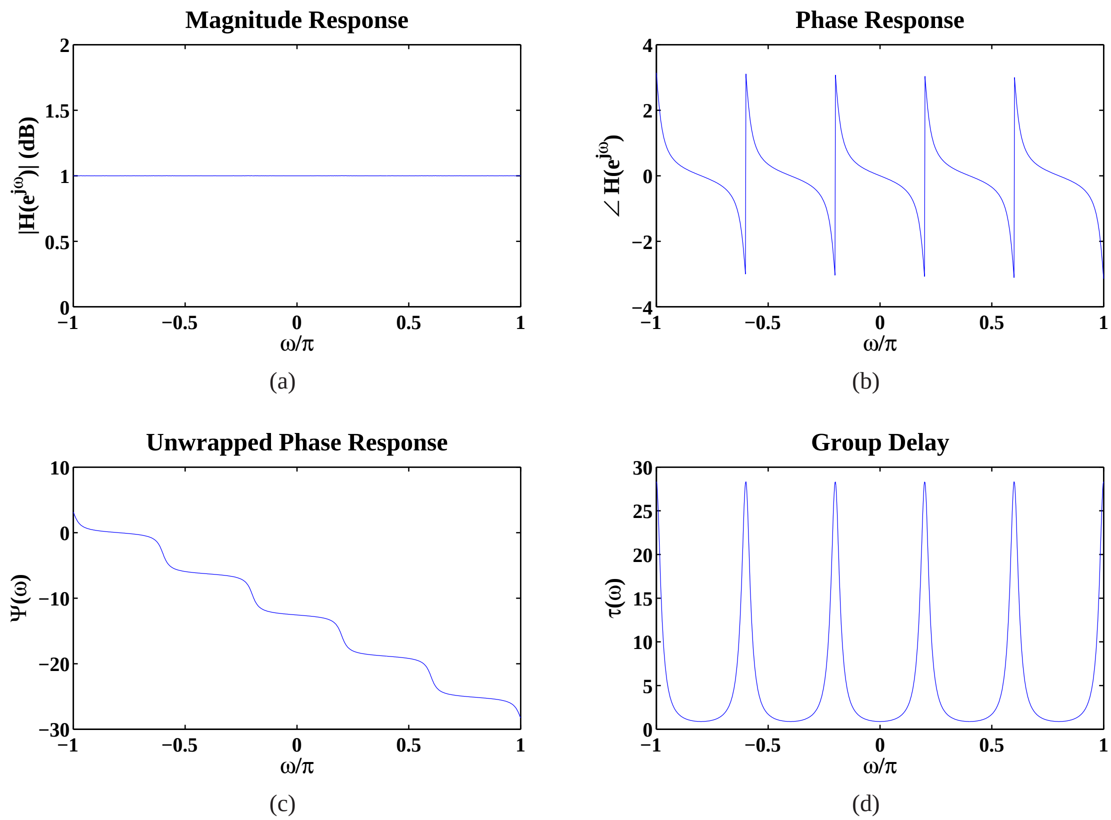


FIGURE 5.51: Magnitude, wrapped-phase, unwrapped-phase, and group-delay responses for $D = 5$ and $a = -0.7$.

Basic Problems

23. (a) Solution:
The frequency response is

$$H(e^{j\omega}) = \frac{b}{1 - 0.8e^{-j\omega} - 0.81e^{-2j\omega}}$$

(b) Solution:
b = 0.1702.

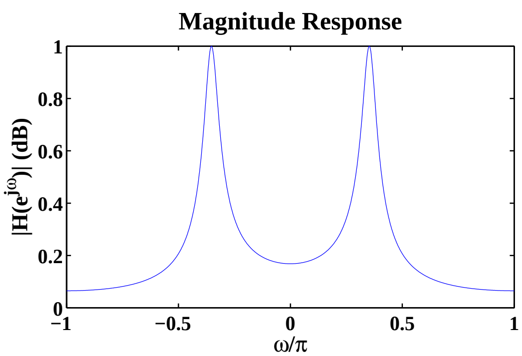


FIGURE 5.52: Magnitude response.

(c) See plot below.

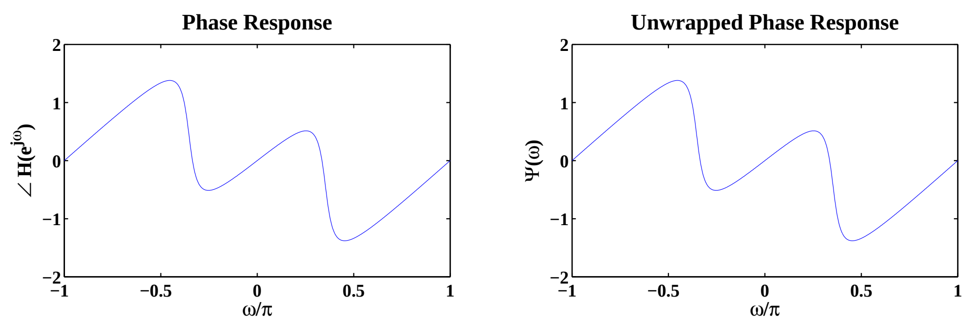


FIGURE 5.53: Wrapped and the unwrapped phase responses.

(d) Solution:

$$x[n] = 2\cos(\frac{\pi n}{3} + \frac{\pi}{4}) = e^{j\frac{\pi}{4}} e^{j\frac{\pi n}{3}} + e^{-j\frac{\pi}{4}} e^{-j\frac{\pi n}{3}}$$

$$
\begin{aligned}
y[n] &= \mathrm{e}^{j\frac{\pi}{4}}\mathrm{e}^{\mathrm{j}\frac{\pi n}{3}}H\big(\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\big) + \mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}\mathrm{e}^{-\mathrm{j}\frac{\pi n}{3}}H\big(\mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\big) \\
&= 2\times 0.0577\cos\left(\frac{\pi n}{3}+\frac{\pi}{4}\right) - 2\times 0.0809\sin\left(\frac{\pi n}{3}+\frac{\pi}{4}\right)
\end{aligned}
$$

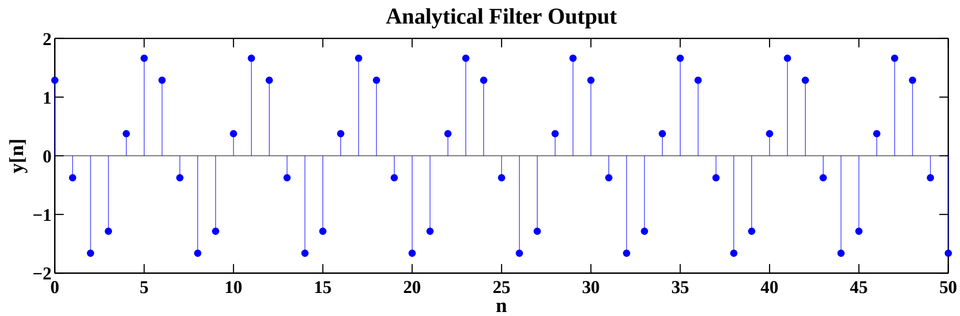


FIGURE 5.54: Analytical response $y[n]$ to the input $x[n] = 2\cos(\pi n/3 + 45°)$.

(e) See plot below.

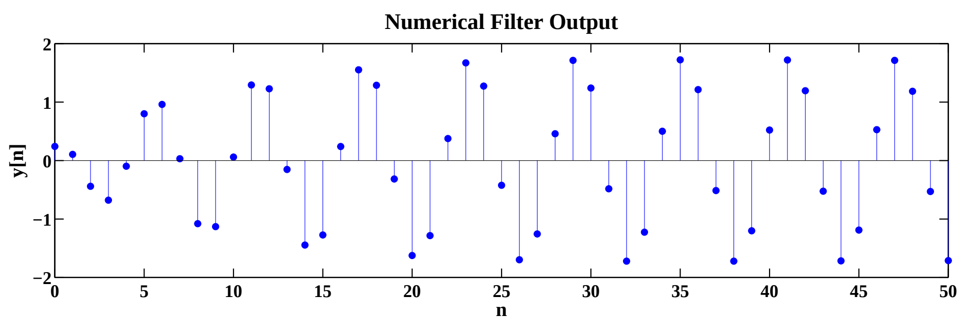


FIGURE 5.55: Numerical response $y[n]$ to the input $x[n] = 2\cos(\pi n/3 + 45°)$.

24. (a) Solution:
The frequency response is:

$$
H\big(\mathrm{e}^{\mathrm{j}\omega}\big) = \frac{Y\big(\mathrm{e}^{\mathrm{j}\omega}\big)}{X\big(\mathrm{e}^{\mathrm{j}\omega}\big)} = \frac{b}{1+a\mathrm{e}^{-2\mathrm{j}\omega}} = \frac{b}{1+a\cos 2\omega - a\,\mathrm{j}\sin 2\omega}
$$

The magnitude response is:

$$
|H\big(\mathrm{e}^{\mathrm{j}\omega}\big)| = \frac{|b|}{\sqrt{(1+a\cos 2\omega)^2 + a^2\sin^2 2\omega}} = \frac{|b|}{\sqrt{1+a^2+2a\cos 2\omega}}
$$

The phase response is:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) = \tan^{-1}\frac{a\sin 2\omega}{1+a\cos 2\omega}$$

In order to constrain the maximum magnitude response equal to 1, we have

$$|b| = 1 - a$$

Hence, for $a = 0.8$, we choose $b = 0.2$.

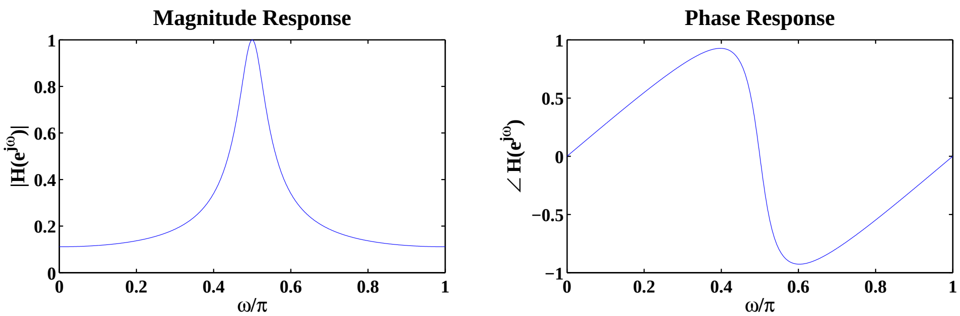


FIGURE 5.56: Magnitude and phase responses for $a = 0.8$.

(b) Solution:

(i) The input is:

$$x[n] = 3\cos(\pi n/2) = \frac{3}{2}(\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n})$$

The output is:

$$\begin{aligned} y[n] &= \frac{3}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n}H(\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}) + \frac{3}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n}H(\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}) \\ &= \frac{3}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n}\frac{b}{1-a} + \frac{3}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n}\frac{b}{1-a} = 3\cos\left(\frac{\pi}{2}n\right) \end{aligned}$$

(ii) The input is:

$$x[n] = 3\sin(\pi n/4) = \frac{3}{2\mathrm{j}}(\mathrm{e}^{\mathrm{j}\frac{\pi}{4}n} - \mathrm{e}^{-\mathrm{j}\frac{\pi}{4}n})$$

The output is:

$$
\begin{aligned}
y[n] &= \frac{3}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}\frac{\pi}{4}n}H(\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}) - \frac{3}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}n}H(\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}) \\
&= \frac{3}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}\frac{\pi}{4}n}\frac{b}{1-a\mathrm{j}} - \frac{3}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}n}\frac{b}{1+a\mathrm{j}} \\
&= \frac{1-a}{1+a^2}3\sin\frac{\pi}{4}n + \frac{a(1-a)}{1+a^2}3\cos\frac{\pi}{4}n
\end{aligned}
$$

(c) Solution:

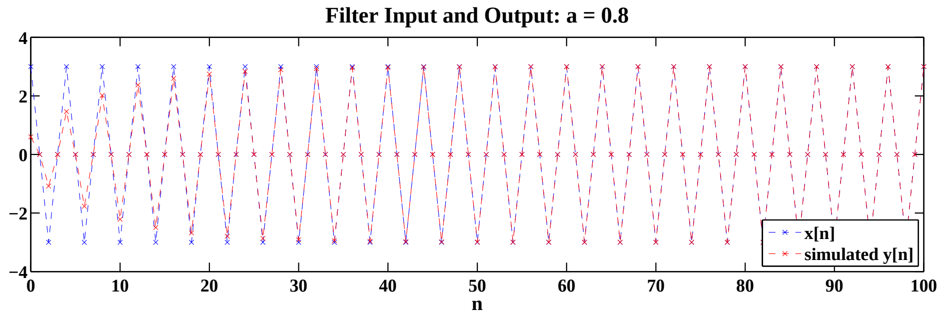


FIGURE 5.57: Plot for input $x[n]$ and output $y[n]$ when $x[n] = 3\cos(\pi n/2)$.

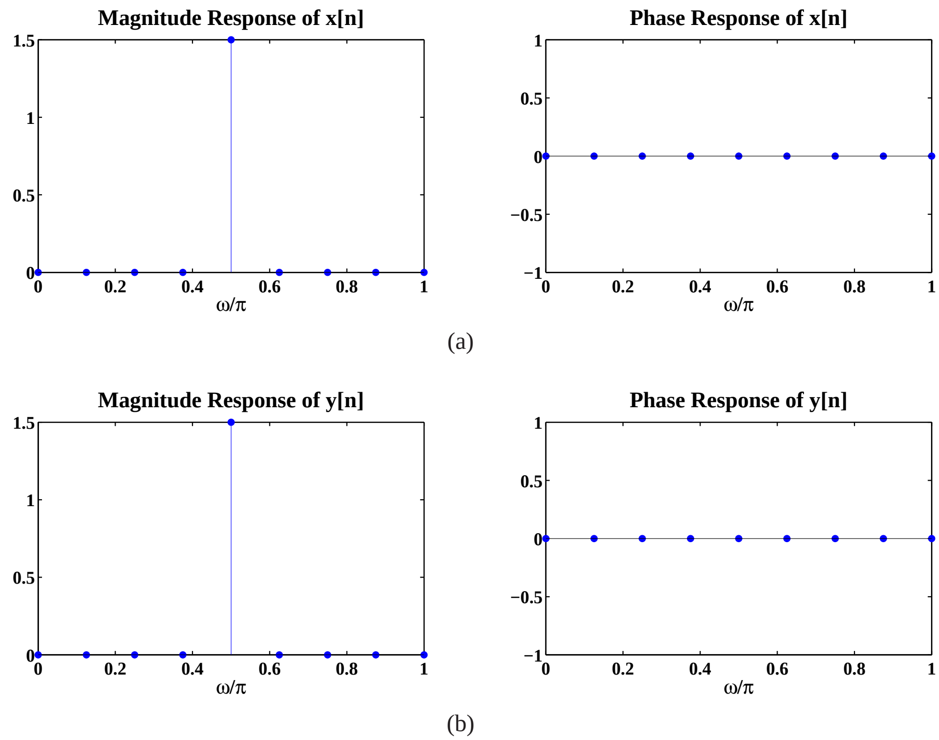


FIGURE 5.58: Magnitude and phase responses of (a) $x[n]$. (b) $y[n]$.

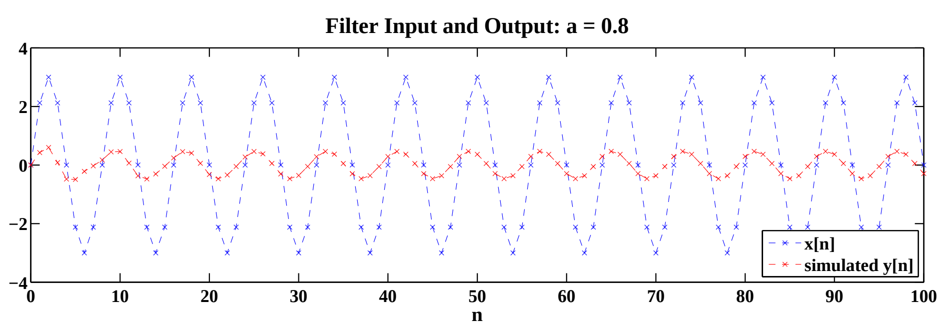


FIGURE 5.59: Plot for input $x[n]$ and output $y[n]$ when $x[n] = 3\sin(\pi n/4)$.

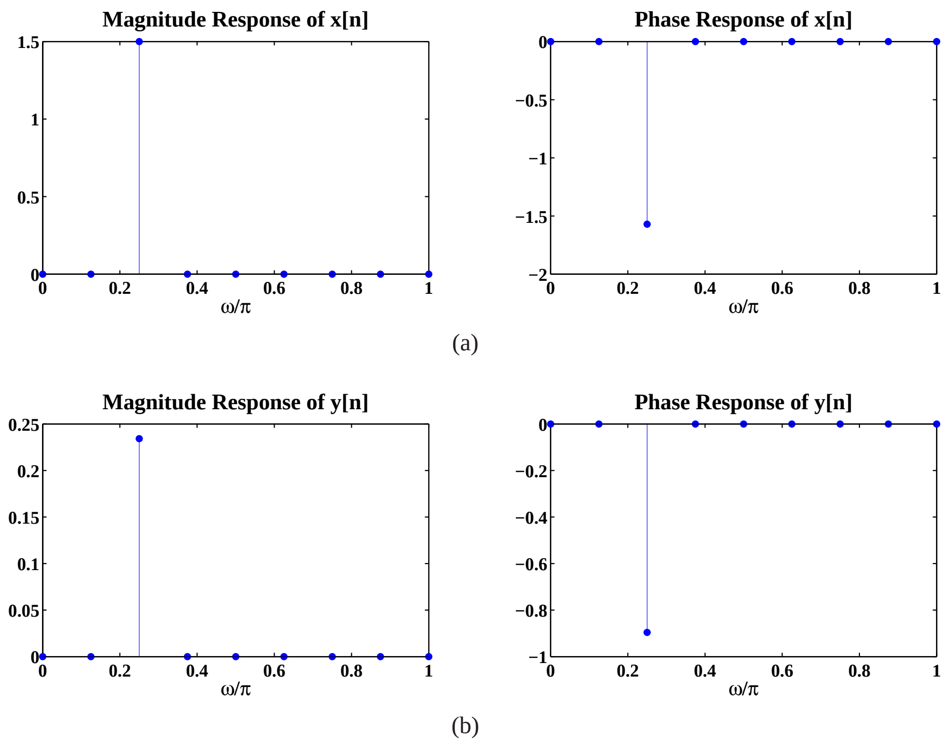


FIGURE 5.60: Magnitude and phase responses of (a) $x[n]$. (b) $y[n]$.

25. (a) Solution:

The frequency response is:

$$H(e^{j\omega}) = \frac{e^{j\frac{\pi}{6}}}{1-0.8e^{-j(\omega-\frac{\pi}{4})}} + \frac{e^{-j\frac{\pi}{6}}}{1-0.8e^{-j(\omega+\frac{\pi}{4})}} + \frac{5}{1+0.9e^{-j\omega}}$$

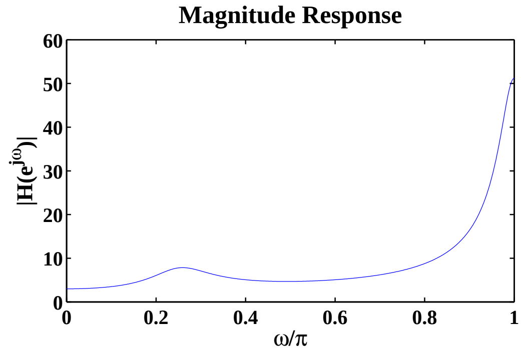


FIGURE 5.61: Magnitude response of the system.

(b) Graph the wrapped and the unwrapped phase responses in one plot.

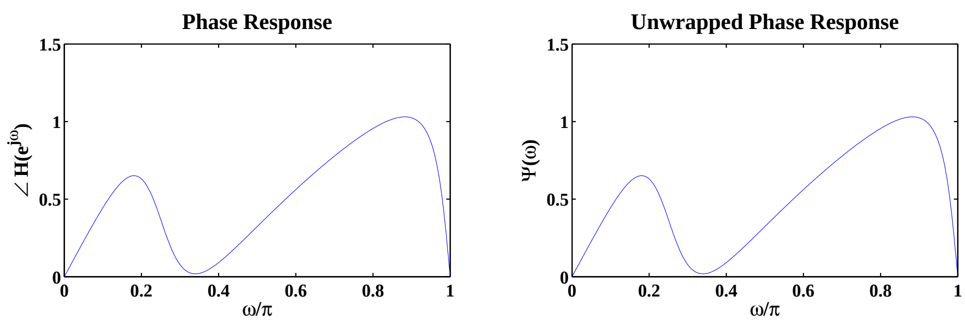


FIGURE 5.62: Wrapped and the unwrapped phase responses of the system.

(c) Solution:

The input is:

$$x[n] = 1e^{j0} + \frac{3}{2}(e^{j\frac{\pi}{6}}e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{6}}e^{-j\frac{\pi}{4}n}) + 5e^{-j\pi n}$$

The output is:

$$y[n] = H(e^{j0}) + \frac{3}{2}e^{j\frac{\pi}{6}}e^{j\frac{\pi}{4}n}H(e^{j\frac{\pi}{4}}) + \frac{3}{2}e^{-j\frac{\pi}{6}}e^{-j\frac{\pi}{4}n}H(e^{-j\frac{\pi}{4}}) + 5e^{-j\pi n}H(e^{-j\pi})$$

(d) Using MATLAB compute the steady-state response to $x[n]$ above and verify your result.

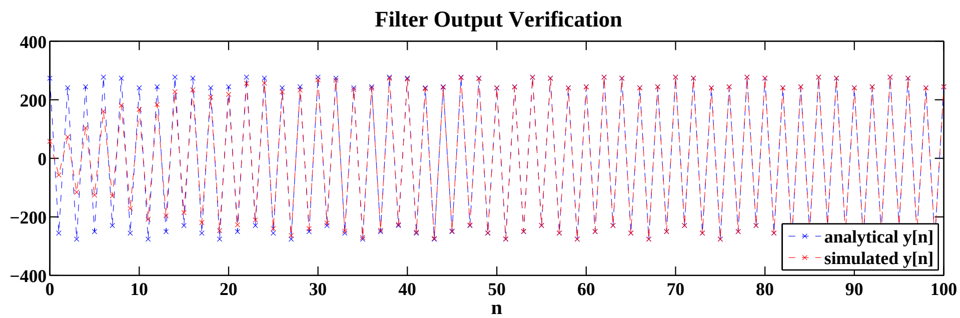


FIGURE 5.63: MATLAB verification of the steady-state response to $x[n]$.

26. (a) Solution:

$$c_k^{(x)} = \frac{1}{10}\sum_{n=0}^{9} 0.8^n \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}kn} = \frac{1}{10}\cdot\frac{1-0.8^{10}}{1-0.8\mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}k}}$$

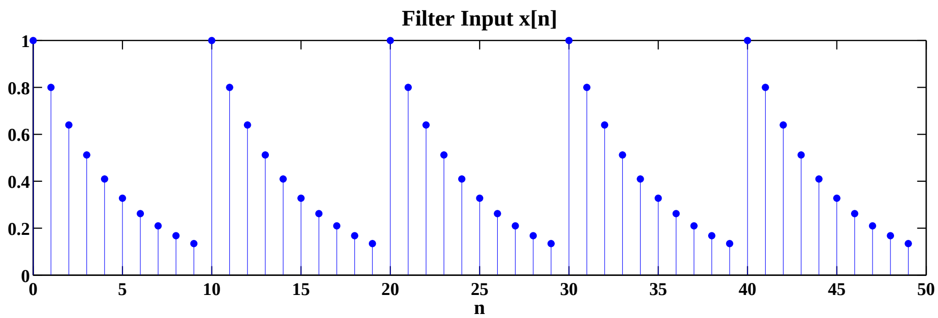
(b) Solution:

$$c_k^{(y)} = c_k^{(x)} H\big(\mathrm{e}^{\mathrm{j}\frac{2\pi}{10}k}\big)$$

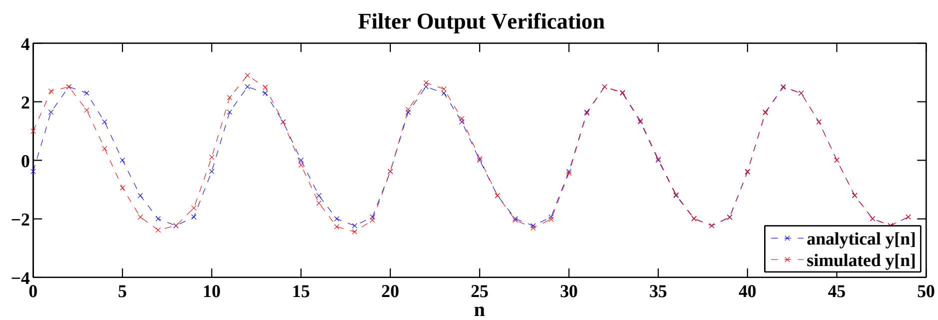
(c) Solution:

$$y_{\text{ss}}[n] = \sum_{n=0}^{9} c_k^{(x)} \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}kn} H\big(\mathrm{e}^{\mathrm{j}\frac{2\pi}{10}k}\big)$$

(d) See plot below.



(a)



(b)

FIGURE 5.64: Signal plots of (a) $x[n]$. (b) $y[n]$.

27. (a) Solution:

$$X(z) = \frac{1}{1-0.25z^{-1}} + \frac{1}{1-0.2z^{-1}}, \quad |z| > 0.25$$

$$Y(z) = \frac{1}{1-0.1z^{-1}}, \quad |z| > 0.1$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}\frac{1-0.45z^{-1}+0.05\mathrm{e}^{-2\mathrm{j}\omega}}{1-0.325z^{-1}+0.0225z^{-2}}, \quad |z| > 0.25$$

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1}{2}\frac{1-0.45\mathrm{e}^{-\mathrm{j}\omega}+0.05\mathrm{e}^{-2\mathrm{j}\omega}}{1-0.325\mathrm{e}^{-\mathrm{j}\omega}+0.0225\mathrm{e}^{-2\mathrm{j}\omega}}$$

The frequency response exists and is unique.

(b) Solution:
The frequency response does NOT exist since the frequency changes.

(c) Solution:
The frequency response exists but is NOT unique.

(d) Solution:

$$y[n] = x[n] - x[n-1] + \delta[n]$$

which is NOT LTI system.

28. (a) Solution:
The frequency response is:

$$\begin{aligned} H\big(e^{j\omega}\big) &= \frac{1}{2}(1 - 2e{-}\,j\omega + 4e{-}2j\omega - 2e{-}3j\omega + e{-}4j\omega) \\ &= \frac{1}{2}(1 - 2\cos\omega + 4\cos 2\omega - 2\cos 3\omega + \cos 4\omega) \\ &\quad + \frac{1}{2}j(2\sin\omega - 4\sin 2\omega + 2\sin 3\omega - \sin 4\omega) \end{aligned}$$

(b) See plot below.

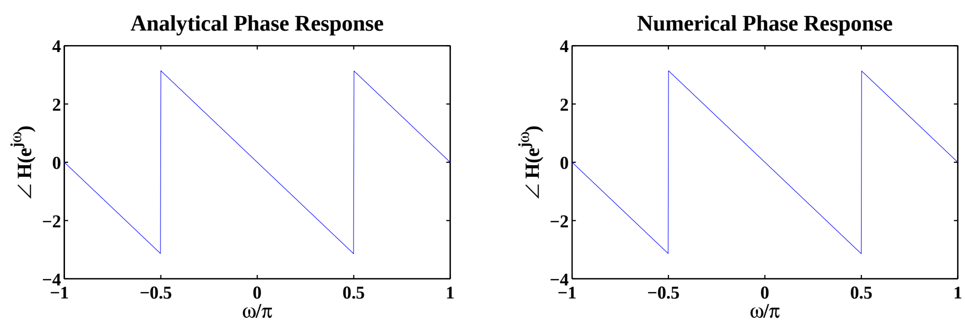


FIGURE 5.65: Phase response plot.

29. (a) Solution:
The frequency response is:

$$\begin{aligned} H\big(e^{j\omega}\big) &= \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} \\ &= e^{-0j\omega} - 2e^{-j\omega} + 3e^{-2j\omega} - 4e^{-3j\omega} + 0 + 4e^{-5j\omega} - 3e^{-6j\omega} + 2e^{-7j\omega} - e^{-8j\omega} \\ &= (1 - 2\cos\omega + 3\cos 2\omega - 4\cos 3\omega + 4\cos 5\omega - 3\cos 6\omega + 2\cos 7\omega - \cos 8\omega) \\ &\quad + j(2\sin\omega - 3\sin 2\omega + 4\sin 3\omega - 4\sin 5\omega + 3\sin 6\omega - 2\sin 7\omega + \sin 8\omega) \end{aligned}$$

The analytical phase response is:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) =$$
$$\tan^{-1}\frac{2\sin\omega - 3\sin 2\omega + 4\sin 3\omega - 4\sin 5\omega + 3\sin 6\omega - 2\sin 7\omega + \sin 8\omega}{1 - 2\cos\omega + 3\cos 2\omega - 4\cos 3\omega + 4\cos 5\omega - 3\cos 6\omega + 2\cos 7\omega - \cos 8\omega}$$

(b) See plot below.

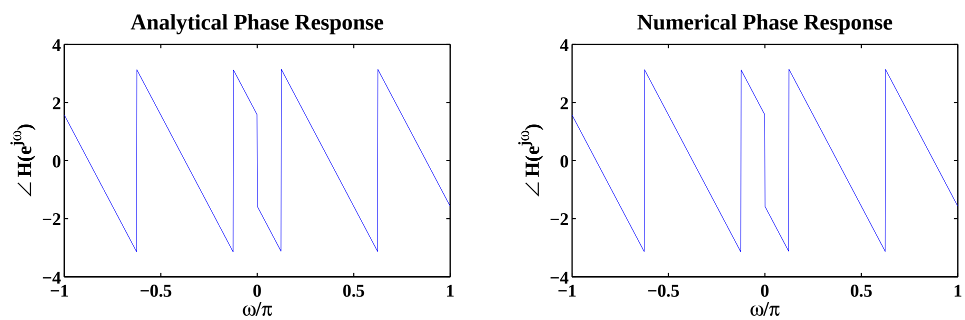


FIGURE 5.66: Phase response plot.

30. (a) See plot below.

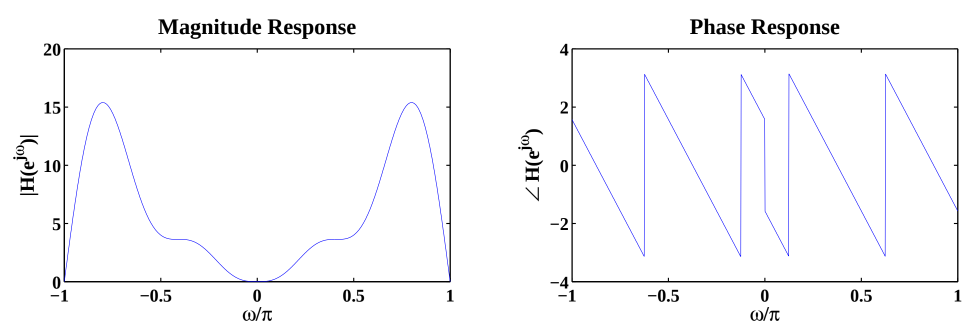


FIGURE 5.67: Magnitude and phase responses of the system.

(b) See plot below.

(c) See plot below.

(d) See plot below.

(e) See plot below.

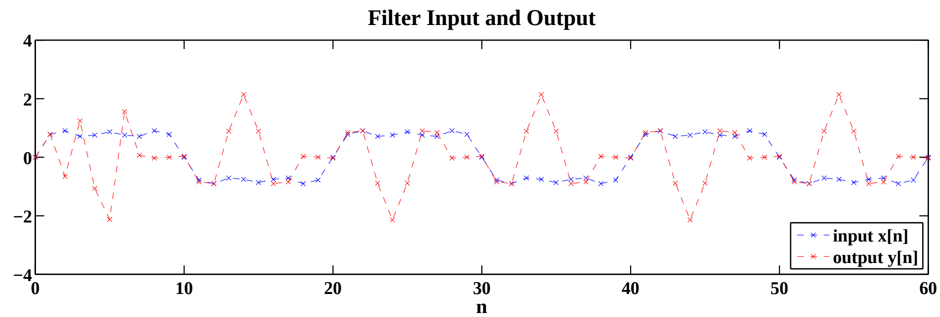


FIGURE 5.68: Plot of the input and steady state response.

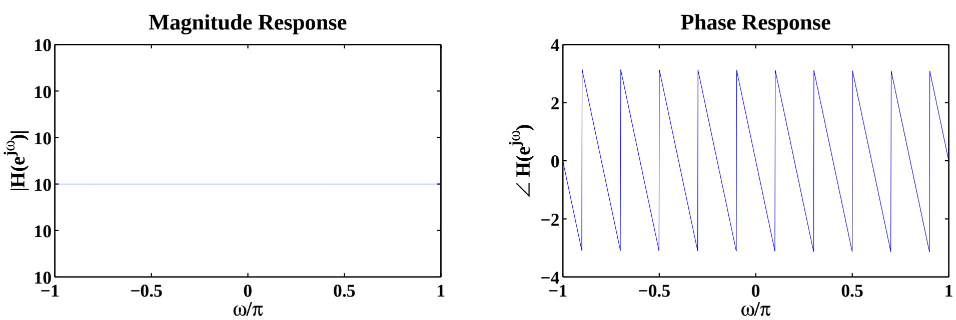


FIGURE 5.69: Magnitude and phase responses of the system.

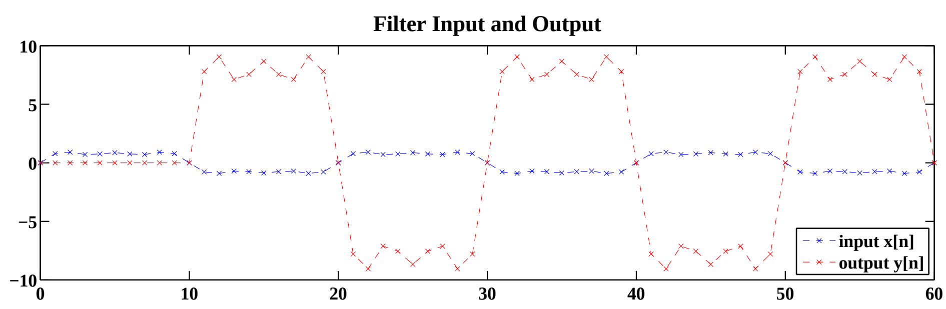


FIGURE 5.70: Plot of the input and steady state response.

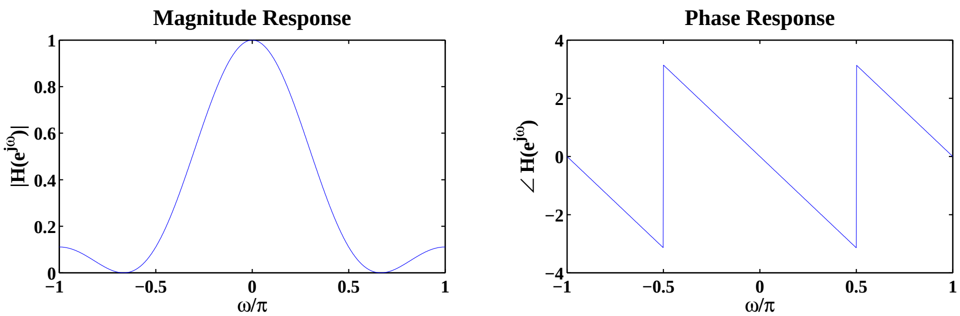


FIGURE 5.71: Magnitude and phase responses of the system.

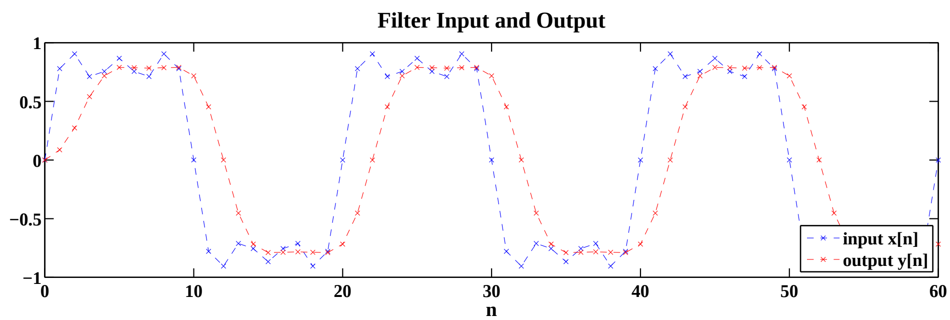


FIGURE 5.72: Plot of the input and steady state response.

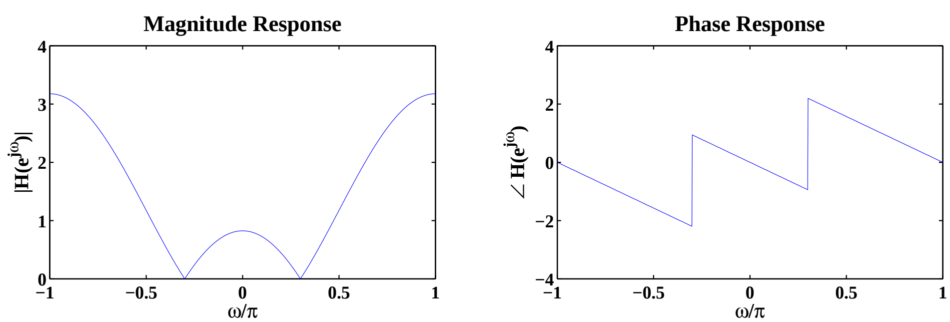


FIGURE 5.73: Magnitude and phase responses of the system.

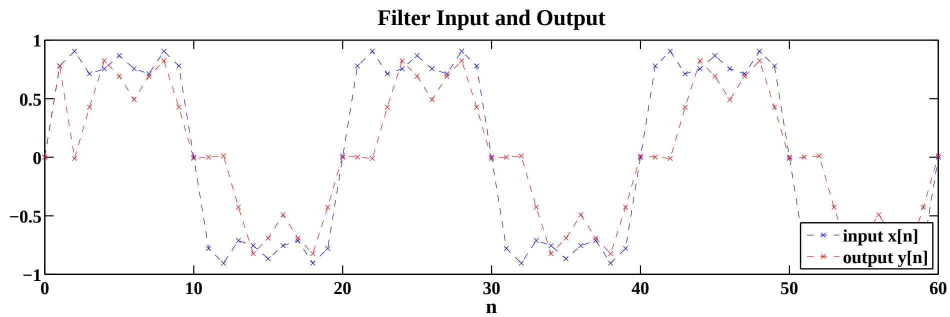


FIGURE 5.74: Plot of the input and steady state response.

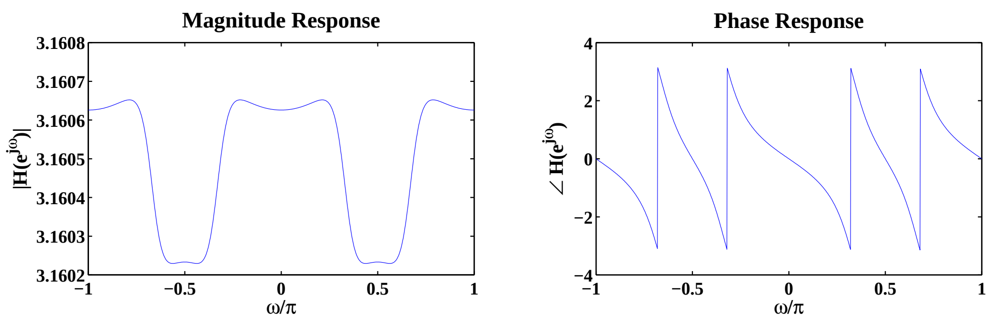


FIGURE 5.75: Magnitude and phase responses of the system.

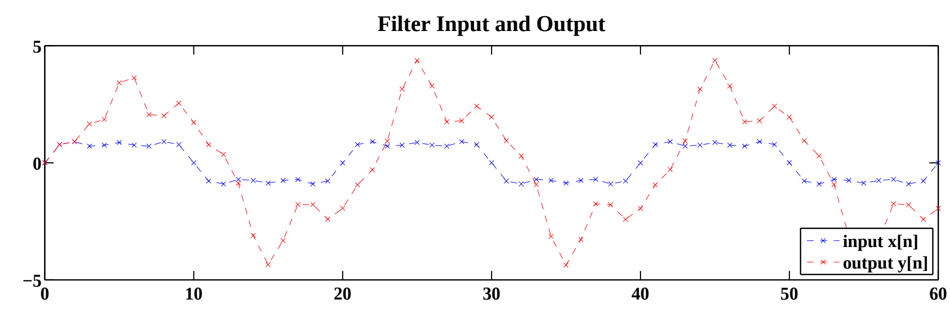


FIGURE 5.76: Plot of the input and steady state response.

31. (a) Solution:
The impulse response of the filter is:

$$
\begin{aligned}
h[n] &= \frac{1}{2\pi}\int_{2\pi} H(\mathrm{e}^{\mathrm{j}\omega})\,\mathrm{e}^{\mathrm{j}\omega n}\mathrm{d}\omega \\
&= \frac{1}{2\pi}\int_{-\frac{7\pi}{8}}^{-\frac{5\pi}{8}} 0.5\mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}\omega n}\mathrm{d}\omega + \frac{1}{2\pi}\int_{-\frac{2\pi}{8}}^{-\frac{\pi}{8}} \mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}\omega n}\mathrm{d}\omega \\
&+ \frac{1}{2\pi}\int_{\frac{\pi}{8}}^{\frac{2\pi}{8}} \mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}\omega n}\mathrm{d}\omega + \frac{1}{2\pi}\int_{\frac{7\pi}{8}}^{\frac{5\pi}{8}} 0.5\mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}\omega n}\mathrm{d}\omega \\
&= \frac{1}{2\pi}\frac{2}{n-n_d}\left[\sin\frac{2\pi}{8}(n-n_d) - \sin\frac{\pi}{8}(n-n_d)\right] \\
&\quad + \frac{1}{2\pi}\frac{1}{n-n_d}\left[\sin\frac{7\pi}{8}(n-n_d) - \sin\frac{5\pi}{8}(n-n_d)\right]
\end{aligned}
$$

(b) See plot below.

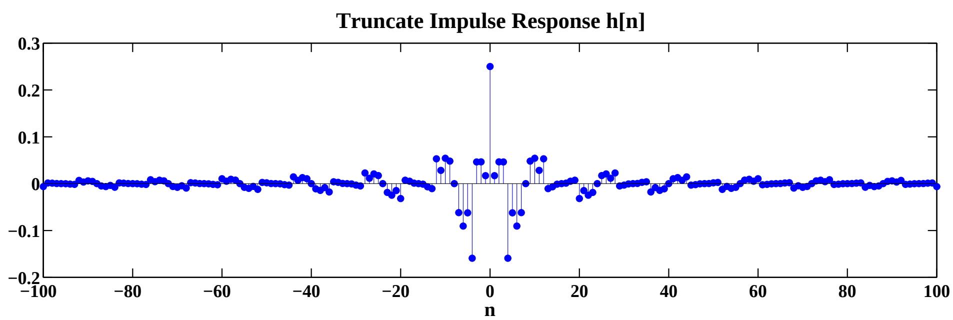


FIGURE 5.77: Impulse response for $n_d = 0$ for $-100 \le n \le 100$.

(c) See plot below.
MATLAB script:

```
% P0531: Multiband Ideal Bandpass Filter
close all; clc
n = -100:100;
nd = 0;
hn = (2./(n-nd).*(sin(2*pi/8*(n-nd))-sin(pi/8*(n-nd)))+...
    1./(n-nd).*(sin(7*pi/8*(n-nd))-sin(5*pi/8*(n-nd))))/2/pi;
hn(n==0) = (2*pi/8+2*pi/8)/2/pi;
w = linspace(-1,1,1000)*pi;
```

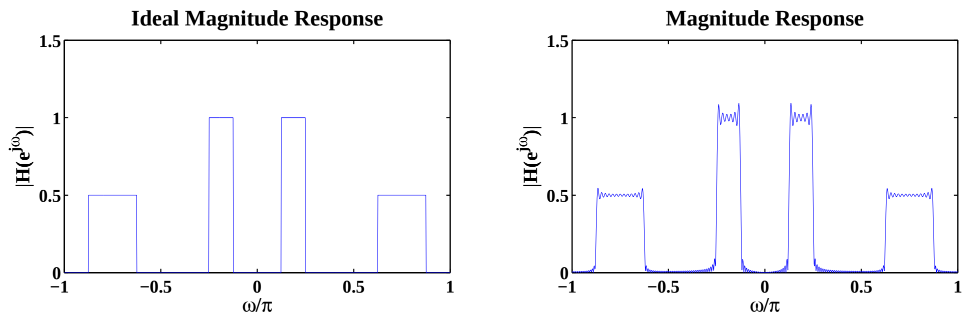


FIGURE 5.78: Magnitude response of the filter using MATLAB and the ideal filter magnitude response.

```
H_i = zeros(size(w));
ind = abs(w)>pi/8 & abs(w)<2*pi/8;
H_i(ind) = exp(-j*w(ind)*nd);
ind = abs(w)>5*pi/8 & abs(w)<7*pi/8;
H_i(ind) = 0.5*exp(-j*w(ind)*nd);
H = freqz(hn,1,w);

%% Plot:
hfa = figconfg('P0531a','long');
stem(n,hn,'filled')
xlabel('n','fontsize',LFS)
title('Truncate Impulse Response h[n]','fontsize',TFS)

hfb = figconfg('P0530b','long');
subplot(121)
plot(w/pi,abs(H_i))
ylim([0 1.5])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Ideal Magnitude Response','fontsize',TFS)
subplot(122)
plot(w/pi,abs(H))
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
```

32. See plot below.

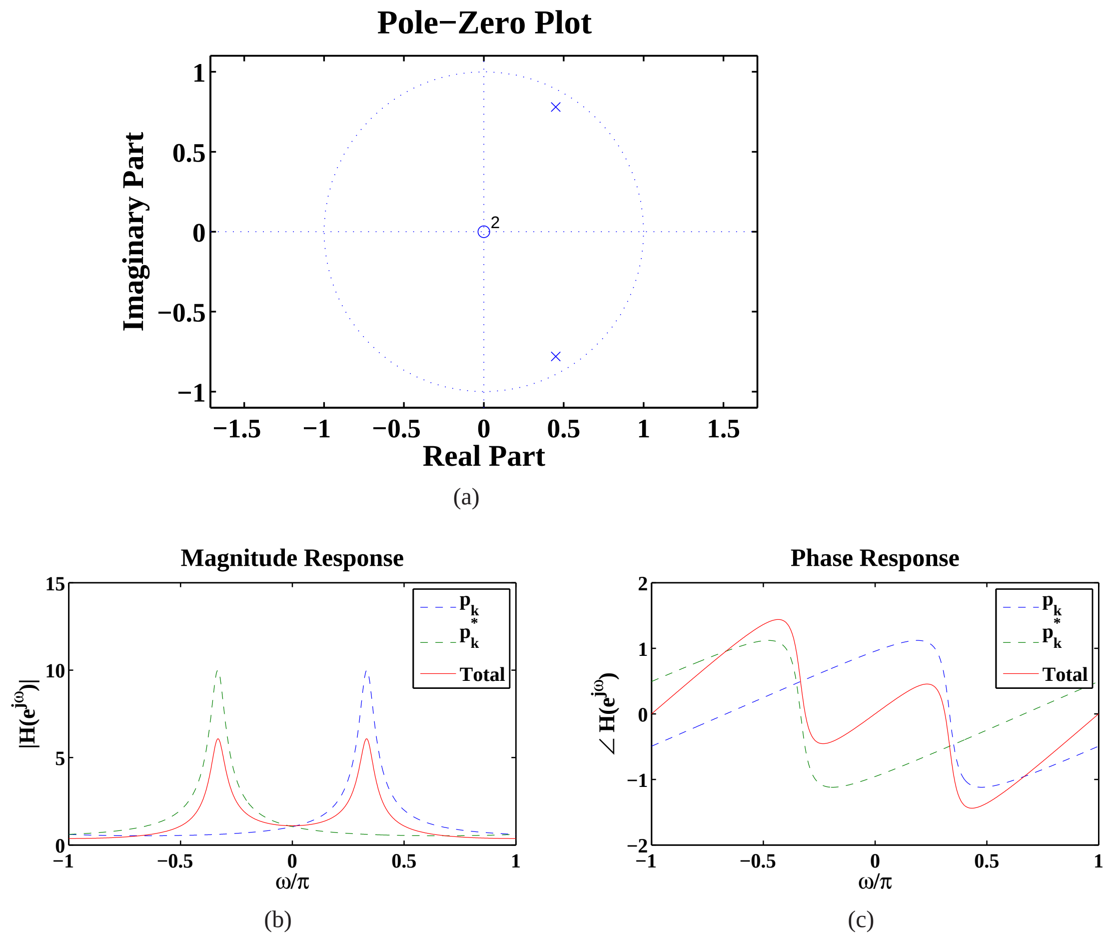


FIGURE 5.79: Figure 5.19 reproduction.

33. (a) See plot below.
    (b) See plot below.
    (c) See plot below.
    MATLAB script:

```
function [mag,pha,omega] = myfreqz(b,a)
% Function implements equations (5.87) and (5.88)
p = roots(a);
p = p(:);
```

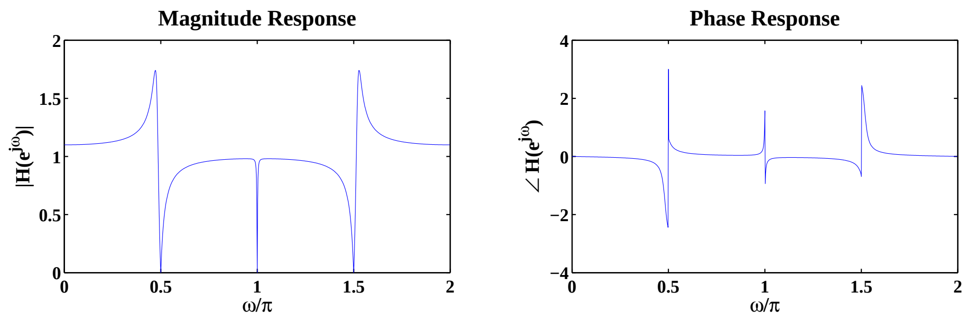


FIGURE 5.80: Magnitude and phase responses of the system using MATLAB function freqz.

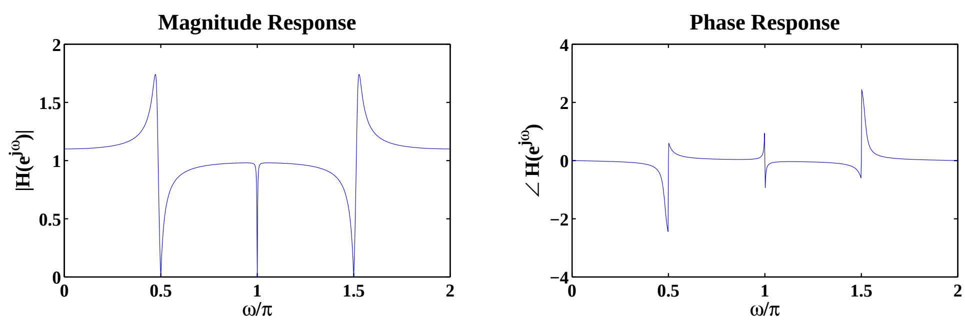


FIGURE 5.81: Magnitude and phase responses of the system using MATLAB function freqz0.

```
N = size(p,1);
r = abs(p);
phi = angle(p);

z = roots(b);
z = z(:);
M = size(z,1);
q = abs(z);
theta = angle(z);

b0 = b(1);

K = 1024;
```

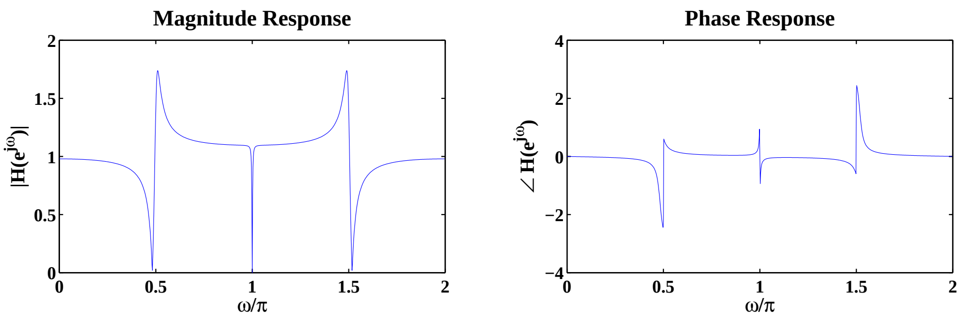


FIGURE 5.82: Magnitude and phase responses of the system using MATLAB function `myfreqz`.

```
omega = (0:K-1)/K*2*pi;

r_epd = repmat(r,1,K);
phi_epd = repmat(phi,1,K);
q_epd = repmat(q,1,K);
theta_epd = repmat(theta,1,K);
temp1 = cos(repmat(omega,N,1)-phi_epd);
temp2 = cos(repmat(omega,M,1)-theta_epd);
temp3 = sin(repmat(omega,N,1)-phi_epd);
temp4 = sin(repmat(omega,M,1)-theta_epd);

mag = abs(b0)*prod(sqrt(1+q_epd.^2-2*q_epd.*temp1))./...
    prod(sqrt(1+r_epd.^2-2*r_epd.*temp2));
pha = angle(b0) + sum(atan2(q_epd.*temp4,1-q_epd.*temp2)) - ...
    sum(atan2(r_epd.*temp3,1-r_epd.*temp1));
```

34. (a) See plot below.
    (b) See plot below.
    (c) See plot below.
    MATLAB script:

```
function [grp,omega] = mygrpdelay(b,a)
% Implement equation (5.89) to compute group delay
p = roots(a);
p = p(:);
N = size(p,1);
r = abs(p);
```

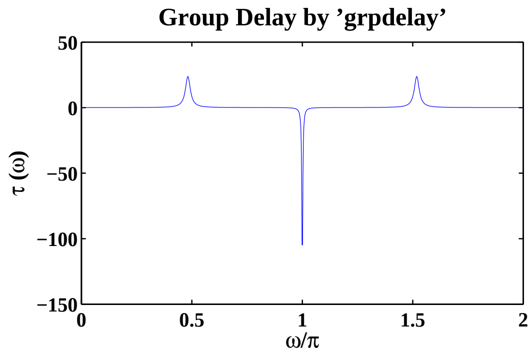


FIGURE 5.83: Group delay of the system using MATLAB function `grpdelay`.

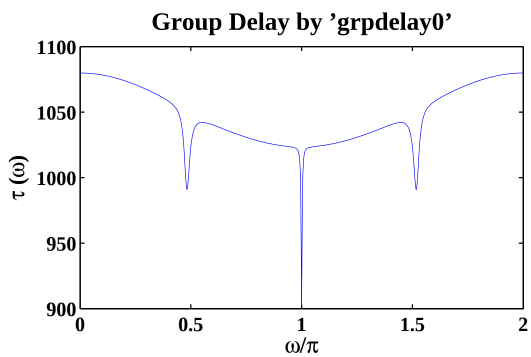


FIGURE 5.84: Group delay of the system using MATLAB function `grpdelay0`.

```
phi = angle(p);

z = roots(b);
z = z(:);
M = size(z,1);
q = abs(z);
theta = angle(z);

K = 1024;
omega = 2*pi*(0:K-1)/K;

r_epd = repmat(r,1,K);
phi_epd = repmat(phi,1,K);
```

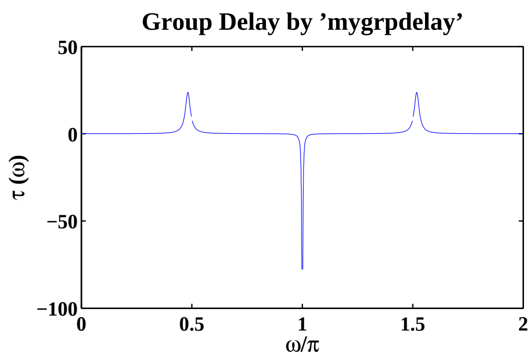


FIGURE 5.85: Group delay of the system using MATLAB function `mygrpdelay`.

```
q_epd = repmat(q,1,K);
theta_epd = repmat(theta,1,K);
temp1 = cos(repmat(omega,N,1)-phi_epd);
temp2 = cos(repmat(omega,M,1)-theta_epd);

grp = sum((r_epd.^2-r_epd.*temp1)./(1+r_epd.^2-2*r_epd.*temp1),1);
grp = -grp + sum((q_epd.^2-q_epd.*temp2)./(1+q_epd.^2-2*q_epd.*temp2),1);
```

35. (a) Solution:

$$H(z) = \frac{Y(z)}{X(z)} = 1 - \alpha \cdot z^{-1}$$

$$H(e^{j\omega}) = 1 - \alpha \cdot e^{-j\omega} = 1 - \alpha\cos\omega + j\alpha\sin\omega$$

$$\angle H(e^{j\omega}) = \tan^{-1}\frac{\alpha\sin\omega}{1-\alpha\cos\omega}$$

$$\begin{aligned}\tau_{\mathrm{gd}}(\omega) &= \frac{-\mathrm{d}\angle H(e^{j\omega}) + 2k\pi}{\mathrm{d}\omega} \\ &= -\frac{1}{1+\left(\frac{\alpha\sin\omega}{1-\alpha\cos\omega}\right)^2} \cdot \frac{\alpha\cos\omega(1-\alpha\cos\omega) - \alpha\sin\omega\cdot\alpha\sin\omega}{(1-\alpha\cos\omega)^2} \\ &= \frac{\alpha^2 - \alpha\cos\omega}{1+\alpha^2 - 2\alpha\cos\omega}\end{aligned}$$

(b) Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\alpha\cdot z^{-1}}$$

$$H\left(e^{j\omega}\right) = \frac{1}{1 - \alpha \cdot e^{-j\omega}} = \frac{1}{1 - \alpha\cos\omega + j\alpha\sin\omega}$$

$$\angle H\left(e^{j\omega}\right) = -\tan^{-1}\frac{\alpha\sin\omega}{1 - \alpha\cos\omega}$$

$$\begin{aligned}
\tau_{\text{gd}}(\omega) &= \frac{-\mathrm{d}\angle H\left(e^{j\omega}\right) + 2k\pi}{\mathrm{d}\omega} \\
&= \frac{1}{1 + \left(\frac{\alpha\sin\omega}{1-\alpha\cos\omega}\right)^2} \cdot \frac{\alpha\cos\omega(1 - \alpha\cos\omega) - \alpha\sin\omega\cdot\alpha\sin\omega}{(1 - \alpha\cos\omega)^2} \\
&= \frac{-\alpha^2 + \alpha\cos\omega}{1 + \alpha^2 - 2\alpha\cos\omega}
\end{aligned}$$

(c) Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 2\alpha\cos\phi z^{-1} + \alpha^2 z^{-2}}$$

$$\begin{aligned}
H\left(e^{j\omega}\right) &= \frac{1}{1 - 2\alpha\cos\phi e^{-j\omega} + \alpha^2 e^{-2j\omega}} \\
&= \frac{1}{(1 - 2\alpha\cos\phi\cos\omega + \alpha^2\cos 2\omega) + j(2\alpha\cos\phi\sin\omega - \alpha^2\sin 2\omega)}
\end{aligned}$$

$$\angle H\left(e^{j\omega}\right) = -\tan^{-1}\frac{2\alpha\cos\phi\sin\omega - \alpha^2\sin 2\omega}{1 - 2\alpha\cos\phi\cos\omega + \alpha^2\cos 2\omega}$$

$$\tau_{\text{gd}}(\omega) = \frac{-\mathrm{d}\angle H\left(e^{j\omega}\right) + 2k\pi}{\mathrm{d}\omega}$$

$$= \frac{(2\alpha\cos\phi\cos\omega - 2\alpha^2\cos 2\omega)(1 - 2\alpha\cos\phi\cos\omega + \alpha^2\cos 2\omega) - (2\alpha\cos\phi\sin\omega - \alpha^2\sin 2\omega)(2\alpha\cos\phi\sin\omega - 2\alpha^2\sin 2\omega)}{(2\alpha\cos\phi\sin\omega - \alpha^2\sin 2\omega)^2 + (1 - 2\alpha\cos\phi\cos\omega + \alpha^2\cos 2\omega)^2}$$

$$= \frac{4\alpha^3\cos\phi\cos\omega - 2\alpha^4 - 4\alpha^2\cos^2\phi + 2\alpha^3\sin\omega\sin 2\omega + 2\alpha\cos\phi\cos\omega + 2\alpha^2\cos\phi\cos\omega\cos 2\omega - 2\alpha^2\cos 2\omega}{4\alpha^2\cos^2\phi + 1 + \alpha^4 - 4\alpha^3\cos\phi\cos\omega - 4\alpha\cos\phi\cos\omega + 2\alpha^2\cos 2\omega}$$

36. (a) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}(1 + z^{-1})$$

The frequency response is:

$$H(e^{j\omega}) = \frac{1}{2}(1 + e^{-j\omega}) = \frac{1}{2}(1 + \cos\omega - j\sin\omega)$$

The magnitude response is:

$$|H(e^{j\omega})| = \frac{1}{2}\sqrt{(1 + \cos\omega)^2 + \sin^2\omega} = |\cos\frac{\omega}{2}|$$

The phase response is:

$$\angle H(e^{j\omega}) = \tan^{-1}\frac{-\sin\omega}{1 + \cos\omega} = -\tan^{-1}\tan\frac{\omega}{2} = -\frac{\omega}{2}$$

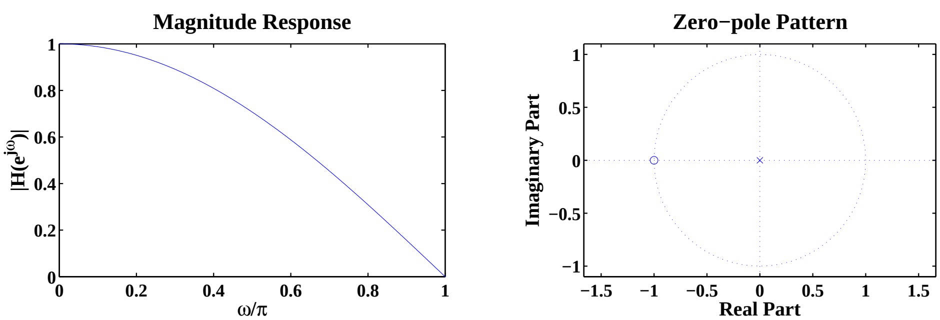


FIGURE 5.86: Pole-zero pattern and magnitude response of the system.

(b) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}(1 + z^{-2})$$

The frequency response is:

$$H(e^{j\omega}) = \frac{1}{2}(1 + e^{-2j\omega}) = \frac{1}{2}(1 + \cos 2\omega - j\sin 2\omega)$$

The magnitude response is:

$$|H(e^{j\omega})| = \frac{1}{2}\sqrt{(1+\cos 2\omega)^2 + \sin^2 2\omega} = |\cos\omega|$$

The phase response is:

$$\angle H(e^{j\omega}) = \tan^{-1}\frac{-\sin 2\omega}{1+\cos 2\omega} = -\tan^{-1}\tan\omega = -\omega$$

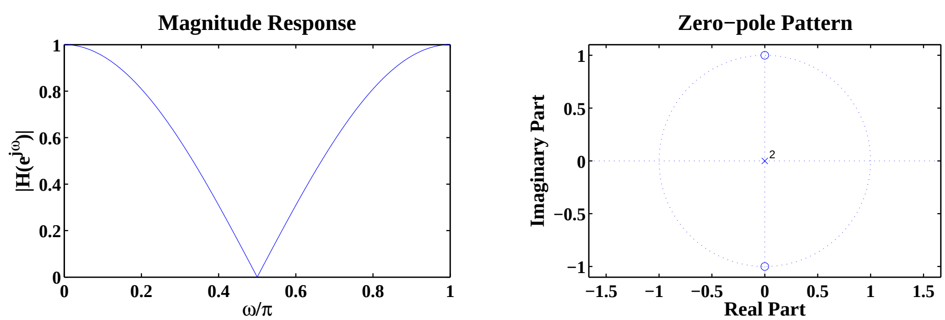


FIGURE 5.87: Pole-zero pattern and magnitude response of the system.

(c) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 - z^{-1} + z^{-2} - z^{-3})$$

The frequency response is:

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{4}(1 - e^{-j\omega} + e^{-2j\omega} - e^{-3j\omega}) \\ &= \frac{1}{4}\left[(1 - \cos\omega + \cos 2\omega - \cos 3\omega) - j(\sin\omega - \sin 2\omega + \sin 3\omega)\right] \end{aligned}$$

The magnitude response is:

$$\begin{aligned} |H(e^{j\omega})| &= \frac{1}{4}\sqrt{(1 - \cos\omega + \cos 2\omega - \cos 3\omega)^2 + (\sin\omega - \sin 2\omega + \sin 3\omega)^2} \\ &= \frac{1}{4}\sqrt{4 - 6\cos\omega + 4\cos 2\omega - 2\cos 3\omega} \end{aligned}$$

The phase response is:

$$\angle H(e^{j\omega}) = \tan^{-1}\frac{\sin\omega - \sin 2\omega + \sin 3\omega}{1 - \cos\omega + \cos 2\omega - \cos 3\omega} = -\tan^{-1}\frac{\sin 2\omega(2\cos\omega - 1)}{1 + \cos 2\omega(1 - 2\cos\omega)}$$

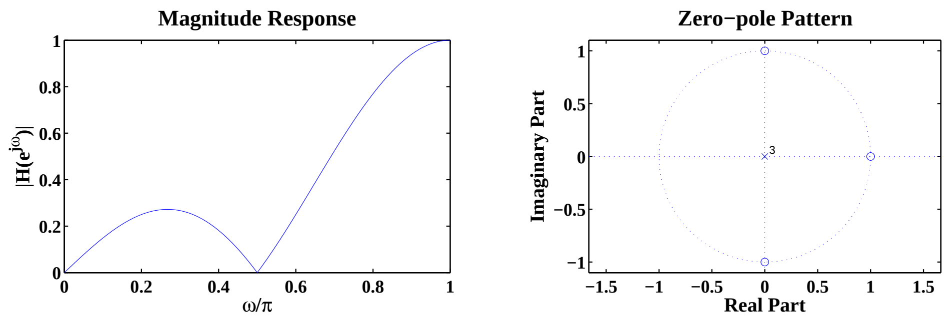


FIGURE 5.88: Pole-zero pattern and magnitude response of the system.

(d) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 - z^{-1} + z^{-3} - z^{-4})$$

The frequency response is:

$$\begin{aligned} H\big(e^{j\omega}\big) &= \frac{1}{4}(1 - e^{-j\omega} + e^{-3j\omega} - e^{-4j\omega}) \\ &= \frac{1}{4}\left[(1 - \cos\omega + \cos 3\omega - \cos 4\omega) - j(\sin\omega - \sin 3\omega + \sin 4\omega)\right] \end{aligned}$$

The magnitude response is:

$$\begin{aligned} |H\big(e^{j\omega}\big)| &= \frac{1}{4}\sqrt{(1 - \cos\omega + \cos 3\omega - \cos 4\omega)^2 + (\sin\omega - \sin 3\omega + \sin 4\omega)^2} \\ &= \frac{1}{4}\sqrt{4 - 4\cos\omega - 2\cos 2\omega + 4\cos 3\omega - 2\cos 4\omega} \end{aligned}$$

The phase response is:

$$\angle H\big(e^{j\omega}\big) = \tan^{-1}\frac{\sin\omega - \sin 3\omega + \sin 4\omega}{1 - \cos\omega + \cos 3\omega - \cos 4\omega}$$

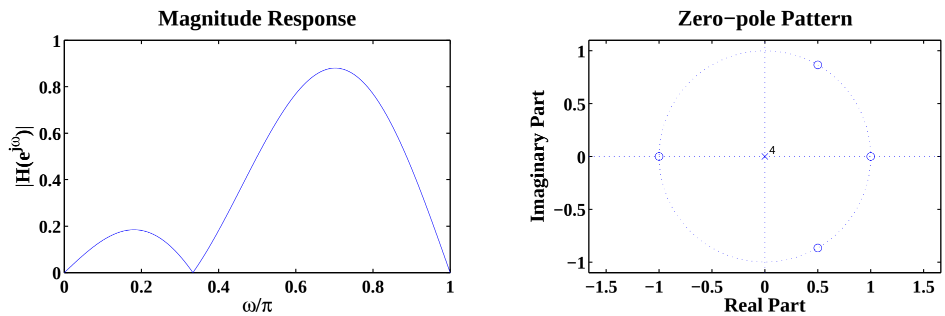


FIGURE 5.89: Pole-zero pattern and magnitude response of the system.

37. (a) Solution:

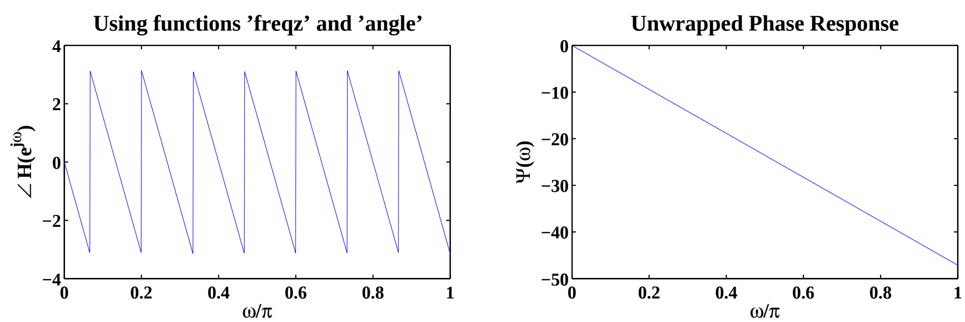


FIGURE 5.90: Phase response of pure delay $y[n] = x[n-15]$ using the functions freqz, angle and unwrap.

(b) Solution:

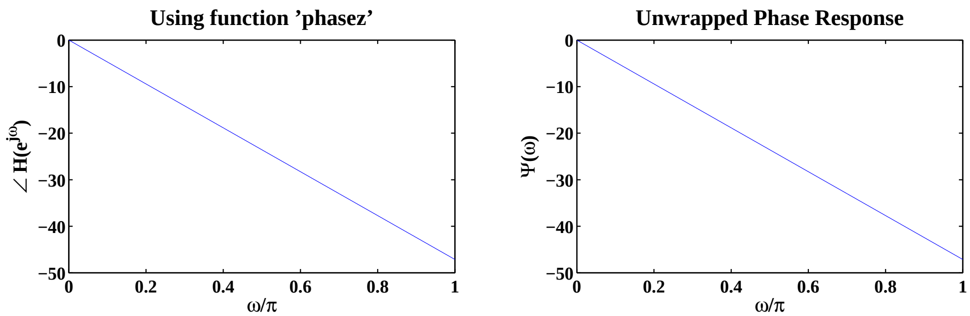


FIGURE 5.91: Phase response of pure delay $y[n] = x[n-15]$ using the function phasez.

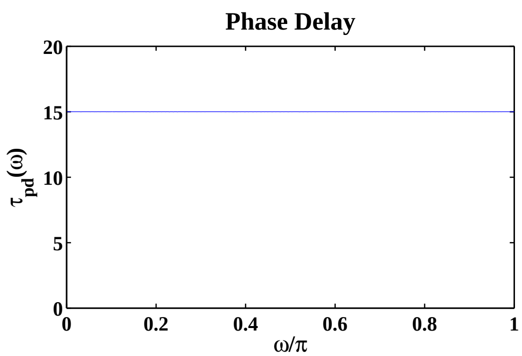


FIGURE 5.92: Phase delay of pure delay $y[n] = x[n-15]$ using the function phasedelay.

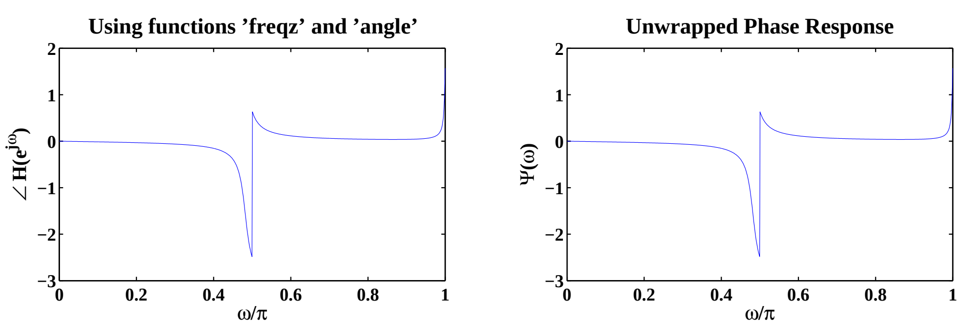


FIGURE 5.93: Phase response of $H(z)$ using the functions freqz, angle and unwrap.

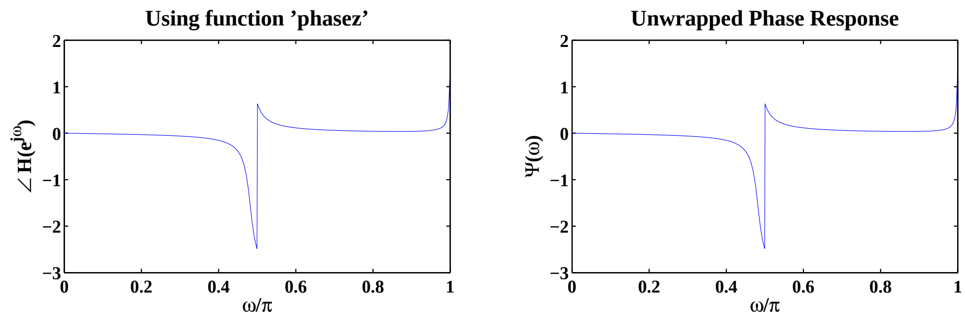


FIGURE 5.94: Phase response of $H(z)$ using the function phasez.

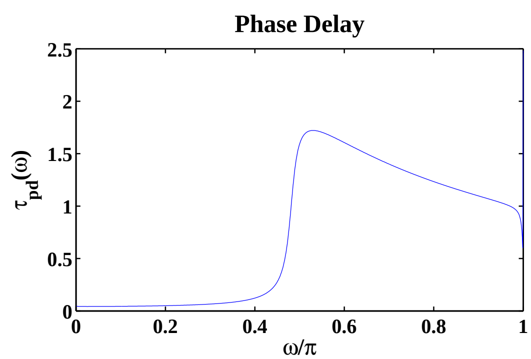


FIGURE 5.95: Phase delay of $H(z)$ using the function phasedelay.

38. (a) Solution:

$$\text{zeros:} \quad z_1 = \mathrm{e}^{\mathrm{j}0} = 1, \quad z_2 = \mathrm{e}^{\mathrm{j}\pi} = -1$$

$$\text{poles:} \quad p_1 = r\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}, \quad p_2 = r\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}, \quad r \in (0,1)$$

The system function is:

$$H(z) = b_0 \frac{(1-z^{-1})(1+z^{-1})}{(1-r\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}z^{-1})(1-r\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}z^{-1})}$$

The frequency response is:

$$H\big(\mathrm{e}^{\mathrm{j}\omega}\big) = b_0 \frac{(1-\mathrm{e}^{-\mathrm{j}\omega})(1+\mathrm{e}^{-\mathrm{j}\omega})}{(1-r\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}\mathrm{e}^{-\mathrm{j}\omega})(1-r\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}\mathrm{e}^{-\mathrm{j}\omega})}$$

Constrain $|H(\mathrm{e}^{\mathrm{j}\omega})|_{\max} = 1$, we have

$$|b_0| = \frac{(1-r)\sqrt{1+r^2}}{\sqrt{2}}$$

Choose $r = 0.95$.

(b) See plot below.

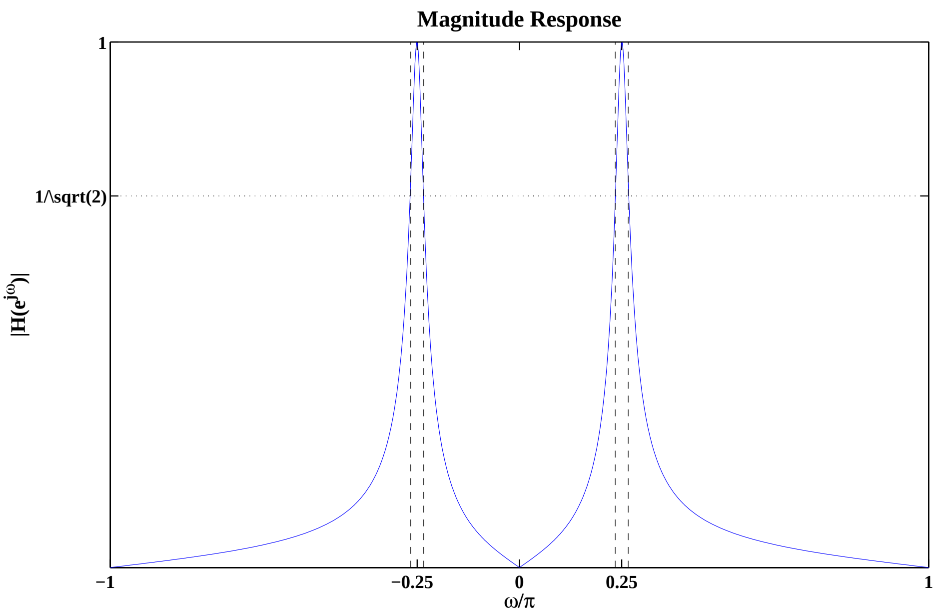


FIGURE 5.96: Magnitude response of the filter.

(c) See plot below.

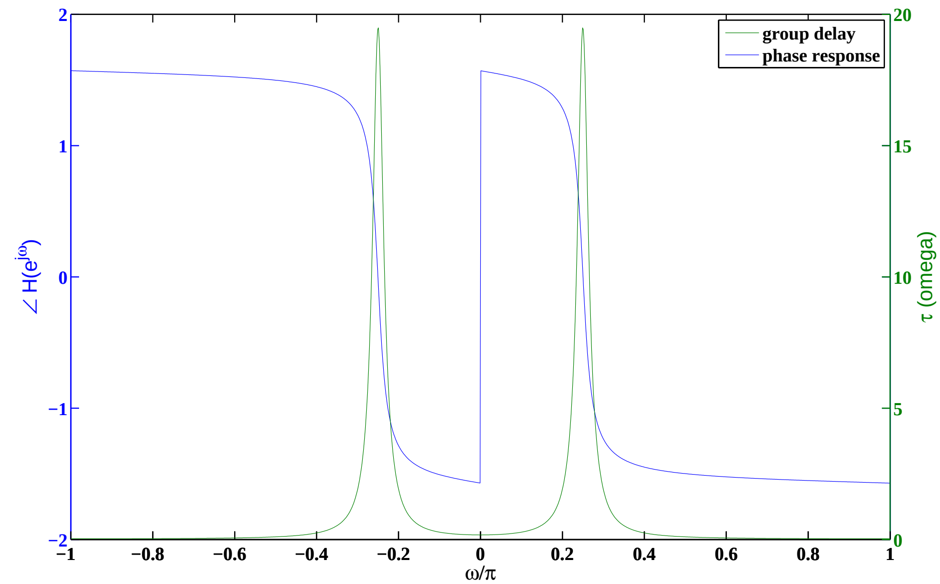


FIGURE 5.97: Phase and group-delay responses of the filter.

39. (a) Solution:

The frequency response is:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = b_0[1-(2r\cos\phi)\mathrm{e}^{-\mathrm{j}\omega}+r^2\mathrm{e}^{-2\mathrm{j}\omega}]$$
$$= b_0[(1-2r\cos\phi\cos\omega+r^2\cos 2\omega)+\mathrm{j}(2r\cos\phi\sin\omega-r^2\sin 2\omega)]$$

The magnitude response is:

$$|H(\mathrm{e}^{\mathrm{j}\omega})| = |b_0|\sqrt{(1-2r\cos\phi\cos\omega+r^2\cos 2\omega)^2+(2r\cos\phi\sin\omega-r^2\sin 2\omega)^2}$$

Constrain $|H(\mathrm{e}^{\mathrm{j}\omega})|_{\mathrm{max}}=1$, we have

$$|b_0| = \frac{1}{\sqrt{1+4r^2\cos^2\phi+r^4+4r^2\cos\phi+4r\cos\phi+2r^2}}$$

(b) See plot below.

(c) Solution:

$$H_k(\mathrm{e}^{\mathrm{j}\omega}) = 1-(2\cos\phi_k)\mathrm{e}^{-\mathrm{j}\omega}+\mathrm{e}^{-2\mathrm{j}\omega}$$
$$= (1-2\cos\phi_k\cos\omega+\cos 2\omega)+\mathrm{j}(2\cos\phi_k\sin\omega-\sin 2\omega)$$

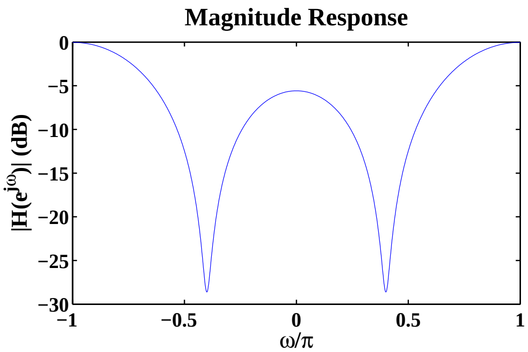


FIGURE 5.98: Magnitude response of the notch filter for $r = 0.95$.

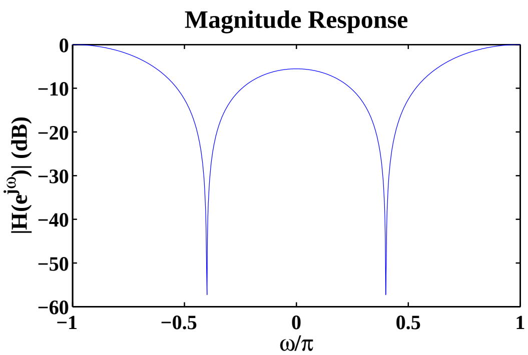


FIGURE 5.99: Magnitude response of the notch filter for $r = 1$.

$$|H_k(\mathrm{e}^{\mathrm{j}\omega})| = \sqrt{2 + 4\cos^2\phi_k - 4\cos\phi_k\cos\omega + 2\cos 2\omega}$$

$$|H_k(\mathrm{e}^{\mathrm{j}\omega})|_{\max} = 2(1 + \cos\phi_k)$$

$$|b_0| = \frac{1}{\prod_{k=-1}^{1} 2(1 + \cos\phi_k)}$$

(d) Solution:

$$|b_0| = \frac{1}{\prod_{k=-2}^{2} 2(1 + \cos\phi_k)}$$

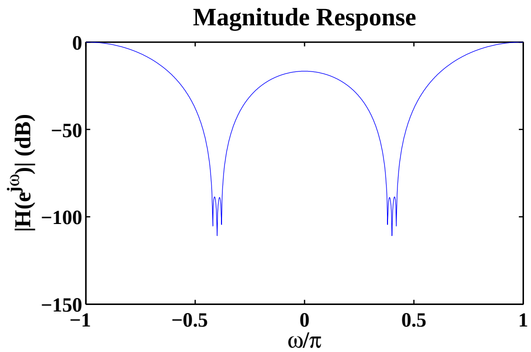


FIGURE 5.100: Magnitude response of cascade of three FIR notch filters for $r = 0.95$.

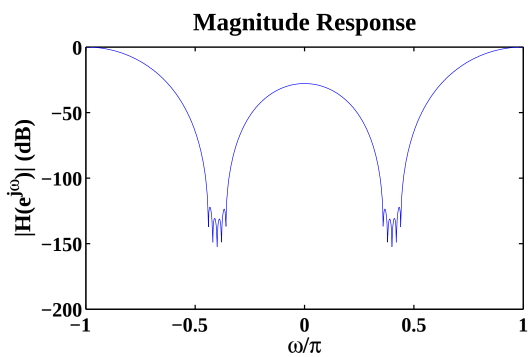


FIGURE 5.101: Magnitude response of cascade of five FIR notch filters for $r = 0.95$.

40. (a) Solution:

$$\text{zeros:}\quad z_1 = \mathrm{e}^{\mathrm{j}\frac{2\pi}{3}},\quad z_2 = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}$$

$$\text{poles:}\quad p_1 = r\mathrm{e}^{\mathrm{j}(\frac{2\pi}{3}+\phi)},\quad p_2 = r\mathrm{e}^{-\mathrm{j}(\frac{2\pi}{3}+\phi)},\quad r \in (0,1)$$

The system function is:

$$H(z) = b_0\frac{(1-\mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}z^{-1})(1-\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}z^{-1})}{(1-r\mathrm{e}^{\mathrm{j}(\frac{2\pi}{3}+\phi)}z^{-1})(1-r\mathrm{e}^{-\mathrm{j}(\frac{2\pi}{3}+\phi)}z^{-1})}$$

The frequency response is:

$$H\big(\mathrm{e}^{\mathrm{j}\omega}\big) = b_0\frac{(1-\mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}\mathrm{e}^{-\mathrm{j}\omega})(1-\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}\mathrm{e}^{-\mathrm{j}\omega})}{(1-r\mathrm{e}^{\mathrm{j}(\frac{2\pi}{3}+\phi)}\mathrm{e}^{-\mathrm{j}\omega})(1-r\mathrm{e}^{-\mathrm{j}(\frac{2\pi}{3}+\phi)}\mathrm{e}^{-\mathrm{j}\omega})}$$

Constrain $|H(e^{j\omega})|_{\max} = 1$, we have

$$|b_0| = \frac{(1-r)|1-re^{-2j(\frac{2\pi}{3}+\phi)}|}{|1-e^{-j\phi}||1-e^{-j(\frac{4\pi}{3}+\phi)}|}$$

Choose $r = 0.9, \phi = 0.01$.

(b) See plot below.

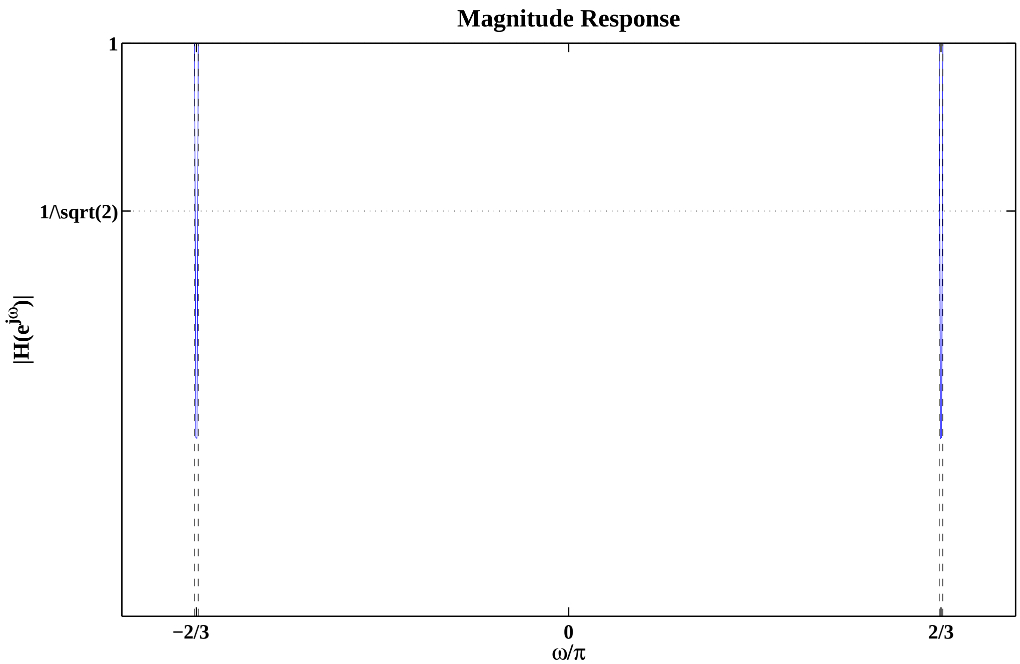


FIGURE 5.102: Magnitude response of the filter.

(c) See plot below.

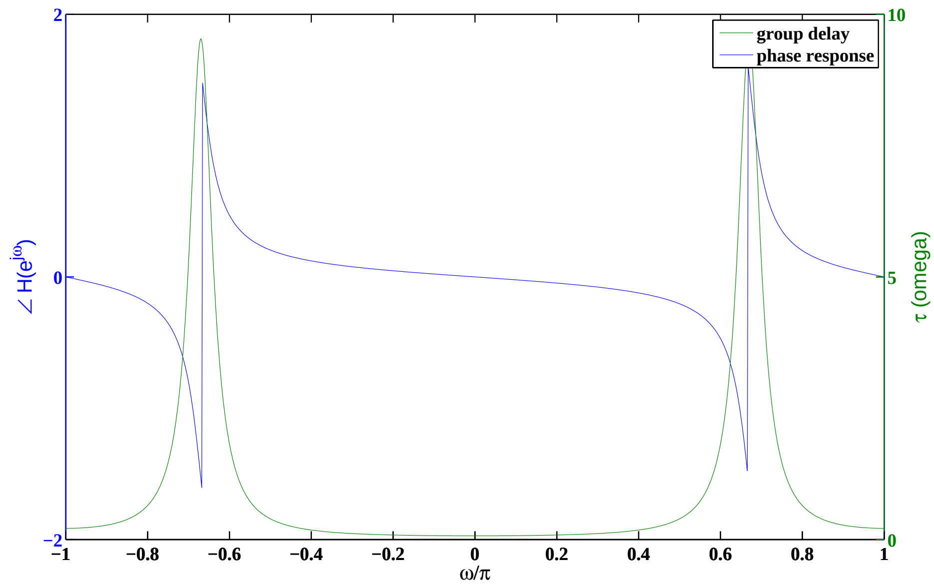


FIGURE 5.103: Phase and group-delay responses of the filter.

41. tba

42. (a) Solution:

$$H(z) = H_{\min}(z) \cdot H_{\text{ap}}(z)$$

$$H_{\min}(z) = \frac{1 + 5.6569z^1 + 16z^2}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$H_{\text{ap}}(z) = \frac{1 + 5.6569z^{-1} + 16z^{-2}}{1 + 5.6569z^1 + 16z^2}$$

(b) See plot below

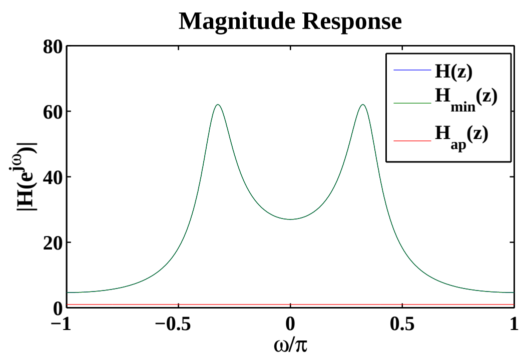


FIGURE 5.104: Magnitude responses of $H(z)$ and its minimum-phase and all-pass components .

(c) See plot below.

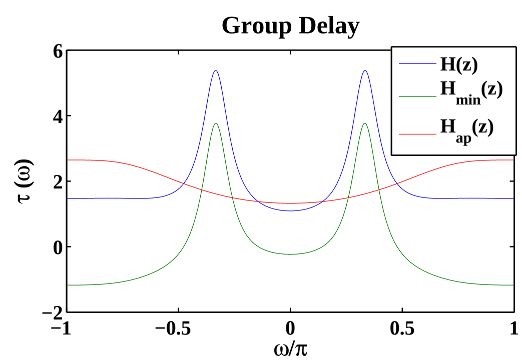


FIGURE 5.105: Group-delays of $H(z)$ and its minimum-phase and all-pass components .

43. (a) Solution:

$$s^2Y(s) + 2sY(s) + 101Y(s) = 10sX(s)$$

The system function is:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{10s}{s^2 + 2s + 101}$$

(b) Solution:
The input signal can be written as:

$$x(t) = 5 - 2(\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}\mathrm{e}^{\mathrm{j}10t} + \mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}\mathrm{e}^{-\mathrm{j}10t}) + \frac{3}{2\mathrm{j}}(\mathrm{e}^{\mathrm{j}20t} - \mathrm{e}^{-\mathrm{j}20t}) + 2\mathrm{e}^{-\mathrm{j}100t}$$

The filter response is:

$$\begin{aligned} y(t) = 5H(0) &- 2\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}\mathrm{e}^{\mathrm{j}10t}H(10) - 2\mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}\mathrm{e}^{-\mathrm{j}10t}H(-10) \\ &+ \frac{3}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}20t}H(20) - \frac{3}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}20t}H(-20) + 2\mathrm{e}^{-\mathrm{j}100t}H(-100) \end{aligned}$$

where

$H(0) = 0, H(-10) = -0.5525, H(10) = 0.4525, H(-20) = -0.4338,$
$H(20) = 0.3697, H(-100) = -0.1010$

44. (a) Solution:

$$H_{\max}(\mathrm{j}\Omega) = \frac{\Omega^2 - 3.4641\Omega + 4}{\Omega^2 - 4\Omega + 5}$$

$$H_{\min}(\mathrm{j}\Omega) = \frac{\Omega^2 + 3.4641\Omega + 4}{\Omega^2 + 4\Omega + 5}$$

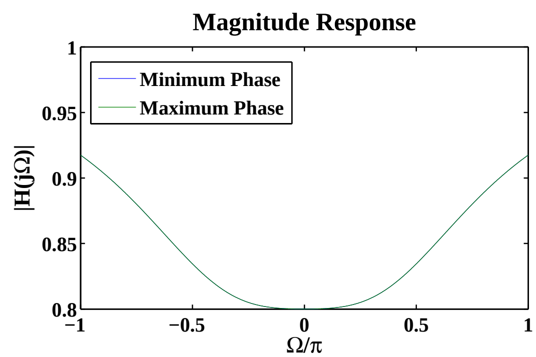
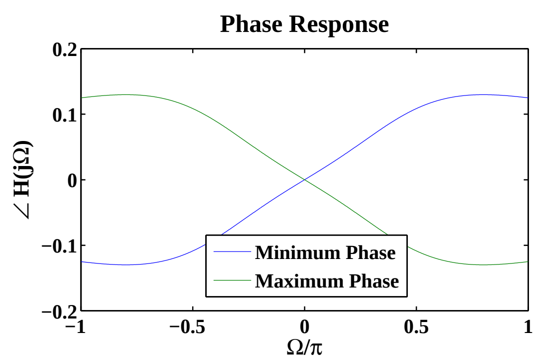


FIGURE 5.106: Magnitude responses of he minimum-phase and maximum-phase system components.



(b)

FIGURE 5.107: Phase responses of he minimum-phase and maximum-phase system components.

MATLAB script:

```
% P0544: Pole-zero investigation of continuous LTI system
close all; clc
b = [1 0 0 0 0 0 64];
a = [-1 0 2 0 -1 0 -100];
```

```
br = roots(b);
ar = roots(a);
bmin = poly(br(real(br)<-1e-10));
amin = poly(ar(real(ar)<-1e-10));
bmax = poly(br(real(br)>1e-10));
amax = poly(ar(real(ar)>1e-10));
w = linspace(-1,1,1000)*pi;
Hmin = freqs(bmin,amin,w);
Hmax = freqs(bmax,amax,w);
%% Plot
hfa = figconfg('P0544a','small');
plot(w/pi,abs(Hmin),w/pi,abs(Hmax))
xlabel('\Omega/\pi','fontsize',LFS)
ylabel('|H(j\Omega)|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
legend('Minimum Phase','Maximum Phase','location','best')

hfb = figconfg('P0544b','small');
plot(w/pi,angle(Hmin),w/pi,angle(Hmax))
xlabel('\Omega/\pi','fontsize',LFS)
ylabel('\angle H(j\Omega)','fontsize',LFS)
title('Phase Response','fontsize',TFS)
legend('Minimum Phase','Maximum Phase','location','best')
```

Assessment Problems

45. (a) Solution:

The frequency response is:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{Y(\mathrm{e}^{\mathrm{j}\omega})}{X(\mathrm{e}^{\mathrm{j}\omega})} = \frac{b - b\mathrm{e}^{-2\mathrm{j}\omega}}{1 + a\mathrm{e}^{-2\mathrm{j}\omega}} = \frac{b(1 - \cos 2\omega + \mathrm{j}\sin 2\omega)}{1 + a\cos 2\omega - a\,\mathrm{j}\sin 2\omega}$$

The magnitude response is:

$$|H(\mathrm{e}^{\mathrm{j}\omega})| = |b|\frac{\sqrt{2 - 2\cos 2\omega}}{\sqrt{1 + a^2 + a\cos 2\omega}}$$

The phase response is:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) = \omega + \tan^{-1}\frac{a\sin 2\omega}{1 + a\cos 2\omega}$$

Constrain the maximum magnitude response is equal to 1, we have

$$|b| = \frac{1-a}{2}$$

For $a = 0.9$, we choose $b = 0.05$.

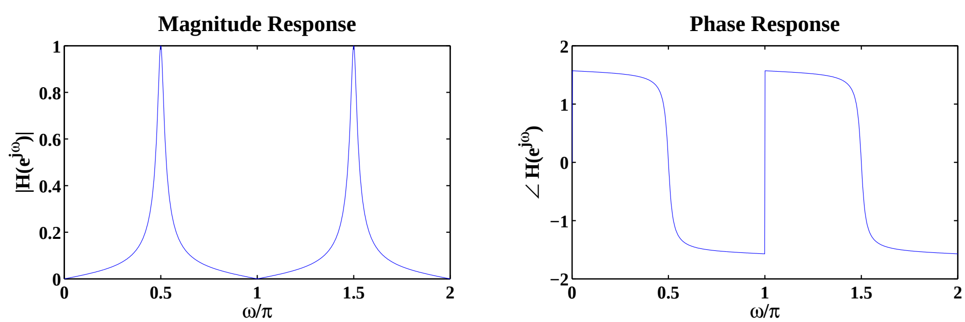


FIGURE 5.108: Magnitude and phase responses of the system for $a = 0.9$.

(b) Solution:

(i) The input sequence is:

$$x[n] = 3\cos(\pi n/2) = \frac{3}{2}(\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n})$$

The output sequence is:

$$\begin{aligned} y[n] &= \frac{3}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n}H(\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}) + \frac{3}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n}H(\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}) \\ &= \frac{3}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n}\frac{2b}{1-a} + \frac{3}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n}\frac{2b}{1-a} = \frac{2b}{1-a}\cdot 3\cos(\pi n/2) \end{aligned}$$

(iI) The input sequence is:

$$x[n] = 3\sin(\pi n/4) = \frac{3}{2\mathrm{j}}(\mathrm{e}^{\mathrm{j}\frac{\pi}{4}n} - \mathrm{e}^{-\mathrm{j}\frac{\pi}{4}n})$$

The output sequence is:

$$\begin{aligned} y[n] &= \frac{3}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}\frac{\pi}{4}n}H(\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}) + \frac{3}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}n}H(\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}) \\ &= \frac{3}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}\frac{\pi}{4}n}\frac{b(1+\mathrm{j})}{1-a\mathrm{j}} + \frac{3}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}n}\frac{b(1-\mathrm{j})}{1+a\mathrm{j}} \\ &= \frac{3b(1-a)}{1+a^2}\sin(\frac{\pi}{4}n) + \frac{3b(1+a)}{1+a^2}\cos(\frac{\pi}{4}n) \end{aligned}$$

(c) See plot below.

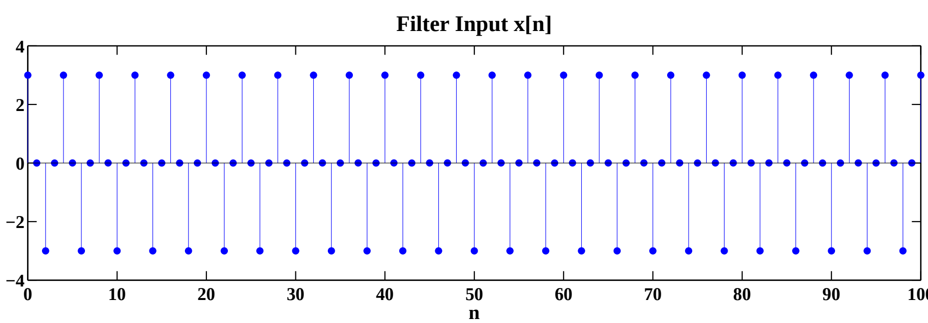


FIGURE 5.109: Input sequence $x[n] = 3\cos(\pi n/2)$.

MATLAB script:

```
% P0545: Second order LTI system
close all; clc
%% Specification:
N = 8;
n = 0:100;
a = 0.9;
```

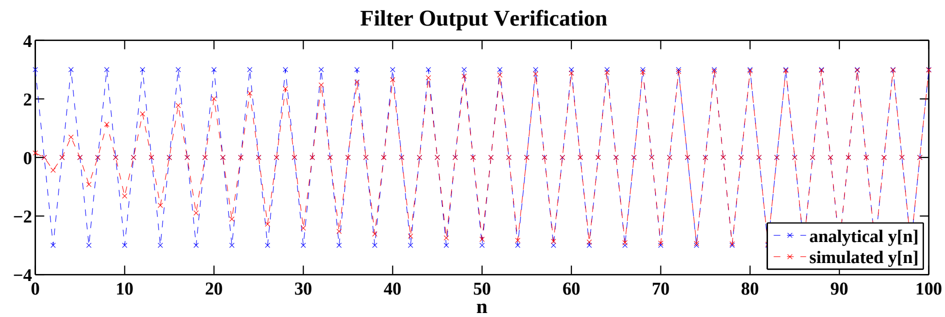


FIGURE 5.110: Output sequence $y[n]$ of $x[n] = 3\cos(\pi n/2)$.

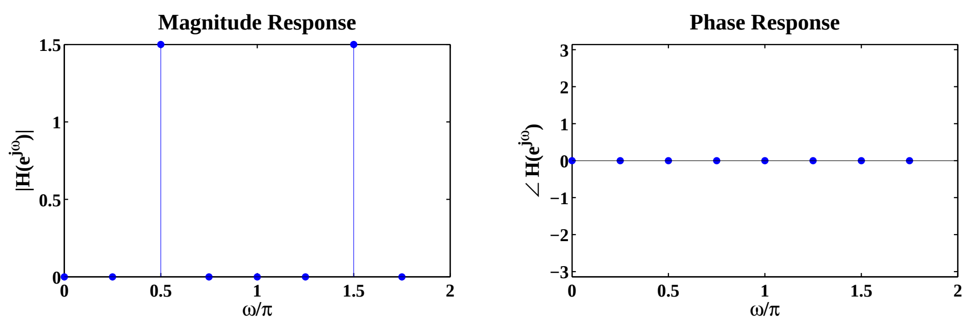


FIGURE 5.111: Magnitude and phase response of $x[n] = 3\cos(\pi n/2)$.

```
b = (1-a)/2;
bh = [b 0 -b];
ah = [1 0 a];
w = linspace(0,2,1000)*pi;
H = freqz(bh,ah,w);
k = 0:N-1;
w2 = 2*pi/N*k;
Hk = b*(1-cos(2*w2)+j*sin(2*w2))./(1+a*cos(2*w2)-a*j*sin(2*w2));

% xn = 3*cos(pi*n/2);
% yn = 3*cos(pi*n/2);
% ckx = 3/2*[0 0 1 0 0 0 1 0];
xn = 3*sin(pi*n/4);
yn = 3*b*(1-a)/(1+a^2)*sin(pi*n/4)+3*b*(1+a)/(1+a^2)*cos(pi*n/4);
ckx = 3/2/j*[0 1 0 0 0 0 0 1];
```

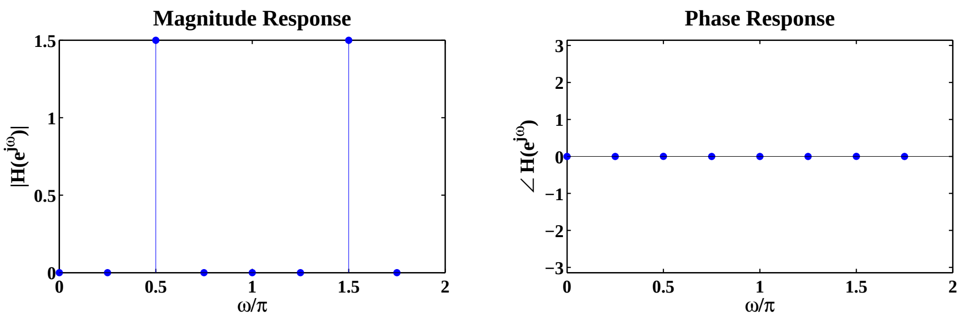


FIGURE 5.112: Magnitude and phase response of $y[n]$ when $x[n] = 3\cos(\pi n/2)$.

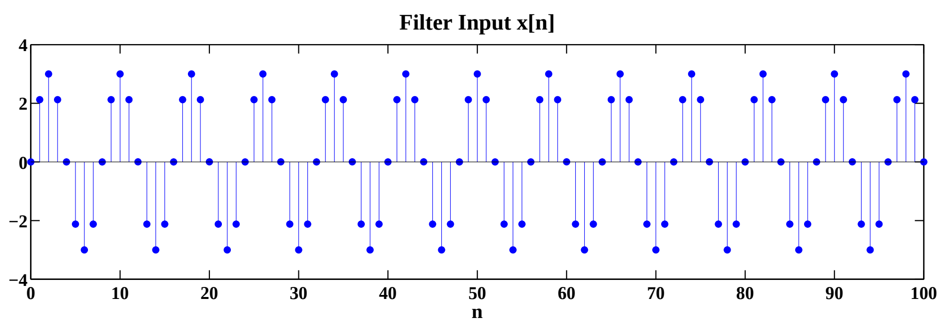


FIGURE 5.113: Input sequence $x[n] = 3\sin(\pi n/4)$.

```
yn_ref = filter(bh,ah,xn);
cky = ckx.*Hk;

%% Plot
hfa = figconfg('P0545a','long');
subplot(121)
plot(w/pi,abs(H))
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(122)
plot(w/pi,angle(H))
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
```

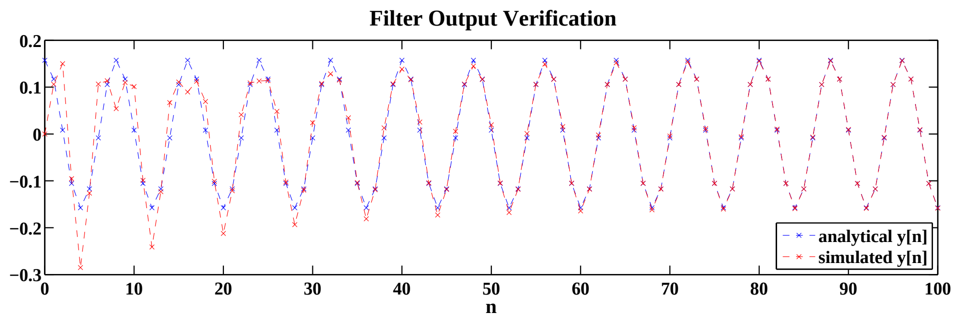


FIGURE 5.114: Output sequence $y[n]$ of $x[n] = 3\sin(\pi n/4)$.

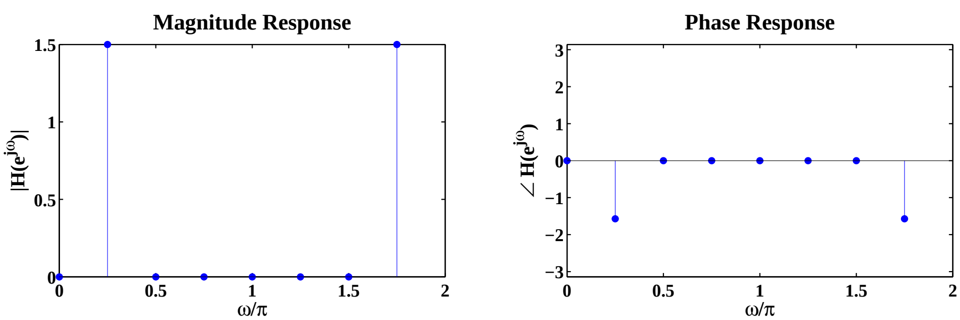


FIGURE 5.115: Magnitude and phase response of $x[n] = 3\sin(\pi n/4)$.

```
title('Phase Response','fontsize',TFS)

hfb = figconfg('P0545b','long');
plot(n,yn,'--xb',n,yn_ref,'--xr')
legend('analytical y[n]','simulated y[n]','Location','Southeast')
xlabel('n','fontsize',LFS)
title('Filter Output Verification','fontsize',TFS)

hfc = figconfg('P0545c','long');
stem(n,xn,'filled')
xlabel('n','fontsize',LFS)
title('Filter Input x[n]','fontsize',TFS)

hfd = figconfg('P0545d','long');
subplot(121)
```

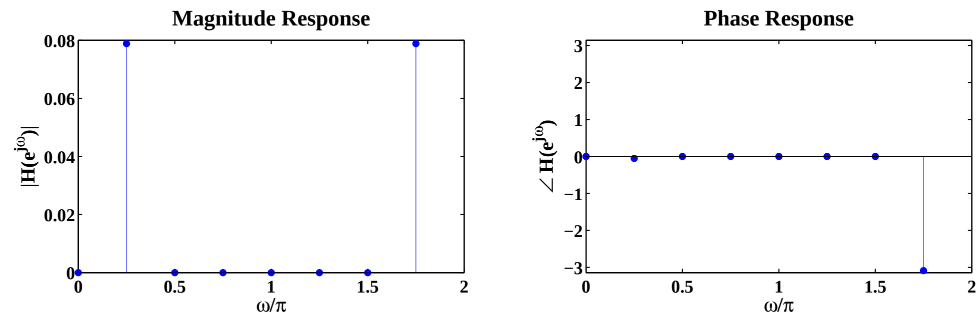


FIGURE 5.116: Magnitude and phase response of $y[n]$ when $x[n] = 3\sin(\pi n/4)$.

```
stem(w2/pi,abs(cky),'filled')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(122)
stem(w2/pi,angle(cky),'filled')
ylim([-pi pi])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title('Phase Response','fontsize',TFS)

hfe = figconfg('P0545e','long');
subplot(121)
stem(w2/pi,abs(ckx),'filled')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(122)
stem(w2/pi,angle(ckx),'filled')
ylim([-pi pi])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title('Phase Response','fontsize',TFS)
```

46. (a) Solution:

The frequency response of the system is:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{Y(\mathrm{e}^{\mathrm{j}\omega})}{X(\mathrm{e}^{\mathrm{j}\omega})} = \frac{1-\mathrm{e}^{-4\mathrm{j}\omega}}{1+0.81\mathrm{e}^{-2\mathrm{j}\omega}+0.6561\mathrm{e}^{-4\mathrm{j}\omega}}$$

(b) See plot below.

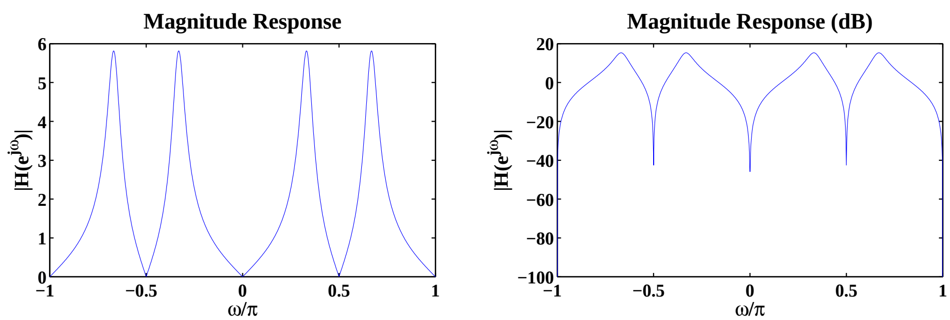


FIGURE 5.117: Magnitude and the dB-Gain responses of the system.

(c) See plot below.

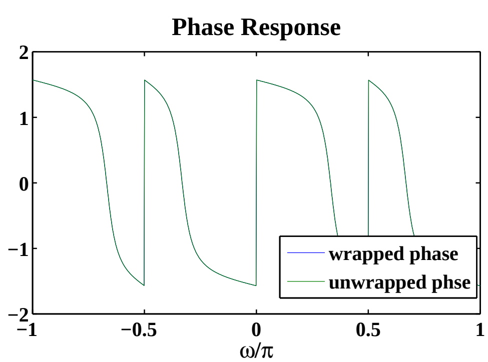


FIGURE 5.118: Wrapped and the unwrapped phase responses of the system.

(d) Solution:

$$x[n] = \mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}n} + \mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}n} + \frac{1}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}\mathrm{e}^{\mathrm{j}\frac{\pi}{4}n} - \frac{1}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}n}$$

$$y[n] = e^{j\frac{\pi}{3}} e^{j\frac{\pi}{2}n} H(e^{j\frac{\pi}{2}}) + e^{-j\frac{\pi}{3}} e^{-j\frac{\pi}{2}n} H(e^{-j\frac{\pi}{2}}) + \frac{1}{2j} e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{4}n} H(e^{j\frac{\pi}{4}}) - \frac{1}{2j} e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{4}n} H(e^{-j\frac{\pi}{4}})$$
$$= (-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j)\frac{1}{0.3439 - j0.81} e^{j\frac{\pi}{4}n} + (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j)\frac{1}{0.3439 + j0.81} e^{-j\frac{\pi}{4}n}$$

(e) See plot below.

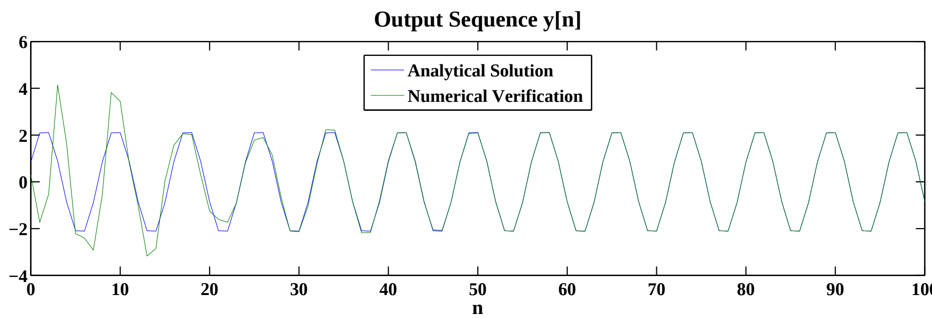


FIGURE 5.119: MATLAB verification of the steady-state response to $x[n]$.

47. (a) Solution:

$$c_k^{(x)} = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} = \frac{1}{5}\sum_{n=1}^{4}(-0.5)^n e^{-j\frac{2\pi}{N}kn}$$
$$= \frac{1}{5}\frac{1-(-1/2)^5}{1+\frac{1}{2}e^{-j\frac{2\pi}{5}k}}, \quad 0 \le k < 5$$

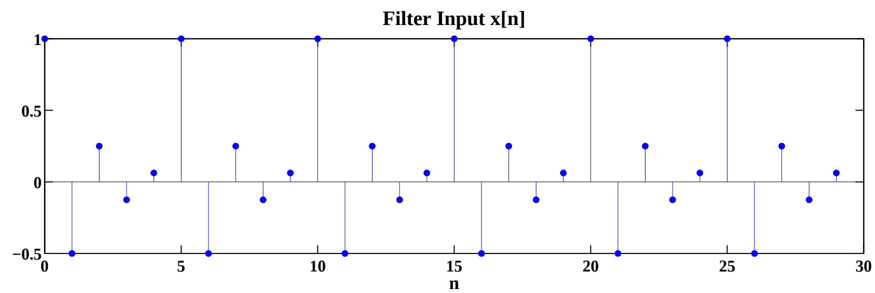
(b) Solution:

$$c_k^{(y)} = c_k^{(x)} H\big(e^{j\frac{2\pi}{5}k}\big)$$

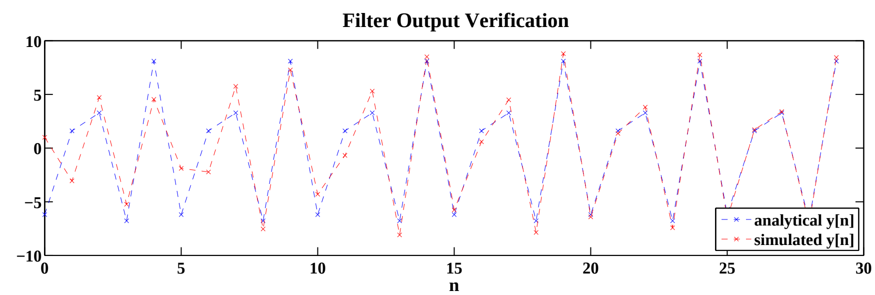
(c) Solution:

$$y_{ss}[n] = \sum_{k=0}^{4} c_k^{(x)} H\big(e^{j\frac{2\pi}{5}k}\big) e^{j\frac{2\pi}{5}kn}$$

(d) See plot below.



(a)



(b)

FIGURE 5.120: Plots of time sequences (a) $x[n]$. (b) $y[n]$ and $y_{\text{ss}}[n]$.

48. (a) See plot below.

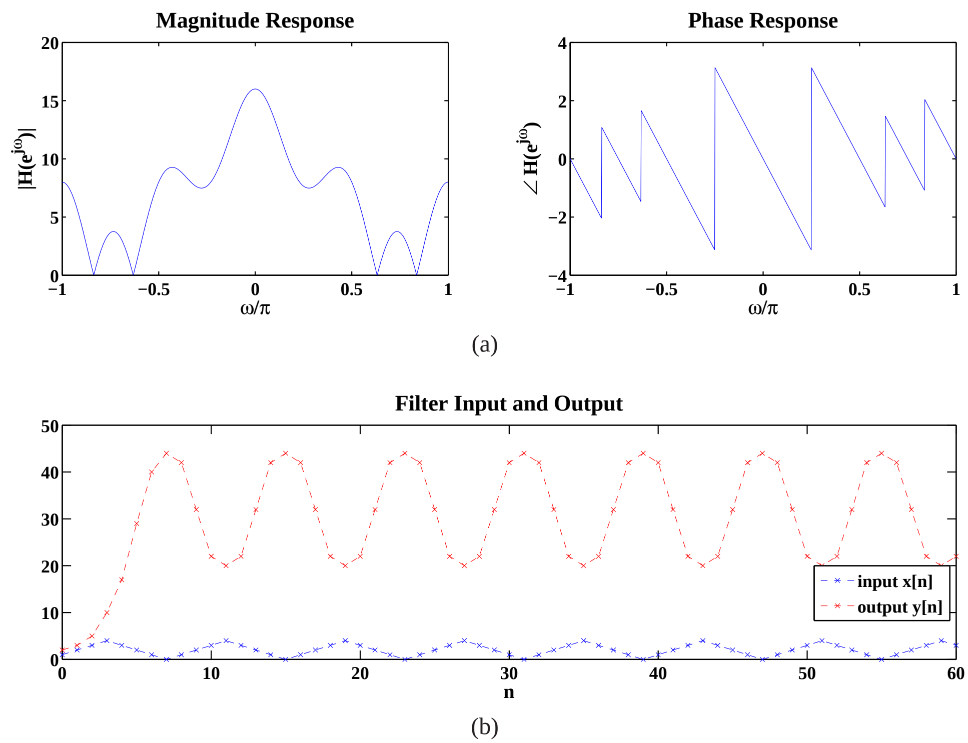
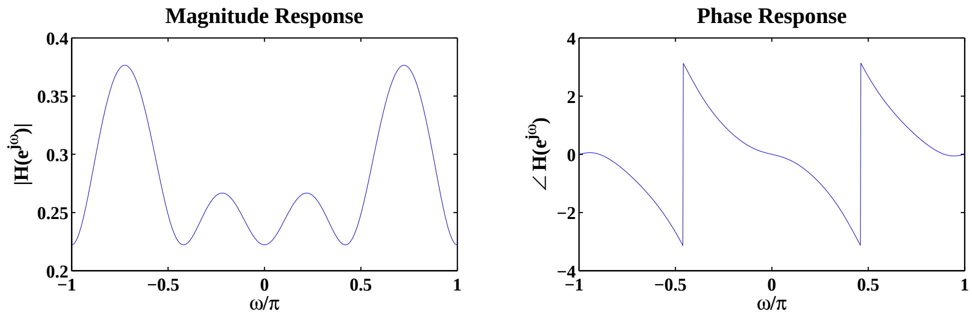
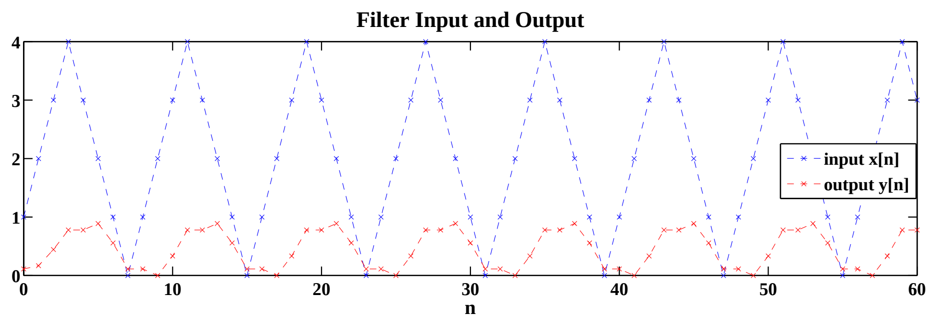


FIGURE 5.121: (a) Magnitude and phase response of system in part (a). (b) Input and steady state response.

(b) See plot below.



(a)



(b)

FIGURE 5.122: (a) Magnitude and phase response of system in part (b). (b) Input and steady state response.

(c) See plot below.

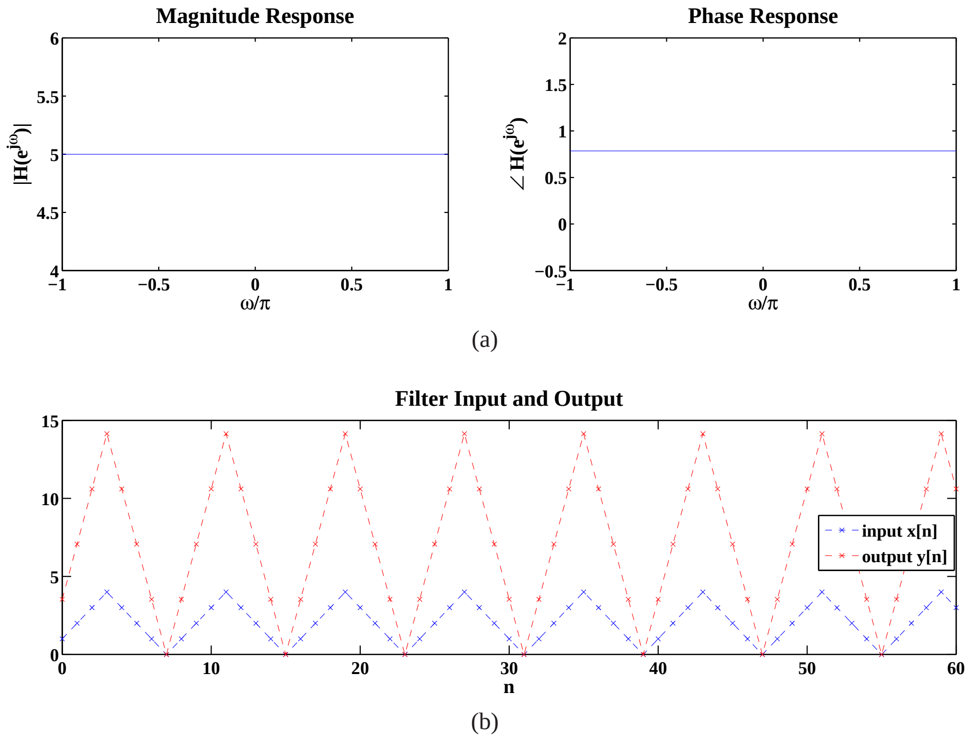
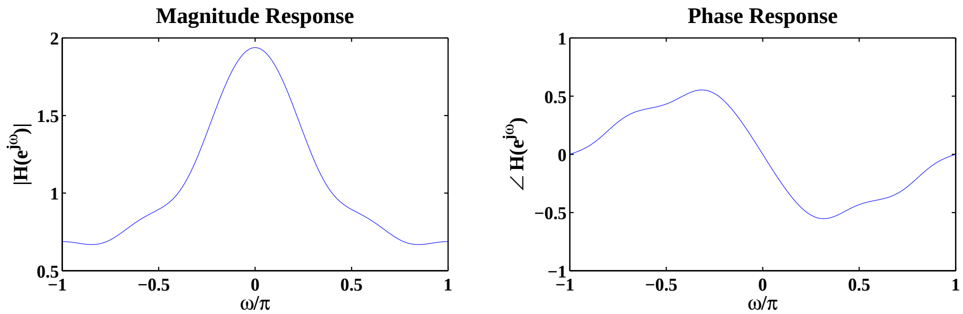
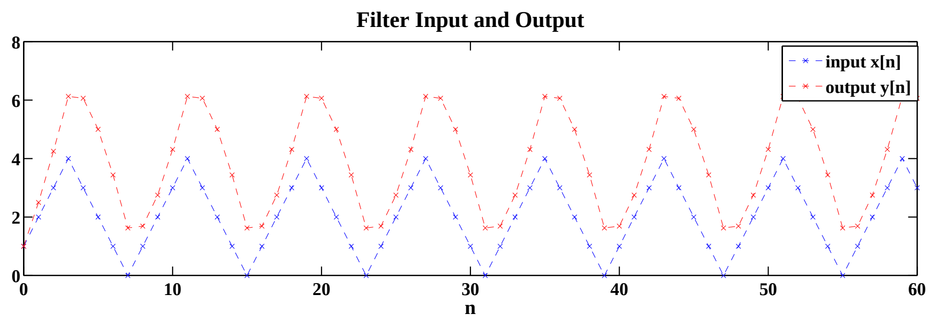


FIGURE 5.123: (a) Magnitude and phase response of system in part (c). (b) Input and steady state response.

(d) See plot below.



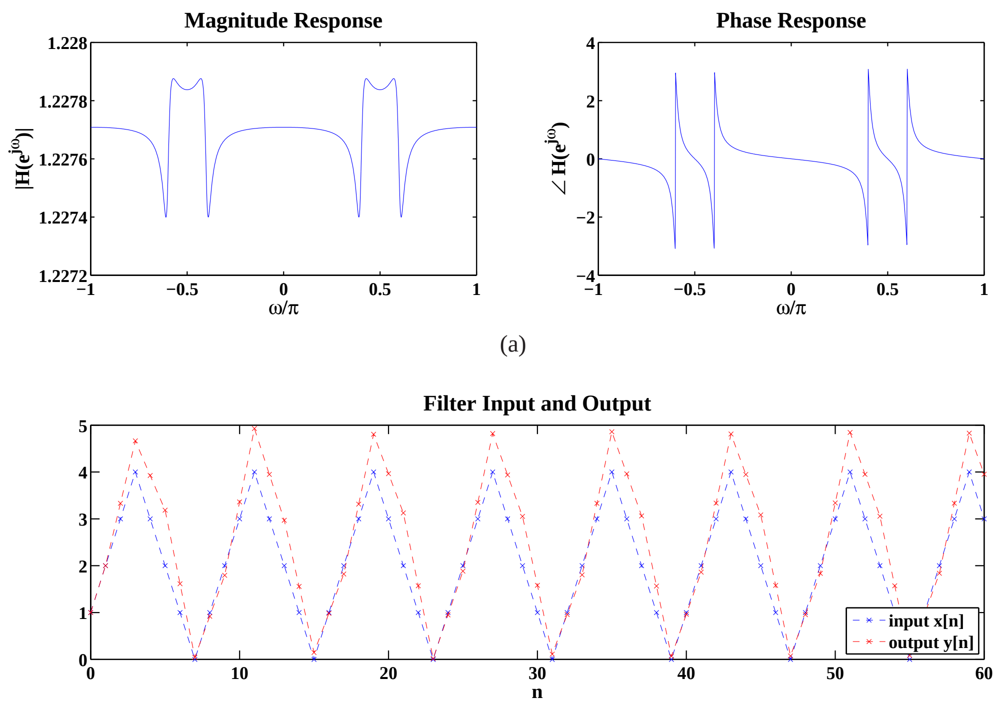
(a)



(b)

FIGURE 5.124: (a) Magnitude and phase response of system in part (d). (b) Input and steady state response.

(e) See plot below.



(a)

(b)

FIGURE 5.125: (a) Magnitude and phase response of system in part (e). (b) Input and steady state response.

49. (a) Solution:
The frequency response is:

$$\begin{aligned}H\left(\mathrm{e}^{\mathrm{j}\omega}\right) &= 5 - 4\mathrm{e}^{-\mathrm{j}\omega} + 3\mathrm{e}^{-2\mathrm{j}\omega} - 3\mathrm{e}^{-3\mathrm{j}\omega} + 4\mathrm{e}^{-4\mathrm{j}\omega} - 5\mathrm{e}^{-5\mathrm{j}\omega} \\ &= (5 - 4\cos\omega + 3\cos 2\omega - 3\cos 3\omega + 4\cos 4\omega - 5\cos 5\omega) \\ &\quad + \mathrm{j}(4\sin\omega - 3\sin 2\omega + 3\sin 3\omega - 4\sin 4\omega + 5\sin 5\omega)\end{aligned}$$

The analytical phase response is:

$$\angle H\left(\mathrm{e}^{\mathrm{j}\omega}\right) = \tan^{-1}\frac{4\sin\omega - 3\sin 2\omega + 3\sin 3\omega - 4\sin 4\omega + 5\sin 5\omega}{5 - 4\cos\omega + 3\cos 2\omega - 3\cos 3\omega + 4\cos 4\omega - 5\cos 5\omega}$$

(b) See plot below.

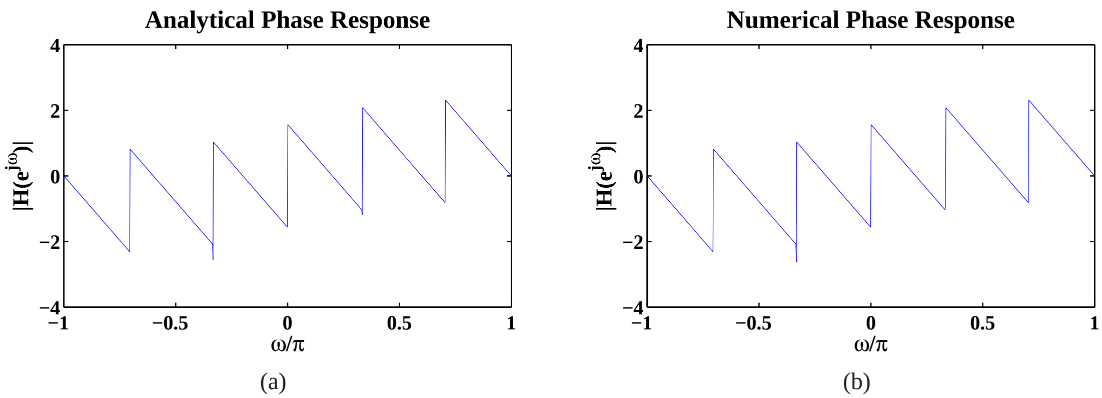


FIGURE 5.126: Phase responses plot (a) Analytical formula. (b) Numerical plot.

50. (a) See plot below.

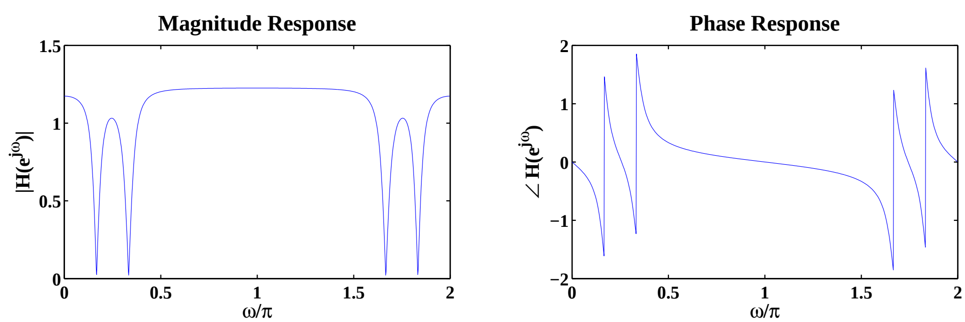


FIGURE 5.127: Magnitude and phase responses of the system using `freqz`.

(b) See plot below.

(c) See plot below.

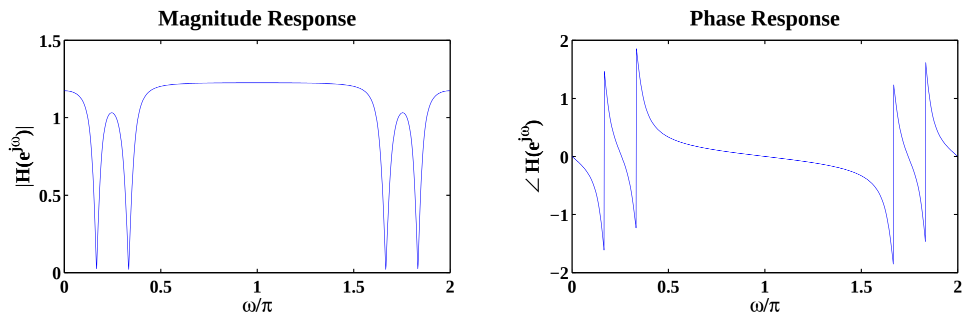


FIGURE 5.128: Magnitude and phase responses of the system using `freqz0`.

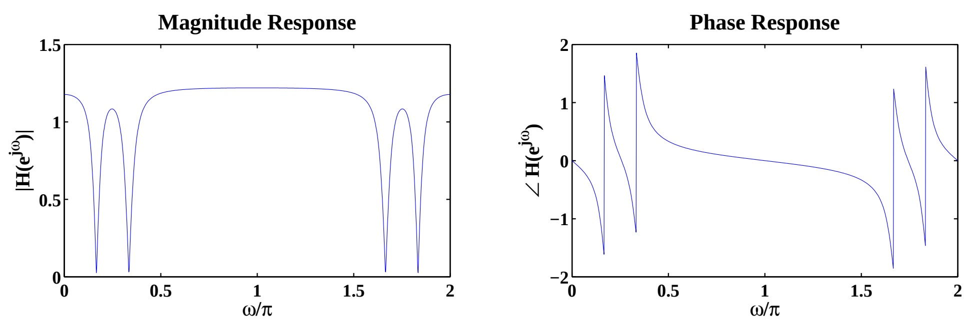


FIGURE 5.129: Magnitude and phase responses of the system using `[mag,pha,omega]=myfreqz(b,a)`.

51. (a) See plot below.
    (b) See plot below.
    (c) See plot below.

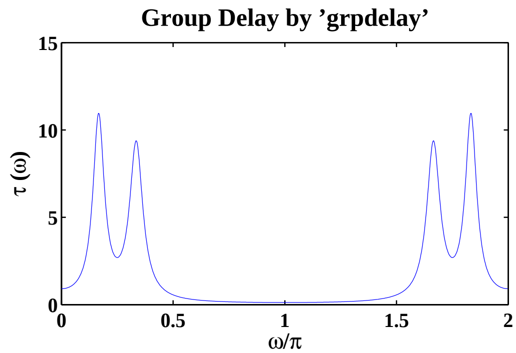


FIGURE 5.130: Group delay of the system using `grpdelay`.

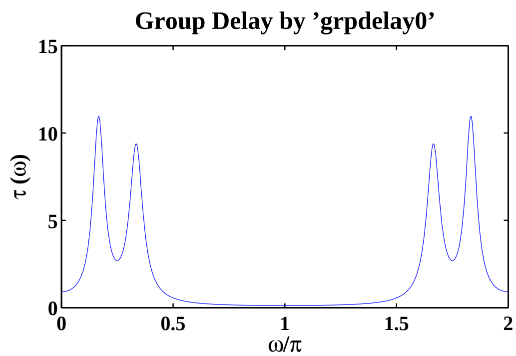


FIGURE 5.131: Group delay of the system using `grpdelay0`.

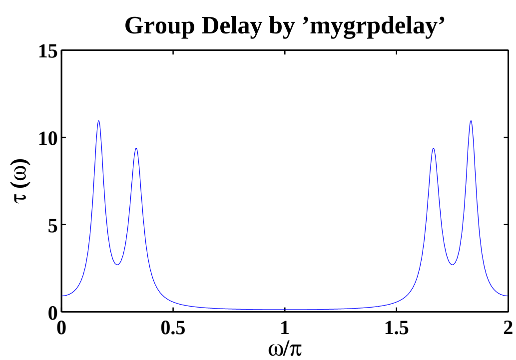


FIGURE 5.132: Group delay of the system using `mygrpdelay`.

52. (a) Solution:

$$y[n] = x[n/2]$$

NOT exists. The above system is not LTI.

(b) Solution:
Exists but NOT unique.

(c) Solution:
Exists and Unique.

$$H\big(e^{j\omega}\big) = 3e^{-j\frac{\pi}{2}}$$

(d) Solution:
Exists and unique.

$$H\big(e^{j\omega}\big) = \frac{3}{2} - \frac{5}{2}e^{-5j\omega}$$

53. (a) See plot below.

(b) Solution:
The frequency response of the system is:

$$H\big(e^{j\omega}\big) = \frac{1}{8}(1 + e^{-2j\omega})^8 = \frac{1}{8}(1 + \cos 2\omega - j\sin 2\omega)^8$$

The analytical phase response is:

$$\angle H\big(e^{j\omega}\big) = 8\tan^{-1}\frac{-\sin 2\omega}{1 + \cos 2\omega} = -8\omega$$

(c) See plot below.

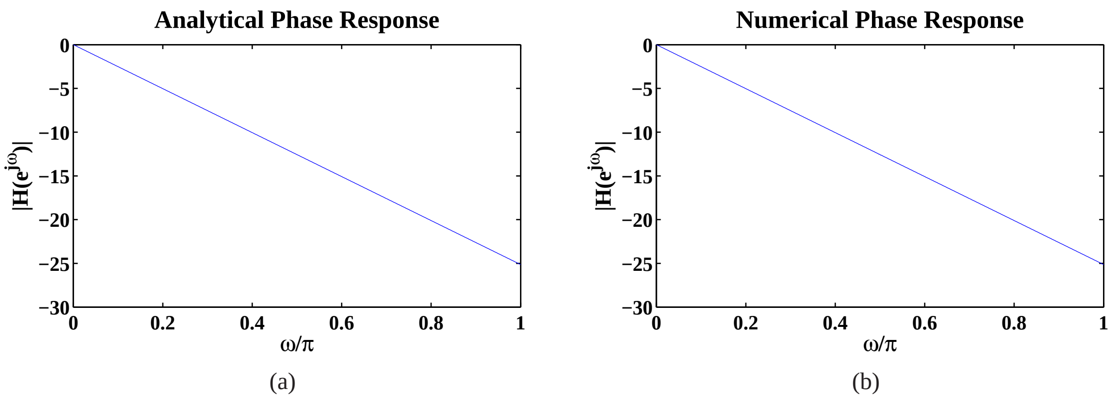


FIGURE 5.133: Phase responses plot (a) Analytical formula. (b) Numerical plot.

54. (a) Solution:

$$\begin{aligned}
h[n] &= \frac{1}{2\pi}\int_{2\pi} H\big(\mathrm{e}^{\mathrm{j}\omega}\big)\,\mathrm{e}^{\mathrm{j}n\omega}\,\mathrm{d}\omega \\
&= \frac{1}{2\pi}\int_{-\pi}^{-\frac{7\pi}{8}} \frac{1}{3}\mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}n\omega}\,\mathrm{d}\omega + \frac{1}{2\pi}\int_{\frac{7\pi}{8}}^{\pi} \frac{1}{3}\mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}n\omega}\,\mathrm{d}\omega \\
&\quad \frac{1}{2\pi}\int_{-\frac{5\pi}{8}}^{-\frac{3\pi}{8}} \frac{2}{3}\mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}n\omega}\,\mathrm{d}\omega + \frac{1}{2\pi}\int_{\frac{3\pi}{8}}^{\frac{5\pi}{8}} \frac{2}{3}\mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}n\omega}\,\mathrm{d}\omega \\
&\quad \frac{1}{2\pi}\int_{-\frac{\pi}{8}}^{0} \mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}n\omega}\,\mathrm{d}\omega + \frac{1}{2\pi}\int_{0}^{\frac{\pi}{8}} \mathrm{e}^{-\mathrm{j}\omega n_d}\mathrm{e}^{\mathrm{j}n\omega}\,\mathrm{d}\omega \\
&= \frac{1}{2\pi}\int_{\frac{7\pi}{8}}^{\pi} \frac{2}{3}\cos\omega(n-n_d)\,\mathrm{d}\omega + \frac{1}{2\pi}\int_{\frac{3\pi}{8}}^{\frac{5\pi}{8}} \frac{4}{3}\cos\omega(n-n_d)\,\mathrm{d}\omega \\
&\quad + \frac{1}{2\pi}\int_{0}^{\frac{\pi}{8}} 2\cos\omega(n-n_d)\,\mathrm{d}\omega \\
&= \frac{1}{3\pi(n-n_d)}\Big[\sin\pi(n-n_d) - \sin\frac{7\pi}{8}(n-n_d)\Big] \\
&\quad + \frac{2}{3\pi(n-n_d)}\Big[\sin\frac{5\pi}{8}(n-n_d) - \sin\frac{3\pi}{8}(n-n_d)\Big] \\
&\quad + \frac{1}{\pi(n-n_d)}\sin\frac{\pi}{8}(n-n_d)
\end{aligned}$$

(b) See plot below.

(c) See plot below.

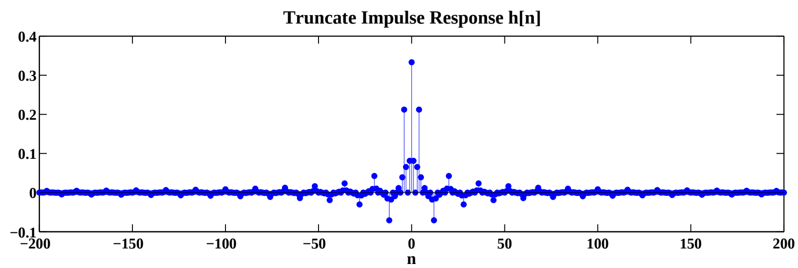


FIGURE 5.134: Impulse response for $n_d = 0$ for $-200 \leq n \leq 200$.

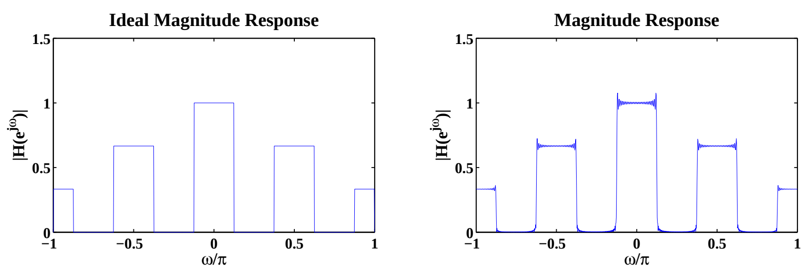


FIGURE 5.135: Magnitude response comparison (a) ideal (b) truncated.

55. (a) See plot below.

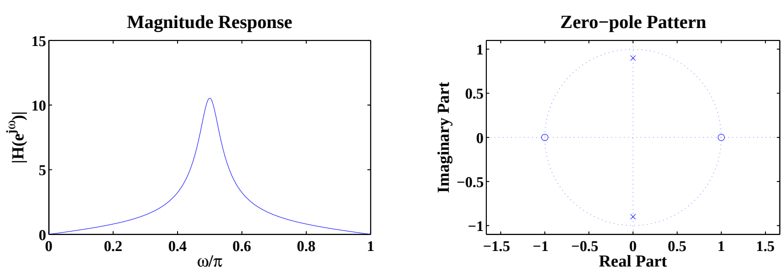


FIGURE 5.136: (a) Magnitude response (b) Pole-zero pattern.

(b) See plot below.

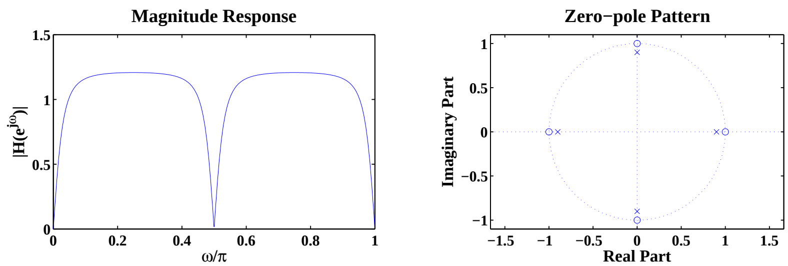


FIGURE 5.137: (a) Magnitude response (b) Pole-zero pattern.

(c) See plot below.

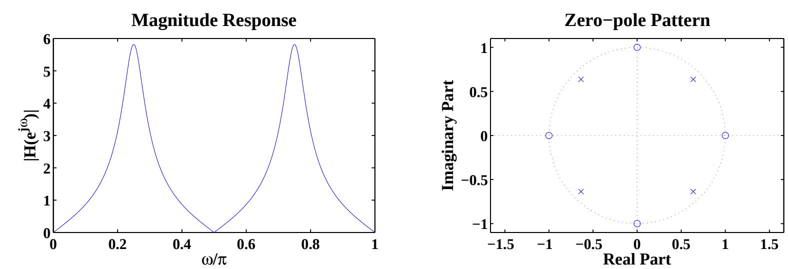


FIGURE 5.138: (a) Magnitude response (b) Pole-zero pattern.

(d) See plot below.

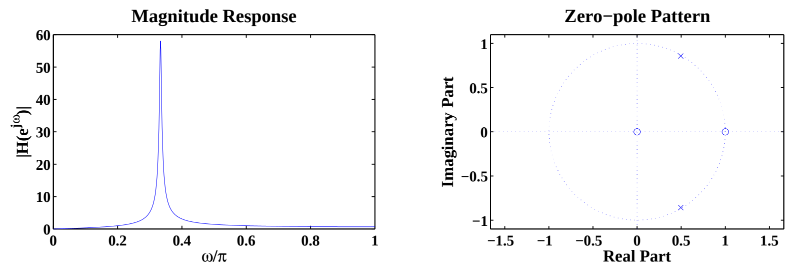


FIGURE 5.139: (a) Magnitude response (b) Pole-zero pattern.

56. (a) Solution:

$$h_{\text{lp}}[n] = \frac{\sin \omega_c (n - n_d)}{\pi (n - n_d)} \quad (5.70)$$

$$h_{\text{bp}}[n] = 2\frac{\sin \omega_c (n - n_d)}{\pi (n - n_d)} \cos \omega_0 n \quad (5.72)$$

$$H_{\text{lp}}\big(e^{j\omega}\big) = \begin{cases} e^{j\omega n_d}, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

$$H_{\text{bp}}\big(e^{j\omega}\big) = H_{\text{lp}}\big(e^{j(\omega-\omega_0)}\big) + H_{\text{lp}}\big(e^{j(\omega+\omega_0)}\big)$$

$$h_{\text{bp}}[n] = e^{j\omega_0 n} h_{\text{lp}}[n] + e^{-j\omega_0 n} h_{\text{lp}}[n] = 2\cos\omega_0 n \cdot h_{\text{lp}}[n]$$

(b) Solution:

$$H_{\text{bp}}\big(e^{j\omega}\big) = H_{\text{lp1}}\big(e^{j\omega}\big) - H_{\text{lp2}}\big(e^{j\omega}\big)$$

where

$$H_{\text{lp1}}\big(e^{j\omega}\big) = \begin{cases} e^{j\omega n_\ell}, & |\omega| < \omega_1 \\ 0, & \omega_1 < |\omega| \le \pi \end{cases}$$

$$H_{\text{lp2}}\big(e^{j\omega}\big) = \begin{cases} e^{j\omega n_d}, & |\omega| < \omega_2 \\ 0, & \omega_2 < |\omega| \le \pi \end{cases}$$

57. (a) Solution:

The frequency response is:

$$H\big(e^{j\omega}\big) = 1 - 0.9e^{-j\omega} = 1 - 0.9\cos\omega + j0.9\sin\omega$$

The phase response is:

$$\Psi(\omega) = \tan^{-1}\frac{0.9\sin\omega}{1 - 0.9\cos\omega} + 2k\pi$$

The group delay is:

$$\tau_{\mathrm{gd}}(\omega) = -\frac{\mathrm{d}\Psi(\omega)}{\mathrm{d}\omega} = \frac{1-0.9\cos\omega}{1+0.9^2-1.8\cos\omega}$$

(b) Solution:
The frequency response is:

$$H\left(\mathrm{e}^{\mathrm{j}\omega}\right) = \frac{1}{1-0.8\mathrm{e}^{-\mathrm{j}\omega}} = \frac{1}{1-0.8\cos\omega+\mathrm{j}0.8\sin\omega}$$

The phase response is:

$$\Psi(\omega) = -\tan^{-1}\frac{0.8\sin\omega}{1-0.8\cos\omega} + 2k\pi$$

The group delay is:

$$\tau_{\mathrm{gd}}(\omega) = -\frac{\mathrm{d}\Psi(\omega)}{\mathrm{d}\omega} = \frac{0.8\cos\omega-0.8^2}{1+0.8^2-1.6\cos\omega}$$

(c) Solution:
The frequency response is:

$$\begin{aligned}H\left(\mathrm{e}^{\mathrm{j}\omega}\right) &= \frac{1}{1-0.7\mathrm{e}^{-\mathrm{j}\omega}+0.49\mathrm{e}^{-2\mathrm{j}\omega}}\\ &= \frac{1}{(1-0.7\cos\omega+0.49\cos2\omega)+\mathrm{j}(0.7\sin\omega-0.49\sin2\omega)}\end{aligned}$$

The phase response is:

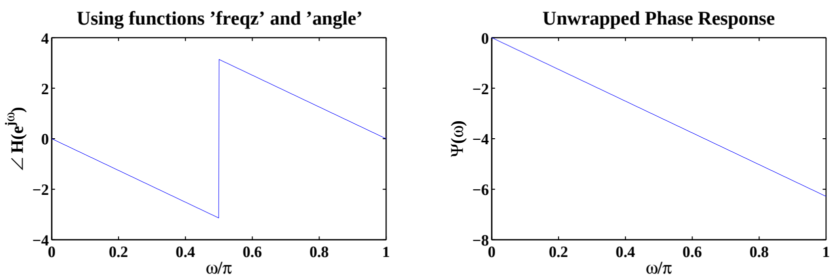
$$\Psi(\omega) = -\tan^{-1}\frac{0.7\sin\omega-0.49\sin2\omega}{1-0.7\cos\omega+0.49\cos2\omega} + 2k\pi$$

The group delay is:

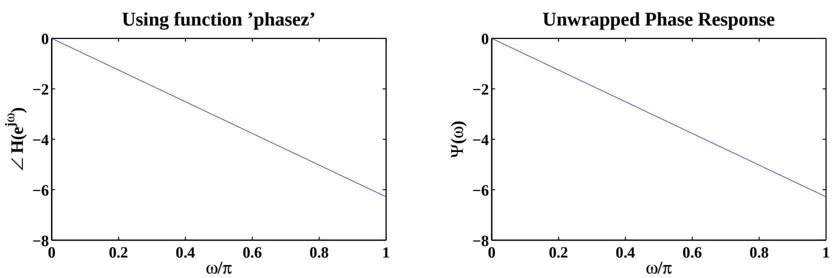
$$\begin{aligned}\tau_{\mathrm{gd}}(\omega) &= -\frac{\mathrm{d}\Psi(\omega)}{\mathrm{d}\omega}\\ &= \frac{(0.7\cos\omega-2\times0.49\cos2\omega)(1-0.7\cos\omega+0.49\cos2\omega)}{(0.7\sin\omega-0.49\sin2\omega)^2+(1-0.7\cos\omega+0.49\cos2\omega)^2}\\ &\quad-\frac{(0.7\sin\omega-0.49\sin2\omega)(0.7\sin\omega-2\times0.49\sin2\omega)}{(0.7\sin\omega-0.49\sin2\omega)^2+(1-0.7\cos\omega+0.49\cos2\omega)^2}\end{aligned}$$

58. tba

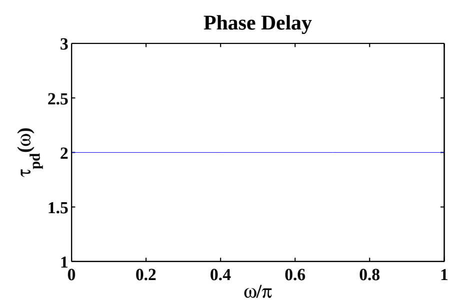
59. (a) See plot below.



(a)



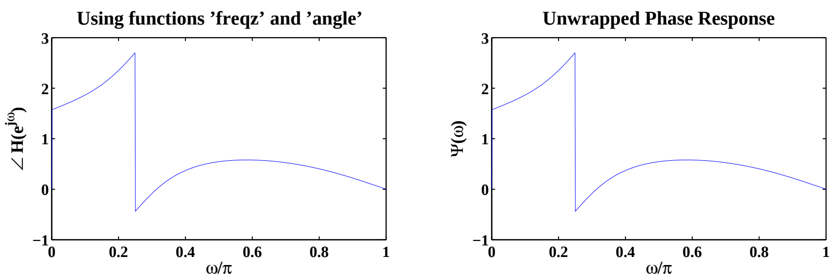
(b)



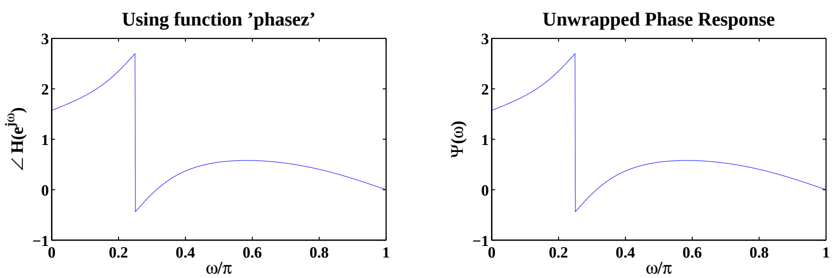
(c)

FIGURE 5.140: Phase response using the functions freqz, angle, phasez, unwrap, and phasedelay of system in part (a).

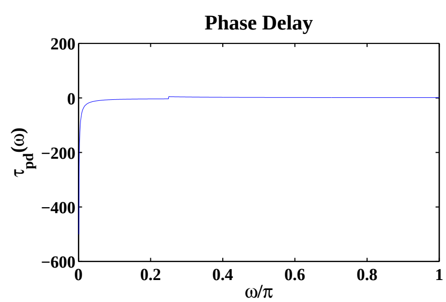
(b) See plot below.



(a)



(b)



(c)

FIGURE 5.141: Phase response using the functions freqz, angle, phasez, unwrap, and phasedelay of system in part (b).

60. Solution:

$$H(z) = \frac{1+5z^{-1}}{1+\frac{1}{2}z^{-1}}, \qquad (5.176)$$

$$H(z) = 5 \cdot H_{\text{min}}(z) \cdot H_{\text{ap}}(z)$$

where

$$H_{\text{min}}(z) = \frac{1+\frac{1}{5}z^{-1}}{1+\frac{1}{2}z^{-1}}, \quad H_{\text{ap}}(z) = \frac{z^{-1}+\frac{1}{5}}{1+\frac{1}{5}z^{-1}}$$

The equalizer is defined by:

$$H_{\text{eq}}(z) = \frac{Gz^{-n_d}}{H_{\text{min}}(z)}$$

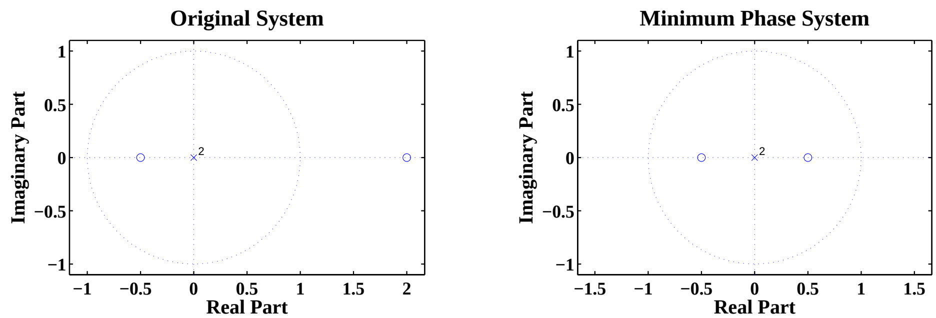
where we have $G = 1$ and $n_d = 0$.

$$H_{\text{eq}}(z) = \frac{1}{H_{\text{min}}(z)} = \frac{1}{5}\frac{1+\frac{1}{2}\mathrm{e}^{-\mathrm{j}\omega}}{1+\frac{1}{5}\mathrm{e}^{-\mathrm{j}\omega}}$$

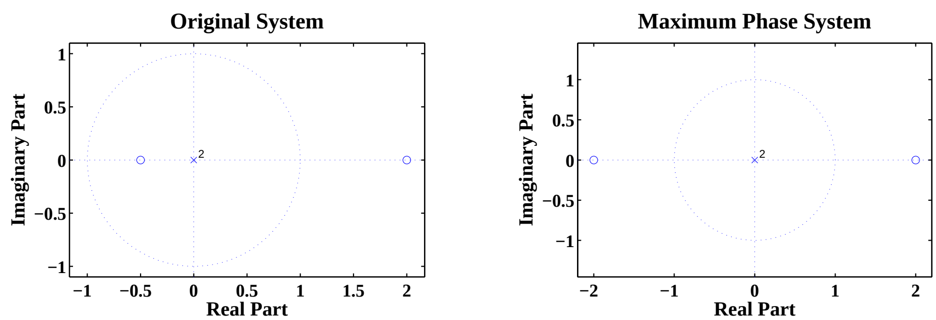
61. (a) See plot below.
    (b) See plot below.
    (c) See plot below.
    (d) See plot below.

    MATLAB script:

```
% P0561: Illustraion of using Matlab function 'polystab'
close all; clc
%% Part a:
% b = [1 -1.5 -1];
% a = 1;

%% Part b:
% b = [1 2 0.75];
% a = [1 -0.5];

%% Part c:
% b = [1 -2 3.2 -2 1];
% a = 1;

%% Part d:
b = [1 -2.4142 2.4142 -1];
```



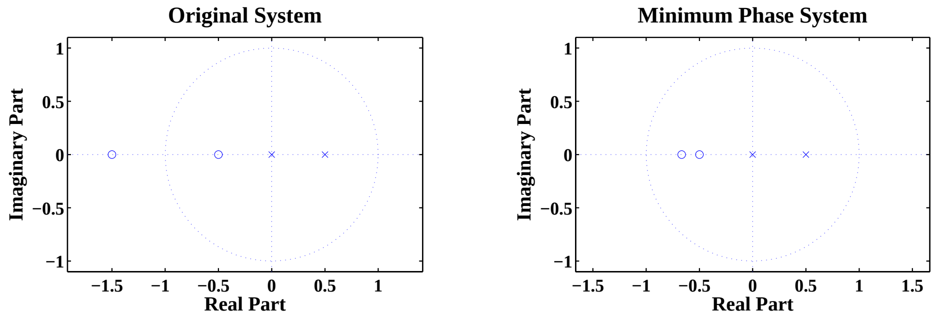
(a)



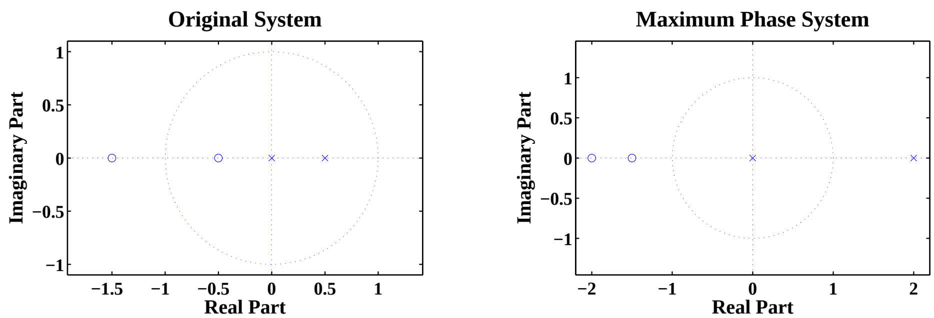
(b)

FIGURE 5.142: MATLAB verification of system in part (a). (a) minimum-phase system (b) maximum-phase system.

```
a = [1 -1.8 1.62 0.729];

% Computation:
bmin = polystab(b)*norm(b)/norm(polystab(b));
bmax = fliplr(bmin);
amin = polystab(a)*norm(a)/norm(polystab(a));
amax = fliplr(amin);
% Plot:
hfa = figconfg('P0561a','long');
subplot(121)
zplane(b,a)
xlabel('Real Part','fontsize',LFS)
ylabel('Imaginary Part','fontsize',LFS)
title('Original System','fontsize',TFS)
subplot(122)
```



(a)



(b)

FIGURE 5.143: MATLAB verification of system in part (b). (a) minimum-phase system (b) maximum-phase system.

```
zplane(bmin,amin)
xlabel('Real Part','fontsize',LFS)
ylabel('Imaginary Part','fontsize',LFS)
title('Minimum Phase System','fontsize',TFS)

hfb = figconfg('P0561b','long');
subplot(121)
zplane(b,a)
xlabel('Real Part','fontsize',LFS)
ylabel('Imaginary Part','fontsize',LFS)
title('Original System','fontsize',TFS)
subplot(122)
zplane(bmax,amax)
xlabel('Real Part','fontsize',LFS)
ylabel('Imaginary Part','fontsize',LFS)
```

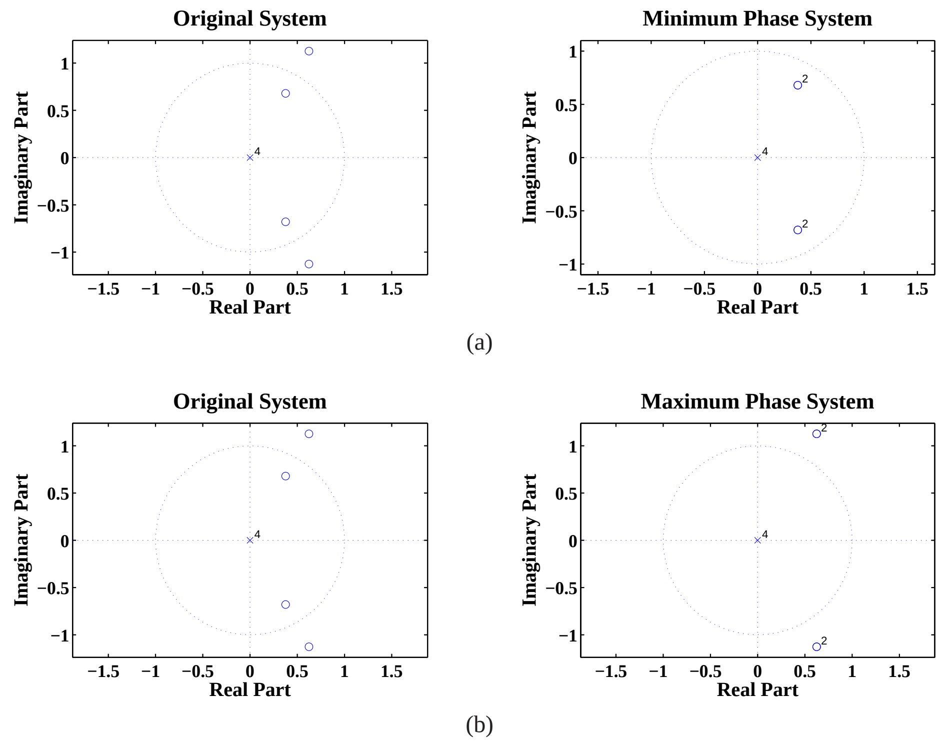
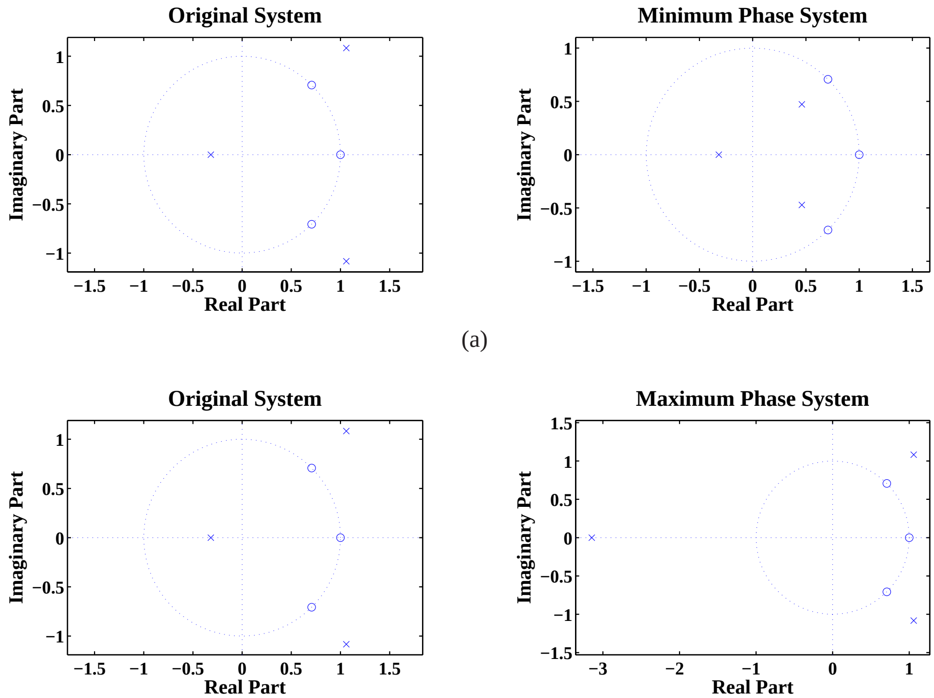


FIGURE 5.144: MATLAB verification of system in part (c). (a) minimum-phase system (b) maximum-phase system.

```
title('Maximum Phase System','fontsize',TFS)
```



(b)

FIGURE 5.145: MATLAB verification of system in part (a). (a) minimum-phase system (b) maximum-phase system.

62. tba

63. (a) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 0.1z^{-5}$$

The impulse response is:

$$h[n] = \delta[n] + 0.1\delta[n-5]$$

The frequency response is:

$$H(e^{j\omega}) = 1 + 0.1e^{-5j\omega} = (1 + 0.1\cos 5\omega) + j(-0.1\sin 5\omega)$$

The magnitude response is:

$$|H(e^{j\omega})| = \sqrt{(1 + 0.1\cos 5\omega)^2 + (-0.1\sin 5\omega)^2} = \sqrt{1 + 0.1^2 + 0.2\cos 5\omega}$$

The phase response is:

$$\angle H(e^{j\omega}) = -\tan^{-1}\frac{0.1\sin 5\omega}{1 + 0.1\cos 5\omega}$$

(b) Solution:
The system function of the inverse system is:

$$H_{\text{inv}}(z) = \frac{1}{H(z)} = \frac{1}{1 + 0.1z^{-5}}$$

The difference equation of the inverse system is

$$y[n] = x[n] - 0.1y[n-5]$$

(c) Solution:
The frequency response of the inverse system is:

$$H_{\text{inv}}(e^{j\omega}) = \frac{1}{1 + 0.1e^{-5j\omega}} = \frac{1}{(1 + 0.1\cos 5\omega) + j(-0.1\sin 5\omega)}$$

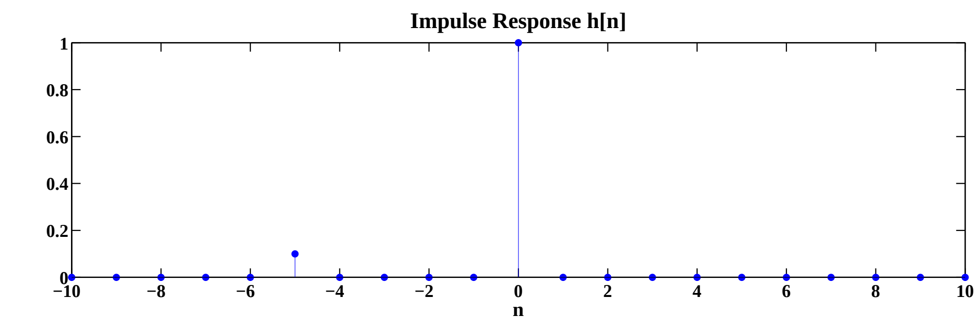
The magnitude response of the inverse system is:

$$|H_{\text{inv}}(e^{j\omega})| = \frac{1}{\sqrt{1 + 0.1^2 + 0.2\cos 5\omega}}$$

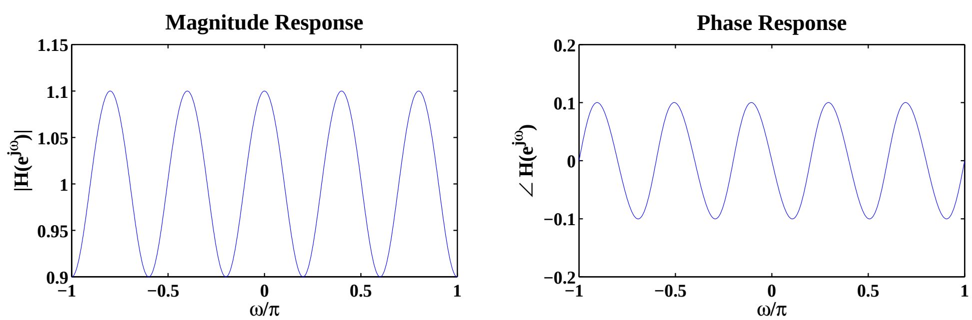
The phase response of the inverse system is:

$$\angle H_{\text{inv}}(e^{j\omega}) = \tan^{-1}\frac{0.1\sin 5\omega}{1 + 0.1\cos 5\omega}$$

(d) See plot below.

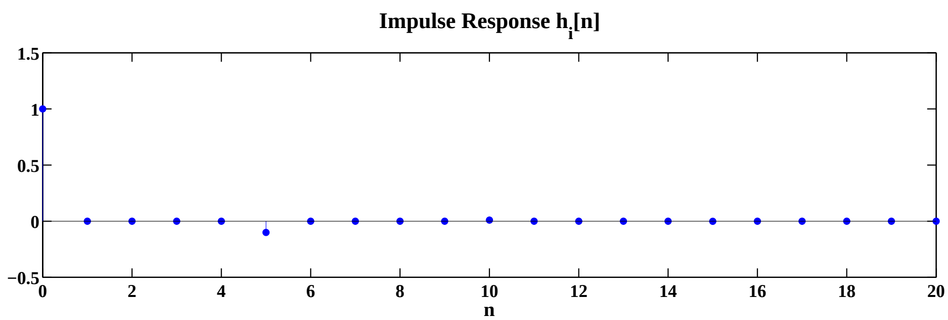


(a)

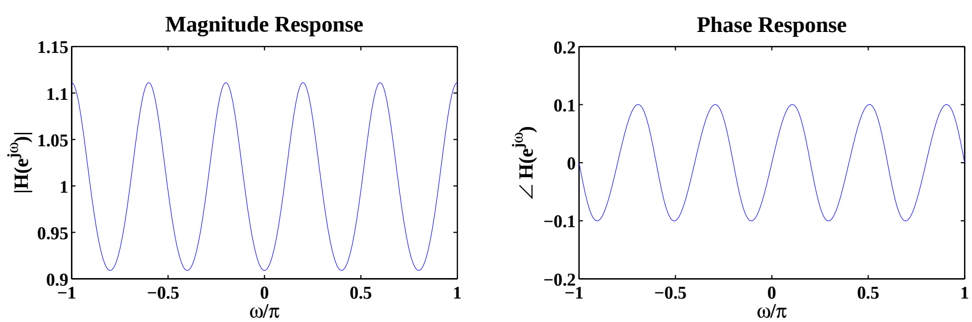


(b)

FIGURE 5.146: (a) Impulse response (b) magnitude and phase responses of the system $H(z)$.



(a)



(b)

FIGURE 5.147: (a) Impulse response (b) magnitude and phase responses of the system $H_{\text{inv}}(z)$.

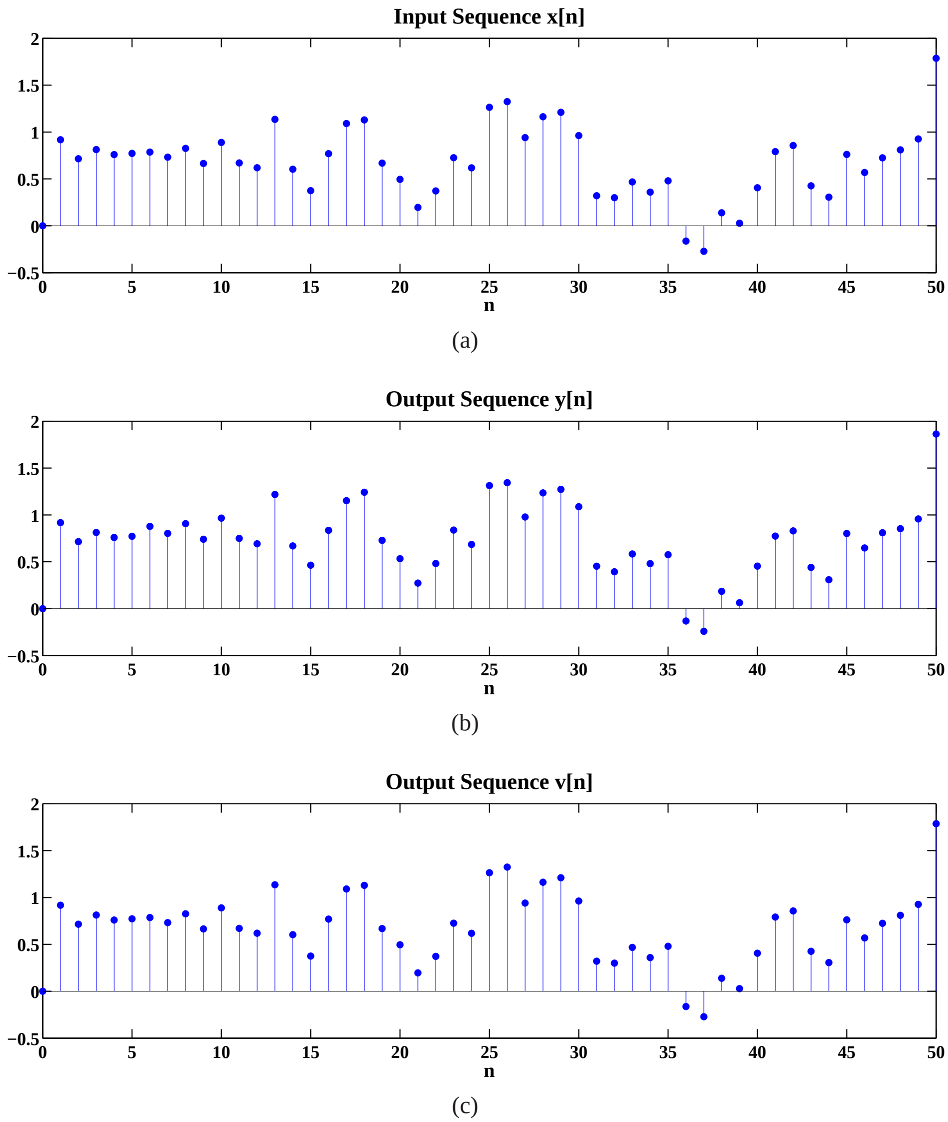


FIGURE 5.148: (a) Input sequence $x[n]$ (b) Input sequence of the inverse system $y[n]$ (c) Output sequence of the inverse system $v[n]$.

64. (a) Solution:
The frequency response is:

$$G(e^{j\omega}) = b_0 \frac{1 - 2\cos\phi e^{-j\omega} + e^{-2j\omega}}{1 - 2r\cos\phi e^{-j\omega} + r^2 e^{-2j\omega}}$$
$$= b_0 \frac{(1 - 2\cos\phi\cos\omega + \cos 2\omega) + j(2\cos\phi\sin\omega - \sin 2\omega)}{(1 - 2r\cos\phi\cos\omega + r^2\cos 2\omega) + j(2r\cos\phi\sin\omega - r^2\sin 2\omega)}$$

(b) See plot below.

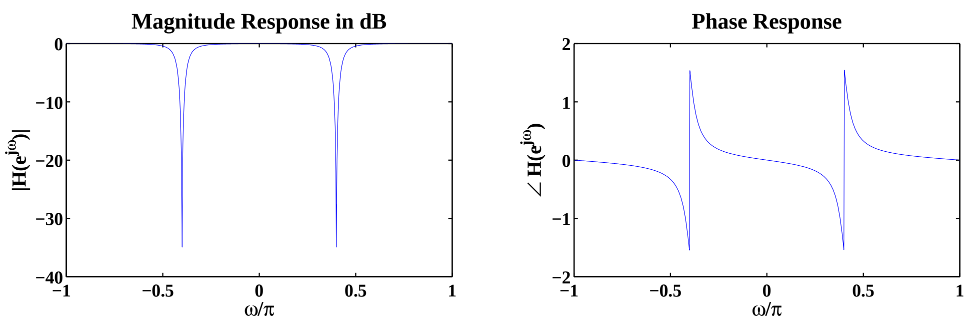


FIGURE 5.149: (a) Magnitude response (b) Phase response when $r = 0.9$ and $\phi = 2\pi/5$.

(c) See plot below.

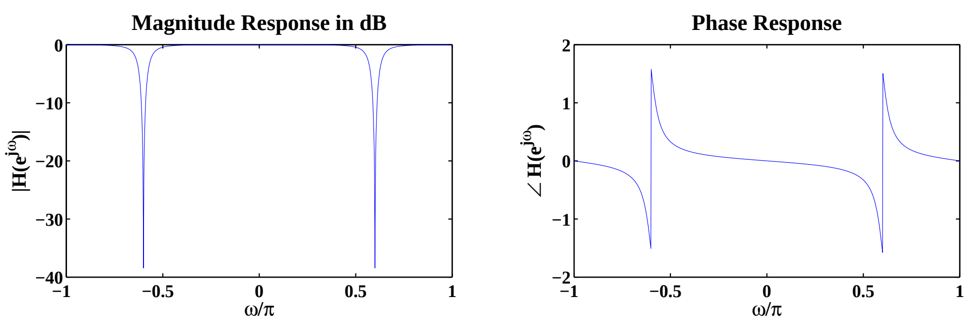


FIGURE 5.150: (a) Magnitude response (b) Phase response when $r = 0.9$ and $\phi = 3\pi/5$.

65. (a) Solution:

$$\text{zeros:} \quad z_1 = \mathrm{e}^{\mathrm{j}\frac{\pi}{2}}, z_2 = \mathrm{e}^{\mathrm{j}\frac{3\pi}{2}}$$
$$\text{poles:} \quad p_1 = r\mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}, p_2 = r\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}$$

The system function is:

$$H(z) = b_0 \frac{(1-\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}z^{-1})(1-\mathrm{e}^{\mathrm{j}\frac{3\pi}{2}}z^{-1})}{(1-r\mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}z^{-1})(1-r\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}z^{-1})}$$

(b) See plot below.

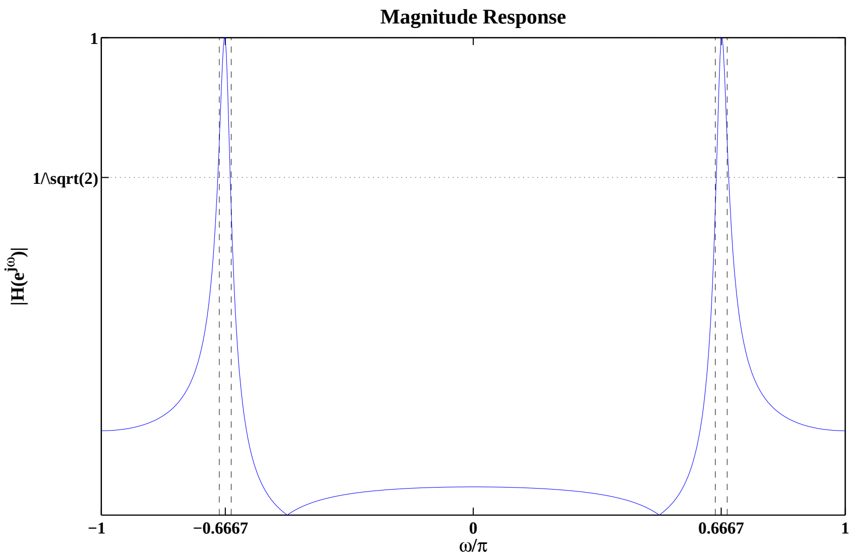


FIGURE 5.151: Magnitude response of the filter.

(c) See plot below.

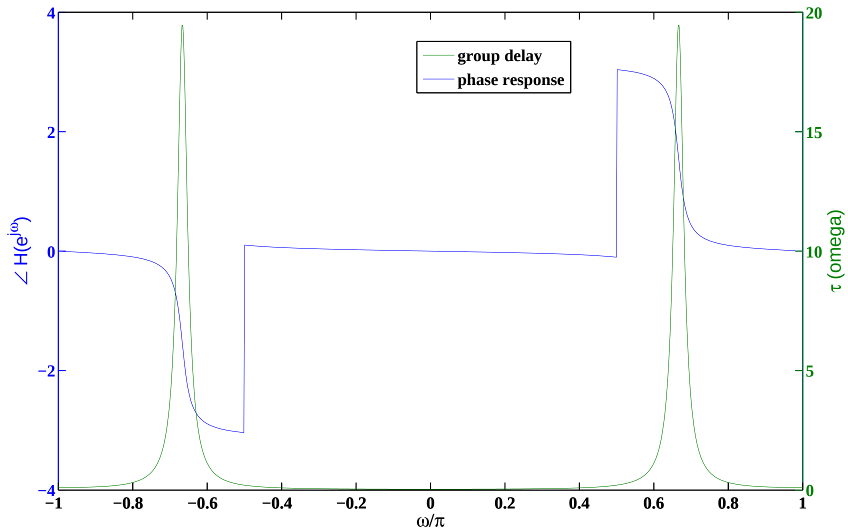


FIGURE 5.152: Phase and group-delay responses of the filter.

66. (a) Solution:

$$H(z) = H_{\text{min}}(z) \cdot H_{\text{ap}}(z)$$

where

$$H_{\text{min}}(z) = \frac{2.7 - 2.1z^{-1} + z^{-2}}{1 + 0.3126z^{-1} + 0.81z^{-2}}, \quad H_{\text{ap}}(z) = \frac{1 - 2.1z^{-1} + 2.7z^{-2}}{2.7 - 2.1z^{-1} + z^{-2}}$$

(b) See plot below.

(c) See plot below.

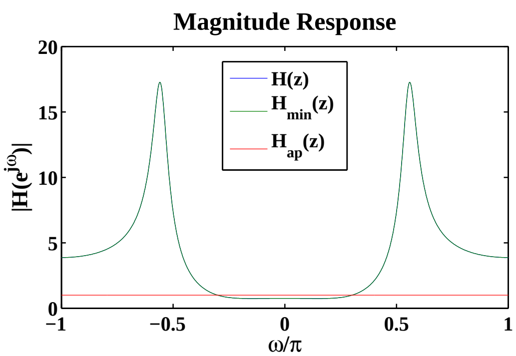


FIGURE 5.153: Magnitude responses of $H(z)$ and its minimum-phase and all-pass components.

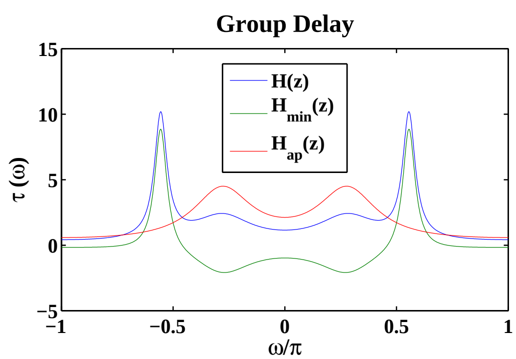


FIGURE 5.154: Group delays of $H(z)$ and its minimum-phase and all-pass components.

67. (a) Solution:

$$h(t) = 2\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}\mathrm{e}^{-0.005t}\mathrm{e}^{\mathrm{j}10\pi t}u(t) + 2\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}\mathrm{e}^{-0.005t}\mathrm{e}^{-\mathrm{j}10\pi t}u(t)$$

The system function is:

$$H(s) = \frac{2\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}}{s + 0.05 - 10\pi} + \frac{2\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}}{s + 0.05 + 10\pi}$$

(b) Solution:

$$x(t) = 4 - \frac{3}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{\mathrm{j}4\pi t} - \frac{3}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{-\mathrm{j}4\pi t} + \frac{5}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}20\pi t} - \frac{5}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}20\pi t}$$

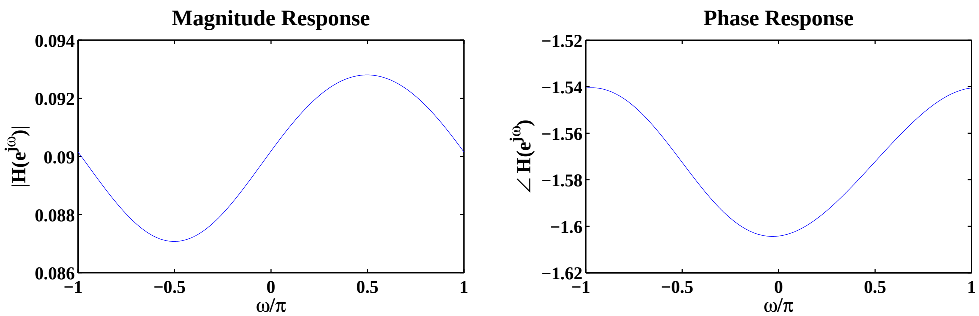


FIGURE 5.155: Magnitude and phase responses of the system.

$$y(t) = 4H(0) - \frac{3}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{\mathrm{j}4\pi t}H(4\pi) - \frac{3}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{-\mathrm{j}4\pi t}H(-4\pi) + \frac{5}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}20\pi t}H(20\pi) - \frac{5}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}20\pi t}H(-20\pi)$$

68. Solution:
Choose $D = 5$.

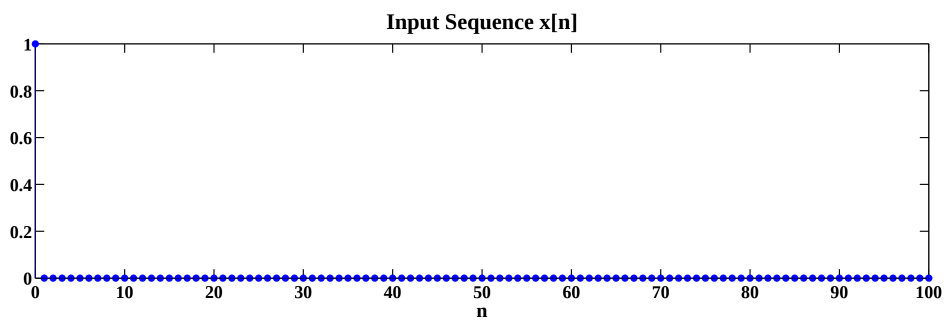


FIGURE 5.156: Input sequence (i).

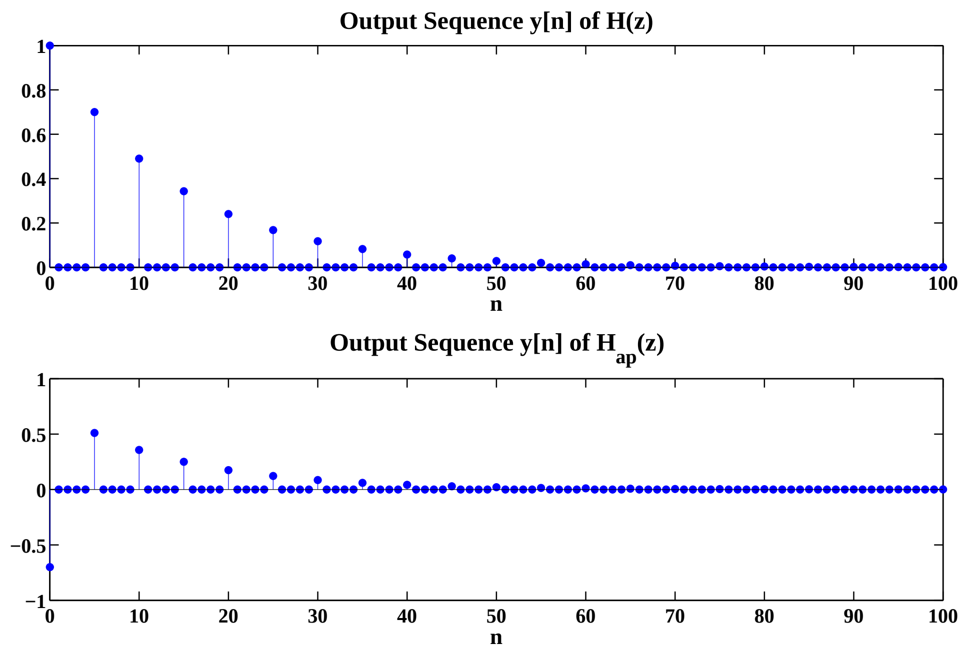


FIGURE 5.157: Output sequences to input (i).

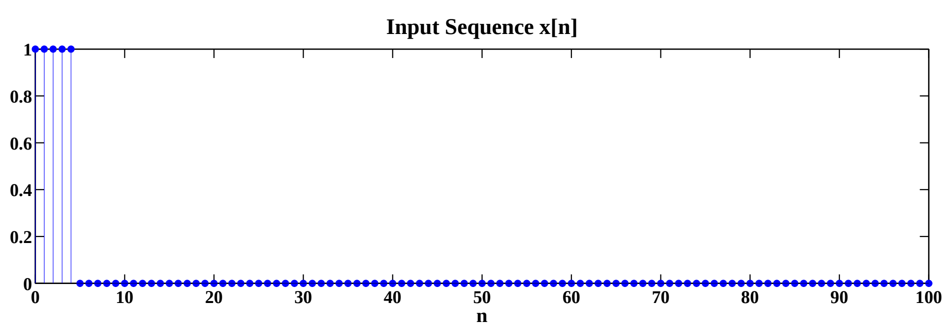


FIGURE 5.158: Input sequence (ii).

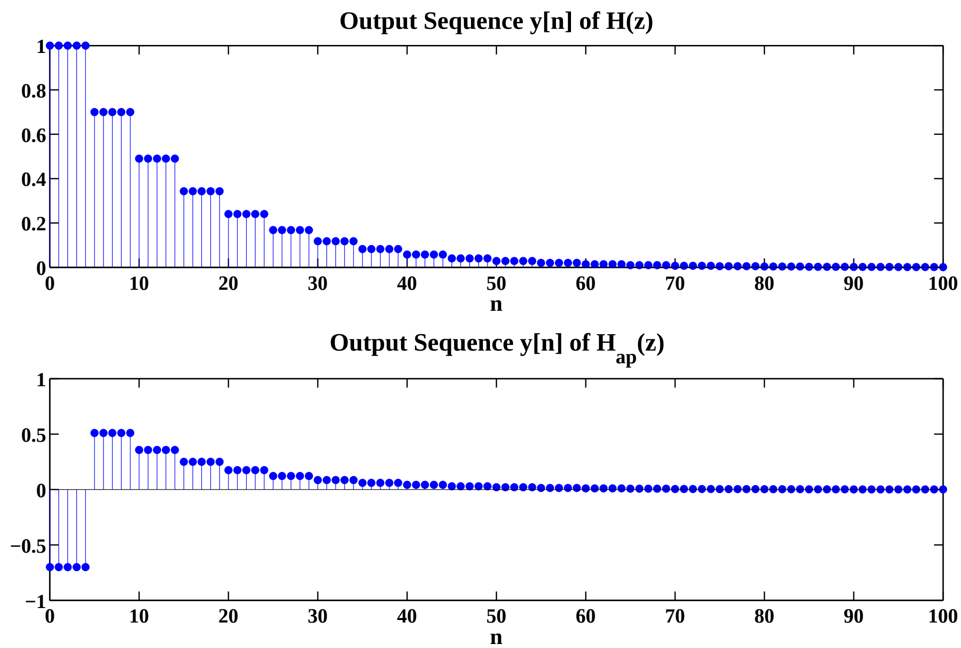


FIGURE 5.159: Output sequences to input (ii).

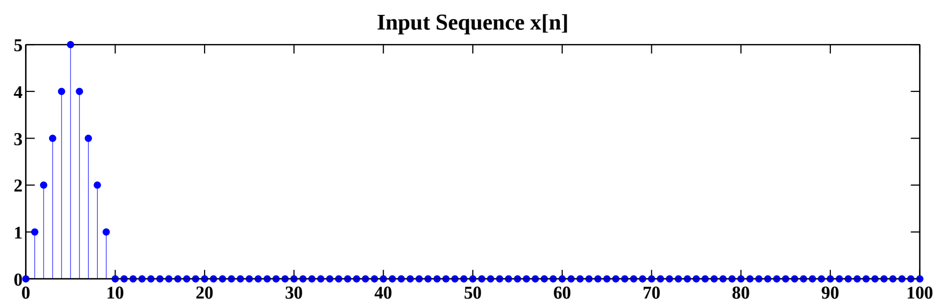


FIGURE 5.160: Input sequence (iii).

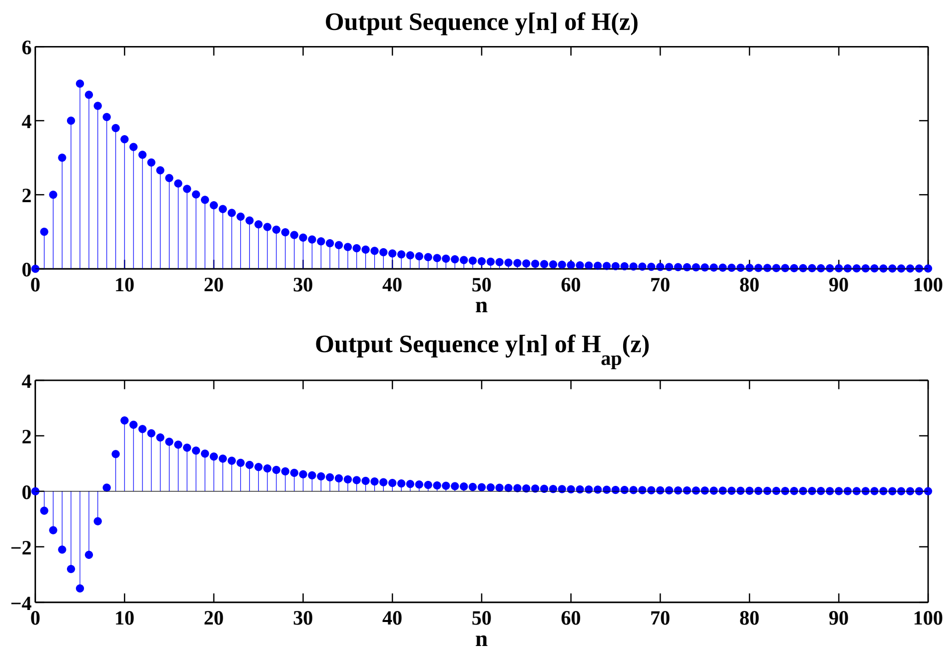


FIGURE 5.161: Output sequences to input (iii).

Review Problems

69. tba

70. (a) Solution:

$$G(z) = \frac{z^4 + 7z^3 + 20z^2 + 29z + 15}{z^5 + 4.1z^3 + 3.63z^2 + 2.015z + 0.63}$$

(b) tba.