CHAPTER 6

Sampling of Continuous-Time Signals

Tutorial Problems

1. (a) Solution:

$$x_{\mathrm{c}}(t) = \frac{5}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{\mathrm{j}200\pi t} + \frac{5}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{-\mathrm{j}200\pi t} + \frac{2}{\mathrm{j}}\mathrm{e}^{\mathrm{j}300\pi t} - \frac{2}{\mathrm{j}}\mathrm{e}^{-\mathrm{j}300\pi t}$$

The spectra of $x_{\mathrm{c}}(t)$ is:

$$Xc(\mathrm{j}\Omega) = \begin{cases} \frac{5}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}, & \Omega = 200\pi \\ \frac{5}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}, & \Omega = -200\pi \\ \frac{2}{\mathrm{j}}, & \Omega = 300\pi \\ -\frac{2}{\mathrm{j}}, & \Omega = -300\pi \\ 0, & \text{elsewhere} \end{cases}$$

The spectra $X\big(\mathrm{e}^{\mathrm{j}\omega}\big)$ of $x[n]$ is:

$$X\big(\mathrm{e}^{\mathrm{j}\omega}\big)\big|_{\omega=\Omega T} = F_s \sum_{k=-\infty}^{\infty} X_c(\mathrm{j}\Omega - \mathrm{j}2\pi k F_s)$$

$$X\big(\mathrm{e}^{\mathrm{j}\omega}\big)\big|_{\omega=2\pi FT} = F_s \sum_{k=-\infty}^{\infty} X_c[\mathrm{j}2\pi(F - kF_s)]$$

The signal can recovered for $x[n]$ if $F_{\mathrm{s}} = 1$ KHz.

(b) Solution:
The signal can recovered for $x[n]$ if $F_{\mathrm{s}} = 500$ Hz.

(c) Solution:
The signal can NOT recovered for $x[n]$ if $F_{\mathrm{s}} = 100$ Hz.

(d) tba.

MATLAB script:

```
% P0601: Illustrates the alias distortion
close all; clc
Fs = 1e3; % Part (a)
% Fs = 500; % Part (b)
% Fs = 100; % Part (c)
T = 1/Fs;
FH = 150;
FL = FH+Fs;
F = -FL:50:FL;
X = zeros(1,length(F));
for k = -1:1;
ind = F == -150+k*Fs; X(ind) = X(ind)-2/j;
ind = F == -100+k*Fs; X(ind) = X(ind)+5/2*exp(-j*pi/6);
ind = F == 100+k*Fs; X(ind) = X(ind)+5/2*exp(j*pi/6);
ind = F == 150+k*Fs; X(ind) = X(ind)+2/j;
end
ind = X==0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0601a');
subplot(211)
stem(F*2*pi*T,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F*2*pi*T,angle(X),'filled')
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('\angle H(e^{j\omega})|','fontsize',LFS)
title('Phase Response','fontsize',TFS)

hfb = figconfg('P0601b');
```

```
subplot(211)
stem(F,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi FT})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F,angle(X),'filled')
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('\angle H(e^{j2\pi FT})','fontsize',LFS)
title('Phase Response','fontsize',TFS)
```

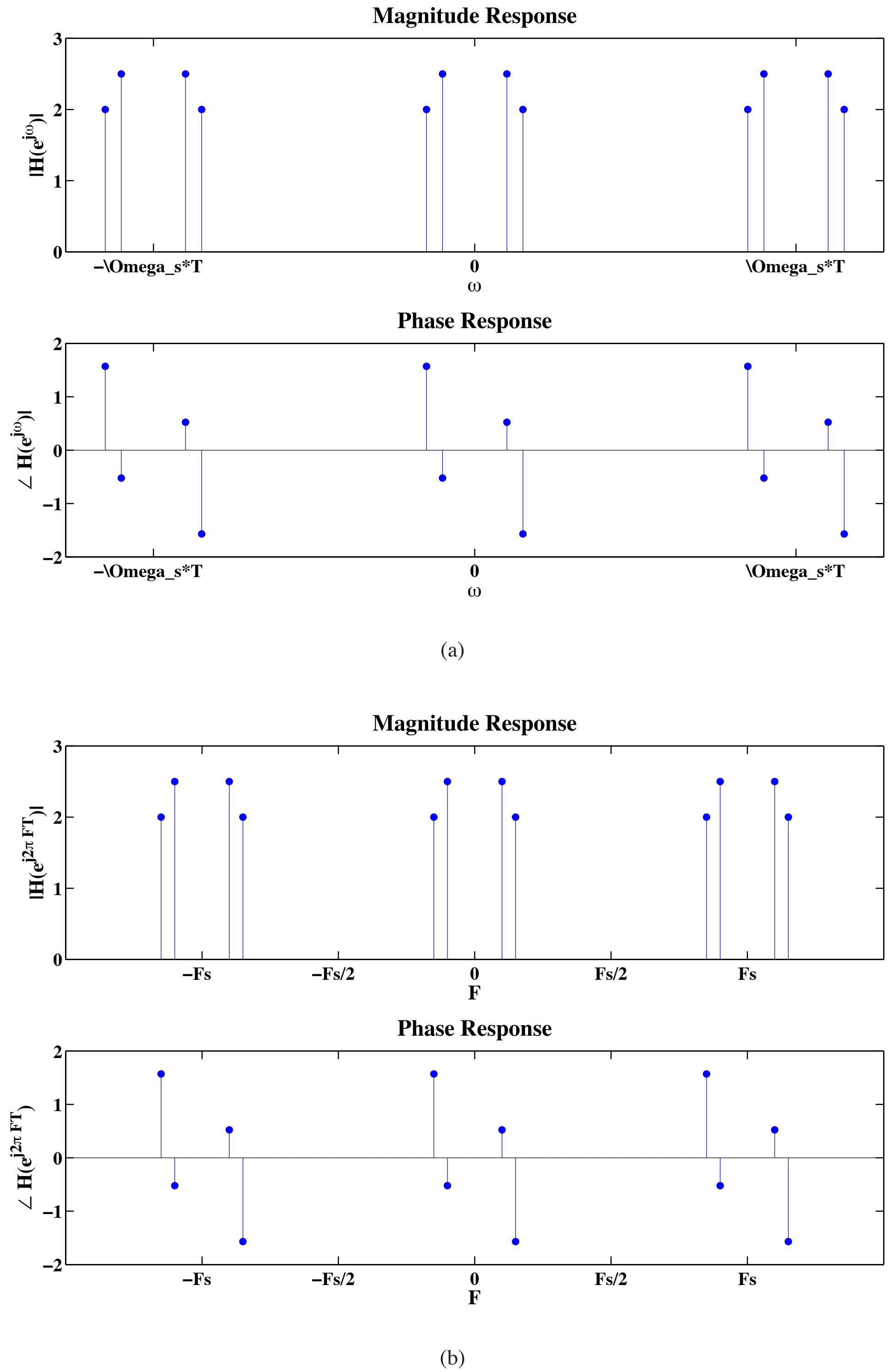
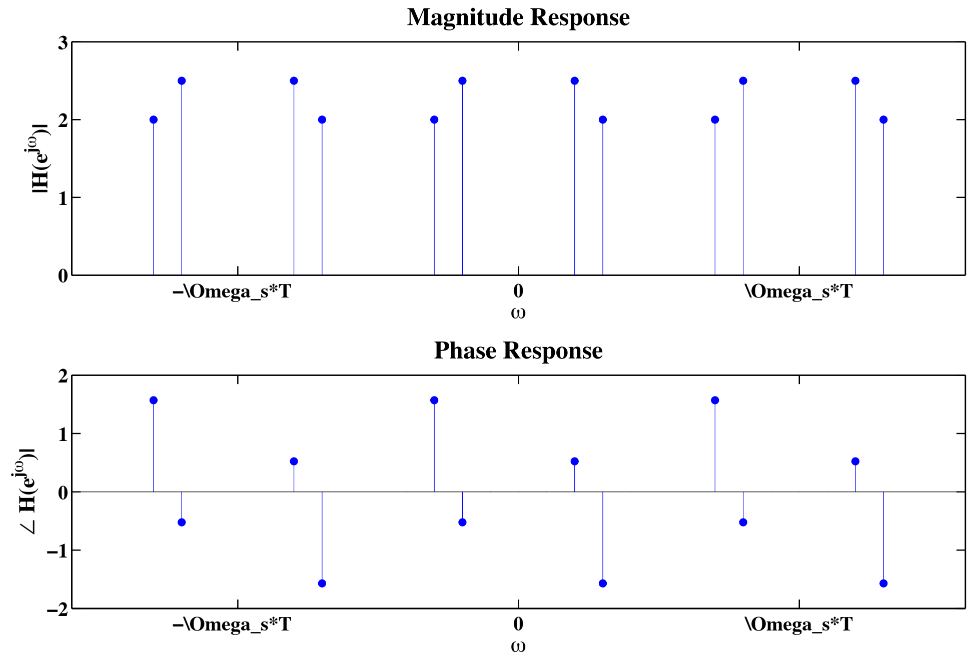
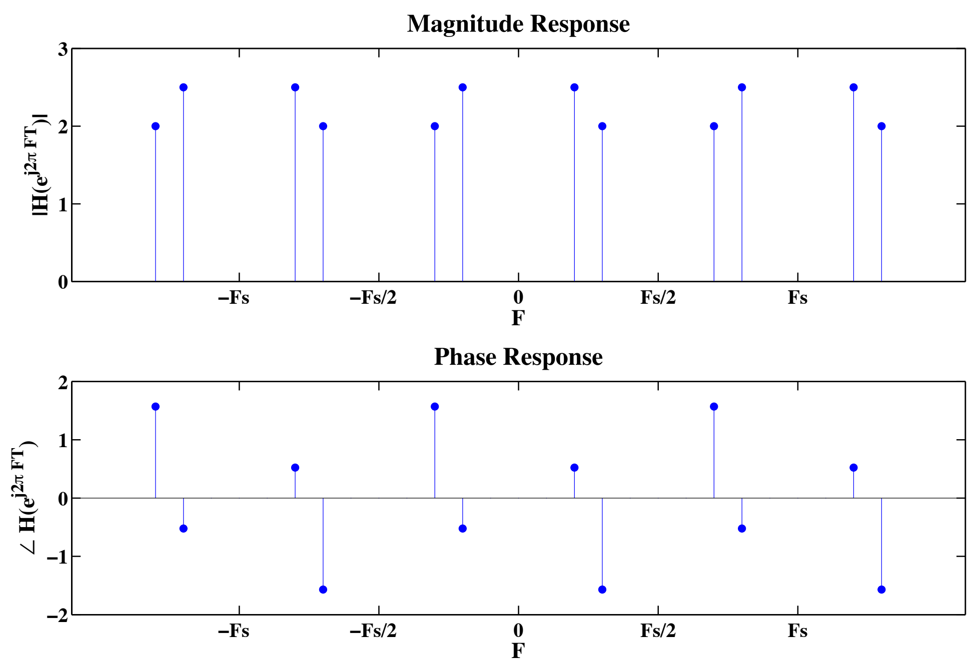


FIGURE 6.1: Spectra of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 1$ KHz.

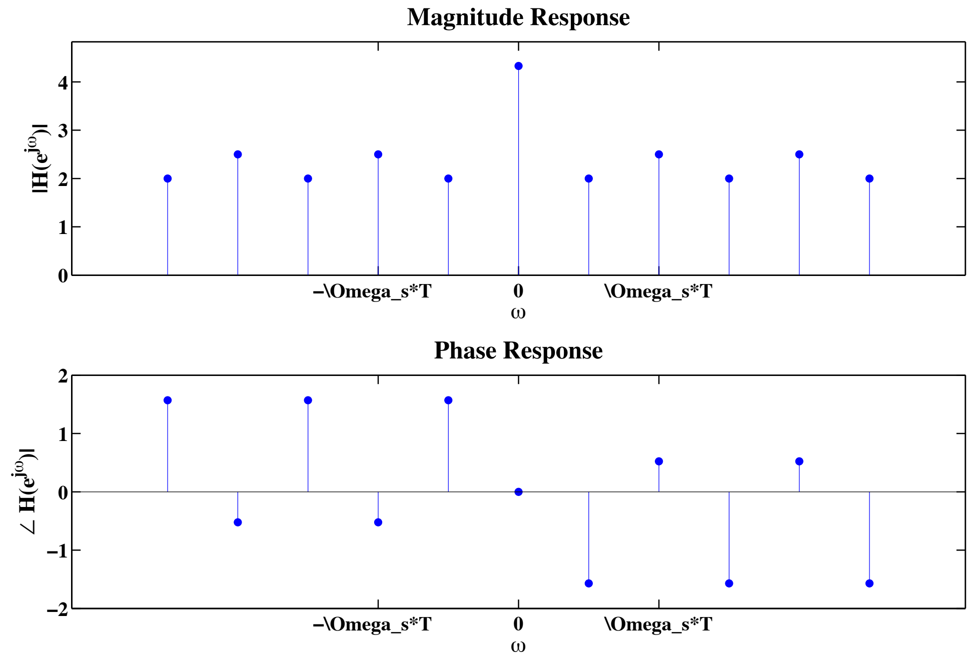


(a)

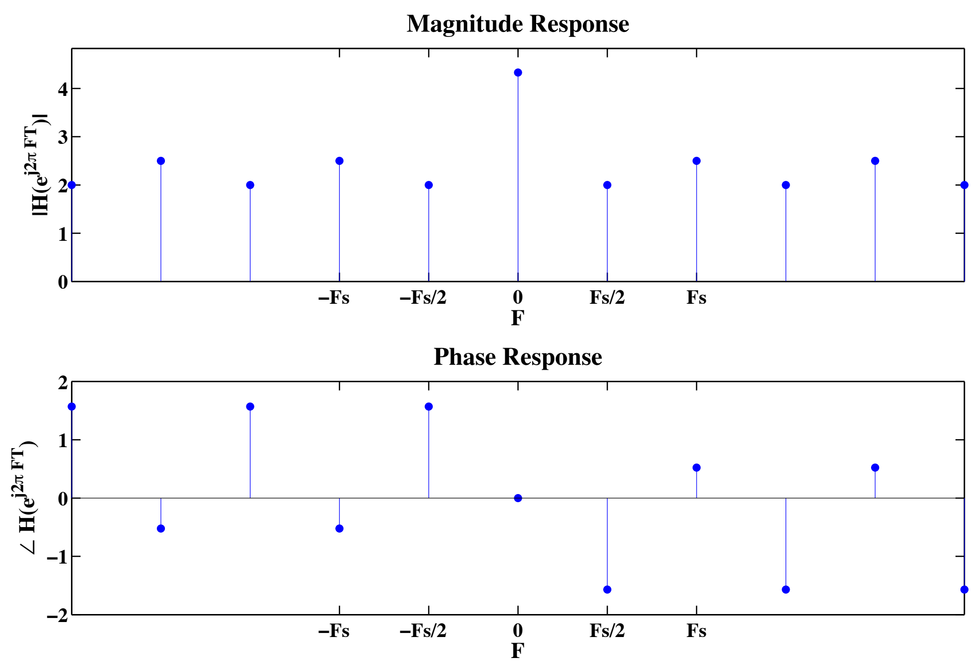


(b)

FIGURE 6.2: Spectra of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 500$ Hz.



(a)



(b)

FIGURE 6.3: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 100$ Hz.

2. (a) Solution:
The continuous signal $x_c(t)$ is:

$$x_c(t) = 5e^{-10|t|}$$

The sampled sequence $x[n]$ is:

$$x[n] = x_c(nT) = 5e^{-10|n|T} = 5a^{|n|}, \qquad \text{define } a = e^{-10T}$$

The spectra $X(e^{j\omega})$ of $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=2\pi FT} = 5 \cdot \frac{1-a^2}{1-2a\cos(2\pi FT)+a^2}$$

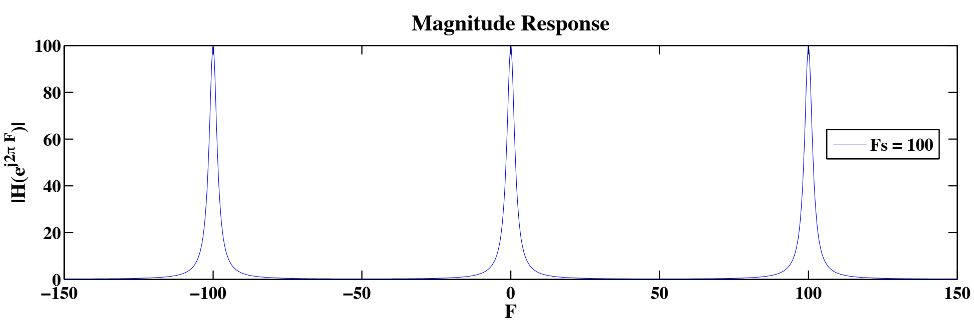


FIGURE 6.4: Magnitude response of $X(e^{j\omega})$ as a function of F in Hz when the sampling rate is $F_s = 100$.

(b) See plot below.

(c) See plot below.

(d) Solution:
For sampling rate $F_s = 100$ Hz, the signal $X_c(t)$ can be reasonably recovered from its samples $x[n]$.

MATLAB script:

```
% P0602: Plot the spectra of sampled sequence
close all; clc
% Fs = 100; % Part a
% Fs = 50; % Part b
Fs = 25; % Part c
T = 1/Fs;
```

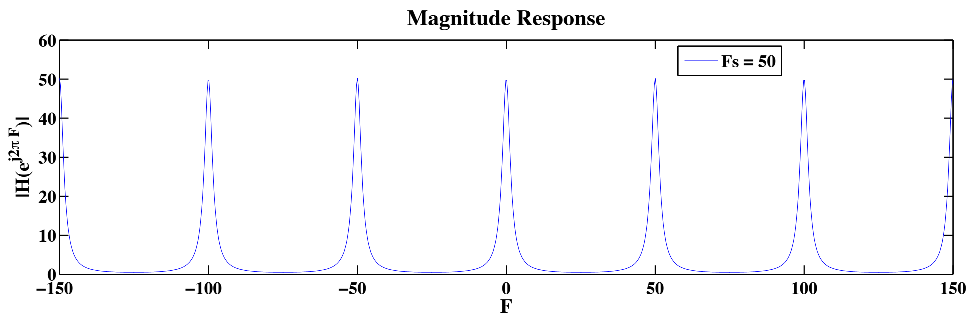


FIGURE 6.5: Magnitude response of $X(e^{j\omega})$ as a function of F in Hz when the sampling rate is $F_s = 50$.

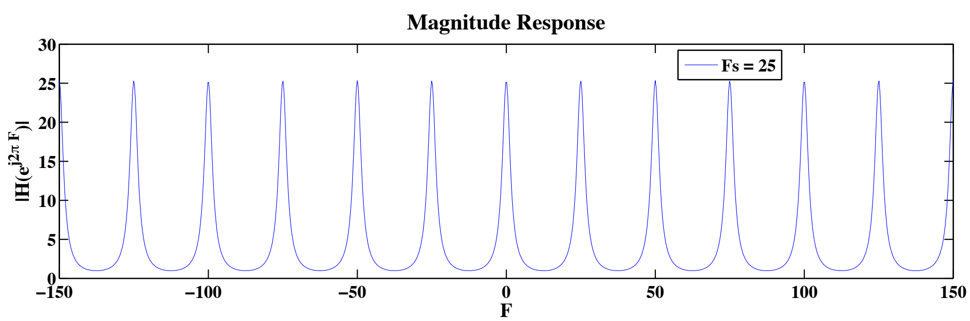


FIGURE 6.6: Magnitude response of $X(e^{j\omega})$ as a function of F in Hz when the sampling rate is $F_s = 25$.

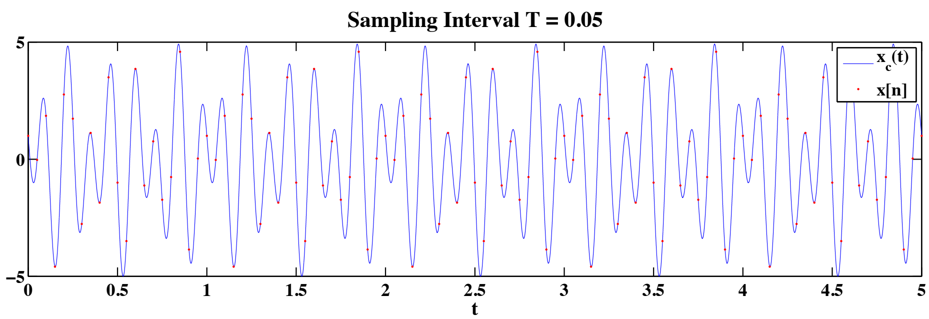
```
a = exp(-10*T);
F = linspace(-150,150,1000);
X = 5*(1-a^2)./(1-2*a*cos(2*pi*F*T)+a^2);
%% Plot:
hfa = figconfg('P0602a','long');
plot(F,abs(X))
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi F})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
legend(['Fs = ',num2str(Fs)],'location','best')
```

3. (a) Solution:

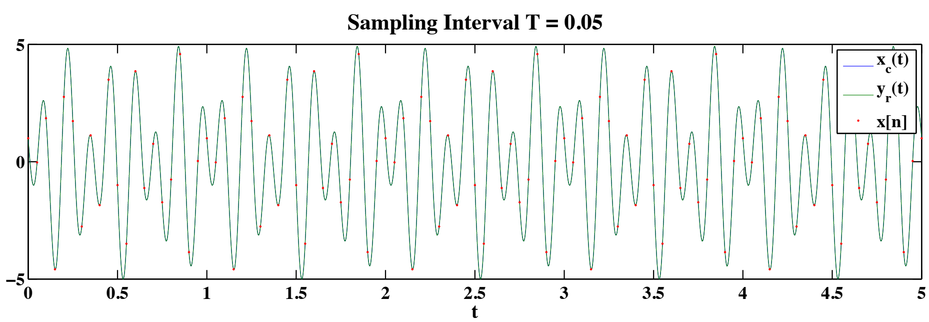
$$x[n] = x_c(0.05n) = 2\cos(0.5\pi n - \frac{\pi}{3}) - 3\sin(0.8\pi n)$$

(b) Solution:

$$y_r(t) = 2\cos(10\pi t - 60^\circ) - 3\sin(16\pi t)$$



(a)



(b)

FIGURE 6.7: (a) Plot of $x[n]$ and $x_c(t)$ and (b) plot of $y_r(t)$ when the continuous signal is sampled at $t = 0.05n$.

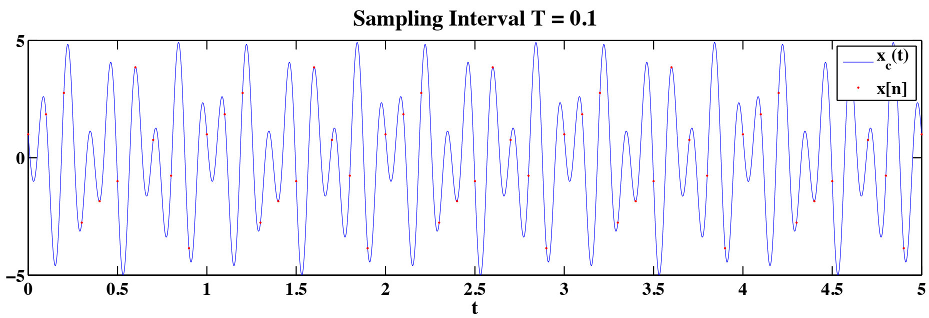
(c) Solution:

$$x[n] = x_c(0.1n) = 2\cos(\pi n - \frac{\pi}{3}) - 3\sin(1.6\pi n)$$

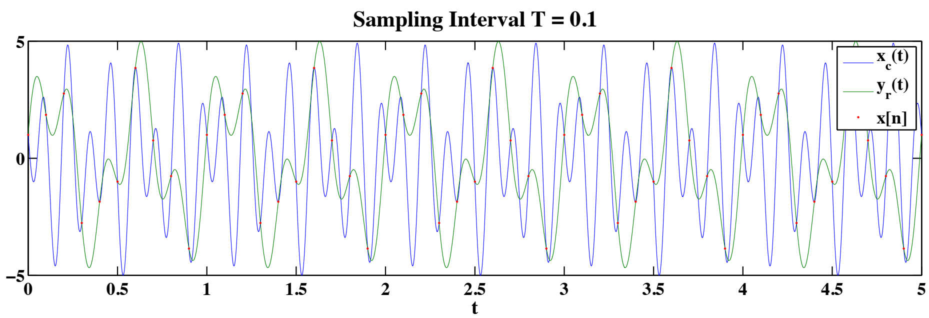
$$y_r(t) = 2\cos(10\pi t - 60^\circ) + 3\sin(4\pi t)$$

(d) Solution:

$$x[n] = x_c(0.5n) = 2\cos(5\pi n - \frac{\pi}{3}) - 3\sin(8\pi n)$$



(a)



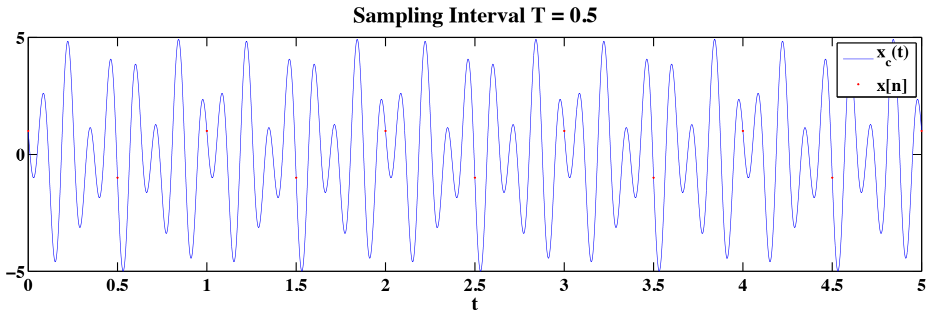
(b)

FIGURE 6.8: (a) Plot of $x[n]$ and $x_{\mathrm{c}}(t)$ and (b) plot of $y_{\mathrm{r}}(t)$ when the continuous signal is sampled at $t = 0.1n$.

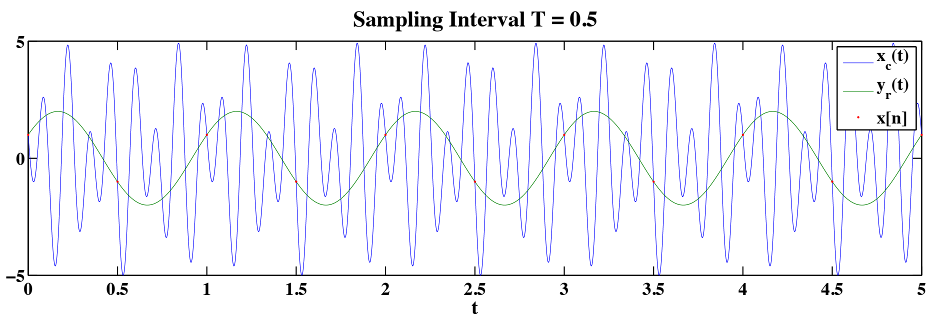
$$y_{\mathrm{r}}(t) = 2\cos(2\pi t - 60^\circ)$$

MATLAB script:

```
% P0603: Illustrate Ideal DAC
close all; clc
t1 = 0; t2 = 5;
dt = 1e-4;
t = t1:dt:t2;
xc = 2*cos(10*pi*t-pi/3)-3*sin(16*pi*t);
%% Part (a) and (b)
T = 0.05;
yr = 2*cos(10*pi*t-pi/3)-3*sin(16*pi*t);
%% Part (c)
% T = 0.1;
% % yr = 2*cos(10*pi*t)+3*sin(4*pi*t);
```



(a)



(b)

FIGURE 6.9: (a) Plot of $x[n]$ and $x_{\mathrm{c}}(t)$ and (b) plot of $y_{\mathrm{r}}(t)$ when the continuous signal is sampled at $t = 0.5n$.

```
% yr = 2*cos(10*pi*t-pi/3)+3*sin(4*pi*t);
%% Part (d)
% T = 0.5;
% yr = 2*cos(2*pi*t-pi/3);

% Sampling:
nT = t1:T:t2;
xn = 2*cos(10*pi*nT-pi/3)-3*sin(16*pi*nT);
%% Plot:
hfa = figconfg('P0603a','long');
plot(t,xc); hold on
plot(nT,xn,'.','color','red')
xlabel('t','fontsize',LFS)
title(['Sampling Interval T = ',num2str(T)],'fontsize',TFS)
legend('x_c(t)','x[n]','location','northeast')
```

```
hfb = figconfg('P0603b','long');
plot(t,xc,t,yr,nT,xn,'.')
xlabel('t','fontsize',LFS)
title(['Sampling Interval T = ',num2str(T)],'fontsize',TFS)
legend('x_c(t)','y_r(t)','x[n]','location','northeast')
```

4. (a) Solution:
   If $F_0 = 10, 20,$ and 40 Hz, $2F_0 < F_s = 100$,

$$y_r(t) = x_c(t) = \cos(2\pi F_0 t + \theta_0)$$

   (b) tba.

   (c) Solution:
   When $2F_0 = F_s$,

$$y_r(t) = 2\cos\theta_0 \cdot \cos 2\pi F_0 t$$

5. The same to last problem P0604

6. MATLAB script:

```
% P0606: Sampling a linear FM signal
close all; clc
B = 10;
Fs = B;
tau = 10;
dt = 1e-4;
t = 0:dt:tau;
xc = sin(pi*B*t.^2/tau);
nT = 0:1/Fs:tau;
xn = sin(pi*B*nT.^2/tau);
ind = B*t/tau > Fs/2;
yr = xc;
yr(ind) = -sin(2*pi*(Fs-B*t(ind)/tau/2).*t(ind));
%% Plot:
hfa = figconfg('P0606a','long');
plot(t,xc)
xlabel('t (sec)','fontsize',LFS)
ylabel('x_c(t)','fontsize',LFS)

hfb = figconfg('P0606b','long');
```

```
plot(nT,xn,'.-');
xlabel('t (sec)','fontsize',LFS)
ylabel('x(nT)','fontsize',LFS)

hfc = figconfg('P0606c','long');
plot(t,yr); hold on
plot(nT,xn,'.')
ylim([-1 1])
xlabel('t (sec)','fontsize',LFS)
ylabel('x_r(t)','fontsize',LFS)
```

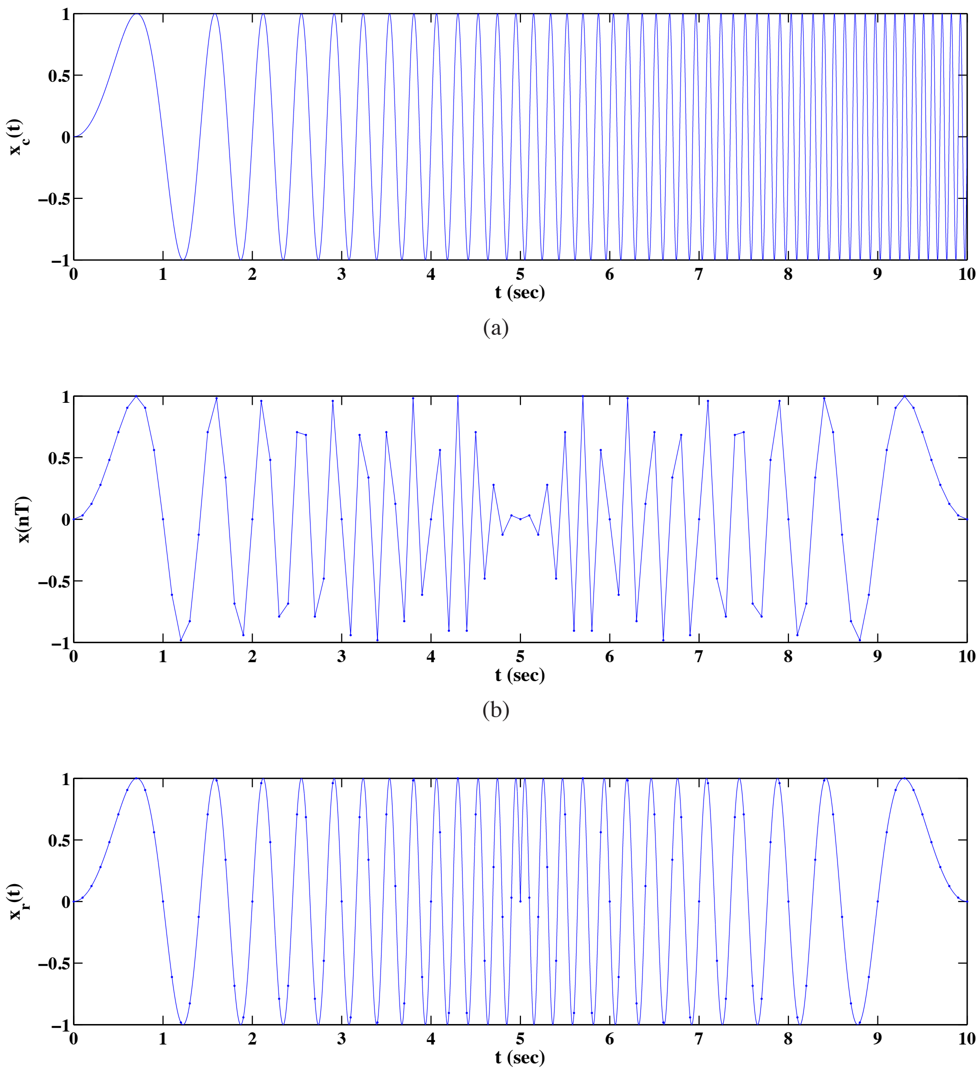


FIGURE 6.10: (a) Continuous signal $x_c(t)$, (b) sampled sequence $x[n]$, and (c) reconstructed signal $x_r(t)$.

7. (a) See plot below.

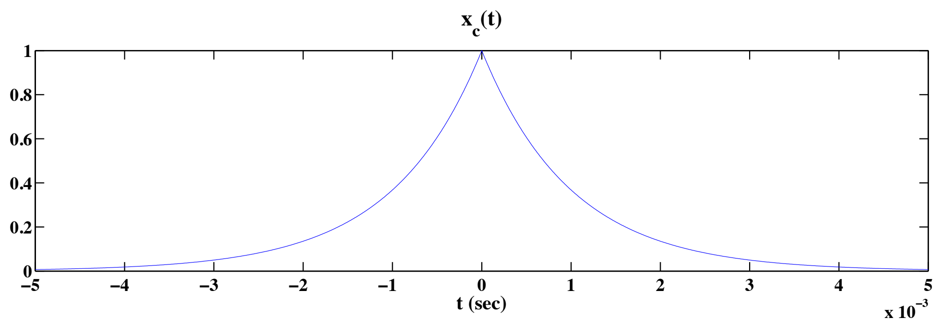


FIGURE 6.11: Plot of the signal $x_{\mathrm{c}}(t)$.

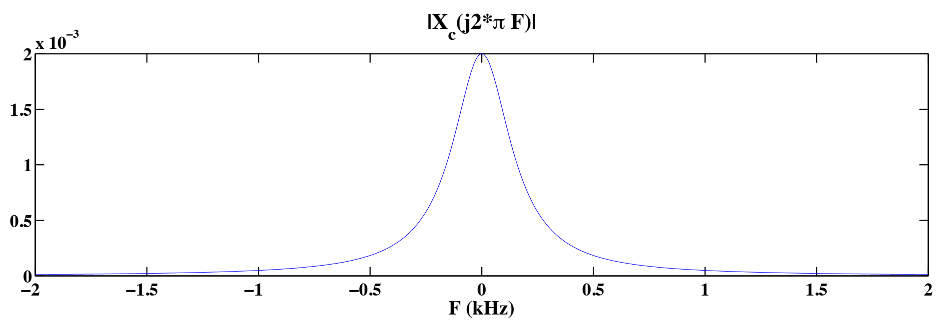


FIGURE 6.12: Plot of the CTFT $X_{\mathrm{c}}(\mathrm{j}2\pi F)$ of signal $x_{\mathrm{c}}(t)$.

(b) See plot below.

(c) See plot below.

(d) See plot below.

(e) See plot below.

MATLAB script:

```
% P0607: Sampling a exponential decaying signal
close all; clc
%% ii = 1, Fs = 1000; ii = 2, Fs = 5000
ii = 2;
t1 = -5e-3; t2 = 5e-3;
dt = 1e-5;
```

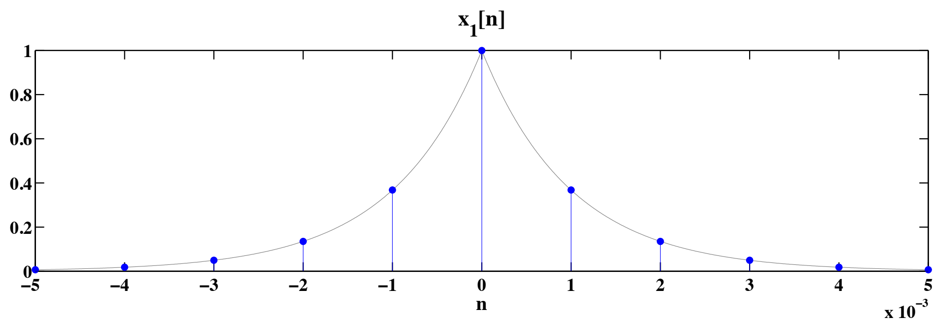


FIGURE 6.13: Plot of sampled signal $x_1[n]$.

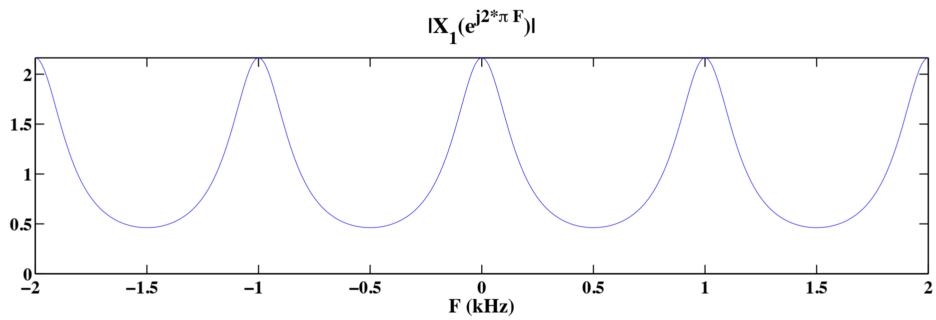


FIGURE 6.14: Plot of the CTFT $X_1(\mathrm{e}^{\mathrm{j}2\pi F})$ of sampled signal $x_1[n]$.

```
t = t1:dt:t2;
xc = exp(-1000*abs(t));
F = linspace(-2e3,2e3,1000);
Xc = 0.002./(1+(0.002*pi*F).^2);
Fs = [1e3 5e3];
nT = t1:1/Fs(ii):t2;
xn = exp(-1000*abs(nT));
a = exp(-1000/Fs(ii));
X = (1-a^2)./(1-2*a*cos(2*pi*F/Fs(ii))+a^2);
[G1,G2] = meshgrid(t,nT);
S = sinc(Fs(ii)*(G1-G2));
yr = xn*S;

%% Plot:
hfa = figconfg('P0607a1','long');
```

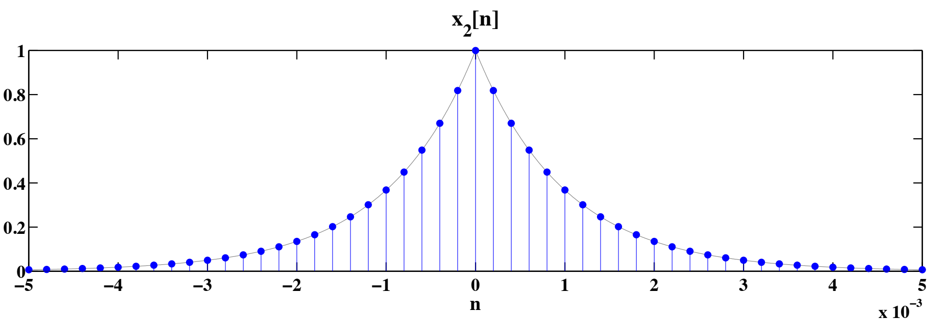


FIGURE 6.15: Plot of sampled signal $x_2[n]$.

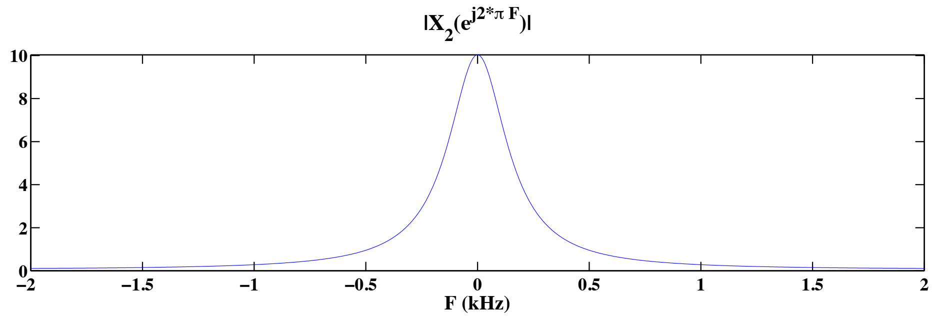


FIGURE 6.16: Plot of the CTFT $X_2(e^{j2\pi F})$ of sampled signal $x_2[n]$.

```
plot(t,xc)
xlabel('t (sec)','fontsize',LFS)
title('x_c(t)','fontsize',TFS)
hfb = figconfg('P0607a2','long');
plot(F/1000,abs(Xc))
xlabel('F (kHz)','fontsize',LFS)
title('|X_c(j2*\pi F)|','fontsize',TFS)

hfc = figconfg('P0607b1','long');
plot(t,xc,'color',[1 1 1]*0.5);hold on
stem(nT,xn,'filled')
xlabel('n','fontsize',LFS)
title(['x_',num2str(ii),'[n]'],'fontsize',TFS)
hfd = figconfg('P0607b2','long');
plot(F/1000,abs(X))
```

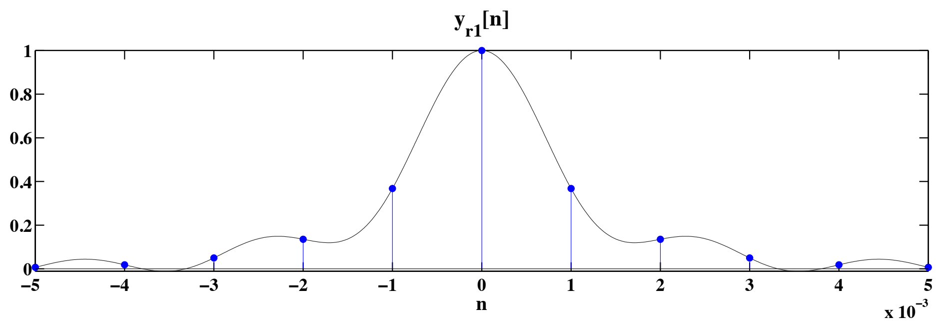


FIGURE 6.17: Reconstructed signal $y_{\mathrm{r}1}(t)$ from samples $x_1[n]$.

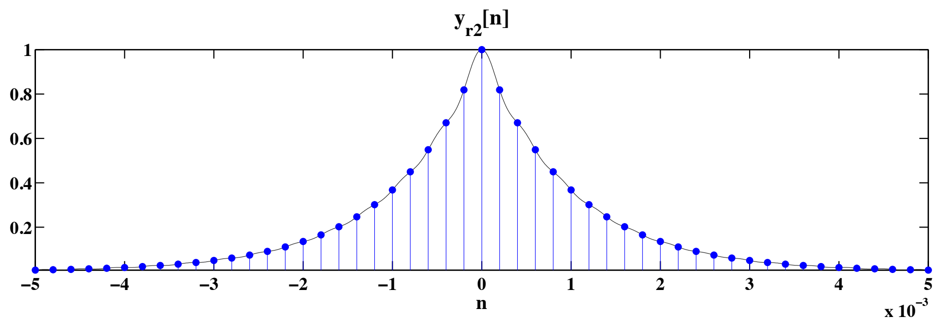


FIGURE 6.18: Reconstructed signal $y_{\mathrm{r}2}(t)$ from samples $x_2[n]$.

```
ylim([0 max(abs(X))])
xlabel('F (kHz)','fontsize',LFS)
title(['|X_',num2str(ii),'(e^{j2*\pi F})|'],'fontsize',TFS)

hfe = figconfg('P0607e','long');
plot(t,yr,'color',[1 1 1]*0.1);hold on
stem(nT,xn,'filled')
ylim([min(yr) max(yr)])
xlabel('n','fontsize',LFS)
title(['y_{r',num2str(ii),'}[n]'],'fontsize',TFS)
```

8. Proof:

$$x_c(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T_0}kt}$$

$$x_c(nT) = x_c(nT_0/N) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T_0}k\frac{nT0}{N}} = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{N}kn}$$

$$x[n] = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn}$$

Since we have $x[n] = x_c(nT)$, we require that

$$\sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{N}kn} = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn}$$

$$\begin{aligned}\sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn} &= \sum_{k=0}^{N-1}\left(\sum_{\ell=-\infty}^{\infty} c_{k-\ell N}\right) e^{j\frac{2\pi}{N}kn} = \sum_{\ell=-\infty}^{\infty}\left(\sum_{k=0}^{N-1} c_{k-\ell N} e^{j\frac{2\pi}{N}kn}\right)\\ &= \sum_{\ell=-\infty}^{\infty}\left(\sum_{k=0}^{N-1} c_{k-\ell N} e^{j\frac{2\pi}{N}kn} e^{j\frac{2\pi}{N}(-\ell N)}\right)\\ &= \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{N}kn}\end{aligned}$$

Hence, we prove that

$$\tilde{c}_k = \sum_{\ell=-\infty}^{\infty} c_{k-\ell N},\ k = 0, \pm 1, \pm 2, \dots$$

9. Proof:

$$\begin{aligned}\sum_{n=-\infty}^{\infty} y[n]\delta[t-nT] * g_{BL}(t) &= \sum_{\tau=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} y[n]\delta[\tau-nT]\cdot g_{BL}(t-\tau)\\ &= \sum_{\tau=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} y[n]\delta[t-\tau-nT]\cdot g_{BL}(\tau)\\ &= \sum_{n=-\infty}^{\infty} y[n]\left(\sum_{\tau=-\infty}^{\infty}\delta[t-\tau-nT]\cdot g_{BL}(\tau)\right)\\ &= \sum_{n=-\infty}^{\infty} y[n]\cdot g_{BL}(t-nT)\end{aligned}$$

10. (a) Solution:

The frequency response of $h_c(t)$ is:

$$H_{\mathrm{c}}(\mathrm{j}2\pi F) = \frac{\Omega_n^2}{\Omega_n^2 - (2\pi F)^2 + \mathrm{j}2\zeta\Omega_n 2\pi F}$$

Its phase response is:

$$\angle H_{\mathrm{c}}(\mathrm{j}2\pi F) = -\tan^{-1}\frac{2\zeta\Omega_n 2\pi F}{\Omega_n^2 - (2\pi F)^2}$$

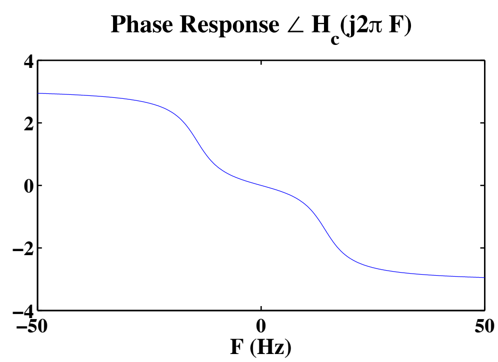


FIGURE 6.19: Plot of phase response $\angle H_{\mathrm{c}}(\mathrm{j}2\pi F)$.

(b) Solution:

The spectra of the sampled sequence is:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{\mathrm{e}^{-\zeta\Omega_n T}\sin(\Omega_n T\sqrt{1-\zeta^2})\mathrm{e}^{-\mathrm{j}\omega}}{1 - 2\mathrm{e}^{-\zeta\Omega_n T}\cos(\Omega_n T\sqrt{1-\zeta^2})\mathrm{e}^{-\mathrm{j}\omega} + \mathrm{e}^{-2\zeta\Omega_n T}\mathrm{e}^{-2\mathrm{j}\omega}}$$

Its phase response is:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) = -\omega - \tan^{-1}\frac{2\mathrm{e}^{-\zeta\Omega_n T}\cos(\Omega_n T\sqrt{1-\zeta^2})\sin\omega - \mathrm{e}^{-2\zeta\Omega_n T}\sin 2\omega}{1 - 2\mathrm{e}^{-\zeta\Omega_n T}\cos(\Omega_n T\sqrt{1-\zeta^2})\cos\omega + \mathrm{e}^{-2\zeta\Omega_n T}\cos 2\omega}$$

The effective phase response is:

$$\angle H_{\mathrm{eff}}(\mathrm{j}2\pi F) = \begin{cases} \angle H(\mathrm{e}^{\mathrm{j}2\pi FT}), & |F| \le \frac{F_s}{2} \\ 0, & |F| > \frac{F_s}{2} \end{cases}$$

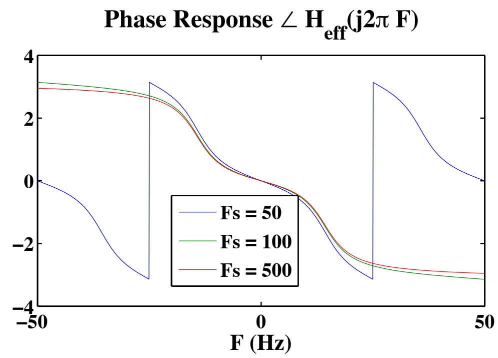


FIGURE 6.20: Plot of the effective phase responses, $\angle H_{\text{eff}}(\mathrm{j}2\pi F)$ for $F_s = 50$, 100, and 500 Hz.

(c) tba.

MATLAB script:

```
% P0610: Example 6.6: Second-order system
close all; clc
Fs = [50 100 500];
zeta = 0.3; Omega_n = 30*pi;
F = linspace(-50,50,1000);
Hc = Omega_n^2./((j*2*pi*F).^2+2*zeta*Omega_n*j*2*pi*F+Omega_n^2);
[FG FsG] = meshgrid(F,Fs);
H = Omega_n/sqrt(1-zeta^2)*exp(-zeta*Omega_n./FsG)...
    .*sin(Omega_n./FsG*sqrt(1-zeta^2)).*exp(-j*2*pi*FG./FsG)...
    ./(1-2*exp(-zeta*Omega_n./FsG).*cos(Omega_n./FsG*sqrt(1-zeta^2))...
    .*exp(-j*2*pi.*FG./FsG)+exp(-2*zeta*Omega_n./FsG)...
    .*exp(-2*j*2*pi.*FG./FsG));
%% Plot:
hfb = figconfg('P0610a','small');
plot(F,angle(Hc));
xlabel('F (Hz)','fontsize',LFS)
title('Phase Response \angle H_c(j2\pi F)','fontsize',TFS)
hfc = figconfg('P0610b','small');
plot(F,angle(H));
xlabel('F (Hz)','fontsize',LFS)
title('Phase Response \angle H_{eff}(j2\pi F)','fontsize',TFS)
legend(['Fs = ',num2str(Fs(1))],['Fs = ',num2str(Fs(2))],...
```

```
['Fs = ',num2str(Fs(3))],'location','best')
```

11. tba

12. (a) Solution:
    The quantizer resolution is:

$$\frac{10\text{v}}{2^8} = 0.0390625\text{v}$$

    (b) Solution:

$$\text{SQNR} = 10\log_{10}\text{SQNR} = 6.02B + 1.76 = 6.02\times 8 + 1.76 = 49.92\text{dB}$$

    (c) Solution:
    The sampling rate is:

$$F_s = \frac{2^{11}}{2^3} = 2^8\text{sam/sec}$$

    The folding frequency is $F_s/2 = 2^7$.
    The Nyquist rate is 500.

    (d) Solution:
    The reconstructed signal $y_c(t)$ is:

$$y_c(t) = 2\cos(200\pi t) - 3\sin(12\pi t)$$

13. Proof:
    (i) Linearity.

$$a_1 \cdot x_{\text{in1}}(nT) + a_2 \cdot x_{\text{in2}}(nT) = a_1 \cdot x_{\text{out1}}(t) + a_2 \cdot x_{\text{out2}}(t)$$

    The S&H system follows the superposition property, and hence is a linear system.
    (ii) Time-variance.

$$x_{\text{out}}(t-\tau) \neq x_{\text{out}}(t), \text{if } t-\tau \notin [nT, (n+1)T]$$

    Hence, the system is time-varying.

14. (a) Solution:

$$g_{\mathrm{SH}}(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases} \xrightarrow{\text{CTFT}} G_{\mathrm{SH}}(\mathrm{j}\Omega) = \frac{2\sin(\Omega T/2)}{\Omega}\mathrm{e}^{-\mathrm{j}\Omega T/2}$$

$$H_{\mathrm{r}}(\mathrm{j}\Omega) = \begin{cases} \frac{\Omega T/2}{\sin(\Omega T/2)} \cdot \mathrm{e}^{\mathrm{j}\Omega T/2}, & |\Omega| < \pi/T \\ 0, & \text{otherwise} \end{cases}$$

The frequency response is:

$$H_{\mathrm{r}}(\mathrm{e}^{\mathrm{j}\omega}) = \begin{cases} \frac{\omega/2}{\sin(\omega/2)\cdot \mathrm{e}^{\mathrm{j}\omega/2}}, & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}$$

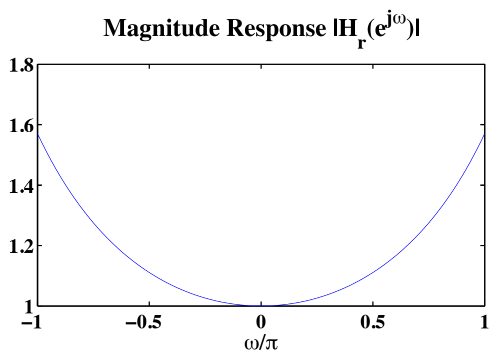


FIGURE 6.21: Magnitude response of ideal digital filter $H_{\mathrm{r}}(\mathrm{e}^{\mathrm{j}\omega})$.

(b) Solution: The magnitude response of $H_{\mathrm{FIR}}(\mathrm{e}^{\mathrm{j}\omega})$ is:

$$\begin{aligned} |H_{\mathrm{FIR}}(\mathrm{e}^{\mathrm{j}\omega})| &= \sqrt{(-\frac{1}{16} + \frac{9}{8}\cos\omega - \frac{1}{16}\cos 2\omega)^2 + (-\frac{9}{8}\sin\omega + \frac{1}{16}\sin 2\omega)^2} \\ &= \sqrt{\frac{1}{16^2} + \frac{9^2}{8^2} + \frac{1}{16^2} - 4\times\frac{9}{8}\times 116\cos\omega + \frac{2}{16^2}\cos 2\omega} \end{aligned}$$

(c) Solution: The magnitude response of $H_{\mathrm{IIR}}(\mathrm{e}^{\mathrm{j}\omega})$ is:

$$|H_{\mathrm{IIR}}(\mathrm{e}^{\mathrm{j}\omega})| = \frac{9}{\sqrt{(8+\cos\omega)^2 + \sin^2\omega}} = \frac{9}{\sqrt{1+8^2+2\cos\omega}}$$

MATLAB script:

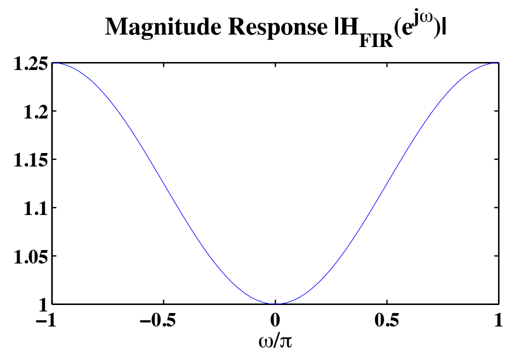


FIGURE 6.22: Magnitude response of low-order FIR filter $H_{\mathrm{FIR}}(\mathrm{e}^{\mathrm{j}\omega})$.

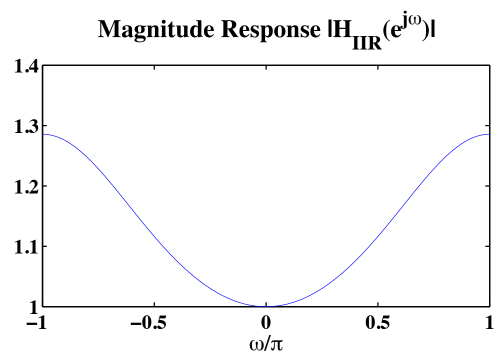


FIGURE 6.23: Magnitude response of low-order IIR filter $H_{\mathrm{IIR}}(\mathrm{e}^{\mathrm{j}\omega})$.

```
% P0614: Investigate droop distortion compensation
close all; clc
w = linspace(-1,1,1000)*pi;
%% Part (a):
Hr = w/2./sin(w/2).*exp(j*w/2);
%% Part (b):
HFIR = -1/16+9/8*exp(-j*w)-1/16*exp(-2*j*w);
%% Part (c):
HIIR = 9./(8+exp(-j*w));
%% Plot:
hfa = figconfg('P0614a','small');
plot(w/pi,abs(Hr));
xlabel('\omega/\pi','fontsize',LFS)
```

```
title('Magnitude Response |H_r(e^{j\omega})|','fontsize',TFS)
hfb = figconfg('P0614b','small');
plot(w/pi,abs(HFIR));
xlabel('\omega/\pi','fontsize',LFS)
title('Magnitude Response |H_{FIR}(e^{j\omega})|','fontsize',TFS)
hfc = figconfg('P0614c','small');
plot(w/pi,abs(HIIR));
xlabel('\omega/\pi','fontsize',LFS)
title('Magnitude Response |H_{IIR}(e^{j\omega})|','fontsize',TFS)
```

15. Solution:

$$g_{\mathrm{r}}(t) = \frac{\sin(\pi Bt)}{\pi Bt}\cos(2\pi F_c t) \tag{6.74}$$

The spectra is:

$$G_{\mathrm{r}}(\mathrm{j}2\pi F) = \begin{cases} 1, & |F| \in [F_L, F_H] \\ 0, & \text{otherwise} \end{cases}$$

Baseband spectra is:

$$G(\mathrm{j}2\pi F) = \begin{cases} T, & |F| \le B/2 \\ 0, & |F| > B/2 \end{cases}$$

The continuous time signal is

$$g(t) = \frac{\sin(\pi tB)}{\pi tB}$$

We can conclude that

$$G_{\mathrm{r}}(\mathrm{j}2\pi F) = G[\mathrm{j}2\pi(F + F_c)] + G[\mathrm{j}2\pi(F - F_c)]$$

Hence,

$$\begin{aligned} g_{\mathrm{r}}(t) &= \frac{\sin(\pi Bt)}{\pi Bt}(\mathrm{e}^{-\mathrm{j}2\pi F_c t} + \mathrm{e}^{\mathrm{j}2\pi F_c t}) \\ &= \frac{\sin(\pi Bt)}{\pi Bt}\cos(2\pi F_c t) \end{aligned}$$

where

$$F_c = \frac{F_H + F_L}{2}, \qquad B = F_H - F_L$$

16. (a) See plot below.

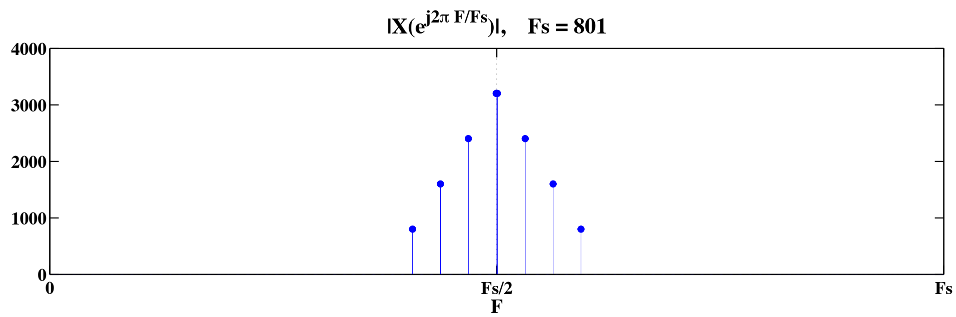


FIGURE 6.24: Spectrum of the sampled signal as a function of F Hz when the sampling rate is $F_s = 801$.

(b) See plot below.

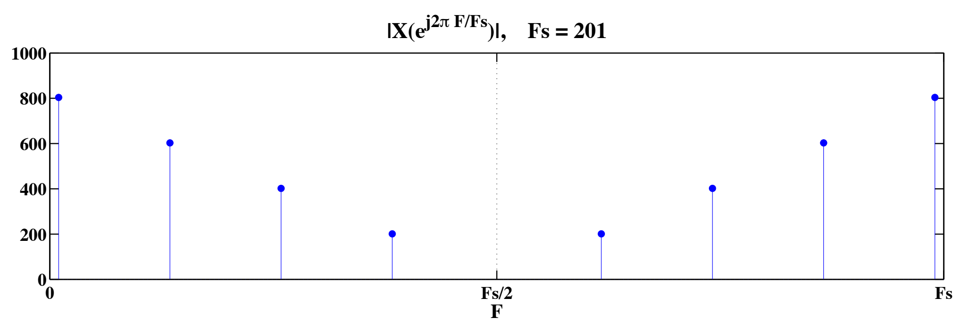


FIGURE 6.25: Spectrum of the sampled signal as a function of F Hz when the sampling rate is $F_s = 201$.

(c) tba.

```
% P0616: Sampling Illustration
close all; clc
%% Part (a):
% Fs = 801;
%% Part (b):
Fs = 201;
dF = 1;
F = 0:dF:Fs;
```

```
X = zeros(size(F));
FcosF = [325 350 375 400];
Fcos = [1 2 3 4];
while any(FcosF > Fs/2)
    ind = FcosF > Fs/2;
    FcosF(ind) = abs(FcosF(ind) - Fs);
end
for jj = -1:1
for ii = 1:length(FcosF)
    ind = abs(F)==abs(FcosF(ii)+jj*Fs);
    X(ind) = X(ind) + Fcos(ii)*Fs;
end
end
ind = X==0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0616a','long');
stem(F,abs(X),'filled');
xlim([0 Fs])
set(gca,'Xtick',[0 Fs/2 Fs])
set(gca,'Xticklabel',{'0','Fs/2','Fs'})
set(gca,'XGrid','on')
xlabel('F','fontsize',LFS)
title(['|X(e^{j2\pi F/Fs})|,    Fs = ',num2str(Fs)],'fontsize',TFS)
```

17. Solution:

$$F'_{\mathrm{L}} = 105 - 5 = 100\text{Hz}, \qquad F'_{\mathrm{H}} = 145 + 5 = 150\text{Hz}$$

The bandwidth is

$$B = F'_{\mathrm{H}} - F'_{\mathrm{L}} = 50\text{Hz}$$

The minimum sampling rate is computed by

$$\min F_{\mathrm{s}} = 2F'_{\mathrm{H}}/\lfloor F'_{\mathrm{H}}/B\rfloor = 100\text{Hz}$$

MATLAB script:

```
% P0617: Sampling Illustration
close all; clc
```

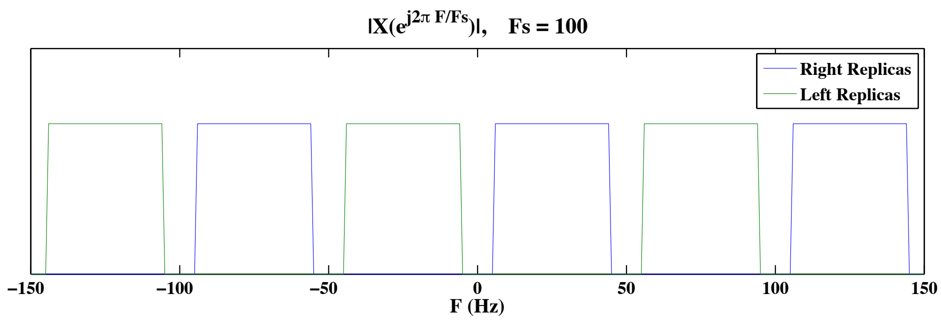


FIGURE 6.26: Spectrum of the sampled signal as a function of F Hz.

```
FL = 105; FH = 145;
dF = 1;
F = -150:dF:150;
Fs = 100;
X = zeros(size(F));
XP = zeros(size(F));
XN = zeros(size(F));
for jj = -10:10
    ind = F > 105+jj*Fs & F < 145+jj*Fs;
    X(ind) = X(ind) + 1;
    XP(ind) = XP(ind) + 1;
    ind = F > -145+jj*Fs & F < -105+jj*Fs;
    X(ind) = X(ind) + 1;
    XN(ind) = XN(ind) + 1;
end
ind = X == 0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0617a','long');
plot(F,XP,F,XN);
ylim([0 1.5])
set(gca,'YTick',-1)
xlabel('F (Hz)','fontsize',LFS)
title(['|X(e^{j2\pi F/Fs})|,    Fs = ',num2str(Fs)],'fontsize',TFS)
legend('Right Replicas','Left Replicas','location','best')
```

18. Proof:

$$p_{\mathrm{c}}(x,y) = \begin{cases} 1/A^2, & |x| < A/2, \quad |y| < A/2 \\ 0, & \text{otherwise} \end{cases} \tag{6.89}$$

$$P_{\mathrm{c}}(F_x,F_y) = \frac{\sin(\pi F_x A)}{\pi F_x A} \times \frac{\sin(\pi F_y A)}{\pi F_y A} \tag{6.90}$$

$$\begin{aligned} P_{\mathrm{c}}(F_x,F_y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{\mathrm{c}}(x,y)\mathrm{e}^{-\mathrm{j}2\pi(xF_x+yF_y)}\,\mathrm{d}x\mathrm{d}y \\ &= \int_{-\frac{A}{2}}^{\frac{A}{2}}\int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{A^2}\mathrm{e}^{-\mathrm{j}2\pi(xF_x+yF_y)}\,\mathrm{d}x\mathrm{d}y \\ &= \left(\int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{A}\mathrm{e}^{-\mathrm{j}2\pi xF_x}\,\mathrm{d}x\right)\cdot\left(\int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{A}\mathrm{e}^{-\mathrm{j}2\pi yF_y}\,\mathrm{d}y\right) \\ &= \frac{\mathrm{e}^{-\mathrm{j}2\pi xF_x}\Big|_{-\frac{A}{2}}^{\frac{A}{2}}}{A\cdot(-\mathrm{j}2\pi F_x)}\cdot\frac{\mathrm{e}^{-\mathrm{j}2\pi yF_y}\Big|_{-\frac{A}{2}}^{\frac{A}{2}}}{A\cdot(-\mathrm{j}2\pi F_y)} \\ &= \frac{\sin(\pi F_x A)}{\pi F_x A} \times \frac{\sin(\pi F_y A)}{\pi F_y A} \end{aligned}$$

19. (a) Solution:

$$s_{\mathrm{c}}(x,y) = 3\cos(2.4\pi x + 2.6\pi y) = 3\cos(2.4\pi x)\cos(2.6\pi y) - 3\sin(2.4\pi x)\sin(2.6\pi y)$$

$$s[m,n] = 3\cos(0.8\pi m + 1.3\pi n)$$

$$s_{\mathrm{r}}(x,y) = 3\cos(1.6\pi x - 2.6\pi y)$$

(b) Solution:

$$s[m,n] = 3\cos(1.2\pi m + 0.8667\pi n)$$

$$s_{\mathrm{r}}(x,y) = 3\cos(2.4\pi x - 1.4\pi y)$$

(c) Solution:

$$s[m,n] = 3\cos(0.8\pi m + 0.8667\pi n)$$

$$s_{\mathrm{r}}(x,y) = 3\cos(2.4\pi x + 2.6\pi y)$$

20. (a) Solution:

$$s_{\mathrm{fa}}[m,n] = \frac{1}{\Delta x \Delta y}\int_{m\Delta x - \frac{\Delta x}{2}}^{m\Delta x + \frac{\Delta x}{2}}\int_{n\Delta y - \frac{\Delta y}{2}}^{n\Delta y + \frac{\Delta y}{2}} s_{\mathrm{c}}(x,y)\mathrm{d}x\mathrm{d}y$$

(b) tba

(c) tba