CHAPTER 3

The z-Transform

Basic Problems

25. (a) Solution:

$$X(z) = \frac{2 - \frac{5}{6}z{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} \quad \text{ROC: } |z| > \frac{1}{2}$$

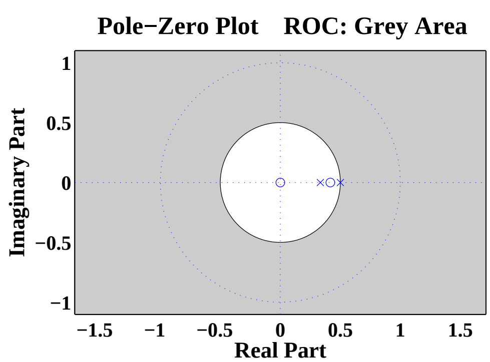

FIGURE 3.1: Pole-zero plot and ROC of $x[n] = (1/2)^n u[n] + (1/3)^n u[n]$.

(b) Solution:

$$X(z) \text{ not exist.}$$

(c) Solution:

$$X(z) = \frac{-\frac{1}{6}z{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} \quad \text{ROC: } \frac{1}{3} < |z| < \frac{1}{2}$$

26.

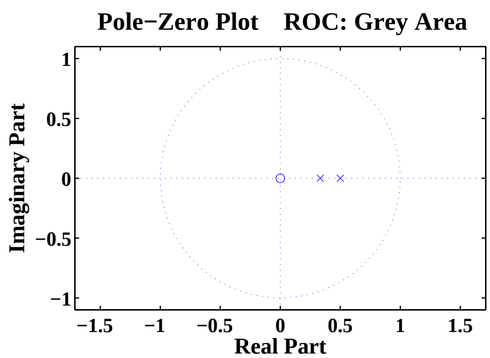

FIGURE 3.2: Pole-zero plot and ROC of $x[n] = (1/2)^n u[n] + (1/3)^n u[-n-1]$.

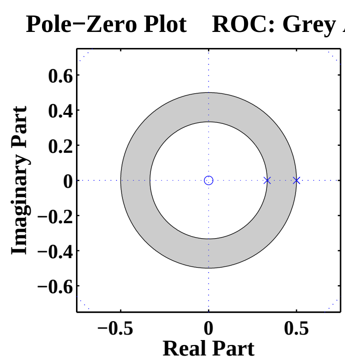

FIGURE 3.3: Pole-zero plot and ROC of $x[n] = (1/3)^n u[n] + (1/2)^n u[-n-1]$.

27. (a)

(b) Solution:

$$Y(z) = \frac{1}{1 - 0.8z^{-2}}, \quad \text{ROC: } |z| > \frac{2}{\sqrt{5}}$$

(c) See plot below.

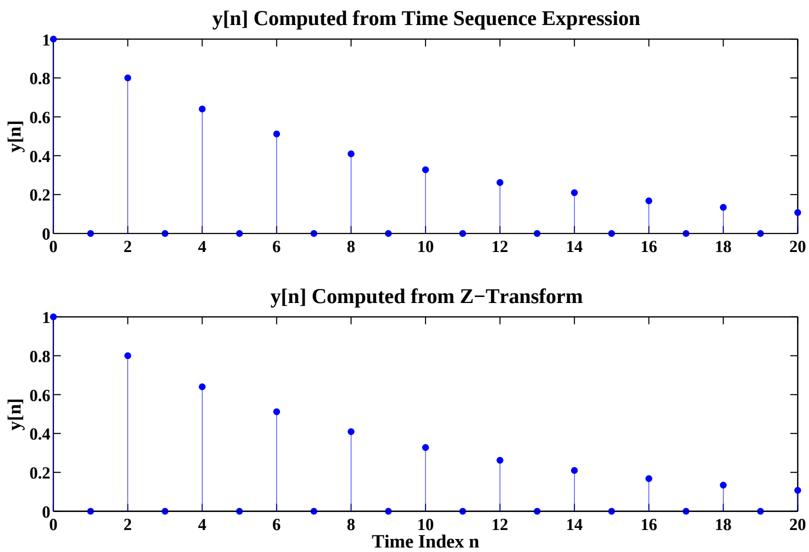

FIGURE 3.4: MATLAB verification of the z-transform of $y[n]$.

28. (a) Solution:

If ROC: $|z| > 3.7443$

$$\begin{aligned} x[n] &= -6.25\delta[n] + 0.5383(3.7443)^n u[n] \\ &\quad + 2 \times 6.6714 \cos(0.9041n + 1.0437)(0.2067)^n u[n] \end{aligned}$$

If ROC: $|z| < 0.2067$

$$\begin{aligned} x[n] &= -6.25\delta[n] - 0.5383(3.7443)^n u[-n-1] \\ &\quad - 2 \times 6.6714 \cos(0.9041n + 1.0437)(0.2067)^n u[-n-1] \end{aligned}$$

If ROC: $0.2067 < |z| < 3.7443$

$$\begin{aligned} x[n] &= -6.25\delta[n] - 0.5383(3.7443)^n u[-n-1] \\ &\quad + 2 \times 6.6714 \cos(0.9041n + 1.0437)(0.2067)^n u[n] \end{aligned}$$

(b) Solution:

$$x[n] = 4(-1)^n u[n] - 4\left(-\frac{1}{2}\right)^n u[n] - 4n\left(-\frac{1}{2}\right)^n u[n]$$

(c) tba

29. (a) Solution:

$$x[n] = 2(0.9)^n \cos(\omega_0 n)u[n] + \frac{20}{\sqrt{14}}(0.9)^n \sin(\omega_0 n)u[n], \ \cos\omega_0 = \frac{5}{9}$$

(b) See plot below.

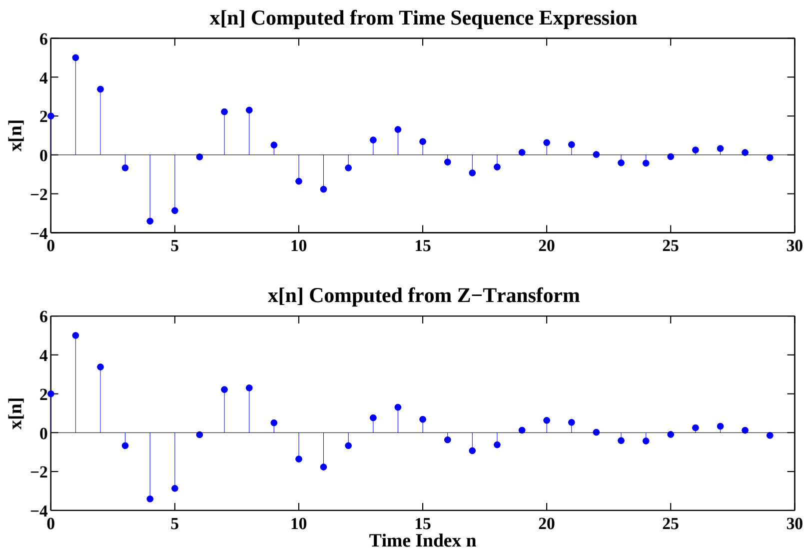

FIGURE 3.5: MATLAB verification of $x[n]$ expression obtained in part (a).

30. (a) Solution:

$$Y(z) = \frac{z^{-2}}{1 - \frac{1}{3}z^{-1}} \quad \text{ROC: } |z| > \frac{1}{3}$$

(b) Solution:

$$Y(z) = \frac{1}{1 - \frac{2}{3}z^{-1}} \quad \text{ROC: } |z| > \frac{2}{3}$$

(c) Solution:

$$Y(z) = \frac{-3}{1 - \frac{10}{3} + z^{-2}} \quad \text{ROC: } \frac{1}{3} < |z| < \frac{2}{3}$$

(d) Solution:

$$Y(z) = \frac{\frac{1}{3}z\left(z + \frac{1}{3}\right)}{z - \frac{1}{3}} \quad \text{ROC: } |z| > \frac{1}{3}$$

(e) Solution:

$$Y(z) = \frac{2z + 3z^{-3}}{1 - \frac{1}{3}z^{-1}} \quad \text{ROC: } |z| > \frac{1}{3}$$

(f) Solution:

$$Y(z) = \frac{z^{-1}}{\left(1 - \frac{1}{3}z^{-1}\right)^2} \quad \text{ROC: } |z| > \frac{1}{3}$$

31. Solution:

   (a) $Y(z) = X(1/z)$
   Solution:
   $$y[n] = 0.8^{-n}u[-n]$$

   (b) $Y(z) = dX(z)/dz$
   Solution:
   $$y[n] = -n0.8^{n-1}u[n-1]$$

   (c) $Y(z) = X^2(z)$
   Solution:
   $$y[n] = 0.8^n(n+1)u[n]$$

32. Solution:
$$Y(z) = \frac{1+a}{1-a}\frac{1}{1-z^{-1}} - \frac{2a}{1-a}\frac{1}{1-az^{-1}} \quad \text{ROC: } |z| > 1$$

33. Solution:
$$Y(z) = 0.2952\frac{z^{-2}+1}{z^4-0.8^4} \quad \text{ROC: } |z| > 0.8$$

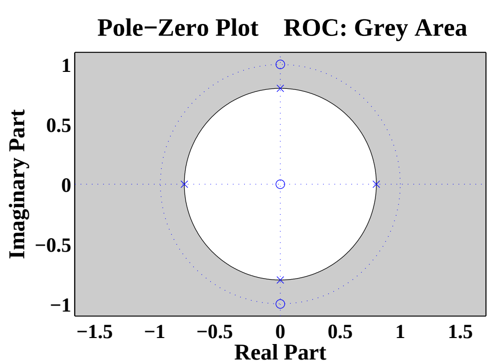

FIGURE 3.6: Pole-zero plot and ROC of signal $y[n]$.

34. Solution:

$$y[n] = \frac{6}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{6}{5} \cdot 3^n u[-n-1]$$

35. (a) Solution:

$$y[n] = 3\left(\frac{4}{5}\right)^n u[n] + 3\left(\frac{6}{5}\right)^n u[-n-1]$$

(b) Solution:

$$y[n] = \frac{4}{3} \cdot 3^n u[-n-1] + \frac{10}{3}\left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{1}{2}\right)^n u[n]$$

(c) Solution:

$$\begin{aligned} y[n] =& \frac{265}{42}\left(\frac{3}{2}\right)^n u[-n-1] - \frac{15}{2}\left(\frac{6}{5}\right)^n u[-n-1] \\ &+ \frac{13}{2}\left(\frac{9}{10}\right)^n u[n] - \frac{41}{7}\left(\frac{4}{5}\right)^n u[n] \end{aligned}$$

36. (a)

(b) Solution:

$$R_{xx}(z) = \frac{1}{(1 - r\mathrm{e}^{\mathrm{j}\theta}z^{-1})(1 - r\mathrm{e}^{-\mathrm{j}\theta}z)} \quad \text{ROC: } r < |z| < r^{-1}$$

(c) Solution:

$$r_{xx}[\ell] = \frac{-\mathrm{e}^{2\mathrm{j}\theta}}{1-\mathrm{e}^{2\mathrm{j}\theta}} r^l \mathrm{e}^{\mathrm{j}\theta l} u[l] + \frac{r^2\mathrm{e}^{2\mathrm{j}\theta}}{1-\mathrm{e}^{2\mathrm{j}\theta}} r^{-l}\mathrm{e}^{\mathrm{j}\theta l} u[-l-1]$$

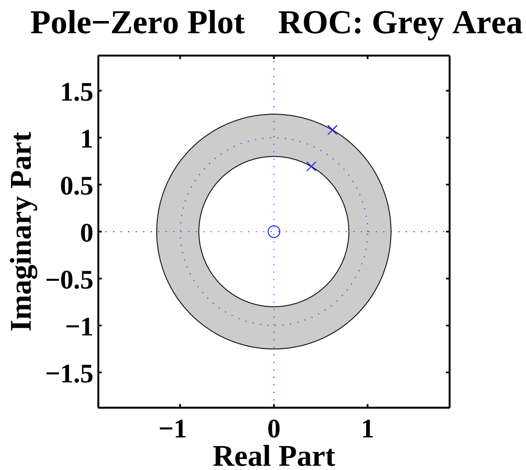

FIGURE 3.7: Pole-zero plot and ROC of $R_{xx}(z)$.

37. Solution:

If ROC: $|z| > 1.1$

$$h[n] = -0.0332 \cdot 1.1^n u[n] + 2 \cdot 0.6087 \cdot 0.9^n \cos(1.0472n - 1.5435)u[n]$$

If ROC: $|z| < 0.9$

$$h[n] = 0.0332 \cdot 1.1^n u[-n-1] - 2 \cdot 0.6087 \cdot 0.9^n \cos(1.0472n - 1.5435)u[-n-1]$$

If ROC: $0.9 < |z| < 1.1$

$$h[n] = 0.0332 \cdot 1.1^n u[-n-1] + 2 \cdot 0.6087 \cdot 0.9^n \cos(1.0472n - 1.5435)u[n]$$

38. (a) $x[n] = \mathrm{e}^{\mathrm{j}(\pi/3)n}, \quad -\infty < n < \infty$

Solution:

$$y[n] = (0.3937 - 8.8648\mathrm{j})\mathrm{e}^{\mathrm{j}(\pi/3)n}$$

(b) $x[n] = \mathrm{e}^{\mathrm{j}(\pi/3)n}u[n]$

Solution:

$$\begin{aligned} y[n] &= (0.3937 - 8.8648i)(0.5000 + 0.8660i)^n u[n] \\ &\quad + (0.1415 + 8.5978i)(0.4000 + 0.8062i)^n u[n] \\ &\quad + (-0.5352 + 0.2670i)(0.4000 - 0.8062i)^n u[n] \end{aligned}$$

(c) $x[n] = 1, \quad -\infty < n < \infty$

Solution:

$$y[n] = 1.9802$$

(d) $x[n] = (-1)^n u[n]$
Solution:

$$y[n] = 2 \cdot 0.6202 \cos(1.1102n - 1.5708)u[n]$$

39. Solution:

$$h[n] = \frac{1}{4}\delta[n+1] + \frac{15}{4}\delta[n] - 2 \cdot \left(\frac{1}{2}\right) u[n]$$

The system is NOT causal but is stable.

40. (a) Solution:

$$h[n] = \frac{5}{3}\delta[n] + \frac{10}{3}\left(\frac{3}{4}\right)^n u[n]$$

(b) Solution:

$$y[n] = -\left(\frac{1}{3}\right)^n u[n] + 6 \cdot \left(\frac{3}{4}\right)^n u[n]$$

(c) See plots below.

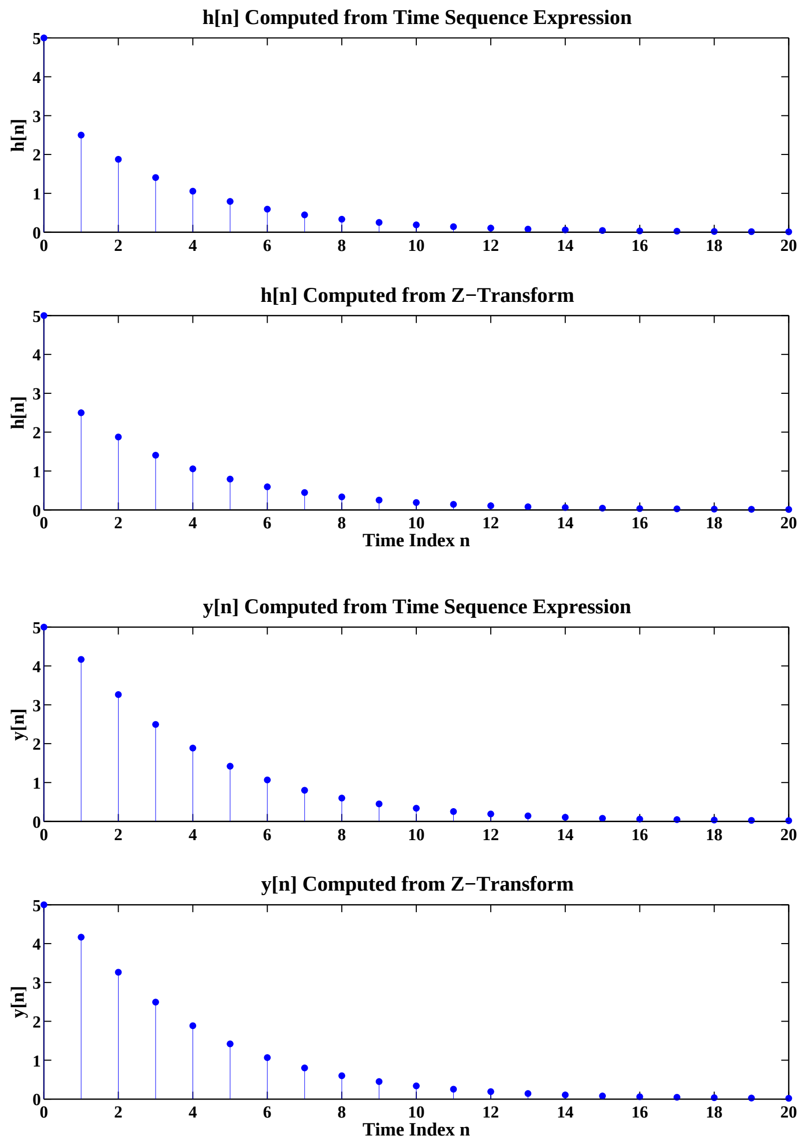

FIGURE 3.8: MATLAB verifications of impulse response $h[n]$ and system response $y[n]$.

41. (a) Solution:

$$h[n] = \frac{8}{3}\delta[n] + \frac{5}{3} \cdot \left(\frac{3}{4}\right)^n u[n]$$

(b) Solution:

$$h[n] = 10 \cdot 2^n u[n] - 7 \cdot \left(\frac{1}{2}\right)^n u[n]$$

42. (a) Solution:

$$X(z) = \frac{3}{1 - \frac{1}{3}z^{-1}} + \frac{4}{1 - 4z^{-1}}, \quad \text{ROC: } \frac{1}{3} < |z| < 4$$

(b) Solution:

$$h[n] = -0.0356 \cdot \delta[n] + 0.075 \cdot \delta[n-1] + 0.1785 \cdot 1.9048^n \cdot u[n]$$

43. (a) Solution:

$$y[n] = 0.96 \cdot 0.9^n u[n] + 0.0038 \cdot (-0.9)^n u[n] + 0.6563 \cdot 0.7^n u[n]$$

(b) See plot below.

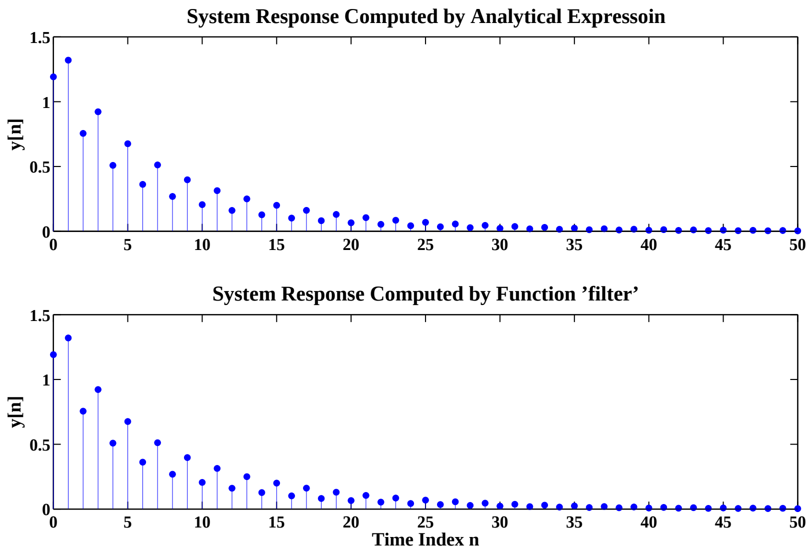

FIGURE 3.9: MATLAB verification of system response $y[n]$.

44. Solution:

zero-input response:

$$\frac{1}{2} \cdot \left(\frac{1}{4}\right).^n u[n]$$

zero-state response

$$(3.0955 - 3.2416\mathrm{j})\mathrm{e}^{\mathrm{j}\pi n/4}u[n] + (-2.0955 + 3.2416\mathrm{j})\left(\frac{1}{4}\right).^n u[n]$$

transient response:

$$\frac{1}{2} \cdot \left(\frac{1}{4}\right).^n u[n] + (-2.0955 + 3.2416\mathrm{j})\left(\frac{1}{4}\right).^n u[n]$$

steady-state response:

$$(3.0955 - 3.2416\mathrm{j})\mathrm{e}^{\mathrm{j}\pi n/4}u[n]$$