CHAPTER 7

The Discrete Fourier Transform

Basic Problems

27. (a) Solution:
The CTFT of $x_{\mathrm{c}}(t)$ is:

$$X_{\mathrm{c}}(\mathrm{j}2\pi F) = \frac{-10\mathrm{j} \times [-(20 + \mathrm{j}2\pi F)^2] + (20\pi)^2}{[(20 + \mathrm{j}2\pi F)^2] + (20\pi)^2]^2}$$

(b) See plot below.

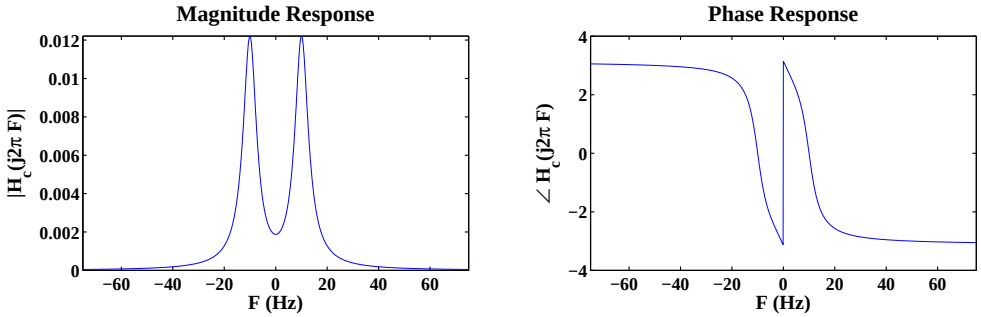


FIGURE 7.1: Magnitude and phase responses of $X_{\mathrm{c}}(\mathrm{j}2\pi F)$ over $-75 \leq F \leq 75$ Hz.

(c) See plot below.

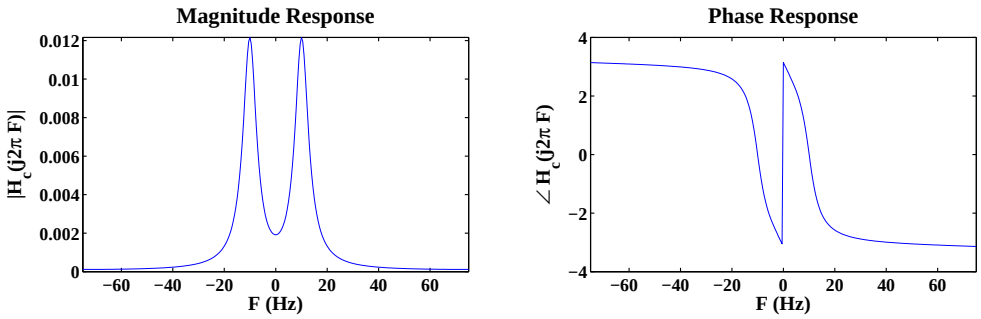


FIGURE 7.2: Approximated magnitude and phase responses of $X_{\mathrm{c}}(\mathrm{j}2\pi F)$ over $-75 \leq F \leq 75$ Hz using `fft` function.

28. (a) Solution:
The CTFS of $\tilde{x}_{\mathrm{c}}(t)$ is:

$$c_k = \frac{1}{5} \times \frac{\mathrm{e}^{-2.5}}{-0.5 - \mathrm{j}k\frac{2\pi}{5}} + \frac{1}{5} \times \frac{1 - \mathrm{e}^{-2.5}}{(0.5 + \mathrm{j}k\frac{2\pi}{5})^2}$$

(b) See plot below.

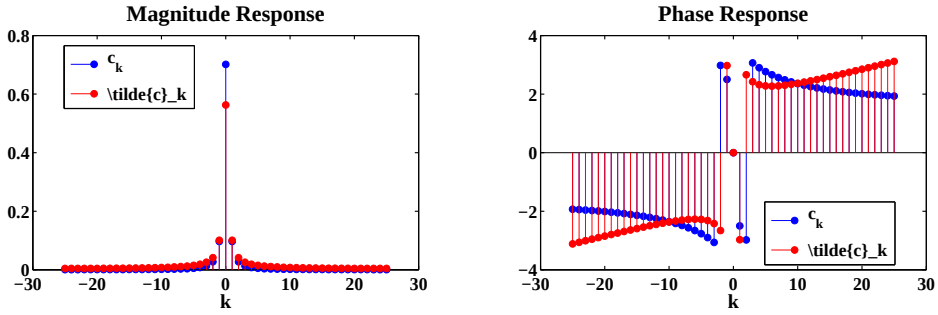


FIGURE 7.3: Magnitude and phase responses of c_k and $\hat{c}_k$ when the sampling interval is $T = 0.1$s.

(c) See plot below.

(d) See plot below.

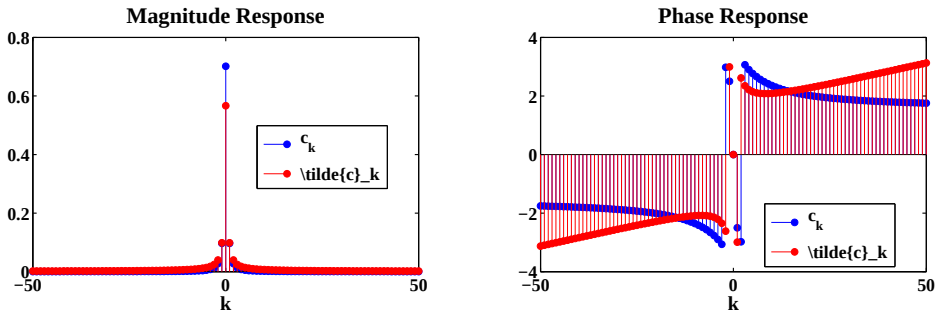


FIGURE 7.4: Magnitude and phase responses of c_k and $\hat{c}_k$ when the sampling interval is $T = 0.05$s.

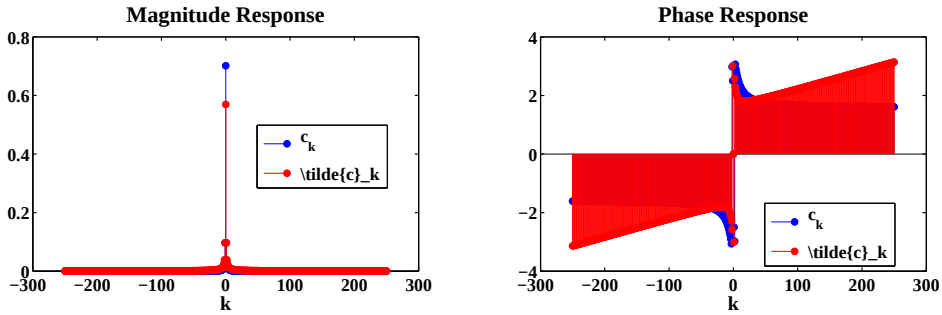


FIGURE 7.5: Magnitude and phase responses of c_k and $\hat{c}_k$ when the sampling interval is $T = 0.01$s.

29. (a) Solution:
The DTFT of $x[n]$ is:

$$\tilde{X}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{5\sin(0.1\pi)\mathrm{e}^{-\mathrm{j}\omega}}{1 - \cos(0.1\pi)\mathrm{e}^{-\mathrm{j}\omega} + 0.25\mathrm{e}^{-2\mathrm{j}\omega}}$$

(b) See plot below.

(c) See plot below.

(d) See plot below.

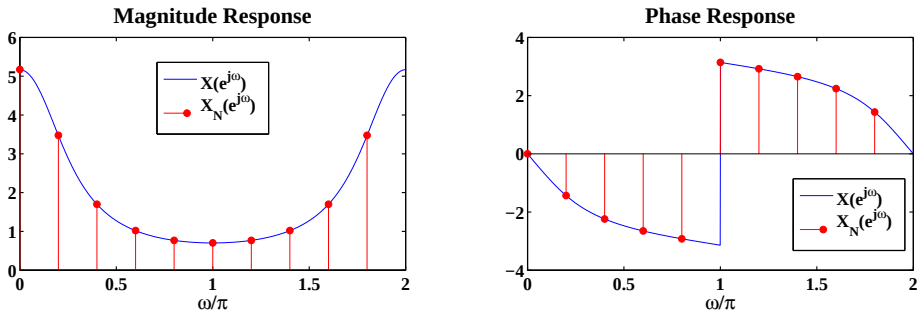


FIGURE 7.6: Magnitude and phase responses of $\tilde{X}\left(\mathrm{e}^{\mathrm{j}\omega}\right)$ and $\tilde{X}_N\left(\mathrm{e}^{\mathrm{j}\omega}\right)$ when $N = 10$.

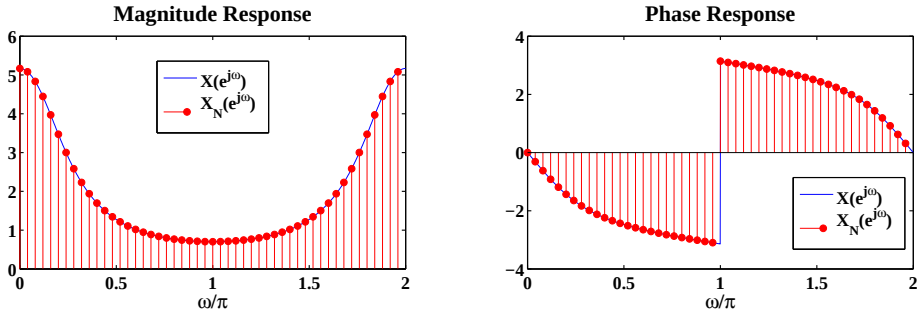


FIGURE 7.7: Magnitude and phase responses of $\tilde{X}\left(\mathrm{e}^{\mathrm{j}\omega}\right)$ and $\tilde{X}_N\left(\mathrm{e}^{\mathrm{j}\omega}\right)$ when $N = 50$.

30. tba

31. (a) See plot below.
    (b) See plot below.
    (c) See plot below.
    (d) See plot below.

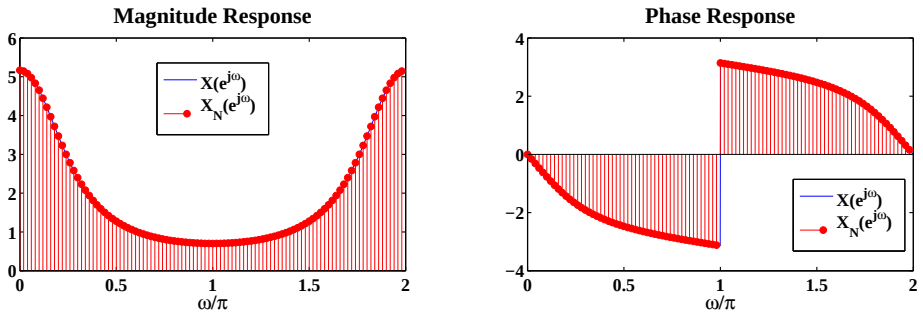


FIGURE 7.8: Magnitude and phase responses of $\tilde{X}(\mathrm{e}^{\mathrm{j}\omega})$ and $\tilde{X}_N(\mathrm{e}^{\mathrm{j}\omega})$ when $N = 100$.

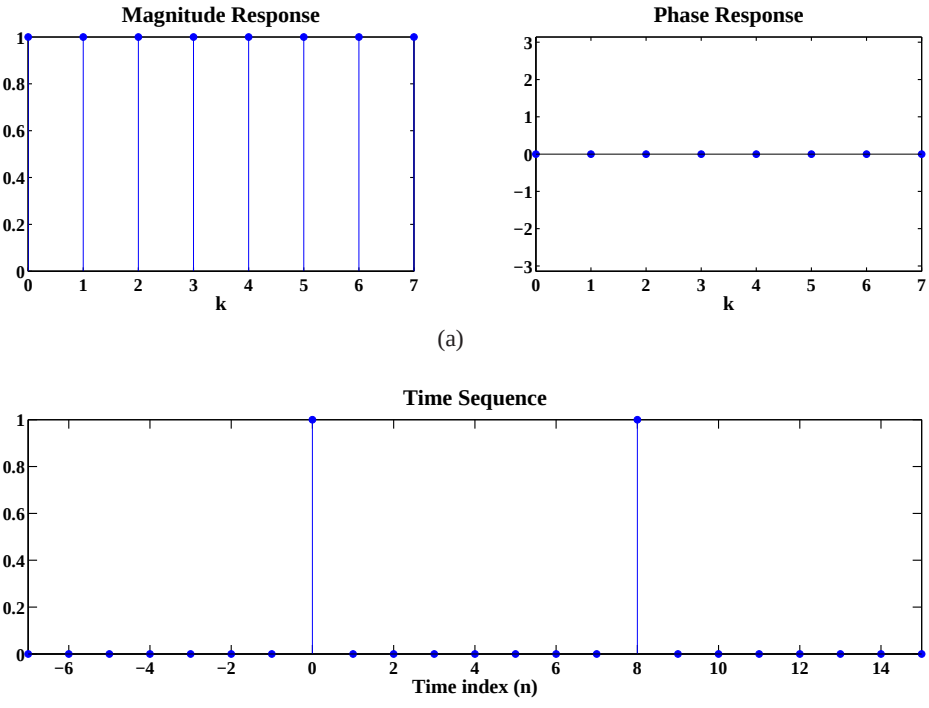
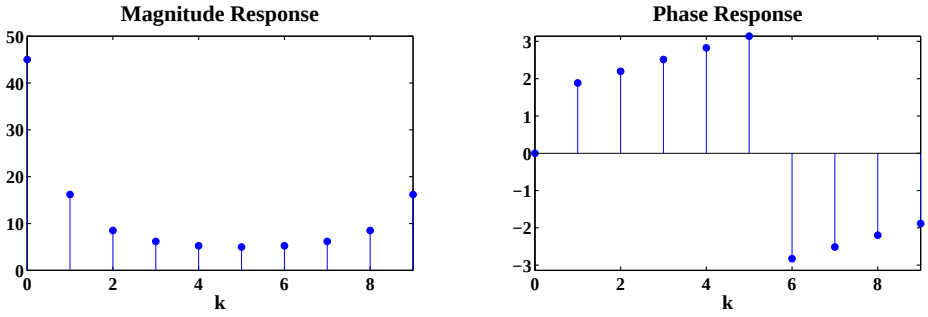
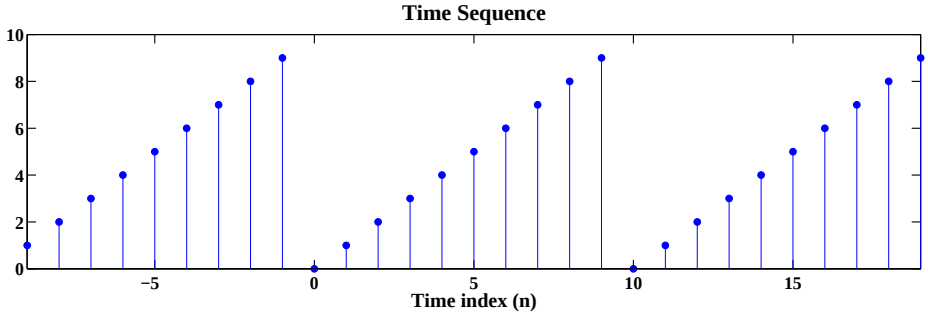


FIGURE 7.9: N-point (a) DFT and (b) IDFT of $x[n] = \delta[n]$, $N = 8$ in the range $-(N-1) \leq n \leq (2N-1)$.



(a)



(b)

FIGURE 7.10: N-point (a) DFT and (b) IDFT of $x[n] = n$, $N = 10$ in the range $-(N-1) \leq n \leq (2N-1)$.

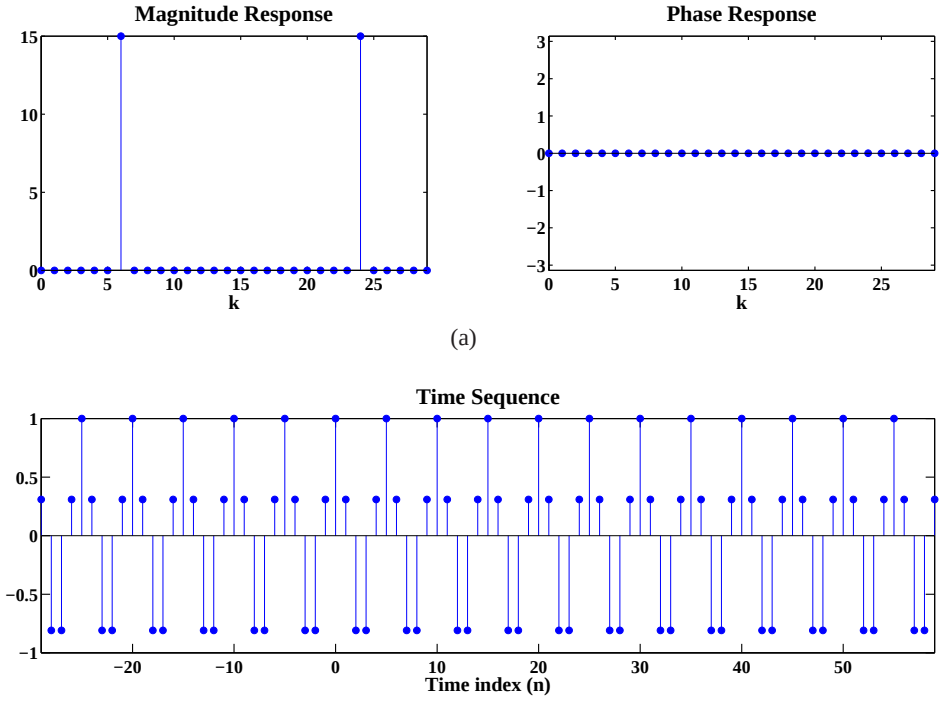
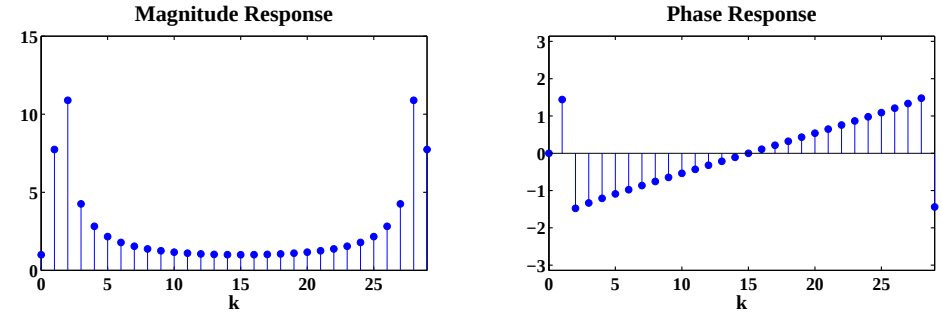
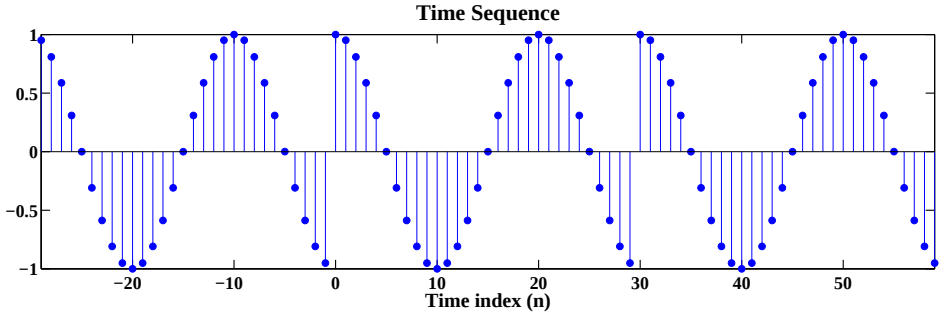


FIGURE 7.11: N-point (a) DFT and (b) IDFT of $x[n] = \cos(6\pi n/15)$, $N = 30$ in the range $-(N-1) \leq n \leq (2N-1)$.



(a)



(b)

FIGURE 7.12: N-point (a) DFT and (b) IDFT of $x[n] = \cos(0.1\pi n)$, $N = 30$ in the range $-(N-1) \leq n \leq (2N-1)$.

32.

33. Solution:
`plot(dftmtx(16))` plots each complex vector $\boldsymbol{w}_k$ within DFT matrix W_{16}. It plots the elements of $\boldsymbol{w}_k$ with real part versus imaginary part and then connect these points from the first one to the last with solid line. Different line color corresponds to different k value.

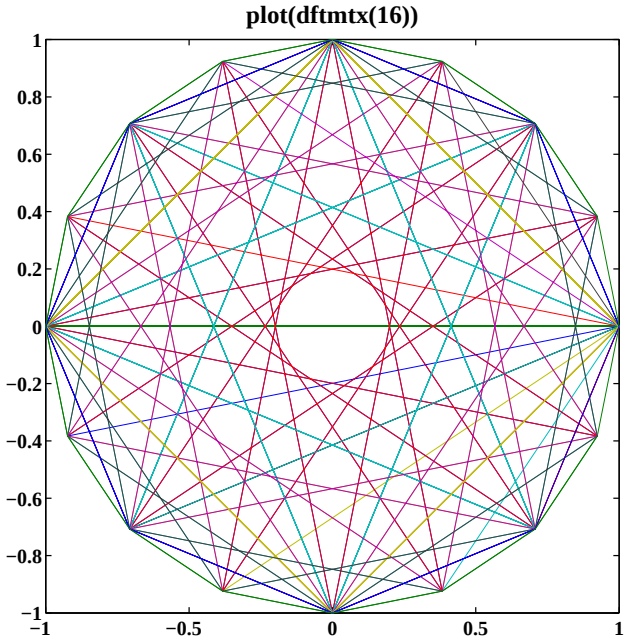


FIGURE 7.13: Plot of `plot(dftmtx(16))`.

34. (a) Solution:
The DTFT of $x[n]$ is:

$$\tilde{X}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1 - 0.8^2}{1 - 2 \cdot 0.8 \cos\omega + 0.8^2}$$

(b) Solution: The DFS of $\tilde{x}[n]$ is:

$$\tilde{X}[k] = \left(1 + 2\sum_{\ell=1}^{\infty} 0.8^{8\ell}\right)\left(\frac{1 - 0.8^8}{1 - 0.8\mathrm{e}^{\mathrm{j}\frac{2\pi}{8}\langle k\rangle_8}} + \frac{1 - 0.8^{-8}}{1 - 0.8^{-1}\mathrm{e}^{\mathrm{j}\frac{2\pi}{8}\langle k\rangle_8}}\right)$$

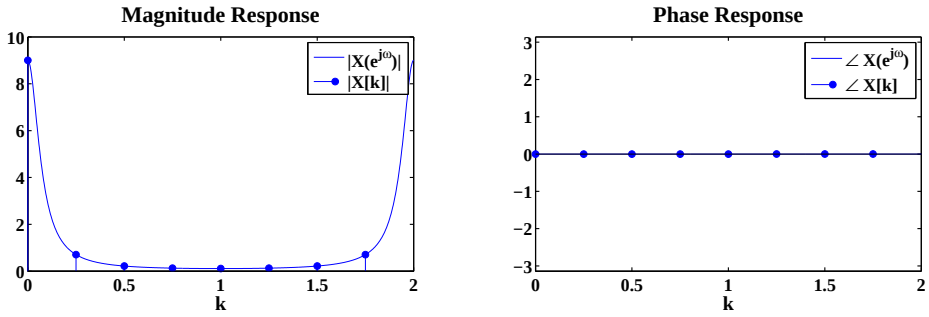


FIGURE 7.14: Plot of DTFT $\tilde{X}(\mathrm{e}^{\mathrm{j}\omega})$ and stem plot of DFS $\tilde{X}[k]$ when $x[n] = (0.8)^{|n|}$.

(c) See plot below.

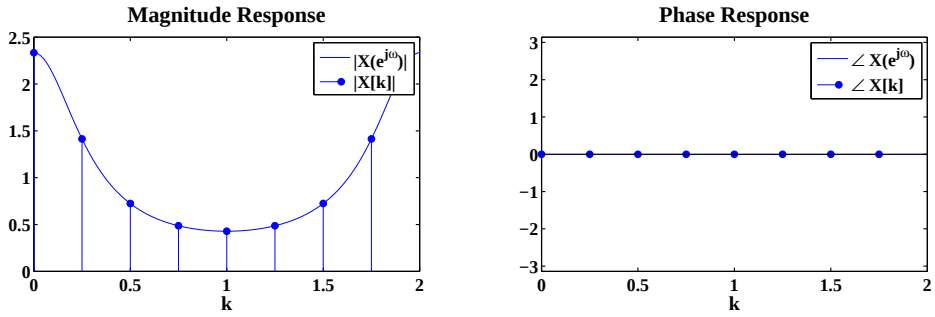


FIGURE 7.15: Plot of DTFT $\tilde{X}(\mathrm{e}^{\mathrm{j}\omega})$ and stem plot of DFS $\tilde{X}[k]$ when $x[n] = (0.4)^{|n|}$.

35. (a) Solution:
The DTFT $\tilde{X}\big(\mathrm{e}^{\mathrm{j}\omega}\big)$ is:

$$\tilde{X}\big(\mathrm{e}^{\mathrm{j}\omega}\big) = \frac{2}{1 - 0.8^2\,\mathrm{e}^{-2\mathrm{j}\omega}}$$

(b) Solution:

$$g[n] = \sum_{\ell=-\infty}^{\infty} x[n - 10\ell]$$

36. Solution:
The ones have a real-valued 8-point DFTs are:

$$x_2[n] = \{5, 2, -9, 4, 7, 4, -9, 2\}$$

$$x_5[n] = \{10, 5, -7, -4, 5, -4, -7, 5\}$$

The ones have 8-point imaginary-valued DFTs are:

$$x_1[n] = \{0, -3, 1, -2, 0, 2, -1, 3\}$$

The ones are complex valued are:

$$x_3[n] = \{8, -3, 1, -2, 6, 2, -1, 3\}$$

$$x_4[n] = \{0, 1, 3, -2, 5, 2, -3, 1\}$$

37. (a) Solution:
The DTFT of $x[n]$ is:

$$\tilde{X}\big(\mathrm{e}^{\mathrm{j}\omega}\big) = \frac{1}{2}\mathrm{e}^{\mathrm{j}\pi/6}\frac{1 - \mathrm{e}^{\mathrm{j}(0.25\pi-\omega)100}}{1 - \mathrm{e}^{\mathrm{j}(0.25\pi-\omega)}} + \frac{1}{2}\mathrm{e}^{-\mathrm{j}\pi/6}\frac{1 - \mathrm{e}^{\mathrm{j}(0.25\pi+\omega)100}}{1 - \mathrm{e}^{\mathrm{j}(0.25\pi+\omega)}}$$

(b) See plot below.

(c) See plot below.

(d) tba.

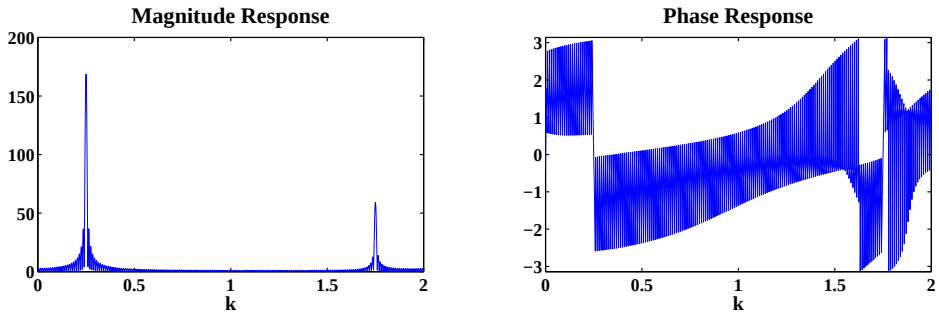


FIGURE 7.16: Plot of the DTFT $\tilde{X}\left(\mathrm{e}^{\mathrm{j}\omega}\right)$ of $x[n]$.

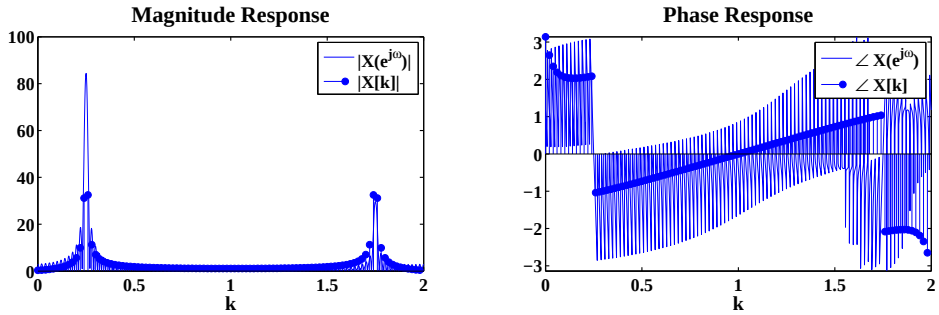


FIGURE 7.17: Plot of 100-point DFT $X[k]$ of $x[n]$ superimposed on the DTFT plot.

38. Solution:
The 9-point DFT $X[k]$ is

$$\{4, 2-\mathrm{j}3, 3+\mathrm{j}2, -4+\mathrm{j}6, 8-\mathrm{j}7, 8+\mathrm{j}7, -4-\mathrm{j}6, 3-\mathrm{j}2, 2+\mathrm{j}3\}$$

(a)
$$X_1[k] = W_9^{-2k} X[k]$$

(b)
$$X_2[k] = 2W_9^{-2k} X^*[k]$$

(c)
$$X_3[k] = |X[k]|^2$$

(d)
$$X_4[k] = \frac{1}{9} X[k] \textcircled{9} X[k]$$

(e)
$$X_5[k] = X[\langle k+2 \rangle_9]$$

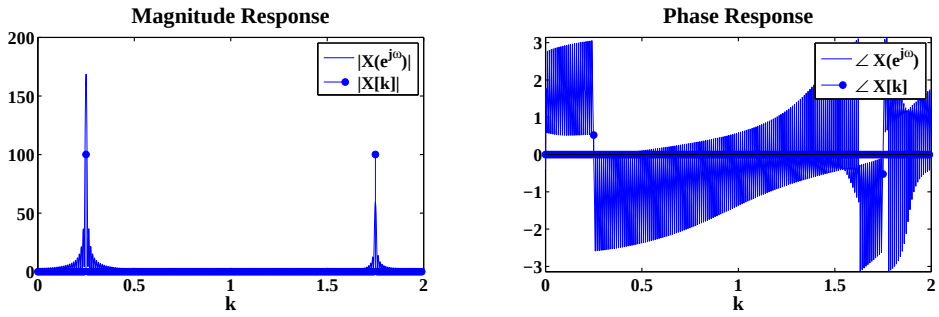


FIGURE 7.18: Plot of 200-point DFT $X[k]$ of $x[n]$ superimposed on the DTFT plot.

39.

40. Solution:

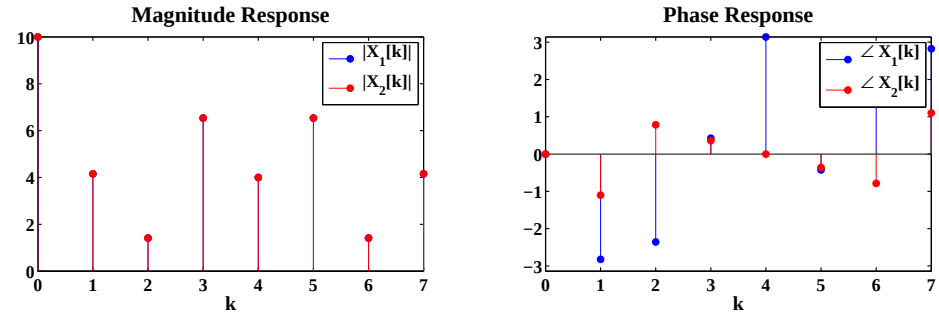


FIGURE 7.19: Verification by choosing $a = 1$, $b = 2$, $c = 3$, and $d = 4$.

41. (a) Solution:

Computing $x_1[n] \textcircled{7} x_2[n]$ using hand calculations:

$$\begin{bmatrix} -2 & 0 & 8 & 6 & -5 & -3 & 1 \\ 1 & -2 & 0 & 8 & 6 & -5 & -3 \\ -3 & 1 & -2 & 0 & 8 & 6 & -5 \\ -5 & -3 & 1 & -2 & 0 & 8 & 6 \\ 6 & -5 & -3 & 1 & -2 & 0 & 8 \\ 8 & 6 & -5 & -3 & 1 & -2 & 0 \\ 0 & 8 & 6 & -5 & -3 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 46 \\ 29 \\ -7 \\ -16 \\ -9 \\ -7 \\ 14 \end{bmatrix}$$

(b)

(c)

42.

43.

44. (a) Solution:
The linear convolution $x_1[n] * x_2[n]$ is:

$$\{\underset{\uparrow}{0}, 2, -5, 4, -7, 7, -8, 4, -12\}$$

(b) Solution:
The circular convolution $x_1[n] \textcircled{6} x_2[n]$ is:

$$\{\underset{\uparrow}{-8}, 6, -17, 4, -7, 7\}$$

(c) Solution:
The smallest value of N so that N-point circular convolution is equal to the linear convolution is:

$$\min N = 9$$

45.

46. (a) See plot below.

(b) See plot below.

(c) See plot below.

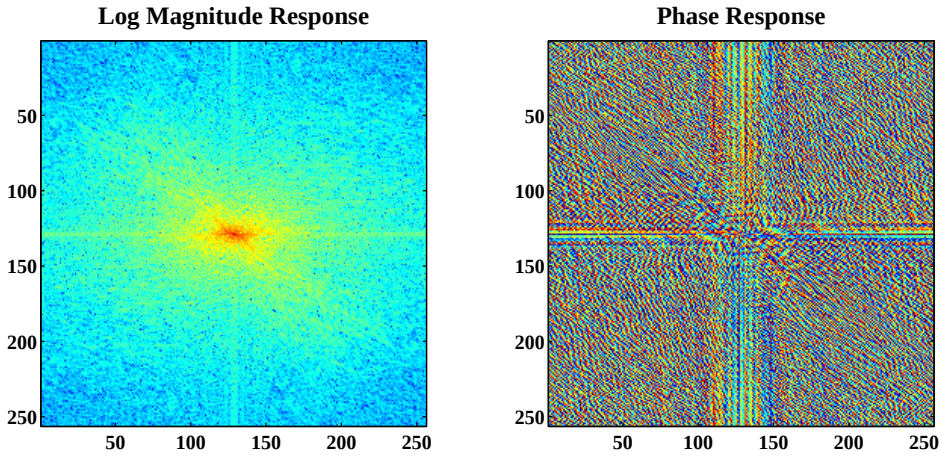


FIGURE 7.20: Plot of log-magnitude and phase as images of 2D-DFT of “Lena” image.

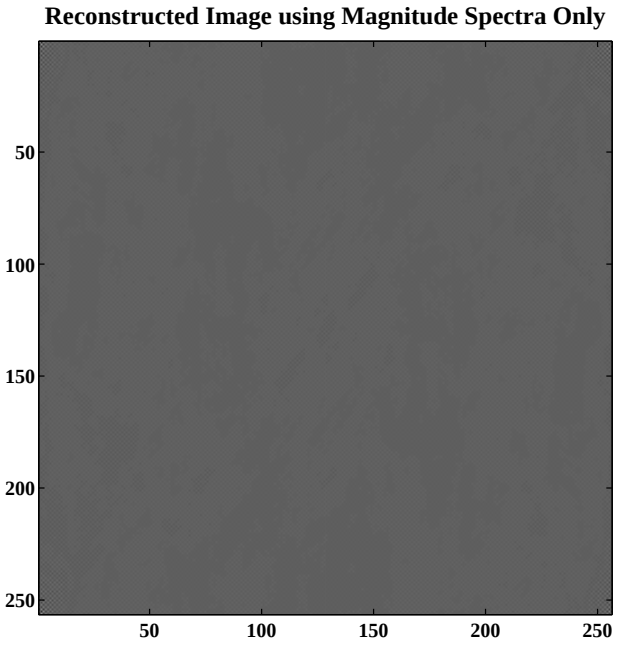


FIGURE 7.21: Plot of reconstructed image using 2D-IDFT of the magnitude array.

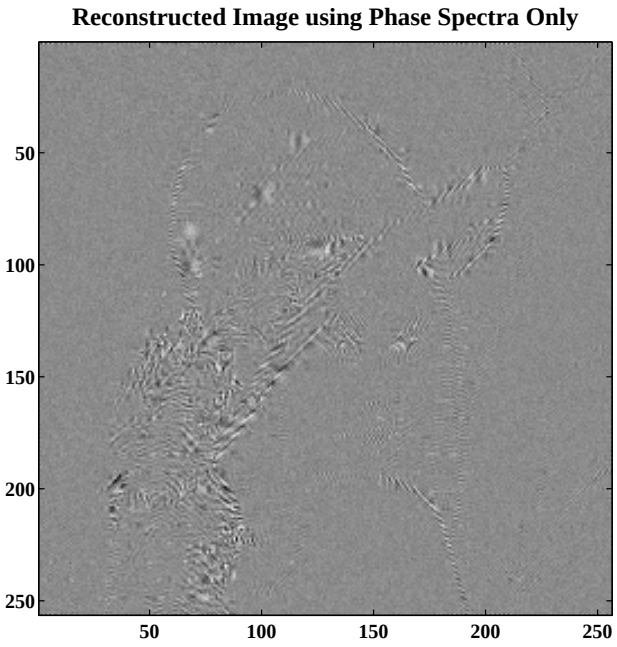


FIGURE 7.22: Plot of reconstructed image using 2D-IDFT of the phase array multiplied by constant magnitude value 128.