CHAPTER 6

Sampling of Continuous-Time Signals

Tutorial Problems

1. (a) Solution:

$$x_c(t) = \frac{5}{2}e^{j\frac{\pi}{6}}e^{j200\pi t} + \frac{5}{2}e^{-j\frac{\pi}{6}}e^{-j200\pi t} + \frac{2}{j}e^{j300\pi t} - \frac{2}{j}e^{-j300\pi t}$$

The spectra of $x_c(t)$ is:

$$Xc(j\Omega) = \begin{cases} \frac{5}{2}e^{j\frac{\pi}{6}}, & \Omega = 200\pi \\ \frac{5}{2}e^{-j\frac{\pi}{6}}, & \Omega = -200\pi \\ \frac{2}{j}, & \Omega = 300\pi \\ -\frac{2}{j}, & \Omega = -300\pi \\ 0, & \text{elsewhere} \end{cases}$$

The spectra $X(e^{j\omega})$ of $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=\Omega T} = F_s \sum_{k=-\infty}^{\infty} X_c(j\Omega - j2\pi k F_s)$$

$$X(e^{j\omega})\big|_{\omega=2\pi FT} = F_s \sum_{k=-\infty}^{\infty} X_c[j2\pi(F - kF_s)]$$

The signal can recovered for $x[n]$ if $F_s = 1$ KHz.

(b) Solution:
The signal can recovered for $x[n]$ if $F_s = 500$ Hz.

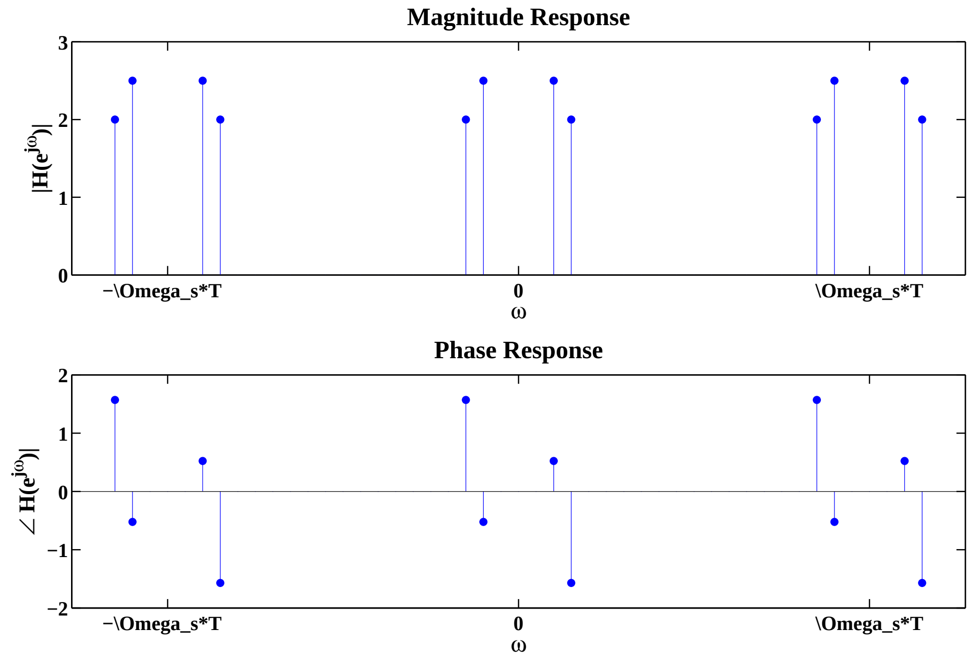
(c) Solution:
The signal can NOT recovered for $x[n]$ if $F_s = 100$ Hz.

(d) tba.

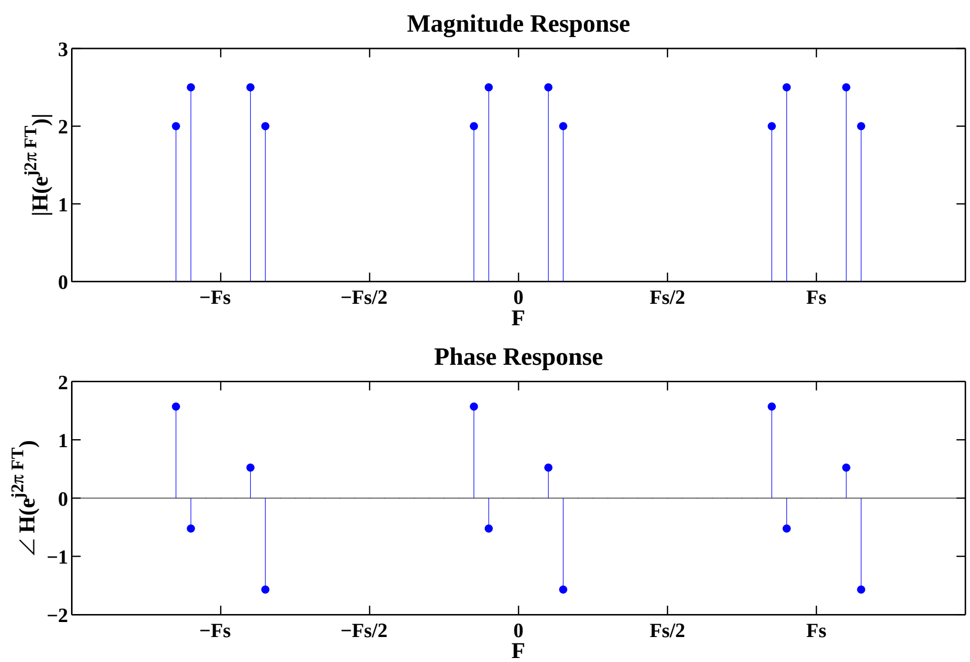
MATLAB script:

```
% P0601: Illustrates the alias distortion
close all; clc
Fs = 1e3; % Part (a)
% Fs = 500; % Part (b)
% Fs = 100; % Part (c)
T = 1/Fs;
FH = 150;
FL = FH+Fs;
F = -FL:50:FL;
X = zeros(1,length(F));
for k = -1:1;
ind = F == -150+k*Fs; X(ind) = X(ind)-2/j;
ind = F == -100+k*Fs; X(ind) = X(ind)+5/2*exp(-j*pi/6);
ind = F == 100+k*Fs; X(ind) = X(ind)+5/2*exp(j*pi/6);
ind = F == 150+k*Fs; X(ind) = X(ind)+2/j;
end
ind = X==0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0601a');
subplot(211)
stem(F*2*pi*T,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F*2*pi*T,angle(X),'filled')
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('\angle H(e^{j\omega})|','fontsize',LFS)
title('Phase Response','fontsize',TFS)

hfb = figconfg('P0601b');
```

```
subplot(211)
stem(F,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi FT})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F,angle(X),'filled')
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('\angle H(e^{j2\pi FT})','fontsize',LFS)
title('Phase Response','fontsize',TFS)
```

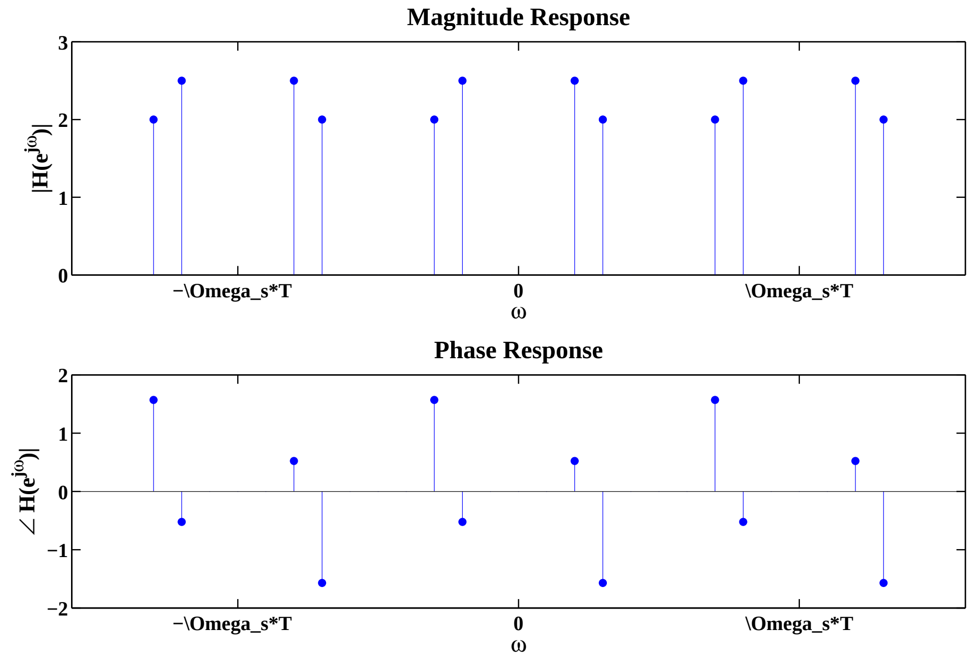


(a)

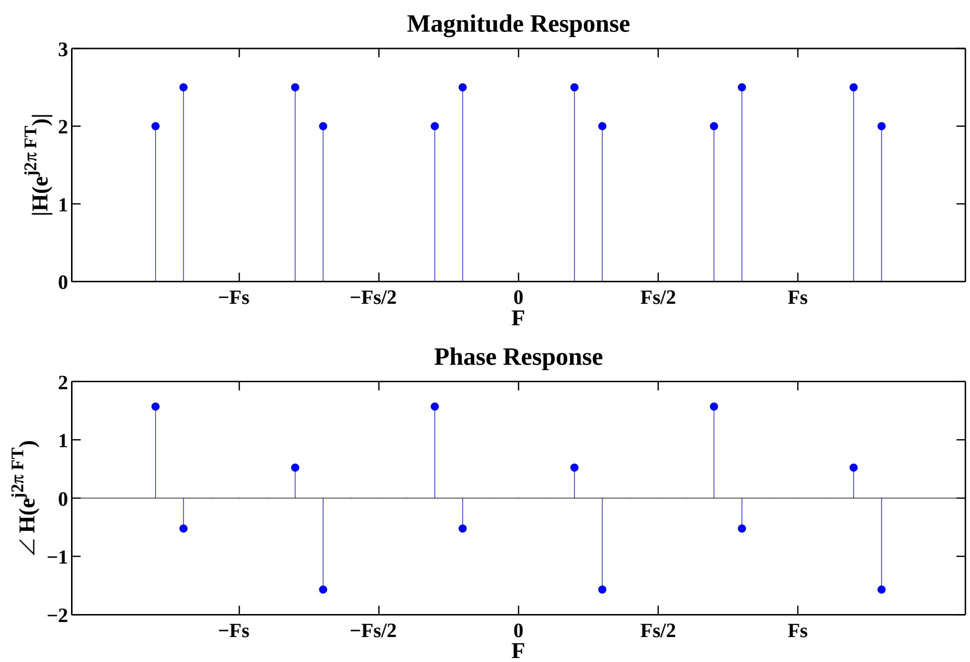


(b)

FIGURE 6.1: Spectra of $X(e^{j\omega})$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 1$ KHz.



(a)



(b)

FIGURE 6.2: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 500$ Hz.

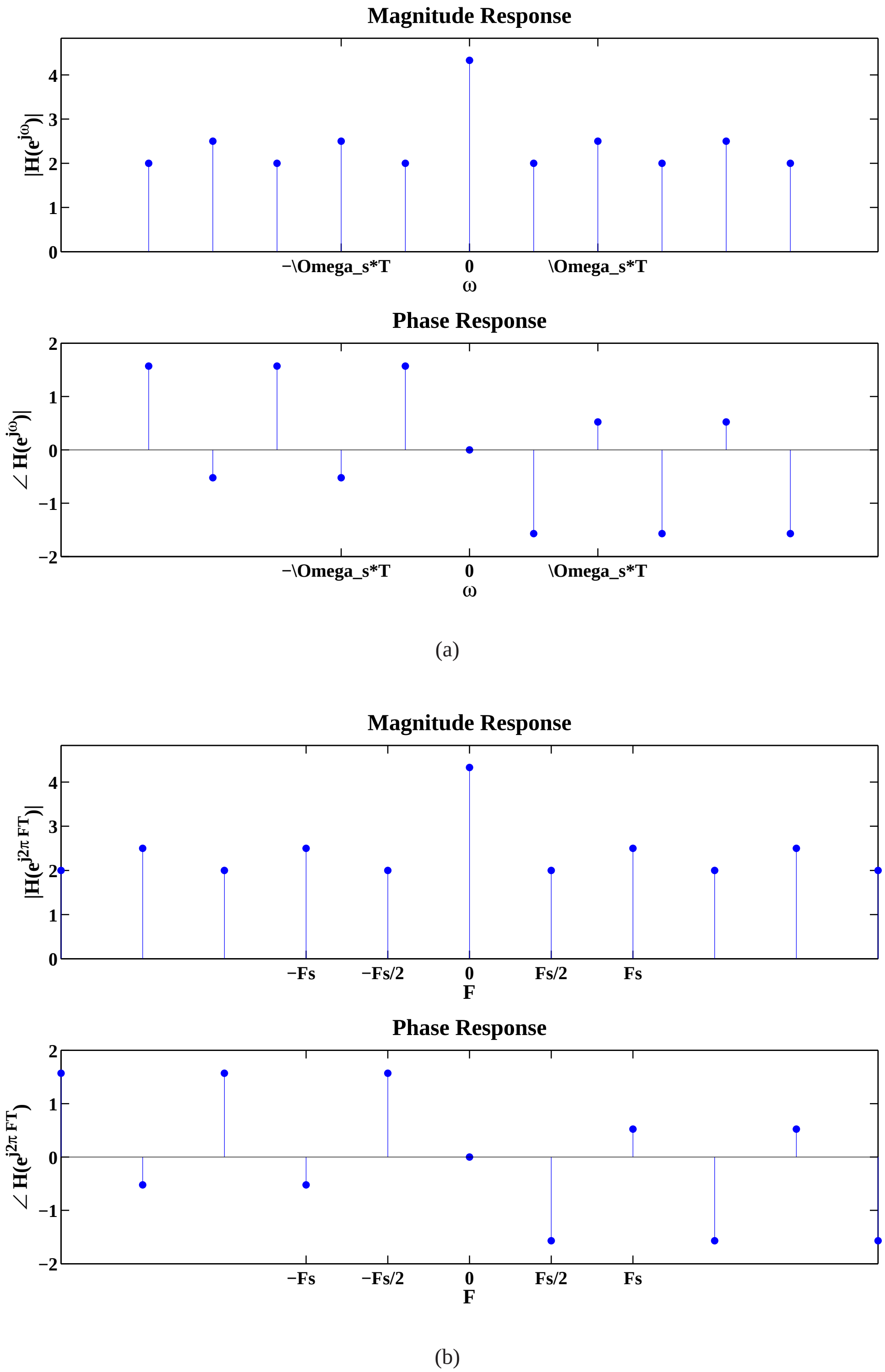


FIGURE 6.3: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 100$ Hz.

2. (a) Solution:
The continuous signal $x_c(t)$ is:

$$x_c(t) = 5e^{-10|t|}$$

The sampled sequence $x[n]$ is:

$$x[n] = x_c(nT) = 5e^{-10|n|T} = 5a^{|n|}, \qquad \text{define } a = e^{-10T}$$

The spectra $X(e^{j\omega})$ of $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=2\pi FT} = 5 \cdot \frac{1-a^2}{1-2a\cos(2\pi FT)+a^2}$$

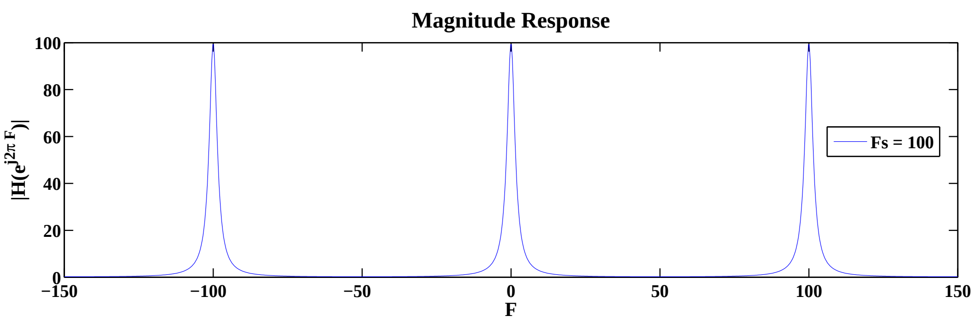


FIGURE 6.4: Magnitude response of $X(e^{j\omega})$ as a function of F in Hz when the sampling rate is $F_s = 100$.

(b) See plot below.

(c) See plot below.

(d) Solution:
For sampling rate $F_s = 100$ Hz, the signal $X_c(t)$ can be reasonably recovered from its samples $x[n]$.

MATLAB script:

```
% P0602: Plot the spectra of sampled sequence
close all; clc
% Fs = 100; % Part a
% Fs = 50; % Part b
Fs = 25; % Part c
T = 1/Fs;
```

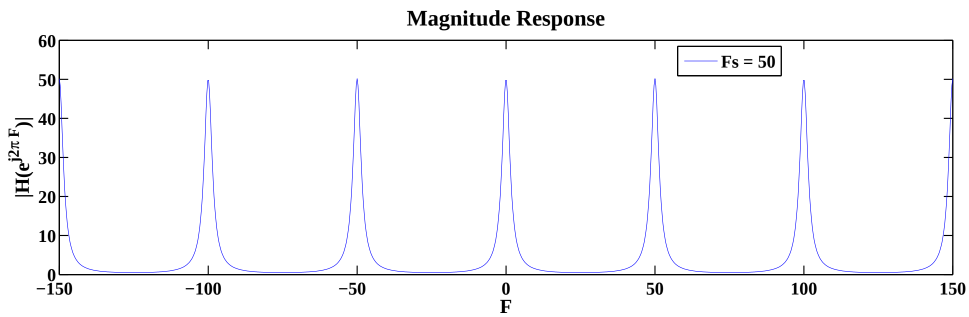


FIGURE 6.5: Magnitude response of $X\left(e^{j\omega}\right)$ as a function of F in Hz when the sampling rate is $F_s = 50$.

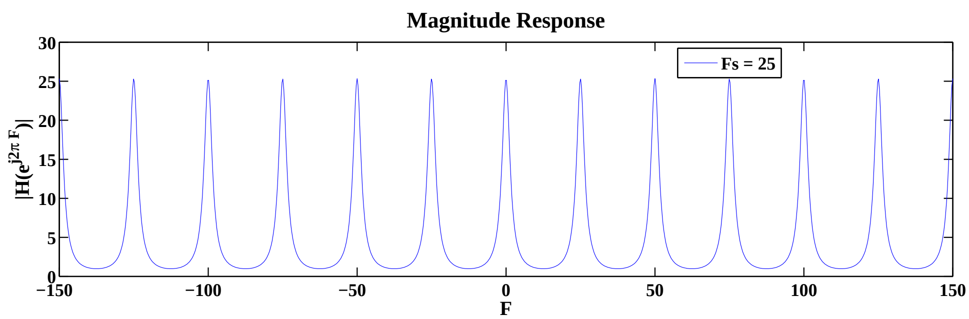


FIGURE 6.6: Magnitude response of $X\left(e^{j\omega}\right)$ as a function of F in Hz when the sampling rate is $F_s = 25$.

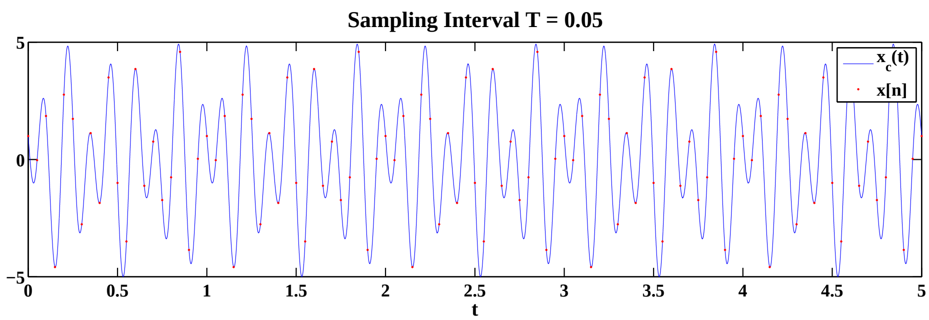
```
a = exp(-10*T);
F = linspace(-150,150,1000);
X = 5*(1-a^2)./(1-2*a*cos(2*pi*F*T)+a^2);
%% Plot:
hfa = figconfg('P0602a','long');
plot(F,abs(X))
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi F})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
legend(['Fs = ',num2str(Fs)],'location','best')
```

3. (a) Solution:

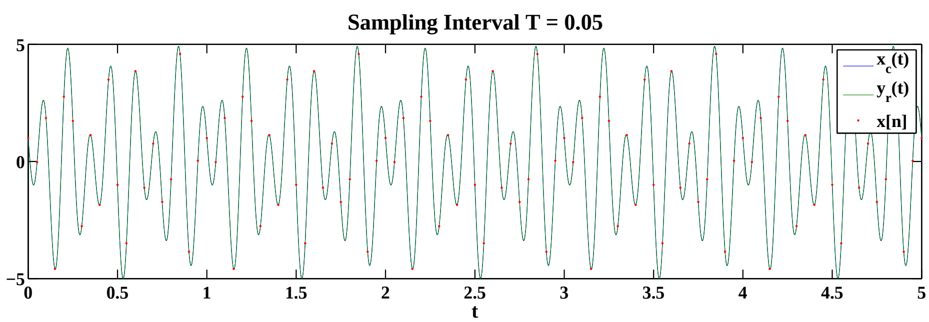
$$x[n] = x_c(0.05n) = 2\cos(0.5\pi n - \frac{\pi}{3}) - 3\sin(0.8\pi n)$$

(b) Solution:

$$y_r(t) = 2\cos(10\pi t - 60^\circ) - 3\sin(16\pi t)$$



(a)



(b)

FIGURE 6.7: (a) Plot of $x[n]$ and $x_c(t)$ and (b) plot of $y_r(t)$ when the continuous signal is sampled at $t = 0.05n$.

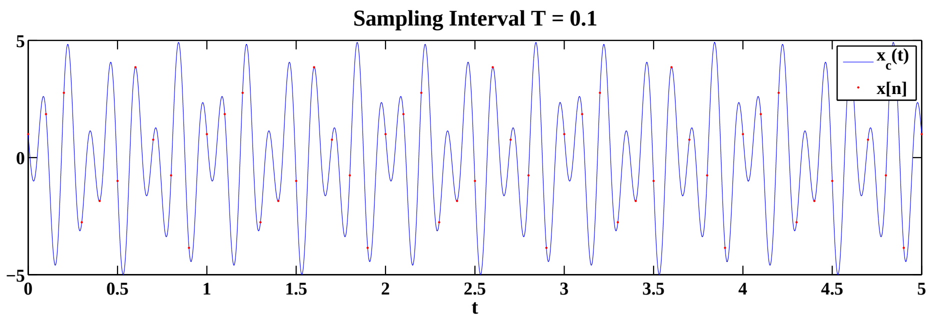
(c) Solution:

$$x[n] = x_c(0.1n) = 2\cos(\pi n - \frac{\pi}{3}) - 3\sin(1.6\pi n)$$

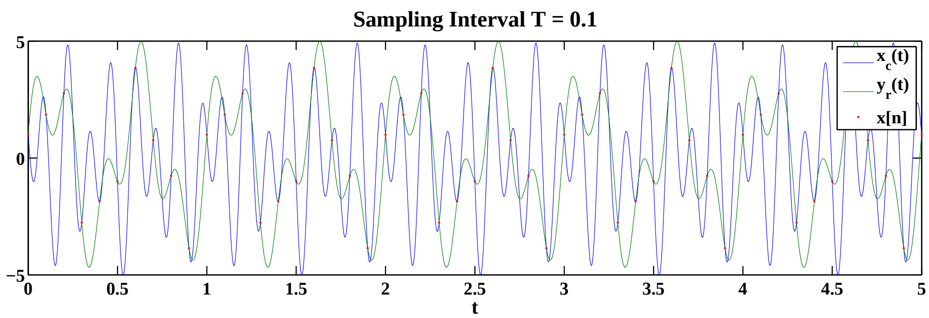
$$y_r(t) = 2\cos(10\pi t - 60^\circ) + 3\sin(4\pi t)$$

(d) Solution:

$$x[n] = x_c(0.5n) = 2\cos(5\pi n - \frac{\pi}{3}) - 3\sin(8\pi n)$$



(a)



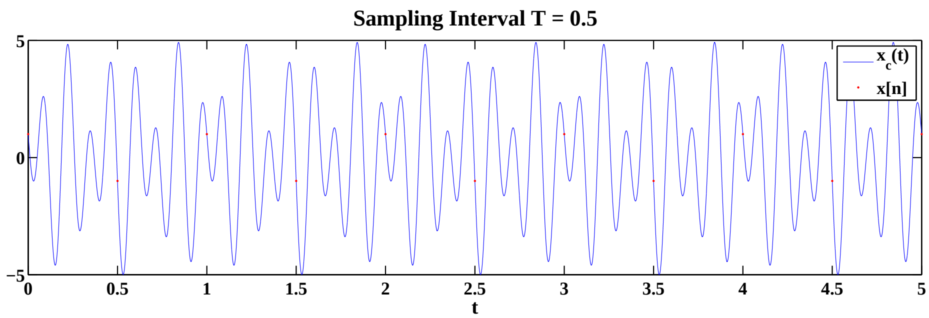
(b)

FIGURE 6.8: (a) Plot of $x[n]$ and $x_{\mathrm{c}}(t)$ and (b) plot of $y_{\mathrm{r}}(t)$ when the continuous signal is sampled at $t = 0.1n$.

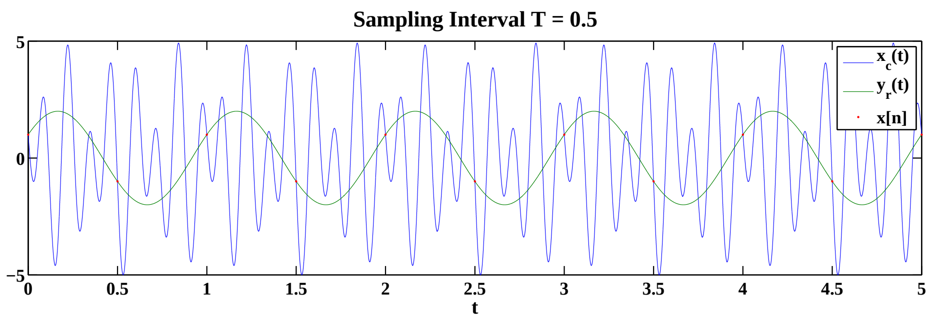
$$y_{\mathrm{r}}(t) = 2\cos(2\pi t - 60^\circ)$$

MATLAB script:

```
% P0603: Illustrate Ideal DAC
close all; clc
t1 = 0; t2 = 5;
dt = 1e-4;
t = t1:dt:t2;
xc = 2*cos(10*pi*t-pi/3)-3*sin(16*pi*t);
%% Part (a) and (b)
T = 0.05;
yr = 2*cos(10*pi*t-pi/3)-3*sin(16*pi*t);
%% Part (c)
% T = 0.1;
% % yr = 2*cos(10*pi*t)+3*sin(4*pi*t);
```



(a)



(b)

FIGURE 6.9: (a) Plot of $x[n]$ and $x_c(t)$ and (b) plot of $y_r(t)$ when the continuous signal is sampled at $t = 0.5n$.

```
% yr = 2*cos(10*pi*t-pi/3)+3*sin(4*pi*t);
%% Part (d)
% T = 0.5;
% yr = 2*cos(2*pi*t-pi/3);

% Sampling:
nT = t1:T:t2;
xn = 2*cos(10*pi*nT-pi/3)-3*sin(16*pi*nT);
%% Plot:
hfa = figconfg('P0603a','long');
plot(t,xc); hold on
plot(nT,xn,'.','color','red')
xlabel('t','fontsize',LFS)
title(['Sampling Interval T = ',num2str(T)],'fontsize',TFS)
legend('x_c(t)','x[n]','location','northeast')
```

```
hfb = figconfg('P0603b','long');
plot(t,xc,t,yr,nT,xn,'.')
xlabel('t','fontsize',LFS)
title(['Sampling Interval T = ',num2str(T)],'fontsize',TFS)
legend('x_c(t)','y_r(t)','x[n]','location','northeast')
```

4. (a) Solution:
   If $F_0 = 10$, 20, and 40 Hz, $2F_0 < F_s = 100$,

$$y_r(t) = x_c(t) = \cos(2\pi F_0 t + \theta_0)$$

   (b) tba.

   (c) Solution:
   When $2F_0 = F_s$,

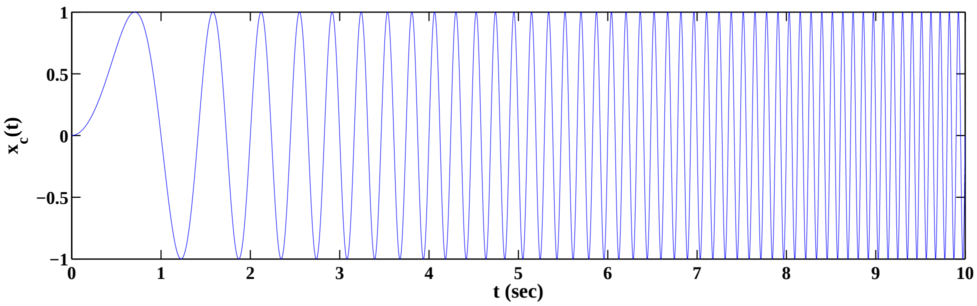
$$y_r(t) = 2\cos\theta_0 \cdot \cos 2\pi F_0 t$$

5. The same to last problem P0604

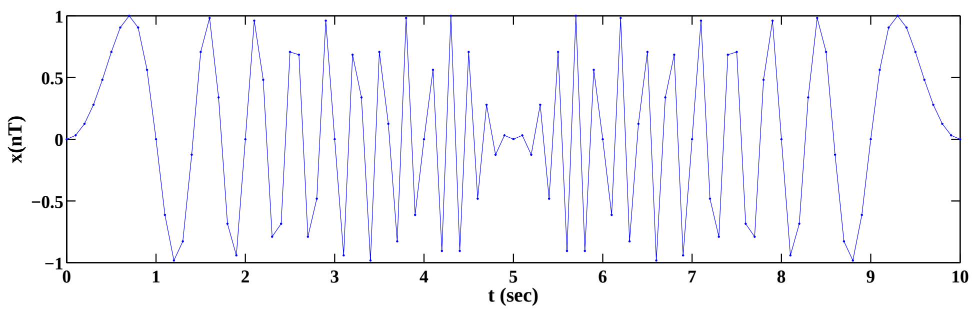
6. MATLAB script:

```
% P0606: Sampling a linear FM signal
close all; clc
B = 10;
Fs = B;
tau = 10;
dt = 1e-4;
t = 0:dt:tau;
xc = sin(pi*B*t.^2/tau);
nT = 0:1/Fs:tau;
xn = sin(pi*B*nT.^2/tau);
ind = B*t/tau > Fs/2;
yr = xc;
yr(ind) = -sin(2*pi*(Fs-B*t(ind)/tau/2).*t(ind));
%% Plot:
hfa = figconfg('P0606a','long');
plot(t,xc)
xlabel('t (sec)','fontsize',LFS)
ylabel('x_c(t)','fontsize',LFS)

hfb = figconfg('P0606b','long');
```

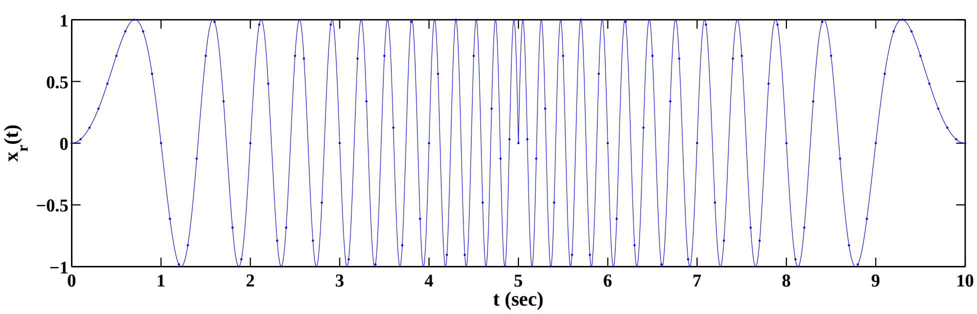
```
plot(nT,xn,'.-');
xlabel('t (sec)','fontsize',LFS)
ylabel('x(nT)','fontsize',LFS)

hfc = figconfg('P0606c','long');
plot(t,yr); hold on
plot(nT,xn,'.')
ylim([-1 1])
xlabel('t (sec)','fontsize',LFS)
ylabel('x_r(t)','fontsize',LFS)
```



(a)



(b)



(c)

FIGURE 6.10: (a) Continuous signal $x_{\mathrm{c}}(t)$, (b) sampled sequence $x[n]$, and (c) reconstructed signal $x_{\mathrm{r}}(t)$.

7. (a) See plot below.

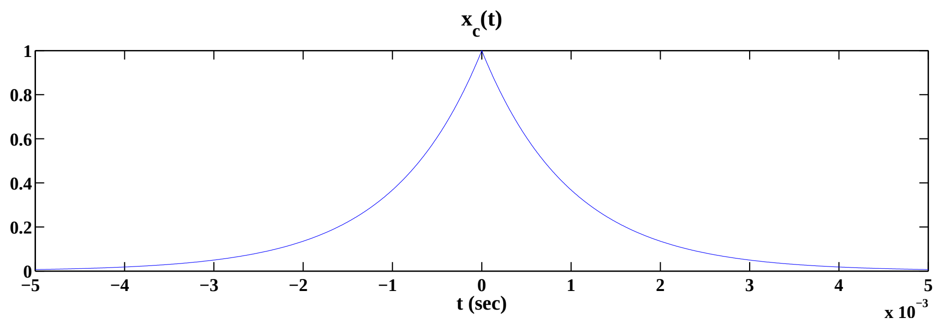


FIGURE 6.11: Plot of the signal $x_{\mathrm{c}}(t)$.

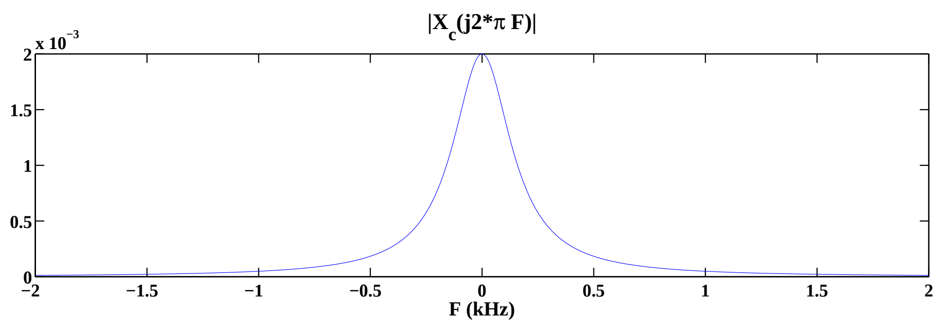


FIGURE 6.12: Plot of the CTFT $X_{\mathrm{c}}(\mathrm{j}2\pi F)$ of signal $x_{\mathrm{c}}(t)$.

(b) See plot below.

(c) See plot below.

(d) See plot below.

(e) See plot below.

MATLAB script:

```
% P0607: Sampling a exponential decaying signal
close all; clc
%% ii = 1, Fs = 1000; ii = 2, Fs = 5000
ii = 2;
t1 = -5e-3; t2 = 5e-3;
dt = 1e-5;
```

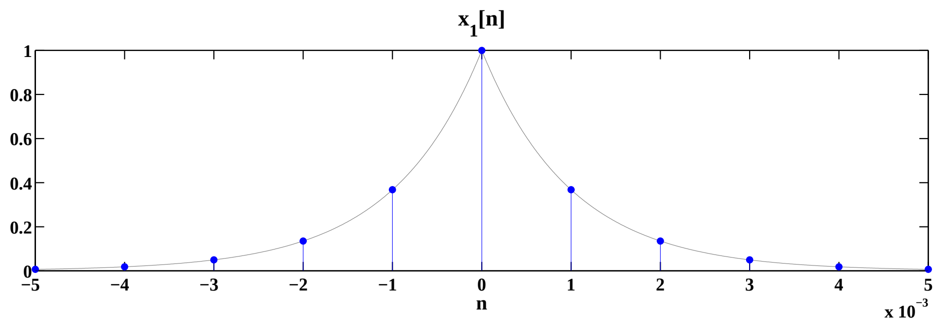


FIGURE 6.13: Plot of sampled signal $x_1[n]$.

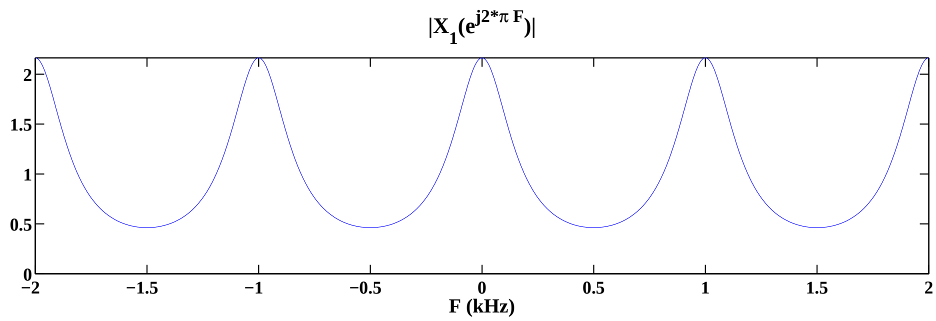


FIGURE 6.14: Plot of the CTFT $X_1(e^{j2\pi F})$ of sampled signal $x_1[n]$.

```
t = t1:dt:t2;
xc = exp(-1000*abs(t));
F = linspace(-2e3,2e3,1000);
Xc = 0.002./(1+(0.002*pi*F).^2);
Fs = [1e3 5e3];
nT = t1:1/Fs(ii):t2;
xn = exp(-1000*abs(nT));
a = exp(-1000/Fs(ii));
X = (1-a^2)./(1-2*a*cos(2*pi*F/Fs(ii))+a^2);
[G1,G2] = meshgrid(t,nT);
S = sinc(Fs(ii)*(G1-G2));
yr = xn*S;

%% Plot:
hfa = figconfg('P0607a1','long');
```

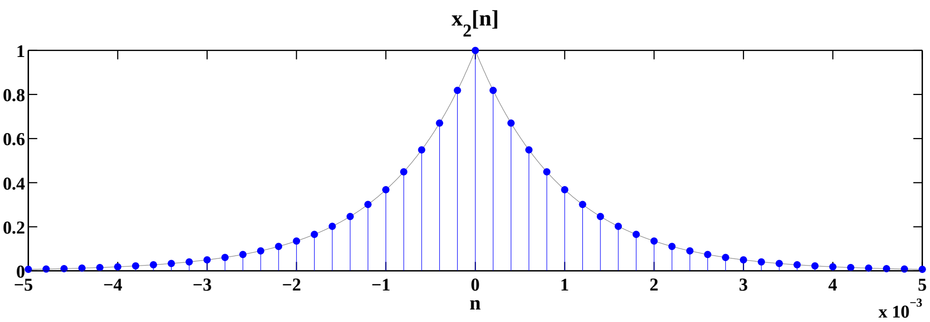


FIGURE 6.15: Plot of sampled signal $x_2[n]$.

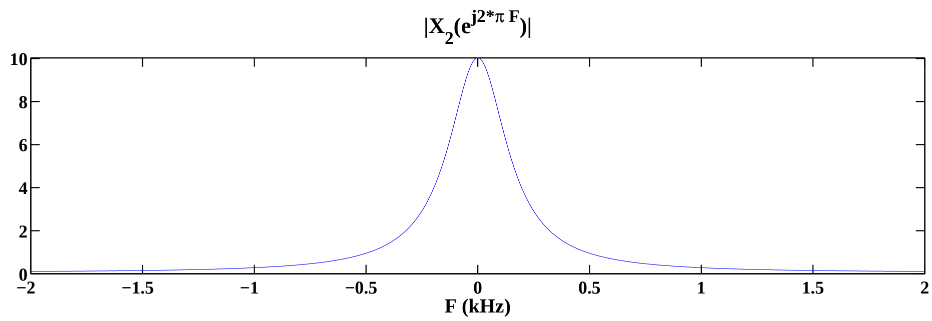


FIGURE 6.16: Plot of the CTFT $X_2\left(e^{j2\pi F}\right)$ of sampled signal $x_2[n]$.

```
plot(t,xc)
xlabel('t (sec)','fontsize',LFS)
title('x_c(t)','fontsize',TFS)
hfb = figconfg('P0607a2','long');
plot(F/1000,abs(Xc))
xlabel('F (kHz)','fontsize',LFS)
title('|X_c(j2*\pi F)|','fontsize',TFS)

hfc = figconfg('P0607b1','long');
plot(t,xc,'color',[1 1 1]*0.5);hold on
stem(nT,xn,'filled')
xlabel('n','fontsize',LFS)
title(['x_',num2str(ii),'[n]'],'fontsize',TFS)
hfd = figconfg('P0607b2','long');
plot(F/1000,abs(X))
```

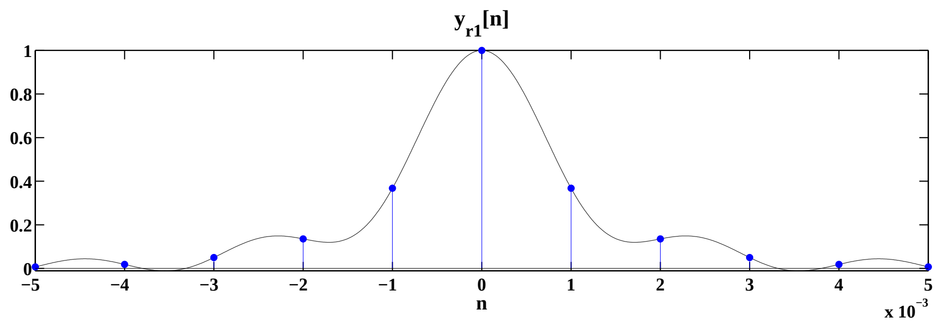


FIGURE 6.17: Reconstructed signal $y_{\mathrm{r}1}(t)$ from samples $x_1[n]$.

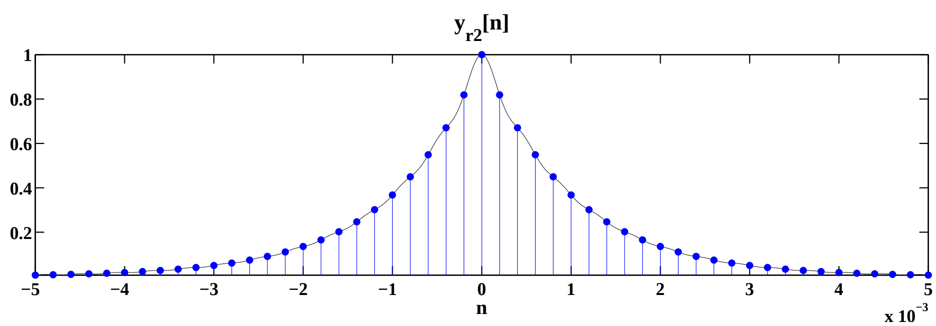


FIGURE 6.18: Reconstructed signal $y_{\mathrm{r}2}(t)$ from samples $x_2[n]$.

```
ylim([0 max(abs(X))])
xlabel('F (kHz)','fontsize',LFS)
title(['|X_',num2str(ii),'(e^{j2*\pi F})|'],'fontsize',TFS)

hfe = figconfg('P0607e','long');
plot(t,yr,'color',[1 1 1]*0.1);hold on
stem(nT,xn,'filled')
ylim([min(yr) max(yr)])
xlabel('n','fontsize',LFS)
title(['y_{r',num2str(ii),'}[n]'],'fontsize',TFS)
```

8. Proof:

$$x_c(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T_0}kt}$$

$$x_c(nT) = x_c(nT_0/N) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T_0}k\frac{nT0}{N}} = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{N}kn}$$

$$x[n] = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn}$$

Since we have $x[n] = x_c(nT)$, we require that

$$\sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{N}kn} = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn}$$

$$\begin{aligned}\sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn} = \sum_{k=0}^{N-1}\left(\sum_{\ell=-\infty}^{\infty} c_{k-\ell N}\right) e^{j\frac{2\pi}{N}kn} &= \sum_{\ell=-\infty}^{\infty}\left(\sum_{k=0}^{N-1} c_{k-\ell N} e^{j\frac{2\pi}{N}kn}\right) \\ &= \sum_{\ell=-\infty}^{\infty}\left(\sum_{k=0}^{N-1} c_{k-\ell N} e^{j\frac{2\pi}{N}kn} e^{j\frac{2\pi}{N}(-\ell N)}\right) \\ &= \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{N}kn}\end{aligned}$$

Hence, we prove that

$$\tilde{c}_k = \sum_{\ell=-\infty}^{\infty} c_{k-\ell N},\ k = 0, \pm 1, \pm 2, \dots$$

9. Proof:

$$\begin{aligned}\sum_{n=-\infty}^{\infty} y[n]\delta[t-nT] * g_{BL}(t) &= \sum_{\tau=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} y[n]\delta[\tau-nT]\cdot g_{BL}(t-\tau) \\ &= \sum_{\tau=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} y[n]\delta[t-\tau-nT]\cdot g_{BL}(\tau) \\ &= \sum_{n=-\infty}^{\infty} y[n]\left(\sum_{\tau=-\infty}^{\infty}\delta[t-\tau-nT]\cdot g_{BL}(\tau)\right) \\ &= \sum_{n=-\infty}^{\infty} y[n]\cdot g_{BL}(t-nT)\end{aligned}$$

10. (a) Solution:
The frequency response of $h_c(t)$ is:

$$H_\mathrm{c}(\mathrm{j}2\pi F) = \frac{\Omega_n^2}{\Omega_n^2 - (2\pi F)^2 + \mathrm{j}2\zeta\Omega_n 2\pi F}$$

Its phase response is:

$$\angle H_\mathrm{c}(\mathrm{j}2\pi F) = -\tan^{-1}\frac{2\zeta\Omega_n 2\pi F}{\Omega_n^2 - (2\pi F)^2}$$

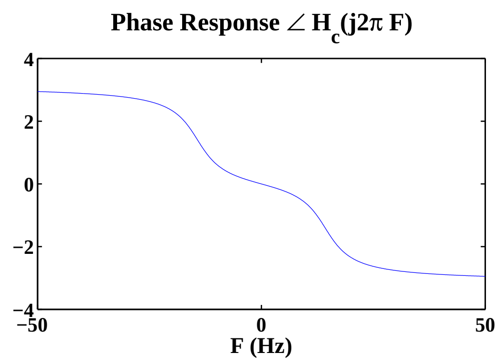


FIGURE 6.19: Plot of phase response $\angle H_\mathrm{c}(\mathrm{j}2\pi F)$.

(b) Solution:
The spectra of the sampled sequence is:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{\mathrm{e}^{-\zeta\Omega_n T}\sin(\Omega_n T\sqrt{1-\zeta^2})\mathrm{e}^{-\mathrm{j}\omega}}{1 - 2\mathrm{e}^{-\zeta\Omega_n T}\cos(\Omega_n T\sqrt{1-\zeta^2})\mathrm{e}^{-\mathrm{j}\omega} + \mathrm{e}^{-2\zeta\Omega_n T}\mathrm{e}^{-2\mathrm{j}\omega}}$$

Its phase response is:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) = -\omega - \tan^{-1}\frac{2\mathrm{e}^{-\zeta\Omega_n T}\cos(\Omega_n T\sqrt{1-\zeta^2})\sin\omega - \mathrm{e}^{-2\zeta\Omega_n T}\sin 2\omega}{1 - 2\mathrm{e}^{-\zeta\Omega_n T}\cos(\Omega_n T\sqrt{1-\zeta^2})\cos\omega + \mathrm{e}^{-2\zeta\Omega_n T}\cos 2\omega}$$

The effective phase response is:

$$\angle H_\mathrm{eff}(\mathrm{j}2\pi F) = \begin{cases} \angle H(\mathrm{e}^{\mathrm{j}2\pi FT}), & |F| \le \frac{F_\mathrm{s}}{2} \\ 0, & |F| > \frac{F_\mathrm{s}}{2} \end{cases}$$

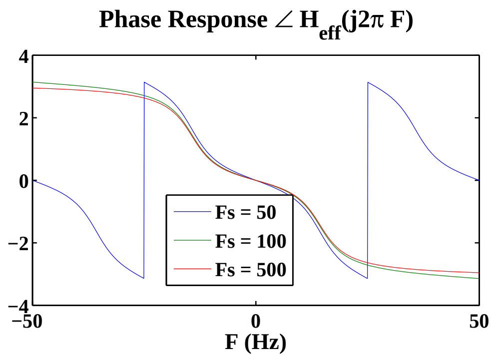


FIGURE 6.20: Plot of the effective phase responses, $\angle H_{\mathrm{eff}}(\mathrm{j}2\pi F)$ for $F_{\mathrm{s}} = 50$, 100, and 500 Hz.

(c) tba.

MATLAB script:

```
% P0610: Example 6.6: Second-order system
close all; clc
Fs = [50 100 500];
zeta = 0.3; Omega_n = 30*pi;
F = linspace(-50,50,1000);
Hc = Omega_n^2./((j*2*pi*F).^2+2*zeta*Omega_n*j*2*pi*F+Omega_n^2);
[FG FsG] = meshgrid(F,Fs);
H = Omega_n/sqrt(1-zeta^2)*exp(-zeta*Omega_n./FsG)...
    .*sin(Omega_n./FsG*sqrt(1-zeta^2)).*exp(-j*2*pi*FG./FsG)...
    ./(1-2*exp(-zeta*Omega_n./FsG).*cos(Omega_n./FsG*sqrt(1-zeta^2))...
    .*exp(-j*2*pi.*FG./FsG)+exp(-2*zeta*Omega_n./FsG)...
    .*exp(-2*j*2*pi.*FG./FsG));
%% Plot:
hfb = figconfg('P0610a','small');
plot(F,angle(Hc));
xlabel('F (Hz)','fontsize',LFS)
title('Phase Response \angle H_c(j2\pi F)','fontsize',TFS)
hfc = figconfg('P0610b','small');
plot(F,angle(H));
xlabel('F (Hz)','fontsize',LFS)
title('Phase Response \angle H_{eff}(j2\pi F)','fontsize',TFS)
legend(['Fs = ',num2str(Fs(1))],['Fs = ',num2str(Fs(2))],...
```

```
['Fs = ',num2str(Fs(3))],'location','best')
```

11. tba

12. (a) Solution:
    The quantizer resolution is:

$$\frac{10\text{v}}{2^8} = 0.0390625\text{v}$$

    (b) Solution:

$$\text{SQNR} = 10\log_{10}\text{SQNR} = 6.02B + 1.76 = 6.02\times 8 + 1.76 = 49.92\text{dB}$$

    (c) Solution:
    The sampling rate is:

$$F_\text{s} = \frac{2^{11}}{2^3} = 2^8\text{sam/sec}$$

    The folding frequency is $F_\text{s}/2 = 2^7$.
    The Nyquist rate is 500.

    (d) Solution:
    The reconstructed signal $y_\text{c}(t)$ is:

$$y_\text{c}(t) = 2\cos(200\pi t) - 3\sin(12\pi t)$$

13. Proof:
    (i) Linearity.

$$a_1 \cdot x_{\text{in1}}(nT) + a_2 \cdot x_{\text{in2}}(nT) = a_1 \cdot x_{\text{out1}}(t) + a_2 \cdot x_{\text{out2}}(t)$$

    The S&H system follows the superposition property, and hence is a linear system.
    (ii) Time-variance.

$$x_{\text{out}}(t-\tau) \neq x_{\text{out}}(t), \text{if } t-\tau \notin [nT, (n+1)T]$$

    Hence, the system is time-varying.

14. (a) Solution:

$$g_{\mathrm{SH}}(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases} \xrightarrow{\mathrm{CTFT}} G_{\mathrm{SH}}(\mathrm{j}\Omega) = \frac{2\sin(\Omega T/2)}{\Omega}\mathrm{e}^{-\mathrm{j}\Omega T/2}$$

$$H_{\mathrm{r}}(\mathrm{j}\Omega) = \begin{cases} \frac{\Omega T/2}{\sin(\Omega T/2)} \cdot \mathrm{e}^{\mathrm{j}\Omega T/2}, & |\Omega| < \pi/T \\ 0, & \text{otherwise} \end{cases}$$

The frequency response is:

$$H_{\mathrm{r}}(\mathrm{e}^{\mathrm{j}\omega}) = \begin{cases} \frac{\omega/2}{\sin(\omega/2)\cdot \mathrm{e}^{\mathrm{j}\omega/2}}, & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}$$

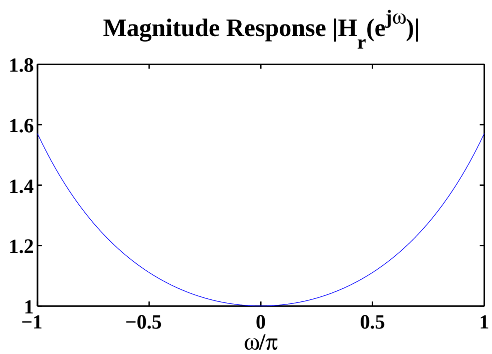


FIGURE 6.21: Magnitude response of ideal digital filter $H_{\mathrm{r}}(\mathrm{e}^{\mathrm{j}\omega})$.

(b) Solution: The magnitude response of $H_{\mathrm{FIR}}(\mathrm{e}^{\mathrm{j}\omega})$ is:

$$\begin{aligned} |H_{\mathrm{FIR}}(\mathrm{e}^{\mathrm{j}\omega})| &= \sqrt{(-\frac{1}{16} + \frac{9}{8}\cos\omega - \frac{1}{16}\cos 2\omega)^2 + (-\frac{9}{8}\sin\omega + \frac{1}{16}\sin 2\omega)^2} \\ &= \sqrt{\frac{1}{16^2} + \frac{9^2}{8^2} + \frac{1}{16^2} - 4 \times \frac{9}{8} \times 116\cos\omega + \frac{2}{16^2}\cos 2\omega} \end{aligned}$$

(c) Solution: The magnitude response of $H_{\mathrm{IIR}}(\mathrm{e}^{\mathrm{j}\omega})$ is:

$$|H_{\mathrm{IIR}}(\mathrm{e}^{\mathrm{j}\omega})| = \frac{9}{\sqrt{(8+\cos\omega)^2 + \sin^2\omega}} = \frac{9}{\sqrt{1 + 8^2 + 2\cos\omega}}$$

MATLAB script:

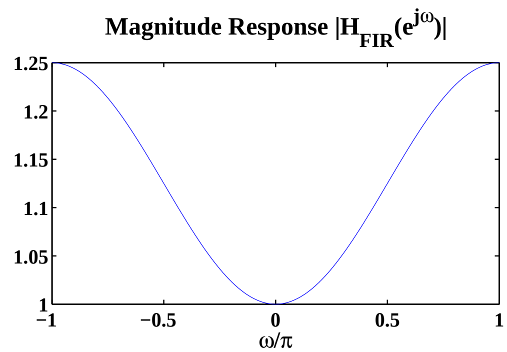


FIGURE 6.22: Magnitude response of low-order FIR filter $H_{\mathrm{FIR}}(\mathrm{e}^{\mathrm{j}\omega})$.

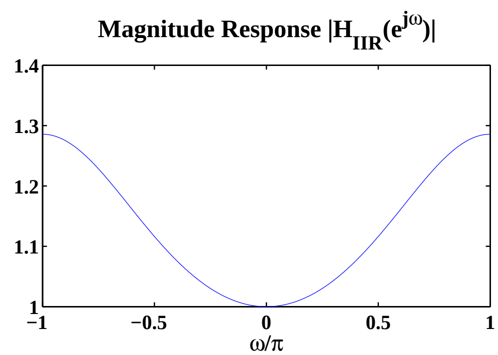


FIGURE 6.23: Magnitude response of low-order IIR filter $H_{\mathrm{IIR}}(\mathrm{e}^{\mathrm{j}\omega})$.

```
% P0614: Investigate droop distortion compensation
close all; clc
w = linspace(-1,1,1000)*pi;
%% Part (a):
Hr = w/2./sin(w/2).*exp(j*w/2);
%% Part (b):
HFIR = -1/16+9/8*exp(-j*w)-1/16*exp(-2*j*w);
%% Part (c):
HIIR = 9./(8+exp(-j*w));
%% Plot:
hfa = figconfg('P0614a','small');
plot(w/pi,abs(Hr));
xlabel('\omega/\pi','fontsize',LFS)
```

```
title('Magnitude Response |H_r(e^{j\omega})|','fontsize',TFS)
hfb = figconfg('P0614b','small');
plot(w/pi,abs(HFIR));
xlabel('\omega/\pi','fontsize',LFS)
title('Magnitude Response |H_{FIR}(e^{j\omega})|','fontsize',TFS)
hfc = figconfg('P0614c','small');
plot(w/pi,abs(HIIR));
xlabel('\omega/\pi','fontsize',LFS)
title('Magnitude Response |H_{IIR}(e^{j\omega})|','fontsize',TFS)
```

15. Solution:

$$g_{\mathrm{r}}(t) = \frac{\sin(\pi Bt)}{\pi Bt}\cos(2\pi F_c t) \tag{6.74}$$

The spectra is:

$$G_{\mathrm{r}}(\mathrm{j}2\pi F) = \begin{cases} 1, & |F| \in [F_L, F_H] \\ 0, & \text{otherwise} \end{cases}$$

Baseband spectra is:

$$G(\mathrm{j}2\pi F) = \begin{cases} T, & |F| \le B/2 \\ 0, & |F| > B/2 \end{cases}$$

The continuous time signal is

$$g(t) = \frac{\sin(\pi tB)}{\pi tB}$$

We can conclude that

$$G_{\mathrm{r}}(\mathrm{j}2\pi F) = G[\mathrm{j}2\pi(F+F_c)] + G[\mathrm{j}2\pi(F-F_c)]$$

Hence,

$$\begin{aligned} g_{\mathrm{r}}(t) &= \frac{\sin(\pi Bt)}{\pi Bt}(\mathrm{e}^{-\mathrm{j}2\pi F_c t} + \mathrm{e}^{\mathrm{j}2\pi F_c t}) \\ &= \frac{\sin(\pi Bt)}{\pi Bt}\cos(2\pi F_c t) \end{aligned}$$

where

$$F_c = \frac{F_H + F_L}{2}, \qquad B = F_H - F_L$$

16. (a) See plot below.

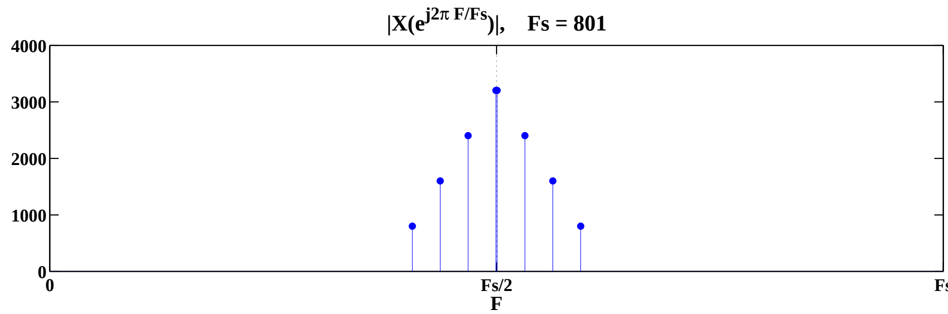


FIGURE 6.24: Spectrum of the sampled signal as a function of F Hz when the sampling rate is $F_s = 801$.

(b) See plot below.

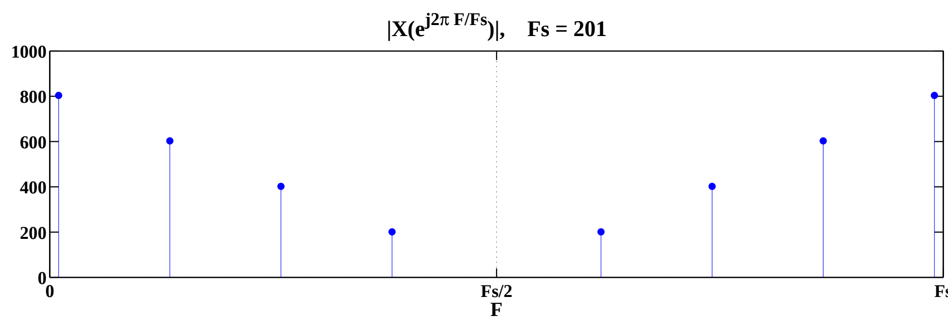


FIGURE 6.25: Spectrum of the sampled signal as a function of F Hz when the sampling rate is $F_s = 201$.

(c) tba.

```
% P0616: Sampling Illustration
close all; clc
%% Part (a):
% Fs = 801;
%% Part (b):
Fs = 201;
dF = 1;
F = 0:dF:Fs;
```

```
X = zeros(size(F));
FcosF = [325 350 375 400];
Fcos = [1 2 3 4];
while any(FcosF > Fs/2)
    ind = FcosF > Fs/2;
    FcosF(ind) = abs(FcosF(ind) - Fs);
end
for jj = -1:1
for ii = 1:length(FcosF)
    ind = abs(F)==abs(FcosF(ii)+jj*Fs);
    X(ind) = X(ind) + Fcos(ii)*Fs;
end
end
ind = X==0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0616a','long');
stem(F,abs(X),'filled');
xlim([0 Fs])
set(gca,'Xtick',[0 Fs/2 Fs])
set(gca,'Xticklabel',{'0','Fs/2','Fs'})
set(gca,'XGrid','on')
xlabel('F','fontsize',LFS)
title(['|X(e^{j2\pi F/Fs})|,    Fs = ',num2str(Fs)],'fontsize',TFS)
```

17. Solution:

$$F'_{\mathrm{L}} = 105 - 5 = 100\mathrm{Hz}, \qquad F'_{\mathrm{H}} = 145 + 5 = 150\mathrm{Hz}$$

The bandwidth is

$$B = F'_{\mathrm{H}} - F'_{\mathrm{L}} = 50\mathrm{Hz}$$

The minimum sampling rate is computed by

$$\min F_{\mathrm{s}} = 2F'_{\mathrm{H}}/\lfloor F'_{\mathrm{H}}/B \rfloor = 100\mathrm{Hz}$$

MATLAB script:

```
% P0617: Sampling Illustration
close all; clc
```

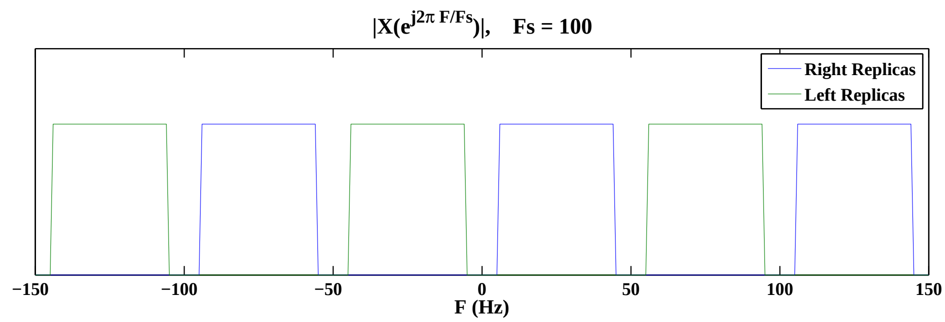


FIGURE 6.26: Spectrum of the sampled signal as a function of F Hz.

```
FL = 105; FH = 145;
dF = 1;
F = -150:dF:150;
Fs = 100;
X = zeros(size(F));
XP = zeros(size(F));
XN = zeros(size(F));
for jj = -10:10
    ind = F > 105+jj*Fs & F < 145+jj*Fs;
    X(ind) = X(ind) + 1;
    XP(ind) = XP(ind) + 1;
    ind = F > -145+jj*Fs & F < -105+jj*Fs;
    X(ind) = X(ind) + 1;
    XN(ind) = XN(ind) + 1;
end
ind = X == 0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0617a','long');
plot(F,XP,F,XN);
ylim([0 1.5])
set(gca,'YTick',-1)
xlabel('F (Hz)','fontsize',LFS)
title(['|X(e^{j2\pi F/Fs})|,    Fs = ',num2str(Fs)],'fontsize',TFS)
legend('Right Replicas','Left Replicas','location','best')
```

18. Proof:

$$p_{\mathrm{c}}(x,y) = \begin{cases} 1/A^2, & |x| < A/2, \quad |y| < A/2 \\ 0, & \text{otherwise} \end{cases} \tag{6.89}$$

$$P_{\mathrm{c}}(F_x,F_y) = \frac{\sin(\pi F_x A)}{\pi F_x A} \times \frac{\sin(\pi F_y A)}{\pi F_y A} \tag{6.90}$$

$$\begin{aligned}
P_{\mathrm{c}}(F_x,F_y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{\mathrm{c}}(x,y)\mathrm{e}^{-\mathrm{j}2\pi(xF_x+yF_y)}\,\mathrm{d}x\mathrm{d}y \\
&= \int_{-\frac{A}{2}}^{\frac{A}{2}}\int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{A^2}\mathrm{e}^{-\mathrm{j}2\pi(xF_x+yF_y)}\,\mathrm{d}x\mathrm{d}y \\
&= \left(\int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{A}\mathrm{e}^{-\mathrm{j}2\pi xF_x}\,\mathrm{d}x\right)\cdot\left(\int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{A}\mathrm{e}^{-\mathrm{j}2\pi yF_y}\,\mathrm{d}y\right) \\
&= \frac{\mathrm{e}^{-\mathrm{j}2\pi xF_x}\Big|_{-\frac{A}{2}}^{\frac{A}{2}}}{A\cdot(-\mathrm{j}2\pi F_x)}\cdot\frac{\mathrm{e}^{-\mathrm{j}2\pi yF_y}\Big|_{-\frac{A}{2}}^{\frac{A}{2}}}{A\cdot(-\mathrm{j}2\pi F_y)} \\
&= \frac{\sin(\pi F_x A)}{\pi F_x A} \times \frac{\sin(\pi F_y A)}{\pi F_y A}
\end{aligned}$$

19. (a) Solution:

$$s_{\mathrm{c}}(x,y) = 3\cos(2.4\pi x + 2.6\pi y) = 3\cos(2.4\pi x)\cos(2.6\pi y) - 3\sin(2.4\pi x)\sin(2.6\pi y)$$

$$s[m,n] = 3\cos(0.8\pi m + 1.3\pi n)$$

$$s_{\mathrm{r}}(x,y) = 3\cos(1.6\pi x - 2.6\pi y)$$

(b) Solution:

$$s[m,n] = 3\cos(1.2\pi m + 0.8667\pi n)$$

$$s_{\mathrm{r}}(x,y) = 3\cos(2.4\pi x - 1.4\pi y)$$

(c) Solution:

$$s[m,n] = 3\cos(0.8\pi m + 0.8667\pi n)$$

$$s_{\mathrm{r}}(x,y) = 3\cos(2.4\pi x + 2.6\pi y)$$

20. (a) Solution:

$$s_{\mathrm{fa}}[m,n] = \frac{1}{\Delta x\Delta y}\int_{m\Delta x-\frac{\Delta x}{2}}^{m\Delta x+\frac{\Delta x}{2}}\int_{n\Delta y-\frac{\Delta y}{2}}^{n\Delta y+\frac{\Delta y}{2}} s_{\mathrm{c}}(x,y)\,\mathrm{d}x\mathrm{d}y$$

(b) tba

(c) tba

Basic Problems

21. Proof:
The sampler is:

$$x_{\text{out}}(t) = x_{\text{in}}(nT); \quad nT \le t < (n+1)T, \ \forall n$$

(i) Memoryless. The current system value is only related to the current time index and is not affected by previous system values. Hence, the sampler is memoryless.
(ii) Linearity.

$$a_1 \cdot x_{\text{in1}}(nT) + a_2 \cdot x_{\text{in2}}(nT) = a_1 \cdot x_{\text{out1}}(t) + a_2 \cdot x_{\text{out2}}(t)$$

The S&H system follows the superposition property, and hence is a linear system.
(iii) Time-variance.

$$x_{\text{out}}(t-\tau) \neq x_{\text{out}}(t), \text{if } t-\tau \notin [nT,(n+1)T]$$

Hence, the system is time-varying.

22. Solution:
The spectra of the continuous signal $x_{\text{c}}(t)$ is

$$X_{\text{c}}(\text{j}2\pi F) = \begin{cases} 3, & F = 0 \\ \text{j}, & F = 8 \\ -\text{j}, & F = -8 \\ 5, & F = 12 \\ 5, & F = -12 \\ 0, & \text{otherwise} \end{cases}$$

The spectra of sampled sequence $x[n]$ is:

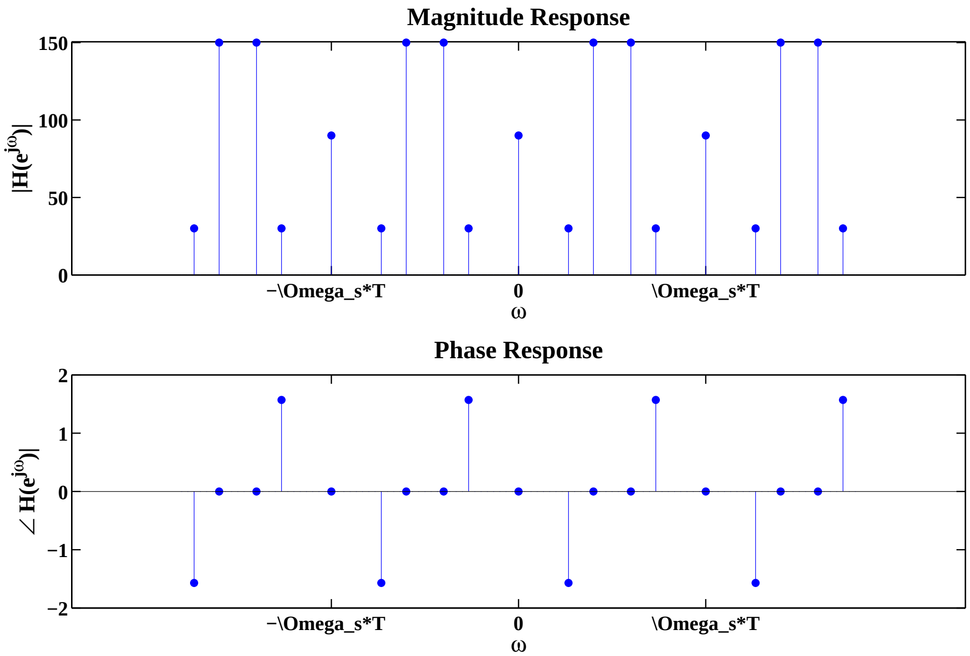
$$X\big(\text{e}^{\text{j}\omega}\big)\big|_{\omega=2\pi F/F_{\text{s}}} = F_{\text{s}} \sum_{n=-\infty}^{\infty} X_{\text{c}}[\text{j}2\pi(F - nF_{\text{s}})]$$

$x_{\text{c}}(t)$ can be recovered if (a) $F_{\text{s}} = 30$ Hz, and can NOT be recovered if (b) $F_{\text{s}} = 20$ Hz, (c) $F_{\text{s}} = 15$ Hz.

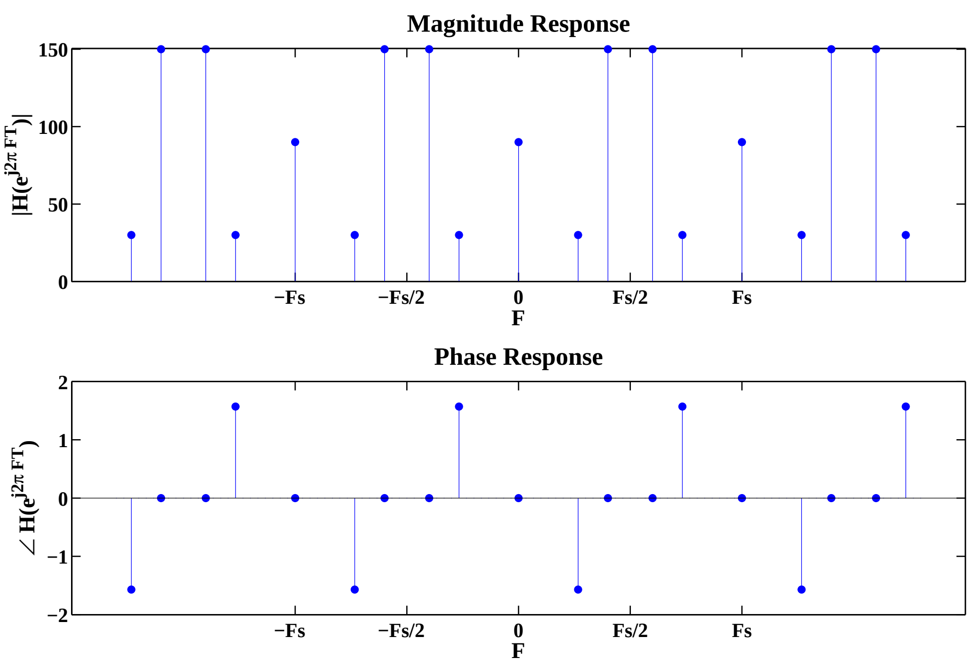
MATLAB script:

```
% P0622: Illustrates the alias distortion
close all; clc
Fs = 30; % Part (a)
% Fs = 20; % Part (b)
% Fs = 15; % Part (c)
T = 1/Fs;
FH = 24;
FL = FH+Fs;
F = -FL:1:FL;
X = zeros(1,length(F));
for k = -5:5;
ind = F == k*Fs; X(ind) = X(ind)+3*Fs;
ind = F == -12+k*Fs; X(ind) = X(ind)+5*Fs;
ind = F == -8+k*Fs; X(ind) = X(ind)-1/j*Fs;
ind = F == 8+k*Fs; X(ind) = X(ind)+1/j*Fs;
ind = F == 12+k*Fs; X(ind) = X(ind)+5*Fs;
end
ind = X==0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0622a');
subplot(211)
stem(F*2*pi*T,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F*2*pi*T,angle(X),'filled')
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('\angle H(e^{j\omega})|','fontsize',LFS)
title('Phase Response','fontsize',TFS)

hfb = figconfg('P0622b');
subplot(211)
stem(F,abs(X),'filled')
```

```
ylim([0 max(abs(X))+0.5])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi FT})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F,angle(X),'filled')
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('\angle H(e^{j2\pi FT})','fontsize',LFS)
title('Phase Response','fontsize',TFS)
```



(a)



(b)

FIGURE 6.27: Spectra of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_{\text{s}} = 30$ KHz.

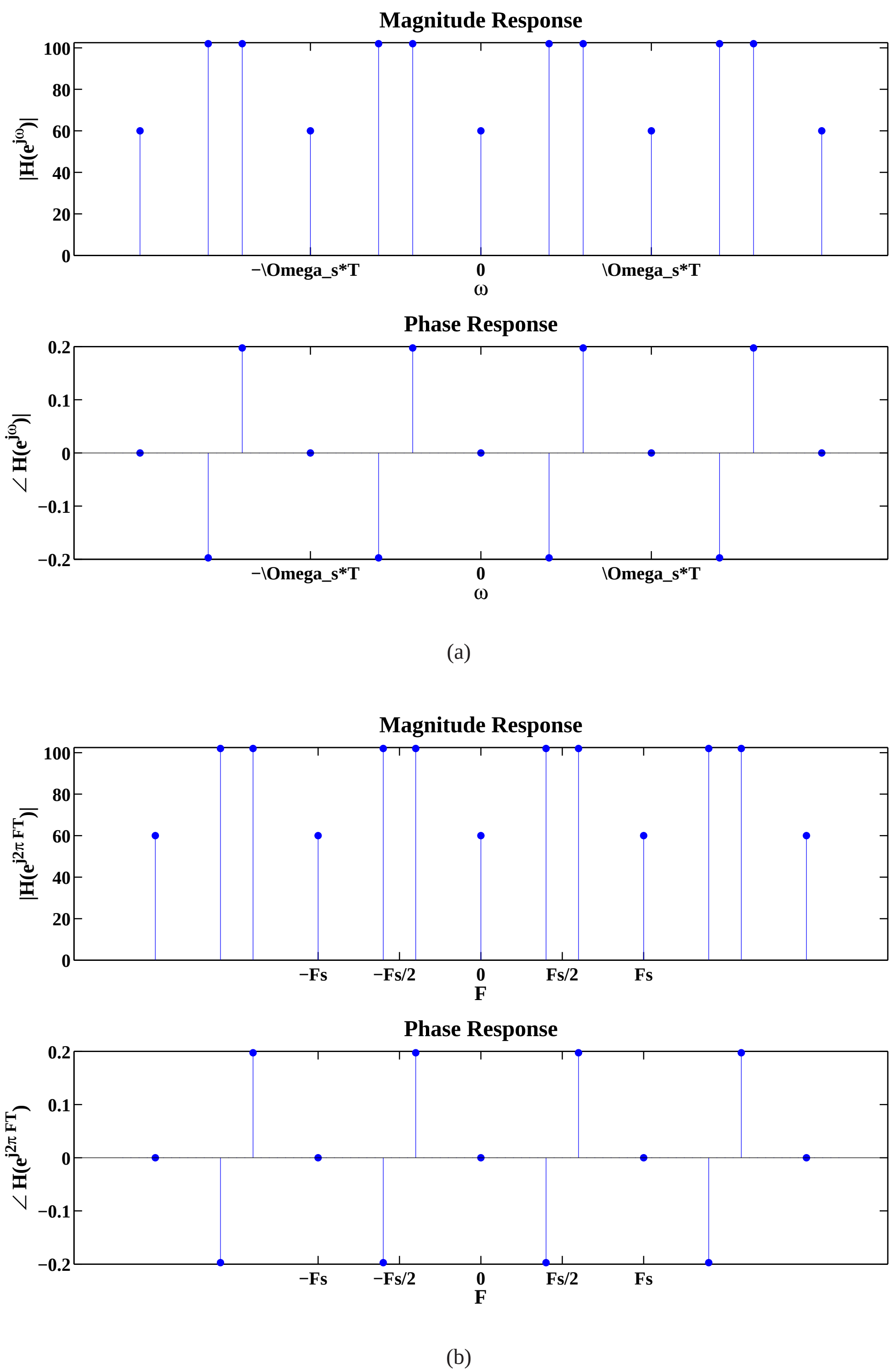
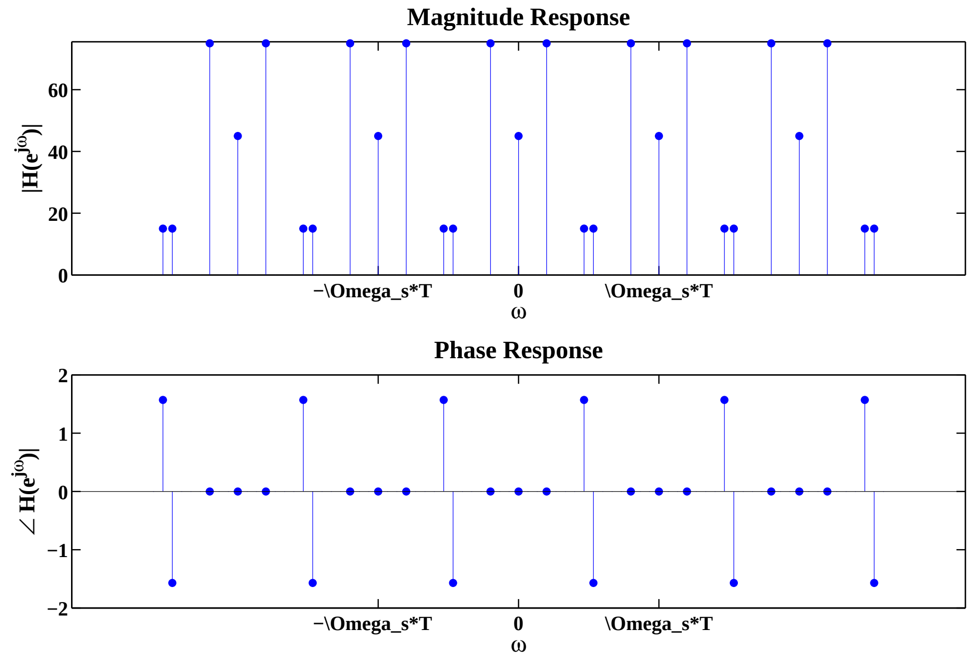
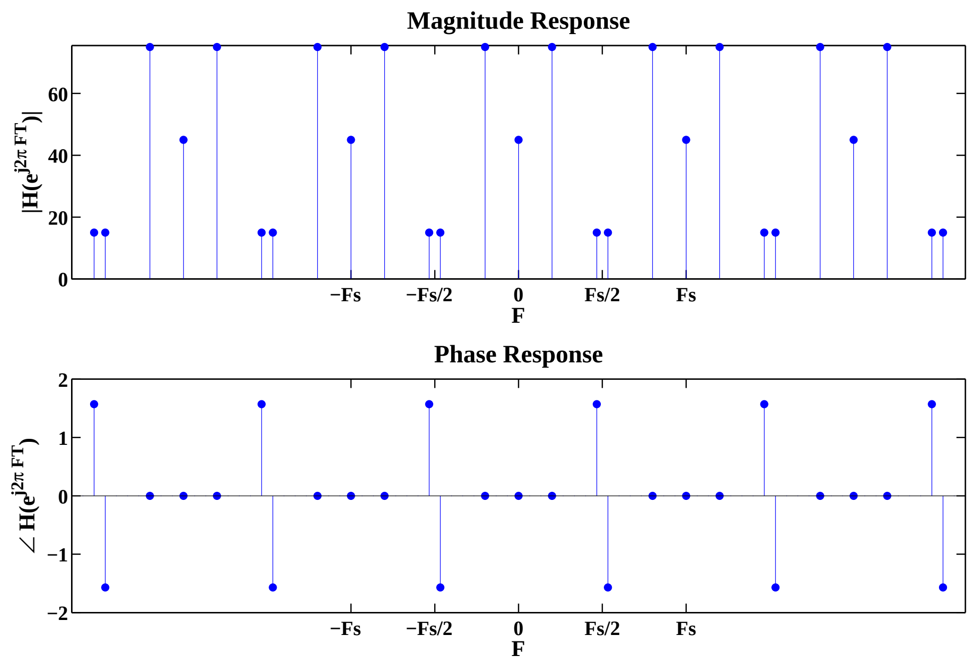


FIGURE 6.28: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 20$ KHz.



(a)



(b)

FIGURE 6.29: Spectra of $X(e^{j\omega})$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 15$ KHz.

23. Solution:
The spectra of the continuous signal $x_{\mathrm{c}}(t)$ is:

$$X_{\mathrm{c}}(\mathrm{j}\Omega) = \begin{cases} 5, & \Omega = 40 \\ 3, & \Omega = -70 \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X(\mathrm{e}^{\mathrm{j}\omega})\big|_{\omega=\Omega T} = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_{\mathrm{r}}[\mathrm{j}(\Omega - k\Omega_s)]$$

The continuous signal $x_{\mathrm{c}}(t)$ can be recovered if the sampling interval is (a) $T = 0.01$, (b) $T = 0.04$, and can NOT be recovered if the sampling interval is (c) $T = 0.1$.

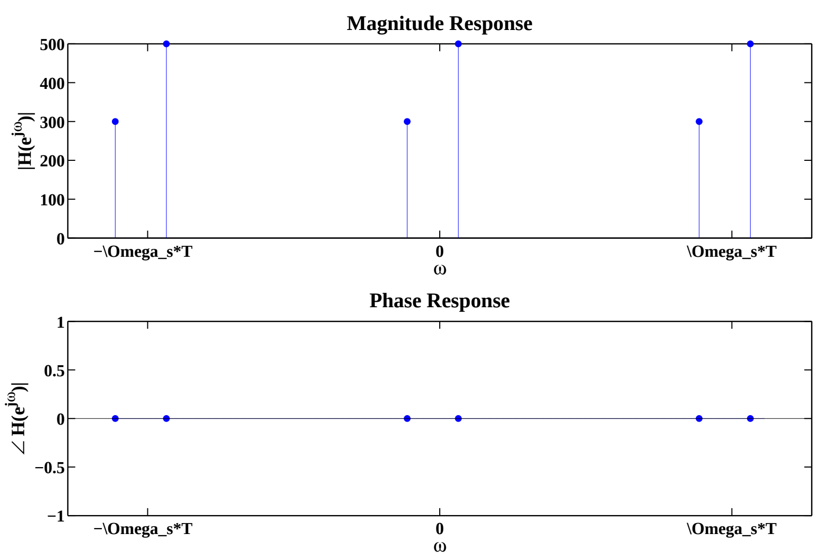


FIGURE 6.30: Magnitude and phase responses of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = 0.01$.

MATLAB script:

```
% P0623: Illustrates the alias distortion
close all; clc
% T = 0.01; % Part (a)
```

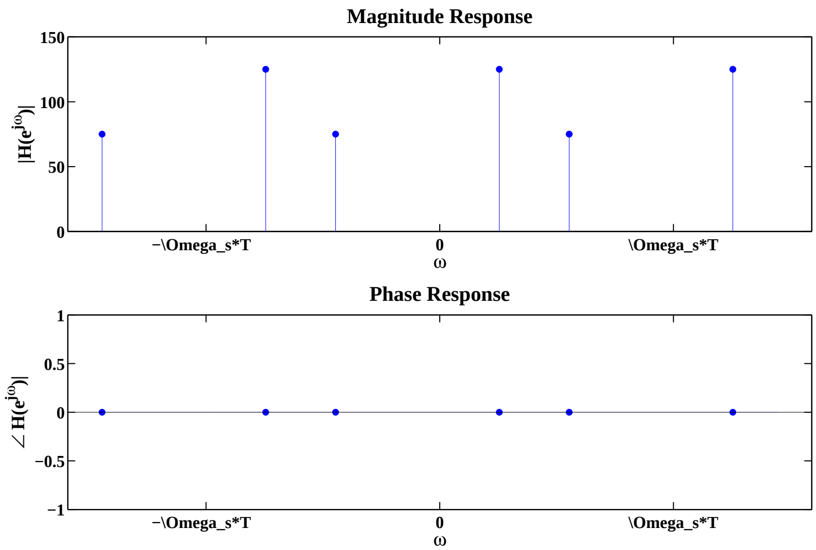


FIGURE 6.31: Magnitude and phase responses of $X\left(e^{j\omega}\right)$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = 0.04$.

```
% T = 0.04; % Part (b)
T = 0.1; % Part (c)
Fs = 1/T;
Omegas = 2*pi*Fs;
OmegaH = 70;
OmegaL = round(OmegaH+Omegas);
Omega = -OmegaL:1:OmegaL;
X = zeros(1,length(Omega));
for k = -10:10;
ind = Omega == round(40+k*Omegas); X(ind) = X(ind)+5*Fs;
ind = Omega == round(-70+k*Omegas); X(ind) = X(ind)+3*Fs;
end
ind = X==0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0623a');
subplot(211)
stem(Omega*T,abs(X),'filled')
```

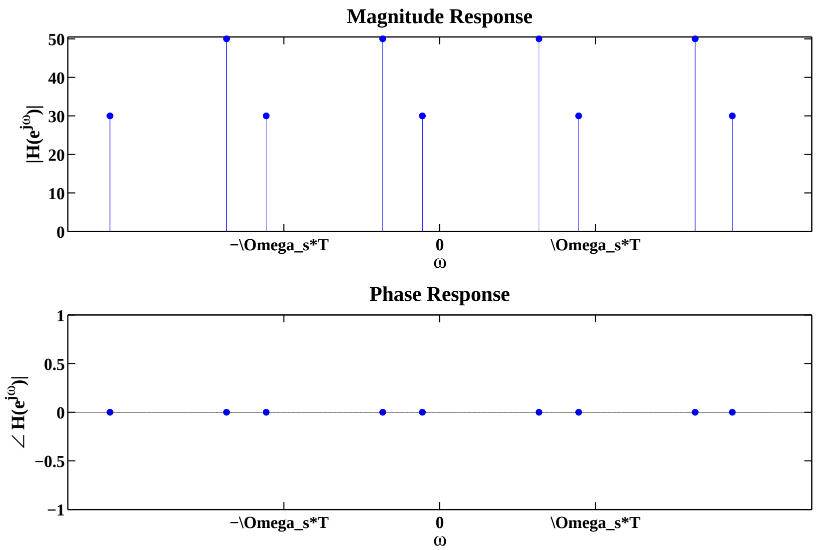


FIGURE 6.32: Magnitude and phase responses of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = 0.1$.

```
ylim([0 max(abs(X))+0.5])
set(gca,'XTick',[-Omegas*T 0 Omegas*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(Omega*T,angle(X),'filled')
set(gca,'XTick',[-Omegas*T 0 Omegas*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('\angle H(e^{j\omega})|','fontsize',LFS)
title('Phase Response','fontsize',TFS)
```

24. Solution:
The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j2\pi F) = \begin{cases} \frac{2}{5}|F| + 2, & |F| \le 5 \\ -\frac{4}{5}|F| + 8, & 5 < |F| \le 10 \\ 0, & \text{otherwise} \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=2\pi F/F_s} = F_s \sum_{k=-\infty}^{\infty} X_c[j2\pi(F - kF_s)]$$

The signal $x_c(t)$ can NOT be recovered from $x[n]$ if the sampling rate is (a) $F_s = 10$ Hz and (b) $F_s = 15$ Hz and can be recovered if the sampling rate is (c) $F_s = 30$ Hz.

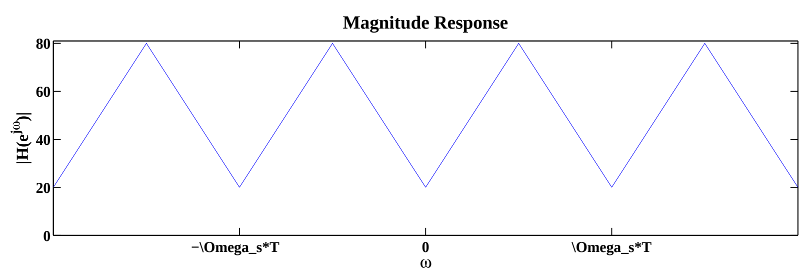


FIGURE 6.33: Magnitude response of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling rate is $F_s = 10$.

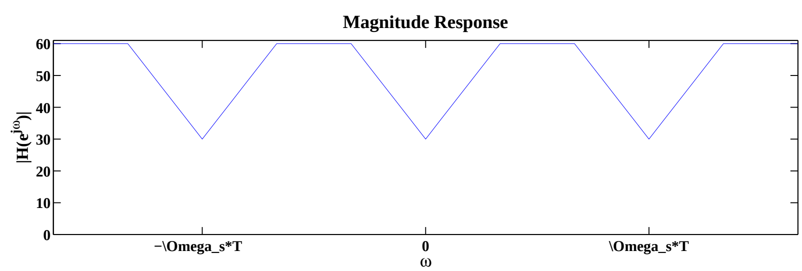


FIGURE 6.34: Magnitude response of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling rate is $F_s = 15$.

MATLAB script:

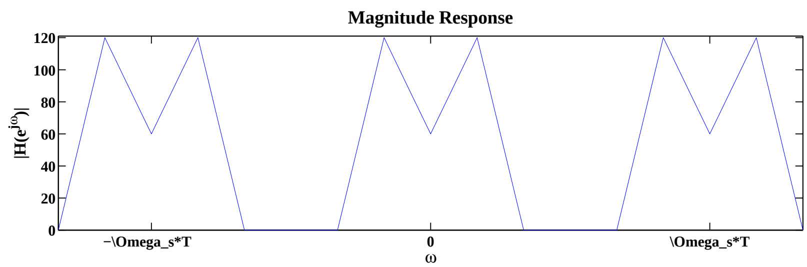


FIGURE 6.35: Magnitude response of $X\left(e^{j\omega}\right)$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling rate is $F_s = 30$.

```
% P0624: Illustrates the alias distortion
close all; clc
Fs = 10; % Part (a)
% Fs = 15; % Part (b)
% Fs = 30; % Part (c)
T = 1/Fs;
FH = 10;
FL = FH+Fs;
F = -FL:1:FL;
X = zeros(1,length(F));
for k = -5:5;
ind = abs(F-k*Fs) <= 5;
X(ind) = X(ind)+(2/5*abs(F(ind)-k*Fs)+2)*Fs;
ind = abs(F-k*Fs) > 5 & abs(F-k*Fs) <= 10;
X(ind) = X(ind)+(-4/5*abs(F(ind)-k*Fs)+8)*Fs;
end
%% Plot:
hfa = figconfg('P0624a','long');
plot(F*2*pi*T,abs(X))
ylim([0 max(abs(X))+1])
xlim([-FL*2*pi*T FL*2*pi*T])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
```

25. Solution:

The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j2\pi F) = \begin{cases} -5|F|+10, & 0 \le |F| \le 1 \\ 5|F|, & 1 \le |F| \le 2 \\ 10, & 2 \le |F| \le 3 \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X\big(e^{j2\pi F/F_s}\big) = F_s \sum_{k=-\infty}^{\infty} X_c[j2\pi(F - kF_s)]$$

The signal $x_c(t)$ can NOT be recovered from $x[n]$ when the sampling interval is (a) $T = 0.2$, (b) $T = 0.25$, (c) $T = 0.5$.

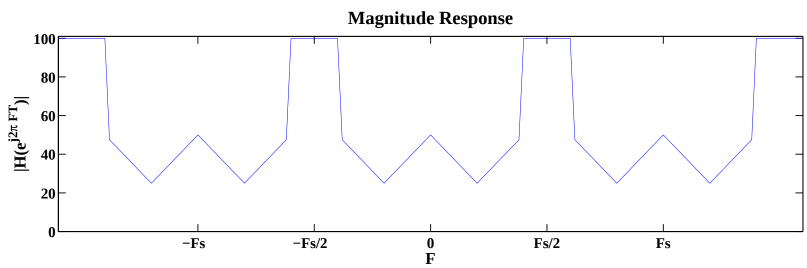


FIGURE 6.36: Magnitude response of $X\big(e^{j\omega}\big)$ as a function of F in Hz when the sampling interval is $T = 0.2$.

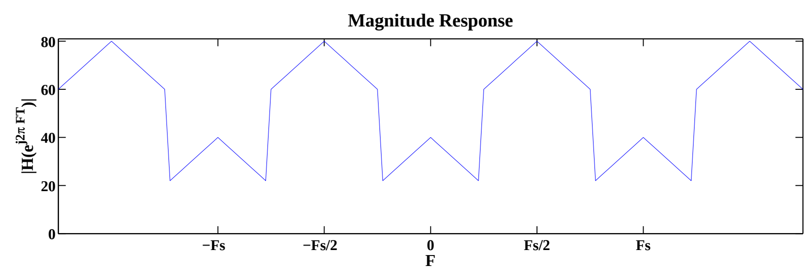


FIGURE 6.37: Magnitude response of $X\big(e^{j\omega}\big)$ as a function of F in Hz when the sampling interval is $T = 0.25$.

MATLAB script:

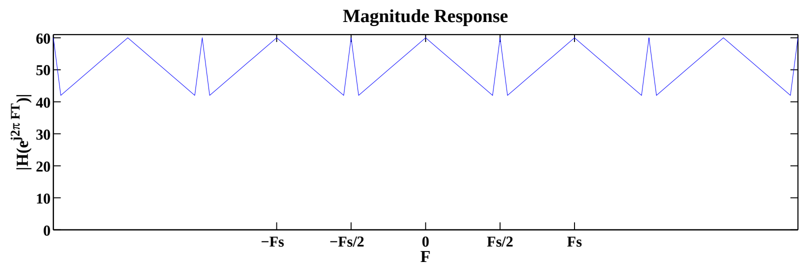


FIGURE 6.38: Magnitude response of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of F in Hz when the sampling interval is $T = 0.5$.

```
% P0625: Illustrates the alias distortion
close all; clc
% T = 0.2; % Part (a)
% T = 0.25; % Part (b)
T = 0.5; % Part (c)
Fs = 1/T;
FH = 3;
FL = FH+Fs;
F = -FL:0.1:FL;
X = zeros(1,length(F));
for k = -5:5;
ind = abs(F-k*Fs) <= 1;
X(ind) = X(ind)+(-5*abs(F(ind)-k*Fs)+10)*Fs;
ind = abs(F-k*Fs) > 1 & abs(F-k*Fs) <= 2;
X(ind) = X(ind)+(5*abs(F(ind)-k*Fs))*Fs;
ind = abs(F-k*Fs) > 2 & abs(F-k*Fs) <= 3;
X(ind) = X(ind)+10*Fs;
end
%% Plot:
hfa = figconfg('P0625a','long');
plot(F,abs(X))
xlim([-FL FL])
ylim([0 max(abs(X))+1])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi FT})|','fontsize',LFS)
```

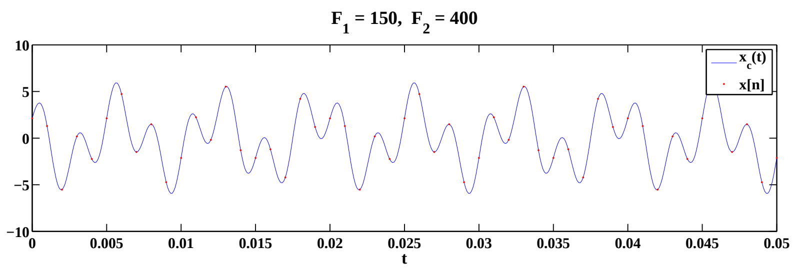
```
title('Magnitude Response','fontsize',TFS)
```

26. (a) Solution:
The sampled sequence $x[n]$ is:

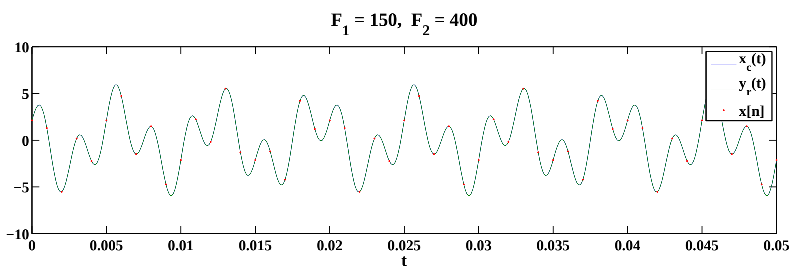
$$x[n] = x_c(nT) = 3\cos(0.3\pi n + \pi/4) + 3\sin(0.8\pi n)$$

(b) Solution:
The constructed signal $y_r(t)$ is:

$$y_r(t) = 3\cos(300\pi t + \pi/4) + 3\sin(800\pi t)$$



(a)



(b)

FIGURE 6.39: (a) Plot of $x[n]$ and $x_c(t)$ and (b) plot of $y_r(t)$ when $F_1 = 150$ Hz and $F_2 = 400$ Hz.

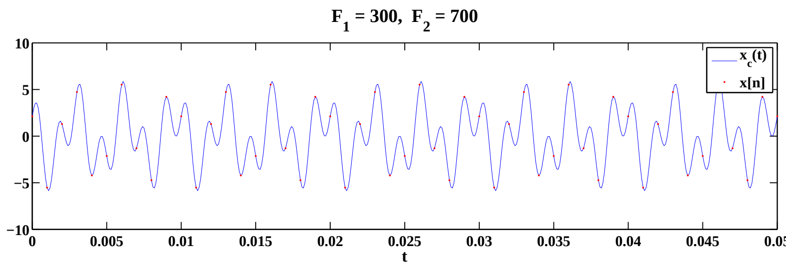
(c) Solution:
The sampled sequence $x[n]$ is:

$$x[n] = x_c(nT) = 3\cos(0.6\pi n + \pi/4) + 3\sin(1.4\pi n)$$

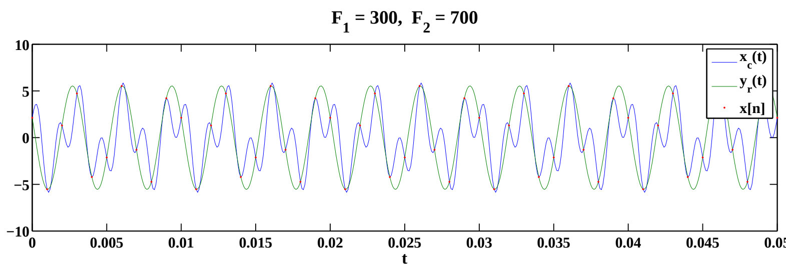
The constructed signal $y_r(t)$ is:

$$y_r(t) = 3\cos(600\pi t + \pi/4) - 3\sin(600\pi t)$$

MATLAB script:



(a)



(b)

FIGURE 6.40: (a) Plot of $x[n]$ and $x_{\mathrm{c}}(t)$ and (b) plot of $y_{\mathrm{r}}(t)$ when $F_1 = 300$ Hz and $F_2 = 700$ Hz.

```
% P0626: Illustrate Ideal DAC
close all; clc
t1 = 0; t2 = 0.05;
dt = 1e-4;
t = t1:dt:t2;
T = 0.001;
Fs = 1/T;
% F1 = 150; F2 = 400; % Part (a) (b)
F1 = 300; F2 = 700; % Part (c)
xc = 3*cos(2*pi*F1*t+pi/4)+3*sin(2*pi*F2*t);
% Sampling:
nT = t1:T:t2;
xn = 3*cos(2*pi*F1*nT+pi/4)+3*sin(2*pi*F2*nT);
F1y = F1;
while F1y > Fs/2
    F1y = F1y-Fs;
end
F2y = F2;
```

```
while F2y > Fs/2
    F2y = F2y-Fs;
end
yr = 3*cos(2*pi*F1y*t+pi/4)+3*sin(2*pi*F2y*t);
%% Plot:
hfa = figconfg('P0626a','long');
plot(t,xc); hold on
plot(nT,xn,'.','color','red')
xlabel('t','fontsize',LFS)
title(['F_1 = ',num2str(F1),',  F_2 = ',num2str(F2)],'fontsize',TFS)
legend('x_c(t)','x[n]','location','northeast')
hfb = figconfg('P0626b','long');
plot(t,xc,t,yr,nT,xn,'.')
xlabel('t','fontsize',LFS)
title(['F_1 = ',num2str(F1),',  F_2 = ',num2str(F2)],'fontsize',TFS)
legend('x_c(t)','y_r(t)','x[n]','location','northeast')
```

27. (a) Solution:

$$B = \frac{64\text{k}}{8\text{k}} = 8\text{bits/sam}$$

The quantizer step is:

$$\frac{10\text{v}}{2^8} = 0.0390625\text{v}$$

(b) Solution:
The SQNR is:

$$\text{SQNR} = 10\log_{10}\text{SQNR} = 6.02B+1.76 = 6.02*B+1.76 = 49.92\text{dB}$$

(c) the folding frequency
Solution:
The folding frequency is $F_s/2 = 4\text{k}$.

(d) Solution:
The reconstructed signal $x_r(t)$ is:

$$x_r(t) = -5\sin[6000\pi t - \pi/2]$$

28. Solution:

$$B = F_H - F_L = 20 - 18.1 = 1.9\text{KHz}$$

The minimum sampling frequency is:

$$\min F_s = 2F_H/\lfloor F_H/B \rfloor = 4\text{KHz}$$

29. Solution:

$$F'_L = F_L - 2\text{KHz} = 1000.5\text{KHz}$$
$$F'_H = F_H + 2\text{KHz} = 1048\text{KHz}$$

The bandwidth is:

$$B = F'_H - F'_L = 47.5\text{KHz}$$

The minimum sampling rate can be computed by

$$\min F_s = 2F'_H/\lfloor F'_H/B \rfloor = 95.2727273\text{KHz}$$

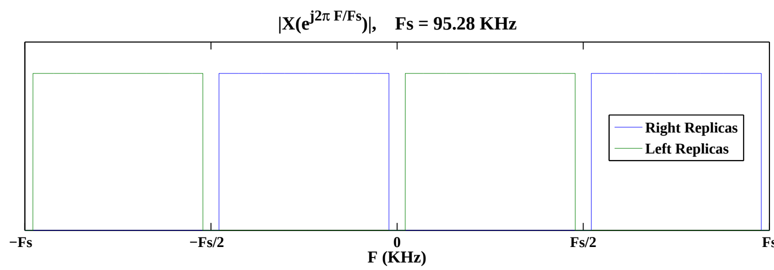


FIGURE 6.41: Baseband signal spectrum after sampling.

MATLAB script:

```
% P0629: Sampling Illustration
close all; clc
FL = 1002.5e3; FH = 1046e3;
dF = 10;
FG = 4e3;
FHH = FH + FG/2;
FLL = FL - FG/2;
Fs = 95.28e3;
F = -1.1*Fs:dF:1.1*Fs;
X = zeros(size(F));
XP = zeros(size(F));
```

```
XN = zeros(size(F));
for jj = -30:30
    ind = F > FL+jj*Fs & F < FH+jj*Fs;
    X(ind) = X(ind) + 1;
    XP(ind) = XP(ind) + 1;
    ind = F > -FH+jj*Fs & F < -FL+jj*Fs;
    X(ind) = X(ind) + 1;
    XN(ind) = XN(ind) + 1;
end
ind = X == 0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0629a','long');
plot(F/1e3,XP,F/1e3,XN);
xlim([-Fs/1e3 Fs/1e3])
ylim([0 1.2])
set(gca,'YTick',-1)
xlabel('F (KHz)','fontsize',LFS)
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs]/1e3)
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
title(['|X(e^{j2\pi F/Fs})|,    Fs = ',num2str(Fs/1e3),' KHz']...
    ,'fontsize',TFS)
legend('Right Replicas','Left Replicas','location','best')
```

30. (a) Solution:

    (i) when $\Delta x = \Delta y = 0.5$ meter, $F_{sx} = F_{sy} = 2$, the reconstructed signal is:

    $$s_{\mathrm{r}}(x, y) = 4\cos(2\pi y)$$

    (ii) when $\Delta x = \Delta y = 0.2$ meter, $F_{sx} = F_{sy} = 5$, the reconstructed signal is:

    $$s_{\mathrm{r}}(x, y) = 4\cos(4\pi x)\cos(4\pi y)$$

    (b) tba

31. tba.

Assessment Problems

32. Solution:

The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j2\pi F) = \begin{cases} 2, & |F| = 2\text{KHz} \\ 3, & |F| = 3\text{KHz} \\ \frac{1}{2j}, & F = 7\text{KHz} \\ -\frac{1}{2j}, & F = -7\text{KHz} \\ 0, & \text{otherwise} \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

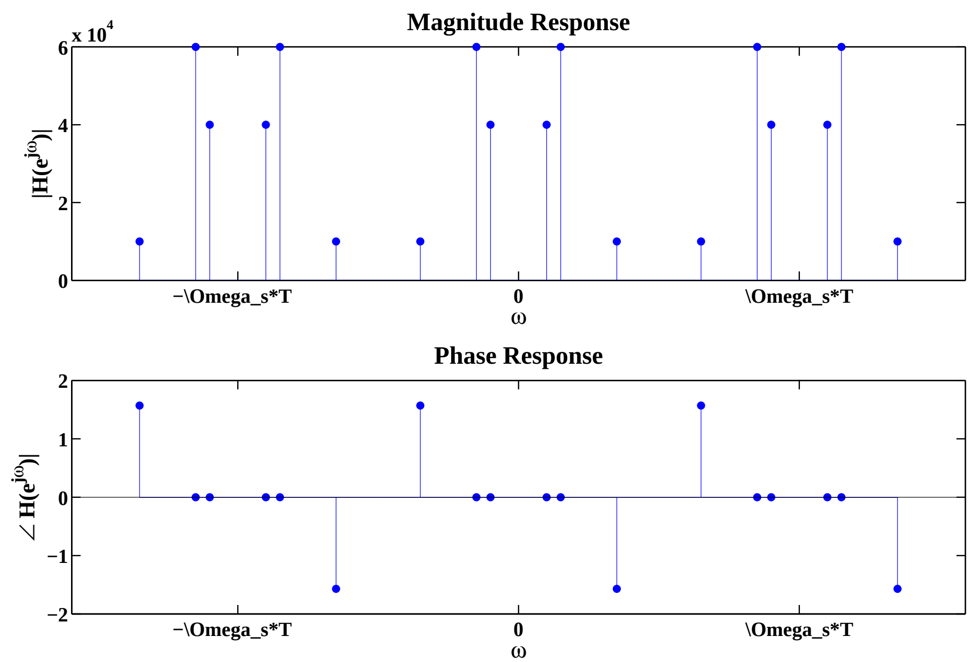
$$X(e^{j\omega})\big|_{\omega=2\pi F/F_s} = F_s \sum_{k=-\infty}^{\infty} X_c[j2\pi(F - kF_s)]$$

The signal $x_c(t)$ can be recovered from $x[n]$ if the sampling rate is (a) $F_s = 20$ and can NOT be recovered if the sampling rate is (b) $F_s = 10$, and (c) $F_s = 5$.

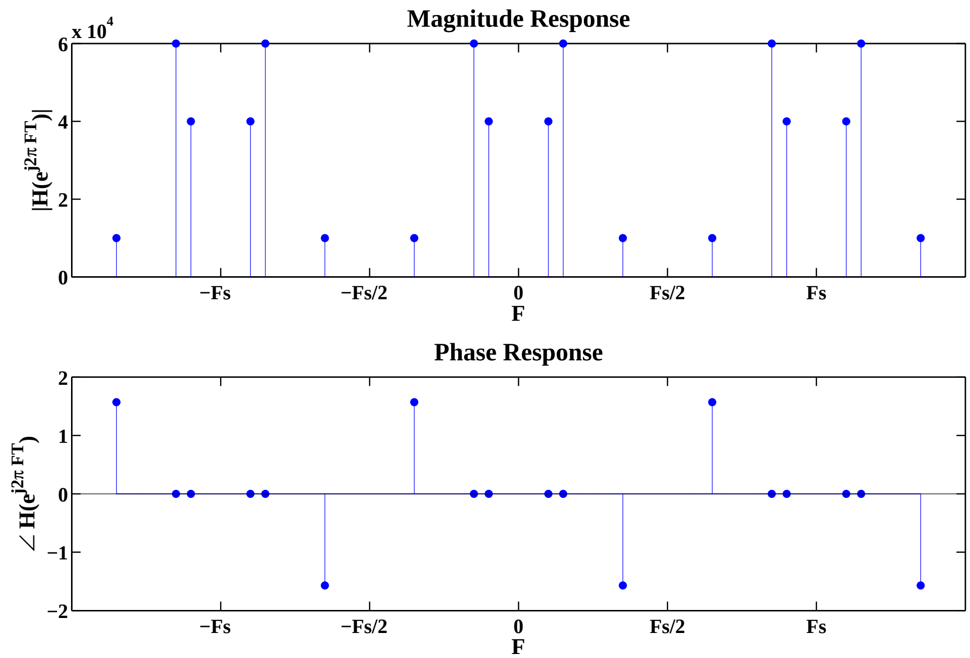
MATLAB script:

```
% P0632: Illustrates the alias distortion
close all; clc
% Fs = 20e3; % Part (a)
% Fs = 10e3; % Part (b)
Fs = 5e3; % Part (c)
T = 1/Fs;
FH = 7e3;
FL = FH+Fs;
F = -FL:10:FL;
X = zeros(1,length(F));
for k = -10:10;
ind = F == -7e3+k*Fs; X(ind) = X(ind)-1/2/j*Fs;
ind = F == -3e3+k*Fs; X(ind) = X(ind)+3*Fs;
ind = F == -2e3+k*Fs; X(ind) = X(ind)+2*Fs;
ind = F == 7e3+k*Fs; X(ind) = X(ind)+1/2/j*Fs;
ind = F == 3e3+k*Fs; X(ind) = X(ind)+3*Fs;
ind = F == 2e3+k*Fs; X(ind) = X(ind)+2*Fs;
end
ind = X==0;
```

```
X(ind) = nan;
%% Plot:
hfa = figconfg('P0632a');
subplot(211)
stem(F*2*pi*T,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
xlim([-FL*2*pi*T FL*2*pi*T])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F*2*pi*T,angle(X),'filled')
% ylim([0 max(abs(X))+0.5])
xlim([-FL*2*pi*T FL*2*pi*T])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('\angle H(e^{j\omega})|','fontsize',LFS)
title('Phase Response','fontsize',TFS)

hfb = figconfg('P0632b');
subplot(211)
stem(F,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
xlim([-FL FL])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi FT})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F,angle(X),'filled')
xlim([-FL FL])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('\angle H(e^{j2\pi FT})','fontsize',LFS)
title('Phase Response','fontsize',TFS)
```

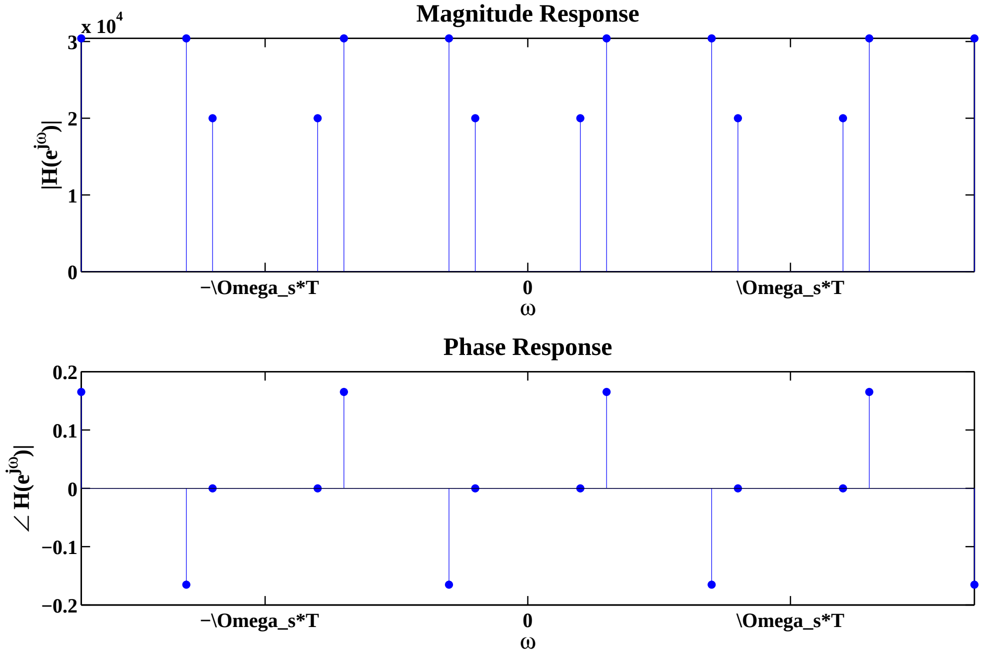


(a)

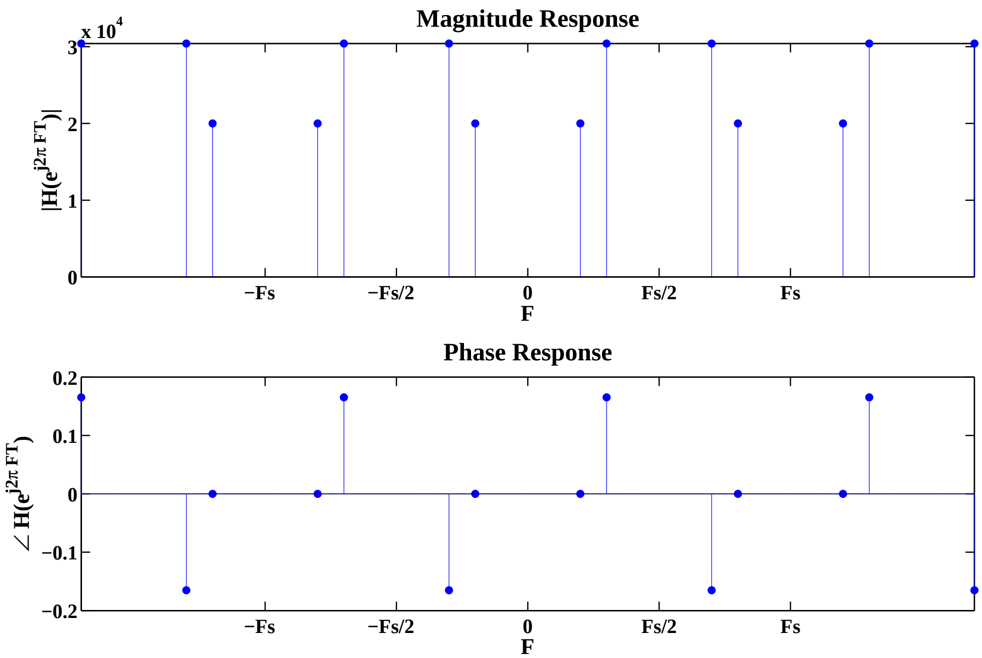


(b)

FIGURE 6.42: Spectra of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_{\mathrm{s}} = 20$ KHz.

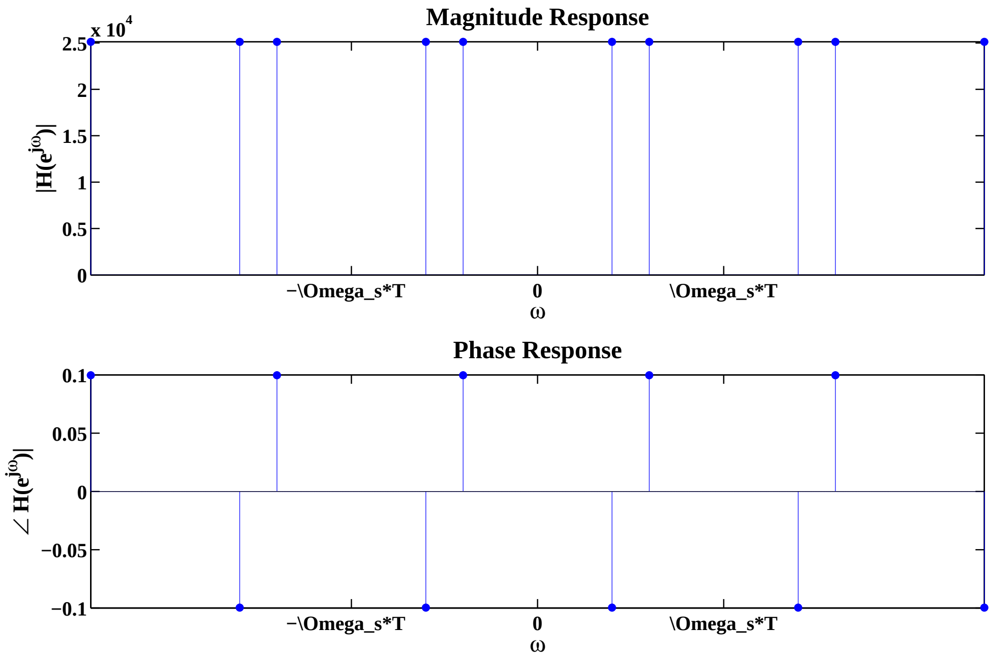


(a)

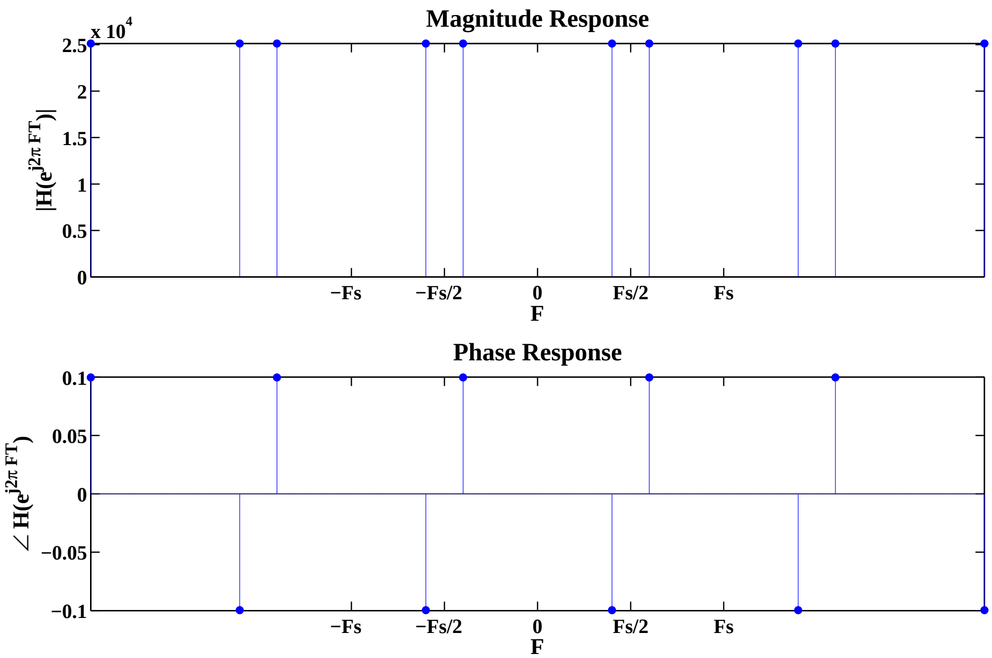


(b)

FIGURE 6.43: Spectra of $X(e^{j\omega})$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 10$ KHz.



(a)



(b)

FIGURE 6.44: Spectra of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 5$ KHz.

33. Solution:

$$x_c(t) = 8+12e^{-j20\pi(t-1)}+7e^{-j40\pi(t+1)} = x_c(t) = 8+12e^{-j20\pi t}+7e^{-j40\pi t}$$

The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j2\pi F) = \begin{cases} 8, & F = 0 \\ 12, & F = -10 \\ 7, & F = -20 \\ 0, & \text{otherwise} \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=2\pi F/F_s} = F_s \sum_{k=-\infty}^{\infty} X_c[j2\pi(F - kF_s)]$$

The signal $x_c(t)$ can be recovered from $x[n]$ if the sampling rate is (a) $F_s = 50$ and can NOT be recovered if the sampling rate is (b) $F_s = 20$, and (c) $F_s = 10$.

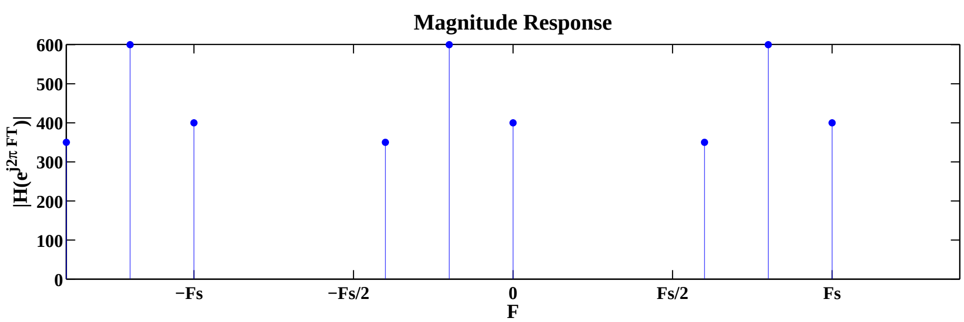


FIGURE 6.45: Magnitude response of $X(e^{j\omega})$ as a function of F in Hz when the sampling rate is $F_s = 50$.

MATLAB script:

```
% P0633: Illustrates the alias distortion
close all; clc
% Fs = 50; % Part (a)
% Fs = 20; % Part (b)
Fs = 10; % Part (c)
```

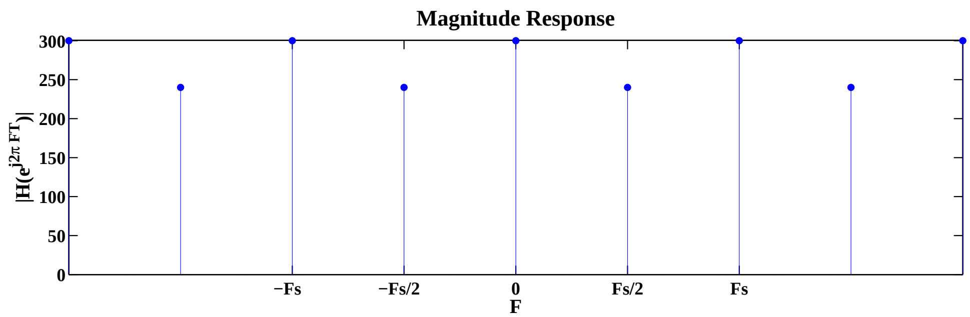


FIGURE 6.46: Magnitude response of $X(e^{j\omega})$ as a function of F in Hz when the sampling rate is $F_s = 20$.

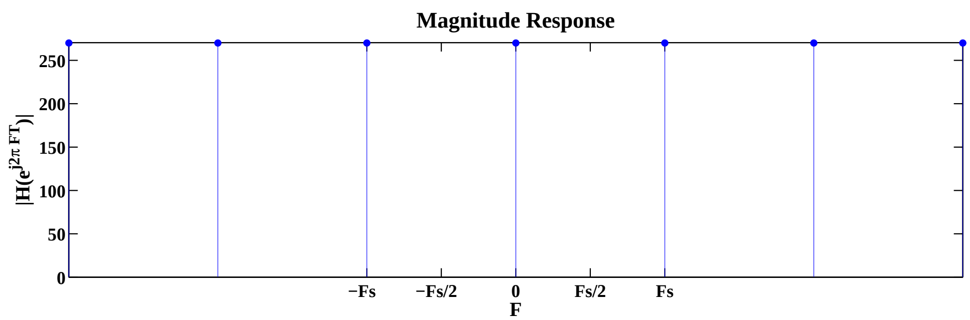


FIGURE 6.47: Magnitude response of $X(e^{j\omega})$ as a function of F in Hz when the sampling rate is $F_s = 10$.

```
T = 1/Fs;
FH = 20;
FL = FH+Fs;
F = -FL:1:FL;
X = zeros(1,length(F));
for k = -10:10;
ind = F == k*Fs; X(ind) = X(ind)+8*Fs;
ind = F == -20+k*Fs; X(ind) = X(ind)+7*Fs;
ind = F == -10+k*Fs; X(ind) = X(ind)+12*Fs;
end
ind = X==0;
X(ind) = nan;
```

```
%% Plot:
hfa = figconfg('P0633a','long');
stem(F,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
xlim([-FL FL])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi FT})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
```

34. Solution:

The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j2\pi F) = \begin{cases} 3, & F = 0 \\ \frac{1}{j}, & F = 8 \\ -\frac{1}{j}, & F = -8 \\ 5, & |F| = 12 \\ 0, & \text{otherwise} \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X(e^{j\omega})\big|_{\omega=2\pi F/F_s} = F_s \sum_{k=-\infty}^{\infty} X_c[j2\pi(F - kF_s)]$$

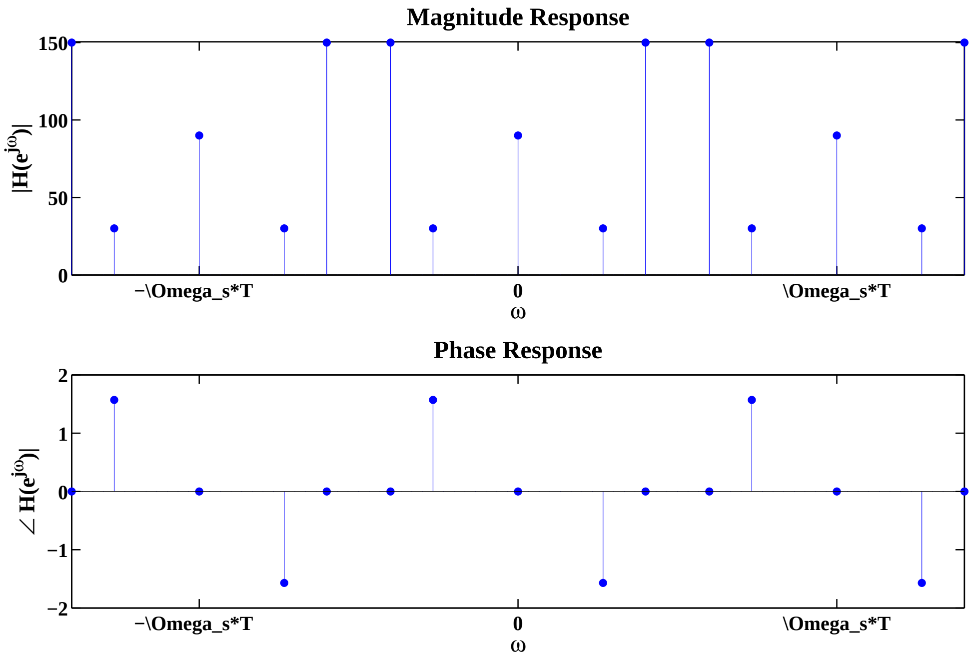
The signal $x_c(t)$ can be recovered from $x[n]$ if the sampling rate is (a) $F_s = 30$ Hz, and can NOT be recovered if the sampling rate is (b) $F_s = 20$ Hz, (c) $F_s = 15$ Hz.

MATLAB script:

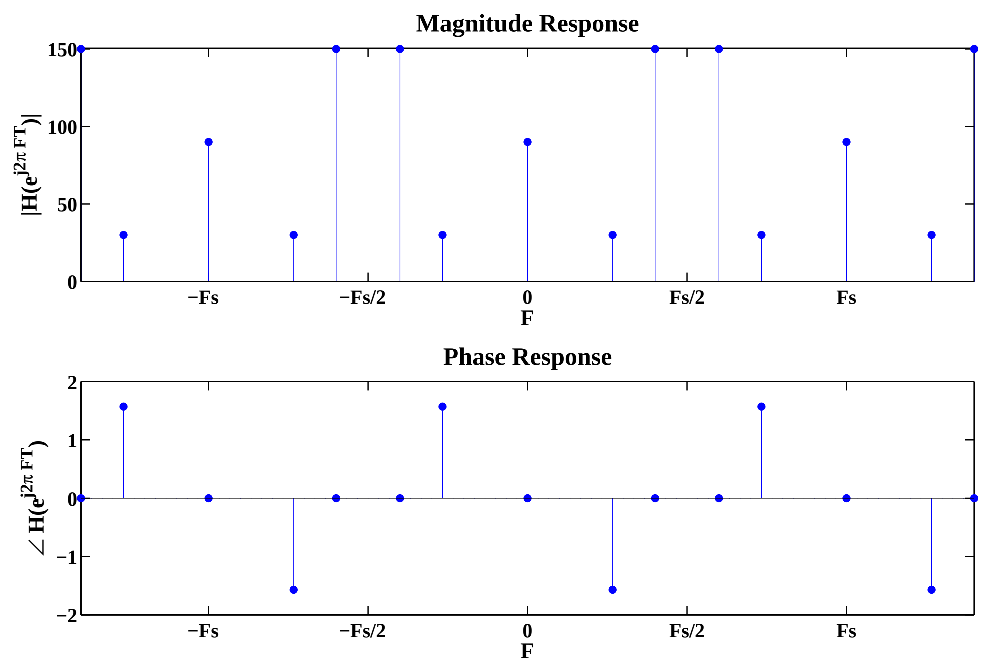
```
% P0634: Illustrates the alias distortion
close all; clc
% Fs = 30; % Part (a)
% Fs = 20; % Part (b)
Fs = 15; % Part (c)
T = 1/Fs;
FH = 12;
FL = FH+Fs;
F = -FL:1:FL;
```

```
X = zeros(1,length(F));
for k = -10:10;
ind = F == k*Fs; X(ind) = X(ind)+3*Fs;
ind = F == -12+k*Fs; X(ind) = X(ind)+5*Fs;
ind = F == -8+k*Fs; X(ind) = X(ind)-1/j*Fs;
ind = F == 12+k*Fs; X(ind) = X(ind)+5*Fs;
ind = F == 8+k*Fs; X(ind) = X(ind)+1/j*Fs;
end
ind = X==0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0634a');
subplot(211)
stem(F*2*pi*T,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
xlim([-FL*2*pi*T FL*2*pi*T])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
subplot(212)
stem(F*2*pi*T,angle(X),'filled')
xlim([-FL*2*pi*T FL*2*pi*T])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega','fontsize',LFS)
ylabel('\angle H(e^{j\omega})|','fontsize',LFS)
title('Phase Response','fontsize',TFS)

hfb = figconfg('P0634b');
subplot(211)
stem(F,abs(X),'filled')
ylim([0 max(abs(X))+0.5])
xlim([-FL FL])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('|H(e^{j2\pi FT})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
```

```
subplot(212)
stem(F,angle(X),'filled')
xlim([-FL FL])
set(gca,'XTick',[-Fs -Fs/2 0 Fs/2 Fs])
set(gca,'XTickLabel',{'-Fs','-Fs/2','0','Fs/2','Fs'})
xlabel('F','fontsize',LFS)
ylabel('\angle H(e^{j2\pi FT})','fontsize',LFS)
title('Phase Response','fontsize',TFS)
```

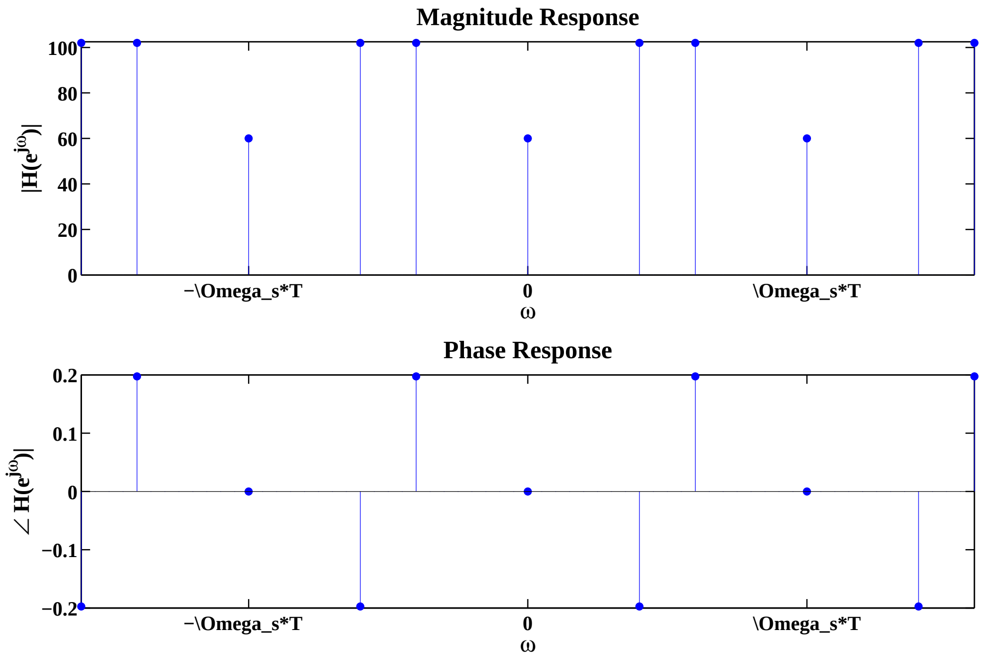


(a)

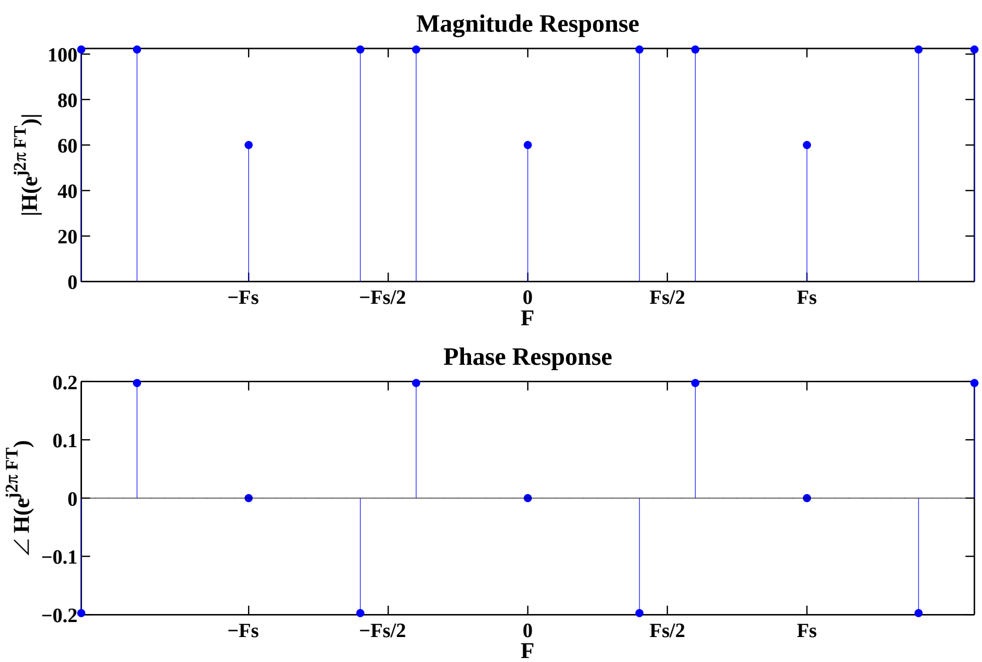


(b)

FIGURE 6.48: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 30$ KHz.

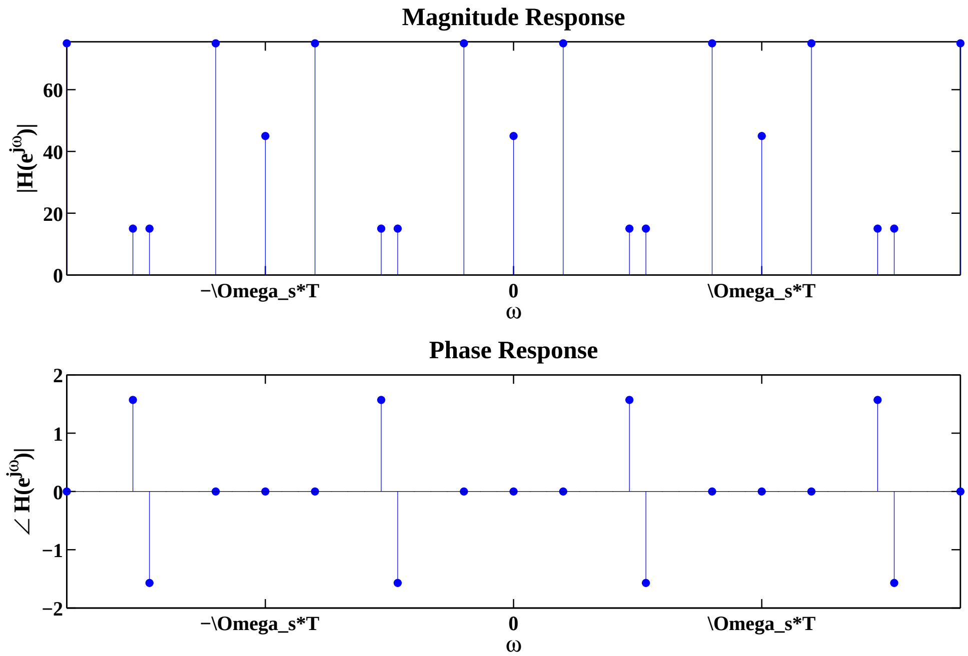


(a)

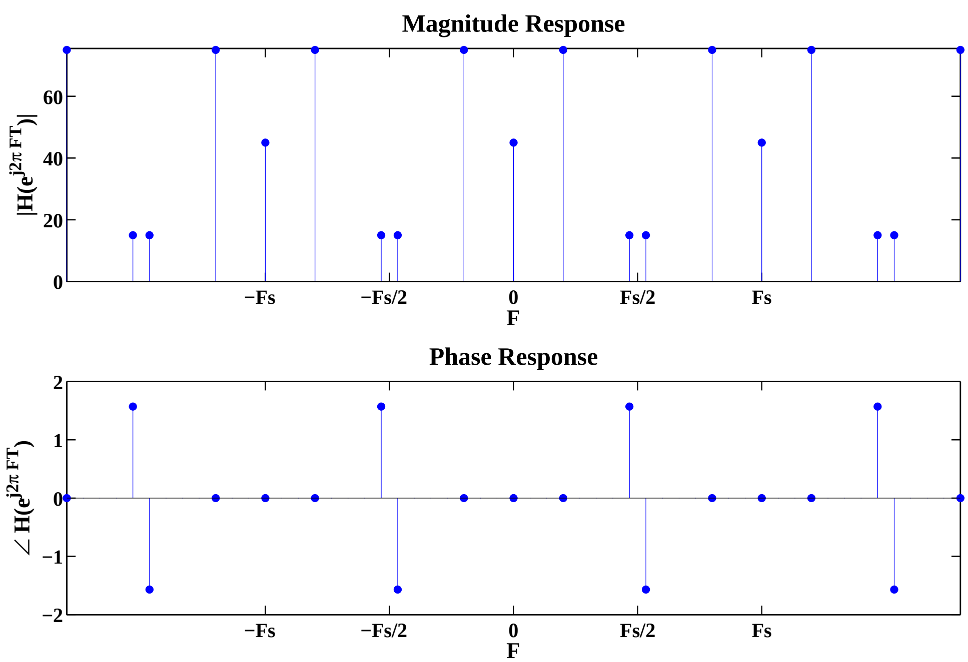


(b)

FIGURE 6.49: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 20$ KHz.



(a)



(b)

FIGURE 6.50: Spectra of $X\left(e^{j\omega}\right)$ as a function of (a) ω in $\frac{\text{rad}}{\text{sam}}$ and (b) F in Hz when the sample rate is $F_s = 15$ KHz.

35. The same as P0623

36. Solution:
The spectra of the sampled sequence $x[n]$ is:

$$X\left(\mathrm{e}^{\mathrm{j}\omega}\right)\big|_{\omega=\Omega/F_s} = F_s \sum_{k=-\infty}^{\infty} X_c[\mathrm{j}(\Omega - 2\pi k F_s)]$$

The signal $x_c(t)$ can NOT be recovered when the sampling interval is (a) $T = \pi$, (b) $T = 0.5\pi$, (c) $T = 0.2\pi$.

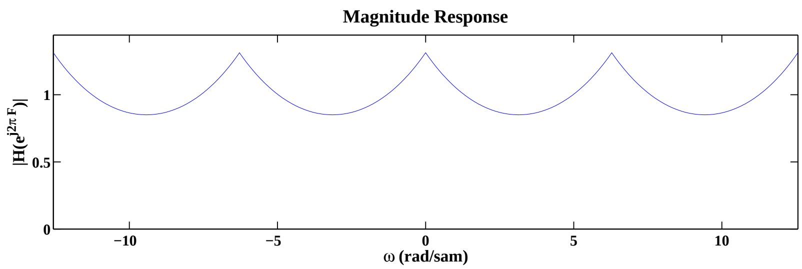


FIGURE 6.51: Magnitude response of $X\left(\mathrm{e}^{\mathrm{j}\omega}\right)$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = \pi$.

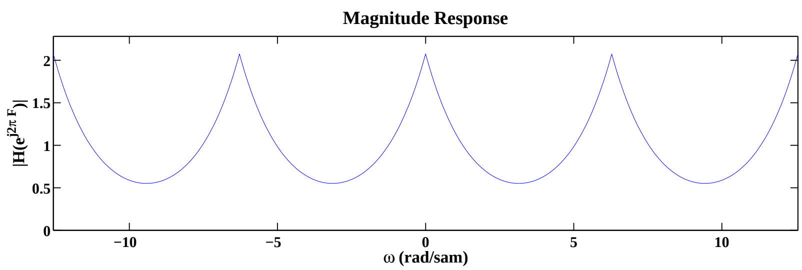


FIGURE 6.52: Magnitude response of $X\left(\mathrm{e}^{\mathrm{j}\omega}\right)$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = 0.5\pi$.

MATLAB script:

```
% P0636: Plot the spectra of sampled sequence
close all; clc
T = pi; % Part a
% T = 0.5*pi; % Part b
```

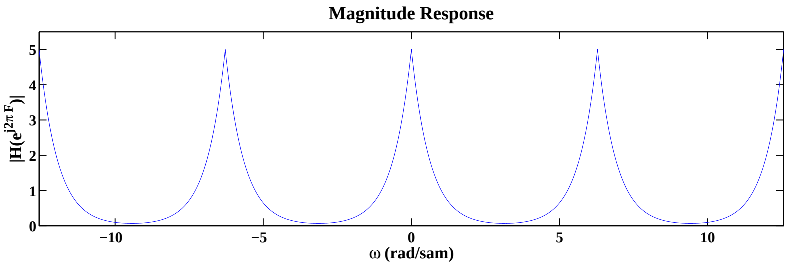


FIGURE 6.53: Magnitude response of $X(\mathrm{e}^{\mathrm{j}\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling interval is $T = 0.2\pi$.

```
% T = 0.2*pi; % Part c
Fs = 1/T;
Omegas = 2*pi/T;
Omega = -2*Omegas:0.01:2*Omegas;
X = zeros(size(Omega));
for k = -10:1:10
X = X + pi*exp(-abs(Omega-k*Omegas))/T;
end
%% Plot:
hfa = figconfg('P0636a','long');
plot(Omega*T,abs(X))
xlabel('\omega (rad/sam)','fontsize',LFS)
ylabel('|H(e^{j2\pi F})|','fontsize',LFS)
ylim([0 max(abs(X))*1.1])
xlim([-2*Omegas*T 2*Omegas*T])
title('Magnitude Response','fontsize',TFS)
```

37. Solution:

The spectra of the continuous signal $x_{\mathrm{c}}(t)$ is:

$$X_{\mathrm{c}}(\mathrm{j}2\pi F) = \begin{cases} 1 - \frac{F^2}{25}, & |F| \le 5 \\ 0, & \text{otherwise} \end{cases}$$

The spectra of the sampled sequence $x[n]$ is:

$$X(\mathrm{e}^{\mathrm{j}\omega})\big|_{\omega=2\pi F/F_{\mathrm{s}}} = F_{\mathrm{s}} \sum_{k=-\infty}^{\infty} X_{\mathrm{c}}[\mathrm{j}2\pi(F - kF_{\mathrm{s}})]$$

The continuous signal $x_c(t)$ can NOT be recovered from the sequence $x[n]$ when the sampling frequency is (a) $F_s = 6$ Hz, (b) $F_s = 4$ Hz, (c) $F_s = 2$ Hz.

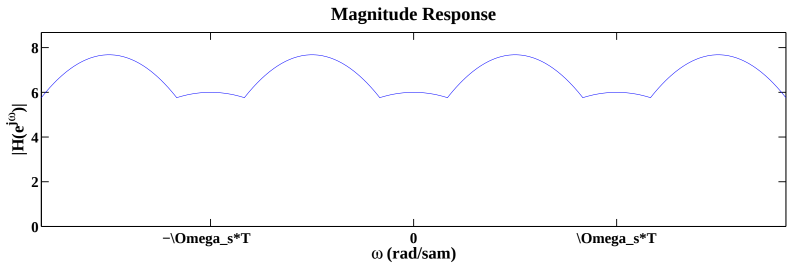


FIGURE 6.54: Magnitude response of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling rate is $F_s = 6$.

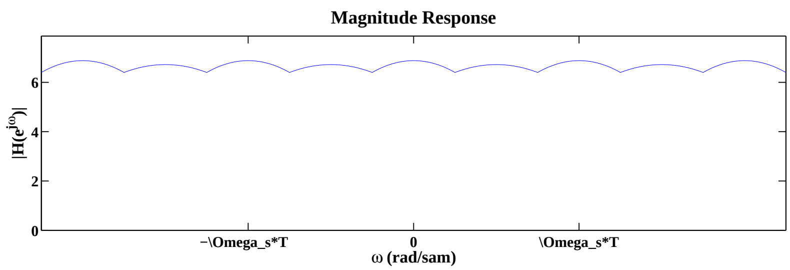


FIGURE 6.55: Magnitude response of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling rate is $F_s = 4$.

MATLAB script:

```
% P0637: Illustrates the alias distortion
close all; clc
Fs = 6; % Part (a)
% Fs = 4; % Part (b)
% Fs = 2; % Part (c)
T = 1/Fs;
FH = 5;
FL = FH+Fs;
F = -FL:0.01:FL;
X = zeros(1,length(F));
for k = -5:5;
```

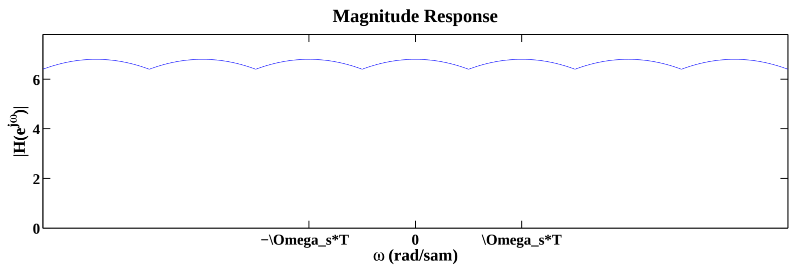


FIGURE 6.56: Magnitude response of $X\big(e^{j\omega}\big)$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ when the sampling rate is $F_s = 2$.

```
ind = abs(F-k*Fs) <= 5;
X(ind) = X(ind)+(1-(F(ind)-k*Fs).^2/25)*Fs;
end
%% Plot:
hfa = figconfg('P0637a','long');
plot(F*2*pi*T,abs(X))
ylim([0 max(abs(X))+1])
xlim([-FL*2*pi*T FL*2*pi*T])
set(gca,'XTick',[-Fs*2*pi*T 0 Fs*2*pi*T])
set(gca,'XTickLabel',{'-\Omega_s*T','0','\Omega_s*T'})
xlabel('\omega (rad/sam)','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
```

38. Proof:
The ideal DAC is represented by

$$y_r(t) = \sum_{n=-\infty}^{\infty} y[n] g_{\mathrm{BL}}(t - nT) \tag{6.43}$$

(i) Non-causality.
when $n < 0$, $y_r(t)$ is related to the future values, hence the system is non-causal.
(ii) Linearity.

when the input is $y[n] = a_1 y_1[n] + a_2 y_2[n]$, the output of the system is:

$$\begin{aligned} y_{\mathrm{r}}(t) &= \sum_{n=-\infty}^{\infty} (a_1 y_1[n] + a_2 y_2[n]) \cdot g_{\mathrm{BL}}(t - nT) \\ &= a_1 \sum_{n=-\infty}^{\infty} y_1[n] g_{\mathrm{BL}}(t - nT) + a_2 \sum_{n=-\infty}^{\infty} y_2[n] g_{\mathrm{BL}}(t - nT) \\ &= a_1 y_{\mathrm{r}1}(t) + a_2 y_{\mathrm{r}2}(t) \end{aligned}$$

which verifies that the system follows superposition property, hence the system is linear.
(iii) Time variance.
In general, we have

$$y_{\mathrm{r}}(t - \tau) = \sum_{n=-\infty}^{\infty} y[n] g_{\mathrm{BL}}(t - \tau - nT) \neq \sum_{n=-\infty}^{\infty} y[n] g_{\mathrm{BL}}(t - nT) = y_{\mathrm{r}}(t)$$

which proves the system is time varying.

39. (a) Solution:
The sampled sequence is:

$$x[n] = x_{\mathrm{c}}(0.01n) = 10 + 3\sin(0.2\pi n + \pi/3) + 5\cos(0.4\pi n)$$

(b) Solution:
The reconstructed signal is:

$$y_{\mathrm{r}}(t) = 10 + 3\sin(20\pi t + \pi/3) + 5\cos(40\pi t)$$

(c) Solution:
The sampled sequence is:

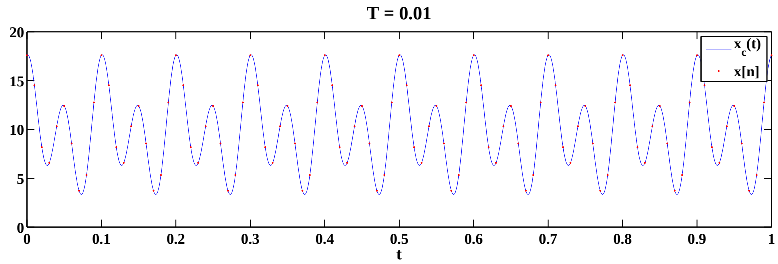
$$x[n] = x_{\mathrm{c}}(0.05n) = 10 + 3\sin(\pi n + \pi/3) + 5\cos(2\pi n)$$

The reconstructed signal is:

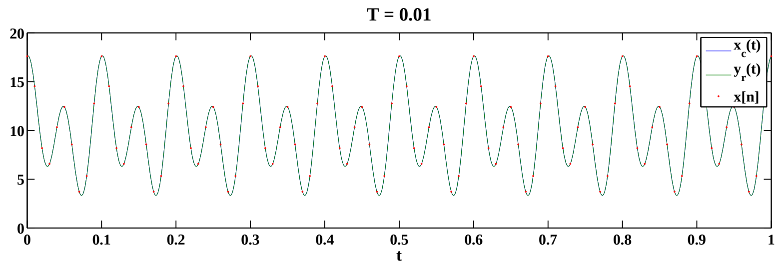
$$y_{\mathrm{r}}(t) = 10 + 3\sin(20\pi t + \pi/3)$$

(d) Solution:
The sampled sequence is:

$$x[n] = x_{\mathrm{c}}(0.1n) = 10 + 3\sin(2\pi n + \pi/3) + 5\cos(4\pi n)$$



(a)



(b)

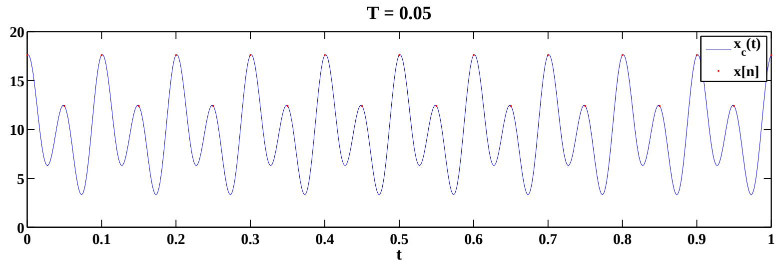
FIGURE 6.57: (a) Plot of $x[n]$ and $x_{\mathrm{c}}(t)$ and (b) plot of $y_{\mathrm{r}}(t)$ when the signal is sampled at $t = 0.01n$.

The reconstructed signal is:

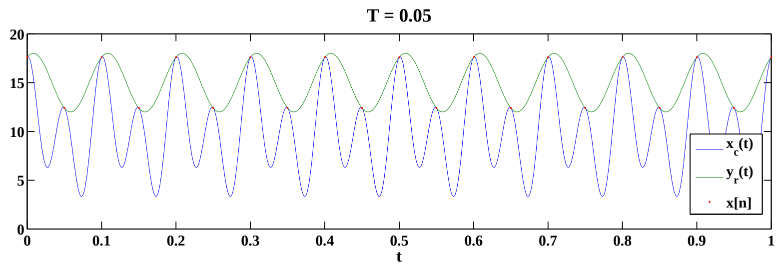
$$y_{\mathrm{r}}(t) = 10 - \frac{3\sqrt{3}}{2} + 5 = 15 - \frac{3\sqrt{3}}{2}$$

MATLAB script:

```
% P0639: Illustrate Ideal DAC
close all; clc
t1 = 0; t2 = 1;
dt = 1e-4;
t = t1:dt:t2;
% T = 0.01; % Part (a) (b)
% T = 0.05; % Part (c)
T = 0.1; % Part (d)
Fs = 1/T;
F1 = 10; F2 = 20;
xc = 10 + 3*sin(2*pi*F1*t+pi/3)+5*cos(2*pi*F2*t);
% Sampling:
```



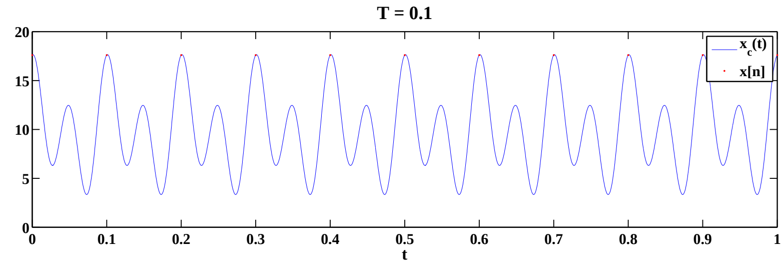
(a)



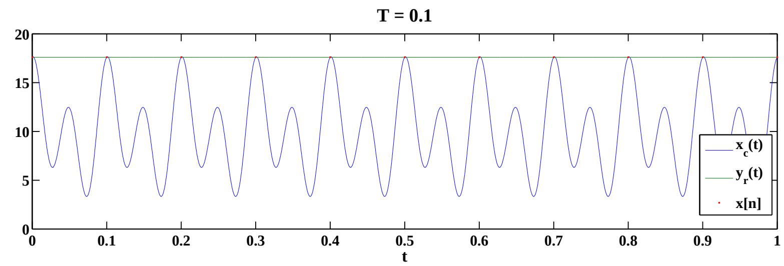
(b)

FIGURE 6.58: (a) Plot of $x[n]$ and $x_c(t)$ and (b) plot of $y_r(t)$ when the signal is sampled at $t = 0.05n$.

```
nT = t1:T:t2;
xn = 10 + 3*sin(2*pi*F1*nT+pi/3)+5*cos(2*pi*F2*nT);
F1y = F1;
while F1y > Fs/2
    F1y = F1y-Fs;
end
F2y = F2;
while F2y > Fs/2
    F2y = F2y-Fs;
end
yr = 10 + 3*sin(2*pi*F1y*t+pi/3)+5*cos(2*pi*F2y*t);
%% Plot:
hfa = figconfg('P0639a','long');
plot(t,xc); hold on
plot(nT,xn,'.','color','red')
xlabel('t','fontsize',LFS)
title(['T = ',num2str(T)],'fontsize',TFS)
legend('x_c(t)','x[n]','location','northeast')
```



(a)



(b)

FIGURE 6.59: (a) Plot of $x[n]$ and $x_c(t)$ and (b) plot of $y_r(t)$ when the signal is sampled at $t = 0.1n$.

```
hfb = figconfg('P0639b','long');
plot(t,xc,t,yr,nT,xn,'.')
xlabel('t','fontsize',LFS)
title(['T = ',num2str(T)],'fontsize',TFS)
legend('x_c(t)','y_r(t)','x[n]','location','northeast')
```

40. Solution:
The sampling frequency interval can be defined as

$$\frac{2F_H}{k} \leq F_s \leq \frac{2F}{k-1}$$

Hence, we compute as $11.2\text{Hz} \leq F_s \leq 12\text{Hz}$.

MATLAB script:

```
% P0640: Sampling Illustration
close all; clc
FL = 24; FH = 28;
dF = 0.01;
```

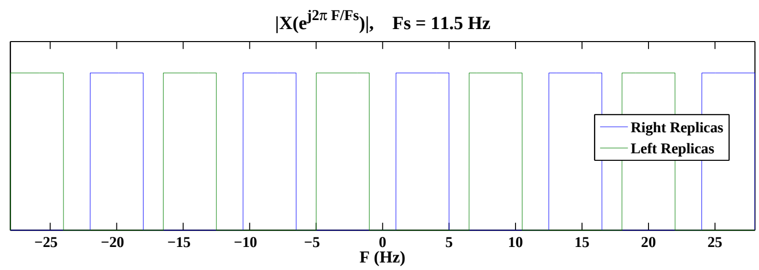


FIGURE 6.60: Spectra of the sampled sequence.

```
% Fs = 11.2;
% Fs = 12;
Fs = 11.5;
F = -FH:dF:FH;
X = zeros(size(F));
XP = zeros(size(F));
XN = zeros(size(F));
for jj = -10:10
    ind = F > FL+jj*Fs & F < FH+jj*Fs;
    X(ind) = X(ind) + 1;
    XP(ind) = XP(ind) + 1;
    ind = F > -FH+jj*Fs & F < -FL+jj*Fs;
    X(ind) = X(ind) + 1;
    XN(ind) = XN(ind) + 1;
end
ind = X == 0;
X(ind) = nan;
%% Plot:
hfa = figconfg('P0640a','long');
plot(F,XP,F,XN);
xlim([-FH FH])
ylim([0 1.2])
set(gca,'YTick',-1)
xlabel('F (Hz)','fontsize',LFS)
title(['|X(e^{j2\pi F/Fs})|,    Fs = ',num2str(Fs),' Hz']...
    ,'fontsize',TFS)
legend('Right Replicas','Left Replicas','location','best')
```

41. Solution:

$$F'_{\mathrm{L}} = 76 - 1 = 75\text{Hz}, \qquad F'_{\mathrm{H}} = 98 + 1 = 99\text{Hz}$$

The bandwidth is

$$B = F'_{\mathrm{H}} - F'_{\mathrm{L}} = 24\text{Hz}$$

The minimum sampling frequency is:

$$\min F_{\mathrm{s}} = 2F'_{\mathrm{H}}/\lfloor F'_{\mathrm{H}}/B \rfloor = 49.5\text{Hz}$$

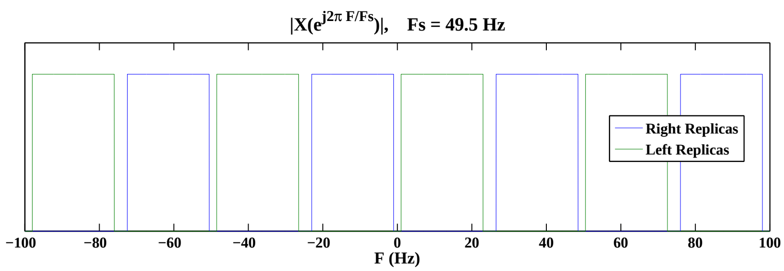


FIGURE 6.61: Plot of the resulting spectrum of the sampled signal over $[-100,\,100]$ Hz range.

MATLAB script:

```
% P0641: Sampling Illustration
close all; clc
FL = 76; FH = 98;
dF = 0.01;
FG = 2;
FHH = FH + FG/2;
FLL = FL - FG/2;
Fs = 49.5;
F = -100:dF:100;
X = zeros(size(F));
XP = zeros(size(F));
XN = zeros(size(F));
for jj = -10:10
    ind = F > FL+jj*Fs & F < FH+jj*Fs;
    X(ind) = X(ind) + 1;
    XP(ind) = XP(ind) + 1;
```

```
        ind = F > -FH+jj*Fs & F < -FL+jj*Fs;
        X(ind) = X(ind) + 1;
        XN(ind) = XN(ind) + 1;
    end
    ind = X == 0;
    X(ind) = nan;
    %% Plot:
    hfa = figconfg('P0641a','long');
    plot(F,XP,F,XN);
    xlim([-100 100])
    ylim([0 1.2])
    set(gca,'YTick',-1)
    xlabel('F (Hz)','fontsize',LFS)
    title(['|X(e^{j2\pi F/Fs})|,    Fs = ',num2str(Fs),' Hz']...
        ,'fontsize',TFS)
    legend('Right Replicas','Left Replicas','location','best')
```

42. (a) Solution:
(i) when $F_{s_x} = 25$ and $F_{s_y} = 50$, the reconstructed signal is

$$s_r(x, y) = 2\cos(2\pi x + 2\pi y)$$

(i) when $F_{s_x} = 100$ and $F_{s_y} = 200$, the reconstructed signal is

$$s_r(x, y) = 2\cos(98\pi x + 198\pi y)$$

(b) tba

Review Problems

43. (a) MATLAB script:

```
function [xhat, t] = DAC1(x,N,T)
% Implementation of function (6.100)
% Input:
%       x:  sample sequence x[n]
%       N:  sample sequence index [0 N]
%       T:  sampling time interval
% Output:
%       xhat: reconstructed time signal
%       t:  time index
dt = T/50;
t = 0:dt:N*T;
n = 0:N;
[Gt Gn] = meshgrid(t,n);
xhat = x*sinc((Gt-Gn*T)/T);
```

(b) See plot below.

(c) See plot below.

(d) tba.

(e) tba.

(f) tba.

MATLAB script:

```
% P0643: Investigate approximate ideal reconstruction
close all; clc
%% Part (b):
ii = 4;
T = 1;
N = 5:5:20;
n = 0:N(ii);
x = ones(1,N(ii)+1);
[xhat t] = DAC1(x,N(ii),T);
xc = ones(size(t));
e1 = xc - xhat;
% Plot:
hfa = figconfg('P0643a','long');
plot(t,[xc;xhat;e1]); hold on
```

```
plot(n*T,x,'.')
xlabel('t','fontsize',LFS)
title(['N = ',num2str(N(ii)),',    T = ',num2str(T)],'fontsize',TFS)
legend('x_c(t)','\hat{x}_c(t)','e_1(t)','location','best')
%% Part (c):
NN = 3:20;
e1size = zeros(size(NN));
for jj = 1:length(NN)
     xtemp = ones(1,NN(jj)+1);
     [xhattemp ttemp] = DAC1(xtemp,NN(jj),T);
     Nthird = round(length(ttemp)/3);
     e1size(jj) = max(abs(1 - xhattemp(Nthird+1:2*Nthird)));
end
% Plot:
hfb = figconfg('P0643b','long');
stem(NN,e1size,'filled')
xlabel('N','fontsize',LFS)
title('Error Size','fontsize',TFS)
```

MATLAB script:

```
% P0643: Investigate approximate ideal reconstruction
close all; clc
%% Part (edf):
F = 1;
theta = 0;
T = 0.1;
ii = 4;
N = 5:5:20;
n = 0:N(ii);
x = cos(2*pi*F*n*T+theta);
[xhat t] = DAC1(x,N(ii),T);
xc = cos(2*pi*F*t+theta);
e1 = xc - xhat;
% Plot:
hfa = figconfg('P0643a','long');
plot(t,[xc;xhat;e1]); hold on
plot(n*T,x,'.')
xlabel('t','fontsize',LFS)
title(['N = ',num2str(N(ii)),',    T = ',num2str(T)],'fontsize',TFS)
```

```
legend('x_c(t)','\hat{x}_c(t)','e_1(t)','location','best')

NN = 3:20;
e1size = zeros(size(NN));
for jj = 1:length(NN)
     nn = 0:NN(jj);
     xtemp = cos(2*pi*F*nn*T+theta);
     [xhattemp ttemp] = DAC1(xtemp,NN(jj),T);
     Nthird = round(length(ttemp)/3);
     e1size(jj) = max(abs(cos(2*pi*F*ttemp(Nthird+1:2*Nthird))...
         - xhattemp(Nthird+1:2*Nthird)));
end
% Plot:
hfb = figconfg('P0643b','long');
stem(NN,e1size,'filled')
xlabel('N','fontsize',LFS)
title('Error Size','fontsize',TFS)
```

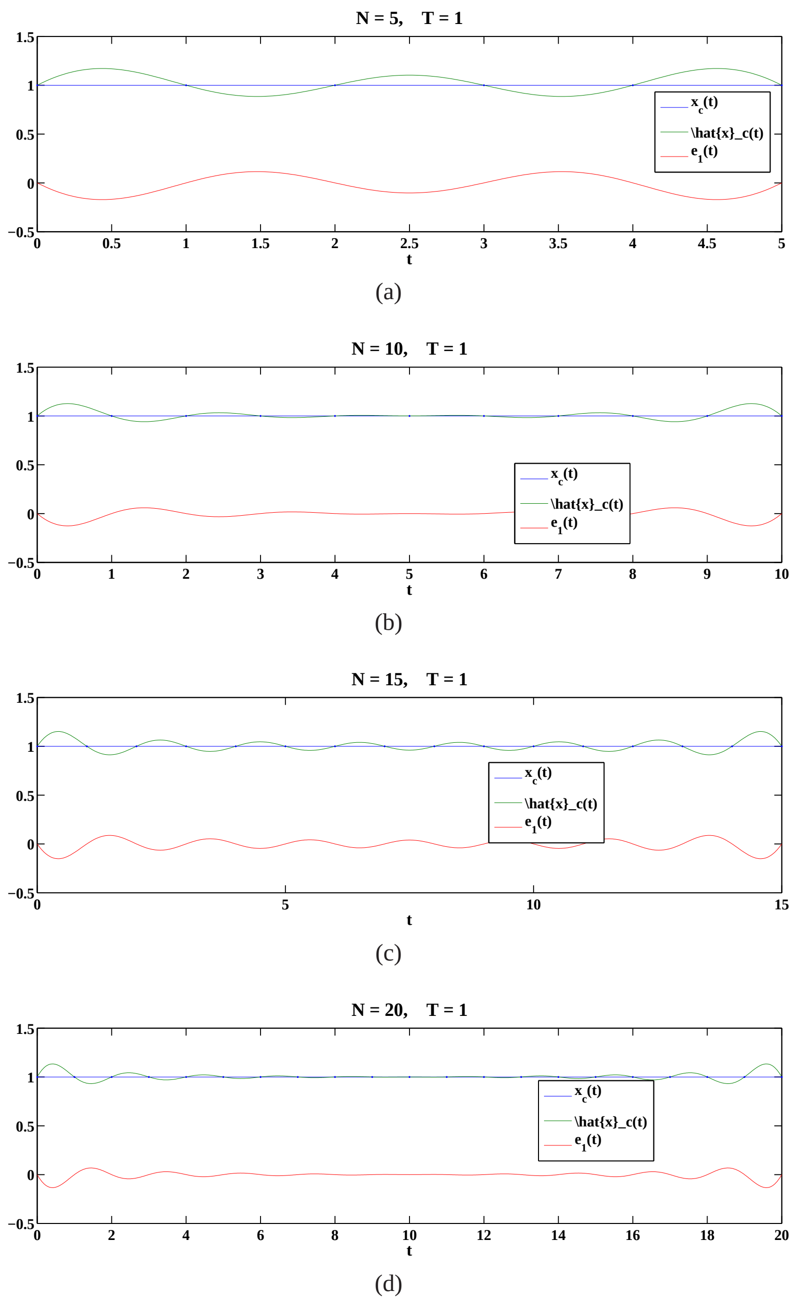


FIGURE 6.62: Plot of $x_c(t)$, $\hat{x}_c(t)$, and $e_1(t)$ for $T = 1$, and (a) $N = 5$, (b) $N = 10$, (c) $N = 15$, (d) $N = 20$.

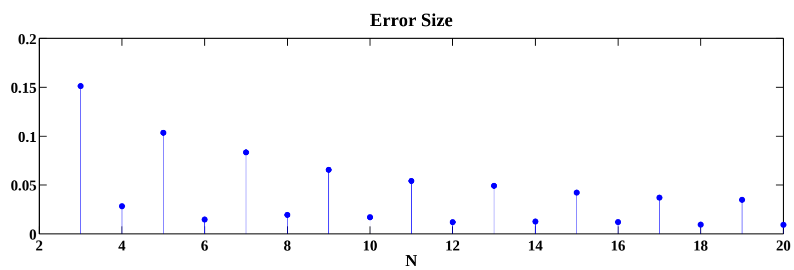


FIGURE 6.63: Plot of the size of error as a function of N.

44. (a) MATLAB script:

```
function [xhat, t] = DAC2(x,N,K,T)
% Implementation of function (6.101) (6.102) and (6.103)
% Input:
%       x:  sample sequence x[n]
%       N:  sample sequence index [0 N]
%       K:  argument employed in (6.101)
%       T:  sampling time interval
% Output:
%       xhat: reconstructed time signal
%       t:  time index
dt = T/50;
t = 0:dt:N*T;
n = 0:N;
[Gt Gn] = meshgrid(t,n);
xhat = x*gk(Gt-Gn*T,K,T);
%% Sub-functions:
function y = gk(t,K,T)
y = sinc(t/T);
ind = abs(t) >= K*T;
y(ind) = 0;
```

(b) See plot below.

(c) See plot below

(d) tba.

(e) tba.

(f) tba.

MATLAB script:

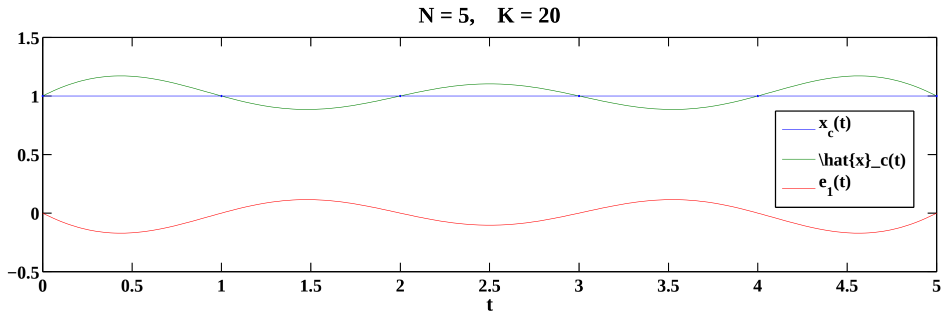
```
% P0644: Investigate approximated ideal reconstruction
close all; clc
%% Part (b):
ii = 1;
K = 20;
T = 1;
N = 5:5:20;
n = 0:N(ii);
x = ones(1,N(ii)+1);
[xhat t] = DAC2(x,N(ii),K,T);
```

```
xc = ones(size(t));
e1 = xc - xhat;
% Plot:
hfa = figconfg('P0644a','long');
plot(t,[xc;xhat;e1]); hold on
plot(n*T,x,'.')
xlabel('t','fontsize',LFS)
title(['N = ',num2str(N(ii)),',    K = ',num2str(K)],'fontsize',TFS)
legend('x_c(t)','\hat{x}_c(t)','e_1(t)','location','best')
%% Part (c):
NN = 3:20;
e1size = zeros(size(NN));
for jj = 1:length(NN)
     xtemp = ones(1,NN(jj)+1);
     [xhattemp ttemp] = DAC2(xtemp,NN(jj),K,T);
     Nthird = round(length(ttemp)/3);
     e1size(jj) = max(abs(1 - xhattemp(Nthird+1:2*Nthird)));
end
% Plot:
hfb = figconfg('P0644b','long');
stem(NN,e1size,'filled')
xlabel('N','fontsize',LFS)
title(['Error Size,     K = ',num2str(K)],'fontsize',TFS)
```

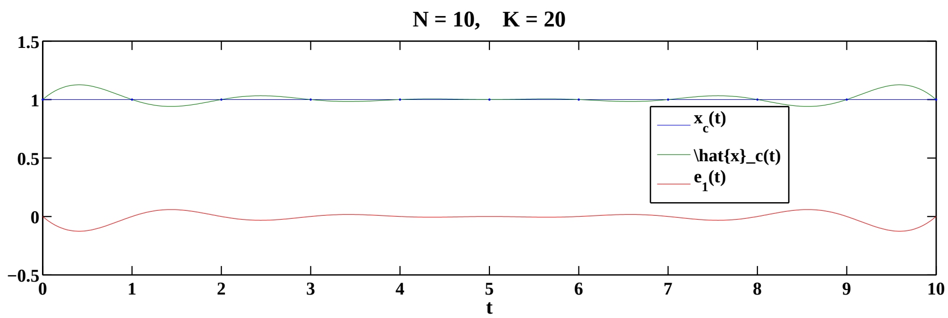
MATLAB script:

```
% P0644: Investigate approximate ideal reconstruction
close all; clc
%% Part (def):
F = 1;
theta = 0;
T = 0.1;
K = 20;
ii = 4;
N = 5:5:20;
n = 0:N(ii);
x = cos(2*pi*F*n*T+theta);
[xhat t] = DAC2(x,N(ii),K,T);
xc = cos(2*pi*F*t+theta);
e1 = xc - xhat;
% Plot:
```

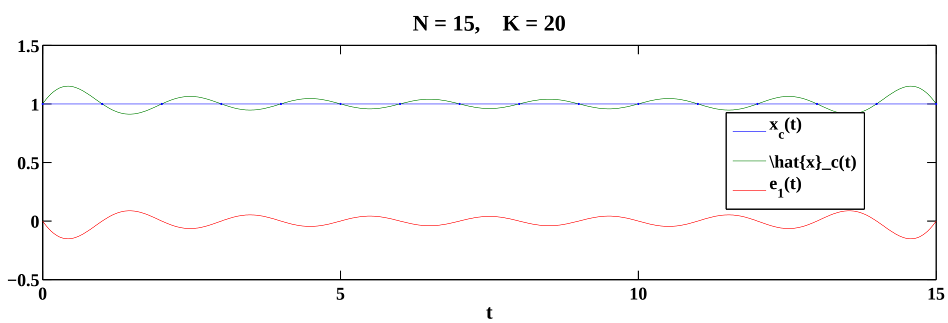
```
hfa = figconfg('P0644a','long');
plot(t,[xc;xhat;e1]); hold on
plot(n*T,x,'.')
xlabel('t','fontsize',LFS)
title(['N = ',num2str(N(ii)),',    T = ',num2str(T)],'fontsize',TFS)
legend('x_c(t)','\hat{x}_c(t)','e_1(t)','location','best')

NN = 3:20;
e1size = zeros(size(NN));
for jj = 1:length(NN)
     nn = 0:NN(jj);
     xtemp = cos(2*pi*F*nn*T+theta);
     [xhattemp ttemp] = DAC2(xtemp,NN(jj),K,T);
     Nthird = round(length(ttemp)/3);
     e1size(jj) = max(abs(cos(2*pi*F*ttemp(Nthird+1:2*Nthird))...
         - xhattemp(Nthird+1:2*Nthird)));
end
% Plot:
hfb = figconfg('P0644b','long');
stem(NN,e1size,'filled')
xlabel('N','fontsize',LFS)
title('Error Size','fontsize',TFS)
```



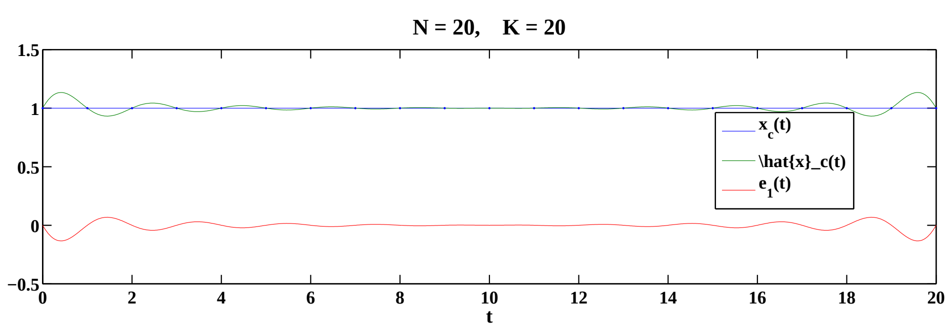
(a)



(b)



(c)



(d)

FIGURE 6.64: Plot of $x_c(t)$, $\hat{x}_c(t)$, and $e_1(t)$ for $T = 1$, and (a) $N = 5$, (b) $N = 10$, (c) $N = 15$, (d) $N = 20$.

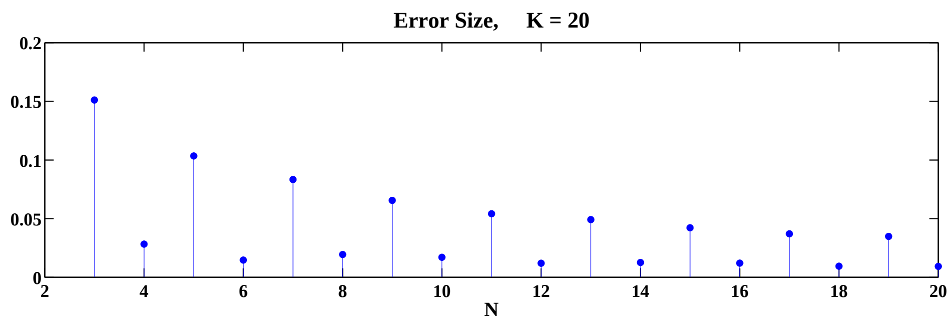


FIGURE 6.65: Plot of the size of error as a function of N.

45. (a) MATLAB script:

```
function [eH, e, eavg, evar] = QuantR(xn,B,N)
% Inputs:
%        xn:    sampled sequence
%        B:     # of bits representing one sample
%        N:     # of samples
% Outputs:
%        eH:    Normalized histogram of e[n]
%        e:     bins of histogram
%        eavg:  mean of e[n]
%        evar:  variance of e[n]
step = 1/(2^B);
xqn = round(xn/step)*step;
en = 2^B*(xn-xqn);
[eH e] = hist(en,100);
eH = eH/N;
eavg = mean(en);
evar = var(en);
```

(b) See plot below.

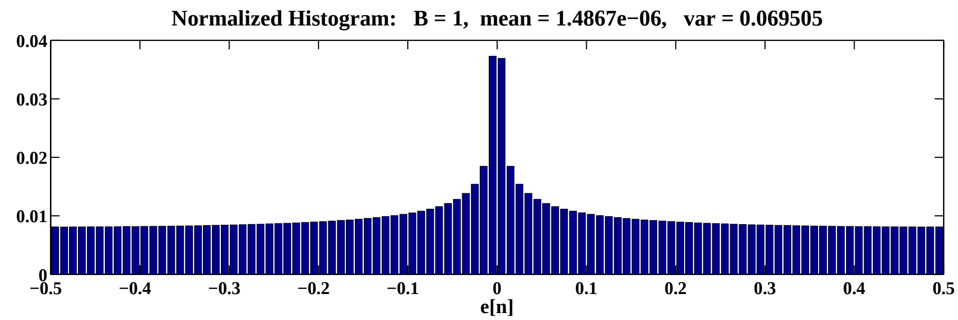
(c) See plot below.

(d) See plot below.

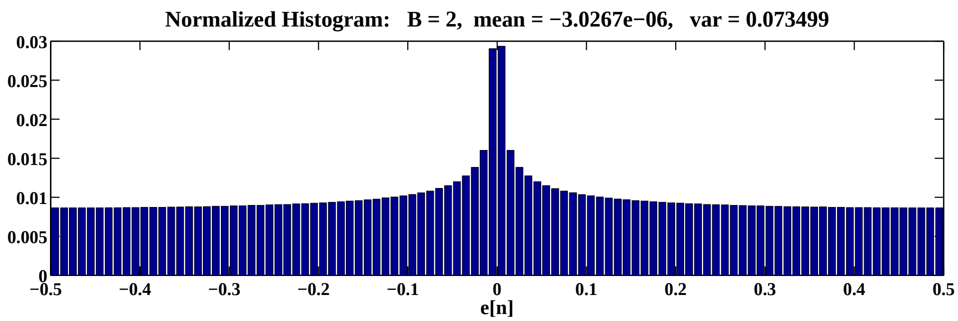
MATLAB script:

```
% P0645: Investigate quantization error
close all; clc
ii = 4;
B = [1 2 4 6];
N = 5e5;
n = 1:N;
xn = cos(n/11); % part (b)
% xn = 1/2*(cos(n/11)+cos(n/17)+sin(n/31)); % part (c)
% xn = rand(size(n)); % part (d)
[eH, e, eavg, evar] = QuantR(xn,B(ii),N);
% Plot:
hfa = figconfg('P0645a','long');
bar(e,eH)
xlabel('e[n]','fontsize',LFS)
title(['Normalized Histogram:   B = ',num2str(B(ii)),...
```

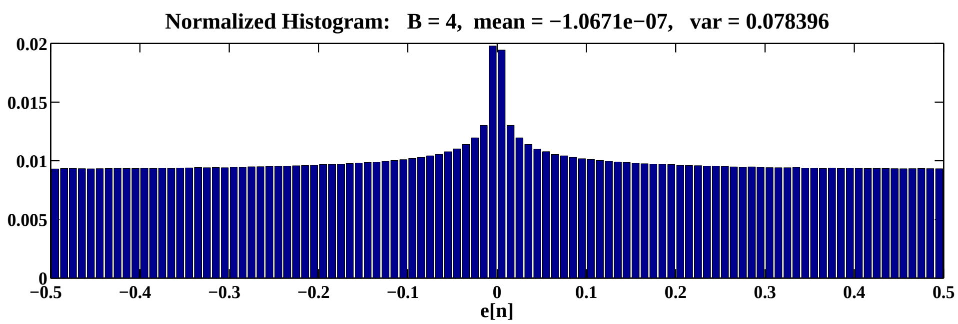
```
',  mean = ',num2str(eavg),',   var = ',...
num2str(evar)],'fontsize',TFS)
```



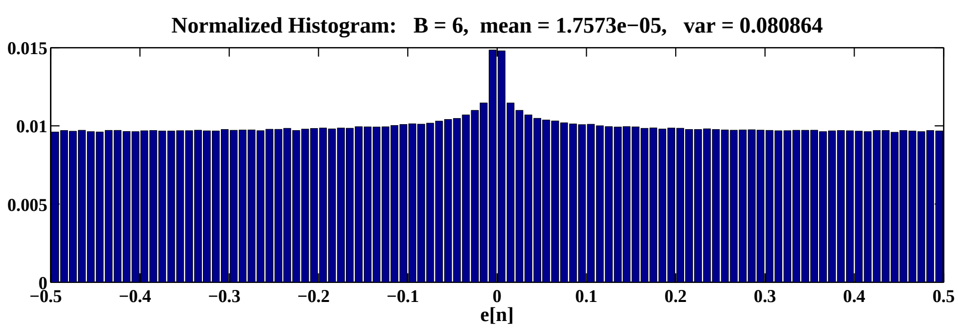
(a)



(b)



(c)



(d)

FIGURE 6.66: Plot the resulting distributions for quantization error $e[n]$ of $x[n]$ in part (a) for (a) 1 bit, (b) 2 bits, (c) 4 bits, (d) 6 bits.

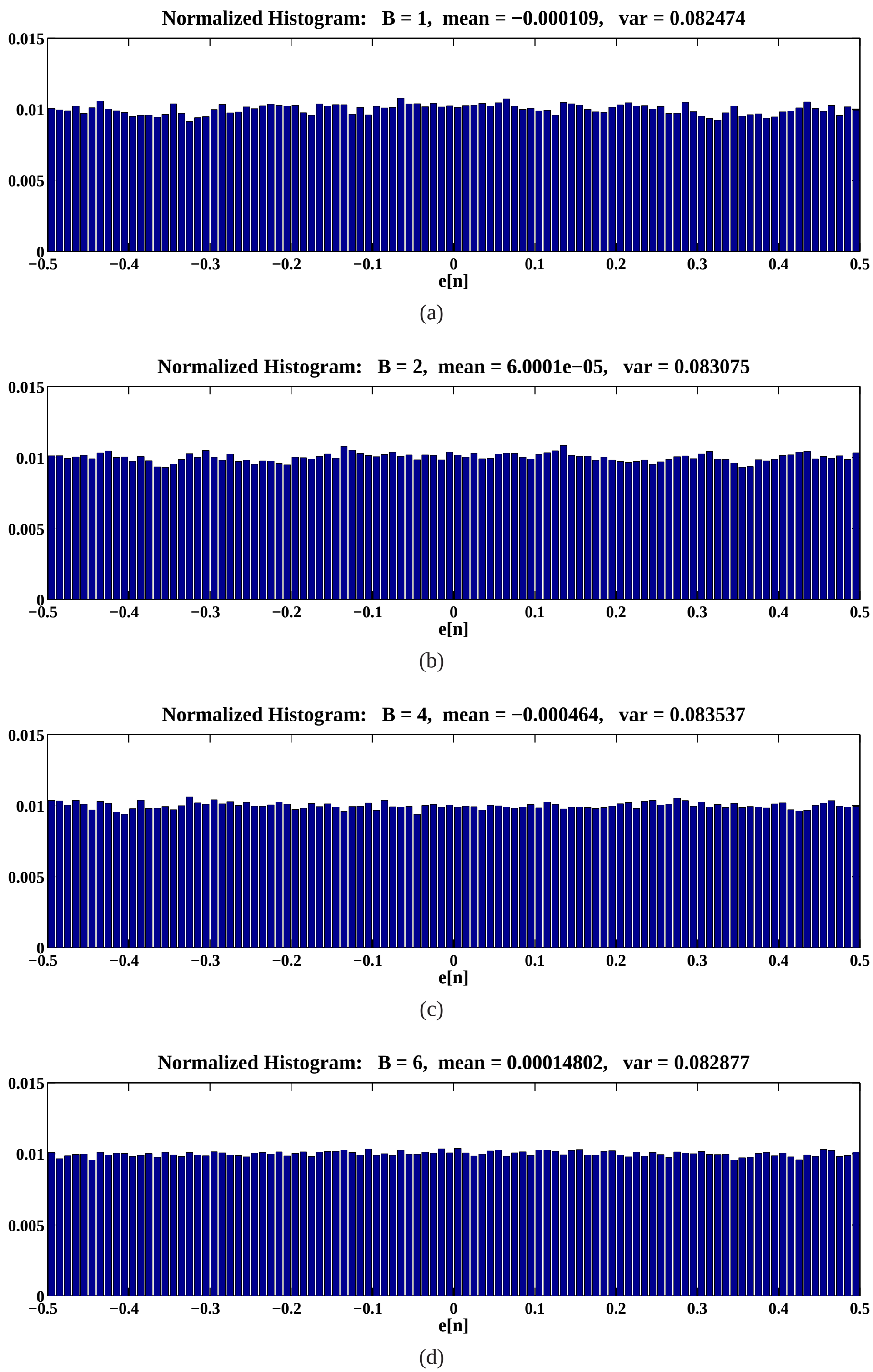


FIGURE 6.67: Plot the resulting distributions for quantization error $e[n]$ of $x[n]$ in part (c) for (a) 1 bit, (b) 2 bits, (c) 4 bits, (d) 6 bits.

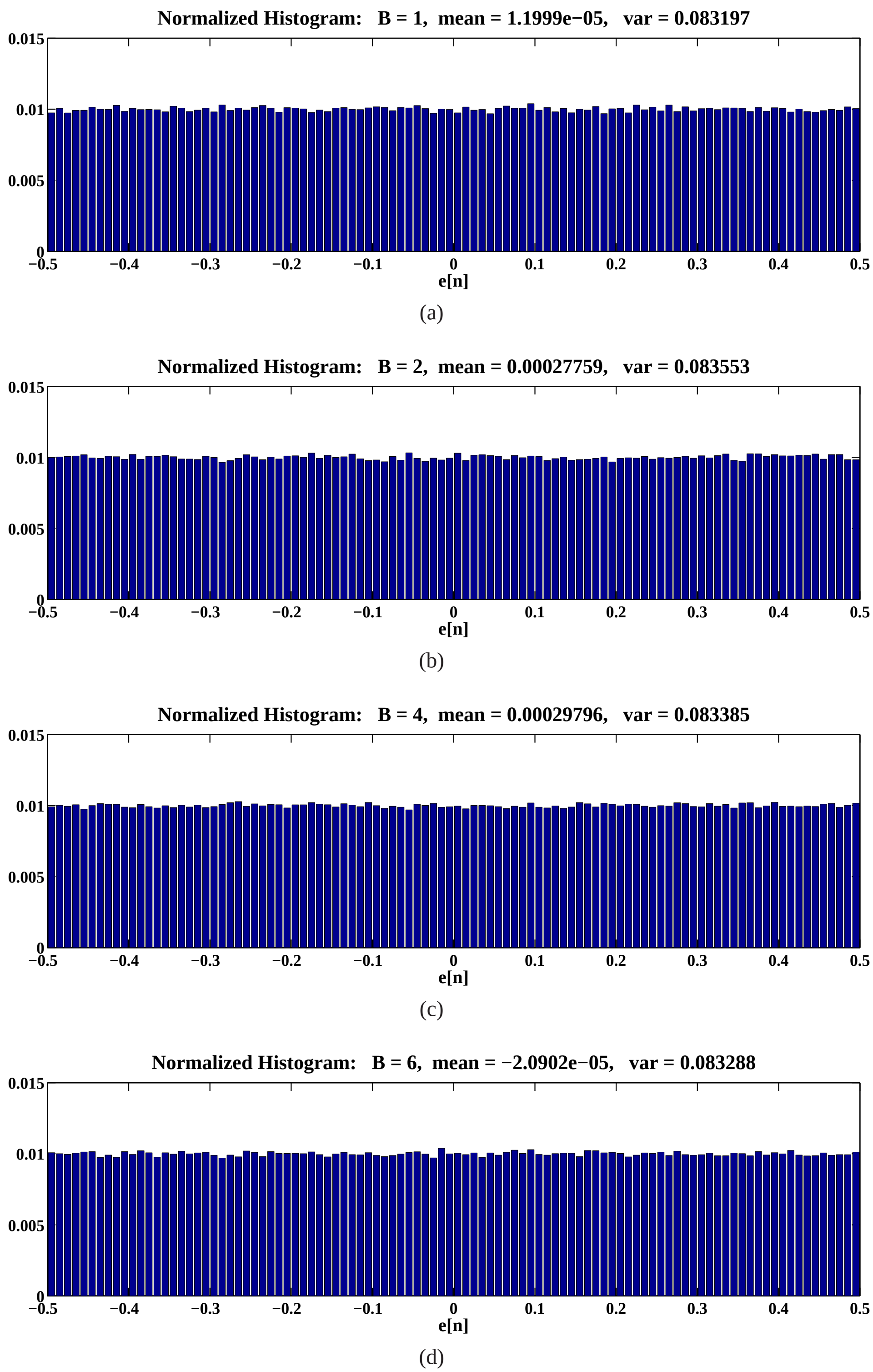


FIGURE 6.68: Plot the resulting distributions for quantization error $e[n]$ of $x[n]$ in part (d) for (a) 1 bit, (b) 2 bits, (c) 4 bits, (d) 6 bits.

46. (a) MATLAB script:

```
function [eH, e, eavg, evar] = QuantF(xn,B,N)
% Inputs:
%       xn:    sampled sequence
%       B:     # of bits representing one sample
%       N:     # of samples
% Outputs:
%       eH:    Normalized histogram of e[n]
%       e:     bins of histogram
%       eavg:  mean of e[n]
%       evar:  variance of e[n]
step = 1/(2^B);
xqn = fix(xn/step)*step;
en = 2^B*(xn-xqn);
[eH e] = hist(en,100);
eH = eH/N;
eavg = mean(en);
evar = var(en);
```

(b) See plot below.

(c) See plot below.

(d) See plot below.

MATLAB script:

```
% P0646: Investigate quantization error
close all; clc
ii = 4;
B = [1 2 4 6];
N = 5e5;
n = 1:N;
% xn = cos(n/11); % part (b)
% xn = 1/2*(cos(n/11)+cos(n/17)+sin(n/31)); % part (c)
xn = rand(size(n)); % part (d)
[eH, e, eavg, evar] = QuantF(xn,B(ii),N);
% Plot:
hfa = figconfg('P064a6','long');
bar(e,eH)
xlabel('e[n]','fontsize',LFS)
title(['Normalized Histogram:   B = ',num2str(B(ii)),...
```

```
',  mean = ',num2str(eavg),',   var = ',...
num2str(evar)],'fontsize',TFS)
```

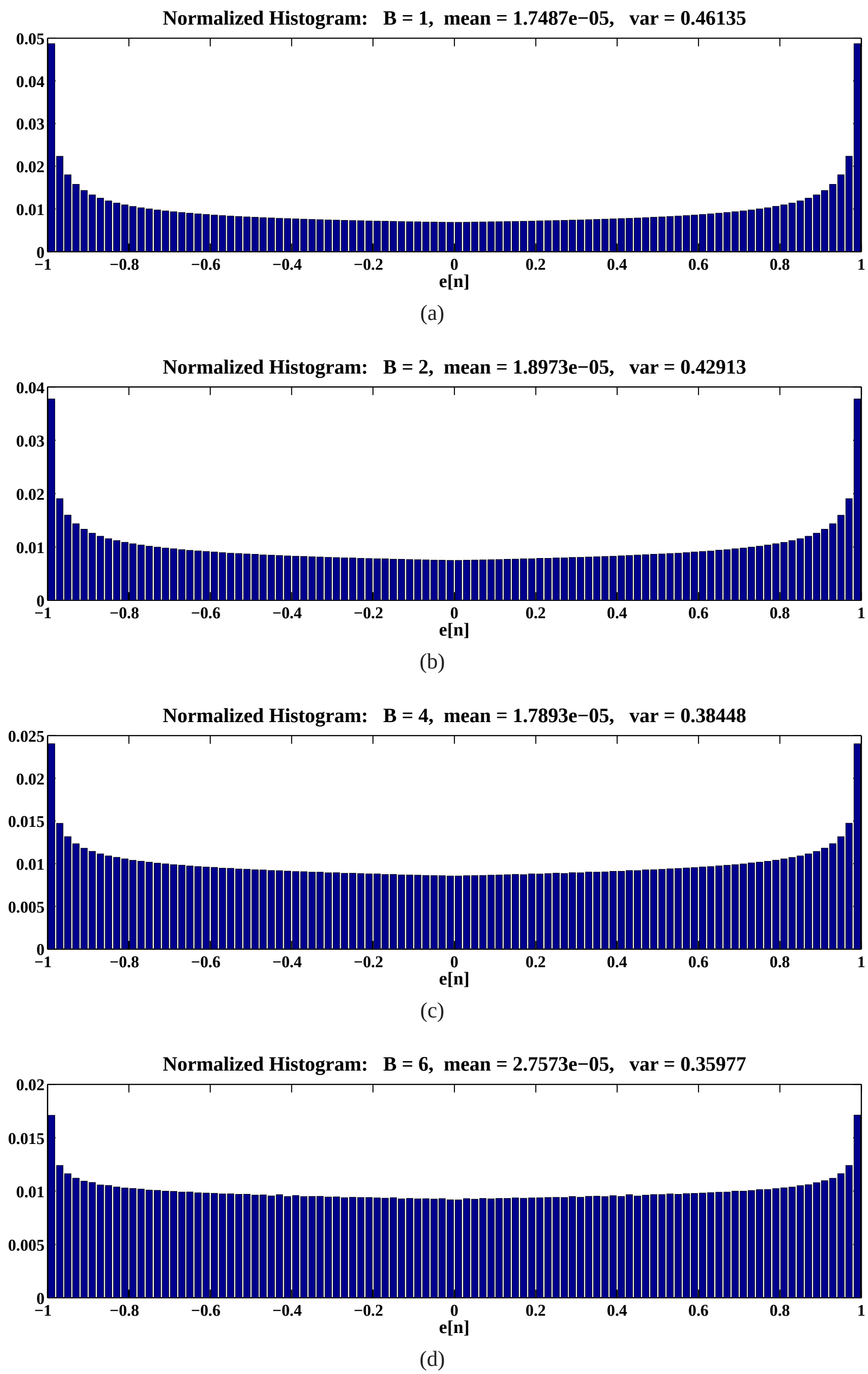
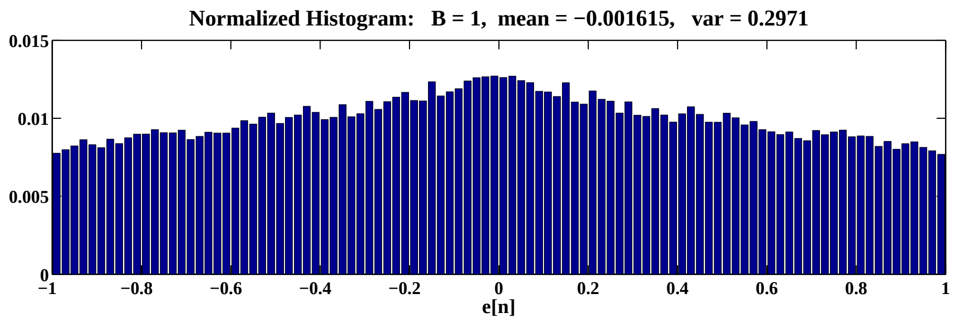
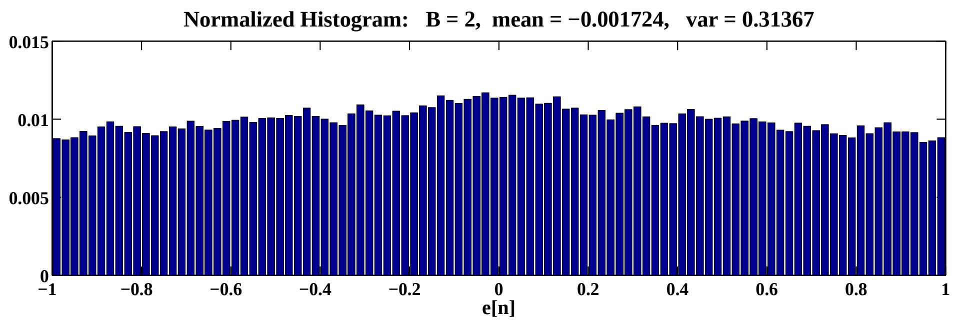


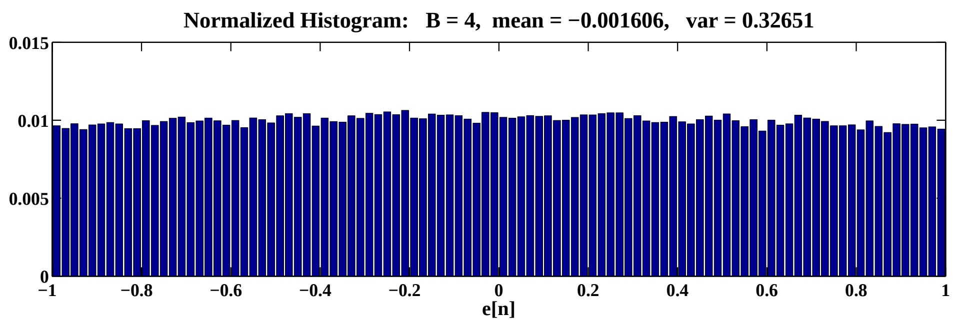
FIGURE 6.69: Plot the resulting distributions for quantization error $e[n]$ of $x[n]$ in part (a) for (a) 1 bit, (b) 2 bits, (c) 4 bits, (d) 6 bits.



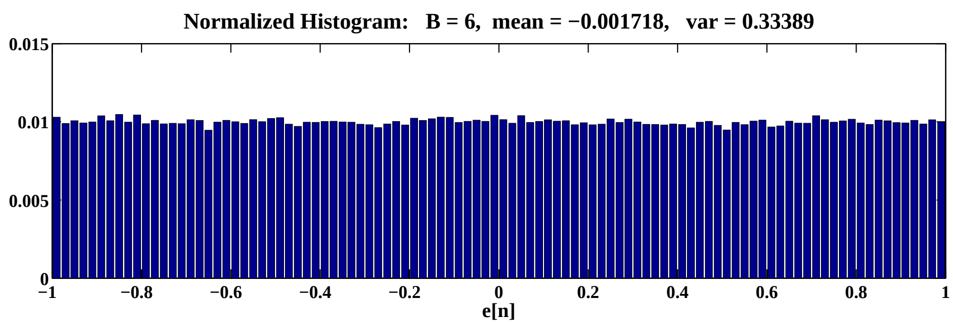
(a)



(b)

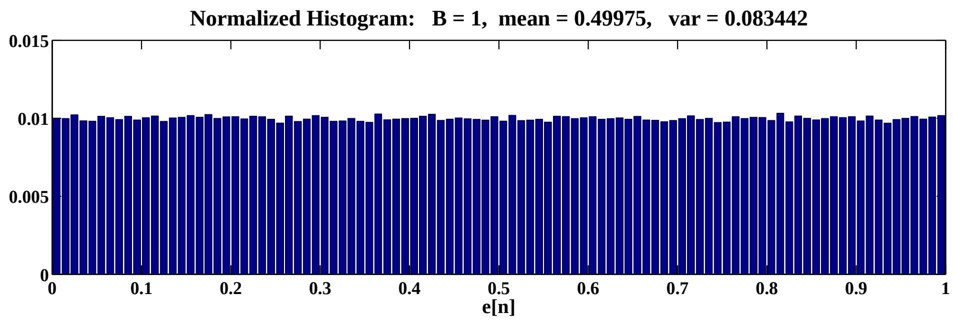


(c)

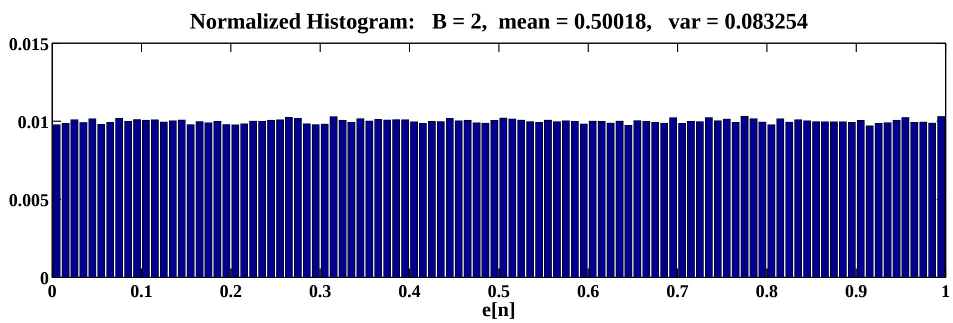


(d)

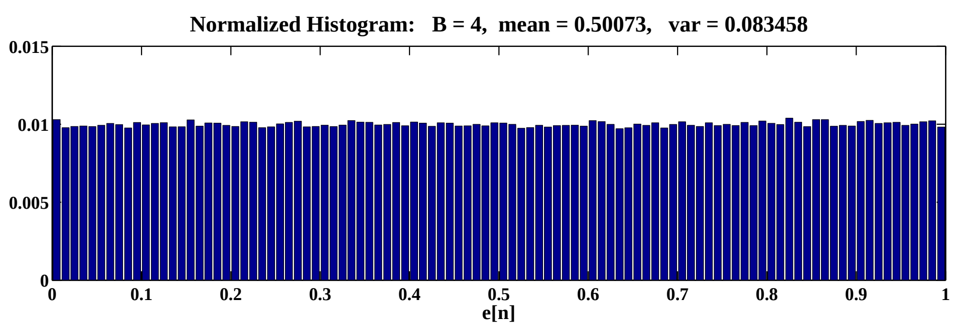
FIGURE 6.70: Plot the resulting distributions for quantization error $e[n]$ of $x[n]$ in part (c) for (a) 1 bit, (b) 2 bits, (c) 4 bits, (d) 6 bits.



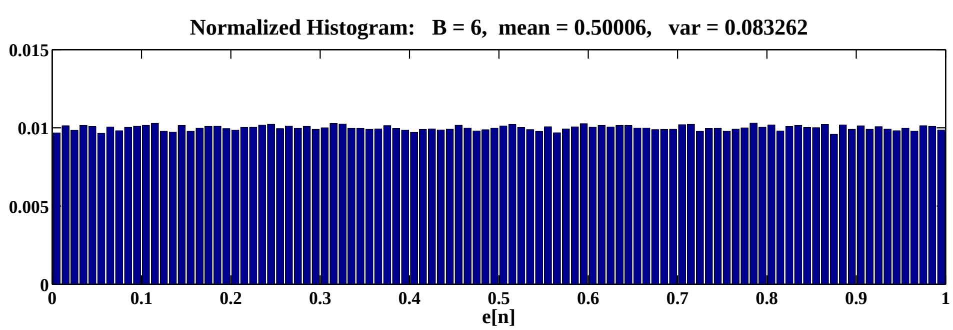
(a)



(b)



(c)



(d)

FIGURE 6.71: Plot the resulting distributions for quantization error $e[n]$ of $x[n]$ in part (d) for (a) 1 bit, (b) 2 bits, (c) 4 bits, (d) 6 bits.

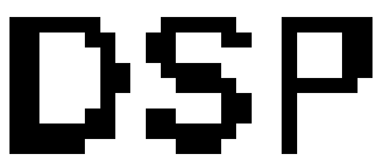
47. (a) See plot below.
    (b) See plot below.
    (c) tba.

MATLAB script:

```
% P0647: Investigate 2D sampling
close all; clc
xc = double(imread('DSP.png'));
%% Part (a):
xc_dim = size(xc);
sind = 5;
indx = mod(1:xc_dim(1),10) == sind;
indy = mod(1:xc_dim(2),10) == sind;
x = xc(indx,indy);
frescale = ones(10);
xs = kron(x,frescale);
% Plot:
hfa1 = figconfg('P0647a1');
imshow(xc)
hfa2 = figconfg('P0647a2');
imshow(x)
hfa3 = figconfg('P0647a3');
imshow(xs)
%% Part (b):
fave = fspecial('average',5); % change filter type and size
% fave = fspecial('gaussian',3);
yc = filter2(fave,xc);
y = yc(indx,indy);
ys = kron(y,frescale);
% Plot:
hfb1 = figconfg('P0647b1');
imshow(yc)
hfb2 = figconfg('P0647b2');
imshow(y)
hfb3 = figconfg('P0647b3');
imshow(ys)
```

(a)

(b)

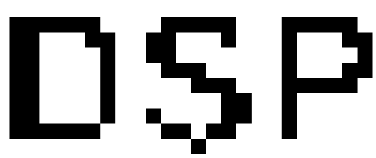


(c)

FIGURE 6.72: Image show of (a) `xc`, (b) `x`, (c) `xs`.

(a)

(b)



(c)

FIGURE 6.73: Image show of (a) `yc`, (b) `y`, (c) `ys`.