CHAPTER 13

Random Signals

Basic Problems

18. (a) Solution:

The sample space S of the experiment is:

$$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

(b) Solution:

$$P(A) = \frac{1}{6}, \quad P(B) = \frac{1}{4}, \quad P(C) = \frac{1}{12}$$

19. (a) Solution:

$$P_1 = \frac{33}{91}$$

(b) Solution:

$$P_2 = \frac{11}{91}$$

(c) Solution:

$$P_3 = \frac{6}{65}$$

20. (a) Solution:

$$K_1 = \frac{1}{2}$$

(b) Solution:

$$f_{Y|X}(y|x) = \frac{x+y}{x+1} \cdot e^{-y} u(y)$$

$$f_{X|Y}(x|y) = \frac{x+y}{y+1} \cdot e^{-x} u(x)$$

(c) Solution:

Random variables X and Y are NOT independent.

21. (a)

   (b)
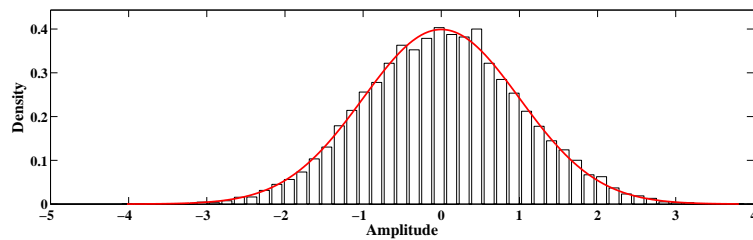
   (c)

   (d) See plots below.

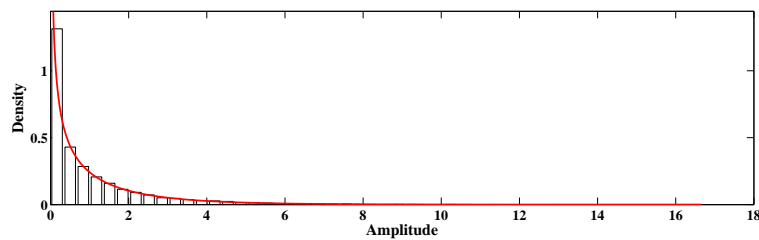FIGURE 13.1: Empirical and theoretical pdfs of X.

FIGURE 13.2: Empirical and theoretical pdfs of Y.

22. (a) Solution:

$$f(x) = 4x^3, \quad 0 \leq x \leq 1$$

$$f(y) = 4y(1 - y^2), \quad 0 \leq y \leq 1$$

$$f(x|y) = \frac{8x}{1 - y^2}, \quad y \leq x \leq 1$$

$$f(y|x) = \frac{2y}{x^2}, \quad 0 \leq y \leq x$$

   (b) Solution:

   X and Y are NOT independent.

23. Solution:
$$E[Y] = 45$$
$$\sigma_Y^2 = 4050$$

24. (a) Solution:
The probability mass function (pmf) for the random variable $x[3]$ is

$$P\{x[3] = 0\} = \frac{1}{2}, \quad P\{x[3] = 1\} = \frac{1}{2}$$

(b) Solution:
$$m_x[n] = 0.5$$

(c) Solution:
$$r_x[m, n] = \frac{1}{4} + \frac{1}{4}\delta[m - n]$$

25. (a) Solution: The mean of $y[n]$ is

$$\mu_y[n] = 0.6n$$

The variance of $y[n]$ is
$$\sigma_y^2[n] = 0.24n$$

(b) Solution:
$$\gamma_y[m, n] = 0.24 \min(m, n)$$

(c) Solution:
$$\sigma_A^2 = 0.24(m - n)$$

26. (a) Solution:
The autocorrelation $r_w[m, n]$ is

$$r_w[m, n] = 11\delta[m - n] + 1$$

(b) Solution:
$$r_v[m, n] = 12\delta[m - n] + 1$$

(c) Solution:
$$r_{w,v}[m, n] = 1$$

(d) Solution:
$$\mu_X[n] = 2$$

(e)