CHAPTER 8

Computation of the Discrete Fourier Transform

Basic Problems

16.

17.

18. Solution:
The resulting trend in the computational complexity of recursive DFT computations is much more efficient and close to linear than direct computation.

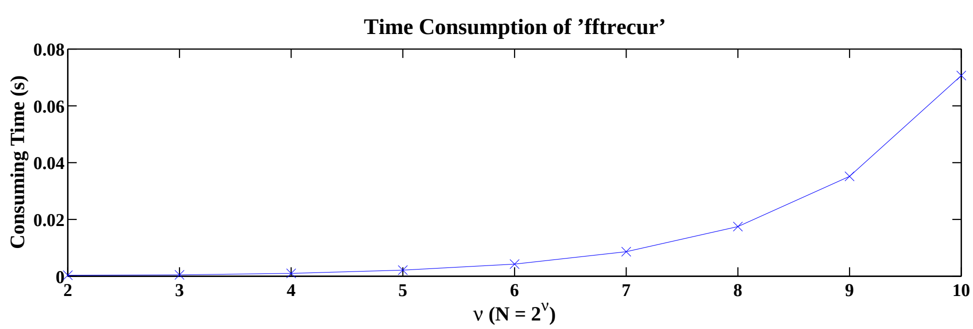


FIGURE 8.1: Plot of computation time for the `fftrecur` function for $N = 2^\nu$ where $2 \leq \nu \leq 10$.

19. (a) Solution:

If we define the following,

$$\begin{cases} A[k] = \sum_{m=0}^{\frac{N}{3}-1} x[3m] W_{\frac{N}{3}}^{km}, & k = 0, 1, \ldots, \frac{N}{3} - 1 \\ B[k] = \sum_{m=0}^{\frac{N}{3}-1} x[3m+1] W_{\frac{N}{3}}^{km}, & k = 0, 1, \ldots, \frac{N}{3} - 1 \\ C[k] = \sum_{m=0}^{\frac{N}{3}-1} x[3m+2] W_{\frac{N}{3}}^{km}, & k = 0, 1, \ldots, \frac{N}{3} - 1 \end{cases}$$

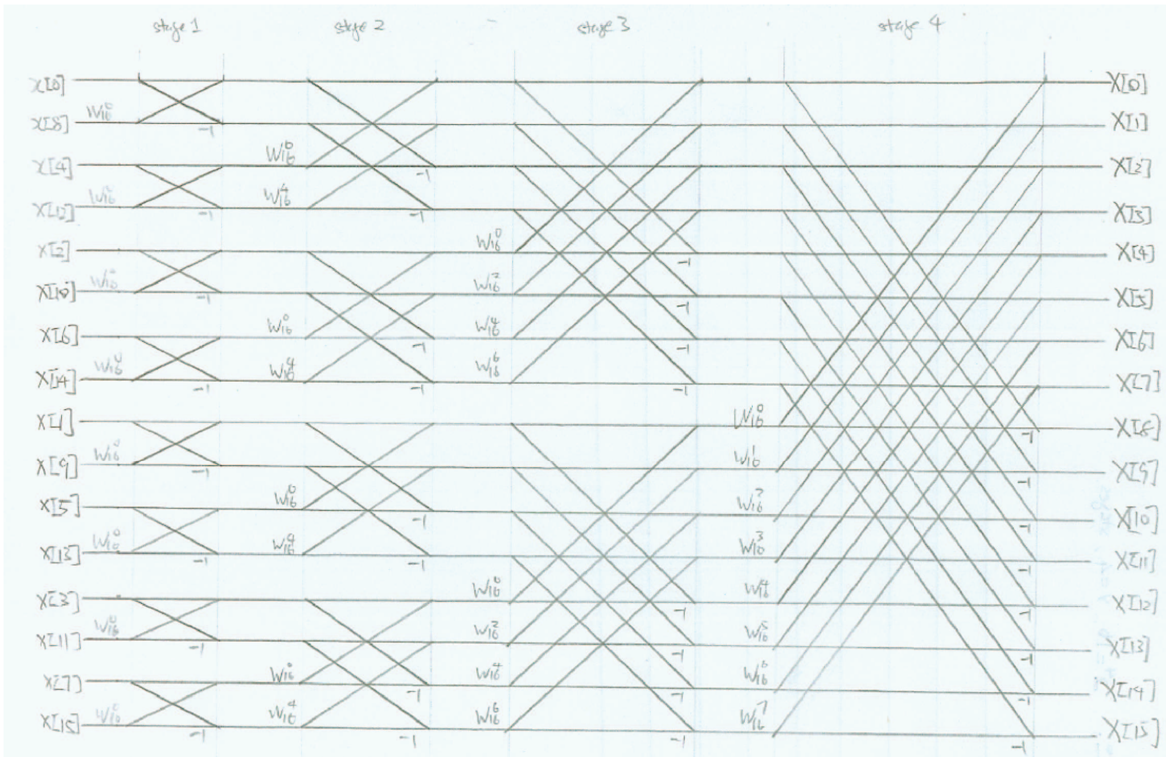
We conclude that:

$$\begin{aligned} X[k] &= A[k] + B[k] W_N^k + C[k] W_N^{2k} \\ X[k + N/3] &= A[k] + B[k] W_N^k W_N^{\frac{N}{3}} + C[k] W_N^{2k} W_N^{\frac{2N}{3}} \\ X[k + 2N/3] &= A[k] + B[k] W_N^k W_N^{\frac{2N}{3}} + C[k] W_N^{2k} W_N^{\frac{N}{3}} \end{aligned}$$

(b) tba

(c) Solution:
The total number of complex multiplications needed to implement is 162.

20. Solution:
The total number of real multiplications is 128 and the total number of real additions is 192.



21. (a)

    (b) Solution:

$$y[n] = \frac{1}{4}(1 + W_8^{7n})$$

22. Solution:

$$\begin{cases} X[k] = A[k] + W_N^k B[k], & n = 0, 1, \ldots, \frac{N}{2} - 1 \\ X[k + N/2] = A[k] - W_N^k B[k], & n = 0, 1, \ldots, \frac{N}{2} - 1 \end{cases} \tag{8.23}$$

If we mistakenly assign $a[n] = x[2n + 1]$ and $b[n] = x[2n]$, we can recover the DFT $X[k]$ as:

$$X'[k] = \frac{X[k] - X[k + \frac{N}{2}]}{2} W_N^{-k} + \frac{X[k] + X[k + \frac{N}{2}]}{2} W_N^k$$
$$X'[k + \frac{N}{2}] = \frac{X[k] - X[k + \frac{N}{2}]}{2} W_N^{-k} - \frac{X[k] + X[k + \frac{N}{2}]}{2} W_N^k$$

23. Solution:
It is a DIT approach.

24. (a) Solution:
There is only one path in the flow-graph begin at the input node $x[1]$ and terminate on the output node $X[2]$.
There is only one path in the flow-graph begin at the input node $x[1]$ and terminate on the output node $x[4]$ to $X[7]$.
The conclusion is that there is only one path from every input node to every output node.

(b) Solution:
The total gain from input node $x[1]$ to output node $X[2]$ is W_8^2.
The total gain from input node $x[4]$ to output node $X[7]$ is $W_8^{28} = -1$.

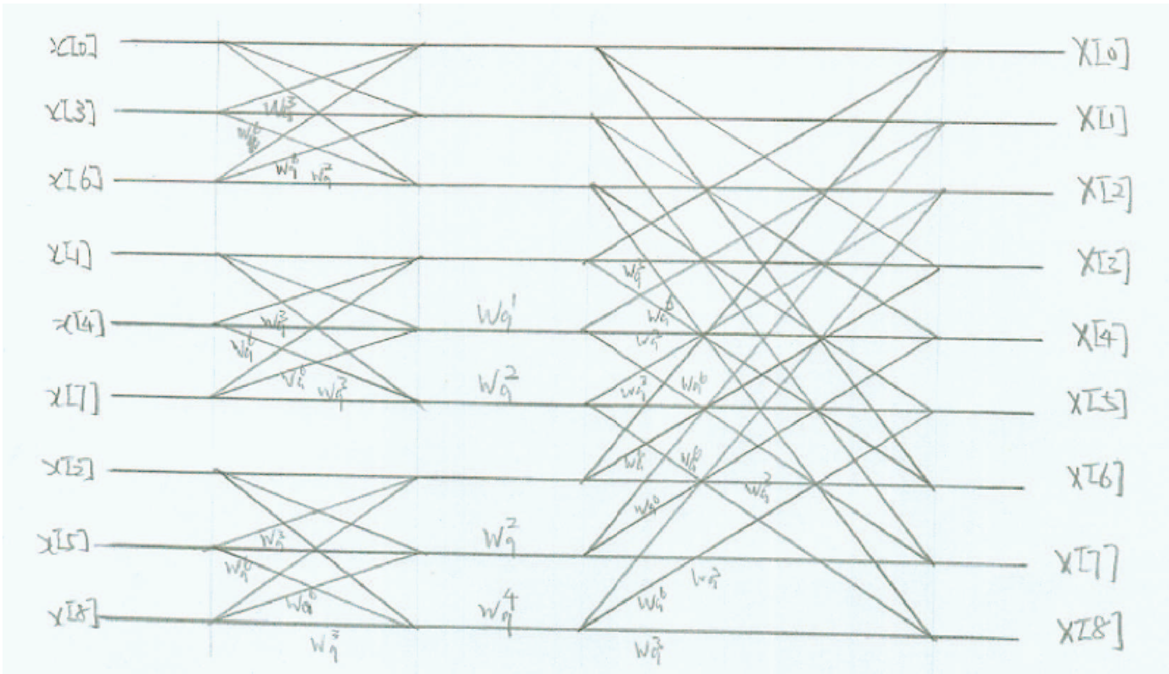
(c) Solution:
$X[4] = \sum_{n=0}^{7} x[n]W_8^{4n}$ can be verified.

25. Solution: If we define the following,

$$\begin{cases} A[k] = \sum_{m=0}^{\frac{N}{3}-1} x[3m]W_{\frac{N}{3}}^{km}, & k = 0, 1, \dots, \frac{N}{3} - 1 \\ B[k] = \sum_{m=0}^{\frac{N}{3}-1} x[3m+1]W_{\frac{N}{3}}^{km}, & k = 0, 1, \dots, \frac{N}{3} - 1 \\ C[k] = \sum_{m=0}^{\frac{N}{3}-1} x[3m+2]W_{\frac{N}{3}}^{km}, & k = 0, 1, \dots, \frac{N}{3} - 1 \end{cases}$$

We conclude that:

$$X[k] = A[k] + B[k]W_N^k + C[k]W_N^{2k}$$
$$X[k + N/3] = A[k] + B[k]W_N^k W_N^{\frac{N}{3}} + C[k]W_N^{2k}W_N^{\frac{2N}{3}}$$
$$X[k + 2N/3] = A[k] + B[k]W_N^k W_N^{\frac{2N}{3}} + C[k]W_N^{2k}W_N^{\frac{N}{3}}$$



26. (a) Solution:

$$\begin{cases} s_1[k] = s_0[k] + s_0[k+4]W_8^0 \\ s_1[k+4] = s_0[k] - s_0[k+4]W_8^0 \end{cases} \qquad k = 0, 1, 2, 3$$

$$\begin{cases} s_2[k] = s_1[k] + s_1[k+2]W_8^0, & k = 0, 1 \\ s_2[k] = s_1[k+2] + s_1[k+4]W_8^2, & k = 2, 3 \\ s_2[k+4] = s_1[k] - s_1[k+2]W_8^0, & k = 0, 1 \\ s_2[k+4] = s_1[k+2] - s_1[k+4]W_8^2, & k = 2, 3 \end{cases}$$

$$\begin{cases} s_3[k] = s_2[2k] + s_2[2k+1]W_8^k \\ s_3[k+4] = s_2[2k] - s_2[2k+1]W_8^k \end{cases} \qquad k = 0, 1, 2, 3$$

(b)

(c) Solution:
The coding complexity of the above function is much larger than that of the `fftditr2` function since the equations are not recursive.

27.

28. (a) Solution:

If we define the following that

$$A[k] = \sum_{n=0}^{2} x[n]W_3^{nk}, \qquad B[k] = \sum_{n=0}^{2} x[n+3]W_3^{nk}$$
$$C[k] = \sum_{n=0}^{2} x[n+6]W_3^{nk}, \qquad D[k] = \sum_{n=0}^{2} x[n+9]W_3^{nk}$$
$$E[k] = \sum_{n=0}^{2} x[n+12]W_3^{nk}$$

Hence, we can conclude that

$$\begin{aligned}
X[5k] &= A[k] + B[k] + C[k] + D[k] + E[k] \\
X[5k+1] &= (A[k] + B[k]W_{15}^{3} + C[k]W_{15}^{6} + D[k]W_{15}^{9} + E[k]W_{15}^{12})W_{15}^{n} \\
X[5k+2] &= (A[k] + B[k]W_{15}^{6} + C[k]W_{15}^{12} + D[k]W_{15}^{3} + E[k]W_{15}^{9})W_{15}^{2n} \\
X[5k+3] &= (A[k] + B[k]W_{15}^{9} + C[k]W_{15}^{3} + D[k]W_{15}^{12} + E[k]W_{15}^{6})W_{15}^{3n} \\
X[5k+4] &= (A[k] + B[k]W_{15}^{12} + C[k]W_{15}^{9} + D[k]W_{15}^{6} + E[k]W_{15}^{3})W_{15}^{4n}
\end{aligned}$$

(b) Solution:

If we define the following that

$$A[k] = \sum_{n=0}^{4} x[n]W_5^{nk}, \qquad B[k] = \sum_{n=0}^{4} x[n+5]W_5^{nk}$$
$$C[k] = \sum_{n=0}^{4} x[n+10]W_5^{nk},$$

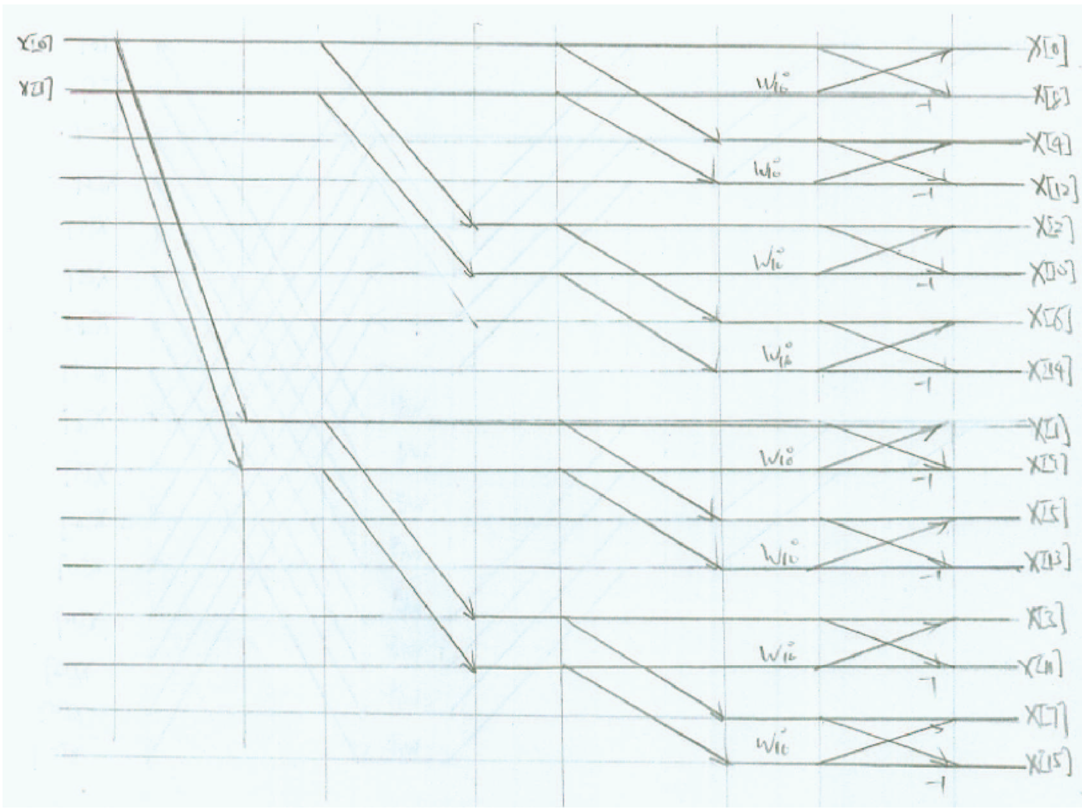
Hence, we can conclude that

$$\begin{aligned}
X[3k] &= A[k] + B[k] + C[k] \\
X[3k+1] &= (A[k] + B[k]W_{15}^{5} + C[k]W_{15}^{10})W_{15}^{n} \\
X[3k+2] &= (A[k] + B[k]W_{15}^{10} + C[k]W_{15}^{5})W_{15}^{2n}
\end{aligned}$$

(c) Solution:

For part (a), the total number of complex multiplications is 90. The total number of complex additions is 90.

For part (b), the total number of complex multiplications is 90. The total number of complex additions is 90.

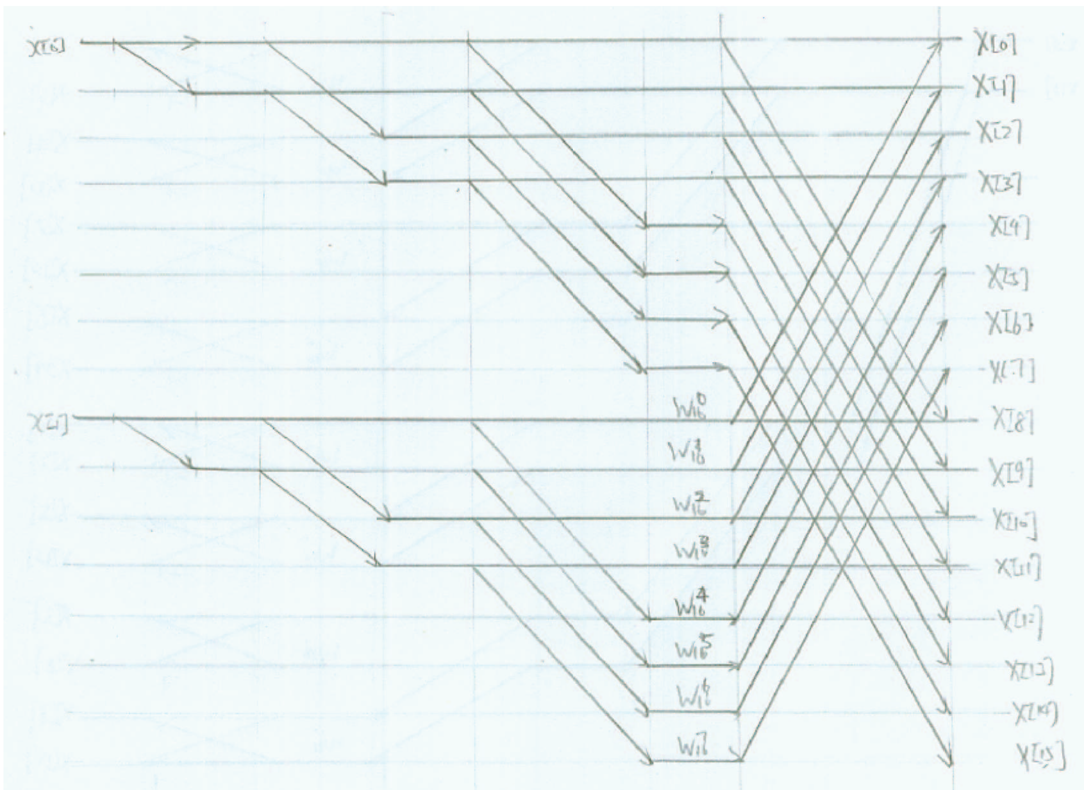


29. (a) See graph below.

    (b) See graph below.

    (c) Solution:
    For part (a), the total number of complex multiplications is 0.
    For part (b), the total number of complex multiplications is 7.

    (d) tba


X[0]
X[1]
X[2]
X[3]
X[4]
X[5]
X[6]
X[7]
X[8]
X[9]
X[10]
X[11]
X[12]
X[13]
X[14]
X[15]
W16^0
W16^1
W16^2
W16^3
W16^4
W16^5
W16^6
W16^7

30.

31.

32. (a) See plot below.

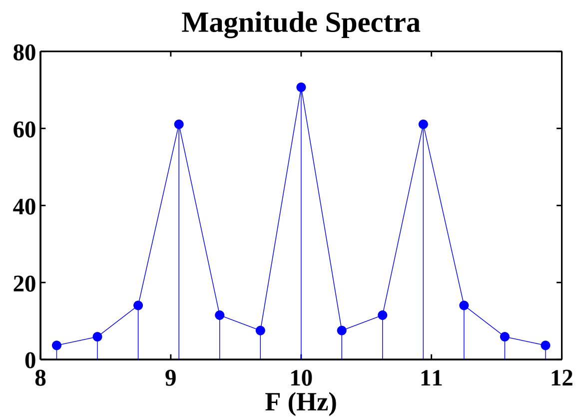


FIGURE 8.2: Magnitude spectra of 128-point FFT of $x[n]$ over $8 \leq F \leq 12$ Hz.

(b) See plot below.

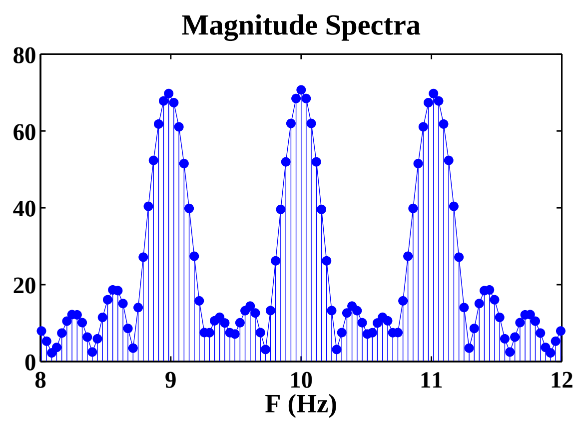


FIGURE 8.3: Magnitude spectra of 1024-point FFT of $x[n]$ over $8 \leq F \leq 12$ Hz.

(c) See plot below.

(d) Solution:
Using `cta` function has the smallest number of computations with a better display.

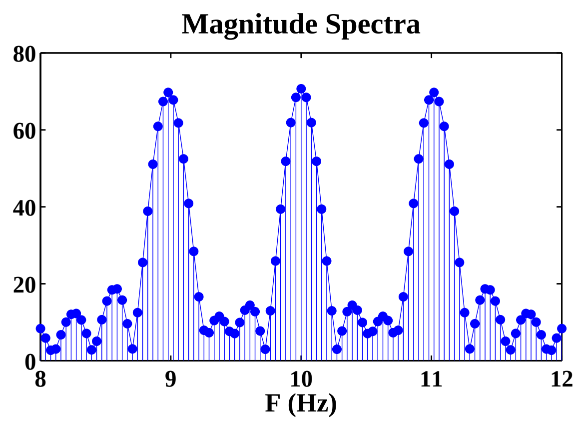


FIGURE 8.4: Magnitude spectra of DFT of $x[n]$ over $8 \leq F \leq 12$ Hz using `cta` function.