CHAPTER 5

Transform Analysis of LTI Systems

Basic Problems

23. (a) Solution:
    The frequency response is

$$H(e^{j\omega}) = \frac{b}{1 - 0.8e^{-j\omega} - 0.81e^{-2j\omega}}$$

    (b) Solution:
    b = 0.1702.

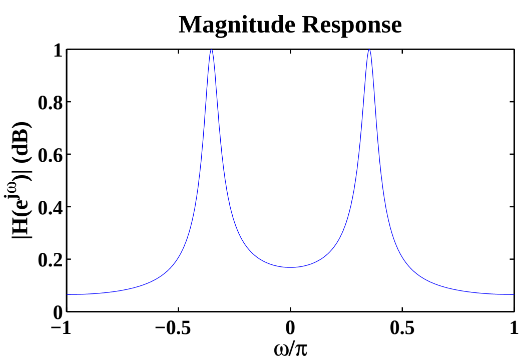


FIGURE 5.1: Magnitude response.

    (c) See plot below.
    (d) Solution:

$$y[n] = 2 \times 0.0577 \cos(\frac{\pi n}{3} + \frac{\pi}{4}) - 2 \times 0.0809 \sin(\frac{\pi n}{3} + \frac{\pi}{4})$$

    (e) See plot below.

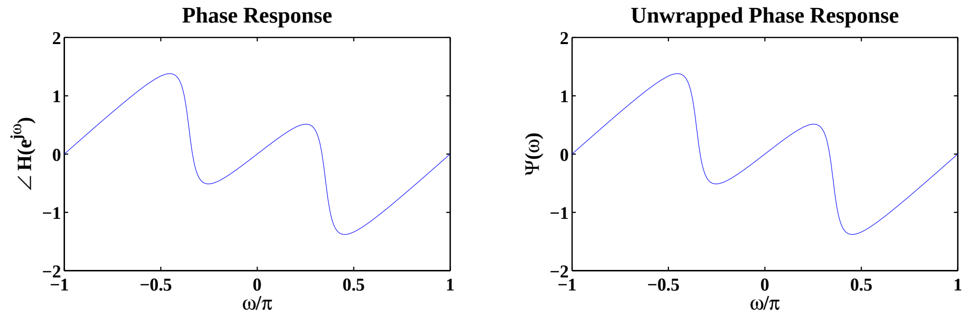


FIGURE 5.2: Wrapped and the unwrapped phase responses.

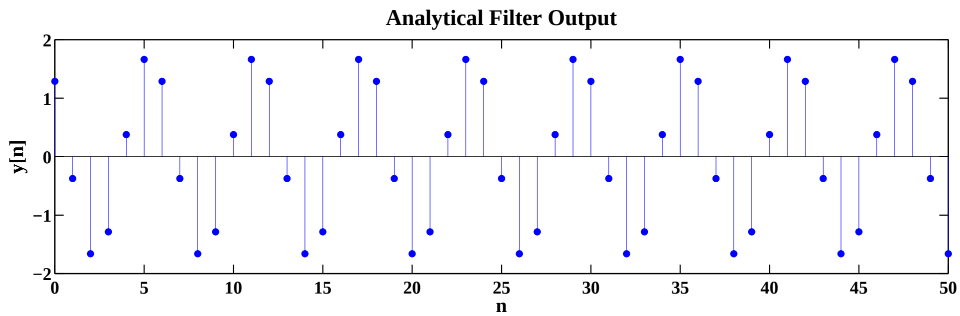


FIGURE 5.3: Analytical response $y[n]$ to the input $x[n] = 2\cos(\pi n/3 + 45°)$.

24. (a) Solution:
The magnitude response is:

$$|H(e^{j\omega})| = \frac{|b|}{\sqrt{1 + a^2 + 2a\cos 2\omega}}$$

The phase response is:

$$\angle H(e^{j\omega}) = \tan^{-1}\frac{a\sin 2\omega}{1 + a\cos 2\omega}$$

In order to constrain the maximum magnitude response equal to 1, we have

$$|b| = 1 - a$$

Hence, for $a = 0.8$, we choose $b = 0.2$.

(b) Solution:
(i) The output is:

$$y[n] = 3\cos\left(\frac{\pi}{2}n\right)$$

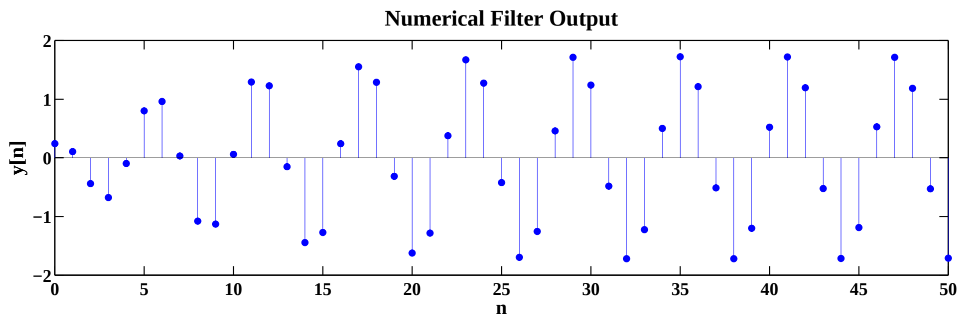


FIGURE 5.4: Numerical response $y[n]$ to the input $x[n] = 2\cos(\pi n/3 + 45°)$.

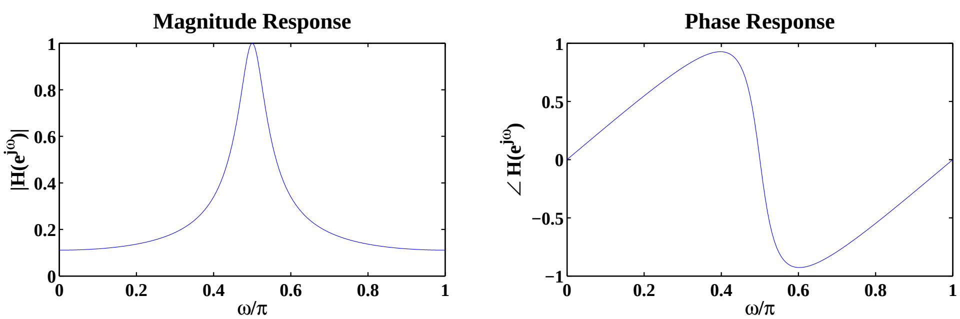


FIGURE 5.5: Magnitude and phase responses for $a = 0.8$.

(ii) The output is:

$$y[n] = \frac{1-a}{1+a^2} 3 \sin \frac{\pi}{4} n + \frac{a(1-a)}{1+a^2} 3 \cos \frac{\pi}{4} n$$

(c) See plots below.

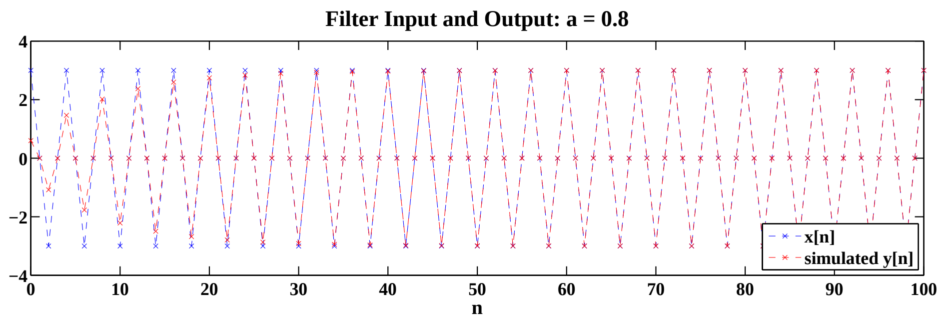
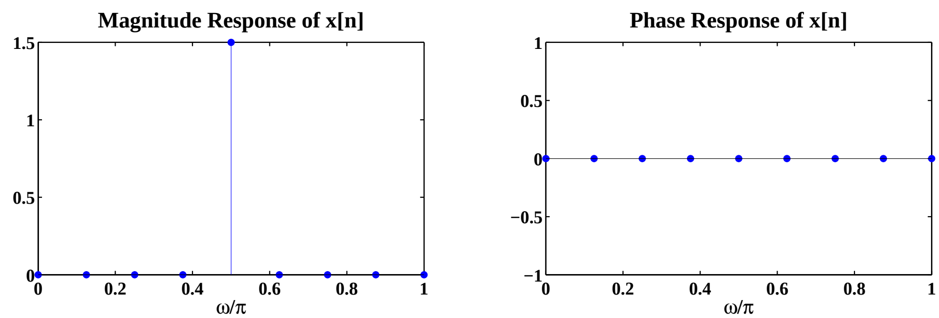
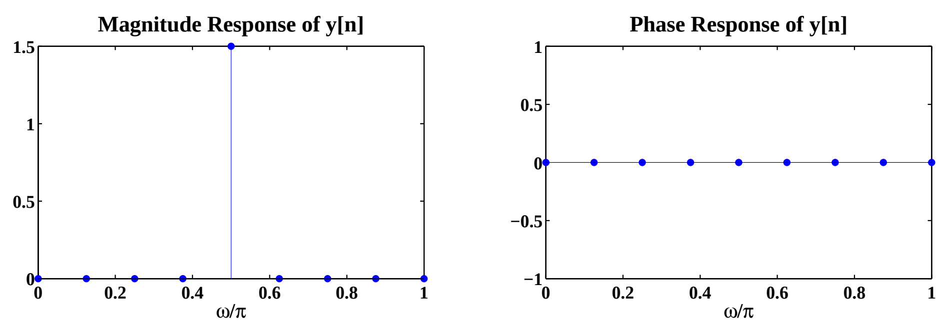


FIGURE 5.6: Plot for input $x[n]$ and output $y[n]$ when $x[n] = 3\cos(\pi n/2)$.



(a)



(b)

FIGURE 5.7: Magnitude and phase responses of (a) $x[n]$. (b) $y[n]$.

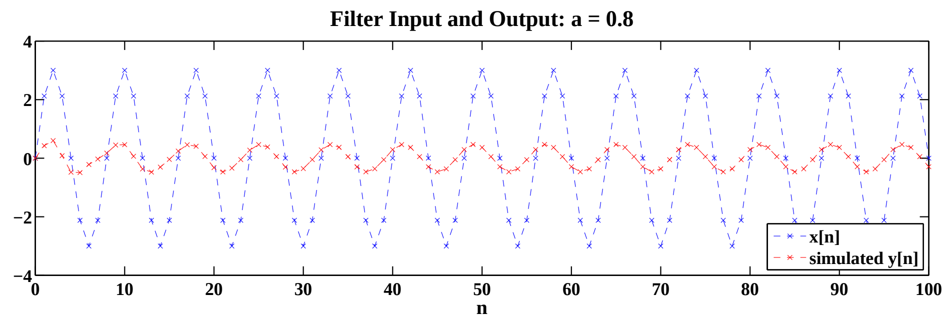


FIGURE 5.8: Plot for input $x[n]$ and output $y[n]$ when $x[n] = 3\sin(\pi n/4)$.

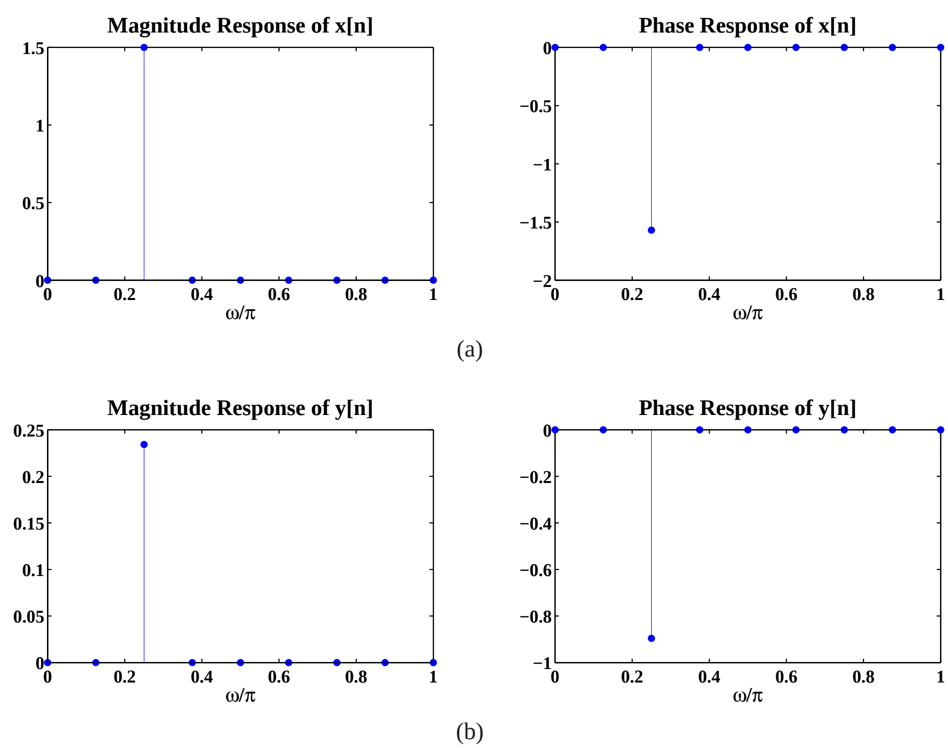


FIGURE 5.9: Magnitude and phase responses of (a) $x[n]$. (b) $y[n]$.

25. (a) Solution:
The frequency response is:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}}{1-0.8\mathrm{e}^{-\mathrm{j}(\omega-\frac{\pi}{4})}} + \frac{\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}}{1-0.8\mathrm{e}^{-\mathrm{j}(\omega+\frac{\pi}{4})}} + \frac{5}{1+0.9\mathrm{e}^{-\mathrm{j}\omega}}$$

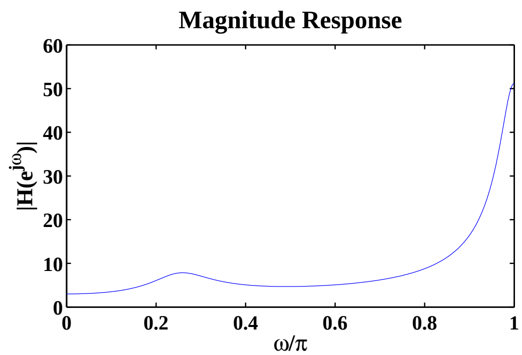


FIGURE 5.10: Magnitude response of the system.

(b) See plot below.

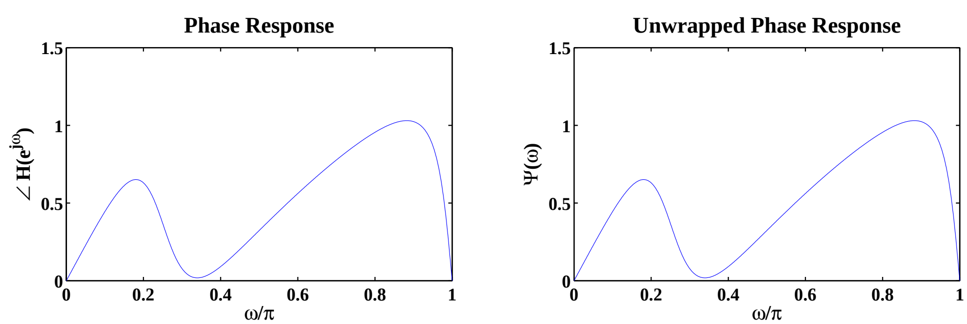


FIGURE 5.11: Wrapped and the unwrapped phase responses of the system.

(c) Solution:
The output is:

$$y[n] = H(\mathrm{e}^{\mathrm{j}0}) + \frac{3}{2}\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{\mathrm{j}\frac{\pi}{4}n}H(\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}) + \frac{3}{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}n}H(\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}) + 5\mathrm{e}^{-\mathrm{j}\pi n}H(\mathrm{e}^{-\mathrm{j}\pi})$$

(d) See plot below.

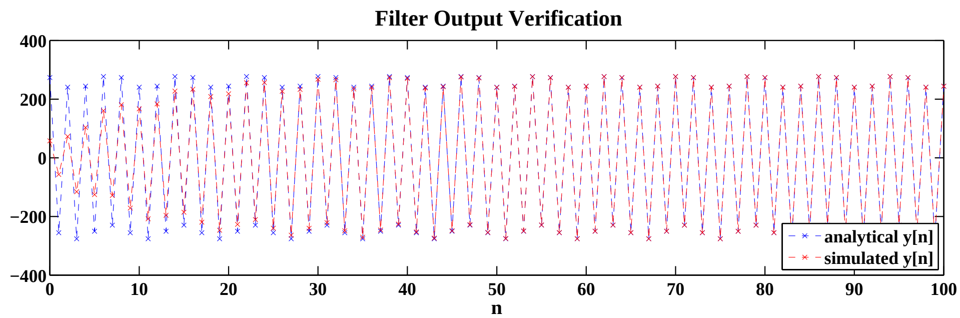


FIGURE 5.12: MATLAB verification of the steady-state response to $x[n]$.

26. (a) Solution:

$$c_k^{(x)} = \frac{1}{10} \cdot \frac{1 - 0.8^{10}}{1 - 0.8\mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}k}}$$

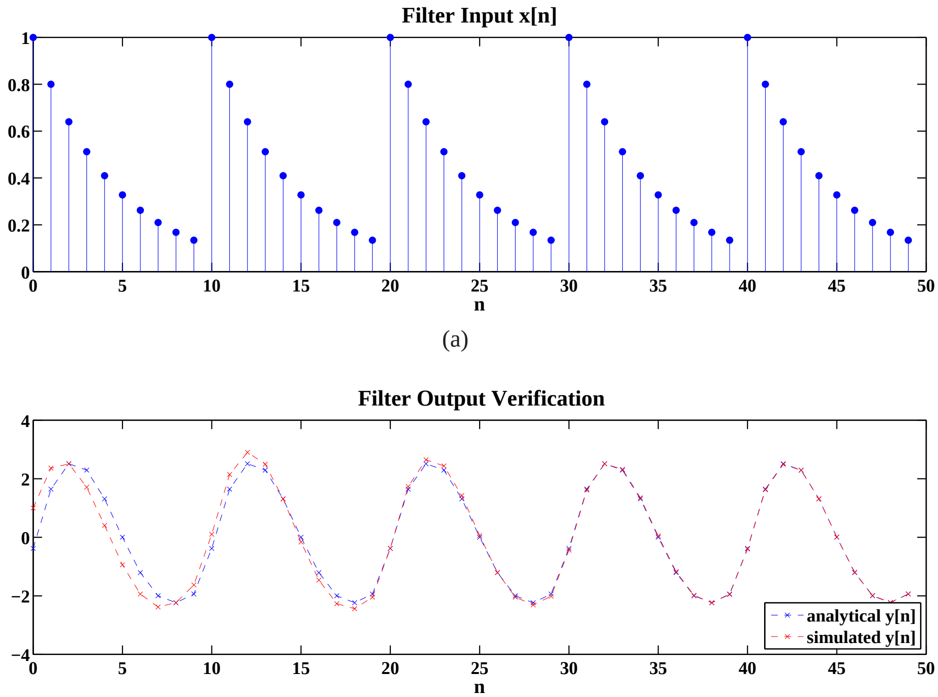
(b) Solution:

$$c_k^{(y)} = c_k^{(x)} H\big(\mathrm{e}^{\mathrm{j}\frac{2\pi}{10}k}\big)$$

(c) Solution:

$$y_{\mathrm{ss}}[n] = \sum_{n=0}^{9} c_k^{(x)} \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}kn} H\big(\mathrm{e}^{\mathrm{j}\frac{2\pi}{10}k}\big)$$

(d) See plot below.



(a)

(b)

FIGURE 5.13: Signal plots of (a) $x[n]$. (b) $y[n]$.

27. (a) Solution:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1}{2}\frac{1 - 0.45\mathrm{e}^{-\mathrm{j}\omega} + 0.05\mathrm{e}^{-2\mathrm{j}\omega}}{1 - 0.325\mathrm{e}^{-\mathrm{j}\omega} + 0.0225\mathrm{e}^{-2\mathrm{j}\omega}}$$

The frequency response exists and is unique.

(b) Solution:
The frequency response does NOT exist since the frequency changes.

(c) Solution:
The frequency response exists but is NOT unique.

(d) Solution:

$$y[n] = x[n] - x[n-1] + \delta[n]$$

which is NOT LTI system.

28. (a) Solution:

The frequency response is:

$$H\left(e^{j\omega}\right) = \frac{1}{2}(1 - 2\cos\omega + 4\cos 2\omega - 2\cos 3\omega + \cos 4\omega) + \frac{1}{2}j(2\sin\omega - 4\sin 2\omega + 2\sin 3\omega - \sin 4\omega)$$

(b) See plot below.

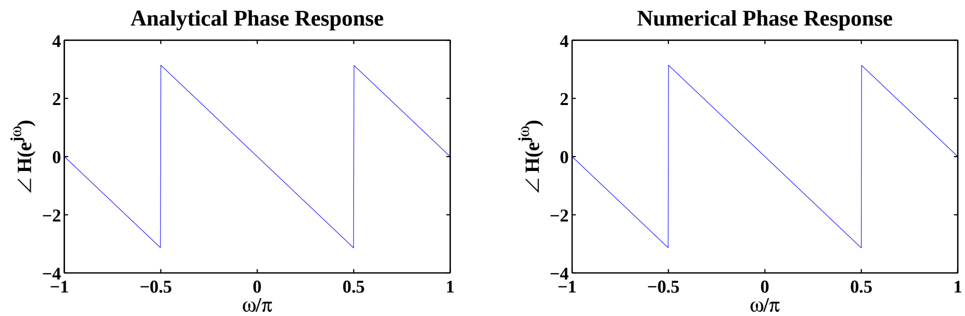


FIGURE 5.14: Phase response plot.

29. (a) Solution:
The frequency response is:

$$H\left(e^{j\omega}\right) = (1 - 2\cos\omega + 3\cos 2\omega - 4\cos 3\omega + 4\cos 5\omega - 3\cos 6\omega + 2\cos 7\omega - \cos 8\omega) + j(2\sin\omega - 3\sin 2\omega + 4\sin 3\omega - 4\sin 5\omega + 3\sin 6\omega - 2\sin 7\omega + \sin 8\omega)$$

The analytical phase response is:

$$\angle H\left(e^{j\omega}\right) = \tan^{-1}\frac{2\sin\omega - 3\sin 2\omega + 4\sin 3\omega - 4\sin 5\omega + 3\sin 6\omega - 2\sin 7\omega + \sin 8\omega}{1 - 2\cos\omega + 3\cos 2\omega - 4\cos 3\omega + 4\cos 5\omega - 3\cos 6\omega + 2\cos 7\omega - \cos 8\omega}$$

(b) See plot below.

30. (a) See plot below.
(b) See plot below.
(c) See plot below.
(d) See plot below.
(e) See plot below.

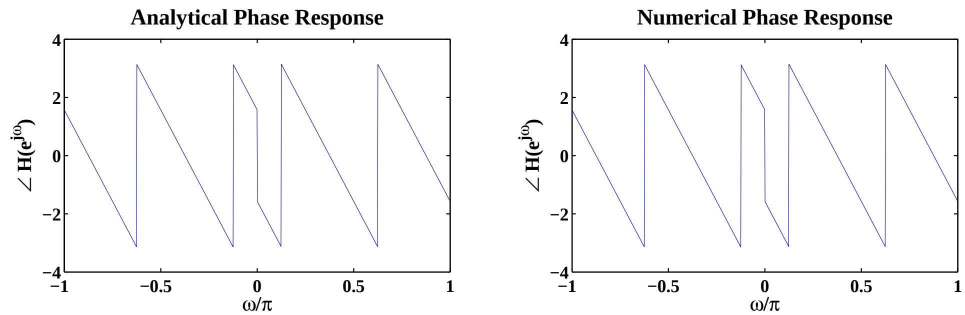


FIGURE 5.15: Phase response plot.

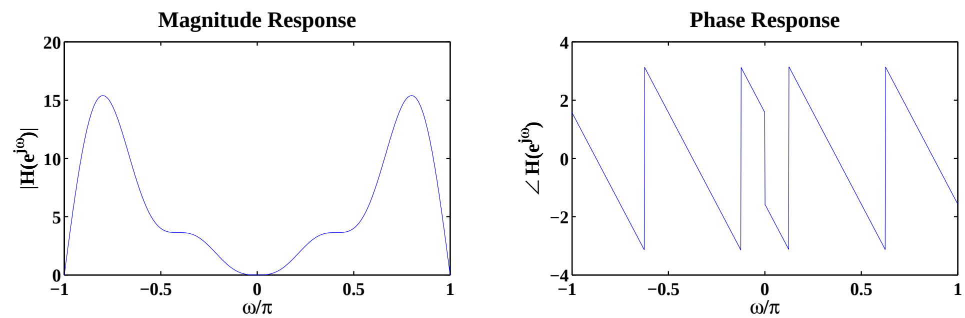


FIGURE 5.16: Magnitude and phase responses of the system.

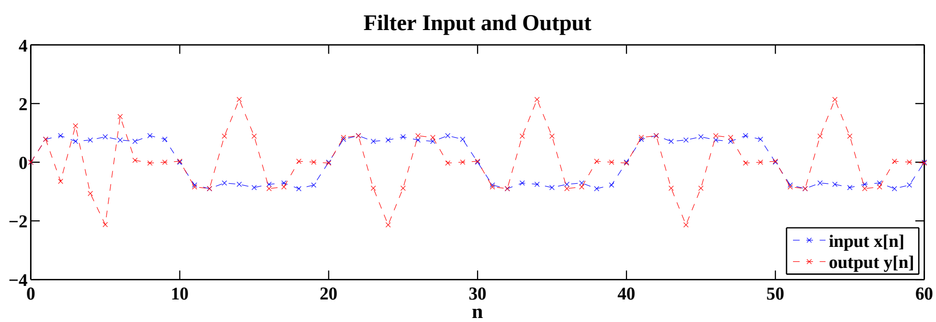


FIGURE 5.17: Plot of the input and steady state response.

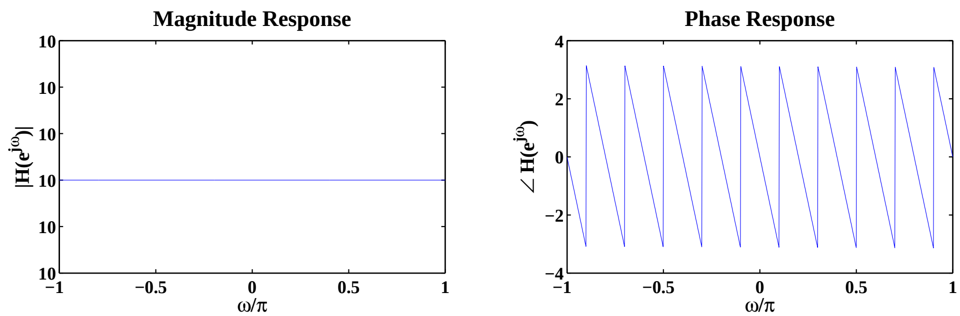


FIGURE 5.18: Magnitude and phase responses of the system.

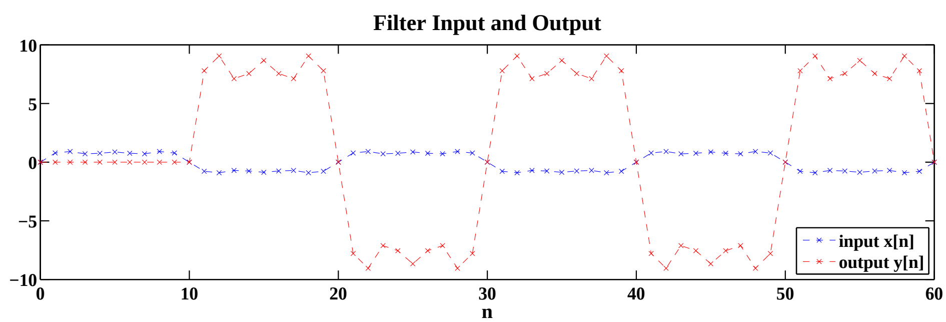


FIGURE 5.19: Plot of the input and steady state response.

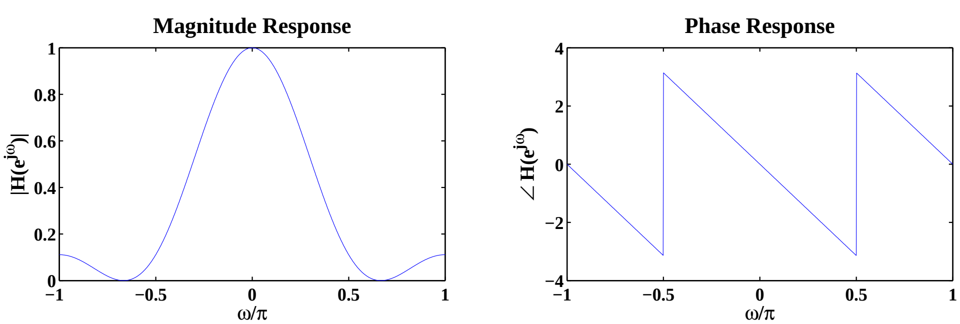


FIGURE 5.20: Magnitude and phase responses of the system.

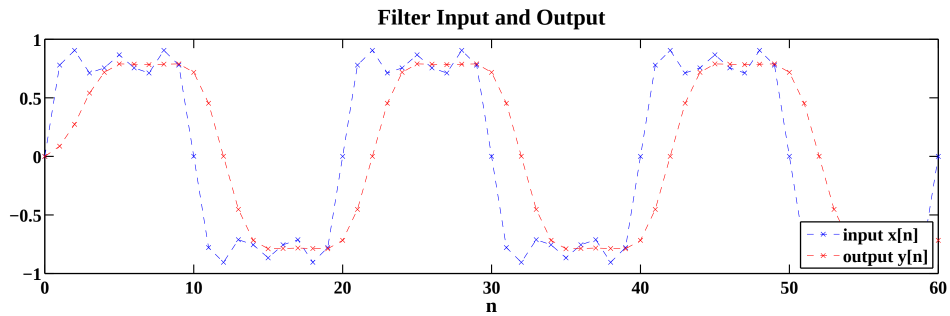


FIGURE 5.21: Plot of the input and steady state response.

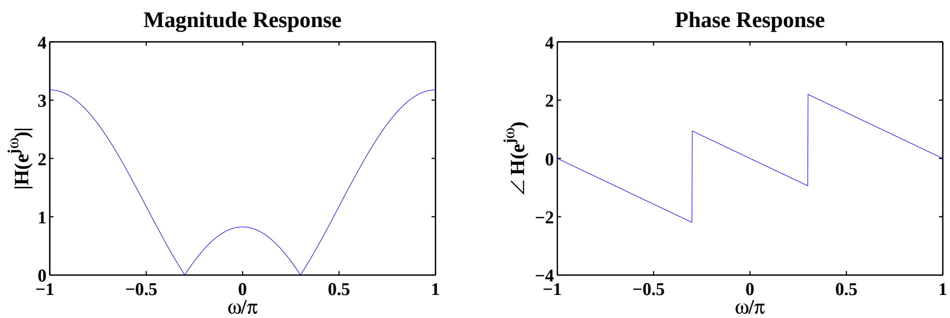


FIGURE 5.22: Magnitude and phase responses of the system.

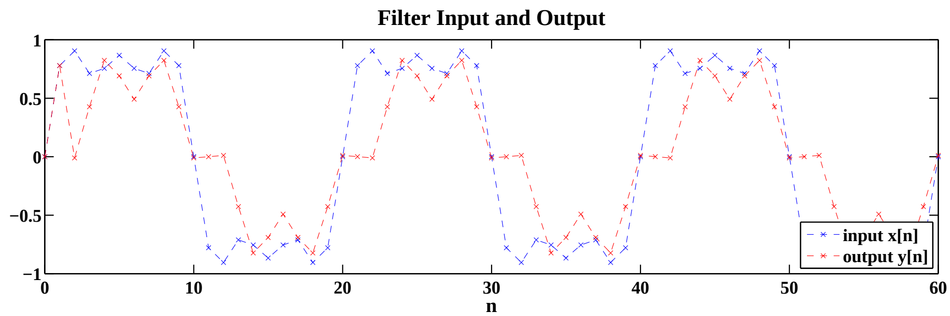


FIGURE 5.23: Plot of the input and steady state response.

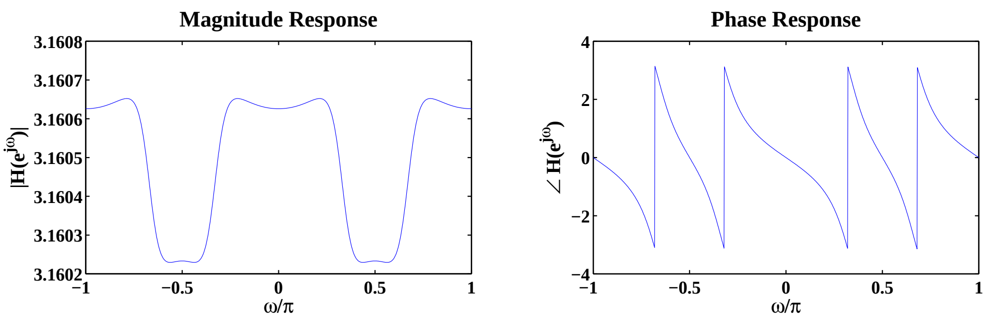


FIGURE 5.24: Magnitude and phase responses of the system.

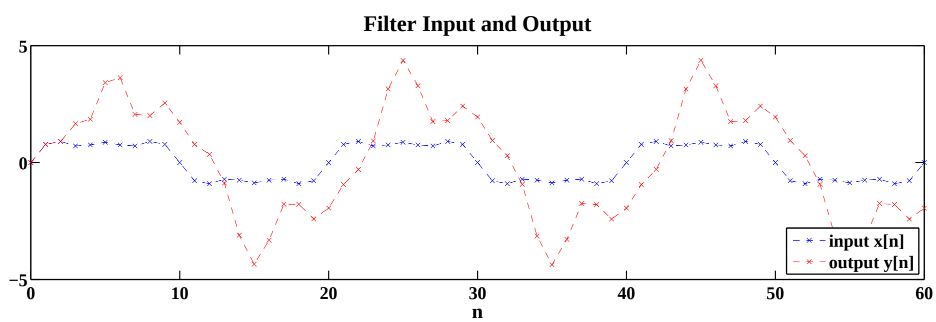


FIGURE 5.25: Plot of the input and steady state response.

31. (a) Solution:
The impulse response of the filter is:

$$h[n] = \frac{1}{2\pi}\frac{2}{n-n_d}\left[\sin\frac{2\pi}{8}(n-n_d) - \sin\frac{\pi}{8}(n-n_d)\right] + \frac{1}{2\pi}\frac{1}{n-n_d}\left[\sin\frac{7\pi}{8}(n-n_d) - \sin\frac{5\pi}{8}(n-n_d)\right]$$

(b) See plot below.

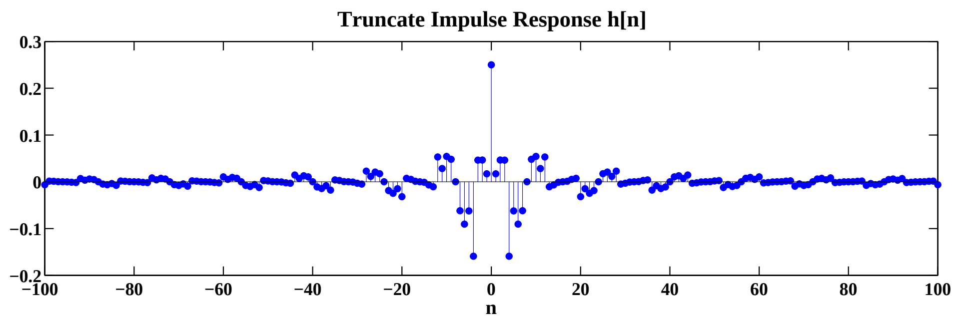


FIGURE 5.26: Impulse response for $n_d = 0$ for $-100 \leq n \leq 100$.

(c) See plot below.

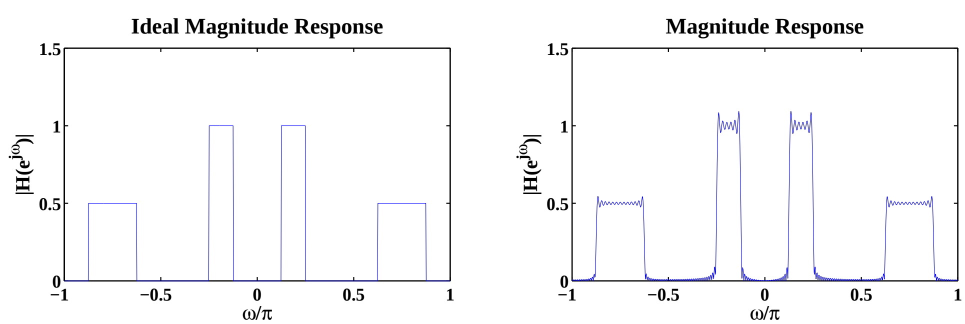
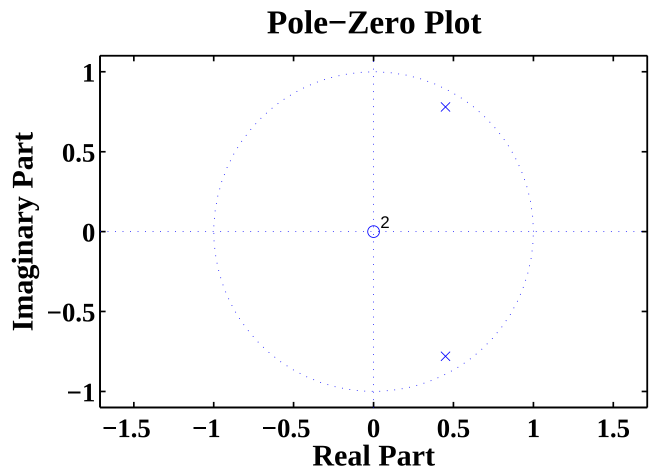
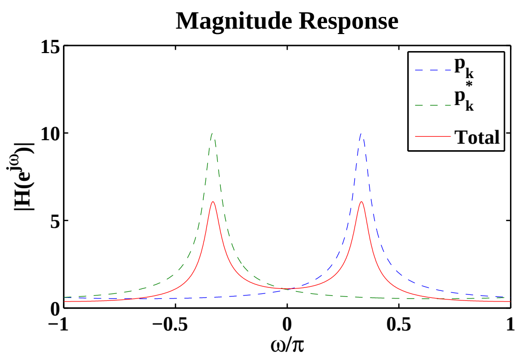


FIGURE 5.27: Magnitude response of the filter using MATLAB and the ideal filter magnitude response.

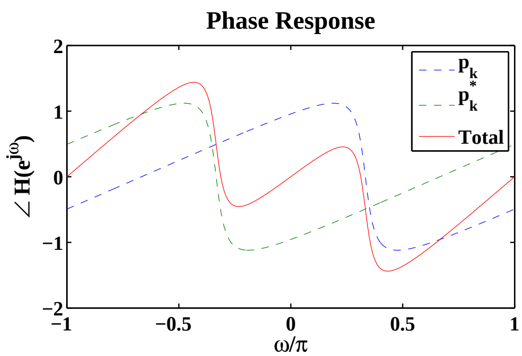
32. See plot below.



(a)



(b)



(c)

FIGURE 5.28: Figure 5.19 reproduction.

33. (a) See plot below.
    (b) See plot below.
    (c) See plot below.

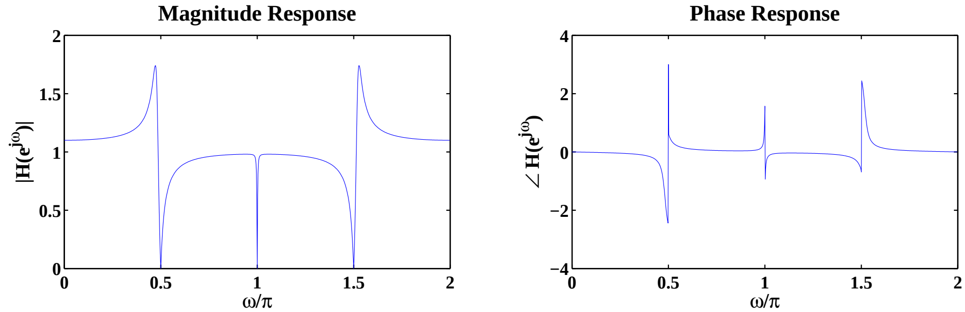


FIGURE 5.29: Magnitude and phase responses of the system using MATLAB function `freqz`.

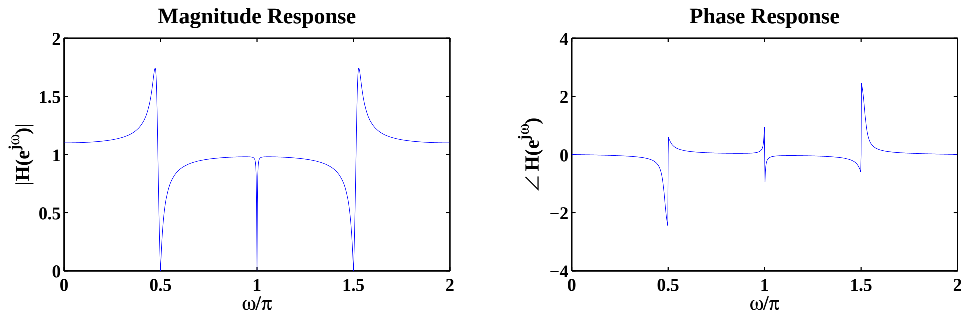


FIGURE 5.30: Magnitude and phase responses of the system using MATLAB function `freqz0`.

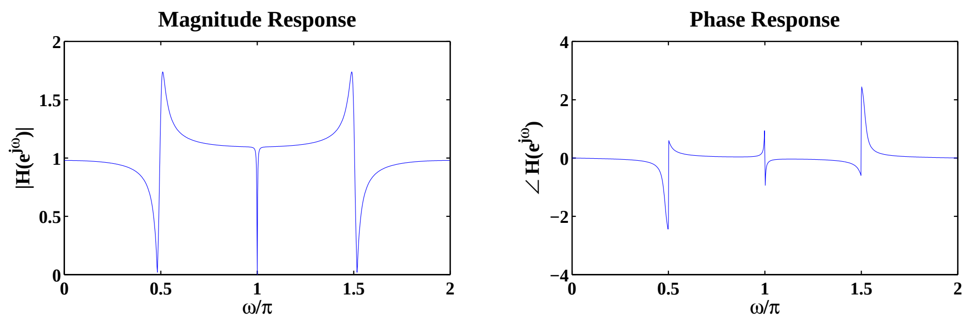


FIGURE 5.31: Magnitude and phase responses of the system using MATLAB function `myfreqz`.

34. (a) See plot below.

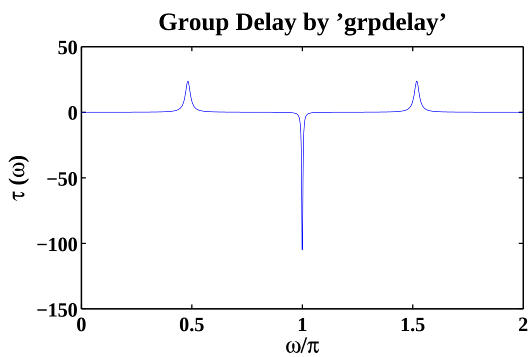


FIGURE 5.32: Group delay of the system using MATLAB function `grpdelay`.

(b) See plot below.

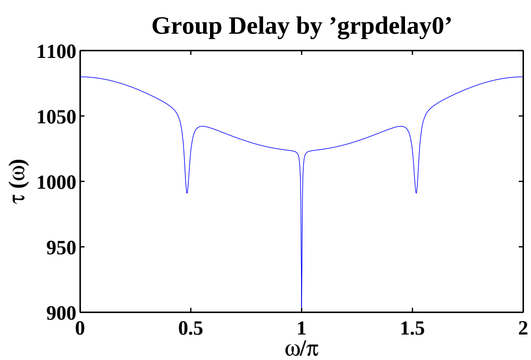


FIGURE 5.33: Group delay of the system using MATLAB function `grpdelay0`.

(c) See plot below.

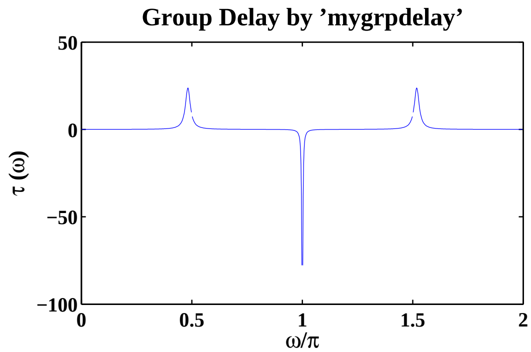


FIGURE 5.34: Group delay of the system using MATLAB function `mygrpdelay`.

35. (a) Solution:

$$\tau_{\text{gd}}(\omega) = \frac{\alpha^2 - \alpha\cos\omega}{1+\alpha^2 - 2\alpha\cos\omega}$$

(b) Solution:

$$\tau_{\text{gd}}(\omega) = \frac{-\alpha^2 + \alpha\cos\omega}{1+\alpha^2 - 2\alpha\cos\omega}$$

(c) Solution:

$$\tau_{\text{gd}}(\omega)$$
$$= \frac{4\alpha^3\cos\phi\cos\omega - 2\alpha^4 - 4\alpha^2\cos^2\phi + 2\alpha^3\sin\omega\sin 2\omega + 2\alpha\cos\phi\cos\omega + 2\alpha^2\cos\phi\cos\omega\cos 2\omega - 2\alpha^2\cos 2\omega}{4\alpha^2\cos^2\phi + 1 + \alpha^4 - 4\alpha^3\cos\phi\cos\omega - 4\alpha\cos\phi\cos\omega + 2\alpha^2\cos 2\omega}$$

36. (a) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}(1 + z^{-1})$$

The magnitude response is:

$$|H(\mathrm{e}^{\mathrm{j}\omega})| = |\cos\frac{\omega}{2}|$$

The phase response is:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) = -\frac{\omega}{2}$$

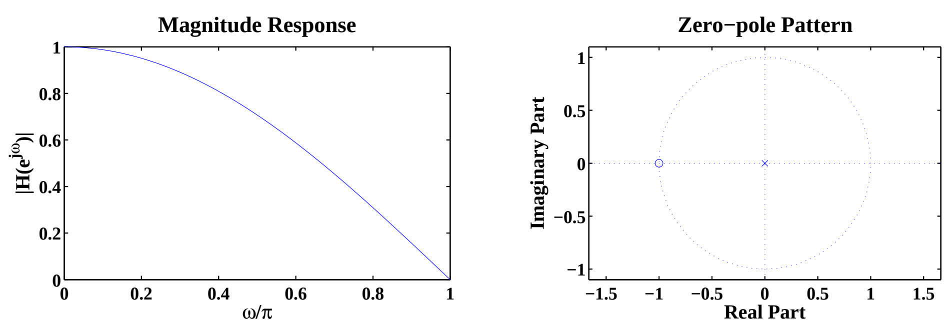


FIGURE 5.35: Pole-zero pattern and magnitude response of the system.

(b) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}(1 + z^{-2})$$

The magnitude response is:

$$|H(\mathrm{e}^{\mathrm{j}\omega})| = |\cos\omega|$$

The phase response is:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) = -\omega$$

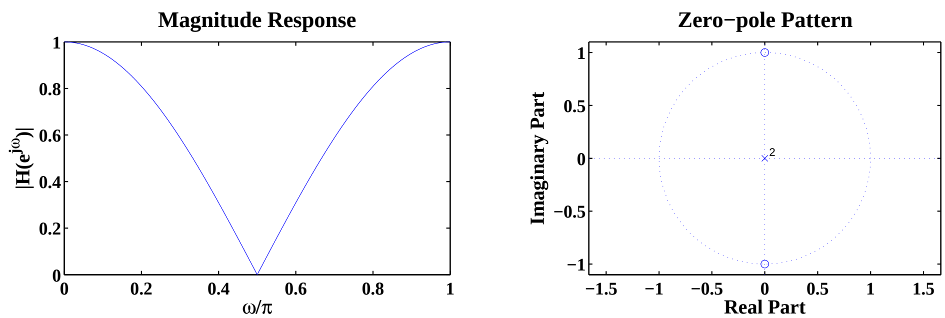


FIGURE 5.36: Pole-zero pattern and magnitude response of the system.

(c) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 - z^{-1} + z^{-2} - z^{-3})$$

The magnitude response is:

$$|H(e^{j\omega})| = \frac{1}{4}\sqrt{4 - 6\cos\omega + 4\cos 2\omega - 2\cos 3\omega}$$

The phase response is:

$$\angle H(e^{j\omega}) = -\tan^{-1}\frac{\sin 2\omega(2\cos\omega - 1)}{1 + \cos 2\omega(1 - 2\cos\omega)}$$

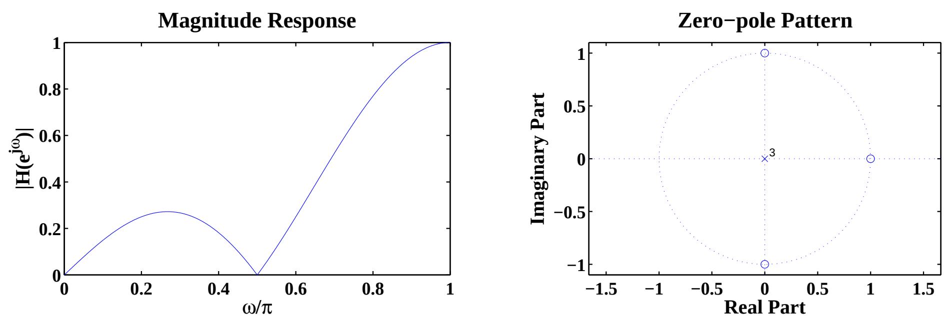


FIGURE 5.37: Pole-zero pattern and magnitude response of the system.

(d) Solution:
The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 - z^{-1} + z^{-3} - z^{-4})$$

The magnitude response is:

$$|H(e^{j\omega})| = \frac{1}{4}\sqrt{4 - 4\cos\omega - 2\cos 2\omega + 4\cos 3\omega - 2\cos 4\omega}$$

The phase response is:

$$\angle H(e^{j\omega}) = \tan^{-1}\frac{\sin\omega - \sin 3\omega + \sin 4\omega}{1 - \cos\omega + \cos 3\omega - \cos 4\omega}$$

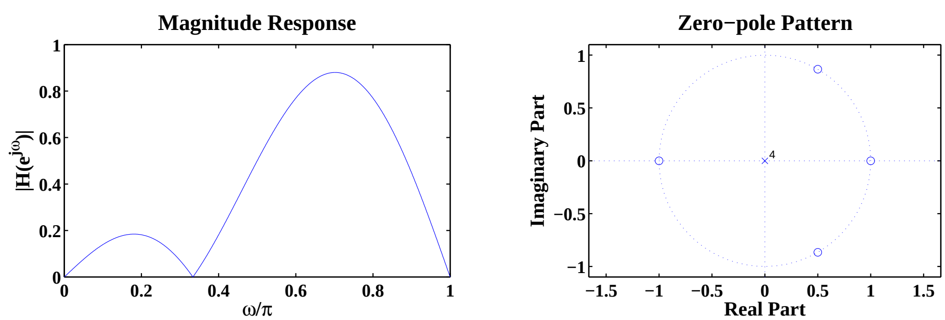


FIGURE 5.38: Pole-zero pattern and magnitude response of the system.

37. (a) Solution:

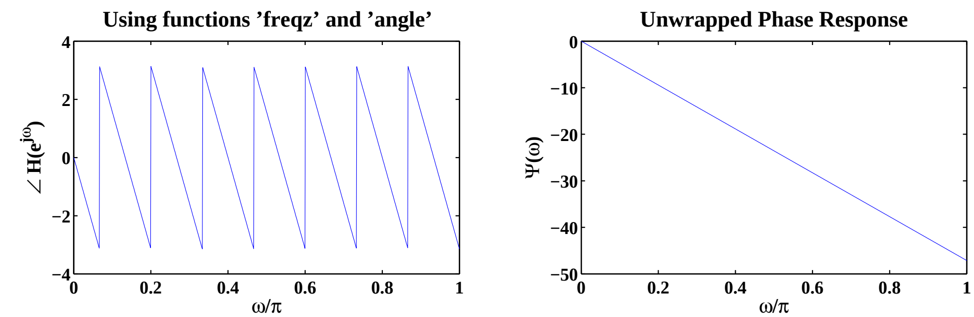


FIGURE 5.39: Phase response of pure delay $y[n] = x[n - 15]$ using the functions freqz, angle and unwrap.

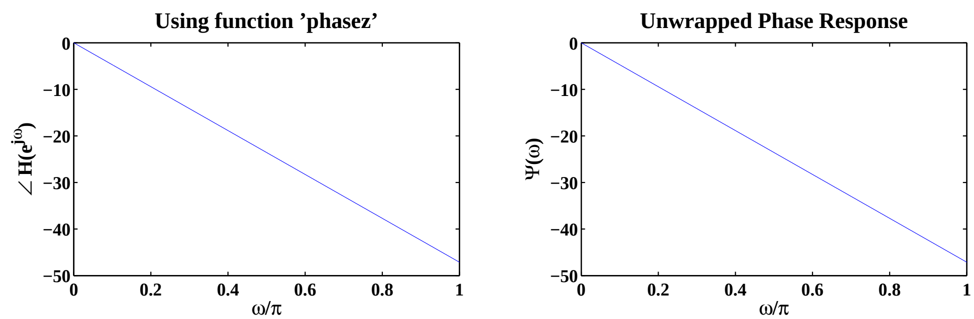


FIGURE 5.40: Phase response of pure delay $y[n] = x[n - 15]$ using the function phasez.

(b) Solution:

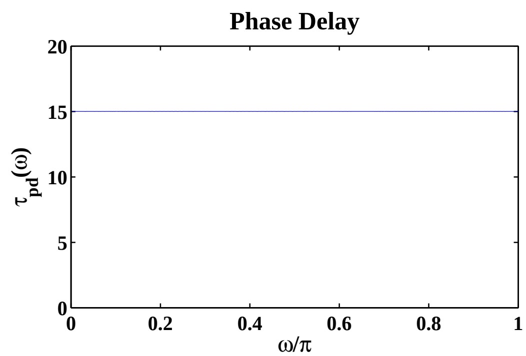


FIGURE 5.41: Phase delay of pure delay $y[n] = x[n-15]$ using the function phasedelay.

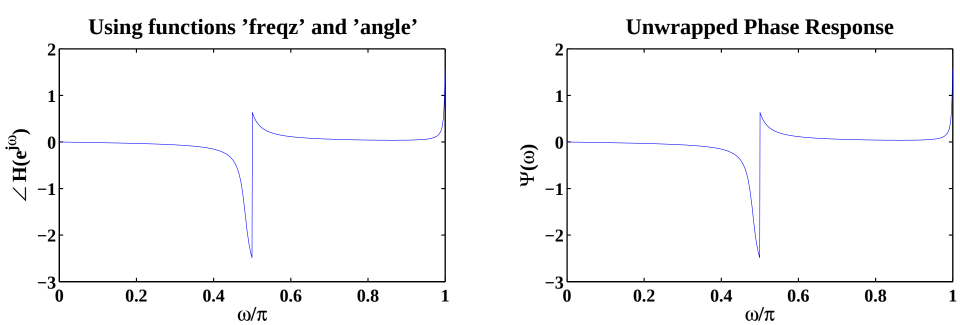


FIGURE 5.42: Phase response of $H(z)$ using the functions freqz, angle and unwrap.

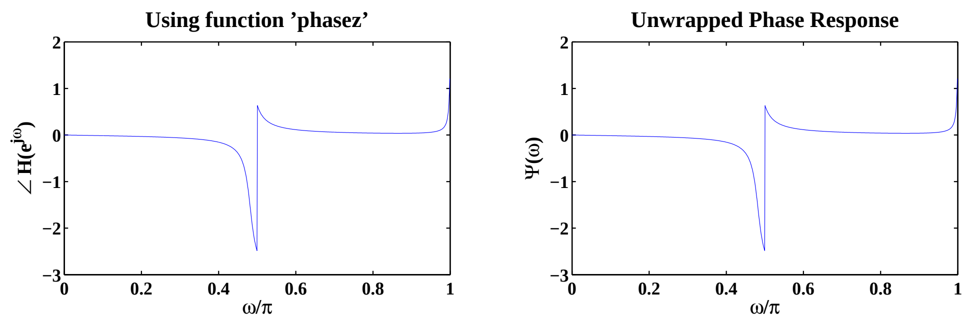


FIGURE 5.43: Phase response of $H(z)$ using the function phasez.

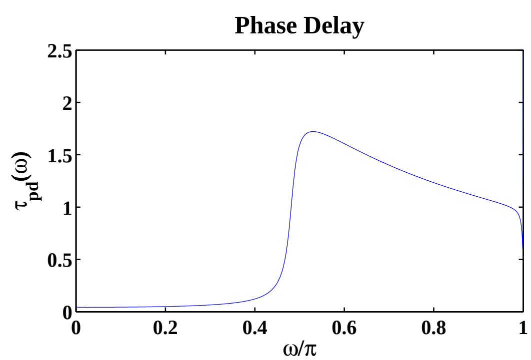


FIGURE 5.44: Phase delay of $H(z)$ using the function phasedelay.

38. (a) Solution:

$$\text{zeros:} \quad z_1 = \mathrm{e}^{\mathrm{j}0} = 1, \quad z_2 = \mathrm{e}^{\mathrm{j}\pi} = -1$$

$$\text{poles:} \quad p_1 = r\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}, \quad p_2 = r\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}, \quad r \in (0, 1)$$

The system function is:

$$H(z) = b_0 \frac{(1 - z^{-1})(1 + z^{-1})}{(1 - r\mathrm{e}^{\mathrm{j}\frac{\pi}{4}} z^{-1})(1 - r\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}} z^{-1})}$$

Choose $r = 0.95$.

(b) See plot below.

(c) See plot below.

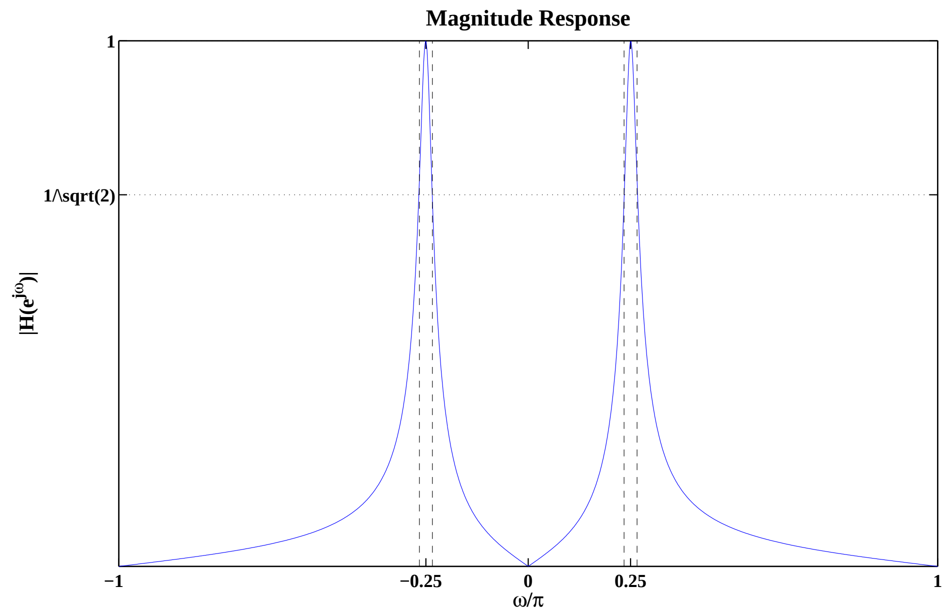


FIGURE 5.45: Magnitude response of the filter.

39. (a) Solution:

$$|b_0| = \frac{1}{\sqrt{1 + 4r^2\cos^2\phi + r^4 + 4r^2\cos\phi + 4r\cos\phi + 2r^2}}$$

(b) See plot below.

(c) Solution:

$$|b_0| = \frac{1}{\prod_{k=-1}^{1} 2(1+\cos\phi_k)}$$

(d) Solution:

$$|b_0| = \frac{1}{\prod_{k=-2}^{2} 2(1+\cos\phi_k)}$$

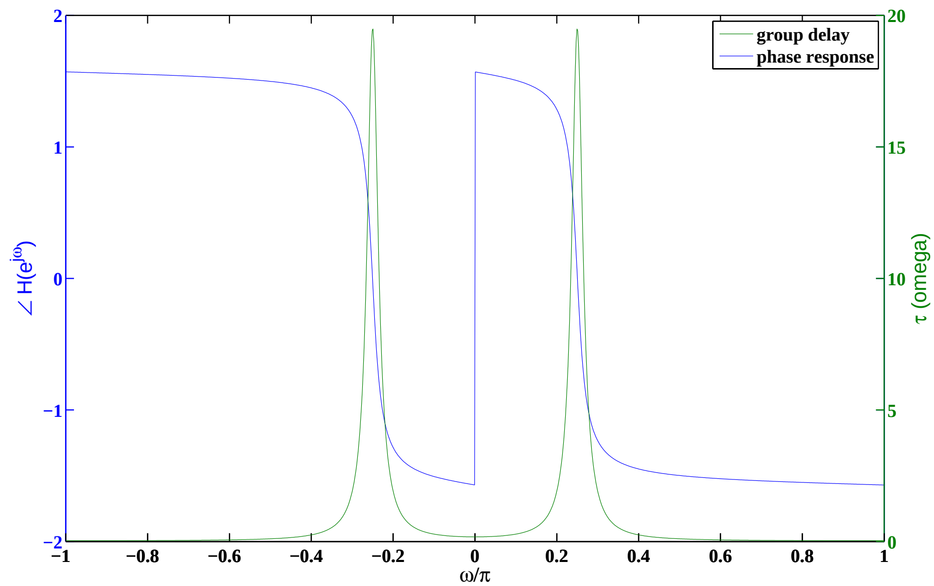


FIGURE 5.46: Phase and group-delay responses of the filter.

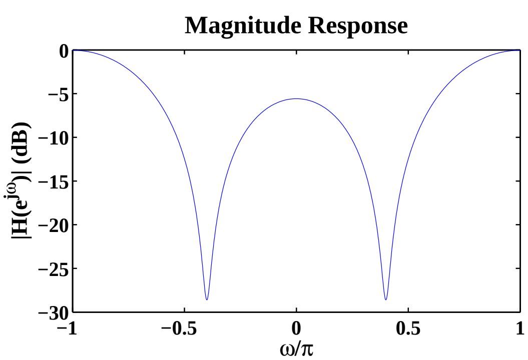


FIGURE 5.47: Magnitude response of the notch filter for $r = 0.95$.

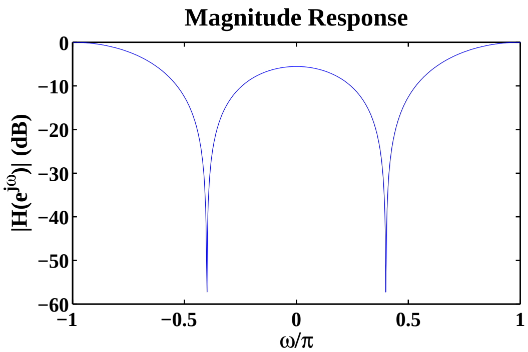


FIGURE 5.48: Magnitude response of the notch filter for $r = 1$.

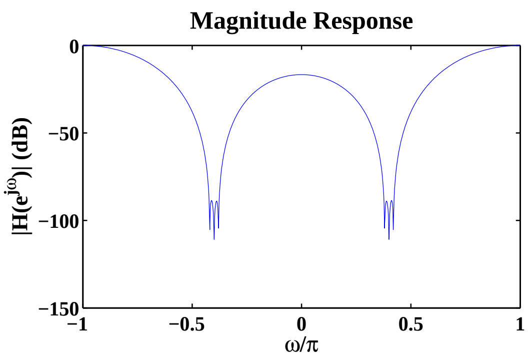


FIGURE 5.49: Magnitude response of cascade of three FIR notch filters for $r = 0.95$.

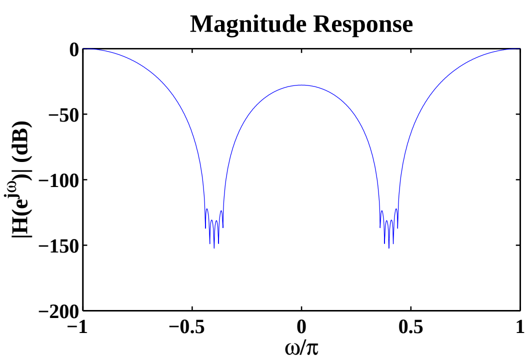


FIGURE 5.50: Magnitude response of cascade of five FIR notch filters for $r = 0.95$.

40. (a) Solution:

$$\text{zeros:} \quad z_1 = \mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}, \quad z_2 = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}$$

$$\text{poles:} \quad p_1 = r\mathrm{e}^{\mathrm{j}(\frac{2\pi}{3}+\phi)}, \quad p_2 = r\mathrm{e}^{-\mathrm{j}(\frac{2\pi}{3}+\phi)}, \quad r \in (0,1)$$

The system function is:

$$H(z) = b_0 \frac{(1-\mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}z^{-1})(1-\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}z^{-1})}{(1-r\mathrm{e}^{\mathrm{j}(\frac{2\pi}{3}+\phi)}z^{-1})(1-r\mathrm{e}^{-\mathrm{j}(\frac{2\pi}{3}+\phi)}z^{-1})}$$

Choose $r = 0.9, \phi = 0.01$.

(b) See plot below.

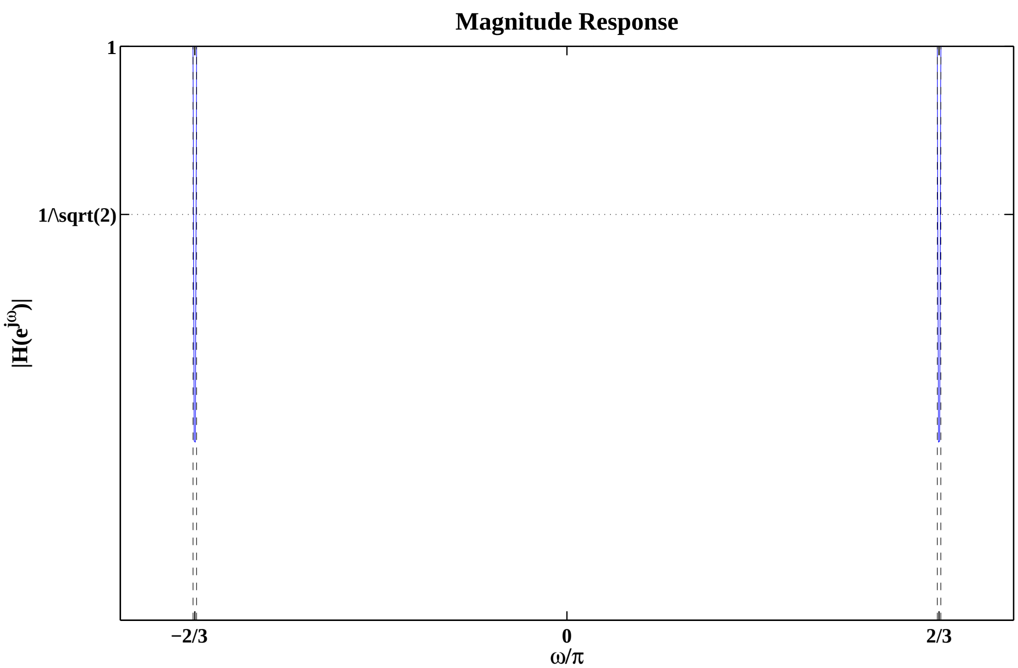


FIGURE 5.51: Magnitude response of the filter.

(c) See plot below.

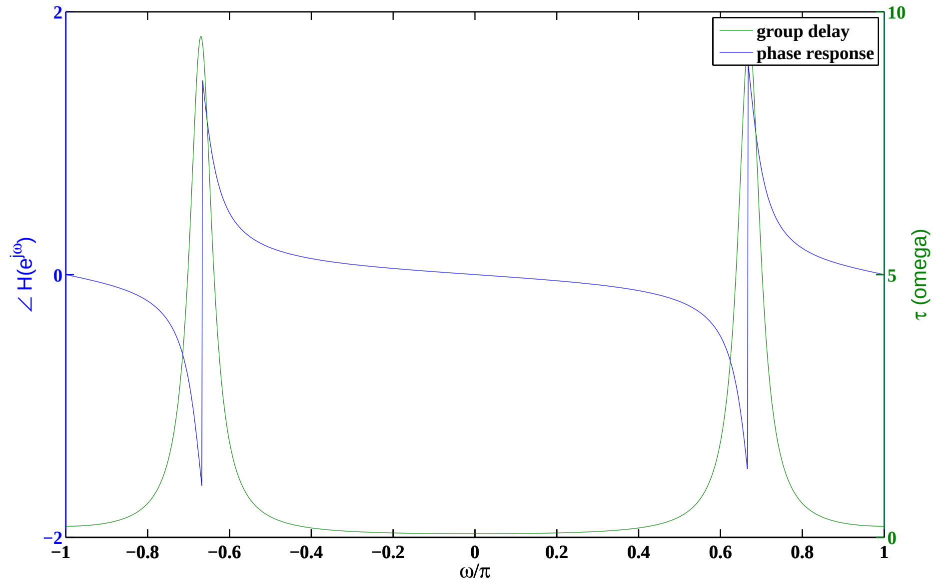


FIGURE 5.52: Phase and group-delay responses of the filter.

41. tba

42. (a) Solution:

$$H(z) = H_{\min}(z) \cdot H_{\text{ap}}(z)$$

$$H_{\min}(z) = \frac{1 + 5.6569z^1 + 16z^2}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$H_{\text{ap}}(z) = \frac{1 + 5.6569z^{-1} + 16z^{-2}}{1 + 5.6569z^1 + 16z^2}$$

(b) See plot below

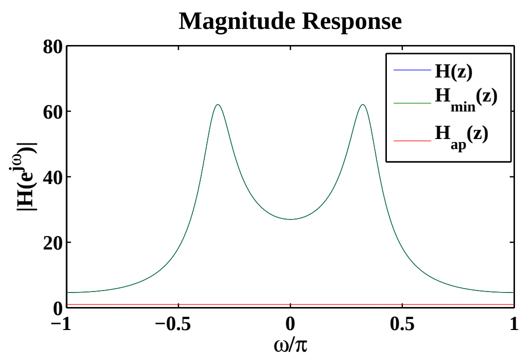


FIGURE 5.53: Magnitude responses of $H(z)$ and its minimum-phase and all-pass components .

(c) See plot below.

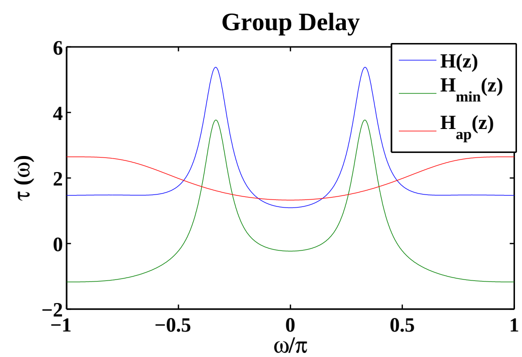


FIGURE 5.54: Group-delays of $H(z)$ and its minimum-phase and all-pass components .

43. (a) Solution

The system function is:

$$H(s) = \frac{10s}{s^2 + 2s + 101}$$

(b) Solution:

$$\begin{aligned} y(t) = {} & 5H(0) - 2\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}\mathrm{e}^{\mathrm{j}10t}H(10) - 2\mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}\mathrm{e}^{-\mathrm{j}10t}H(-10) \\ & + \frac{3}{2\mathrm{j}}\mathrm{e}^{\mathrm{j}20t}H(20) - \frac{3}{2\mathrm{j}}\mathrm{e}^{-\mathrm{j}20t}H(-20) + 2\mathrm{e}^{-\mathrm{j}100t}H(-100) \end{aligned}$$

where

$H(0) = 0, H(-10) = -0.5525, H(10) = 0.4525, H(-20) = -0.4338,$
$H(20) = 0.3697, H(-100) = -0.1010$

44. (a) Solution:

$$H_{\max}(\mathrm{j}\Omega) = \frac{\Omega^2 - 3.4641\Omega + 4}{\Omega^2 - 4\Omega + 5}$$

$$H_{\min}(\mathrm{j}\Omega) = \frac{\Omega^2 + 3.4641\Omega + 4}{\Omega^2 + 4\Omega + 5}$$

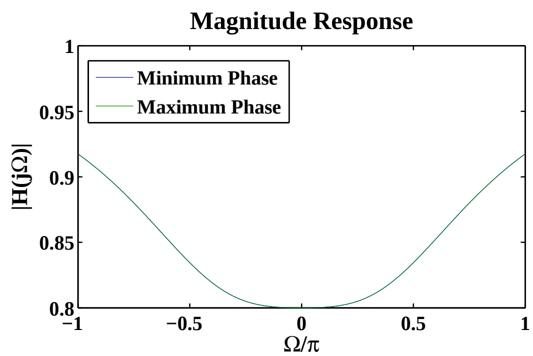


FIGURE 5.55: Magnitude responses of he minimum-phase and maximum-phase system components.

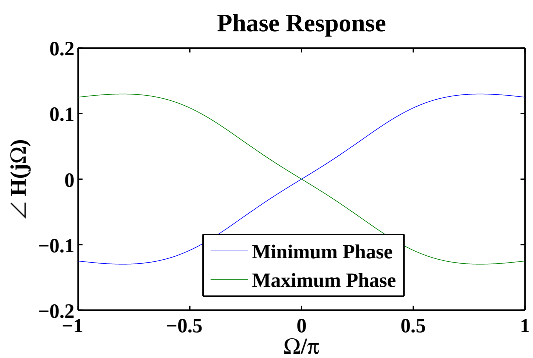


FIGURE 5.56: Phase responses of he minimum-phase and maximum-phase system components.

(b)