CHAPTER 10

Design of FIR Filters

Tutorial Problems

1. (a) Solution:
   The relative specifications are:

$$A_{\mathrm{p}} = 20\log_{10}\left(\frac{1+\delta_{\mathrm{p}}}{1-\delta_{\mathrm{p}}}\right) = 0.1737\mathrm{dB}$$

$$A_{\mathrm{s}} = 20\log_{10}\left(\frac{1+\delta_{\mathrm{p}}}{\delta_{\mathrm{s}}}\right) = 60.0864\mathrm{dB}$$

   The analog filter specifications are:

$$\epsilon = \sqrt{10^{(-0.1A_{\mathrm{p}})}-1} = 0.2020$$

$$A = 10^{(0.05A_{\mathrm{s}})} = 1010$$

   (b) Solution:
   The relative specifications are:

$$A_{\mathrm{p}} = 20\log_{10}(\sqrt{1+\epsilon^2}) = 0.2633\mathrm{dB}$$

$$A_{\mathrm{s}} = 20\log_{10} A = 46.0206\mathrm{dB}$$

   The absolute specifications are:

$$A_{\mathrm{p}} = 20\log_{10}\left(\frac{1+\delta_{\mathrm{p}}}{1-\delta_{\mathrm{p}}}\right) \implies \delta_{\mathrm{p}} = 0.0152$$

$$A_{\mathrm{s}} = 20\log_{10}\left(\frac{1+\delta_{\mathrm{p}}}{\delta_{\mathrm{s}}}\right) \implies \delta_{\mathrm{s}} = 2.4395\times 10^{-4}$$

2. Proof:

$$h[n] = 2h_e[n]u[n] - h_e[0]\delta[n] \tag{10.14}$$

$$H_I(\mathrm{e}^{\mathrm{j}\omega}) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} H_R(\mathrm{e}^{\mathrm{j}\theta})\cot\left(\frac{\omega-\theta}{2}\right)\mathrm{d}\theta \tag{10.16}$$

First, we have

$$\frac{1}{2}\cot(\frac{x}{2}) = \frac{1}{x} + \sum_{n=1}^{\infty}\frac{1}{x+2n\pi} - \frac{1}{2n\pi}$$

$$\mathrm{DTFT}(u[n]) = U(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1}{1-\mathrm{e}^{-\mathrm{j}\omega}} + \sum_{k=-\infty}^{\infty}\pi\delta(\omega - 2\pi k)$$

Hence,

$$H(\mathrm{e}^{\mathrm{j}\omega}) = 2\times\frac{1}{2\pi}\int_{-\pi}^{\pi} H_R(\mathrm{e}^{\mathrm{j}\theta})U(\mathrm{e}^{\mathrm{j}(\omega-\theta)})\mathrm{d}\theta - h[0]$$

where

$$\begin{aligned} U(\mathrm{e}^{\mathrm{j}\omega}) &= \sum_{k=-\infty}^{\infty}\pi\delta(\omega-2\pi k) + \frac{1}{1-\mathrm{e}^{\mathrm{j}\omega}} \\ &= \sum_{k=-\infty}^{\infty}\pi\delta(\omega-2\pi k) + \frac{1}{2} - \frac{\mathrm{j}}{2}\cot(\frac{\omega}{2}) \end{aligned}$$

$$\begin{aligned} X(\mathrm{e}^{\mathrm{j}\omega}) &= X_R(\mathrm{e}^{\mathrm{j}\omega}) + \mathrm{j}X_I(\mathrm{e}^{\mathrm{j}\omega}) \\ &= X_R(\mathrm{e}^{\mathrm{j}\omega}) + \frac{1}{2\pi}\int_{-\pi}^{\pi} X_R(\mathrm{e}^{\mathrm{j}\theta})\mathrm{d}\theta - \frac{1}{2\pi}\int_{-\pi}^{\pi} X_R(\mathrm{e}^{\mathrm{j}\theta})\mathrm{d}\theta \\ &\quad - \frac{\mathrm{j}}{2\pi}\int_{-\pi}^{\pi} X_R(\mathrm{e}^{\mathrm{j}\theta})\cot\left(\frac{\omega-\theta}{2}\right)\mathrm{d}\theta \end{aligned}$$

Hence, we proved that

$$H_I(\mathrm{e}^{\mathrm{j}\omega}) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} H_R(\mathrm{e}^{\mathrm{j}\theta})\cot\left(\frac{\omega-\theta}{2}\right)\mathrm{d}\theta$$

3. (a) Proof:

$$H(e^{j\omega}) = \sum_{k=1}^{(M+1)/2} b[k]\cos[\omega(k-\frac{1}{2})]\cdot e^{-j\omega M/2} \triangleq A(e^{j\omega})\cdot e^{-j\omega M/2} \tag{10.29}$$

$$b[k] = 2h[(M+1)/2-k], \quad k=1,2,\ldots,(M+1)/2 \tag{10.30}$$

$$\begin{aligned}
H(e^{j\omega}) &= \sum_{k=0}^{M} h[k]\cdot e^{-jk\omega} = \sum_{k=0}^{\frac{M-1}{2}} h[k]e^{-jk\omega} + \sum_{k=\frac{M+1}{2}}^{M} h[k]e^{-jk\omega} \\
&= \sum_{k=0}^{\frac{M-1}{2}} \left(h[k]e^{-jk\omega} + h[k+\frac{M+1}{2}]e^{-j(\frac{M+1}{2}+k)\omega} \right) \\
&= \sum_{k=0}^{\frac{M-1}{2}} \left(h[k]e^{j\omega(\frac{M}{2}-k)} + h[k+\frac{M+1}{2}]e^{-j(\frac{1}{2}+k)\omega} \right)\cdot e^{-j\omega M/2} \\
&= \sum_{k=0}^{\frac{M-1}{2}} \left(h[k]e^{j\omega(\frac{M}{2}-k)} + h[\frac{M-1}{2}-k]e^{-j(\frac{1}{2}+k)\omega} \right)\cdot e^{-j\omega M/2} \\
&= \sum_{k=0}^{\frac{M-1}{2}} \left(h[\frac{M-1}{2}-k]e^{j\omega(\frac{1}{2}+k)} + h[\frac{M-1}{2}-k]e^{-j(\frac{1}{2}+k)\omega} \right)\cdot e^{-j\omega M/2} \\
&= \left(\sum_{k=0}^{\frac{M-1}{2}} 2h[\frac{M-1}{2}-k]\cos\omega(k+\frac{1}{2}) \right)\cdot e^{-j\omega M/2} \\
&= \left(\sum_{k=1}^{\frac{M+1}{2}} 2h[\frac{M+1}{2}-k]\cos\omega(k-\frac{1}{2}) \right)\cdot e^{-j\omega M/2}
\end{aligned}$$

(b) Proof:

$$A(e^{j\omega}) = \cos(\frac{\omega}{2}) \sum_{k=0}^{(M-1)/2} \tilde{b}[k]\cos\omega k \tag{10.31}$$

$$b[k] = \begin{cases} \frac{1}{2}(\tilde{b}[1]+2\tilde{b}[0]), & k=1 \\ \frac{1}{2}(\tilde{b}[k]+\tilde{b}[k-1]), & 2\le k\le (M-1)/2 \\ \frac{1}{2}\tilde{b}[(M-1)/2], & k=(M+1)/2 \end{cases} \tag{10.32}$$

$$
\begin{aligned}
A(e^{j\omega}) &= \cos(\frac{\omega}{2}) \sum_{k=0}^{(M-1)/2} \tilde{b}[k] \cos \omega k \\
&= \frac{1}{2} \sum_{k=0}^{(M-1)/2} \tilde{b}[k] \left[\cos \omega (k + \frac{1}{2}) + \cos \omega (k - \frac{1}{2}) \right] \\
&= \sum_{k=0}^{(M-1)/2} \left(\frac{1}{2} \tilde{b}[k] \cos \omega (k + \frac{1}{2}) + \frac{1}{2} \tilde{b}[k] \cos \omega (k - \frac{1}{2}) \right) \\
&= (\tilde{b}[0] + \frac{1}{2} \tilde{b}[1]) \cos \frac{\omega}{2} + \sum_{k=2}^{(M-1)/2} \frac{1}{2} (\tilde{b}[k] + \tilde{b}[k-1]) \cos \omega (k - \frac{1}{2}) \\
&\quad + \frac{1}{2} \tilde{b}[(M-1)/2] \cos \frac{M}{2} \omega
\end{aligned}
$$

4. (a) Solution:

The DTFT of $h[n]$ is:

$$
\begin{aligned}
H(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} = \sum_{n=0}^{3} e^{-j\omega n} = 1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega} \\
&= (1 + \cos \omega + \cos 2\omega + \cos 3\omega) - j(\sin \omega + \sin 2\omega + \sin 3\omega)
\end{aligned}
$$

Hence, the magnitude response is:

$$
|H(e^{j\omega})| = \sqrt{(1 + \cos \omega + \cos 2\omega + \cos 3\omega)^2 + (\sin \omega + \sin 2\omega + \sin 3\omega)^2}
$$

(b) Solution:

$$
\begin{aligned}
A(e^{j\omega}) &= \sum_{k=1}^{2} b[k] \cos[\omega(k - \frac{1}{2})], \quad b[k] = 2h[2-k] \\
A(e^{j\omega}) &= b[1] \cos \frac{1}{2}\omega + b[2] \cos \frac{3}{2}\omega = 2 \cos \frac{1}{2}\omega + 2 \cos \frac{3}{2}\omega
\end{aligned}
$$

(c) Solution:

$$
\angle H(e^{j\omega}) = -\tan^{-1} \frac{\sin \omega + \sin 2\omega + \sin 3\omega}{1 + \cos \omega + \cos 2\omega + \cos 3\omega}
$$

(d) Solution:

$$
\Psi(e^{j\omega}) = -\omega M/2 = -\frac{3}{2}\omega
$$

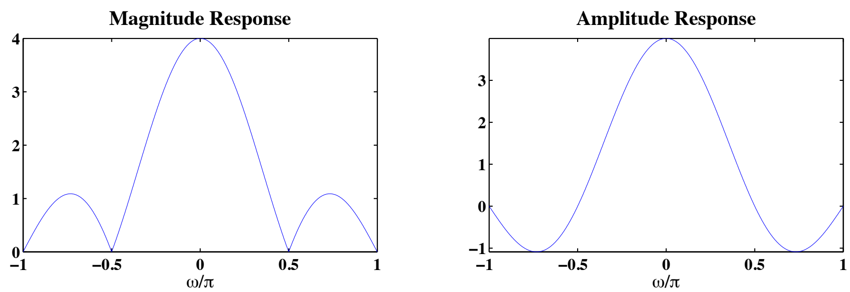

FIGURE 10.1: Plots of magnitude and amplitude responses.

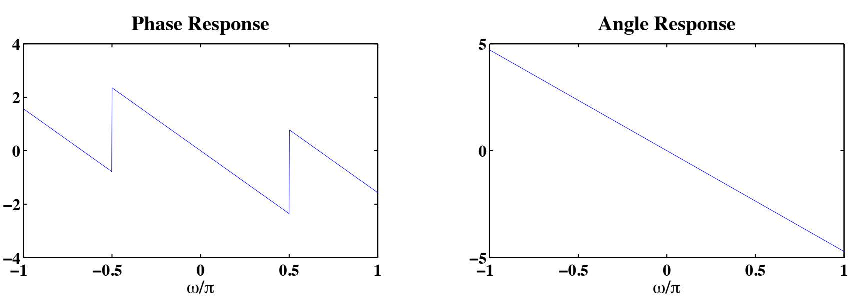

FIGURE 10.2: Plots of phase and angle responses.

5. Proof:
   Part I: Prove expression for $A\big(\mathrm{e}^{\mathrm{j}\omega}\big)$ for $M = 6$ type-III linear-phase FIR filter.
   We have

$$h[n] = -h[M-n], \quad 0 \le n \le M, \quad M = 6, \quad h[3] = 0$$

$$\begin{aligned}
H\big(\mathrm{e}^{\mathrm{j}\omega}\big) &= \sum_{k=0}^{6} h[k] \cdot \mathrm{e}^{-\mathrm{j}k\omega} = \sum_{k=0}^{3} h[k]\mathrm{e}^{-\mathrm{j}\omega k} - h[3]\mathrm{e}^{-\mathrm{j}\omega 3} + \sum_{k=3}^{6} h[k]\mathrm{e}^{-\mathrm{j}k\omega} \\
&= \sum_{k=0}^{3} \Big(h[k]\mathrm{e}^{-\mathrm{j}\omega k} + h[k+3]\mathrm{e}^{-\mathrm{j}\omega(k+3)}\Big) \\
&= \sum_{k=0}^{3} \Big(h[k]\mathrm{e}^{-\mathrm{j}\omega(k-3)} + h[k+3]\mathrm{e}^{-\mathrm{j}\omega k}\Big) \cdot \mathrm{e}^{-\mathrm{j}\omega 3} \\
&= \sum_{k=0}^{3} \Big(h[3-k]\mathrm{e}^{\mathrm{j}\omega k} - h[6-k-3]\mathrm{e}^{-\mathrm{j}\omega k}\Big) \cdot \mathrm{e}^{-\mathrm{j}\omega 3} \\
&= \sum_{k=0}^{3} (h[3-k] \cdot 2\mathrm{j}\sin\omega k) \cdot \mathrm{e}^{-\mathrm{j}\omega 3} \\
&= \sum_{k=1}^{3} (h[3-k] \cdot 2\sin\omega k) \cdot \mathrm{j}\mathrm{e}^{-\mathrm{j}\omega 3} + 2h[3]\,\mathrm{j}\sin(0\omega)\mathrm{e}^{-\mathrm{j}3\omega} \\
&= \left(\sum_{k=1}^{3} c[k] \cdot \sin\omega k\right) \cdot \mathrm{e}^{\mathrm{j}(\frac{\pi}{2}-3\omega)}
\end{aligned}$$

Part II: Prove expression for $A\left(e^{j\omega}\right)$ for $M = 5$ type-IV linear-phase FIR filter.
We have

$$h[n] = -h[M-n], \quad M = 5$$

$$\begin{aligned}
A\left(e^{j\omega}\right) &= \sum_{k=0}^{5} h[k] \cdot e^{-j\omega k} = \sum_{k=0}^{2} \left(h[k]e^{-j\omega k} + h[k+3]e^{-j\omega(k+3)}\right) \\
&= \sum_{k=0}^{2} \left(h[k]e^{-j\omega(k-\frac{5}{2})} + h[k+3]e^{-j\omega(k+\frac{1}{2})}\right) \cdot e^{-j\omega\frac{5}{2}} \\
&= \sum_{k=1}^{3} \left(h[3-k]e^{j\omega(k-\frac{1}{2})} + h[3-k]e^{-j\omega(k+\frac{1}{2})}\right) \cdot e^{-j\omega\frac{5}{2}} \\
&= \sum_{k=1}^{3} \left(2h[3-k]\,j\sin[\omega(k-\frac{1}{2})]\right) \cdot e^{-j\omega\frac{5}{2}} \\
&= \left(\sum_{k=1}^{3} 2h[3-k]\sin[\omega(k-\frac{1}{2})]\right) \cdot je^{-j\omega\frac{5}{2}} \\
&= \left(\sum_{k=1}^{3} d[k]\sin[\omega(k-\frac{1}{2})]\right) e^{j(\frac{\pi}{2}-\frac{5}{2}\omega)}
\end{aligned}$$

6. (a) See plot below.

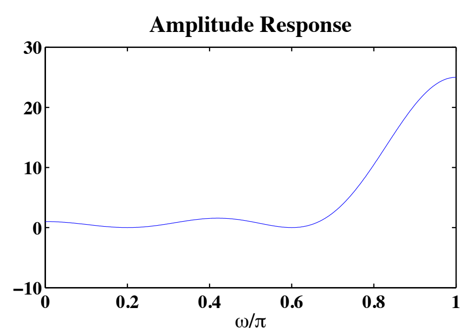

FIGURE 10.3: Plot of amplitude response from $h_1[n]$.

(b) See plot below.
(c) See plot below.
(d) See plot below.

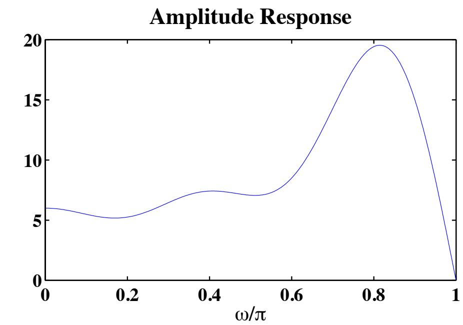

FIGURE 10.4: Plot of amplitude response from $h_2[n]$.

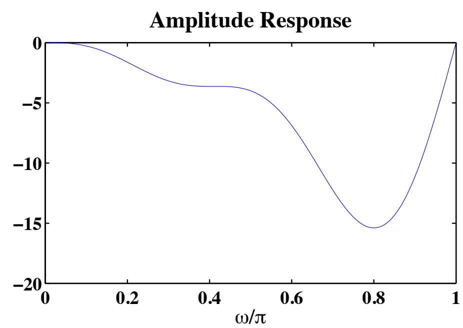

FIGURE 10.5: Plot of amplitude response from $h_3[n]$.

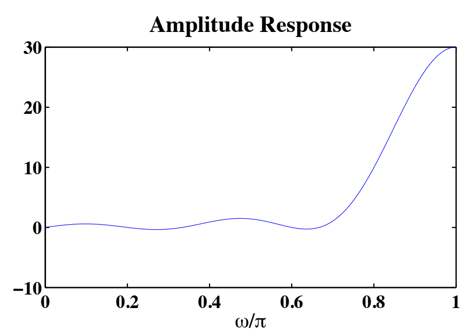

FIGURE 10.6: Plot of amplitude response from $h_4[n]$.

7. (a) See plot below.

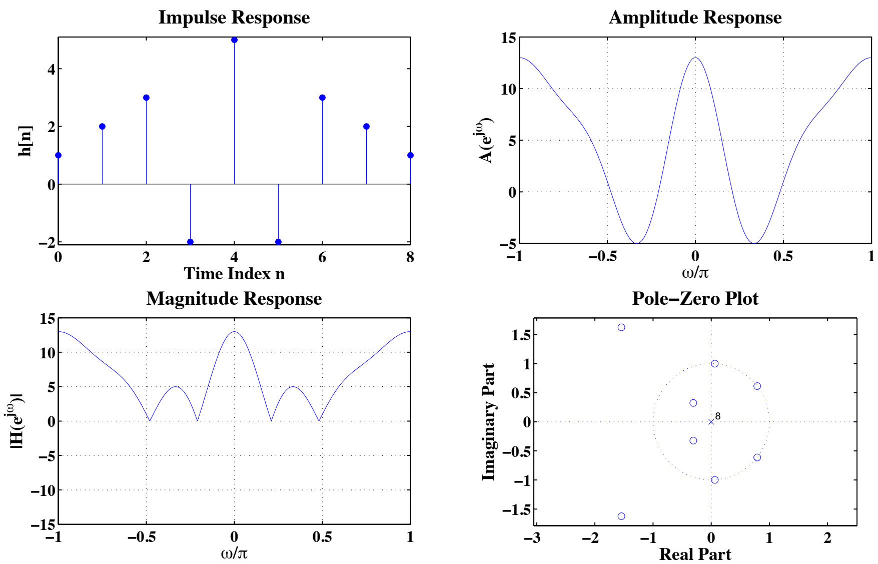

FIGURE 10.7: Plots of impulse response, amplitude response, magnitude response and pole-zero distribution in part (a).

(b) See plot below.

(c) See plot below.

(d) See plot below.

(e) tba.

MATLAB script:

```
% P1007: Reproduce Figures 10.4 and 10.5
close all; clc
%% Part a: Type-I
% hn = [1 2 3 -2 5 -2 3 2 1];
% w = linspace(-1,1,1000)*pi;

%% Part b: Type-II
% hn = [1 2 3 -2 -2 3 2 1];
% w = linspace(-2,2,1000)*pi;

%% Part c: Type-III
% hn = [1 2 3 -2 0 2 -3 -2 -1];
```

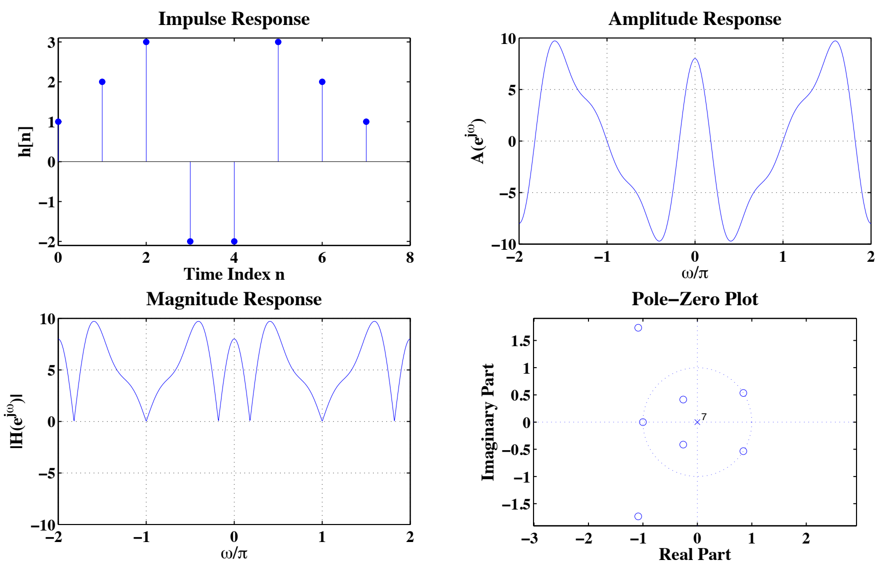

FIGURE 10.8: Plots of impulse response, amplitude response, magnitude response and pole-zero distribution in part (b).

```
% w = linspace(-1,1,1000)*pi;

%% Part d: Type-IV
hn = [1 2 3 -2 2 -3 -2 -1];
w = linspace(-2,2,1000)*pi;

H = freqz(hn,1,w);
Hmag = abs(H);
Hangle = angle(H);
n = 0:length(hn)-1;
[Hr w P L] = amplresp(hn,w);
% Plot:
hfa = figconfg('P1007a','small');
stem(n,hn,'filled');
ylim([min([hn 0])-.1 max(hn)+.1])
xlabel('Time Index n','fontsize',LFS)
ylabel('h[n]','fontsize',LFS)
title('Impulse Response','fontsize',TFS)

hfb = figconfg('P1007b','small');
```

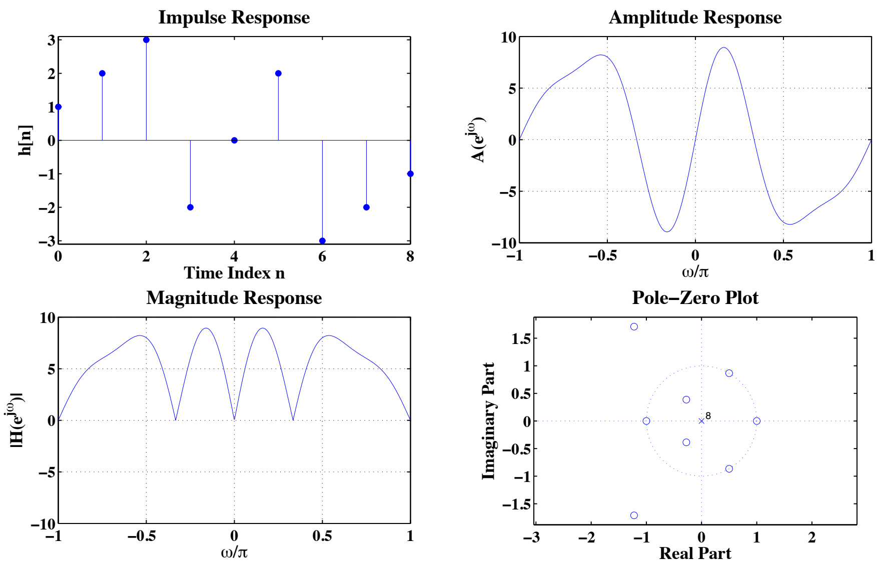

FIGURE 10.9: Plots of impulse response, amplitude response, magnitude response and pole-zero distribution in part (c).

```
zplane(hn,1)
xlabel('Real Part','fontsize',LFS)
ylabel('Imaginary Part','fontsize',LFS)
title('Pole-Zero Plot','fontsize',TFS)

hfc = figconfg('P1007c','small');
plot(w/pi,Hmag); grid on
yl = ylim;
ylim([-yl(2) yl(2)])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)

hfd = figconfg('P1007d','small');
plot(w/pi,Hr); grid on
xlabel('\omega/\pi','fontsize',LFS)
ylabel('A(e^{j\omega})','fontsize',LFS)
title('Amplitude Response','fontsize',TFS)
```

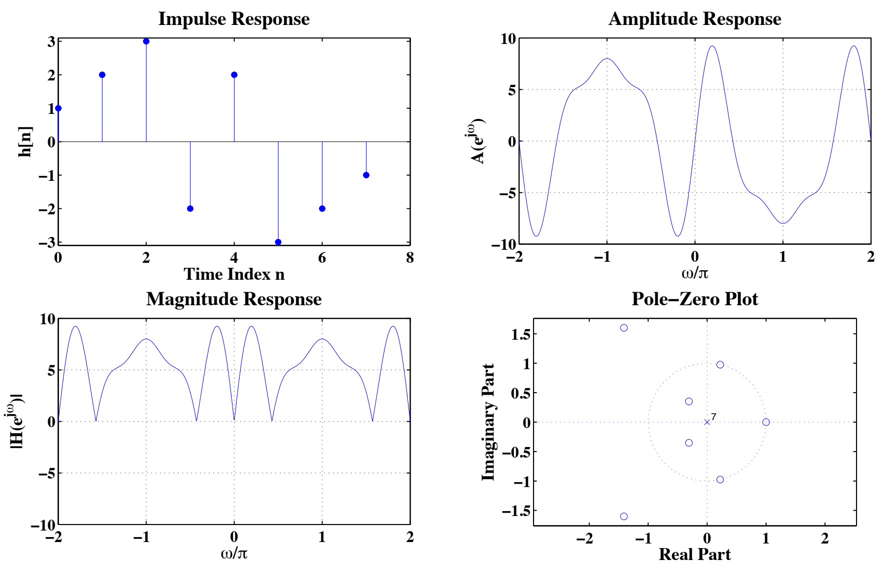

FIGURE 10.10: Plots of impulse response, amplitude response, magnitude response and pole-zero distribution in part (d).

8. (a) See plot below.

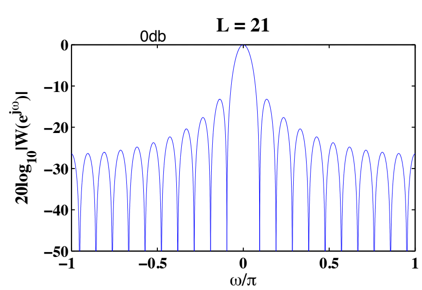

FIGURE 10.11: Plot of the log-magnitude response in dB over $-\pi \leq \omega \leq \pi$ when $L = 21$.

(b) See plot below.

(c) See plot below.

MATLAB script:

```
% P1008: Stude fixed window magnitude response
%        its peak of first side-lobe and transition bandwidth
```

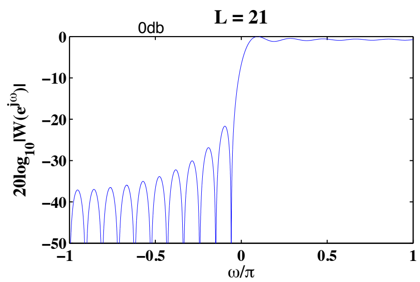

FIGURE 10.12: Plot of the accumulated amplitude response in dB over $-\pi \leq \omega \leq \pi$ when $L = 21$.

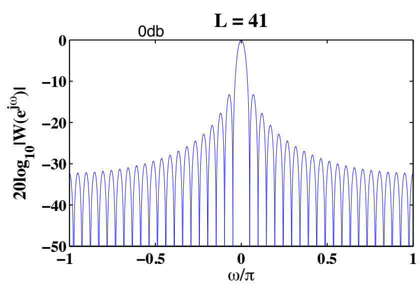

FIGURE 10.13: Plot of the log-magnitude response in dB over $-\pi \leq \omega \leq \pi$ when $L = 41$.

```
%        Rectangular Window
close all; clc
% L = 21; % Part a
L = 41; % Part b
hw = rectwin(L)';
Nw = 10000;
w = linspace(-1,1,Nw)*pi;
H = freqz(hw,1,w);
Hmag = abs(H);
Hmagdb = 20*log10(Hmag/max(Hmag));
[Ha w2 P2 L2] = amplresp(hw,w);
Hac = abs(cumsum(Ha));
Hacdb = 20*log10(Hac/max(Hac));

%% Find Peak Values:
[peakH peakHind] = findpeak(Hmagdb);
```

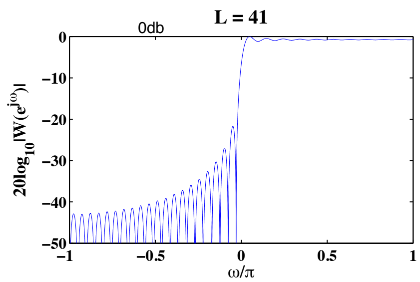

FIGURE 10.14: Plot of the accumulated amplitude response in dB over $-\pi \le \omega \le \pi$ when $L = 41$.

```
[peakHac peakHacind] = findpeak(Hacdb);
Lh = floor(length(peakH)/2);
sidlobeH = max(peakH(1:Lh));
sidlobeHac = max(peakHac(1:Lh));
bandwid = w(peakHacind(Lh+1)) - w(peakHacind(Lh));
bandwid/pi*L

%% Plot:
hfa = figconfg('P1008a','small');
plot(w/pi,Hmagdb);hold on
plot(w/pi,sidlobeH*ones(1,Nw),'--k')
text(w(Nw/5)/pi,sidlobeH,[num2str(sidlobeH,3),'db'],...
    'fontsize',LFS-2,'verticalalignment','bottom')
ylim([-50 0])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('20log_{10}|W(e^{j\omega})|','fontsize',LFS)
title(['L = ',num2str(L)],'fontsize',TFS)

hfb = figconfg('P1008b','small');
plot(w/pi,Hacdb); hold on
plot(w/pi,sidlobeHac*ones(1,Nw),'--k')
text(w(Nw/5)/pi,sidlobeHac,[num2str(sidlobeHac,3),'db'],...
    'fontsize',LFS-2,'verticalalignment','bottom')
ylim([-50 0])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('20log_{10}|W(e^{j\omega})|','fontsize',LFS)
title(['L = ',num2str(L)],'fontsize',TFS)
```

9.

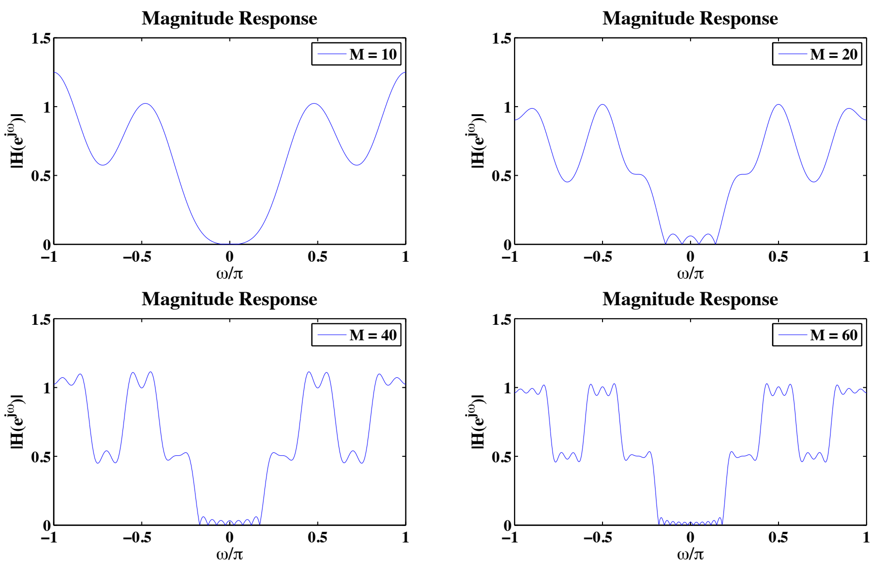

FIGURE 10.15: Amplitude plots of linear-phase FIR filter using the rectangular window of order $M = 10$, $M = 20$, $M = 40$ and $M = 60$.

MATLAB script:

```
% P1009: Design multiband filter using rectangular window
%        change window length for performance comparison
close all; clc
% M = 10;
% M = 20;
% M = 40;
M = 60;
h1 = 0.5*fir1(M,0.2,'high',rectwin(M+1));
h2 = 0.5*fir1(M,[0.4 0.6 0.8],'DC-0',rectwin(M+1));
h = h1 + h2;
w = linspace(-1,1,1000)*pi;
H = freqz(h,1,w);
%% Plot:
hfa = figconfg('P1008a','small');
plot(w/pi,abs(H));
ylim([0 1.5])
xlabel('\omega/\pi','fontsize',LFS)
```

```
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
legend(['M = ',num2str(M)],'location','northeast')
```

10. (a) See plot below.

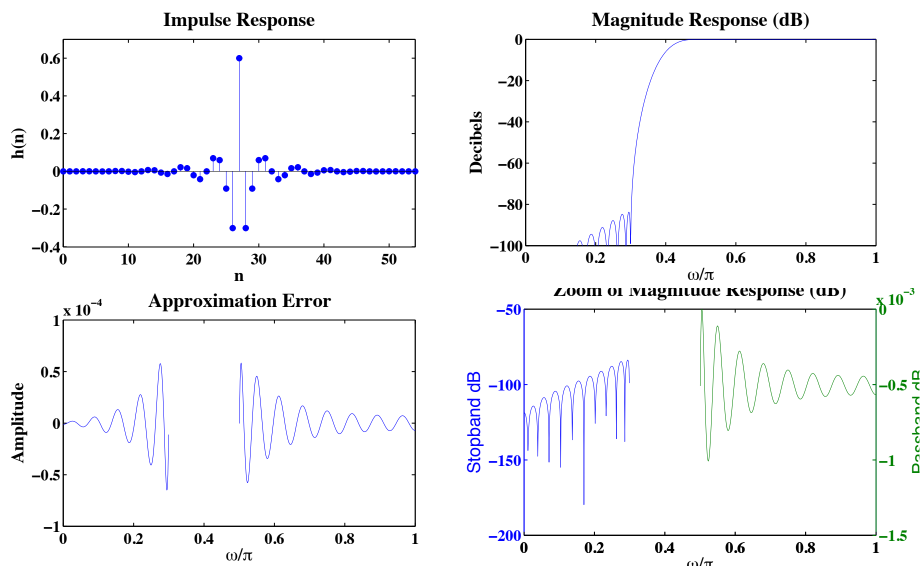

FIGURE 10.16: Impulse response, magnitude response, approximation error and zoom of magnitude response of the highpass FIR filter using fixed window design.

(b) See plot below.

MATLAB script:

```
% P1010: Design highpass filter using appropriate window
close all; clc
ws = 0.3*pi; wp = 0.5*pi;
As = 50; Ap = 0.001;
[deltap, deltas] = spec_convert(Ap,As,'rel','abs');
delta = min([deltap,deltas]);
A = -20*log10(delta);
[M,wn,beta,ftype] = kaiserord([0.3 0.5],[0 1],[deltas,deltap]);
%% Part (a)
wc = (ws+wp)/2;
% h = ideallp(pi,M) - ideallp(wc,M);
```

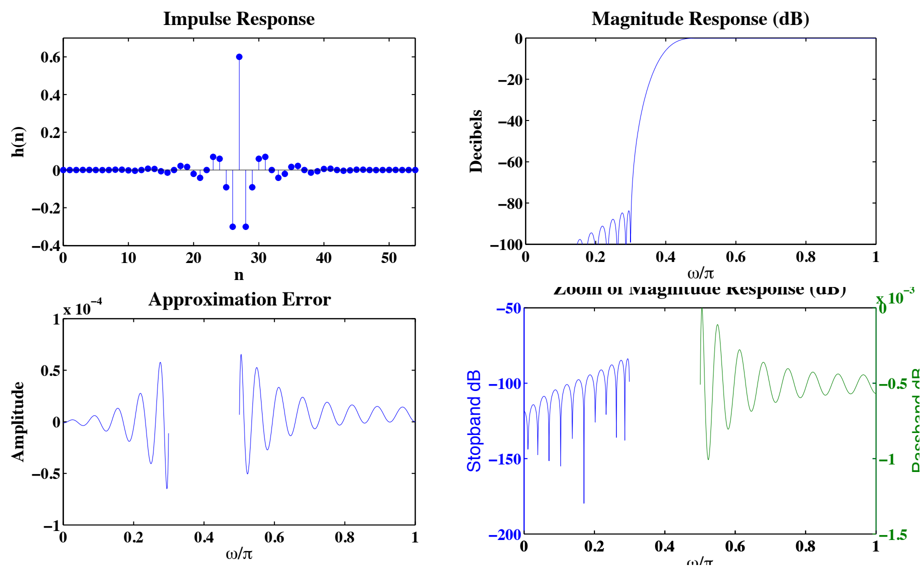

FIGURE 10.17: Impulse response, magnitude response, approximation error and zoom of magnitude response of the highpass FIR filter using fixed window design by fir1 function.

```
% h = h.*kaiser(M+1,beta);

%% Part (b)
h = fir1(M,wn,ftype,kaiser(M+1,beta));

w = linspace(0,1,1000)*pi;
H = freqz(h,1,w);
Hmag = abs(H);
Hdb = 20*log10(Hmag./max(Hmag));
[Ha w2 P2 L2] = amplresp(h(:)',w);
aerr = nan(1,length(w));
magz1 = nan(1,length(w));
magz2 = nan(1,length(w));
ind = w <= ws;
aerr(ind) = Ha(ind);
magz1(ind) = Hdb(ind);
ind = w >= wp;
aerr(ind) = Ha(ind) - 1;
magz2(ind) = Hdb(ind);
```

```
%% Plot:
hfa = figconfg('P1010a','small');
stem(0:M,h,'filled');
xlim([0 M])
ylim([min(h)-0.1 max(h)+0.1])
xlabel('n','fontsize',LFS)
ylabel('h(n)','fontsize',LFS)
title('Impulse Response','fontsize',TFS)

hfb = figconfg('P1010b','small');
plot(w/pi,Hdb);
ylim([-100 0])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Decibels','fontsize',LFS)
title('Magnitude Response (dB)','fontsize',TFS)

hfc = figconfg('P1010c','small');
plot(w/pi,aperr);
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Amplitude','fontsize',LFS)
title('Approximation Error','fontsize',TFS)

hfd = figconfg('P1010d','small');
[AX hf1 hf2] = plotyy(w/pi,magz1,w/pi,magz2);
xlabel('\omega/\pi','fontsize',LFS)
title('Zoom of Magnitude Response (dB)','fontsize',TFS)
set(get(AX(1),'Ylabel'),'string','Stopband dB','fontsize',LFS)
set(get(AX(2),'Ylabel'),'string','Passband dB','fontsize',LFS)
```

11. (a) See plot below.
    (b) See plot below.
    (c) See plot below.

    MATLAB script:

```
% P1011: Study Frequency Sampling Technique of
%        different number of samples
close all; clc
% L = 20; % Part a
L = 400; % Part b & c
```

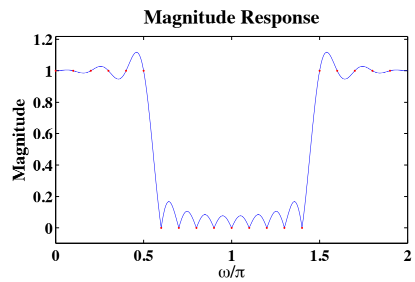

FIGURE 10.18: Magnitude response when $L = 20$ in part (a).

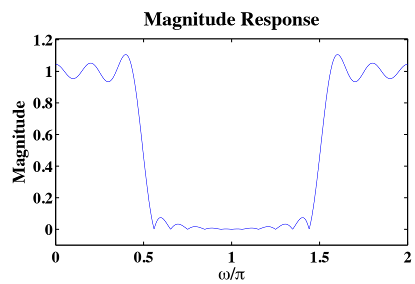

FIGURE 10.19: Magnitude response when $L = 400$ in part (b).

```
M = L - 1;
wc = pi/2;

Dw = 2*pi/L;
k1 = floor(wc/Dw);
Ad = [ones(1,k1+1),zeros(1,L-2*k1-1),ones(1,k1)];
alpha = M/2; Q = floor(alpha);
psid = -alpha*2*pi/L*[(0:Q),-(L-(Q+1:M))];
Hd = Ad.*exp(j*psid);
hd = real(ifft(Hd));
h = hd(L/2-9:L/2+10); % Part a & b
% h = hd(L/2-9:L/2+10).*hamming(20)'; % Part c

w = linspace(0,2,1000)*pi;
H = freqz(h,1,w);
Hmag = abs(H);

%% Plot:
```

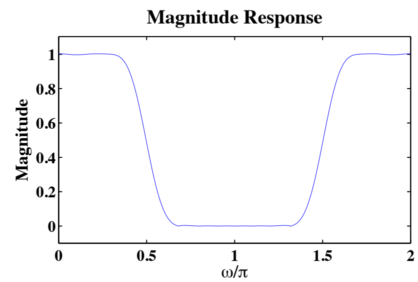

FIGURE 10.20: Magnitude response when $L = 400$ in part (c).

```
hf = figconfg('P1011','small');
plot(w/pi,Hmag);hold on
% plot((0:L-1)/L*2,Ad,'.r')
ylim([min(Hmag)-0.1 max(Hmag)+0.1])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Magnitude','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
```

12. (a) See plot below.

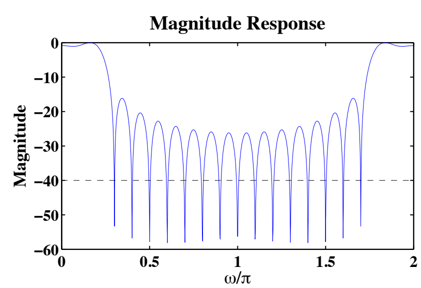

FIGURE 10.21: Magnitude response when $L = 20$ in part (a).

(b) See plot below.

(c) See plot below.

MATLAB script:

```
% P1012: Lowpass filter design by frequency sampling
close all; clc
% L = 20; % Part a
L = 40; % Part b & c
```

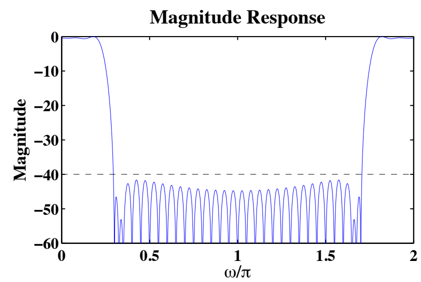

FIGURE 10.22: Magnitude response when $L = 40$ in part (b).

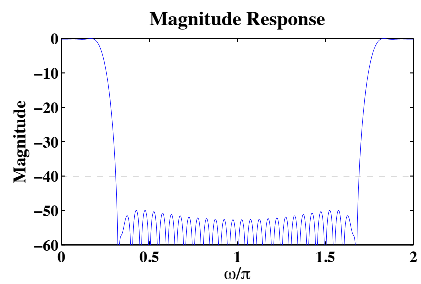

FIGURE 10.23: Magnitude response when $L = 40$ in part (c).

```
M = L - 1;
wp = 0.2*pi; ws = 0.3*pi; Ap = 0.2; As = 40;
wc = (wp+ws)/2;
Dw = 2*pi/L;
alpha = M/2; Q = floor(alpha);
psid = -alpha*2*pi/L*[(0:Q),-(L-(Q+1:M))];
T1 = 0.37897949;

%% Part a:
% k = floor(wc/Dw);
% Ad = [ones(1,k+1),zeros(1,L-2*k-1),ones(1,k)];
% Hd = Ad.*exp(j*psid);
% hd = real(ifft(Hd));
% h = hd.*rectwin(L)';

%% Part b:
% k1 = floor(wp/Dw); k2 = ceil(ws/Dw);
% Ad = [ones(1,k1+1),T1,zeros(1,L-2*k2+1),T1,ones(1,k1)];
```

```
% Hd = Ad.*exp(j*psid);
% hd = real(ifft(Hd));
% h = hd.*rectwin(L)';

%% Part c:
h = fir2(M,[0 wp/pi wc/pi ws/pi 1],[1 1 T1 0 0],rectwin(L));

w = linspace(0,2,1000)*pi;
H = freqz(h,1,w);
Hmag = abs(H);
Hdb = 20*log10(Hmag/max(Hmag));

%% Plot:
hf = figconfg('P1012','small');
plot(w/pi,Hdb);hold on
plot(w/pi,-40*ones(1,length(w)),'--','color','k')
ylim([-60 0])
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Magnitude','fontsize',LFS)
title('Magnitude Response','fontsize',TFS)
```

13. (a) Proof:

$$\cos[(n+1)\omega] = \cos n\omega \cos\omega - \sin n\omega \sin\omega$$
$$\cos[(n-1)\omega] = \cos n\omega \cos\omega + \sin n\omega \sin\omega$$

which implies that

$$\cos[(n+1)\omega] + \cos[(n-1)\omega] = 2\cos n\omega \cos\omega$$

that is

$$\cos[(n+1)\omega] = 2\cos(\omega)\cos(n\omega) - \cos[(n-1)\omega]$$

(b) Proof:
Define $\theta = \cos^{-1} x$, we have

$$\begin{aligned} T_{n+1}(x) = \cos[(n+1)\omega] &= 2\cos\theta\cos n\theta - \cos[(n-1)\theta] \\ &= 2x \cdot \cos(n\theta) - \cos(n-1)\theta \\ &= 2xT_n(x) - T_{n-1}(x) \end{aligned}$$

(c) Proof:

$$T_0(x) = 1, \quad T_1(x) = 1$$

$$n = 2, \quad T_2(x) = 2xT_1(x) - T_0(x) = 2x^2 - 1$$

$$n = 3, \quad T_3(x) = 2xT_2(x) - T_1(x) = 2x(2x^2 - 1) - x = 4x^3 - 3x$$

$$n = 4, \quad T_4(x) = 2xT_3(x) - T_2(x) = 2x(4x^3 - 3x) - (2x^2 - 1) = 8x^4 - 8x^2 + 1$$

$$n = 5, \quad T_5(x) = 2xT_4(x) - T_3(x) = 2x(8x^4 - 8x^2 + 1) - (4x^3 - 3x) = 16x^5 - 20x^3 + 5x$$

14. (a) See plot below.

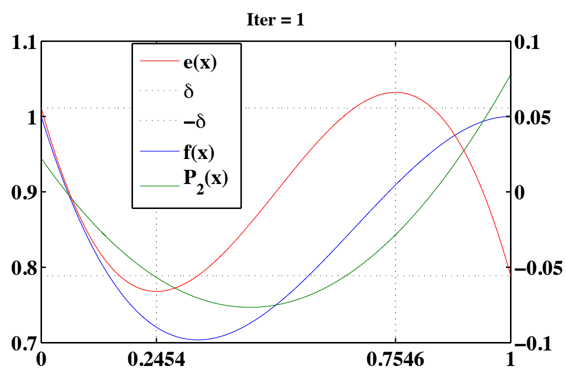

FIGURE 10.24: Graph of $f(x)$, the resulting $P_2(x)$, and $e(x)$.

(b) See plot below.

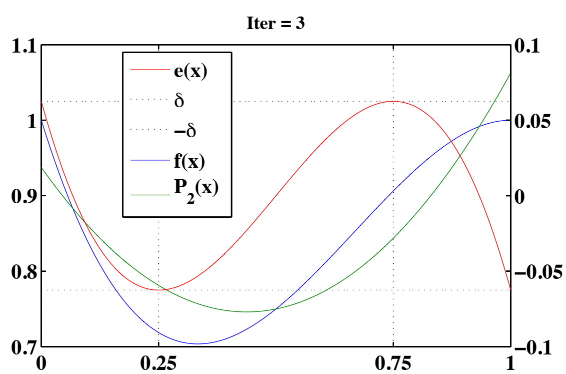

FIGURE 10.25: Graph of final $f(x)$, the resulting $P_2(x)$, and $e(x)$.

MATLAB script:

```
% P1014: Illustration of realization of alternation theorem
function [coeff] = main_1014
close all; clc
fx = @(x) 1-2*x+4*x.^2-2*x.^3;
x_init = [0 1/3 2/3 1];
x_loc = x_init;
ii = 0;
thresh_x = 1;
thresh_ex = 1;
while thresh_x > 1e-10 && thresh_ex > 1e-5
    ii = ii + 1;
    coeff = coeff_solver(x_loc);
    N = 10000;
    xp = linspace(0,1,N);
    a0 = coeff(1); a1 = coeff(2); a2 = coeff(3); delta = coeff(4);
    Px = @(x) a0+a1*x+a2*x.^2;
    ex = fx(xp)-Px(xp);

    hf = figconfg('P1014','small');
    % plot:
    [Ax hf1 hf2] = plotyy(xp,[fx(xp);Px(xp)],xp,...
        [ex;delta*ones(1,N);-delta*ones(1,N)]);
    set(hf2(2),'color','k','Linestyle',':')
    set(hf2(3),'color','k','Linestyle',':')
    legend('e(x)','\delta','-\delta','f(x)','P_2(x)','location','best')

    [extrema extrema_loc] = find_extrema(ex,xp);
    set(Ax(1),'Xtick',extrema_loc,'Xgrid','on')
    set(Ax(2),'Xtick',[],'Xgrid','on')
    title(['Iter = ',num2str(ii)],'fontsize',14)

    thresh_x = max(abs(x_loc-extrema_loc));
    [X Y] = meshgrid(abs(extrema),abs(extrema));
    thresh_ex = max(abs(X(:)-Y(:)));
    x_loc = extrema_loc;
    thresh_x, thresh_ex

end

end
```

```
%% Subfunctions:
function coeff = coeff_solver(x)
% Given guessed nodes, solve for coefficients a_k and delta
% Input:
%       x = [zeta_0 zeta_1 zeta_2 zeta_3];
% Output:
%       coeff = [a_0;a_1;a_2;delta];
x = x(:);
n = length(x);
A = [ones(n,1),x,x.^2];
B = [A,x.^3];
c = (-1).^(0:n-1);
A = [A,c(:)];
coeff = inv(A)*B*[1;-2;4;-2];
end

function [extrema extrema_loc] = find_extrema(fx,x)
% Find the locations of extrema of fx
% Inputs:
%        x: independent variable, value between 0 and 1
%        fx: dependent values
% Output:
%        extrema: values of x where fx is an extrema
fx = fx(:)';
a1 = [0,diff(fx)];
a2 = fliplr([0,diff(fliplr(fx))]);
a = abs(sign(a1) + sign(a2));
ind = find(a>0);
extrema_loc = x(ind);
extrema = fx(ind);
end
```

15. tba

16. MATLAB script:

```
% P1016: Design highpass FIR filter using Parks-McClellan
close all; clc
```

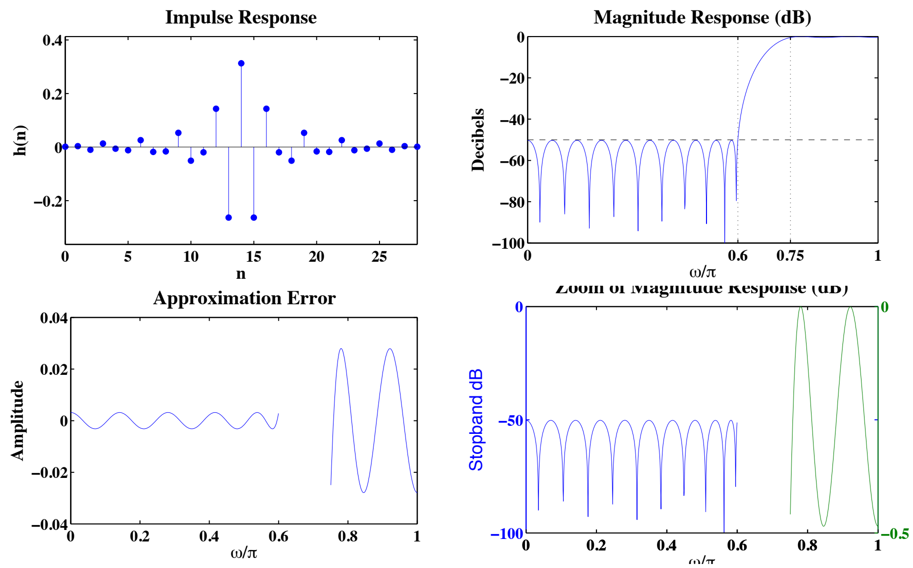

FIGURE 10.26: Graph of impulse response, magnitude response, approximation error and zoom magnitude response.

```
%% Specification:
ws = 0.6*pi; wp = 0.75*pi; As = 50; Ap = 0.5;
%% Passband and Stopband Ripple Calculation:
deltap = (10^(Ap/20)-1)/(10^(Ap/20)+1);
deltas = (1+deltap)/(10^(As/20));
%% Estimated Filter order using FIRPMORD function:
[M,fo,ao,W] = firpmord([ws,wp]/pi,[0,1],[deltas,deltap]);
M = M + 2
%% Filter Design using FIRPM function:
[h,delta] = firpm(M,fo,ao,W);
delta,
deltap,

w = linspace(0,1,1000)*pi;
H = freqz(h,1,w);
Hmag = abs(H);
Hdb = 20*log10(Hmag./max(Hmag));
[Ha w2 P2 L2] = amplresp(h(:)',w);
aperr = nan(1,length(w));
magz1 = nan(1,length(w));
```

```
magz2 = nan(1,length(w));
ind = w <= ws;
aperr(ind) = Ha(ind);
magz1(ind) = Hdb(ind);
ind = w >= wp;
aperr(ind) = Ha(ind)-1;
magz2(ind) = Hdb(ind);
%% Plot:
hfa = figconfg('P1016a','small');
stem(0:M,h,'filled');
xlim([0 M])
ylim([min(h)-0.1 max(h)+0.1])
xlabel('n','fontsize',LFS)
ylabel('h(n)','fontsize',LFS)
title('Impulse Response','fontsize',TFS)

hfb = figconfg('P1016b','small');
plot(w/pi,Hdb);hold on
plot(w/pi,-As*ones(1,length(w)),'--','color','k')
ylim([-100 0])
set(gca,'XTick',[0 ws wp pi]/pi,'Xgrid','on')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Decibels','fontsize',LFS)
title('Magnitude Response (dB)','fontsize',TFS)

hfc = figconfg('P1016c','small');
plot(w/pi,aperr);
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Amplitude','fontsize',LFS)
title('Approximation Error','fontsize',TFS)

hfd = figconfg('P1016d','small');
[AX hf1 hf2] = plotyy(w/pi,magz1,w/pi,magz2);
xlabel('\omega/\pi','fontsize',LFS)
title('Zoom of Magnitude Response (dB)','fontsize',TFS)
set(get(AX(2),'Ylabel'),'string','Passband dB','fontsize',LFS)
set(get(AX(1),'Ylabel'),'string','Stopband dB','fontsize',LFS)
set(AX(1),'Ytick',[-100 -50 0],'ylim',[-100 0])
```

17.

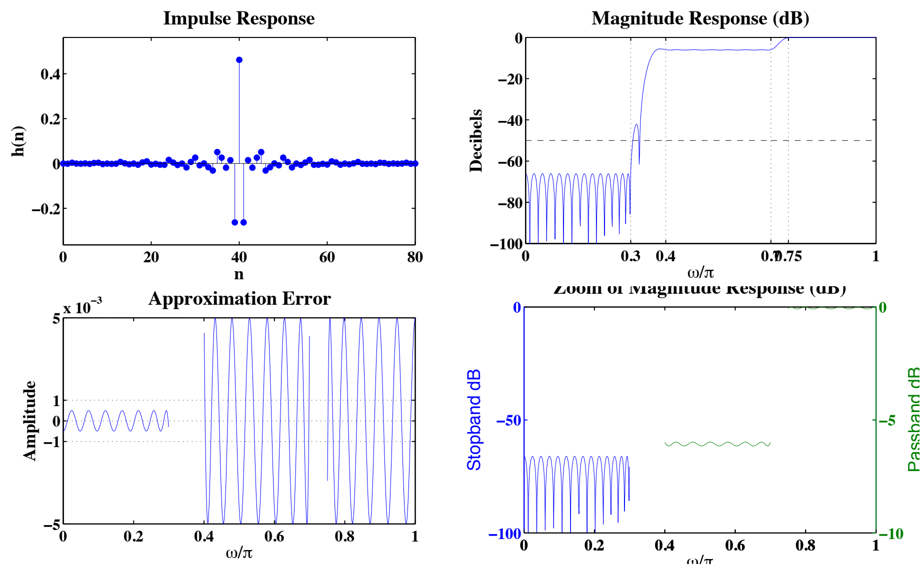

FIGURE 10.27: Graph of impulse response, magnitude response, approximation error and zoom magnitude response.

MATLAB script:

```
% P1017: Design multiband FIR filter using Parks-McClellan
close all; clc
%% Specification:
ws = 0.3*pi; wp1 = 0.4*pi; wp2 = 0.7*pi; wp3 = 0.75*pi;
deltas = 0.001; deltap1 = 0.005; deltap2 = 0.01;
%% Estimated Filter order using FIRPMORD function:
[M,fo,ao,W] = ...
firpmord([ws,wp1,wp2,wp3]/pi,[0,0.5,1],[deltas,deltap1,deltap2]);
M = M + 2
%% Filter Design using FIRPM function:
[h,delta] = firpm(M,fo,ao,W);
delta,
deltap1,

w = linspace(0,1,1000)*pi;
H = freqz(h,1,w);
Hmag = abs(H);
```

```
Hdb = 20*log10(Hmag./max(Hmag));
[Ha w2 P2 L2] = amplresp(h(:)',w);
aperr = nan(1,length(w));
magz1 = nan(1,length(w));
magz2 = nan(1,length(w));
ind = w <= ws;
aperr(ind) = Ha(ind);
magz1(ind) = Hdb(ind);
ind = w >= wp1 & w <= wp2;
aperr(ind) = Ha(ind)-0.5;
magz2(ind) = Hdb(ind);
ind = w >= wp3;
aperr(ind) = Ha(ind)-1;
magz2(ind) = Hdb(ind);
%% Plot:
hfa = figconfg('P1017a','small');
stem(0:M,h,'filled');
xlim([0 M])
ylim([min(h)-0.1 max(h)+0.1])
xlabel('n','fontsize',LFS)
ylabel('h(n)','fontsize',LFS)
title('Impulse Response','fontsize',TFS)

hfb = figconfg('P1017b','small');
plot(w/pi,Hdb);hold on
plot(w/pi,-As*ones(1,length(w)),'--','color','k')
ylim([-100 0])
set(gca,'XTick',[0 ws wp1 wp2 wp3 pi]/pi,'Xgrid','on')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Decibels','fontsize',LFS)
title('Magnitude Response (dB)','fontsize',TFS)

hfc = figconfg('P1017c','small');
plot(w/pi,aperr);
set(gca,'Ytick',[-deltap1 -deltas 0 deltas deltap1],'Ygrid','on')
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Amplitude','fontsize',LFS)
title('Approximation Error','fontsize',TFS)

hfd = figconfg('P1017d','small');
```

```
[AX hf1 hf2] = plotyy(w/pi,magz1,w/pi,magz2);
xlabel('\omega/\pi','fontsize',LFS)
title('Zoom of Magnitude Response (dB)','fontsize',TFS)
set(get(AX(2),'Ylabel'),'string','Passband dB','fontsize',LFS)
set(get(AX(1),'Ylabel'),'string','Stopband dB','fontsize',LFS)
set(AX(1),'Ytick',[-100 -50 0],'ylim',[-100 0])
```

18.

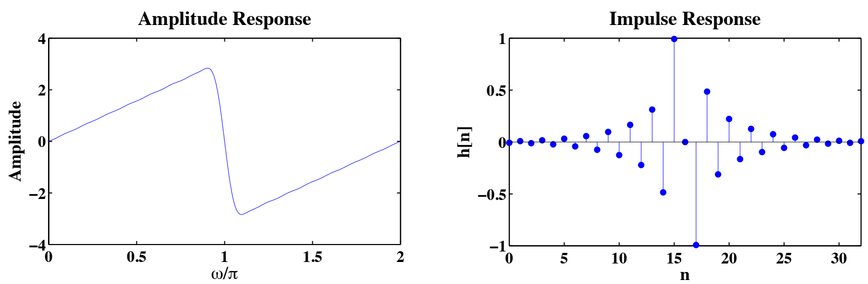

FIGURE 10.28: Graph of amplitude response and impulse response of the designed differentiator.

MATLAB script:

```
% P1018: Design a wideband type-III differentiator
%        using frequency sampling approach
close all; clc
M = 32;
L = M + 1;
Dw = 2*pi/L;
om = (0:L-1)*Dw;
ind = om >= pi;
om(ind) = om(ind) - 2*pi;
alpha = M/2;
H = j*om.*exp(-j*om*alpha);
hd = real(ifft(H));
h = hd.*hamming(L)';

w = linspace(0,2,1000)*pi;
[Ha wt P2 L2] = amplresp(h(:)',w);

%% Plot:
```

```
hfa = figconfg('P1018a','small');
plot(w/pi,Ha);hold on
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Amplitude','fontsize',LFS)
title('Amplitude Response','fontsize',TFS)

hfb = figconfg('P1018b','small');
stem(0:M,h,'filled')
xlim([0 M])
xlabel('n','fontsize',LFS)
ylabel('h[n]','fontsize',LFS)
title('Impulse Response','fontsize',TFS)
```

19.

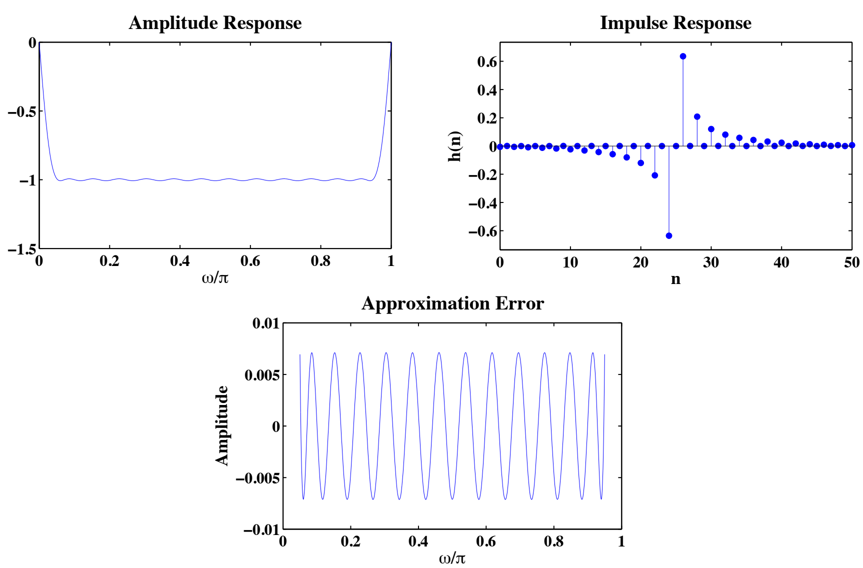

FIGURE 10.29: Graph of amplitude response, impulse response and the approximation error of the designed Hilbert transformer.

MATLAB script:

```
% P1019: Design Hilbert transformer using Parks-McClellan
close all; clc
L = 51; M = L-1;
w1 = 0.05*pi; w2 = 0.95*pi;
```

```
[h,delta] = firpm(M,[w1 w2]/pi,[1 1],'hilbert');

w = linspace(0,1,1000)*pi;
H = freqz(h,1,w);
[Ha wa P2 L2] = amplresp(h(:)',w);
aperr = nan(1,length(w));
ind = (w >= w1 & w <= w2);
aperr(ind) = Ha(ind)+1;

%% Plot:
hfa = figconfg('P1019a','small');
stem(0:M,h,'filled');
xlim([0 M])
ylim([min(h)-0.1 max(h)+0.1])
xlabel('n','fontsize',LFS)
ylabel('h(n)','fontsize',LFS)
title('Impulse Response','fontsize',TFS)

hfb = figconfg('P1019b','small');
plot(w/pi,Ha);hold on
xlabel('\omega/\pi','fontsize',LFS)
title('Amplitude Response','fontsize',TFS)

hfc = figconfg('P1019c','small');
plot(w/pi,aperr);
xlabel('\omega/\pi','fontsize',LFS)
ylabel('Amplitude','fontsize',LFS)
title('Approximation Error','fontsize',TFS)
```

20. tba