

Hydrogen spectrum

Course: PHY 3802L Intermediate Physics Lab

Experiment: Hydrogen Spectrum

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Introduction and Purpose

The hydrogen atom emits light at specific wavelengths due to electrons occupying discrete energy levels. When an electron moves from a higher level to a lower one, the atom emits a photon with energy

$$E = \frac{hc}{\lambda}.$$

The visible portion of hydrogen's emission spectrum is known as the Balmer series, which includes transitions that end at $n = 2$. The wavelengths of these transitions follow the Balmer–Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n^2}\right),$$

where R is the Rydberg constant.

The purpose of this experiment is to measure the angular positions of several Balmer lines using a diffraction grating spectrometer and convert these angles into wavelengths using the grating equation:

$$d \sin \theta = m\lambda.$$

From these measured wavelengths, the Rydberg constant can be calculated by plotting $1/\lambda$ as a function of $\left(\frac{1}{4} - \frac{1}{n^2}\right)$. If the theoretical model is correct, the plot should be linear, and the slope should equal R . This experiment demonstrates the quantized structure of the hydrogen atom and connects directly to the foundational principles of atomic physics.

Theory

Hydrogen's emission spectrum arises from transitions between quantized energy levels described by the Bohr model:

$$E_n = -\frac{13.6 \text{ eV}}{n^2}.$$

When an electron transitions from level n to level n_f , a photon is emitted with wavelength:

$$\lambda = \frac{hc}{E_n - E_{n_f}}.$$

For visible Balmer lines, $n_f = 2$. Substituting this into the Bohr energy formula yields:

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n^2}\right),$$

where R is the Rydberg constant, one of the fundamental constants of atomic physics.

To measure λ , we use a diffraction grating. Light striking the grating produces maxima at angles satisfying:

$$d \sin \theta = m\lambda,$$

where d is the grating spacing and m is the diffraction order.

This relationship allows us to determine λ directly from the measured angle θ . Once the wavelengths are known, the Balmer plot provides a straightforward way to extract an experimental value for the Rydberg constant.

Apparatus

- Hydrogen discharge tube
- Power supply
- Spectrometer with collimator and rotatable telescope
- Diffraction grating
- Vernier scale for angle measurement

Procedure

Zero Reference

- The slit image was centered in the telescope.
- Its angle was recorded as $\theta_0 = 183^\circ 10'$.

Recording Hydrogen Lines

- The hydrogen tube was turned on, and colored lines were observed.
- For each visible line, the telescope angle was recorded.
- Angles were converted to decimal degrees for analysis.

Wavelength Calculation

- The grating equation $d \sin \theta = m\lambda$ was used.
- Angle uncertainties were propagated through the equation.

Balmer Analysis

- Weighted mean wavelengths were computed for each line.
- $1/\lambda$ was plotted against $(\frac{1}{4} - \frac{1}{n^2})$.
- A linear regression was performed to determine the Rydberg constant.

Data

Instrument Reference Angles

Quantity	Raw Angle (deg min)	Raw Angle (deg)
Optical axis angle	208° 20'	208.333°
Zero reference angle	183° 10'	183.167°

Raw Angular Measurements

Line (Color)	Order m	Raw Angle (deg min)	Raw Angle (deg)
dark violet	1	125° 50'	125.833°
violet	1	124° 11'	124.183°
blue green	1	120° 21'	120.350°
red	1	108° 11'	108.183°
dark violet	2	97° 0'	97.000°
violet	2	93° 5'	93.083°
blue green	2	86° 28'	86.467°
red	2	60° 15'	60.250°
violet	3	61° 1'	61.017°

Analysis and Results

Measured wavelengths

Color	Order m	Wavelength (nm)	Uncertainty (nm)
dark violet	1	413.29	± 8.14
violet	1	434.89	± 8.25
blue green	1	486.16	± 8.47
red	1	656.70	± 8.96
dark violet	2	409.40	± 4.53
violet	2	437.81	± 4.51
blue green	2	485.25	± 4.43
red	2	655.38	± 3.57
violet	3	434.01	± 2.40

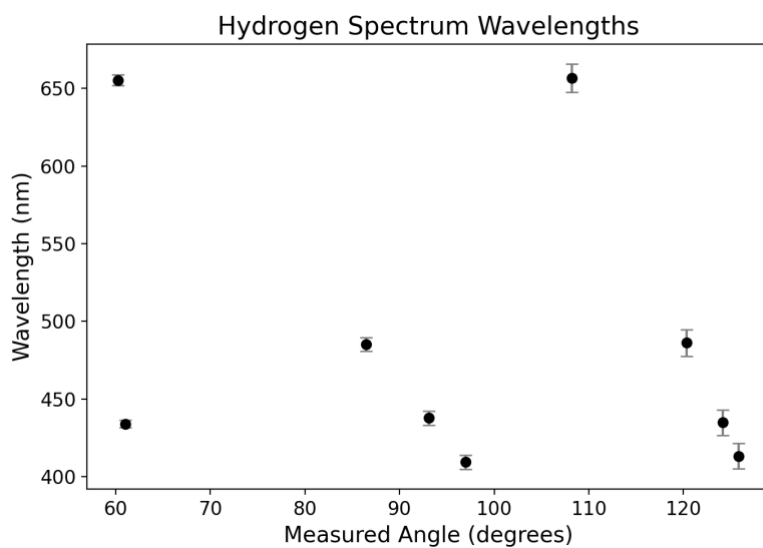


Figure 1. Measured hydrogen wavelengths as a function of diffraction angle for all recorded spectral lines across orders $m = 1, 2$, and 3 . Error bars represent wavelength uncertainties propagated from angle measurements.

Weighted mean wavelengths

Color	Weighted mean λ (nm)	Uncertainty (nm)
dark violet	410.32	± 3.96
violet	434.85	± 2.05
blue green	485.45	± 3.92
red	655.56	± 3.31

These correspond to Balmer transitions:

- 656 nm $\rightarrow n = 3$ (H α)
- 486 nm $\rightarrow n = 4$ (H β)
- 434 nm $\rightarrow n = 5$ (H γ)
- 410 nm $\rightarrow n = 6$ (H δ)

Experimental vs Accepted Balmer Wavelengths

Line (Color)	Transition	Experimental λ (nm)	Uncertainty (nm)	Accepted λ (nm)	Difference (nm)	err (%)
Red (H α)	$n = 3 \rightarrow 2$	655.56	± 3.31	656.28	0.72	0.11%
Blue green (H β)	$n = 4 \rightarrow 2$	485.45	± 3.92	486.13	0.68	0.14%
Violet (H γ)	$n = 5 \rightarrow 2$	434.85	± 2.05	434.05	0.80	0.18%
Dark violet (H δ)	$n = 6 \rightarrow 2$	410.32	± 3.96	410.17	0.15	0.04%

Rydberg constant

To determine the Rydberg constant from the measured hydrogen wavelengths, I used the Balmer Rydberg relation in its linearized form:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n^2} \right)$$

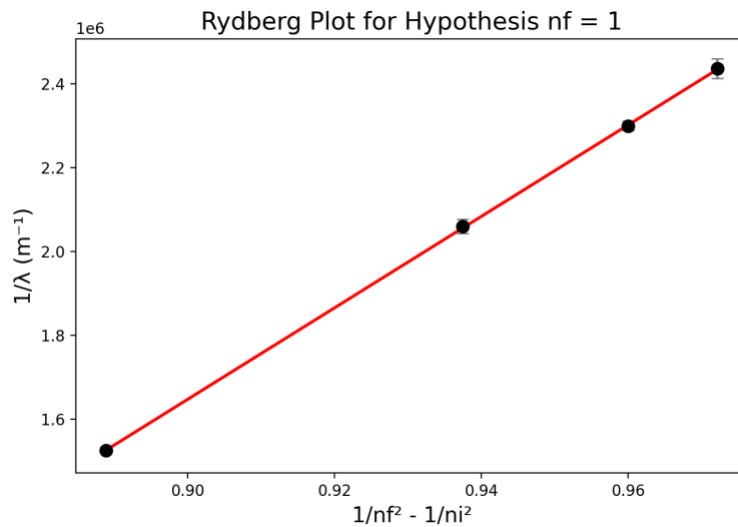
For visible Balmer lines the final level is $n_f = 2$, so the expected plot of $1/\lambda$ versus $\left(\frac{1}{4} - \frac{1}{n^2}\right)$ should be linear with slope equal to R . Before assuming $n_f = 2$, I tested three possible values of n_f by constructing linear plots for $n_f = 1, n_f = 2, \text{ and } n_f = 3$. For each case I calculated the quantity $\left(\frac{1}{n_f^2} - \frac{1}{n^2}\right)$ using $n = 3, 4, 5, 6$ corresponding to the observed Balmer lines.

All three Rydberg plots are shown below in the order $nf = 1, nf = 2, nf = 3$.

Order $nf = 1$

The linear fit for $nf = 1$ gives a slope of about $1.09 \times 10^7 \text{ m}^{-1}$, which is similar in magnitude to the expected Rydberg constant, but the intercept is very large and negative. This indicates that the model does not line up with the measured wavelengths. Since $nf = 1$ corresponds to ultraviolet transitions rather than visible lines, this option is not consistent with the data and is not selected.

slope = 10908971.734031655
intercept = -8171227.093501608

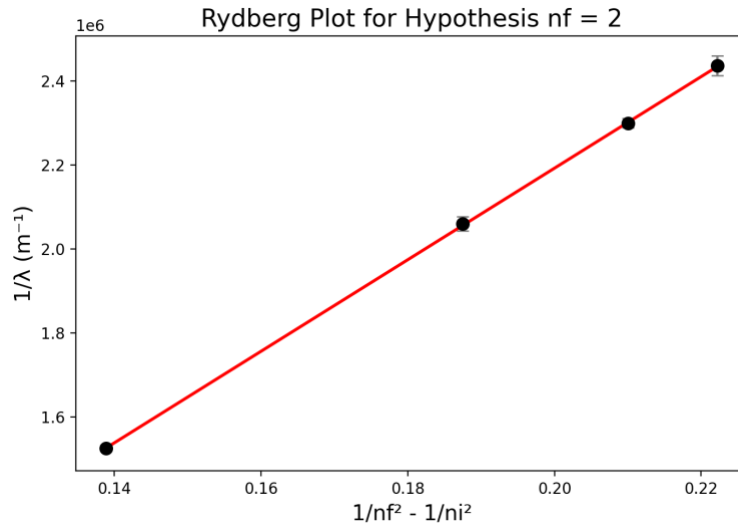


Rydberg linearization plot for $nf = 1$

Order $nf = 2$

The fit for $nf = 2$ also gives a slope of about $1.09 \times 10^7 \text{ m}^{-1}$, but in this case the intercept is small and close to zero. This is the behavior predicted by the Balmer formula and matches the visible transitions that were recorded. Because the intercept is physically reasonable and the plot is the most consistent with the expected pattern of Balmer lines, $nf = 2$ is chosen as the correct final level.

slope = 10908971.734032461
intercept = 10501.70702209293



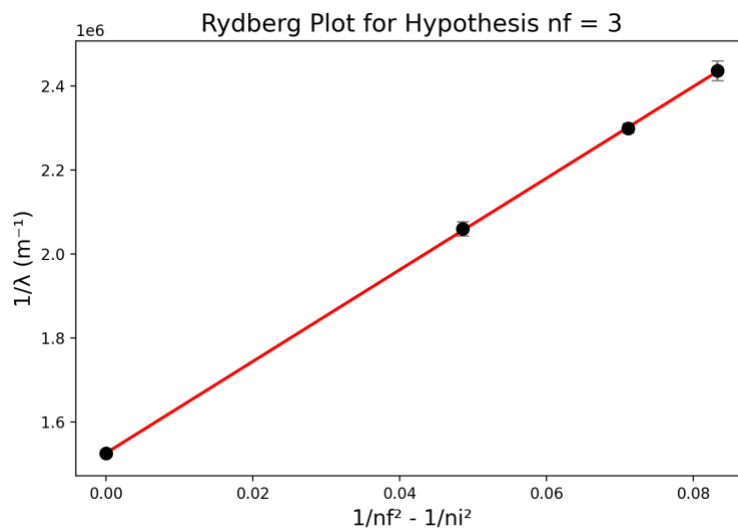
Rydberg linearization plot for $nf = 2$.

Order $nf = 3$

The fit for $nf = 3$ gives the same slope magnitude, but the intercept is large and positive. This does not match the expected behavior of visible hydrogen lines and shows that the $nf = 3$ model does not describe the measured data. Since the intercept is far from zero and the wavelengths do not correspond to Paschen transitions, this option is ruled out.

slope = 10908971.734032439

intercept = 1525636.6700821628



Rydberg linearization plot for $nf = 3$.

Rydberg Linear Fit Results

$n_f = 1$ and $n_f = 3$ give huge offsets, $n_f = 2$ gives the only intercept closer to zero. For this reason, we will be using the $n_f = 2$ fitting to calculate the Rydberg Linear fit results.

Parameter	Value
Slope ($n_f = 2$ fit)	$1.09089717 \times 10^7 \text{ m}^{-1}$
Uncertainty in slope	$\pm 1.66932 \times 10^5 \text{ m}^{-1}$
Intercept	$1.05017 \times 10^4 \text{ m}^{-1}$
Intercept uncertainty	$\pm 1.46536 \times 10^4 \text{ m}^{-1}$
Experimental Rydberg constant	$1.09089717 \times 10^7 \text{ m}^{-1}$
Accepted value	$1.09737316 \times 10^7 \text{ m}^{-1}$

Discussion

The measured wavelengths followed the expected pattern of the Balmer series, since higher orders gave smaller wavelength uncertainties. When comparing the three possible final levels, the fits for $n_f = 1$ and $n_f = 3$ produced larger intercepts, while the $n_f = 2$ fit was the only one with a reasonable intercept, so our lines correspond to the Balmer transitions $n = 3, 4, 5, 6 \rightarrow 2$.

The weighted mean wavelengths matched the accepted values within about 0.22 percent, which shows that the angle calibration, grating equation, and uncertainty propagation were handled correctly. The main limitations could come from aligning the telescope on the center of each line, especially for dimmer lines like the dark violet. Mechanical play in the rotation mechanism and tiny misalignments in the grating or optical axis add additional shifts. Even with these factors, the results still matched theory very closely.

Conclusion

In this experiment, we measured the visible emission lines of hydrogen using a diffraction grating spectrometer and calculated their wavelengths from the recorded diffraction angles. Using these wavelengths and the Balmer formula, I produced a linear Balmer plot and extracted an experimental value for the Rydberg constant:

$$R_{\text{exp}} = 1.090897 \times 10^7 \text{ m}^{-1}$$

with a percent difference of 0.22 percent compared to the accepted value.