Two-Slit Interference: Young's Experiment

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What the experiment shows

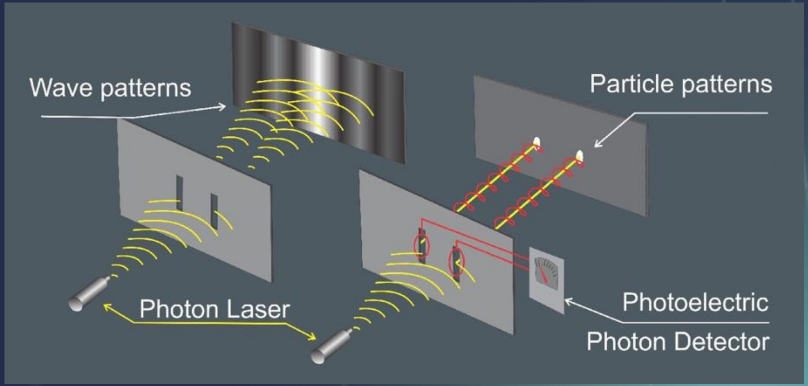
Light was considered a particle, with an ongoing debate of it's also a wave

In 1801, Thomas Young proved, with the double slit experiment, that light behaves exactly like waves showing an interference pattern

This experiment is the starting point for wave particle duality

Young's double slit demonstrates a single idea: **light behaves like a wave.**

Any wave passing through two openings produces a pattern of constructive and destructive interference on a screen.



Setup and experiment

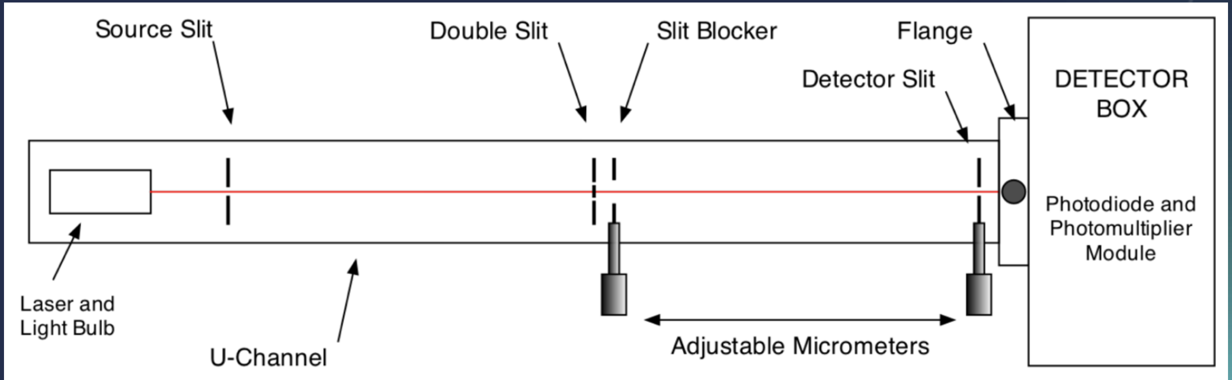
Light Source
→ λ = 670 nm Laser (5 mW) - monochromatic light

Detection (V)
→ Photodiode - converts light → electrical signal
→ Digital Multimeter - reads voltage (∝ intensity)

Measured:
→ Position (mm) vs intensity (mV)

Positioning
→ Micrometers - scan detector position (±10 μm resolution)

Optical
→ Single slit (D)
→ Double slit (S)
→ Distance to detector: 500mm (L>>>S)



What happens with **one** slit?

Difraction

- A single slit of width D acts like a small wave source
- The wave spreads out instead of going straight.
- This spreading produces a diffraction pattern.

- Its intensity is given by the Fraunhofer model:

$$I_1(x) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2$$

$I_1(x)$ = intensity on the screen at position x
I_0 = peak intensity (scaling factor for the brightness)
β = phase difference across the width of the slit

- β the phase parameter:

$$\beta = \frac{\pi D}{\lambda} \frac{x}{L}$$

D = slit width
λ = wavelength of the laser (λ = 670 nm).
x = position of the detector slit relative to the center.
L = distance from the slit to the detector (~500 mm).

What you see on the screen:

- One big bright region in the center
- Dimmer regions on the sides
- Dark zones where the wave cancels itself

Even a single slit creates a series of bright and dark bands, but the pattern is wide and smooth.

It comes from the wave inside the slit interfering with itself.

What happens with **two** slits?

Interference

- Two wave sources, each slit produces its own wave
- They travel at different distances, so they arrive with a phase difference

- Path Difference

$$\Delta = S\sin\theta \approx S\frac{x}{L}$$

- Phase Difference

$$\phi = \frac{2\pi}{\lambda}\Delta$$

- Its intensity is given by:

$$I_2(x) = I_0\cos^2\left(\frac{\phi}{2}\right)$$ or $$I_2(x) = I_0\cos^2\left(\frac{\pi S}{\lambda}\frac{x}{L}\right)$$

→ This is what gives the bright and dark fringes

At each point they either:

- Add if their peaks align
- Cancel if a peak meets a dip

Pattern:

- Many narrow bright lines
- With equally spaced dark lines in between

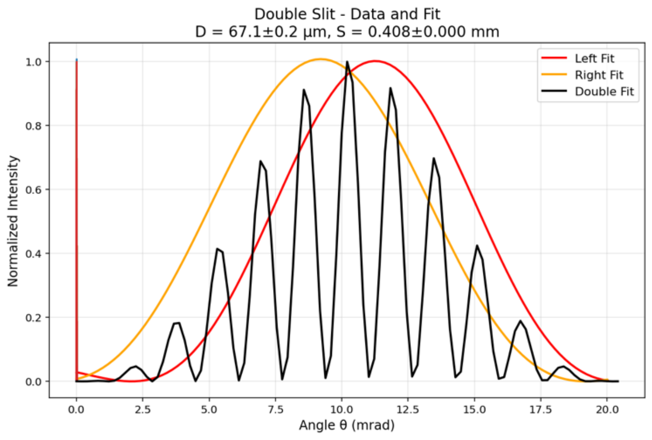
Double Slit Intensity

- The real double slit intensity is the product of diffraction and interference
- These two effects happen simultaneously, not independently.

$$I(x) = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\left(\frac{\phi}{2}\right)$$

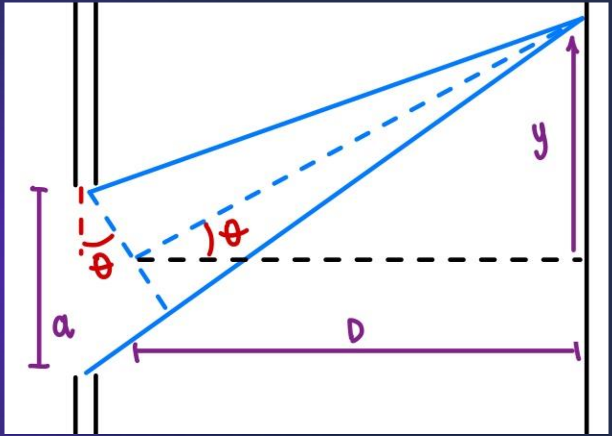
With two slits, you still get the single slit diffraction shape, but inside it you get a regular set of bright and dark stripes created by the interference of the two slits.

So:
- One slit gives a broad diffraction envelope.
- Two slits give that envelope plus many fine interference fringes inside it.



Where bright and dark fringes appear

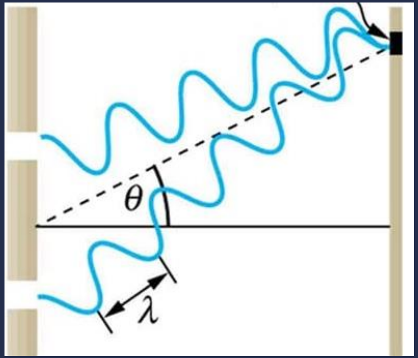
Pathway differences



- a = slit width
- D = distance from slit to screen
- y = point on the screen at height y
- θ = angle

Pathway difference: $\Delta = a \sin\theta$

Constructive interference: bright fringes



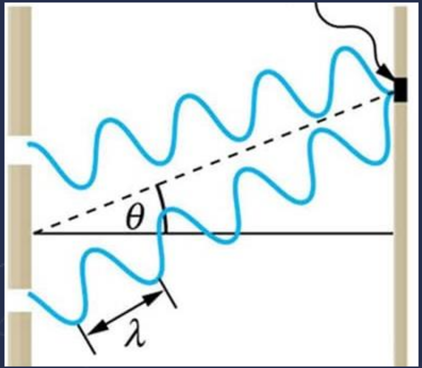
The path difference equals whole wavelengths:

$$\Delta = m\lambda$$

$$S \sin\theta = m\lambda$$

$$x_m = \frac{m\lambda L}{S}$$

Destructive interference: dark fringes



One wave arrives half a wavelength out of phase.

$$\Delta = \left(m + \frac{1}{2}\right)\lambda$$

each point on the slit arrives with a different phase → slit geometry

Slit Geometry: Fourier Transformation

Slits are rectangular → Fourier transform is a sinc function $$\mathrm{sinc}(x) = \frac{\sin x}{x}$$

Single Slit:

In Fraunhofer the diffraction pattern is the Fourier transform of that aperture.

Two Slits:

The slits are shifted, so, their Fourier transform gains a phase shift that appears as the cosine term

So, the Fourier transform is:

- A **sinc** shaped envelope from each slit's width
- A **cosine phase factor** from the separation

$$I(x) = I_0 \overbrace{\left(\frac{\sin\beta}{\beta}\right)^2}^{\text{single slit}} \overbrace{\cos^2\left(\frac{\phi}{2}\right)}^{\text{double slit}}$$

Raw Measurements

- Photodiode voltage readings (mV)
- Zero offset voltage = 8.5 mV (baseline noise)
- Micrometer positions (mm)
- Position offsets (to center the pattern)

  Left slit – x_0 = 4.512 mm
  Right Slit - x_0 = 3.099 mm
  Double slit - x_0 = 4.229 mm

Data Normalization

- Subtracted the 8.5 mV offset (laser off reading).
- Divided all intensities by the maximum value so the peak becomes 1.
  - This allows clean comparison to theoretical curves.

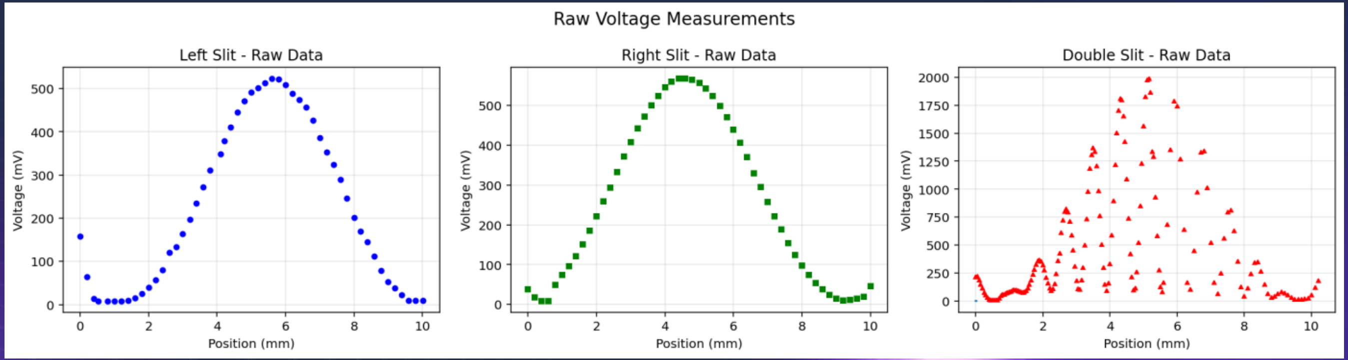
Data transformation

Voltage → Intensity

Voltage is proportional to intensity

Position → Angle

mm to θ using $\theta = x/L$ (L = 500 mm)



Fitting Process

Single-Slit Fitting

For each single slit, the data was fitted to the Fraunhofer diffraction model:

```
I(x) = I₀ × [sin(φ/2)/(φ/2)]²

where φ = (2π/λ) × D × sin(θ)
      θ = (x - x₀)/L
```

D (slit width) - the key unknown
I_0 (peak intensity) - amplitude scaling
x_0 (position offset) - centering correction
left – x_0 = 4.512 mm (position offset)
right - x_0 = 3.099 mm (position offset)

Double-Slit Fitting

The interference pattern was fitted to the combined model:

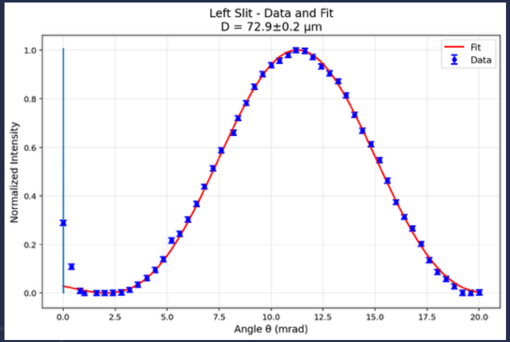
```
I(x) = I₀ × [sin(φ/2)/(φ/2)]² × cos²(ψ/2)

where φ = (2π/λ) × D × sin(θ)
      ψ = (2π/λ) × S × sin(θ)
      θ = (x - x₀)/L
```

D - determines diffraction envelope width
S (separation) - determines fringe spacing
I_0 and x_0 as before
x_0 = 4.229 mm (position offset)

Results

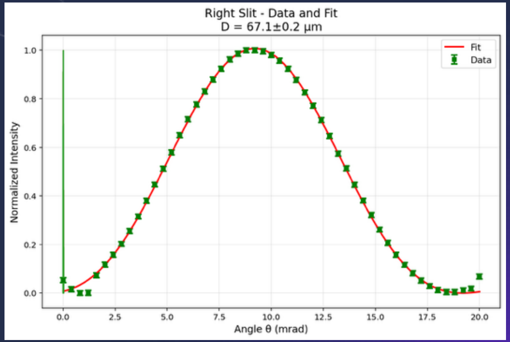
Slit Width (D):

- Left slit: D = 72.9 ± 0.2 µm



x_0 = 4.512 mm (position offset)

- Right slit: 67.1 ± 0.2 µm

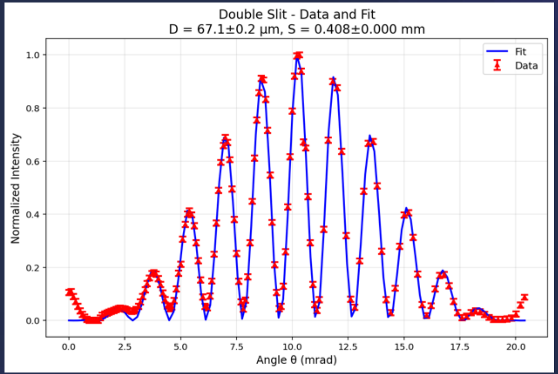


x_0 = 3.099 mm (position offset)

Slit Separation (S):

S = 408.5 ± 0.2 µm (= 16.1 mils)

- D = 67.1 ± 0.2 µm
- x_0 = 4.229 mm (position offset)



- Determined the interference fringe spacing
- Verified which double-slit assembly was used (14, **16≈ 406 µm**, or 18 mil options)

Wavelength

Used known λ = 670 nm in fitting

Results

Quality of Fits

Chi-squared (χ^2) values evaluated fit quality:

- Left slit: χ^2 = 17.1 (poor fit)
- Right slit: χ^2 = 2.33 (good fit)
- Double slit: χ^2 = 8.91 (moderate fit)

The right slit data was the most reliable. The left slit had issues. The double slit fit worked well enough to extract the slit separation and width.

Using the full range from 0 to 10 mm

Key Physics Relationships Used

Number of fringes: $2S/D \approx 12.2$

Fringe spacing: $\Delta x \approx 0.82$ mm

- $\Delta x = \lambda L/S$ (linear)
- $\Delta\theta = \lambda/S$ (angular)

These equations were used to confirm S.

The analysis used the known wavelength and measured intensity patterns to work backwards through the diffraction/interference equations to extract the geometric parameters (D and S) of the slit system

Uncertainties

How Uncertainties Were Determined

The model fit (nonlinear least squares)
- the computer adjusts D, S, x_0, and I_0 until the theory curve matches the data

Covariance matrix (statistical uncertainty)
- tells us how precise those fitted numbers are, based on noise in the data.

Fitted Parameters

Width: D ± 0.2 μm
Separation: S ± 0.2 μm
Angular offset: x_0 = ± 10 μrad

Error sources

Micrometer positioning: ~10 μm resolution
- The position cannot be 100% precise

Alignment and mechanical vibrations
- Slight misalignments can change the interference contrast and shift the pattern

The detector slit small width
- It averages the light over that area and slightly blurs the pattern