EM Ratio Lab Report

Catalina Cisneros

PID: 6517771

Dr. Lei Guo

Intermediate physics lab

September 22nd, 2025

Introduction and theory

The purpose of this experiment is to measure the electron's charge-to-mass ratio (e/m) in units of C/kg. determine how the motion of accelerated electrons in a magnetic field are used to calculate this constant. constant is important because it links two fundamental properties of the electron.

This is determined by accelerating electrons through a potential difference (V), which gives them kinetic energy. They then travel in a circular path under the magnetic field (B) produced by the Helmholtz coils.

The balance of forces is:

$$e\ v\ B = \frac{mv^2}{r}$$

and the kinetic energy relation is:

$$\frac{1}{2}mv^2 = eV$$

which can be simplified to

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

This relationship helps us link the electron's radius (r) along the circular path, the voltage (V), the magnetic field (B), the current (I), the diameter (D), to finally obtain charge to mass ratio (e/m)

In this experiment, the Helmholtz coils generate the magnetic field. The field at the center is given by:

$$B = \mu_0 \cdot \frac{N\ I}{R} \cdot (\frac{4}{5})^{3/2}$$

where N is the number of turns, R is the coil radius and μ_0 is the constant of permeability.

Since the voltages used (200-400V) keep the electrons notably below the speed of light, relativistic corrections are unnecessary, and the classical methods are valid.

The objective is to calculate the e/m ratio from experimental data and compare it with the accepted value

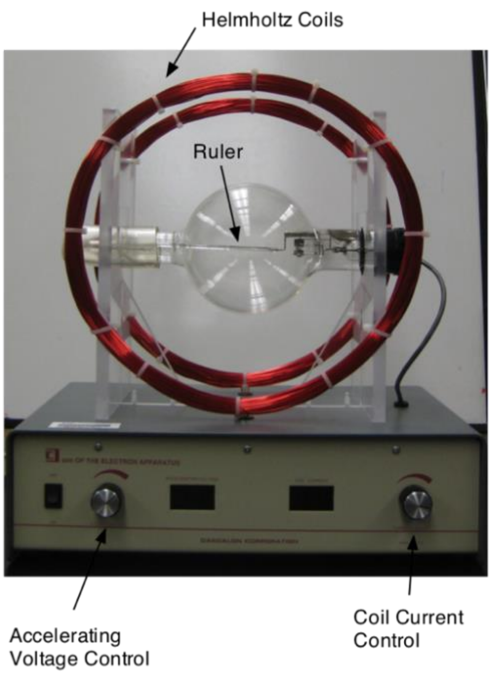
$$\frac{e}{m} = 1.758820\ \times 1011\ \mathrm{C/kg}$$

Apparatus and methods

Apparatus:

1. Electron beam tube, matched Helmholtz coils, voltage supply, current control, ruler in the tube.
2. Ruler to measure coil specifications.
3. Software for analyzing and plotting data (Python libraries LT.box and NumPy)

Coil specifications: $N = 132\ turns$, coil diameter $D_c = 30.50\ cm$, so $R = 15.25\ cm$



Methods

<u>What I measured:</u>

Before starting any data collection, the diameters and distances were measured with a ruler. Then, at a fixed voltage V, I varied the coil current and recorded the beam diameter D with an observed uncertainty dD. The run was repeated for $V = 200V, V = 250V, V = 300V$, and $V = 400V$. The data stores I in A, D in cm, and dD in cm as well. I converted $r = D/2$ and $dr = dD/2$

<u>Uncertainties:</u>

Voltage: $V \pm dV; dV = 1.00V$

Current: $I \pm dI; dI = 0.01A$

Diameter: $D \pm dD$; in cm

Turns (N): exact in this setup = 132

Data

For each accelerating voltage ($V = 200V$, $V = 250V$, $V = 300V$, and $V = 400V$), I collected the data points in tables, to then be plotted into graphs of the e/m value against coil current. Each table, titled with the constant voltage used, includes the current, diameter of the electron beam, and the uncertainty of such diameter. The fitted straight lines are nearly flat, and the scatter is within the uncertainties.

For $V = 200V$: Coil current (I), beam diameter (D), and diameter uncertainty (dD):

I (A)	D (cm)	dD (cm)
2.19	5.0	0.12
2.00	5.5	0.12
1.84	6.0	0.20
1.72	6.5	0.20
1.61	7.0	0.25
1.49	7.5	0.27
1.41	8.0	0.30
1.31	8.5	0.40
1.25	9.0	0.45
1.19	9.5	0.50

Table 1: V = 200V (dV = 1.0V, dI = 0.01A)

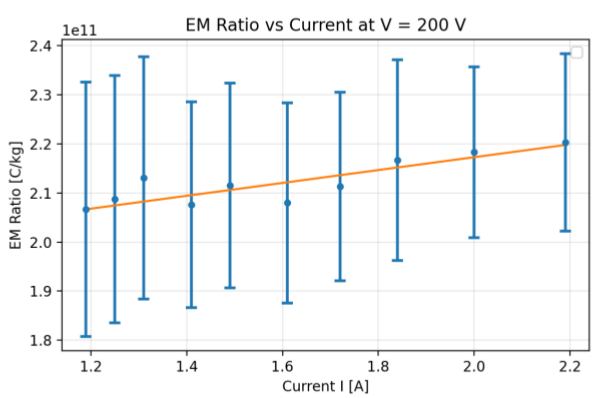


Figure 1: e/m vs current at 200V. Values close to 2.1×10^{11} C/kg; slope is small, no current dependence.

For $V = 250V$: Coil current (I), beam diameter (D), and diameter uncertainty (dD):

I (A)	D (cm)	dD (cm)
2.43	5.0	0.15
2.30	5.5	0.20
2.11	6.0	0.25
1.95	6.5	0.27
1.83	7.0	0.30
1.72	7.5	0.35
1.61	8.0	0.40
1.51	8.5	0.45
1.44	9.0	0.50
1.34	9.5	0.50

Table 2: V = 250V (dV = 1.0V, dI = 0.01A)

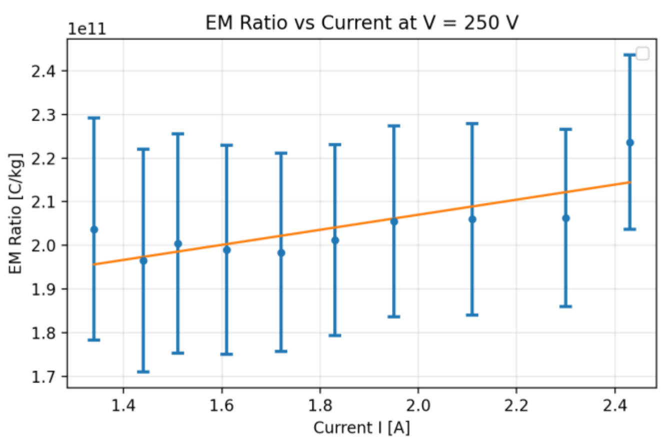


Figure 2: e/m vs current at 300 V. Data stable around 2.0×10^{11} C/kg; slope nearly flat.

For $V = 300V$: Coil current (I), beam diameter (D), and diameter uncertainty (dD):

I (A)	D (cm)	dD (cm)
2.79	5.0	0.12
2.49	5.5	0.20
2.35	6.0	0.20
2.15	6.5	0.25
2.03	7.0	0.25
1.88	7.5	0.30
1.78	8.0	0.30
1.67	8.5	0.40
1.59	9.0	0.40
1.51	9.5	0.50

Table 3: V = 300V (dV = 1.0V, dI = 0.01A)

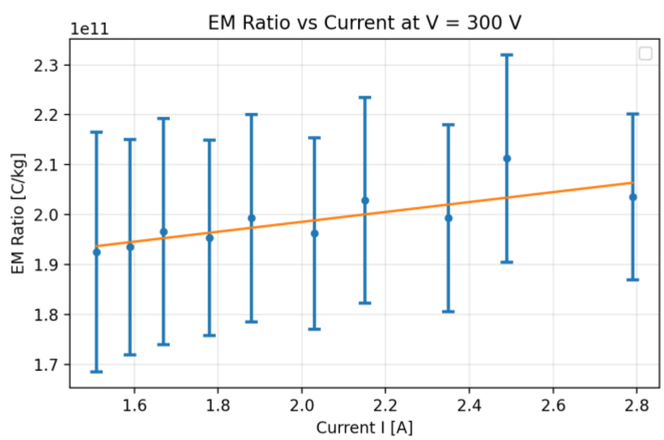


Figure 3: e/m vs current at 300 V. Data stable around 2.0×10^{11} C/kg; slope nearly flat.

For $V = 400V$: Coil current (I), beam diameter (D), and diameter uncertainty (dD):

I (A)	D (cm)	dD (cm)
2.98	5.5	0.20
2.76	6.0	0.20
2.53	6.5	0.25
2.37	7.0	0.25
2.22	7.5	0.27
2.09	8.0	0.30
1.95	8.5	0.30
1.85	9.0	0.32
1.77	9.5	0.40
1.68	10.0	0.45

Table 4: V = 400V (dV = 1.0V, dI = 0.01A)

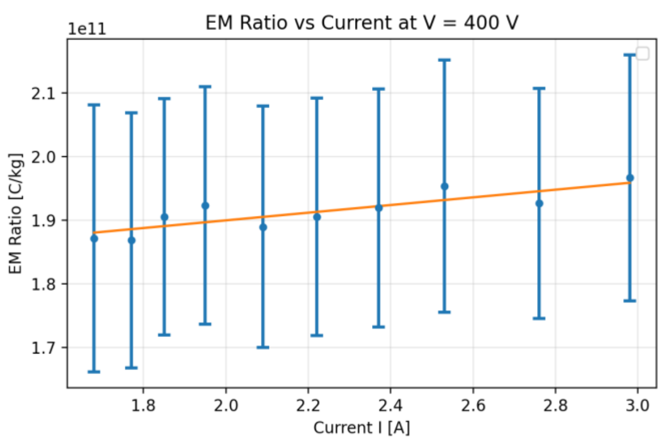


Figure 4: e/m vs current at 400 V. Mean just under 2.0×10^{11} C/kg; fit line consistent with constancy.

Analysis and Results

Weighted means

For each voltage, the charge to mass ratio was computed using

$$\frac{e}{m} = \frac{2Vm}{(kI)^2 r^2}$$

with

$$k = \mu_0 \cdot \frac{N}{R} \cdot \left(\frac{4}{5}\right)^{3/2} = \frac{8\,\mu_0\,N}{\sqrt{125}\,R}$$

Weighted means were calculated with

$$\bar{x} = \frac{\sum w_i\,x_i}{w_i}, \qquad w_i = \frac{1}{\sigma_i^2}, \qquad \sigma_{\bar{x}} = \frac{1}{\sqrt{\sum w_i}}$$

From the slope fits, I obtained the following values:

- At 200 V: e/m = $2.122 \times 10^{11} \pm 4.549 \times 10^{8}$ C/kg (20.7% diff)
- At 250 V: e/m = $2.041 \times 10^{11} \pm 7.280 \times 10^{8}$ C/kg (16.0% diff)
- At 300 V: e/m = $1.991 \times 10^{11} \pm 5.338 \times 10^{8}$ C/kg (13.2% diff)
- At 400 V: e/m = $1.913 \times 10^{11} \pm 3.027 \times 10^{8}$ C/kg (8.8% diff)

The accepted value is 1.76×10^{11} C/kg. My results are slightly higher, but they are within about 10–20% of the known value.

Taking all the measurements together and using a weighted mean, I got an overall weighted mean of about

$$1.985 \times 10^{11} \pm 2.175 \times 108\, C/kg$$

which is a 12.9% difference. This is close enough to the expected value when uncertainties are considered.

Fitting parameters

The data was also analyzed using the relation

$$\frac{1}{r^2} = \frac{e}{m} \cdot \frac{k^2}{2V} \cdot I^2 + b, \qquad b \approx 0$$

so the slope of the relation is

$$m_{slope} = \frac{e}{m} \cdot \frac{k^2}{2V}$$

from this, we can conclude that

$$\frac{e}{m} = \frac{k^2}{2V} \cdot m_{slope}$$

Uncertainties were calculated from the slope and k

$$\Delta(\frac{e}{m}) = \sqrt{(\frac{2V}{k^2}\Delta m)^2 + (\frac{4Vm}{k^3}\Delta k)^2}$$

Discussion

The experimental values of e/m are higher than the accepted value, but this difference is reasonable given the uncertainties. Possible reasons include:

- The magnetic field created by the coils is not perfectly uniform, and the formula used is only exact at the center. This can reduce accuracy on the results
- Other present magnetic fields on Earth (like the same Earth's field or nearby electronics) can affect from the Helmholtz coil's field
- The beam diameter measurement is not completely accurate, since it had a finite thickness and parallax error, which could also increase the uncertainty in r.
- The assumption that electrons start at rest before turning on the voltage could be false. Sometimes leftover kinetic energy lowers potential energy gained.

Relativistic considerations,

At $V = 400$, the estimated electron speed is

$$\beta = \frac{v}{c} = = \frac{\sqrt{2\left(\frac{\mathrm{e}}{\mathrm{m}}\right)_{acc}\mathrm{V}}}{c} \approx 0.04$$

This is far below the speed of light, which is why classical equations are being used.

Consistency,

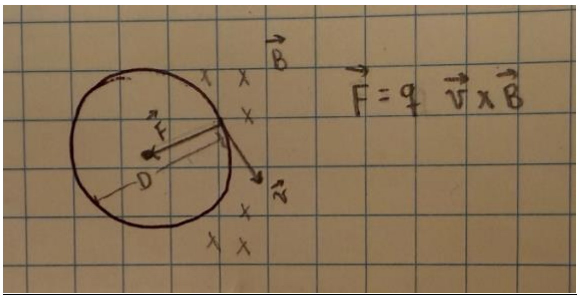
The slopes and intercepts came out as expected. Intercepts were basically zero, which backs up the linear model. Even though numbers weren't exactly the accepted e/m value, the results are close enough. The values also make sense when scaling them with voltage.

Experiment-Specifics

Magnetic field direction: The right-hand rule shows that the coil creates a magnetic field perpendicular to the electron velocity creating a centripetal force

Since $\beta < 0.05$, relativistic corrections are not necessary

Vector sketch:



Conclusion

The experiment successfully measured the charge to mass ratio of the electron. The final weighted mean value achieved from the collected data was approximately:

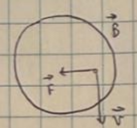
$$\frac{e}{m} = 1.985 \times\ 10^{11}\ \pm\ 2.175\ \times 10^{8}\ \text{C/kg}$$

which is 12.9% of difference in value. While uncertainties caused the difference in accuracy, the results confirm the order of magnitude and validate the classical nonrelativistic method to determine e/m

Notes from class

E/M Ratio concepts

1) direction of $\vec{B}, \vec{F}, \vec{v}$
2) $\vec{B}$
3) eV as unit of energy
4) rel. non-relativistic



$\vec{B}$ Idirec

$\vec{F} = (q\vec{v}) \times \vec{B}$

B.S law (Ampere's law)

$$B = \frac{\mu_0}{4\pi} \int \frac{\hat{r} \times I d\vec{l}}{r^2} \qquad \hat{r} = \frac{\vec{r}}{|r|}$$

$$B = \frac{\mu_0 N I}{r} \left(\frac{4}{5}\right)^{3/2}$$

$K = (\gamma - 1) mc^2 =$

e.g.

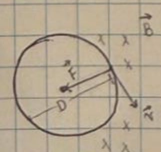
$\cancel{\gamma} = \gamma - 1 = \frac{K}{mc^2} = \frac{100}{511 \times 10^3}$

Data:

diameter coil: 30.5 ± 0.5 cm
distance between coils: 16 ± 1 cm

V = 200, 250, 300, 400

① V = 200V ±1: I = 2.19 ± 2.5
d = 5



$\vec{F} = q\ \vec{v} \times \vec{B}$

Notes from class (recoding of data)

V = 200 ± 0.5V

I	d
2.19	5 ± 0.125
1.84	6 ± 0.2
1.61	7 ± 0.25
1.41	8 ± 0.3
1.25	9 ± 0.45

V = 250 ± 0.5V

I	d
2.43	5 ± 0.15
2.11	6 ± 0.25
1.83	7 ± 0.3
1.61	8 ± 0.40
1.44	9 ± 0.5

V = 300 ± 0.5V

I	d
2.79	5 ± 0.12
2.35	6 ± 0.2
2.03	7 ± 0.25
1.78	8.0 ± 0.3
1.59	9.0 ± 0.4

④ V = 400 ± 0.5 V

I	d
2.98	5.5 ± 0.20
2.76	6.0 ± 0.20
2.53	6.5 ± 0.25
2.37	7.0 ± 0.25
2.22	7.5 ± 0.27
2.09	8.0 ± 0.30
1.95	8.5 ± 0.30
1.85	9.0 ± 0.32
1.77	9.5 ± 0.40
1.68	10.0 ± 0.45

① V = 200 ± 0.5V

I	d
2.19	5.0 ± 0.12
~~1.84~~ 2.00	5.5 ± 0.12
1.84	6.0 ± 0.20
1.72	6.5 ± 0.20
1.61	7.0 ± 0.25
1.49	7.5 ± 0.27
1.41	8.0 ± 0.3
1.31	8.5 ± 0.4
1.25	9.0 ± 0.45
1.19	9.5 ± 0.50

② V = 250 ± 0.5V

I	d
2.43	5.0 ± 0.15
2.30	5.5 ± 0.20
2.11	6.0 ± 0.25
1.95	6.5 ± 0.27
1.83	7.0 ± 0.30
1.72	7.5 ± 0.35
1.61	8.0 ± 0.40
1.51	8.5 ± 0.45
1.44	9.0 ± 0.5
1.34	9.5 ± 0.5

③ V = 300 ± 0.5

I	d
2.79	5.0 ± 0.12
2.49	5.5 ± 0.20
2.35	6.0 ± 0.20
2.15	6.5 ± 0.25
2.03	7.0 ± 0.25
1.88	7.5 ± 0.30
1.78	8.0 ± 0.30
1.67	8.5 ± 0.40
1.59	9.0 ± 0.40
1.51	9.5 ± 0.50

diameter of coil: 30.5 ± 0.50
distance between coils: 16 ± 1
beam thickness: 0.25
1.7

References

Boeglin, W. U. The e/m ratio — Modern lab experiments documentation. Florida International University. Retrieved September 22, 2025, from https://wanda.fiu.edu/boeglinw/courses/Modern_lab_manual3/em_ratio.html