Counting Statistics

Course: PHY 3802L Intermediate Physics Lab

Experiment: Counting Statistics

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Introduction and Purpose

We investigate the randomness of radioactive decay using a Sr-90 source and a GM counter to see Poisson behaviours at low counts and Gaussian behaviour at high counts. This reveals how statistical fluctuations ser uncertainty, guide stable detector operations through the plateau region, and confirms the inverse square falloff for a point source in practical radiation measurements.

We recorded counts in fixed intervals at low and high rates, measured the background "noise" by setting it over a long interval, scanned counts vs detector voltage for the plateau, and took counts vs distance for geometry tests. From the background corrected rate plotted against $1/r^2$ we could determine the source strength or activity.

Theory

The governing relation for the corrected counting rate is

$$R_{\text{corr}}(r) = S_{\text{Sr}} \cdot \frac{\pi d^2}{4} \cdot \frac{1}{4\pi r^2} \cdot (3.7 \times 10^{10}),$$

which comes from an isotropic point source, geometric capture by a circular detector window of diameter d, and activity in curies converted by 3.7×10^{10} s^{-1} Ci^{-1}.

Any linearization fitted to the data takes the form

$$R_{\text{corr}} = K \cdot \frac{1}{r^2} + b,$$

where b should be near zero after background subtraction. The fit slope is

$$K = S_{\text{Sr}} \cdot \frac{d^2}{16} \cdot (3.7 \times 10^{10}).$$

From this relation, the activity of the source is derived as

$$S_{\text{Sr}} = \frac{16K}{(3.7 \times 10^{10})d^2}.$$

The error in the activity, propagated from uncertainties in the slope K and detector diameter d, is

$$\sigma_{S_{\text{Sr}}} = S_{\text{Sr}} \sqrt{(\frac{\sigma_K}{K})^2 + (\frac{2\sigma_d}{d})^2}.$$

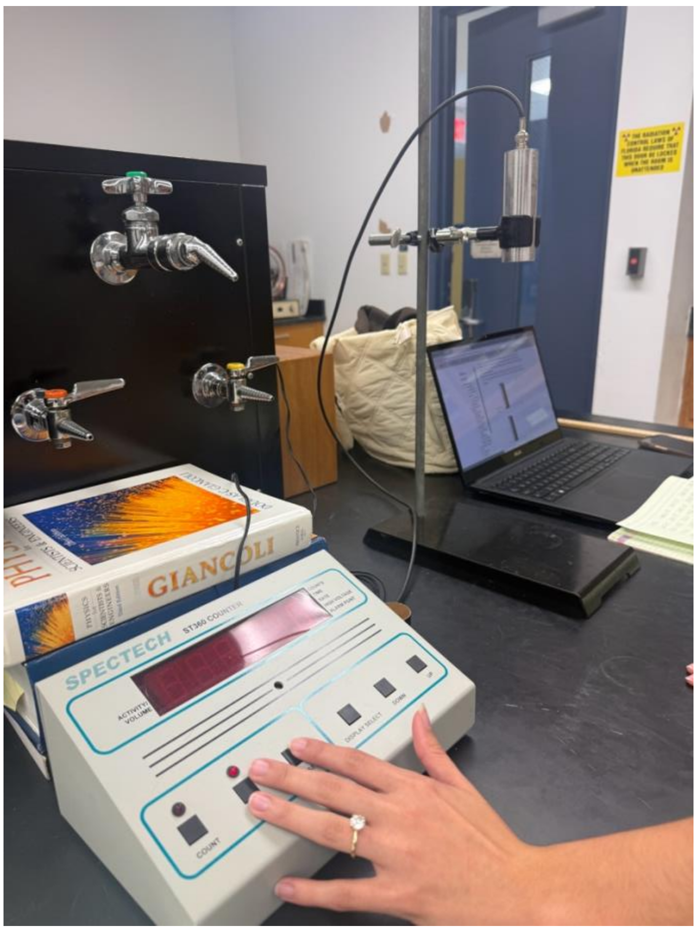
Definitions and units:

- R_{corr}: corrected count rate [s^{-1}] after subtracting measured background
- r: distance from source to detector window plane [cm]
- d: detector window diameter [cm]
- K: slope from the fit of R_{corr} versus $1/r^2$ [counts·cm²/s]
- b: fitted intercept [s $^{-1}$]
- S_{Sr}: activity of the Sr-90 source [Ci]
- Constant 3.7×10^{10}: converts Ci to decays per second [s^{-1} Ci^{-1}]

Assumptions: isotropic point source, circular detector aperture of area $\pi d^2/4$ at distance r, constant intrinsic efficiency over the relevant beta energies, negligible dead time, accurate axial alignment, and a stable background that was measured and subtracted.

Apparatus

We used a Sr-90 beta source on a track facing a GM tube with a thin end window, connected to a high-voltage supply and a digital scaler/timer. We set the HV on the counter's plateau, then recorded 200 low-count runs and 200 high-count runs in fixed time windows. We measured source-to-window distance along the detector axis at six positions with a ruler and took a 300 s background run with the source removed. For geometry, we used the detector window diameter (1in) $d\ =\ 2.54$ for the circular aperture area in the solid-angle model.



Procedure

Plateau

The GM tube was connected to the counter, a 10 s gate was used, and the high voltage was increased in small steps while counts per gate were recorded until a clear plateau was observed. An operating voltage within the flat region was selected and was held fixed for all subsequent runs.

Low counts

With the operating voltage fixed, the source to detector geometry was adjusted to yield only a few counts per window. The integration time was kept constant and 200 consecutive measurements were recorded without changing any settings.

High counts

The geometry was then moved closer to raise the rate to roughly a hundred counts per window. The same gate time and electronics settings were kept, and another set of 200 measurements was collected.

Distance dependence

With the operating voltage fixed at the plateau value, six source to window separations along the detector axis were set and measured. At each distance the alignment was checked and nearby scatterers were kept clear, then one measurement was taken using a gate time chosen to keep reasonable statistics while all other settings were held constant.

Background

The source was removed from the setup while the operating voltage and timing were left unchanged. A single 300 s background run was recorded for later subtraction.

Data

Plateau study counts vs voltage

t = 10 s

V [V]	N [counts]
700	0
720	980
740	1846
760	1838
780	1989
800	2116
820	2091
840	2172

Low-count runs (one-second windows)

t=1 s

V=780

d=18cm

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	5	5	5	6	2	5	3	3	8	1	1	2	6	4	4	4	5	3
1	2	6	1	2	4	3	5	3	2	4	2	2	4	6	3	4	4	2	4
3	3	1	5	4	2	5	4	6	7	3	4	8	0	5	2	6	2	5	6
2	4	2	3	7	3	3	5	2	1	3	5	1	3	5	7	2	5	3	2
3	1	4	8	2	2	3	2	6	2	0	1	5	2	3	1	4	1	3	3
1	3	5	2	7	3	6	2	1	1	4	3	4	2	1	6	3	1	1	2
2	6	5	2	6	2	6	3	2	0	1	1	1	1	8	4	1	3	2	0
1	4	3	4	4	4	1	3	1	2	5	4	3	1	4	4	1	1	4	1
3	4	8	4	1	4	3	1	2	4	3	2	1	2	4	4	2	1	4	3
4	2	6	0	2	5	3	1	2	4	3	2	2	3	2	0	1	3	2	3

High-count runs (one-second windows)

t=1 s

V=780

d=3cm

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
112	119	125	111	122	123	122	127	112	107	107	125	109	116	136	102	136	111	115	110
120	101	121	116	125	131	114	129	125	122	106	130	129	128	100	120	102	113	129	126
138	122	119	107	108	123	112	104	117	110	103	130	125	120	111	121	117	111	106	122
129	112	123	100	119	115	121	113	120	117	126	108	102	118	119	127	126	123	112	113
131	117	112	133	118	115	120	118	120	114	98	121	114	115	113	120	98	138	105	118
107	127	120	105	114	122	125	110	126	133	122	113	126	129	98	118	121	102	107	112
137	124	108	114	132	116	105	136	127	113	98	140	113	121	127	86	113	105	131	106
129	108	112	93	120	117	114	130	126	102	109	119	119	116	129	105	101	124	112	116
106	109	117	116	123	109	114	120	128	121	124	130	124	128	125	119	131	123	124	129
111	114	123	116	91	139	115	113	115	96	111	104	118	117	111	125	99	118	119	113

Distance dependence (single measurement at each position)

V=780

d [cm]	t [s]	N [counts]
10.0	20	227
15.0	30	209
22.0	65	202
26.0	90	200
29.0	119	202
32.0	130	199

Background 'noise'

t=300s (5min)

N(counts)=120

Corresponding to $R_B = 0.400 \pm 0.036\ \mathrm{s}^{-1}$

Outliers excluded:

None.

Analysis

Figure 1. **Plateau study**

Counts rose with voltage and then flattened between about 740 V and 840 V. We chose the point where 780 V so small voltage drifts would not change the rate much. This confirms the GM tube was run on its plateau.

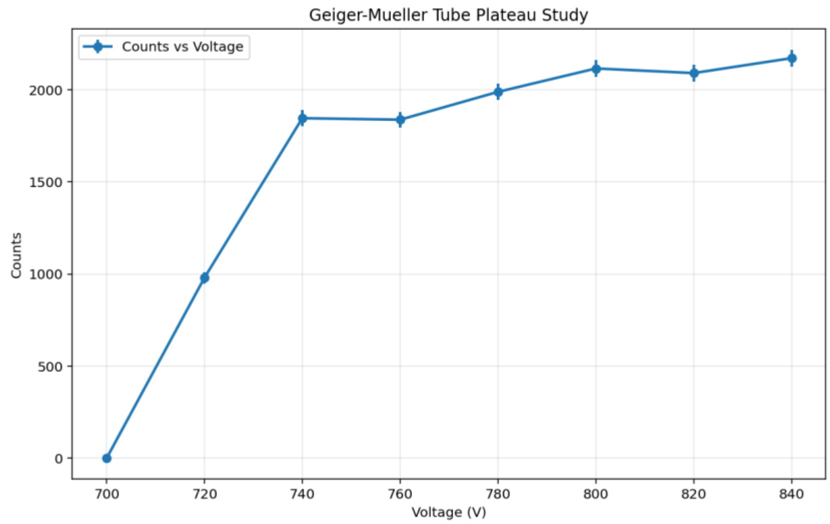


Figure 2. **Low-statistics distribution**

For 200 one-second windows at low rate, the histogram peaks near 3 counts and is well described by a Poisson model with mean about 3.12. The variance is about equal to the mean, so $\sigma^2/\mu \approx 1.065$. This is the expected behavior when events are independent.

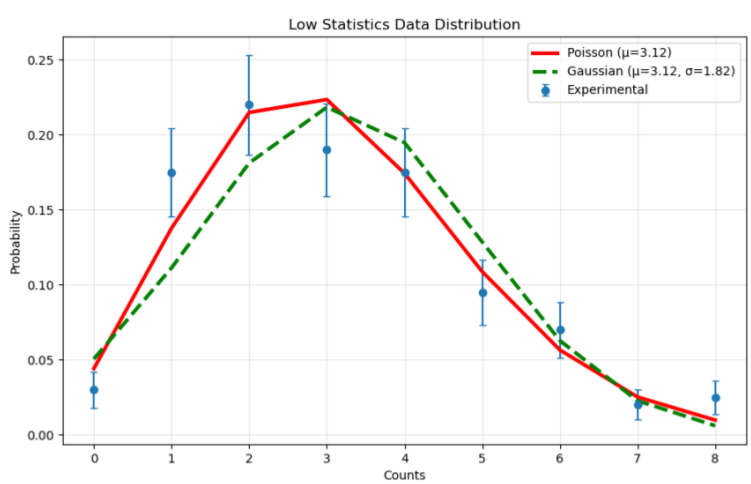


Figure 3. **High-statistics distribution**

For 200 one-second windows at high rate, the histogram centers near 117 counts and follows a Gaussian shape. The variance is about 99.7 and $\sigma^2/\mu \approx 0.851$, consistent with the normal approximation that emerges at large means.

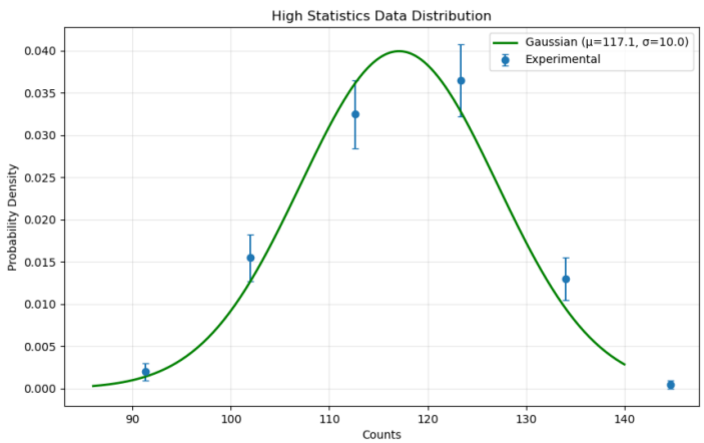
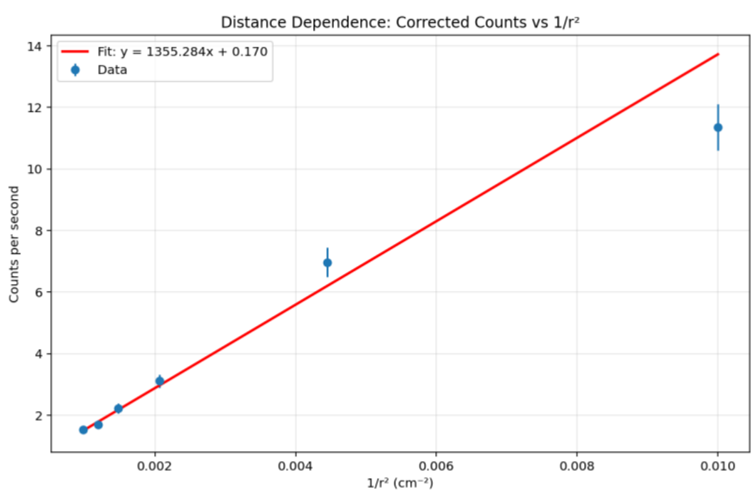


Figure 4. **Distance dependence with fit**

Rates were corrected using the background run (N = 120 counts in 300 s, R_B = 0.400 ± 0.036 s^{-1}). Plotting corrected rate against $1/r^2$ gave an approximately linear trend. A weighted linear fit yielded slope K = 1355 ± 71 counts·cm²/s and intercept b = 0.170 ± 0.119 s^{-1}, with reduced $\chi^2 \approx$ 3.40. The positive slope supports the $1/r^2$ geometric dependence. The small intercept is statistically compatible with zero within uncertainty.



The background measurement gave $N = 120$ counts in 300 s, corresponding to $R_B = 0.400 \pm 0.036\ \mathrm{s}^{-1}$. This value was subtracted from all geometry runs before fitting.

Discussion

The manual states that low count statistics follow Poisson with $\sigma^2 = \mu$, high count data approach a Gaussian for large μ, and an isotropic point source gives $R \propto 1/r^2$ so the linearized model after background removal is $R_{\text{corr}} = K(1/r^2) + b$ with $b = 0$ expected. Our low statistics set gave $\sigma^2/\mu = 1.065$, which is a +6.5% deviation from the Poisson prediction and consistent within counting scatter for small means. Our high statistics set gave $\sigma^2/\mu = 0.851$, a −14.9% deviation from 1.000, still with a histogram well modeled by a Gaussian as expected at large means. The inverse square fit yielded $K = 1355 \pm 71$ counts·cm²/s and $b = 0.170 \pm 0.119$ s $^{-1}$; the intercept differs from zero by 1.43 σ and its 95% interval includes zero, so it is statistically consistent with the accepted expectation $b = 0$ despite a reduced $\chi^2 = 3.40$ that points to extra scatter from small systematics such as distance or alignment

The plots show expected behavior: counts level off from 740 to 840 V, the low rate histogram follows Poisson with $\sigma^2/\mu = 1.065$, the high rate histogram follows a Gaussian with $\sigma^2/\mu < 1$, and the rate is linear in $1/r^2$ with slope K = 1355 ± 71 counts·cm² s^{-1} and intercept 0.170 ± 0.119 s^{-1}, consistent with zero.

Conclusion

We verified Poisson counting statistics at low rates, Gaussian behavior at high rates, and the inverse-square dependence of radiation intensity from a Sr-90 source using a GM tube. From the linear fit of corrected rate versus $1/r^2$, we obtained a slope $K = 1355 \pm 71$ counts\cm²/s and intercept $b = 0.170 \pm 0.119$ s^{-1}, statistically consistent with zero. The measured background rate was $R_B = 0.400 \pm 0.036$ s^{-1}. Using the fitted slope and detector geometry ($A_{\text{det}} = 5.07$ cm^2), the source activity was estimated as

$$A \approx \frac{1355}{5.07} \approx 267\ \text{Bq} \approx 7.2 \times 10^{-9}\ \text{Ci},$$

assuming near-unit detection efficiency. These results align with theoretical expectations. To improve accuracy in future runs, a rigid distance fixture and repeated background runs before and after the series should be used to minimize alignment and systematic errors.