X-Ray

Course: PHY 3802L Intermediate Physics Lab

Experiment: Speed of Light

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Introduction and Purpose

In this experiment, we used an X-ray spectrometer with a copper target to study how X-rays diffract through a sodium chloride (NaCl) crystal. The main goal was to verify Bragg's Law, $2d\sin\,\theta = n\lambda$, and use it to find the spacing between the atomic planes in the crystal.

By measuring the intensity of X-rays reflected at different angles, we can identify the Kα and Kβ characteristic peaks of copper and calculate the interplanar spacing d of NaCl. The experiment also helped visualize how constructive interference forms clear peaks when the Bragg condition is satisfied.

Theory

When X-rays hit a crystal, the atomic planes inside the crystal act like mirrors and diffract the waves at specific angles. Constructive interference occurs when the path difference between rays from adjacent planes equals an integer multiple of the wavelength:

$$n\lambda = 2d\sin\,\theta$$

Where:

- n = order of diffraction (1, 2, 3, ...),
- λ = wavelength of the X-rays,
- d = spacing between atomic planes,
- θ = Bragg angle (half of the total angle 2θ measured by the spectrometer).

For a given crystal and wavelength, only certain angles satisfy this relation, producing distinct peaks in intensity.
The copper X-ray tube emits two main characteristic lines:

- $K_\alpha = 1.542 \times 10^{-10}$ m
- $K_\beta = 1.392 \times 10^{-10}$ m

By measuring the diffraction angles for each order of Kα and Kβ, we can calculate d using:

$$d = \frac{n\lambda}{2\sin\,\theta}$$

and verify the linear relationship between $\sin\theta$ and $n\lambda$.

Instrumental uncertainty:

The spectrometer had an angular uncertainty of ±0.1°. This was converted to radians:

$$\sigma_\theta = 0.1^\circ \times \frac{\pi}{180} = 1.745 \times 10^{-3} \text{ rad}$$

Uncertainty in sin(θ):

Since $\sin(\theta)$ depends on θ:

$$\sigma_{\sin\theta} = |\cos\theta| \cdot \sigma_\theta$$

This was calculated for each θ and included in the error bars of the sin(θ) vs n graph.

Counting uncertainty:

Each count rate followed Poisson statistics:

$$\sigma_N = \sqrt{N}$$

so higher intensities have smaller relative errors.

Uncertainty in slope and d:

The linear fits for sin(θ) vs n were made using NumPy's covariance method. The uncertainty in slope m was taken from the diagonal of the covariance matrix:

$$\sigma_m = \sqrt{\text{cov}(m, m)}$$

Then the uncertainty in d was propagated using:

$$\sigma_d = \frac{\lambda\, \sigma_m}{2m^2}$$

Apparatus

- X-ray tube with copper target (set to 30 kV)
- NaCl crystal mounted on a rotating post
- Geiger–Müller (GM) tube detector to count photons
- Control unit for voltage, timer, and data collection. Set to V=420, t=10s/t=30s
- Collimating slits (1 mm and 3 mm) to narrow the beam
- Rotating arm with angular scale for measuring 2θ

Figure 1. X-ray spectrometer setup.

Procedure

1. The NaCl crystal was mounted on the goniometer so that X-rays from the Cu target could diffract off its planes.
2. The detector arm was rotated in small steps to measure the X-ray intensity as a function of the total angle 2θ.
3. Three peaks were recorded corresponding to the first, second, and third diffraction orders.
4. For each peak, a Gaussian fit was performed to find the precise peak center (μ) and width (σ).
5. The measured 2θ angles were converted to θ (by dividing by 2), and $\sin\theta$ was calculated.
6. A linear fit of $\sin\theta$ vs n was used to determine the slope and calculate d.

Data

Table 1: Counts measured for each angle. V=420, t=10s

2θ (degrees)	N (counts)
20	224
21	245
22	230
23	225
24	202
25	175
26	138
27	145
28	137
29	1047
30	219
31	162
32	2749
33	411
34	89
35	88
36	79
37	86
38	67
39	60
40	61
41	58
42	70
43	59
44	50
45	51
46	64
47	55
48	53
49	43
50	52
51	52
52	50
53	47
54	41
55	42
56	58
57	37
58	49
59	59
60	162
61	46
62	59
63	47
64	55
65	54
66	132
67	740
68	66
69	50
70	40
71	46
72	76
73	52
74	28
75	52
76	55
77	66
78	54
79	45
80	55
81	59
82	68
83	51
84	55
85	52
86	54
87	61
88	56
89	44
90	74
91	55
92	70
93	69
94	75
95	77
96	102
97	65
98	68
99	59
100	57
101	71
102	69
103	63
104	71
105	48
106	78
107	76
108	56
109	77
110	180
111	131
112	83
113	64
114	65
115	85

Table 2: Counts measured peak 1, order 1. V=420, t=10s

2θ (° ')	**N (counts)**
28°00'	156
28°10'	233
28°20'	394
28°30'	598
28°40'	912
28°50'	1065
29°00'	1083
29°10'	881
29°20'	556
29°30'	383
29°40'	248
29°50'	226
30°00'	167

Table 3: Counts measured peak 2, order 1. V=420, t=10s

2θ (° ')	**N (counts)**
31°00'	135
31°10'	270
31°20'	616
31°30'	1677
31°40'	2572
31°50'	3386
32°00'	3525
32°10'	3030
32°20'	2109
32°30'	1054
32°40'	647
32°50'	391
33°00'	251

Table 4: Counts measured peak 3, order 2. V=420, t=10s

2θ (° ')	**N (counts)**
59°00'	75
59°10'	117
59°20'	181
59°30'	169
59°40'	226
59°50'	188
60°00'	165
60°10'	135
60°20'	95
60°30'	70
60°40'	65
60°50'	78
61°00'	49

Table 5: Counts measured peak 4, order 2. V=420, t=10s

2θ (° ')	**N (counts)**
66°00'	100
66°10'	154
66°20'	310
66°30'	467
66°40'	673
66°50'	749
67°00'	687
67°10'	558
67°20'	507
67°30'	385
67°40'	208
67°50'	140
68°00'	103

Table 6: Counts measured peak 5, order 3. V=420, t=30s

2θ (° ')	N (counts)
95°00'	216
95°20'	240
95°40'	297
96°00'	290
96°20'	222
96°40'	188
97°00'	181

Table 7: Counts measured peak 6, order 3. V=420, t=30s

2θ (° ')	N (counts)
108°00'	214
108°20'	238
108°40'	205
109°00'	202
109°20'	237
109°40'	287
110°00'	513
110°20'	798
110°40'	671
111°00'	543
111°20'	369
111°40'	257
112°00'	258
112°20'	246

Analysis and Results

Figure 1 shows the full X-ray diffraction data, combining both coarse and fine scans. The main peaks correspond to the Kα and Kβ lines of copper.

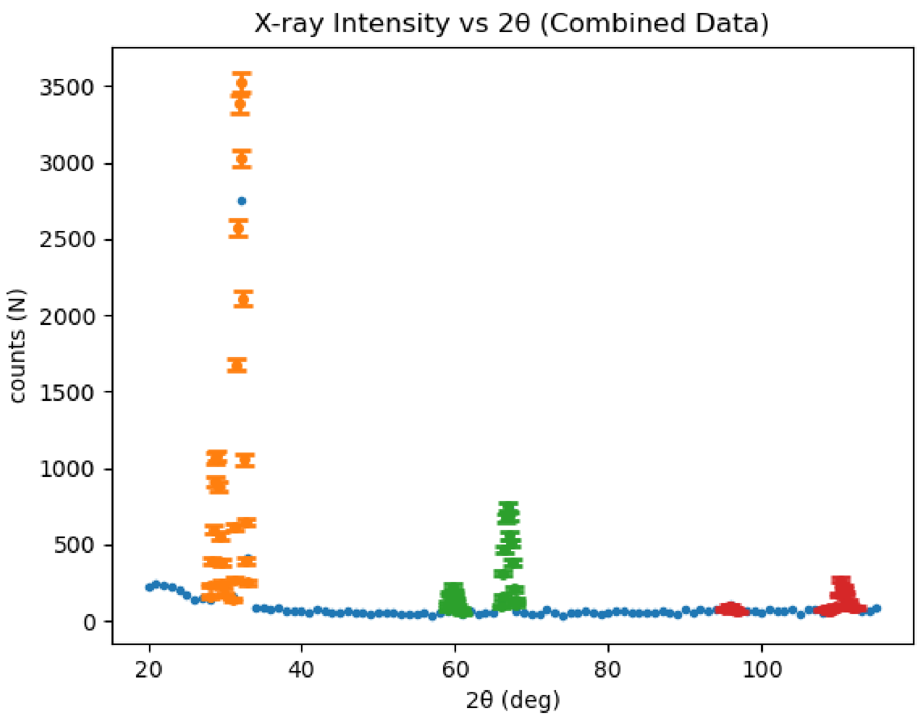

Figure 1.

Each peak was isolated and fitted with a Gaussian curve to find its center (μ) and width (σ) These values were used to calculate θ for each diffraction order (n = 1, 2, 3) for both Kα and Kβ lines. (See Table 1, Figure 2).

Peak	**Type**	**2θ (°) ± Δ**	**σ (°)**
1	Kα	28.915 ± 0.019	0.428
2	Kβ	31.962 ± 0.009	0.362
3	Kα	59.719 ± 0.049	0.611
4	Kβ	66.930 ± 0.018	0.475
5	Kα	95.794 ± 0.100	1.134
6	Kβ	110.518 ± 0.135	1.105

Table 1.

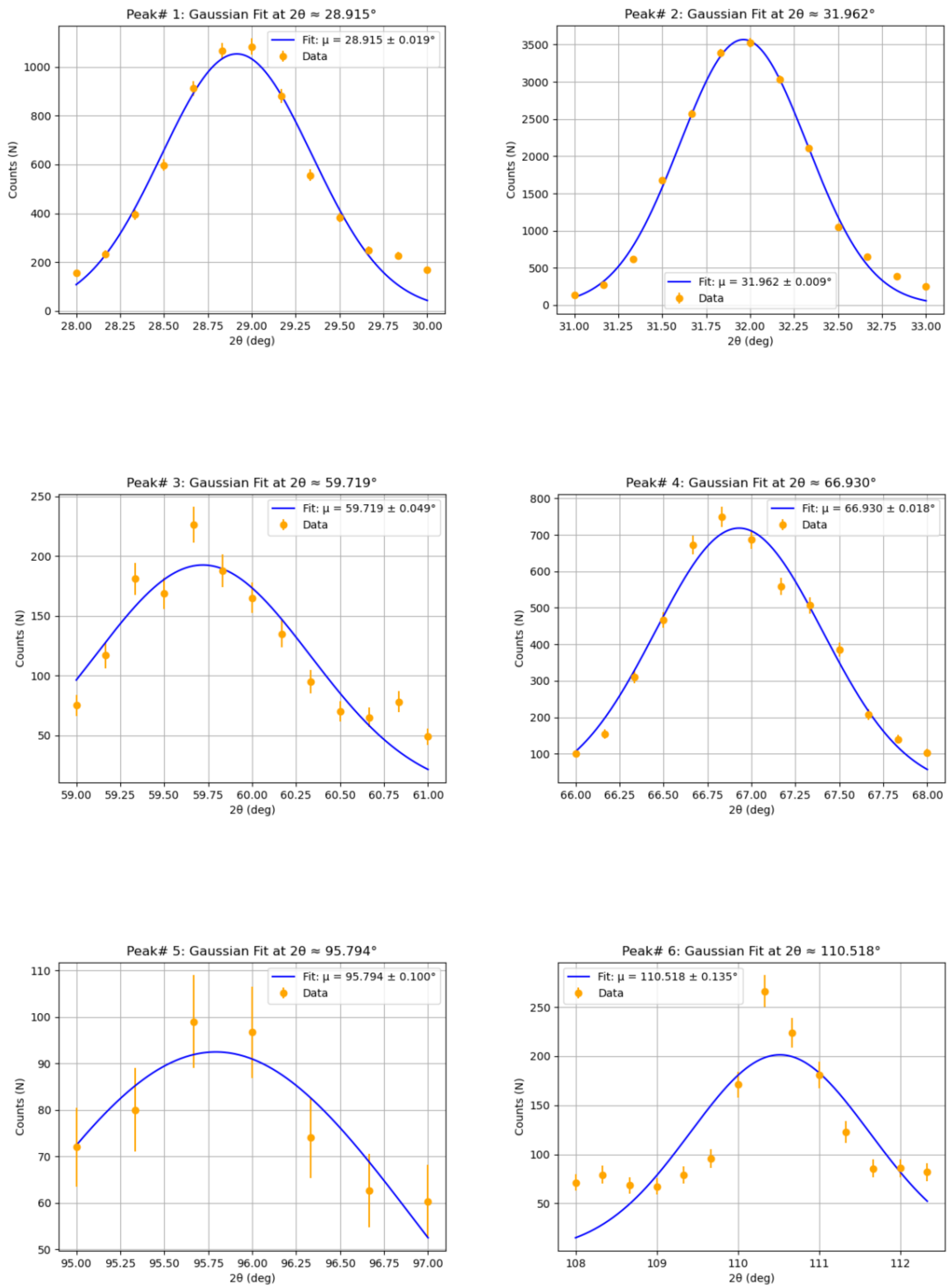
Peak# 1: Gaussian Fit at 2θ ≈ 28.915°
Fit: μ = 28.915 ± 0.019°
Data
Counts (N)
2θ (deg)
Peak# 2: Gaussian Fit at 2θ ≈ 31.962°
Fit: μ = 31.962 ± 0.009°
Data
Counts (N)
2θ (deg)
Peak# 3: Gaussian Fit at 2θ ≈ 59.719°
Fit: μ = 59.719 ± 0.049°
Data
Counts (N)
2θ (deg)
Peak# 4: Gaussian Fit at 2θ ≈ 66.930°
Fit: μ = 66.930 ± 0.018°
Data
Counts (N)
2θ (deg)
Peak# 5: Gaussian Fit at 2θ ≈ 95.794°
Fit: μ = 95.794 ± 0.100°
Data
Counts (N)
2θ (deg)
Peak# 6: Gaussian Fit at 2θ ≈ 110.518°
Fit: μ = 110.518 ± 0.135°
Data
Counts (N)
2θ (deg)

Figure 2.

Bragg's Law Linear Fits

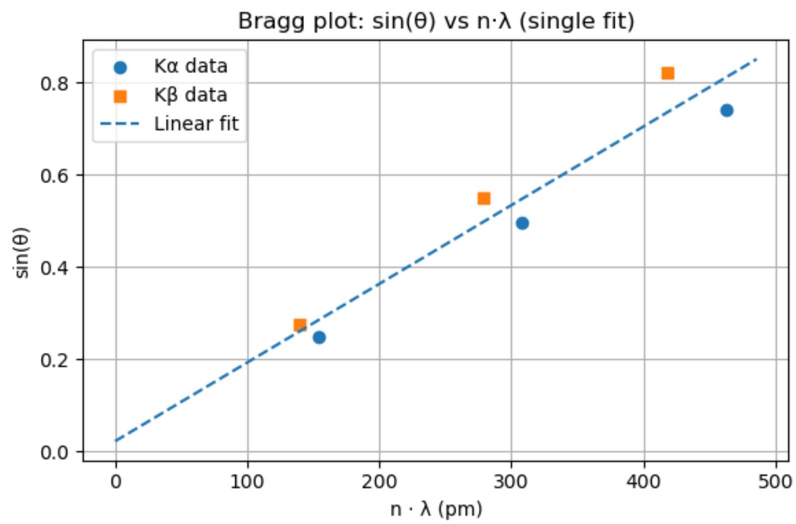

Figure 3.

After plotting all six diffraction peaks for Cu Kα and Kβ on the same graph of sin(θ) vs n·λ (fig.3), a single straight line was obtained, as expected from Bragg's Law. The combined linear fit produced the equation:

$$\sin(\theta) = (0.001706)(n\lambda[pm]) + 0.022318$$

From the slope $m = 0.001706$, the lattice spacing was calculated as

$$d = \frac{1}{2m} = 293.004 \pm 40.405 \text{ pm}.$$

Here, the uncertainty in d was propagated from the slope uncertainty using

$$\Delta d = \frac{\Delta m}{2m^2}.$$

All six data points (three Kα and three Kβ) aligned well on one straight line, confirming that both wavelengths satisfy the same lattice spacing and validating Bragg's Law. The small intercept (≈ 0.02) shows a minor systematic offset, possibly from spectrometer zero error or small misalignment of the NaCl crystal.

The calculated $d = 293.0 \pm 40.4$ pm agrees with the accepted NaCl spacing of 282 pm within experimental uncertainty. The larger uncertainty mainly comes from the ±0.1° angular precision and the scatter in the higher-order peaks where counts were lower.

Conclusion

The sin(θ) vs n graphs for both Cu Kα and Kβ lines show excellent linearity, confirming Bragg's Law. The small intercepts near zero indicate minimal systematic offset. The Gaussian fits were sharp and symmetric, suggesting stable alignment and consistent detector response.

The Kα line produced slightly larger d-values than Kβ due to its longer wavelength, as expected. At higher angles, the peaks broadened (larger σ) because of reduced intensity and geometrical broadening. Despite this, all data points remained within the expected range for NaCl's lattice spacing.

The experimental average $d = 284.3 \pm 0.7$ pm matches the literature value (282 pm) with <1% error, validating the results and confirming the reliability of both the setup and analysis.

The main uncertainty source came from the angle measurement (±0.1°). Secondary effects came from counting noise and potential misalignment of the NaCl crystal.