National University of Computer & Emerging Sciences, Karachi
Spring-2025 FAST School of Computing
Probability and Statistics

Assignment 2

1. A tobacco company produces blends of tobacco, with each blend containing various proportions of Turkish, domestic, and other tobaccos. The proportions of Turkish and domestic in a blend are random variables with joint density function (X = Turkish and Y = domestic):

$$f(x, y) = 24xy, \quad 0 \leq x, y \leq 1, \quad x + y \leq 10, \text{ elsewhere.}$$

 (a) Find the probability that in a given box the Turkish tobacco accounts for over half the blend.
 (b) Find the marginal density function for the proportion of the domestic tobacco.
 (c) Find the probability that the proportion of Turkish tobacco is less than 1/8 if it is known that the blend contains 3/4 domestic tobacco.

2. We are given the joint probability density function (PDF) of two electronic components' lifetimes, X and Y:

$$f(x, y) = \begin{cases} ye^{-y(1+x)}, & x, y \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

 We will determine:

 (a) The marginal density functions of both random variables.
 (b) The probability that both components last more than 2 hours, i.e. $P(X > 2, Y > 2)$.

3. Consider the following joint probability density function of the random variables X and Y.

$$\begin{cases} \dfrac{3x - y}{9} & 1 < x < 3\,, 1 < y < 2 \\ 0 & else\ where \end{cases}$$

 (a) Find the marginal density functions of X and Y.
 (b) Are X and Y independent?
 (c) Find $P(X > 2)$.

4. An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let X be the number of months between successive payments. The cumulative distribution function of X is given by:

$$F(x) = \begin{cases} 0, & x < 1 \\ 0.4, & 1 \le x < 3 \\ 0.6, & 3 \le x < 5 \\ 0.8, & 5 \le x < 7 \\ 1.0, & x \ge 7 \end{cases}$$

(a) What is the probability mass function (PMF) of X?
(b) Compute $P(4 < X \le 7)$.

5. Consider the random variables X and Y that represent the number of vehicles that arrive at two separate street corners during a certain 2-minute period. These street corners are close together, so it is important that traffic engineers deal with them jointly if necessary. The joint distribution of X and Y is known to be:

$$f(x,y) = \frac{9}{16} \cdot \frac{1}{4^{(x+y)}}, \quad \text{for } x = 0, 1, 2, \ldots \text{ and } y = 0, 1, 2, \ldots$$

(a) Are the two random variables X and Y independent? Explain why or why not.
(b) What is the probability that during the time period in question fewer than 4 vehicles arrive at the two street corners?

6. A chemical process results in a batch of final product that contains impurities. The proportion of impurities in a batch follows the probability density function:

$$f(y) = \begin{cases} 10(1-y)^9, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Show that the given function is a valid probability density function (PDF) by verifying that it satisfies the two conditions of a PDF.
(b) A batch is considered unacceptable if the proportion of impurities exceeds 60%. What is the probability that a randomly selected batch is unacceptable?

7. There is one error in one of five blocks of a program. To find the error, we test three randomly selected blocks. Let **X** be the number of errors in these three blocks. Compute **E(X)** and **Var(X).**

8. Consider the joint density function

$$f(x,y) = \begin{cases} \frac{16y}{x^3}, & x > 2,\ 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Compute correlation coefficient ρ_{xy} .

9. According to a study published by a group of University of Massachusetts sociologists, approximately 60% of the Valium users in the state of Massachusetts first took Valium for psychological problems. Find the probability that among the next 8 users from this state who are interviewed:

(a) Exactly 3 began taking Valium for psychological problems.
(b) At least 5 began taking Valium for problems that were not psychological.

10. On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection
(a) Exactly 5 accidents will occur?
(b) Fewer than 3 accidents will occur?
(c) At least 2 accidents will occur?

11. A technician plans to test a certain type of resin developed in the laboratory to determine the nature of the time required before bonding takes place. It is known that the mean time to bonding is 3 hours and the standard deviation is 0.5 hours. The resin will be considered an undesirable product if the bonding time is either less than 1 hour or more than 4 hours.

(a) Comment on the utility of the resin.
(b) How often would its performance be considered undesirable? Assume that the time to bonding is normally distributed.

12. The IQs of 600 applicants to a certain college are approximately normally distributed with a mean of 115 and a standard deviation of 12. If the college requires an IQ of at least 95, how many of these students will be rejected on this basis of IQ, regardless of their other qualifications? Note that IQs are recorded to the nearest integers.
