Propagation of errors

Experimental measures of the number of cases gives us an uncertainty interval $\sigma_n(t)$ for each measure on a given day $n(t)$. Uncertainty of other epidemiological measures can be computed from propagation of errors theory. Given a magnitude z that is a function of two other variables x and y we can compute the uncertainty in z, σ_z, as a function of uncertainty in x, σ_x, and y, σ_y:

$$z = f(x,y) \rightarrow \sigma_z^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2. \tag{1}$$

The epidemiological indicators which we are interested in are: average case count based on last 7 days (n_7), 14-day cumulative incidence (A_{14}), empirical reproduction number (ρ_7) and Effective Potential Growth (EPG).

$$n_7(t) = \frac{1}{7}\sum_{t-6}^{t} n(t) \tag{2}$$

$$A_{14}(t) = \frac{10^5}{population}\sum_{t-13}^{t} n_7(t) \tag{3}$$

$$\rho_7(t) = \frac{n_7(t) + n_7(t-1) + n_7(t-2)}{n_7(t-5) + n_7(t-6) + n_7(t-7)} \tag{4}$$

$$EPG(t) = \rho_7(t) \cdot A_{14}(t) \tag{5}$$

Using equation 1 there can be computed uncertainty intervals for different epidemiological indices:

$$\sigma_{n_7}(t) = \frac{1}{7}\sqrt{\sum_{t-6}^{t} \sigma_n(t)^2}, \tag{6}$$

$$\sigma_{A_{14}}(t) = \frac{10^5}{population}\sqrt{\sum_{t-13}^{t} \sigma_{n_7}(t)^2}, \tag{7}$$

$$\sigma_{\rho_7}(t) = \frac{1}{n_7(t-5) + n_7(t-6) + n_7(t-7)}\sqrt{\sum_{t-2}^{t} \sigma_{n_7}(t)^2 + {\rho_7}^2 \sum_{t-7}^{t-5} \sigma_{n_7}(t)^2}, \tag{8}$$

$$\sigma_{EPG}(t) = \sqrt{A_{14}(t)^2 \cdot \sigma_{\rho_7}(t)^2 + \rho_7(t)^2 \cdot \sigma_{A_{14}}(t)^2}. \tag{9}$$

This can also be performed for proportionality coefficient (p) between EMR cases and PCR cases.

$$p = \frac{EMR}{PCR} \tag{10}$$

$$\sigma_p = \frac{1}{PCR}\sqrt{{\sigma_{EMR}}^2 + p^2 \cdot {\sigma_{PCR}}^2} \tag{11}$$