

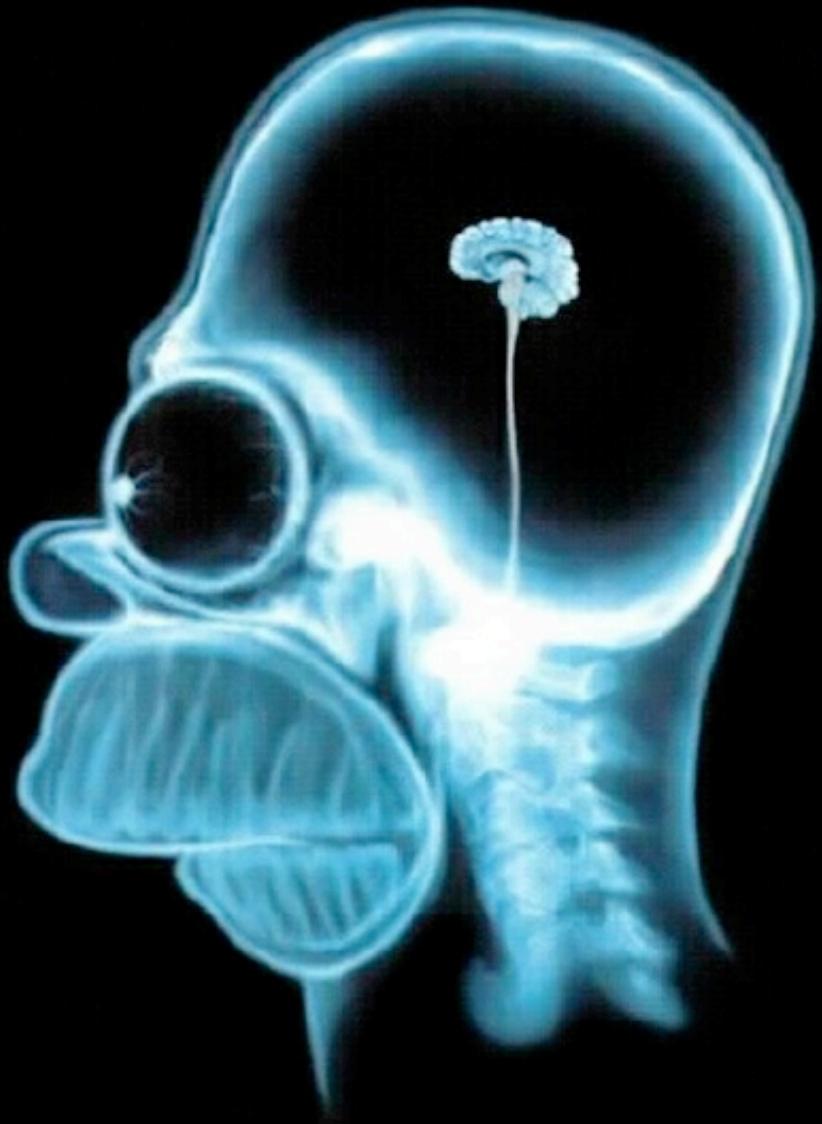
Type-checking Zero-knowledge

Cătălin Hrițcu

Saarland University, Saarbrücken, Germany

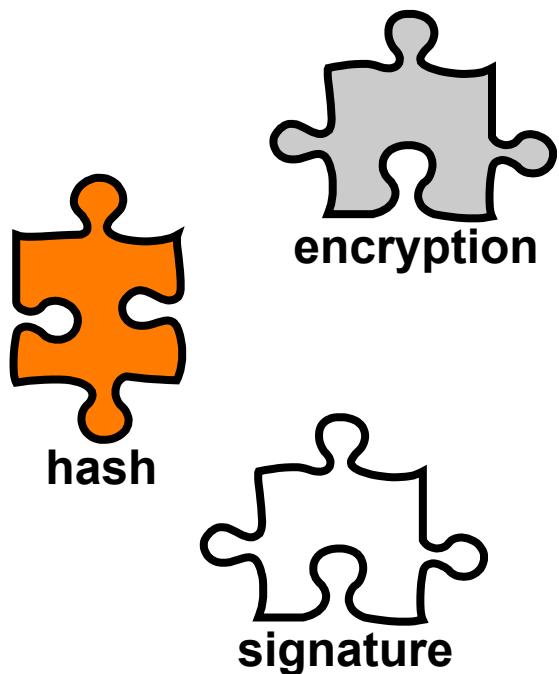
Joint work with: Michael Backes and Matteo Maffei

Zero-knowledge



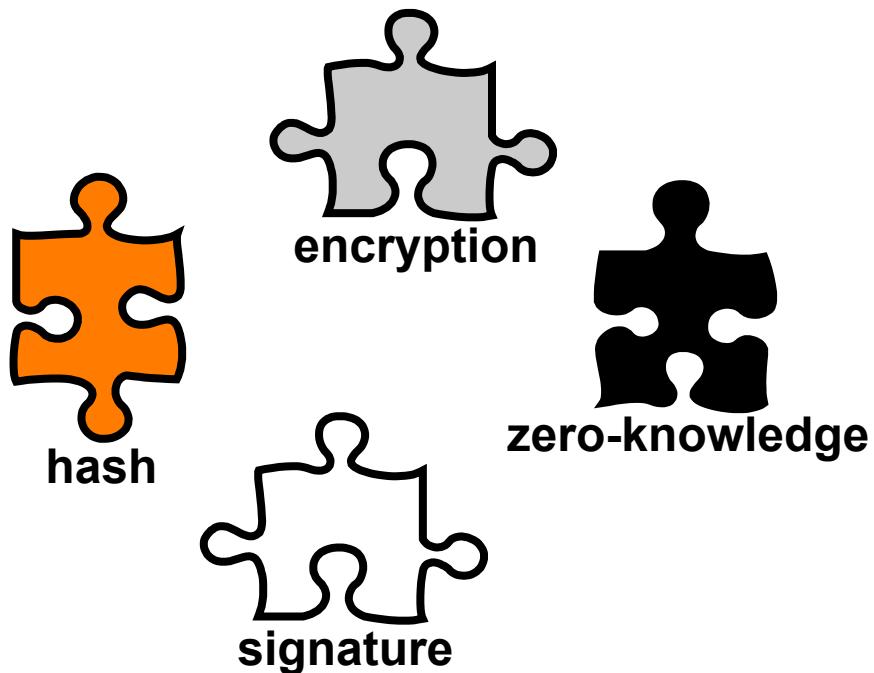
Security Protocols

designer's toolbox



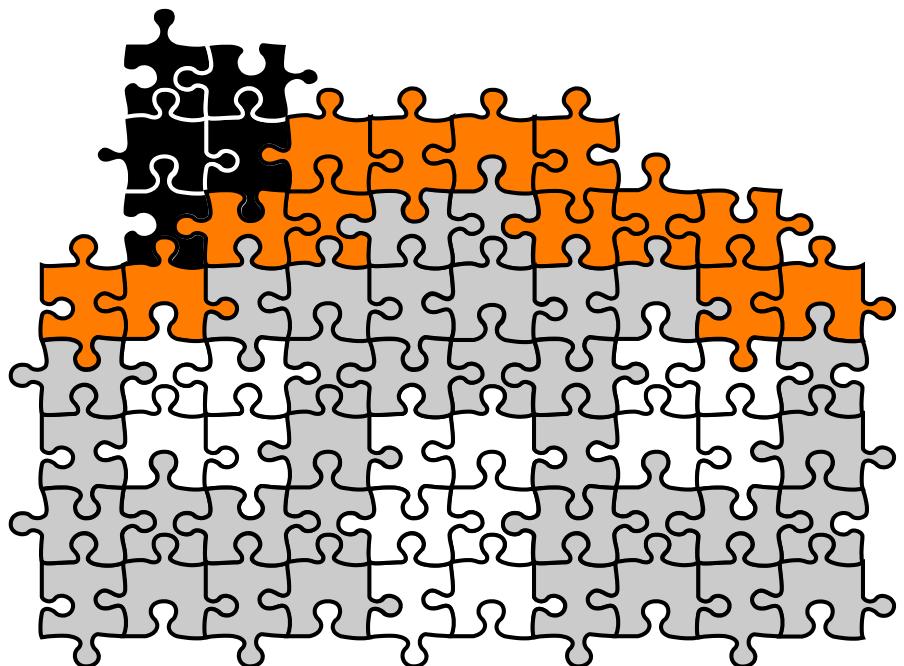
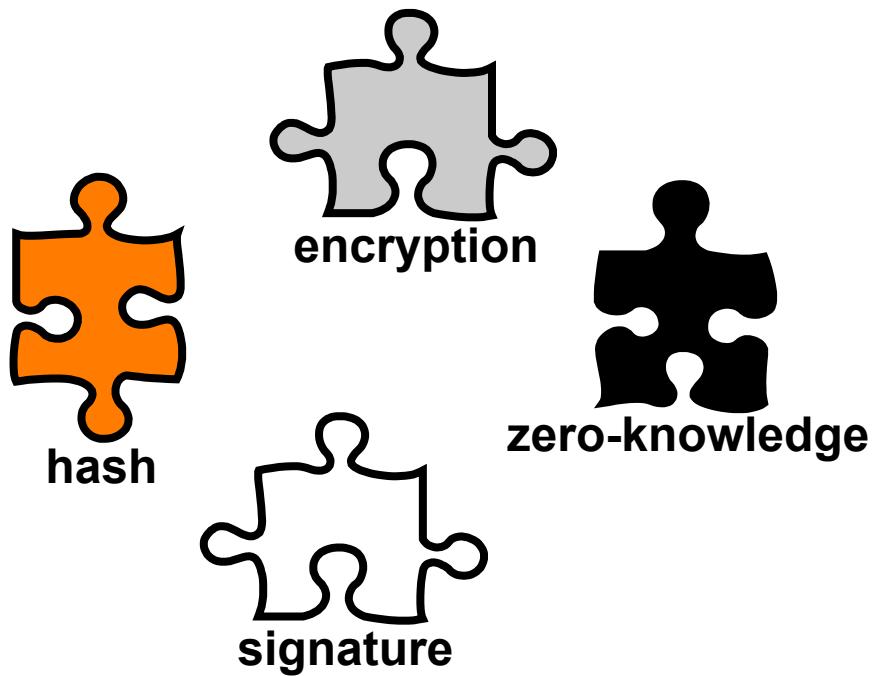
Security Protocols

designer's toolbox



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Zero-knowledge Proofs



▶ Powerful primitives

- Prove the validity of a statement without revealing anything else
 - For instance, prove to know an object with certain properties without revealing this witness

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 - Very limited practical impact

Zero-knowledge Proofs



- ▶ Powerful primitives
 - Prove the validity of a statement without revealing anything else
 - For instance, prove to know an object with certain properties without revealing this witness
- ▶ Early constructions general, but terribly inefficient
 - Very limited practical impact
- ▶ More recent research provided
 - Efficient constructions for special classes of statements
 - Constructions for non-interactive zero-knowledge



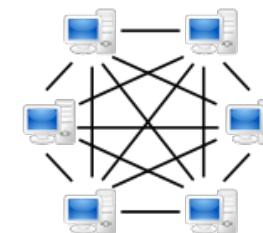
e-voting (Civitas)

Applications that use ZK

unique security features of ZK allows
designing protocols fulfilling seemingly
conflicting requirements



“trusted” computing (DAA)



p2p (PseudoTrust)



privacy



verifiability



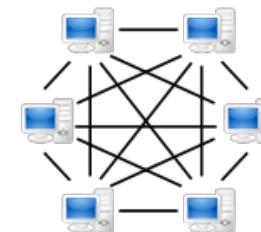
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anonymity

+

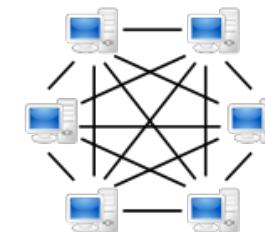


remote attestation

→



“trusted” computing (DAA)



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remote attestation



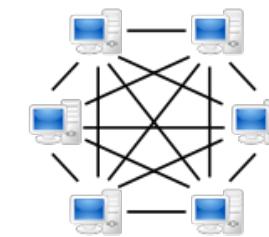
“trusted” computing (DAA)



pseudonymity



trust



p2p (PseudoTrust)

Verifying Protocols Using ZK

- ▶ No verification tool for protocols using ZK

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 - *To automatically analyze protocols using ZK in an efficient and scalable way*

Verifying Protocols Using ZK

- ▶ No verification tool for protocols using ZK
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- ▶ Automated verification can prevent errors
- ▶ Our goal
 - *To automatically analyze protocols using ZK in an efficient and scalable way*
- ▶ We built first *type system for zero-knowledge*

Type System

- ▶ Type-checking fully automated and efficient

Type System

- ▶ Type-checking fully automated and efficient
- ▶ Compositional therefore scalable



This code is safe

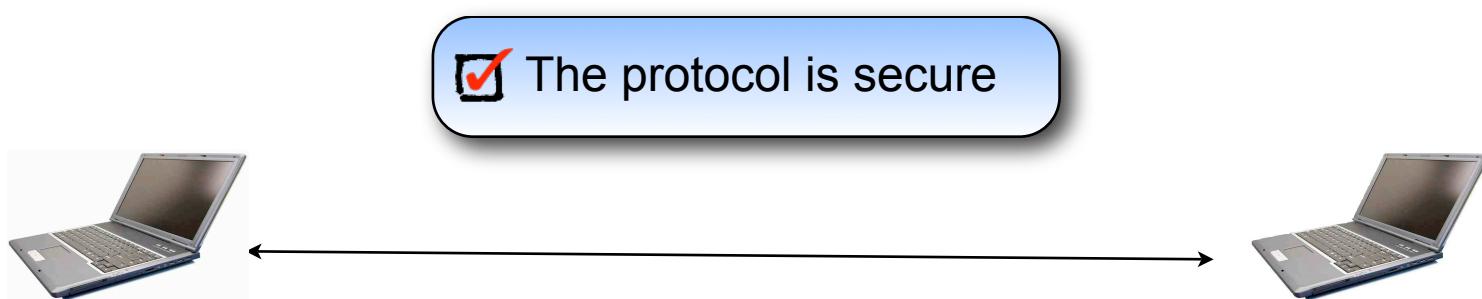


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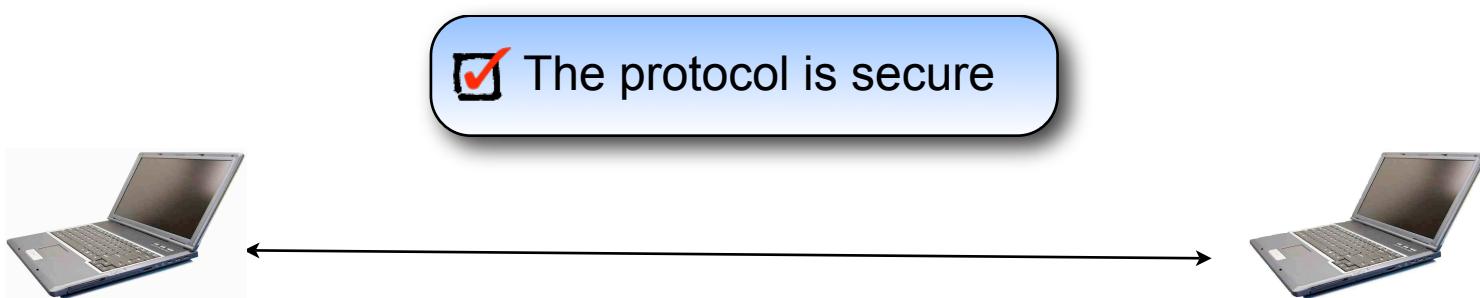
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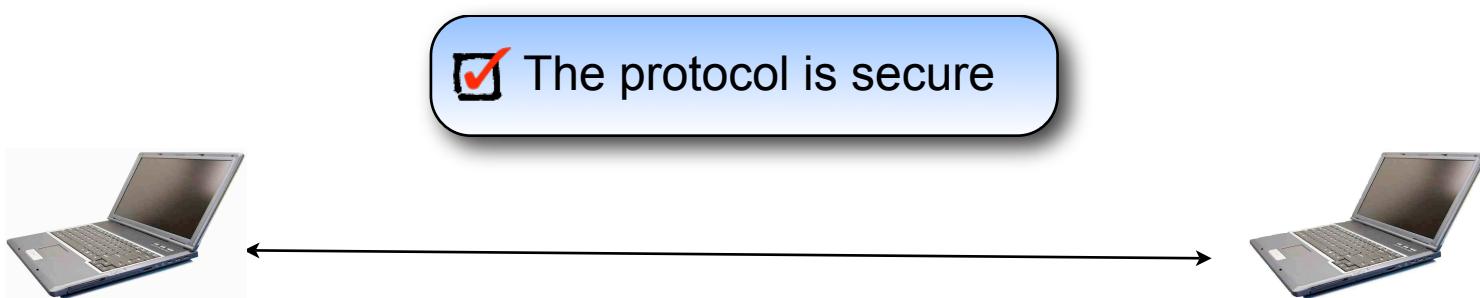
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- ▶ Predictable termination behavior

Type System

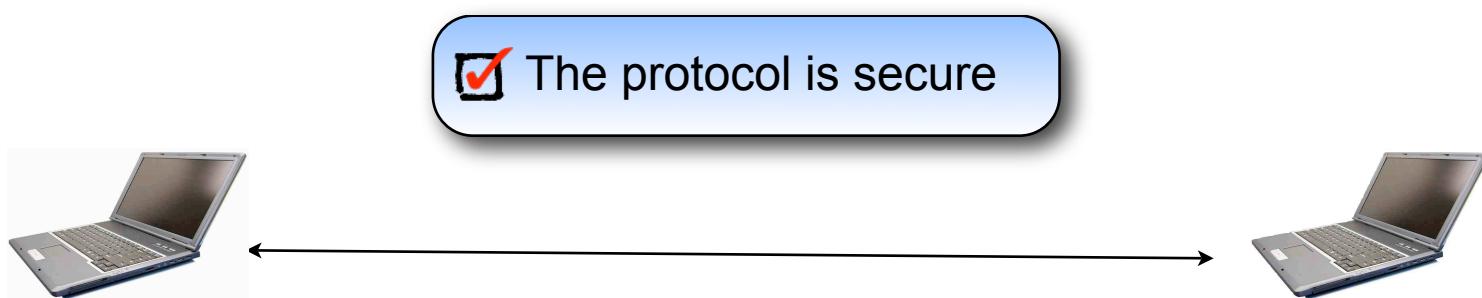
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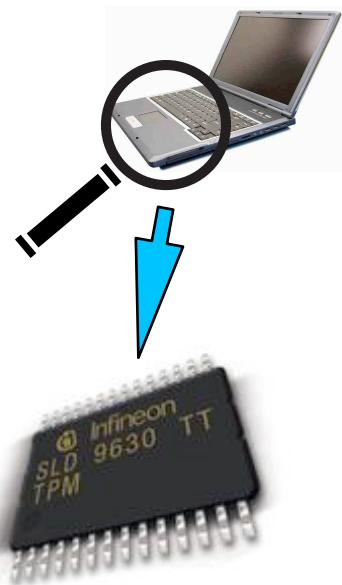


- ▶ Predictable termination behavior
- ▶ User needs to provide annotations (no free lunch)
 - ▶ in certain cases these can be automatically inferred

Type-checking DAA

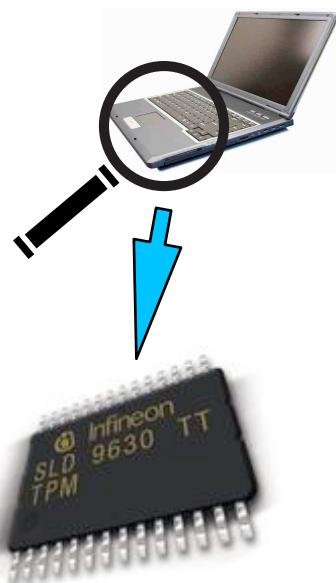
(Direct Anonymous Attestation)

DAA (Direct Anonymous Attestation)



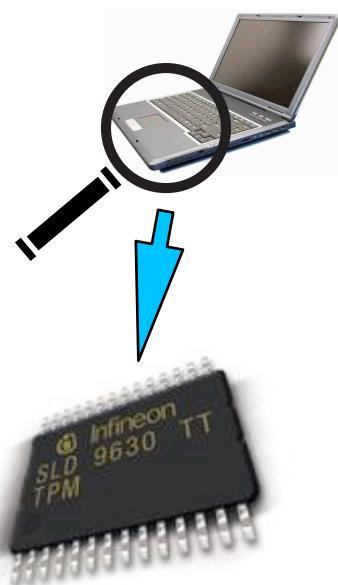
[Brickell, Camenisch & Chen, CCS 2004]

DAA (Direct Anonymous Attestation)



The user wants to authenticate a message m by proving that her platform has a valid TPM inside (*attestation*) ...

DAA (Direct Anonymous Attestation)



The user wants to authenticate a message m by proving that her platform has a valid TPM inside (*attestation*) ...

... but no other party should learn *which* TPM is used to authenticate m (*anonymity*)

Direct Anonymous Attestation (DAA)



f_{tpm}
(secret TPM identifier)

Issuer



Direct Anonymous Attestation (DAA)



Joining Protocol

The user receives a certificate of f_{TPM} from the issuer

Direct Anonymous Attestation (DAA)



sign(f_{tpm}, k_I)

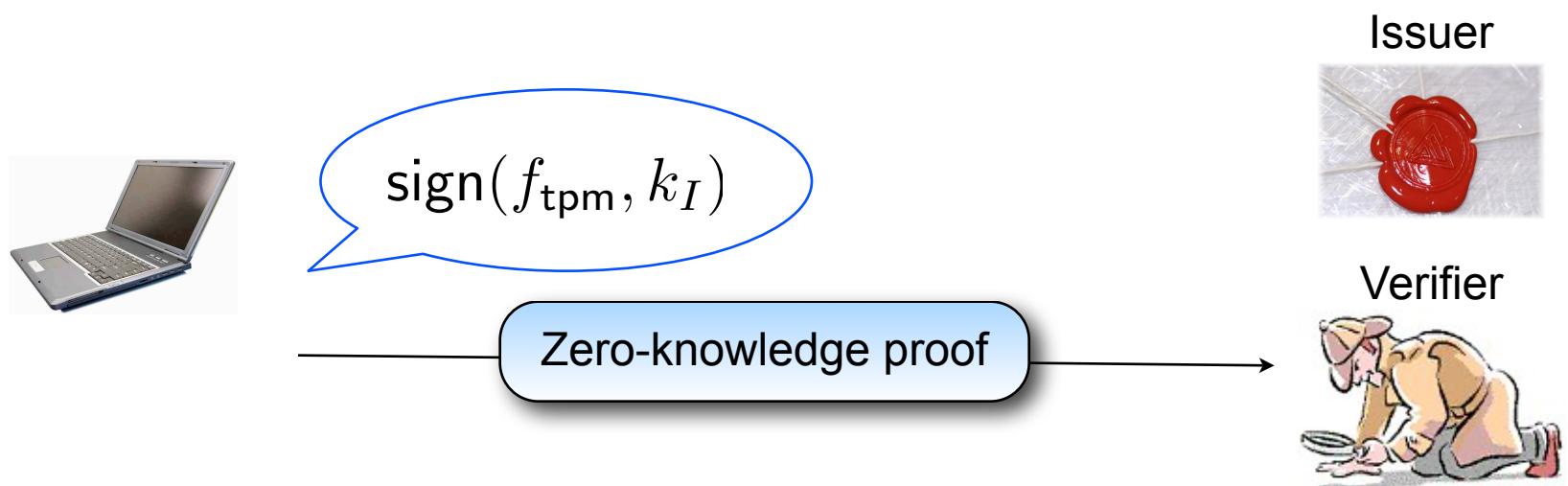
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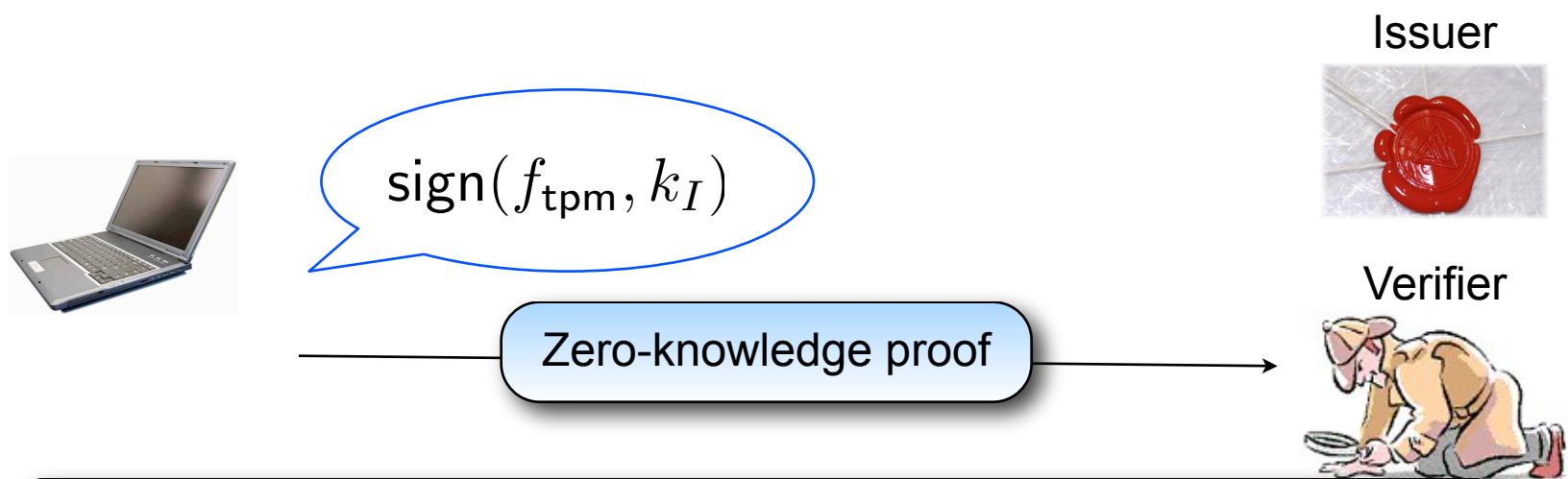
Direct Anonymous Attestation (DAA)



Signing Protocol

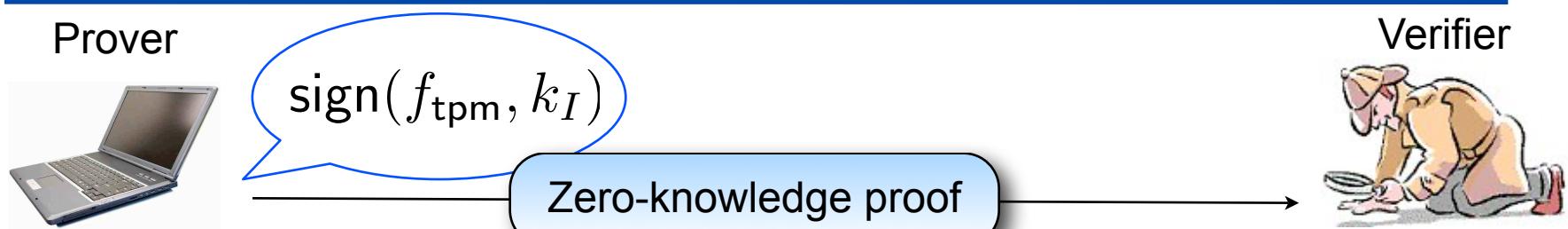
The user proves the knowledge of a certificate for his secret TPM identifier f_{TPM} ... without revealing it!

Direct Anonymous Attestation (DAA)



“the user knows a secret identifier and a certificate, and the certificate is a valid signature made by the issuer on the identifier”

Idealization of Zero-knowledge



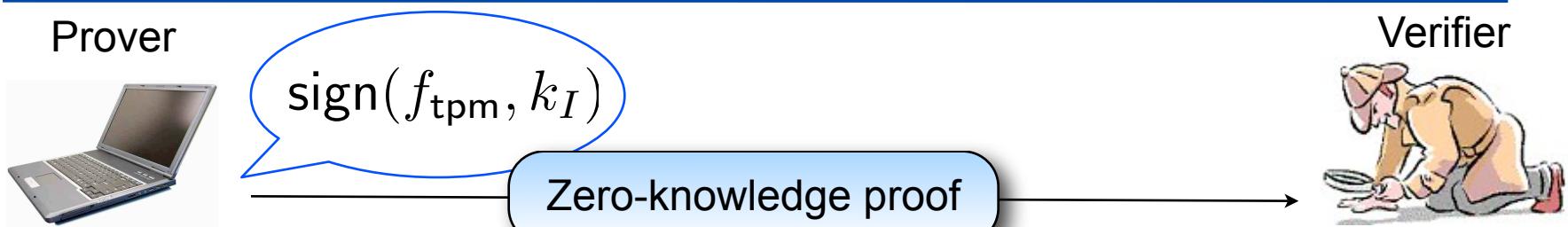
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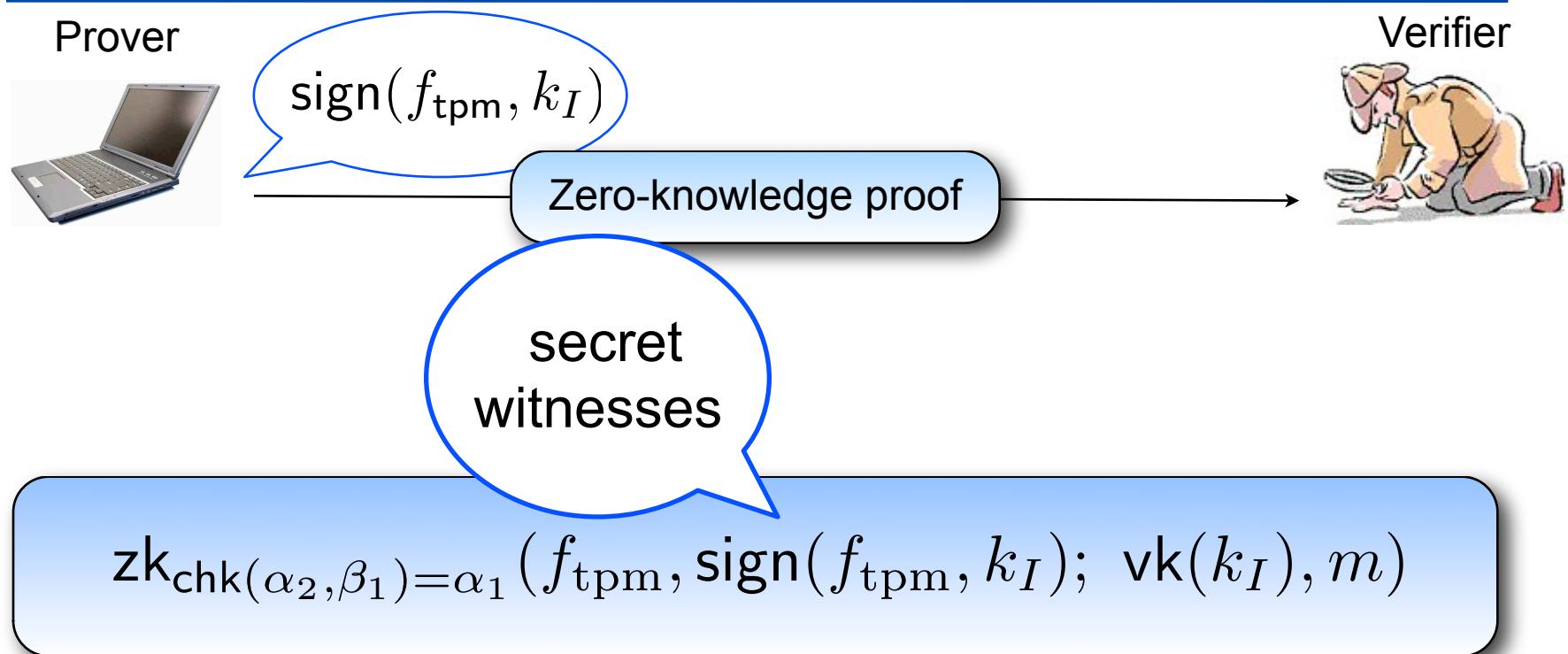


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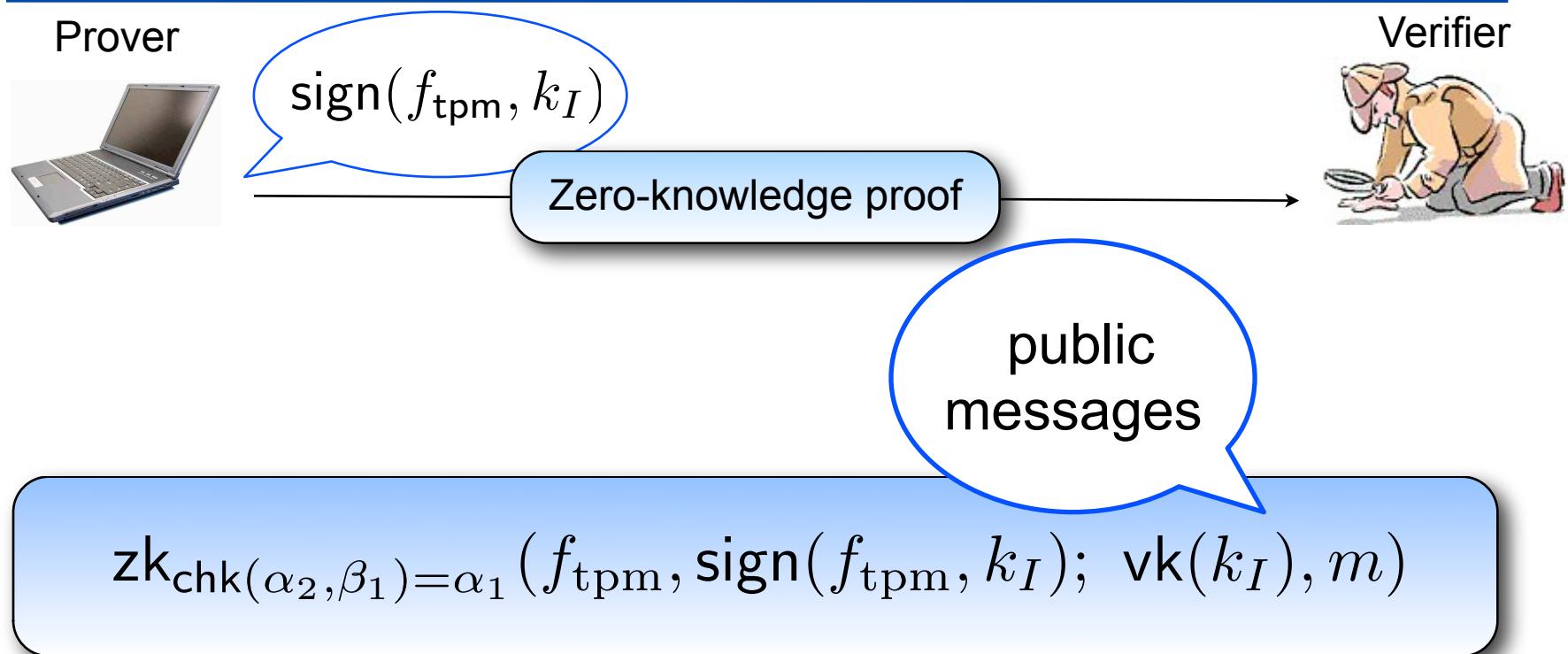

$$\text{zk}_{\text{chk}(\alpha_2, \beta_1)} = \alpha_1(f_{\text{TPM}}, \text{sign}(f_{\text{TPM}}, k_I); \text{vk}(k_I), m)$$

Idealization of Zero-knowledge



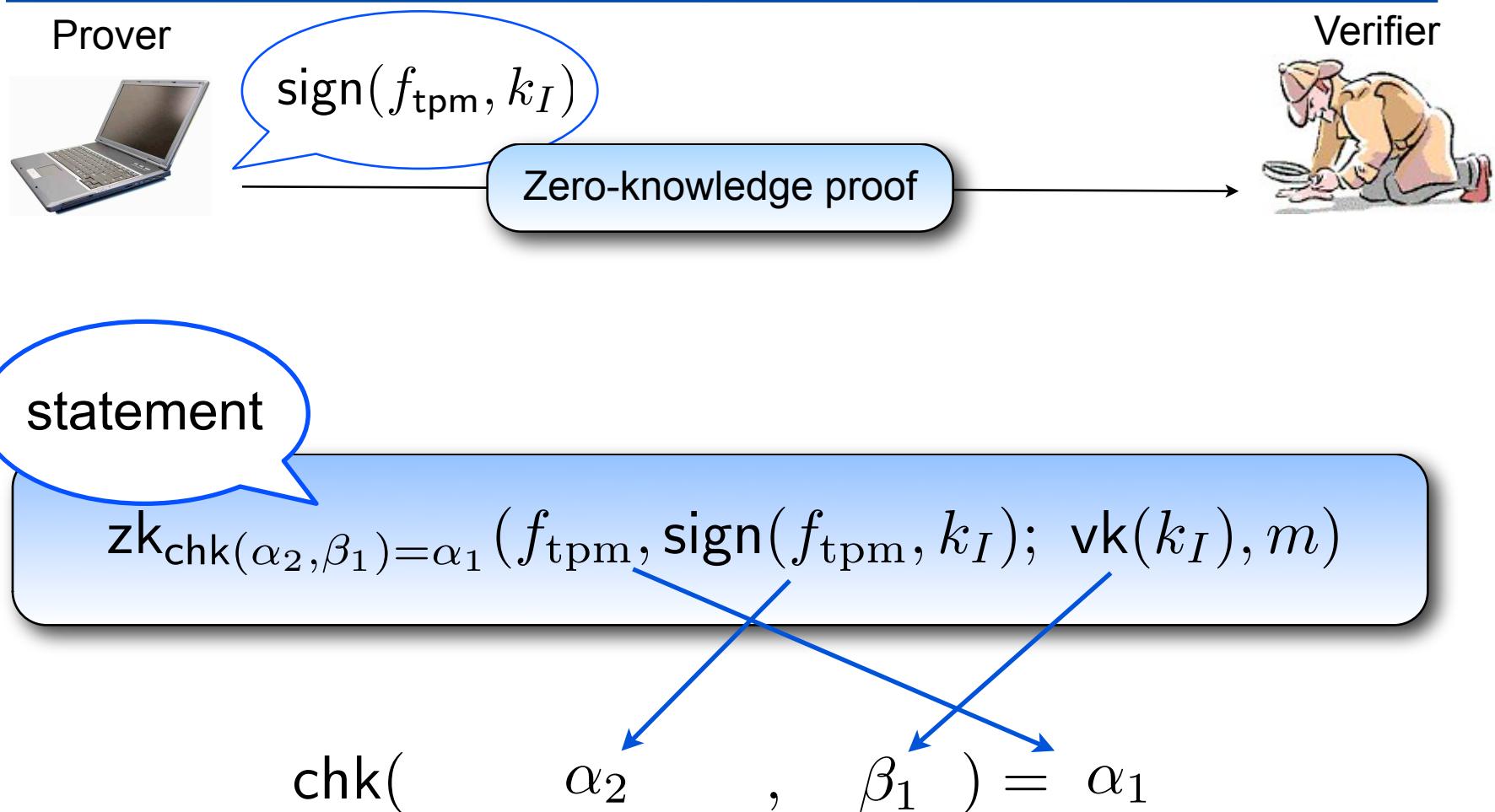
[Backes, Maffei & Unruh, S&P 2008] [Backes & Unruh, CSF 2008]

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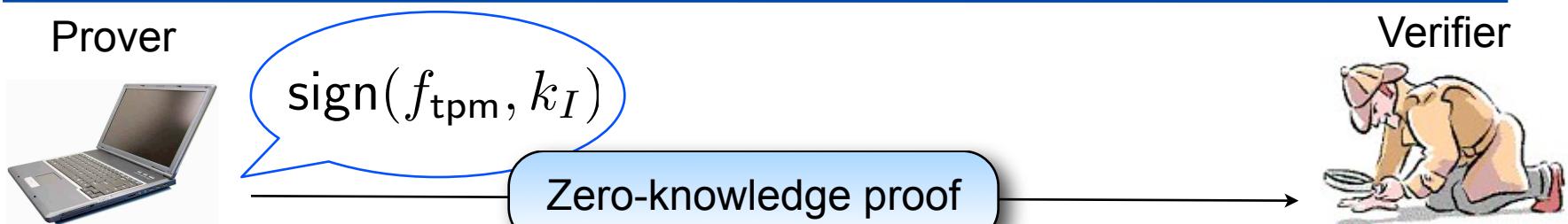
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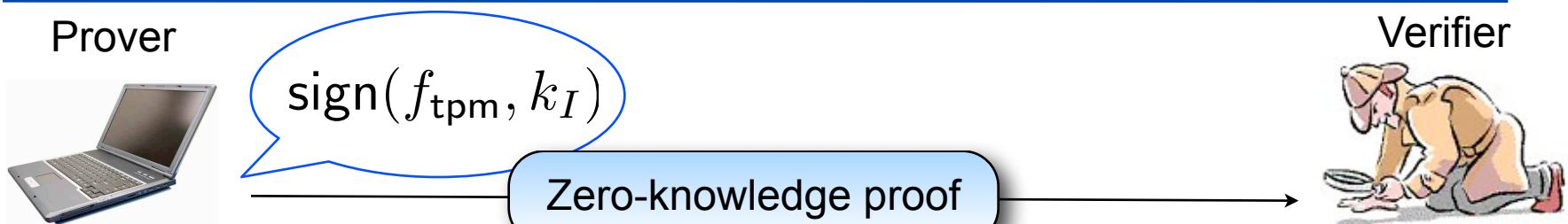
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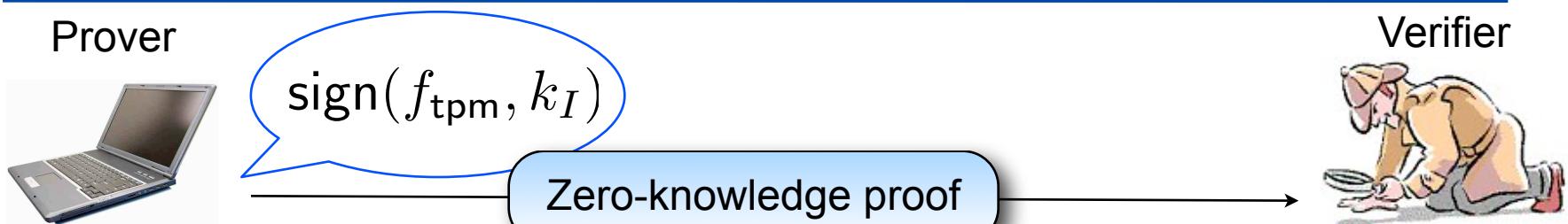
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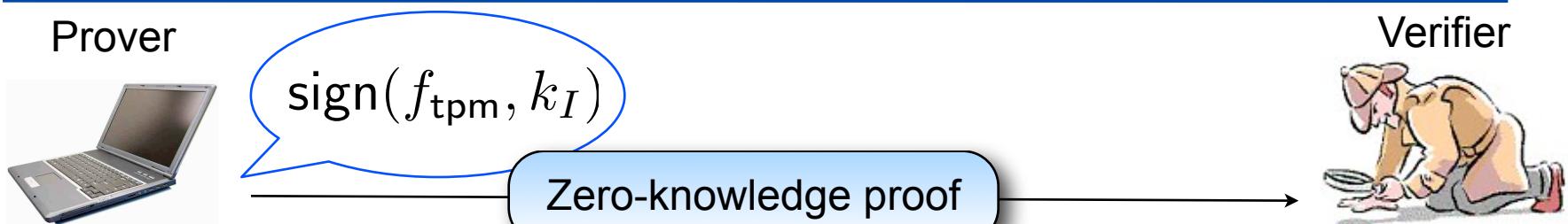
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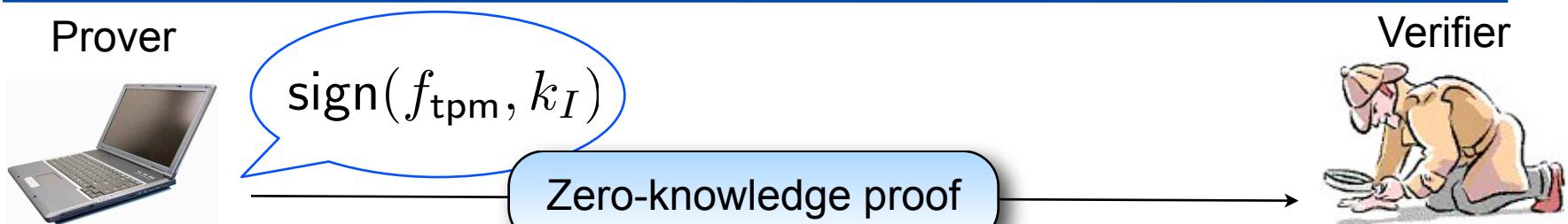
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DAA Signing Protocol (simplified)

Prover



$\text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$

Verifier



new k_I

new f_{tpm}

Prover | Verifier | Issuer

Prover = new m



out($c, \text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$)

Verifier = in(c, x).



let $y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I))$ then

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matching will help typing

Security Annotations

Prover


 $\text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$

Verifier



`new k_I`

`new f_{tpm}`

Prover | Verifier | Issuer

Prover = `new m`



`assume Send(f_{tpm}, m) |
out($c, \text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$)`

Verifier = `in(c, x).`



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Verifier



assume $\forall m. (\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f) \Rightarrow \text{Authenticate}(m)) \mid$

new k_I

new f_{tpm}

Prover | Verifier | Issuer

authorization policy
(in some logic)

Prover = new m



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assert $\text{Authenticate}(y_m)$

Security Annotations

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new k_I

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Prover | Verifier | Issuer

Prover = new m


assume $\text{Send}(f_{\text{tpm}}, m) \mid$
 $\text{out}(c, \text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

Verifier = $\text{in}(c, x).$


let $y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I))$
assert $\text{Authenticate}(y_m)$

authorization policy
(in some logic)

Safety
 Asserts are entailed by
 the current assumes

Basic Types

assume $\forall m. (\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f) \Rightarrow \text{Authenticate}(m)) \mid$
new k_I
new f_{tpm}
Prover | Verifier | Issuer

Prover = new m

assume $\text{Send}(f_{\text{tpm}}, m) \mid$
 $\text{out}(c, \text{zk}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

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assert $\text{Authenticate}(y_m)$

Basic Types

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new k_I
new f_{tpm}
Prover | Verifier | Issuer

Prover = new $m: \text{Un}$

assume $\text{Send}_{\text{out}}(c, \text{zk}_{\text{chk}}(\dots, \alpha_1, \beta_1, \dots, \alpha_I, \beta_I); \text{vk}(k_I), m)$

Verifier = in(c, x).

let $y_m = \text{ver}_{\text{chk}}(\alpha_2, \beta_1) - \alpha_1, \dots, \beta_I)$ then
assert $\text{Authenticate}(y_m)$

Type of
messages known to
the attacker

Basic Types

assume $\forall m. (\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f) \Rightarrow \text{Authenticate}(m)) \mid$
new k_I

new f_{tpm} : **Private**

Prover | Verifier



Prover = new n assume Send
out($c, \text{zk}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(J_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$)

Verifier = $\text{in}(c, x).$

let $y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(x; \text{vk}(k_I))$ then
assert $\text{Authenticate}(y_m)$

Type of messages
unknown to the attacker



Basic Types

assume $\forall m. (\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m)$ |
new k_I : **SigKey**($\langle x_f : \text{Private} \rangle \{\text{OkTPM}(x_f)\}$)

new f_{tpm} : **Private**

Prover | Verifier | Issuer

Prover = new m : **Un**
assume $\text{Send}(f_{\text{tpm}}, m) \mid$
 $\text{out}(c, \text{zk}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

Verifier = $\text{in}(c, x).$
let $y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(x; \text{vk}(k_I))$ then
assert $\text{Authenticate}(y_m)$

Type of keys used
to sign Private messages for which
OkTPM holds



Basic Types



The type of the key allows us to “transfer” predicates from the prover to the verifier!

assume $\forall m. (\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m)$ |
new k_I : **SigKey**($\langle x_f : \text{Private} \rangle \{\text{OkTPM}(x_f)\}$)
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Prover | Verifier | Issuer

Prover = new m : **Un**

assume $\text{Send}(f_{\text{tpm}}, m)$ |
 $\text{out}(c, \text{zk}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

Verifier = $\text{in}(c, x)$.

let $y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(x; \text{vk}(k_I))$ then
assert $\text{Authenticate}(y_m)$

Basic Types

But, the verifier can't use the key to check a certificate he never receives.

assume $\forall m. (\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m)$ |
new k_I : **SigKey**($\langle x_f : \text{Private} \rangle \{\text{OkTPM}(x_f)\}$)
new f_{tpm} : **Private**
Prover | Verifier | Issuer

Prover = new m : **Un**

assume $\text{Send}(f_{\text{tpm}}, m)$ |
 $\text{out}(c, \text{zk}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

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assert $\text{Authenticate}(y_m)$

Basic Types

But, the verifier can't use the key to check a certificate he never receives.

Worse, ZK don't necessarily rely on keys!



assume $\forall m. (\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m)$

new k_I : $\text{SigKey}(\langle x_f : \text{Private} \rangle \{\text{OkTPM}(x_f)\})$

new f_{tpm} : Private

Prover | Verifier | Issuer

Prover = new m : 

assume $\text{Send}(f_{\text{tpm}}, m) \mid$
 $\text{out}(c, \text{zk}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

Verifier = 

$\text{in}(c, x).$
 $\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1)=\alpha_1}(x; \text{vk}(k_I)) \text{ then}$
 $\text{assert } \text{Authenticate}(y_m)$

Typing Zero-knowledge Proofs

Our solution:

User gives a type to each statement proved by ZK

Typing Zero-knowledge Proofs

Prover


$$\text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

Verifier



$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} (\langle y_k : \text{VerKey}(\dots), y_m : \text{Un} \rangle \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \})$

Typing Zero-knowledge Proofs

Prover



Verifier



$zk_{chk(\alpha_2, \beta_1)=\alpha_1}(f_{tpm}, \text{sign}(f_{tpm}, k_I); vk(k_I), m)$

Type of
public messages

$ZK_{chk(\alpha_2, \beta_1)=\alpha_1} (\langle y_k : \text{VerKey}(\dots), y_m : \text{Un} \rangle \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \})$

Typing Zero-knowledge Proofs

Prover


$$\text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

Verifier


$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} (\langle y_k : \text{VerKey}(\dots), y_m : \text{Un} \rangle \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \})$$

Logical formula where
the secret witnesses are
existentially quantified

Type-checking the Prover

$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

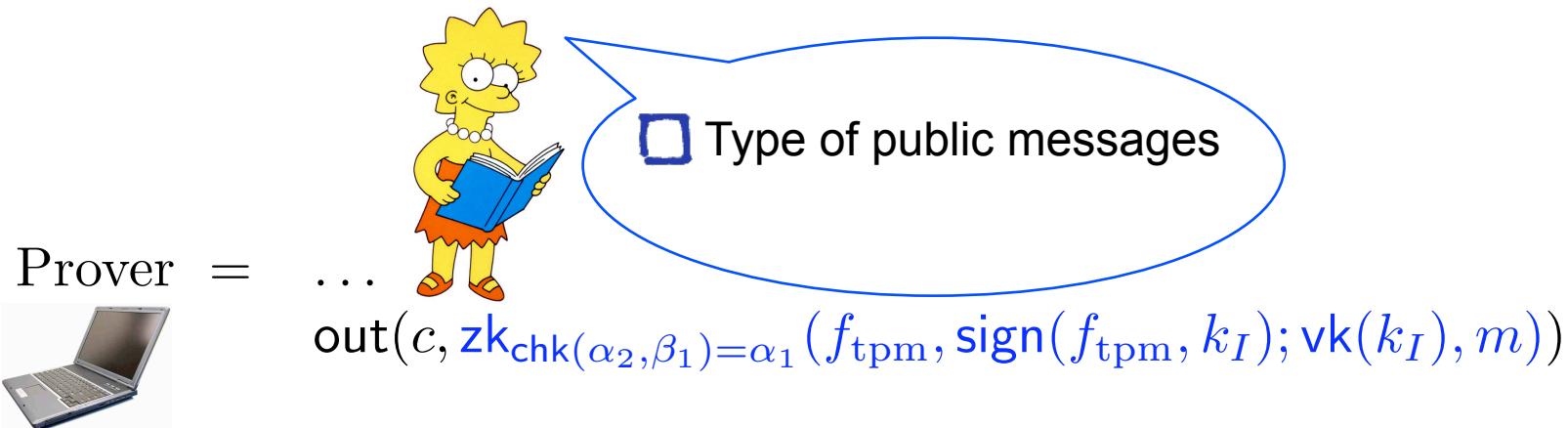
Prover = ...



$\text{out}(c, \text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

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$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$



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$\Gamma = \dots$

$k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}),$
 $m : \text{Un},$

\dots

$\text{OkTPM}(f_{\text{tpm}}),$
 $\text{Send}(f_{\text{tpm}}, m)$



Type of public messages

Prover = ...



$\text{out}(c, \text{zk}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

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- Type of public messages
- Logical formula entailed

Prover = ...



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Type of public messages
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Type-checking the Verifier

$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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 $k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}),$
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 $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m)),$

Verifier = $\text{in}(c, x).$



$\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I)) \text{ then}$
 $\text{assert Authenticate}(y_m)$

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If verification succeeds, can we assume the formula in ZK type?

Verifier = $\text{in}(c, x).$



```
let  $y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I))$  then
  assert  $\text{Authenticate}(y_m)$ 
```

Type-checking the Verifier

$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

$\Gamma = \dots$
 $k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}),$
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If verification succeeds, can we assume the formula in ZK type?

In general not, the proof can come from untyped adversary!

Verifier = $\text{in}(c, x).$



$\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I)) \text{ then}$
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Conceptual issue

How can we know whether the zero-knowledge proof comes from an honest participant or from the adversary?!

Type-checking the Verifier

$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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\dots

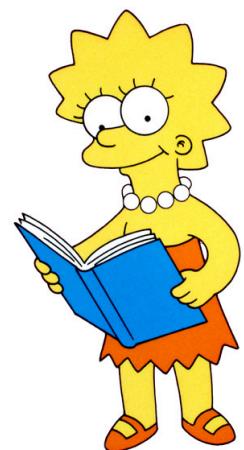
$\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m)),$



Verifier = $\text{in}(c, x).$

$\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I)) \text{ then}$

assert $\text{Authenticate}(y_m)$



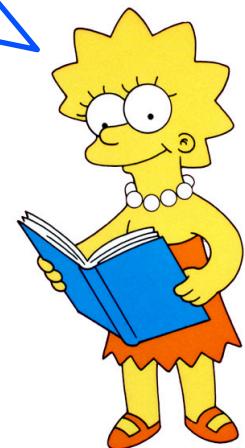
Type-checking the Verifier

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 \dots
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 $\text{chk}(\alpha_2, \beta_1) = \alpha_1$

The statement instantiated with the matched messages is valid (by the semantics)

Verifier = $\text{in}(c, x).$
 $\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I)) \text{ then}$
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$$\begin{aligned} \Gamma = & \dots \\ & k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM} \}) \\ & \dots \\ & \forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \wedge \dots) \\ & \text{chk}(\alpha_2, \beta_1) = \alpha_1 \end{aligned}$$

The statement instantiated with the matched messages is valid (by the semantics)

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Type-checking the Verifier

$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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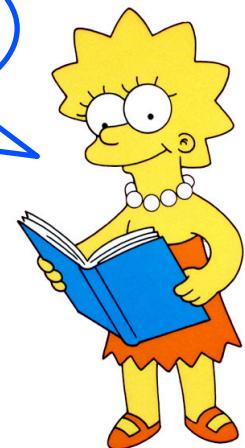
Type-checking the Verifier

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$\Gamma = \dots$
 $k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}),$
 \dots
 $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \rightarrow \text{Authenticate}(m)),$
 $\text{chk}(x_s, \text{vk}(k_I)) = x_f$
 $x_f : \text{Private}$

The type of $\text{vk}(k_I)$ gives us the type of x_f (existentially quantified)

Verifier = $\text{in}(c, x).$
 $\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I)) \text{ then}$
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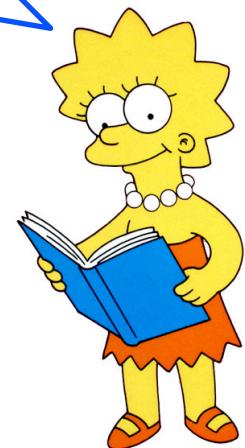
Type-checking the Verifier

$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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 \dots
 $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \wedge$
 $\text{chk}(x_s, \text{vk}(k_I)) = x_f$
 $x_f : \text{Private}$

The prover is honest, since he knows a message of type Private!

Verifier = $\text{in}(c, x).$
 $\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I)) \text{ then}$
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Type-checking the Verifier

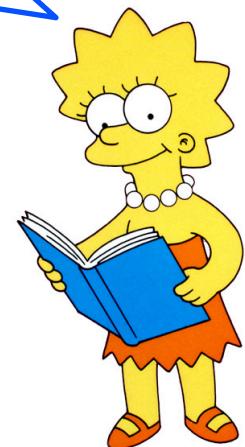
$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

$\Gamma = \dots$
 $k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \})$
 \dots
 $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \wedge \text{chk}(x_s, \text{vk}(k_I)) = x_f)$
 $x_f : \text{Private}$

We can now exploit the formula in the ZK type

Verifier = $\text{in}(c, x).$

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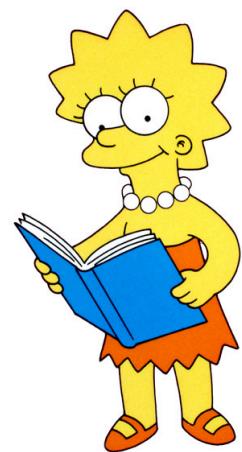
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Verifier = $\text{in}(c, x).$

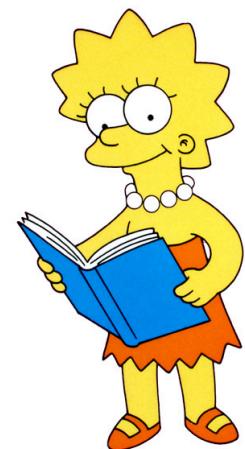
 $\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I)) \text{ then}$
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Type-checking the Verifier

$$\text{ZK}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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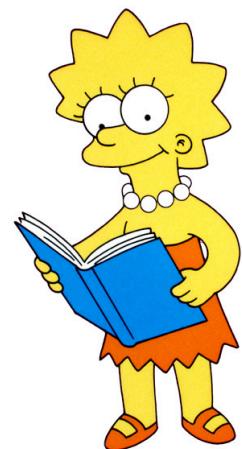
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Type-checking the Verifier

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 $x_f : \text{Private}$
 $\exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f)$

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✓ $\text{let } y_m = \text{ver}_{\text{chk}(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I)) \text{ then}$
 $\text{assert } \text{Authenticate}(y_m)$



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- ▶ ZK proofs are given *dependent types* where the witnesses are *existentially quantified*



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- ▶ *The prover* can only prove statements for which the formula in the ZK type holds
- ▶ *The verifier* can assume the formula in the ZK type
 - if the formula is entirely derived from the proved statement (most often much too weak)



Take Home

- ▶ ZK proofs are given *dependent types* where the witnesses are *existentially quantified*
- ▶ *The prover* can only prove statements for which the formula in the ZK type holds
- ▶ *The verifier* can assume the formula in the ZK type
 - if the formula is entirely derived from the proved statement (most often much too weak)
 - if he can somehow infer that the proof was constructed by an *honest prover (type-checked)*



Implementation

- ▶ Type-checker written in O'Caml (~5000 LOC)
- ▶ Uses automatic prover for discharging FOL formulas
- ▶ Extensible - very easy to add arbitrary primitives + types
- ▶ Efficient - the complete analysis of DAA takes 0.7s
- ▶ Available under the Apache License:
<http://www.infsec.cs.uni-sb.de/projects/zk-typechecker/>
- ▶ Kudos to Stefan Lorenz, Kim Pecina and Thorsten Tarrach



Ongoing Work

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- Remote electronic voting system
[Clarkson, Chong & Myers, S&P 2008]



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THANK YOU!