

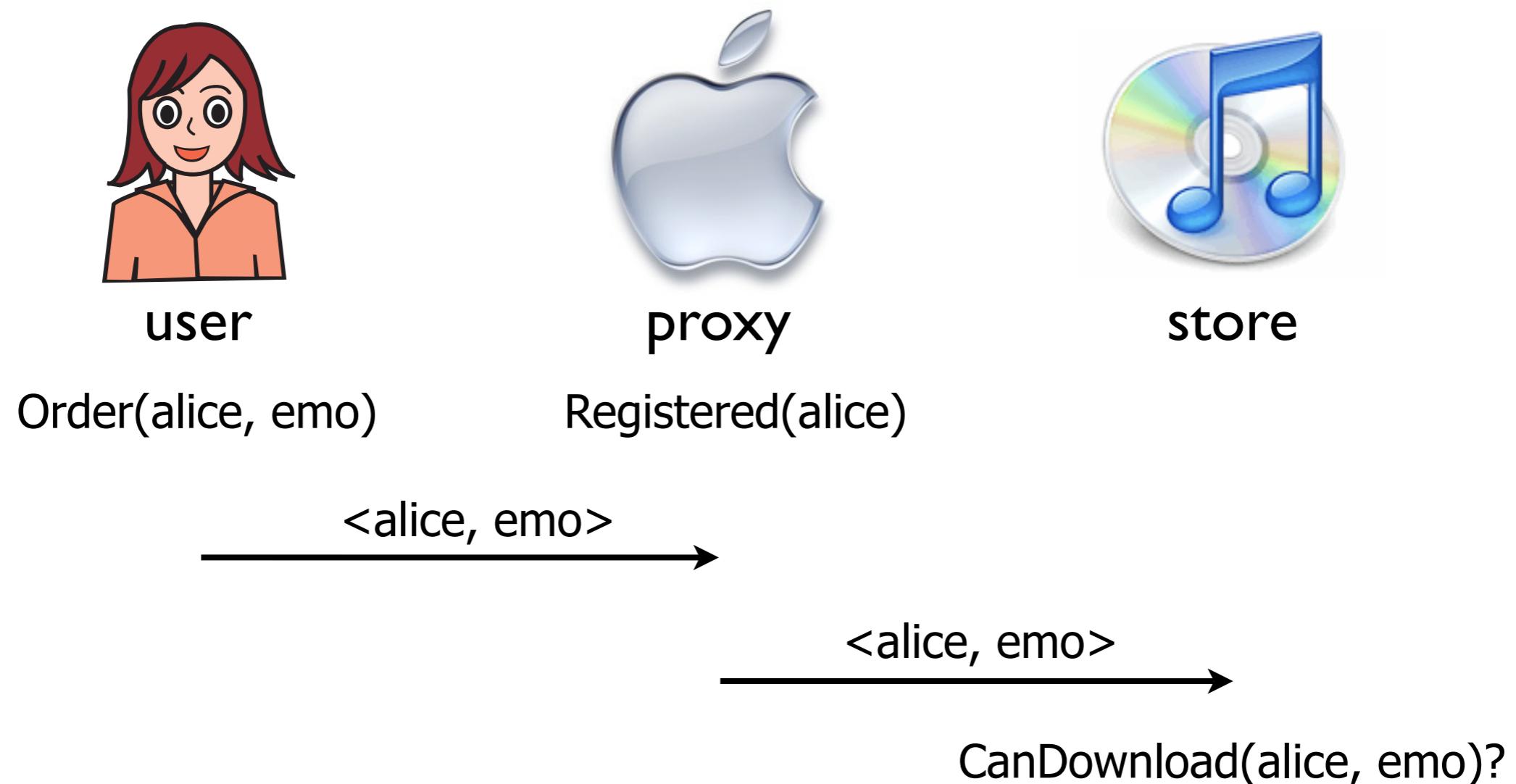
# A Type Discipline for Authorization in Distributed Systems

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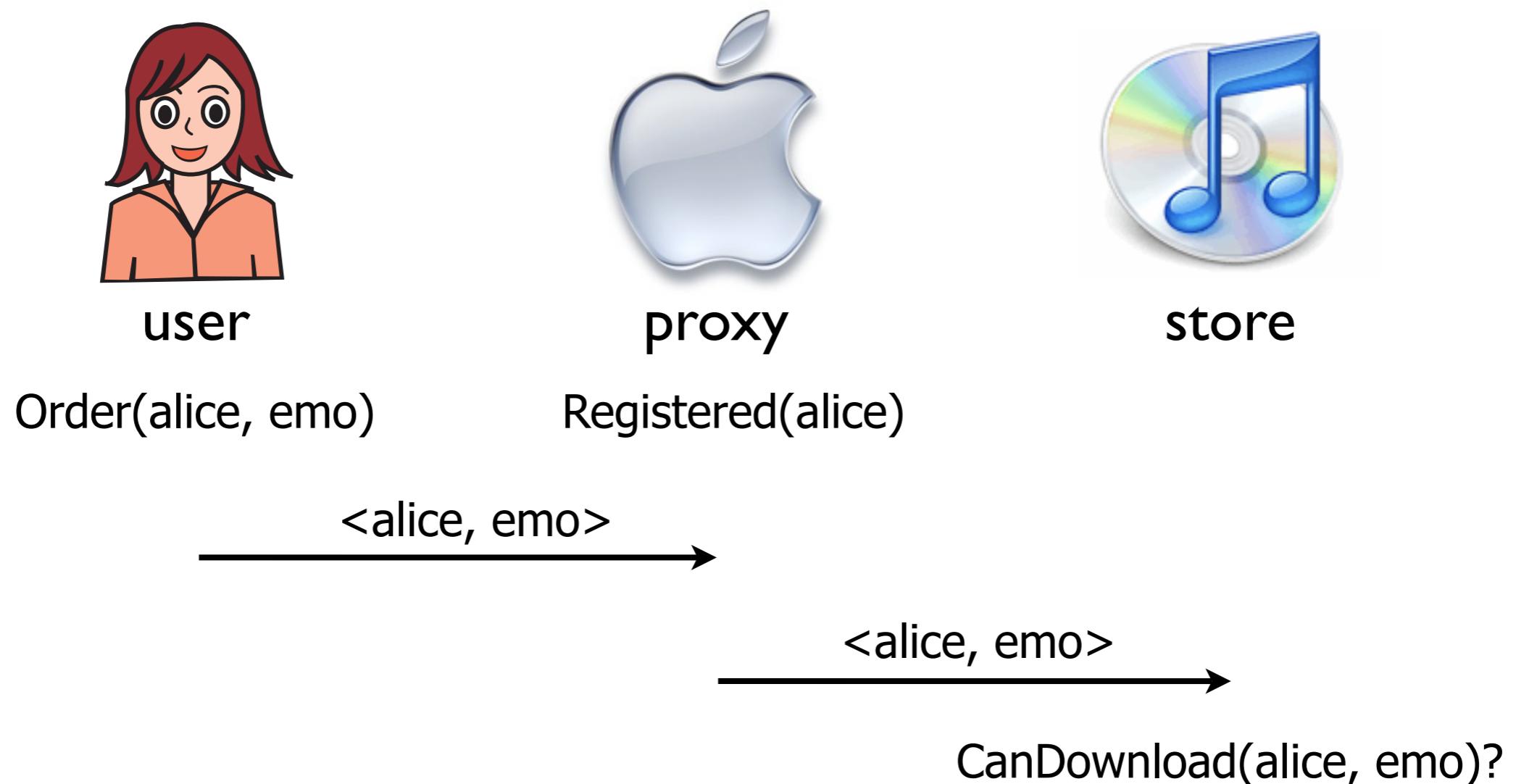
**Cédric Fournet, Andrew D. Gordon, and Sergio Maffeis**  
**20th IEEE Computer Security Foundations Symposium (CSF), 2007**

**Presenter: Cătălin Hrițcu**  
**Information Security & Cryptography Group**

# A simple example

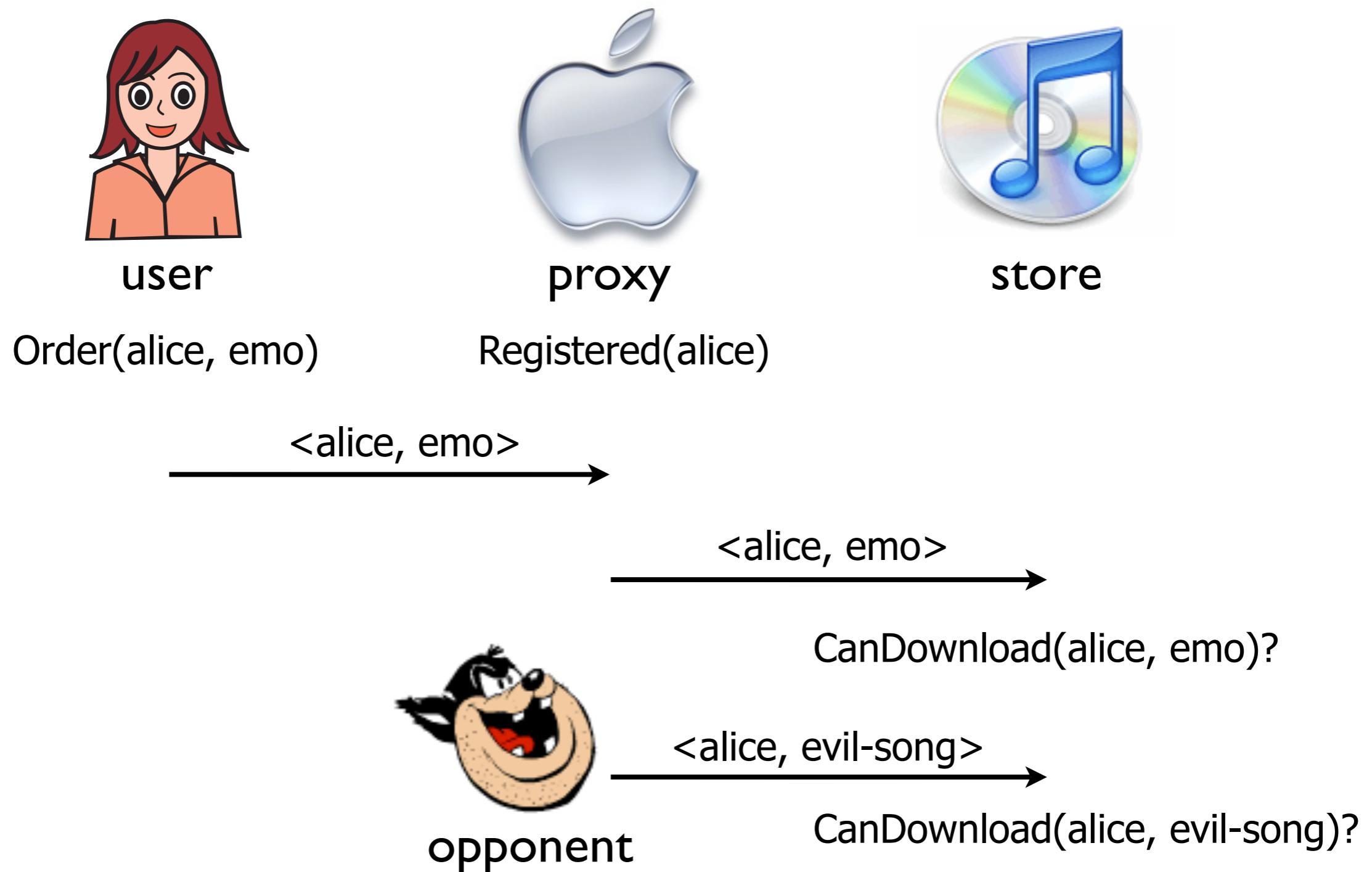


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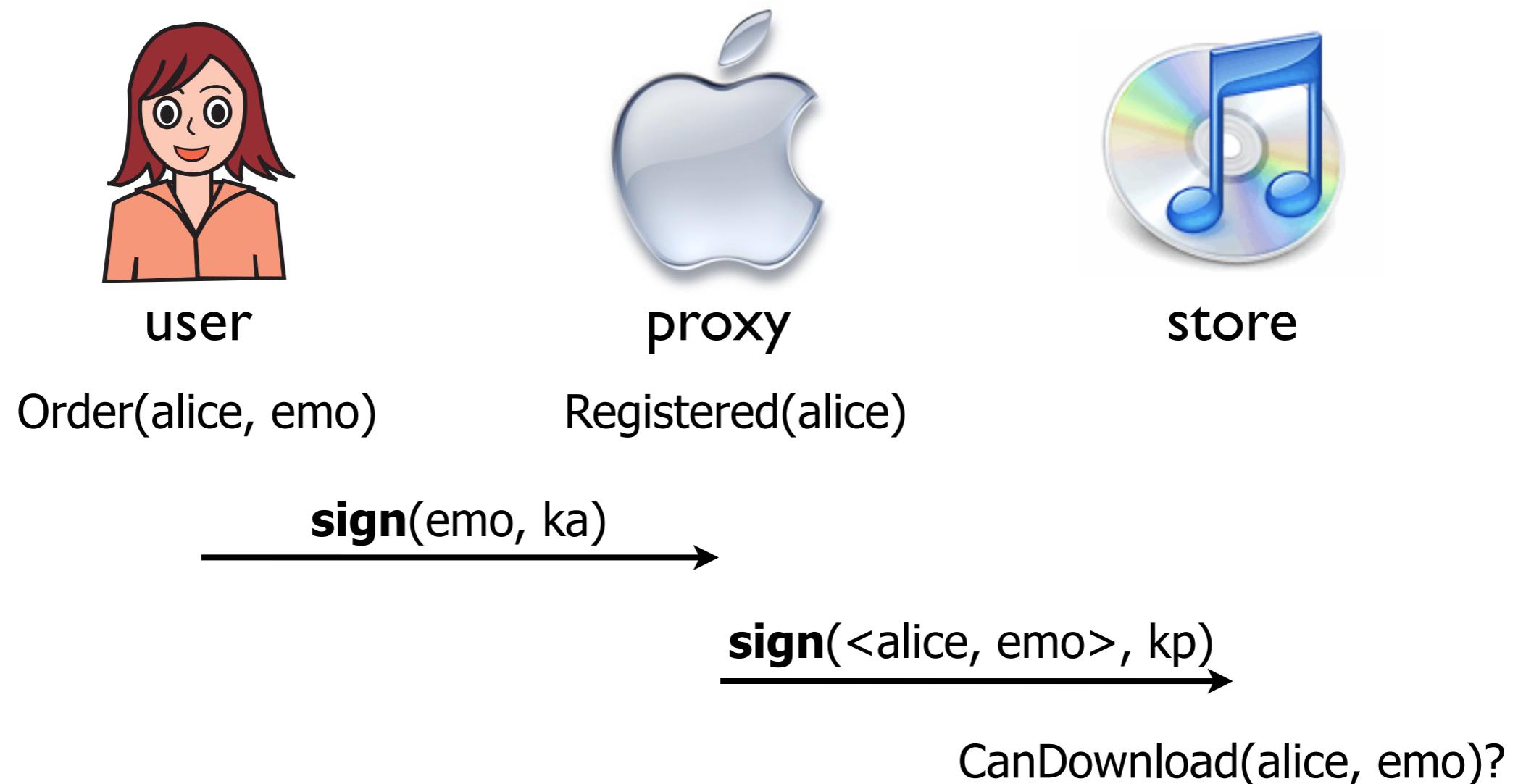
policy =  $\forall \text{user}, \forall \text{song} (\text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \Rightarrow \text{CanDownload}(\text{user}, \text{song}))$

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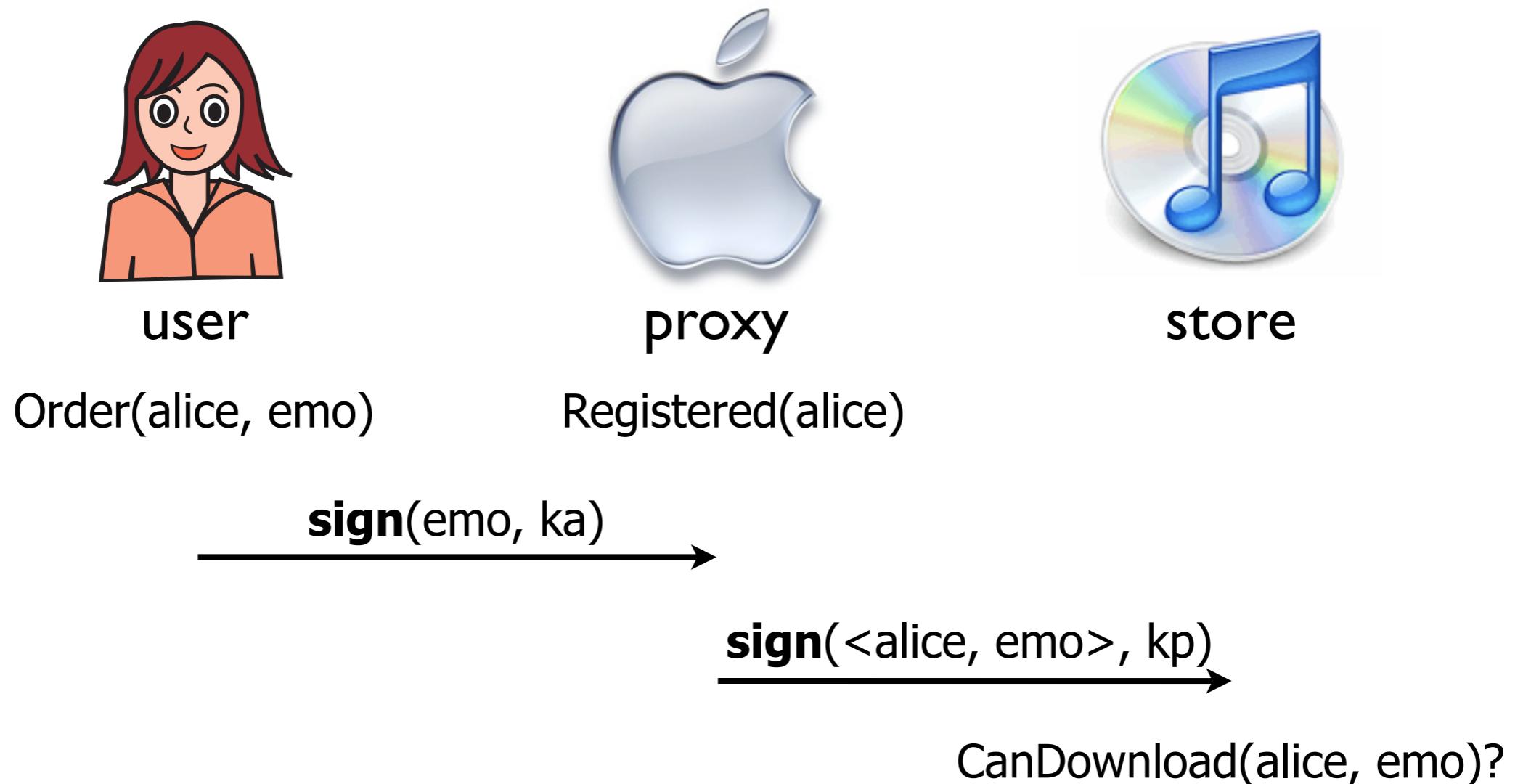
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# A simple example



- Is the authorization policy respected in all executions? (safety)
  - ... even in the presence of an arbitrary opponent? (robust safety)
  - ... even in the presence of corrupted participants? (safety despite compromise)

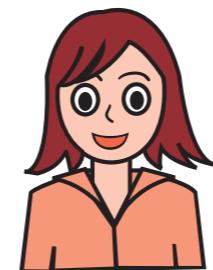
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# Outline

- Modeling protocols and policies (formally)
  - the process calculus: terms, destructors, processes, semantics
  - authorization logic: some assumptions (otherwise parametric)
  - desired security guarantees:
    - safety, robust safety, safety despite compromise
- 5 minutes break
- Type system
- Illustrating the proof technique
  - Subject-reduction  $\Rightarrow$  Safety
  - Safety  $\wedge$  Opponent Typability  $\Rightarrow$  Robust Safety
- Safety despite compromised participants
- Related and future work

# Modeling protocols and policies

# Modeling the example



user  
Order(alice, emo)



proxy  
Registered(alice)



store

# Modeling the example



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Order(alice, emo)



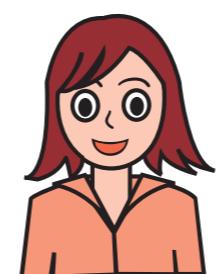
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let user = assume Order(alice, emo) |
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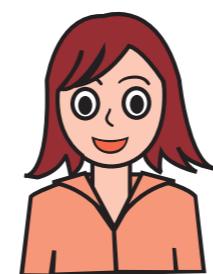
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sign(emo, ka) →

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let user = assume Order(alice, emo) |  
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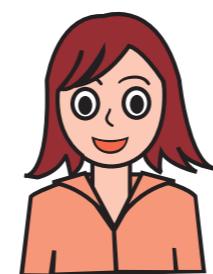
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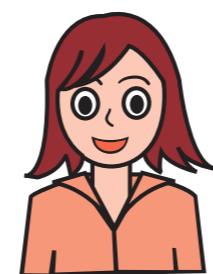
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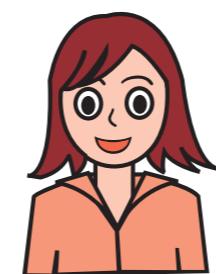
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$$f ::= \text{pk}^1, \text{enc}^2, \text{vk}^1, \text{sign}^2, \text{hash}^1, \dots$$

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- Destructors

$$g ::= \text{dec}^2, \text{check}^2, \dots$$

- Reduction rules for destructors

$$\text{dec}(\text{enc}(M, \text{pk}(K)), K) \Downarrow M$$

$$\text{check}(\text{sign}(M, K), \text{vk}(K)) \Downarrow M$$

# Processes

$P, Q ::= \mathbf{out}(M, N).P$	output
$\mathbf{in}(M, x).P$	input
$\mathbf{new } a : T.P$	restriction
$!P$	replication
$P \mid Q$	parallel composit.
$0$	null process
$\mathbf{let } x = g(M_1, \dots, M_n) \mathbf{ then } P \mathbf{ else } Q$	destructor applic.
$\mathbf{let } \langle x_1, \dots, x_n \rangle = M \mathbf{ in } P$	pair splitting
$\mathbf{assume } C$	assume formula $C$
$\mathbf{assert } C$	expect $C$ to hold

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- Evaluation context

$$\mathcal{E} ::= \mathbf{new}~a_1 : T_1 \dots \mathbf{new}~a_n : T_n. [] ~|~ P$$

# Semantics

- Structural equivalence

$$\begin{aligned}
 P \mid 0 &\equiv P \\
 P \mid Q &\equiv Q \mid P \\
 (P \mid Q) \mid R &\equiv P \mid (Q \mid R) \\
 !P &\equiv !P \mid P \\
 \mathbf{new} \ a : T.(P \mid Q) &\equiv P \mid \mathbf{new} \ a.Q, \text{ if } a \notin fn(P) \\
 \mathbf{new} \ a_1.\mathbf{new} \ a_2.P &\equiv \mathbf{new} \ a_2.\mathbf{new} \ a_1.P, \text{ if } a_1 \neq a_2 \\
 \mathcal{E}[P] &\equiv \mathcal{E}[Q], \text{ if } P \equiv Q
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{out}(a, M).P \mid \mathbf{in}(a, x).Q &\rightarrow P \mid Q\{M/x\} \\
 \mathbf{let} \ x = g(M_1, \dots, M_n) \ \mathbf{then} \ P \ \mathbf{else} \ Q &\rightarrow P\{N/x\}, \text{ if } g(M_1, \dots, M_n) \Downarrow N \\
 \mathbf{let} \ x = g(M_1, \dots, M_n) \ \mathbf{then} \ P \ \mathbf{else} \ Q &\rightarrow Q, \text{ if } g(M_1, \dots, M_n) \not\models \\
 \mathbf{let} \ \langle x_1, \dots, x_n \rangle = \langle M_1, \dots, M_n \rangle \ \mathbf{in} \ P &\rightarrow P\{M_i/x_i\} \\
 \mathcal{E}[P] &\rightarrow \mathcal{E}[Q], \text{ if } P \rightarrow Q \\
 P &\rightarrow Q, \text{ if } P \equiv P', P' \rightarrow Q', Q' \equiv Q
 \end{aligned}$$

# Semantics

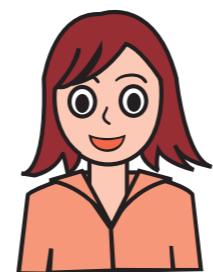
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- Internal reduction

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# Executing the simple example



user  
Order(alice, emo)



proxy  
Registered(alice)



store

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;  
(user | proxy | store | policy)
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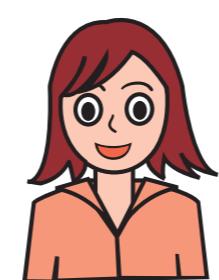
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**let** <s> = check(sign(<emo>, ka), vk(ka)) **in**  
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**out(c, sign(<alice, s>, kp))]**.

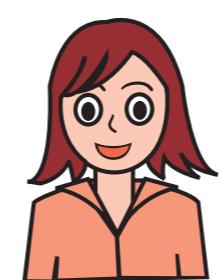
**check(sign(M, K), vk(K)) ↓ M**

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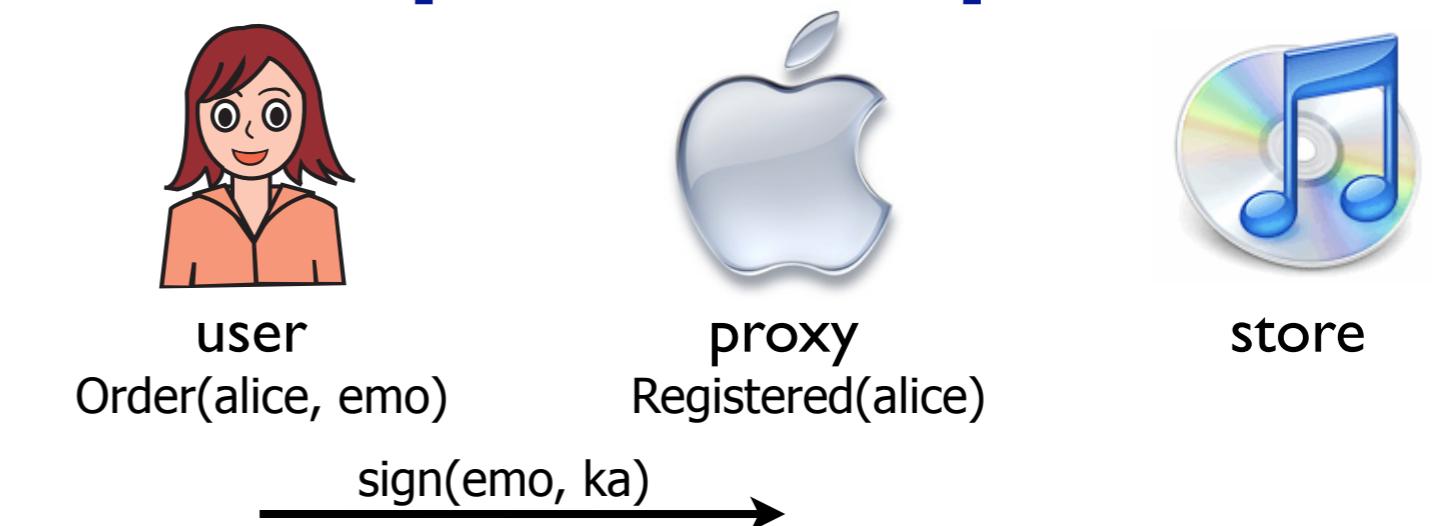
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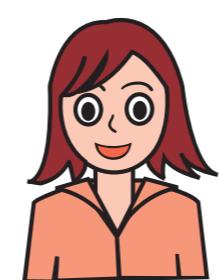
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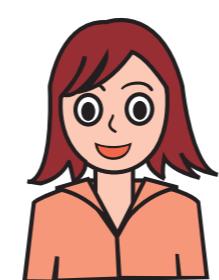
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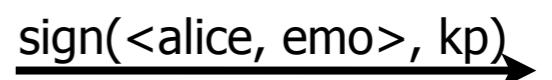
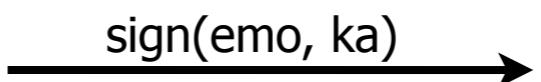
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**let** proxy = **assume** Registered(alice).

**let** store =

**assert** CanDownload(alice, emo).

**let** policy = **assume**  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .

**process** **new** ka; **new** kp;  
(user | proxy | store | policy)

# Executing the simple example



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka) →

sign(<alice, emo>, kp) →

CanDownload(alice, emo)?

**let** user = **assume** Order(alice, emo).

**let** proxy = **assume** Registered(alice).

**let** store =

**assert** CanDownload(alice, emo)

**let** policy = **assume**  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .

**process** **new** ka; **new** kp;  
(user | proxy | store | policy)

# Executing the simple example



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka) →

sign(<alice, emo>, kp) →

CanDownload(alice, emo)?

let user = **assume** Order(alice, emo).

let proxy = **assume** Registered(alice).

let store =

**assert** CanDownload(alice, emo)

let policy = **assume**  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .

**process new** ka; **new** kp;  
(user | proxy | store | policy)

# Executing the simple example



user  
Order(alice, emo)



proxy  
Registered(alice)



store

**let user = assume Order(alice, emo).**

sign(emo, ka) →

sign(<alice, emo>, kp) →

**let proxy = assume Registered(alice).**

CanDownload(alice, emo)?

**let store =** {Order(alice, emo), Registered(alice),  
 $\forall u, \forall s (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ } |= CanDownload(alice, emo)



**assert CanDownload(alice, emo)** ✓

**let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .**

**process new ka; new kp;  
(user | proxy | store | policy)**

# Authorization logic

- Any authorization logic - framework is “parametric”

- Conditions:

**Expansivity:**  $C \in A$  implies that  $A \models C$ ;

**Monotonicity:**  $A \models C$  and  $A \subseteq A'$  then  $A' \models C$ ;

**Idempotence:**  $A \models A'$  and  $A \cup A' \models C$  then  $A \models C$ ;

**Substitution:**  $A \models C$  then  $A\sigma \models C\sigma$ ;

- Examples: Datalog, CDD (more later), DCC, FOL, etc.

# Safety

- All assertions hold in all executions
- Definition (Safety). A process  $P$  is safe iff

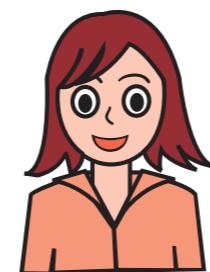
$$\begin{aligned} P \rightarrow^* \text{new } a_1 : T_1 \dots \text{new } a_n : T_n. (\text{assert } C \mid Q) \Rightarrow \\ \exists \mathcal{E} = \text{new } b_1 : U_1 \dots \text{new } b_m : U_m. [ ] \mid Q' \wedge \\ Q \equiv \mathcal{E}[\text{assume } C_1 \mid \dots \mid \text{assume } C_k] \wedge \\ \{C_1, \dots, C_k\} \models C \wedge fn(C) \cap \{b_1, \dots, b_m\} = \emptyset \end{aligned}$$

# Robust safety

- Safety in the presence of an arbitrary attacker
- Definition (Opponent). A process  $O$  is an opponent iff
  - it does not contain any **assert**
  - it only uses the special type  $\text{Un}$  (i.e. opponents are untyped)
- This corresponds to the usual Dolev-Yao attacker
- Definition (Robust Safety). Process  $P$  is robustly safe iff process  $P / O$  is safe for all opponents  $O$ .



# Is our protocol robustly safe?



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | proxy | store | policy)
```

# Is our protocol robustly safe?



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

- Probably, but with what I have shown you so far we have no easy way to prove that a protocol is robustly safe

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | proxy | store | policy)
```

# Is our protocol robustly safe?



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | proxy | store | policy)
```

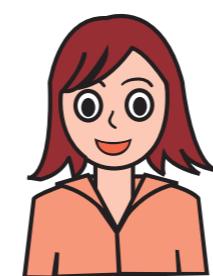
- Probably, but with what I have shown you so far we have no easy way to prove that a protocol is robustly safe
- The type system will do the job

# Safety despite compromise

- Safety when part of the protocol is compromised
- Definition attempt (Safety Despite Compromise).  
A process  $\mathcal{E}[Q]$  is safe despite the compromise of  $Q$   
(where  $\mathcal{E} = \text{new } c_1 : T_1 \dots \text{new } c_n : T_n. [ ] \mid Q^*$ ) iff  
 $\mathcal{E}[\text{out}(c, c_1) \mid \dots \mid \text{out}(c, c_n)]$  is robustly safe



# Compromising as little as possible



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |
  out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |
  in(c,m);
  let <s> = check(m, vk(ka)) in
  out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);
  let <user, song> = check(x, vk(kp)) in
  assert CanDownload(user, song).
```

```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;
  (user | proxy | store | policy)
```

# Compromising as little as possible



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | proxy | store | policy | [0])
```

Compromise this

# Compromising as little as possible



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

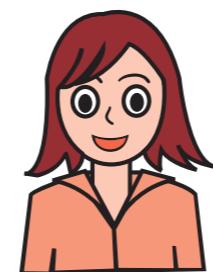
```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | proxy | store | policy | [0])
```

- In this case safety despite compromise reduces to robust safety

# Compromising as much as possible



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

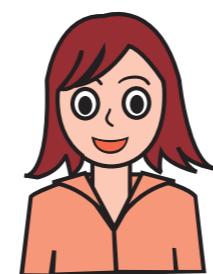
```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;  
[user | proxy | store | policy]
```

# Compromising as much as possible



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |
    out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |
    in(c,m);
    let <s> = check(m, vk(ka)) in
    out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);
    let <user, song> = check(x, vk(kp)) in
    assert CanDownload(user, song).
```

```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;
    [out(c, ka) | out(c, kp) | 0 | 0]
```

# Compromising as much as possible

```
process new ka; new kp;  
[out(c, ka) | out(c, kp) | 0 | 0]
```

# Compromising as much as possible

```
process new ka; new kp;  
[out(c, ka) | out(c, kp) | 0 | 0]
```

- Compromising the whole protocol is always (vacuously) safe - no asserts

# Compromising the user



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |
    out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |
    in(c,m);
    let <s> = check(m, vk(ka)) in
    out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);
    let <user, song> = check(x, vk(kp)) in
    assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;
    [user] | proxy | store | policy
```

# Compromising the user



proxy  
Registered(alice)



store

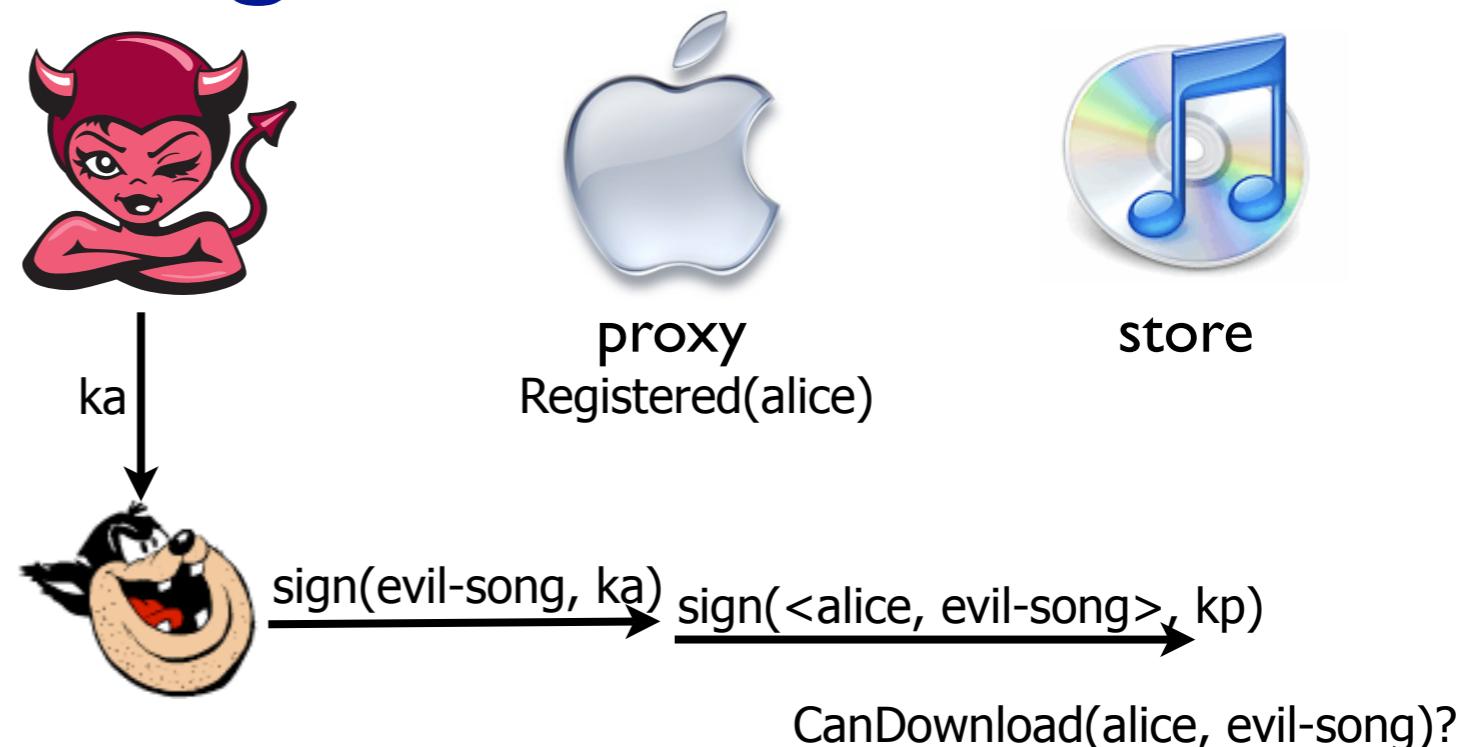
```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka; new kp;  
[out(c, ku)] | proxy | store | policy
```

# Compromising the user



```
let proxy = assume Registered(alice) |  

in(c,m);  

let <s> = check(m, vk(ka)) in  

out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  

let <user, song> = check(x, vk(kp)) in  

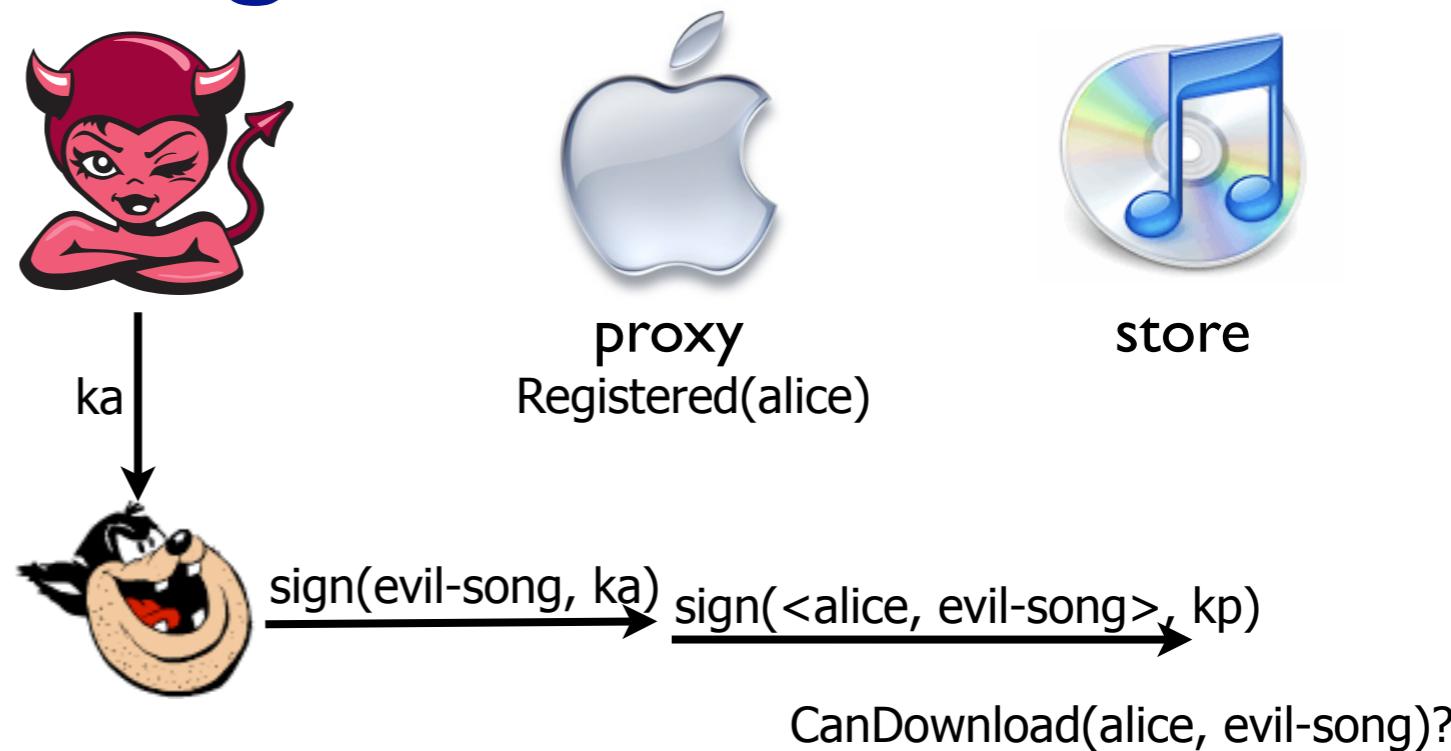
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka; new kp;  

[out(c, ku)] | proxy | store | policy
```

# Compromising the user



```
let proxy = assume Registered(alice) |  

in(c,m);  

let <s> = check(m, vk(ka)) in  

out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  

let <user, song> = check(x, vk(kp)) in  

assert CanDownload(user, song).
```

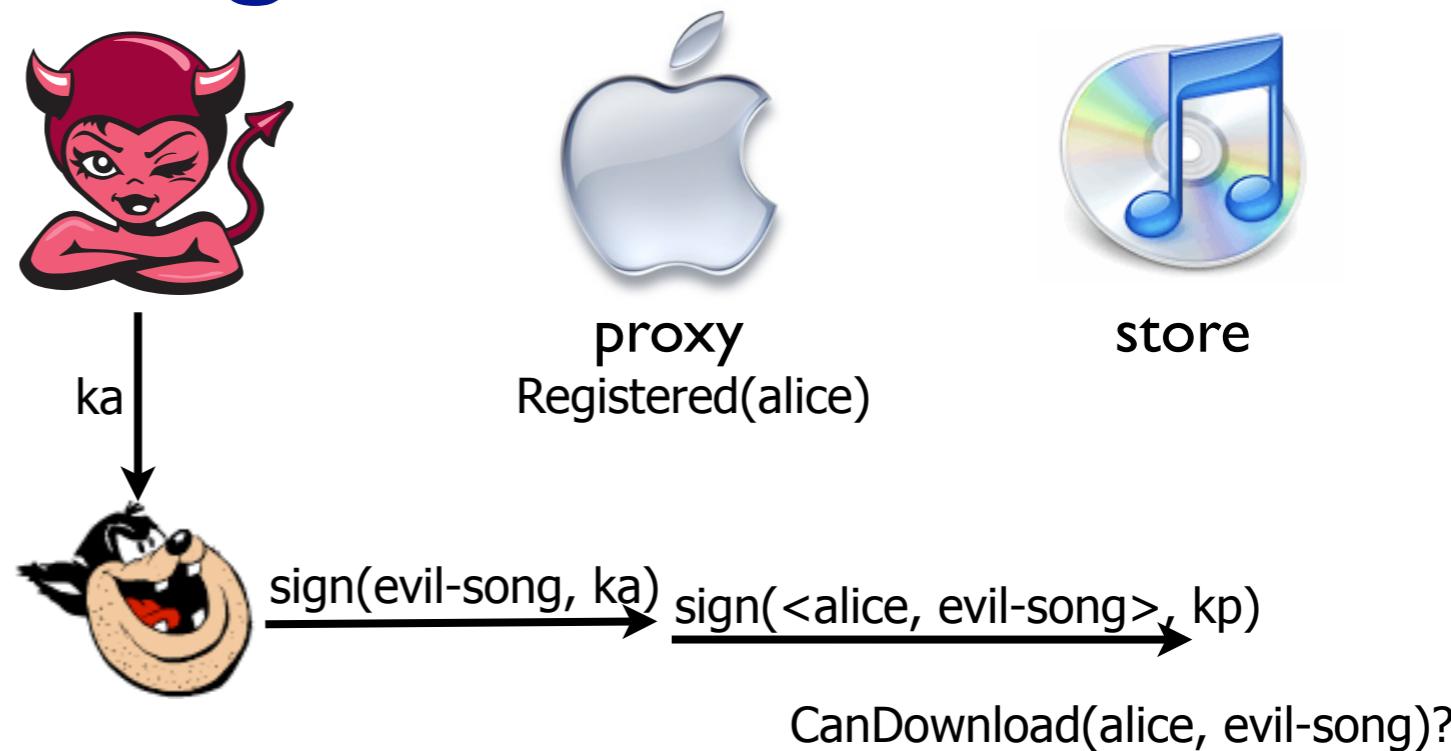
```
let policy = assume  $\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka; new kp;  

[out(c, ku)] | proxy | store | policy
```

- The store accepts the download even if this does not respect the policy (the user never asked for the download)

# Compromising the user



```
let proxy = assume Registered(alice) |  

in(c,m);  

let <s> = check(m, vk(ka)) in  

out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  

let <user, song> = check(x, vk(kp)) in  

assert CanDownload(user, song).
```

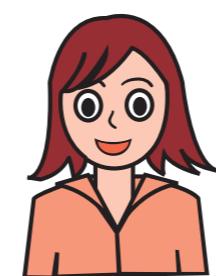
```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  

[out(c, ku)] | proxy | store | policy
```

- The store accepts the download even if this does not respect the policy (the user never asked for the download)
- This breaks robust safety, so the protocol is not safe despite the compromise of the user (not surprising)

# Compromising the proxy



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |
  out(c, sign(<emo>, ka)).
```

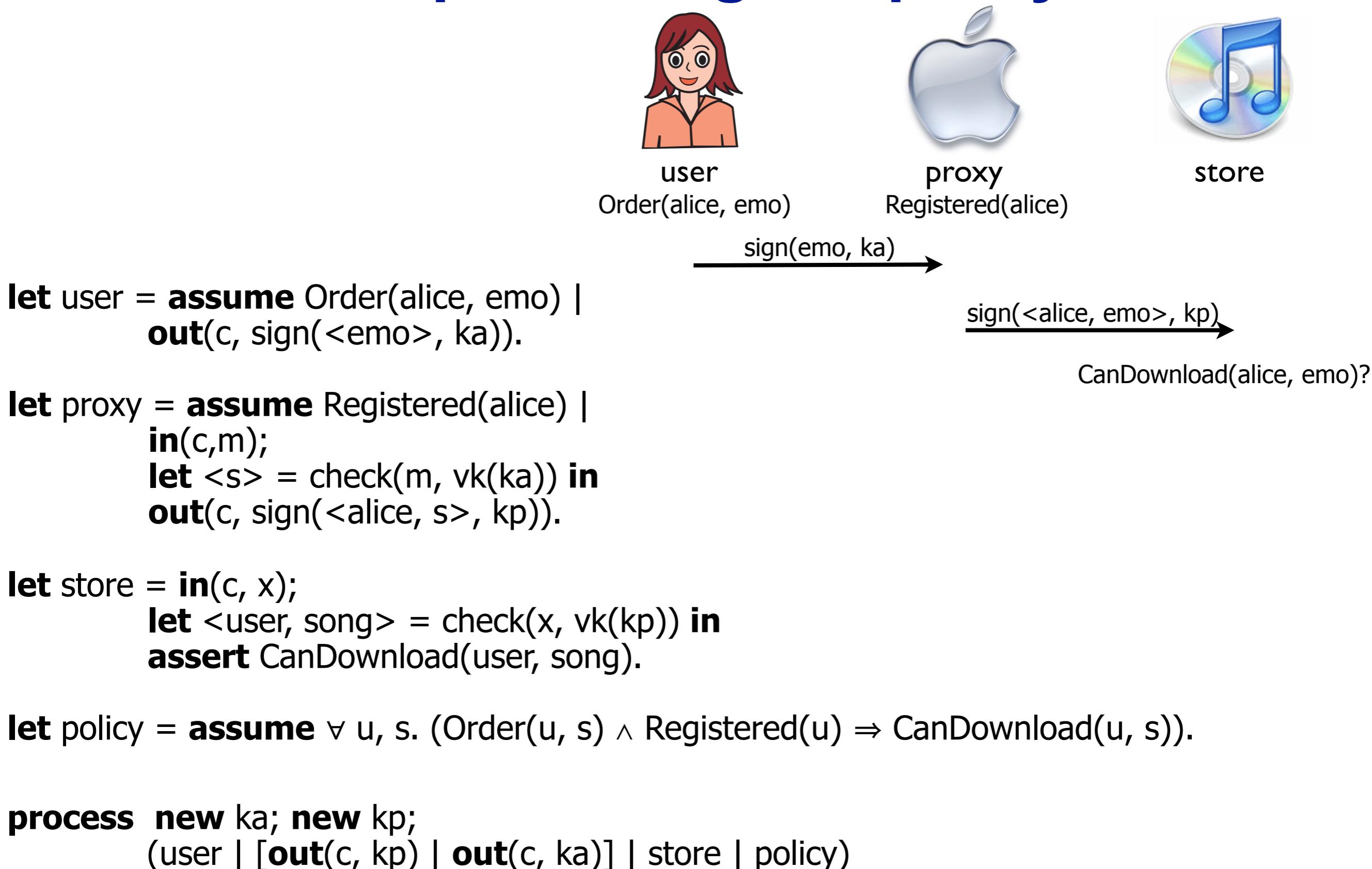
```
let proxy = assume Registered(alice) |
  in(c,m);
let <s> = check(m, vk(ka)) in
  out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);
let <user, song> = check(x, vk(kp)) in
  assert CanDownload(user, song).
```

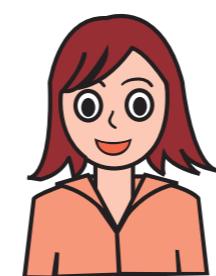
```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;
  (user | [proxy] | store | policy)
```

# Compromising the proxy



# Compromising the proxy



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;  
(user | [out(c, kp) | out(c, ka)] | store | policy)
```

Not the right thing to compromise

# Safety despite compromise



- The real definition now :)
- Definition (Safety Despite Compromise).  
A process  $\mathcal{E}[Q]$  is safe despite the compromise of  $Q$   
(where  $\mathcal{E} = \mathbf{new}~c_1 : T_1 \dots \mathbf{new}~c_n : T_n.~[ ] \mid Q^*$ ) iff
  - $\exists Q', \sigma \quad (1)~Q = Q'\sigma$  with  $fn(Q') \cap \{c_1, \dots, c_n\} = \emptyset$
  - $(2)~\mathcal{E}[\prod_{x \in dom(\sigma)} \mathbf{out}(c, x\sigma)]$  is robustly safe  
(where  $c \notin \{c_1, \dots, c_n\}$ )

# Compromising the proxy



user  
Order(alice, emo)



proxy  
Registered(alice)



store

sign(emo, ka)

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

```
let user = assume Order(alice, emo) |
  out(c, sign(<emo>, ka)).
```

```
let proxy = assume Registered(alice) |
  in(c,m);
  let <s> = check(m, vk(ka)) in
  out(c, sign(<alice, s>, kp)).
```

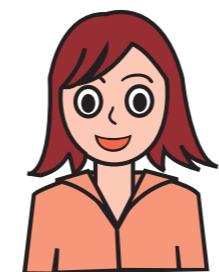
```
let store = in(c, x);
  let <user, song> = check(x, vk(kp)) in
  assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;
  (user | [out(c, kp) | out(c, ka)] | store | policy)
```

Not the right thing to compromise

# Compromising the proxy



user  
Order(alice, emo)



store

sign(emo, ka) → kp, vk(ka)

```
let user = assume Order(alice, emo) |  
out(c, sign(<emo>, ka)).
```

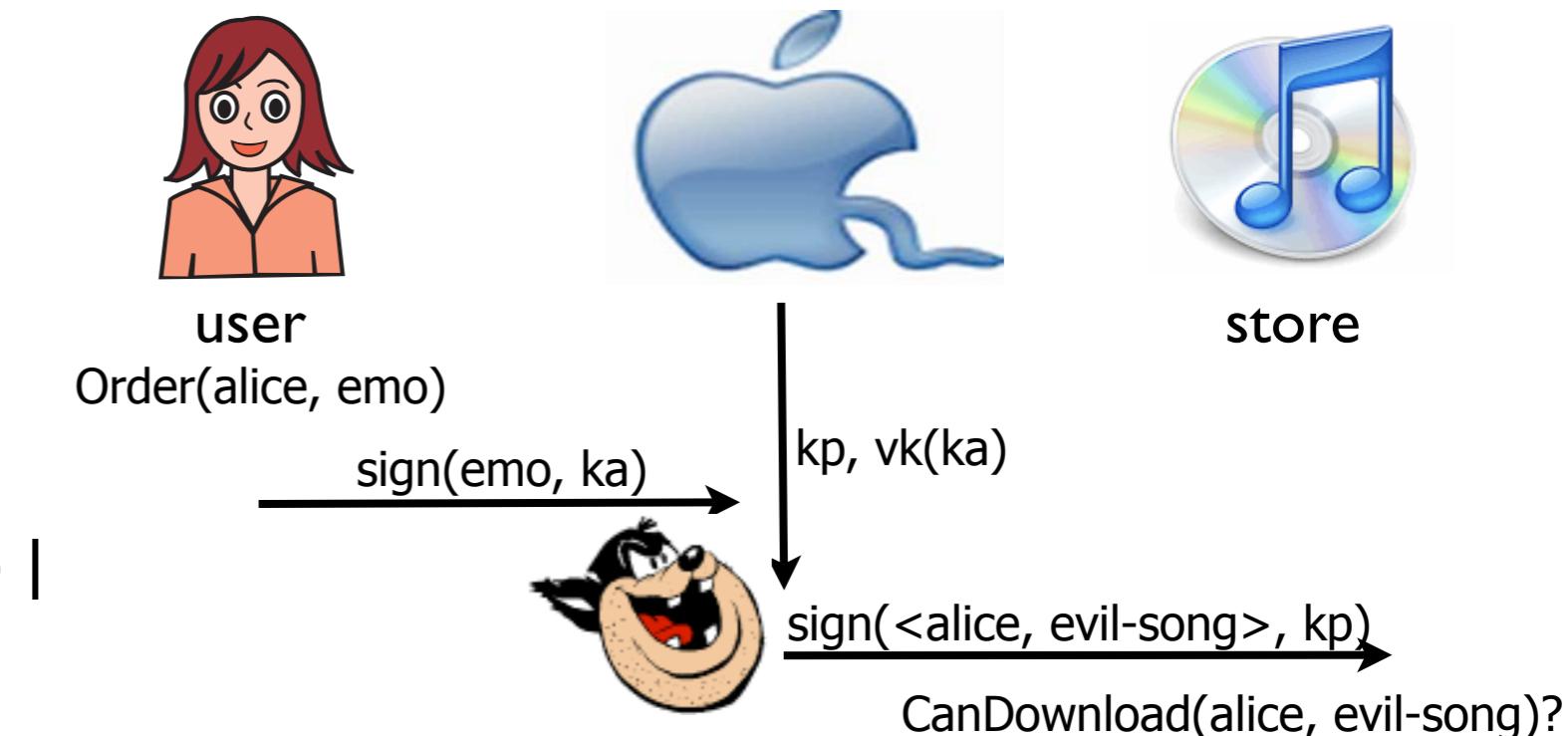
```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;  
(user | [out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Compromising the proxy

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



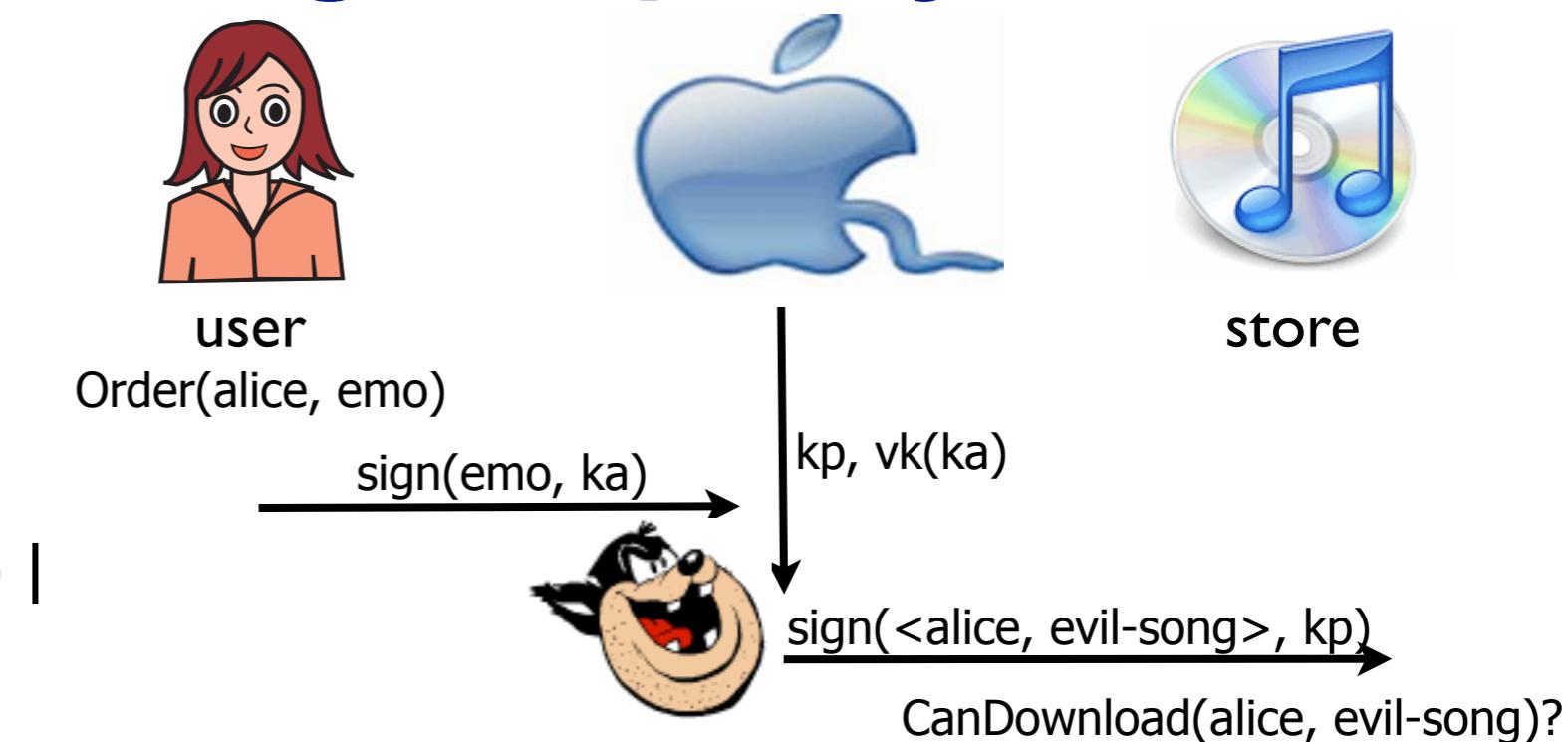
```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | [out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Compromising the proxy

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



- The protocol is not safe despite the compromise of the proxy

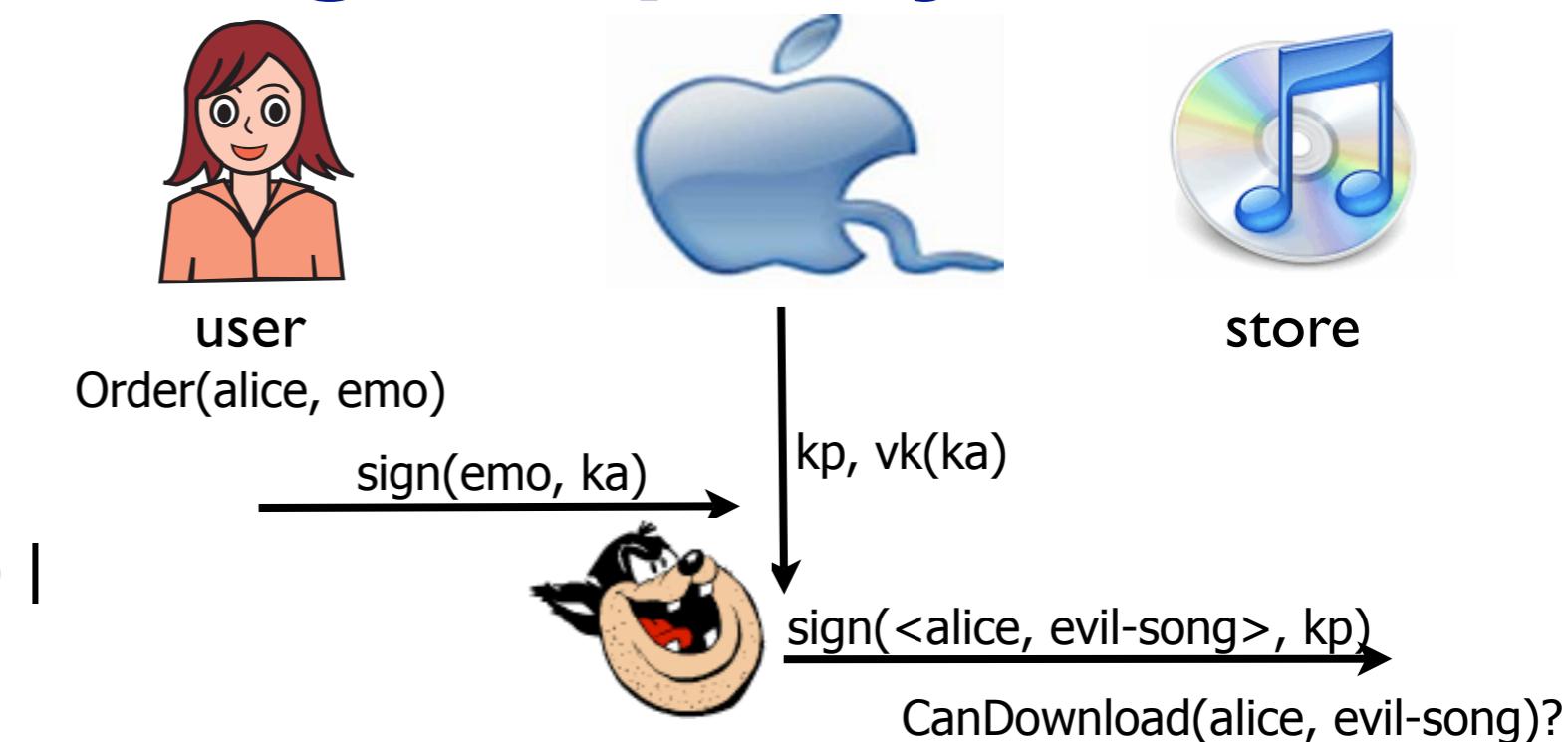
```
let store
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | [out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Compromising the proxy

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



- The protocol is not safe despite the compromise of the proxy
- Still, the policy does not even talk about the proxy ... this is a bug

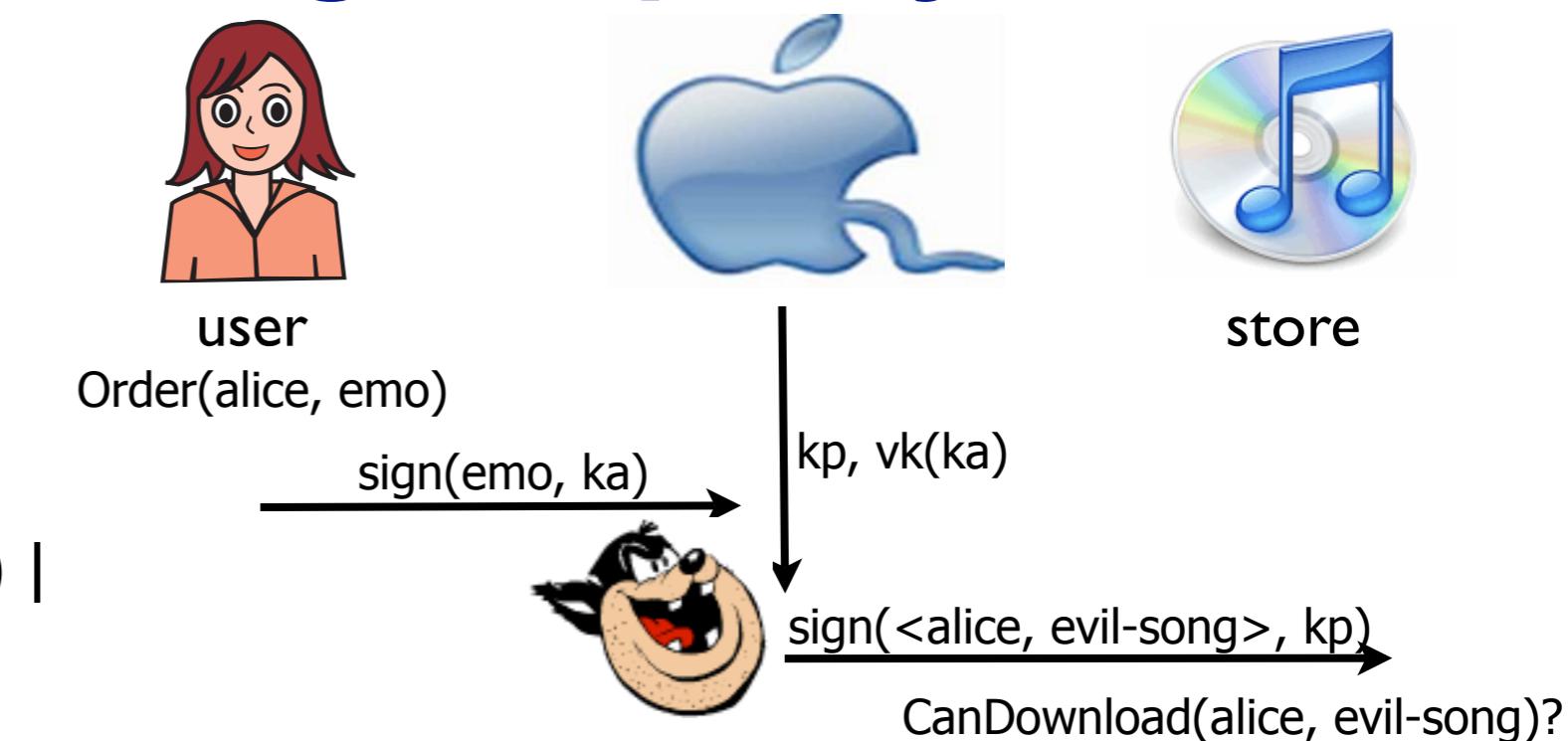
```
let store
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | [out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Compromising the proxy

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



- The protocol is not safe despite the compromise of the proxy
- Still, the policy does not even talk about the proxy ... this is a bug
- Two ways to fix this:

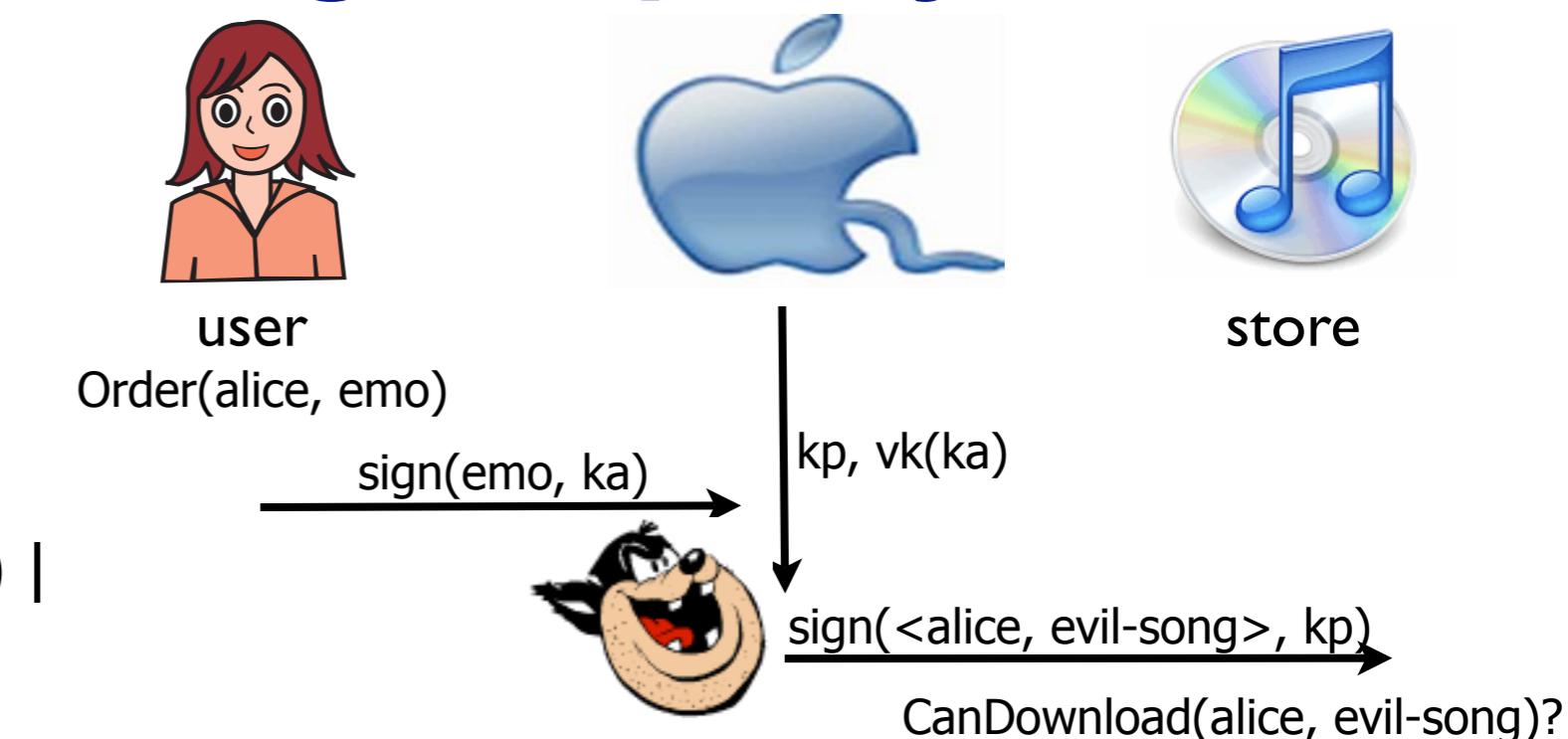
```
let store
```

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | [out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Compromising the proxy

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



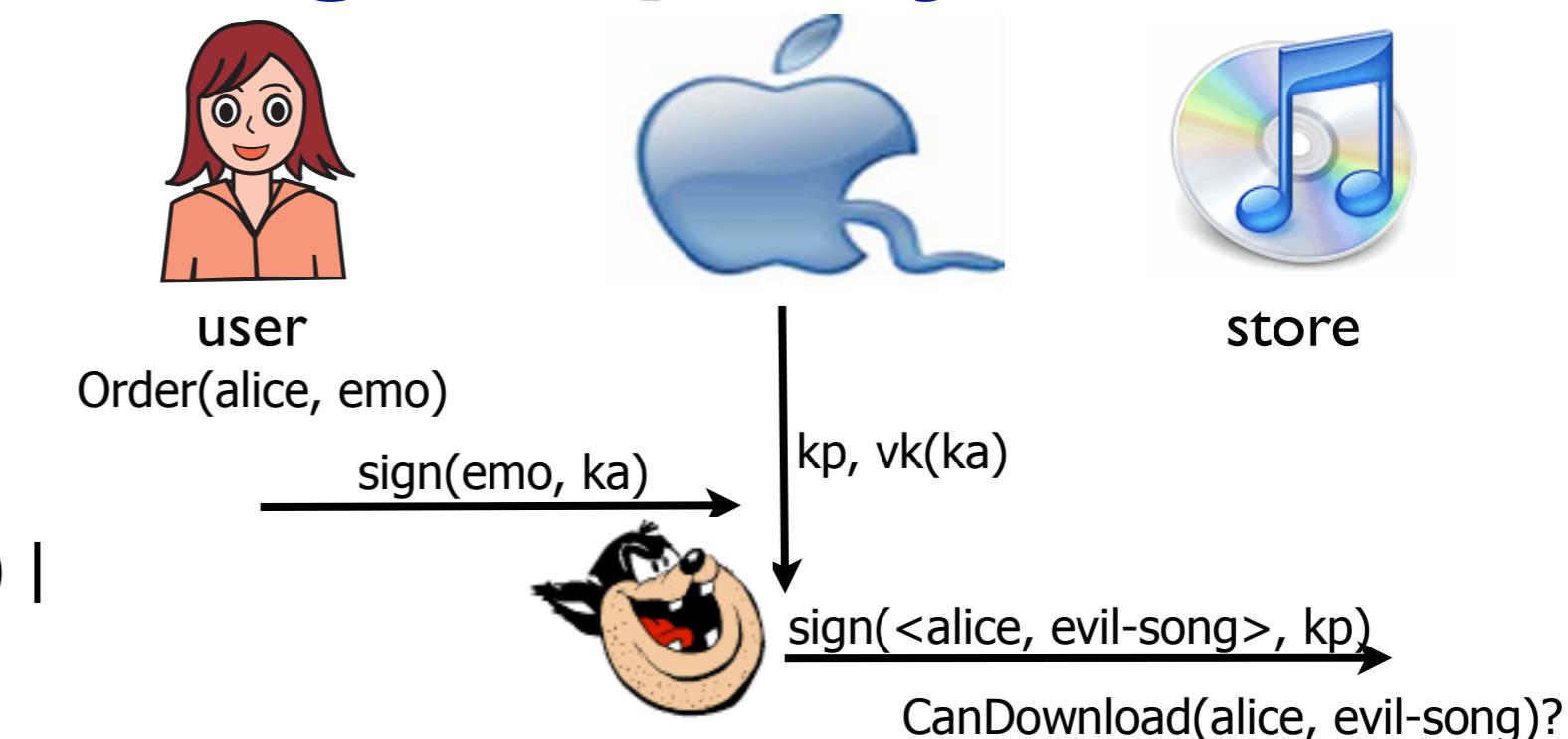
- The protocol is not safe despite the compromise of the proxy
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- Two ways to fix this:
  - document the dependency by weakening the property and making the policy more explicit (what the authors do in this paper)

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | [out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Compromising the proxy

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



- The protocol is not safe despite the compromise of the proxy
- Still, the policy does not even talk about the proxy ... this is a bug
- Two ways to fix this:

```
let store
```

- document the dependency by weakening the property and making the policy more explicit (what the authors do in this paper)
- strengthen the protocol (more about this at the end of the talk)

```
let policy = assume  $\forall u, s. (Order(u, s) \wedge Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;  
(user | [out(c, kp) | out(c, vk(ka))] | store | policy)
```

# 5 minutes break

# Type system

# Types

$T, U ::=$	Un	untrusted
	Ch( $T$ )	channel
	$\langle x_1 : T_1, \dots, x_n : T_n \rangle \{C\}$	tuple (dependent)
	PubKey( $T$ )	encryption key
	PrivKey( $T$ )	decryption key
	PubEnc( $T$ )	encrypted message
	SigKey( $T$ )	signing key
	VerKey( $T$ )	verification key
	Signed( $T$ )	signed message
	Hash( $T$ )	hashed message

# Types

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	SigKey( $T$ )	signing key
	VerKey( $T$ )	verification key
	Signed( $T$ )	signed message
	Hash( $T$ )	hashed message

- Typing environment
  - a list of type bindings ( $u : T$ ) and logical formulas  $C$

# Subtyping

- All messages sent over or received from the untrusted network have type  $\text{Un}$
- For flexibility we introduce a special subtyping relation
  - $T <: \text{Un} \Rightarrow T$  is **public**
    - its elements can be sent over the untrusted network
  - $\text{Un} <: T \Rightarrow T$  is **tainted**
    - elements can be received from the untrusted network
- Subtyping = preorder on types (i.e. weak)

# Typing terms

ENVIRONMENT

$$\frac{\Gamma \vdash \diamond \quad u : T \in \Gamma}{\Gamma \vdash u : T}$$

SUBSUMPTION

$$\frac{\Gamma \vdash M : T \quad \Gamma \vdash T <: T'}{\Gamma \vdash M : T'}$$

TUPLE

$$\frac{\forall i \in [1, n]. \Gamma \vdash M_i : T_i \quad forms(\Gamma) \models C\{M_i/x_i\}}{\Gamma \vdash \langle M_1, \dots, M_n \rangle : \langle x_1 : T_1, \dots, x_n : T_n \rangle \{C\}}$$

CONSTRUCTOR

$$\frac{f : (T_1, \dots, T_n) \mapsto T \quad \forall i \in [1, n]. \Gamma \vdash M_i : T_i}{\Gamma \vdash f(M_1, \dots, M_n) : T}$$

- Typing constructors

$$pk : (\text{PrivKey}(T)) \mapsto \text{PubKey}(T)$$

$$vk : (\text{SigKey}(T)) \mapsto \text{VerKey}(T)$$

$$enc : (T, \text{PubKey}(T)) \mapsto \text{PubEnc}(T)$$

$$sign : (T, \text{SigKey}(T)) \mapsto \text{Signed}(T)$$

$$hash : (T) \mapsto \text{Hash}(T)$$

# Typing processes

OUTPUT

$$\frac{\Gamma \vdash M : \text{Ch}(T) \quad \Gamma \vdash N : T \quad \Gamma \vdash P}{\Gamma \vdash \mathbf{out}(M, N).P}$$

INPUT

$$\frac{\Gamma \vdash M : \text{Ch}(T) \quad \Gamma, x : T \vdash P}{\Gamma \vdash \mathbf{in}(M, x).P}$$

NEW

$$\frac{T \in \{\text{Un}, \text{Ch}(U), \text{SigKey}(U), \text{PrivKey}(U)\} \quad \Gamma, a : T \vdash P}{\Gamma \vdash \mathbf{new } a : T.P}$$

REPLICATION

$$\frac{\Gamma \vdash P}{\Gamma \vdash !P}$$

NULL

$$\frac{\Gamma \vdash \diamond}{\Gamma \vdash \mathbf{0}}$$

# Typing destructor applications

DESTRUCTOR

$$\frac{g : (T_1, \dots, T_n) \mapsto T \quad \forall i \in [1, n]. \Gamma \vdash M_i : T_i \quad \Gamma, x : T \vdash P \quad \Gamma \vdash Q}{\Gamma \vdash \text{let } x = g(M_1, \dots, M_n) \text{ then } P \text{ else } Q}$$

- Typing destructors

$$\text{dec} : (\text{PubEnc}(T), \text{PrivKey}(T)) \mapsto T$$

$$\text{check} : (\text{Signed}(T), \text{VerKey}(T)) \mapsto T$$

# Typing parallel compositions

PARALLEL

$$\frac{P \rightsquigarrow \Gamma_P \quad \Gamma, \Gamma_P \vdash Q \quad Q \rightsquigarrow \Gamma_Q \quad \Gamma, \Gamma_Q \vdash P}{\Gamma \vdash P \mid Q}$$

- Environment extraction relation

assume  $C \rightsquigarrow C$

$$\frac{P \rightsquigarrow \Gamma_P}{\mathbf{new}~a : T.P \rightsquigarrow a : T, \Gamma_P}$$

$$\frac{P \rightsquigarrow \Gamma_P \quad Q \rightsquigarrow \Gamma_Q}{P \mid Q \rightsquigarrow \Gamma_P, \Gamma_Q}$$

everything else  $\rightsquigarrow \emptyset$

# Typing splits, assumes, asserts

SPLIT

$$\frac{\Gamma \vdash M : \langle y_1 : T_1, \dots, y_n : T_n \rangle \{ C \} \quad \Gamma, x_1 : T_1, \dots, x_n : T_n, \langle x_1, \dots, x_n \rangle = M, C \{ x_i / y_i \} \vdash P}{\Gamma \vdash \text{let } \langle x_1, \dots, x_n \rangle = M \text{ in } P}$$

ASSUME

$$\frac{\Gamma, C \vdash \diamond}{\Gamma \vdash \text{assume } C}$$

ASSERT

$$\frac{\Gamma \vdash \diamond \quad \text{forms}(\Gamma) \models C}{\Gamma \vdash \text{assert } C}$$

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



```
let proxy = assume Registered(alice) |  
        in(c,m);  
        let <s> = check(m, vk(ka)) in  
        out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
        let <user, song> = check(x, vk(kp)) in  
        assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka :  $\text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \})$ ;  
new kp :  $\text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$ ;  
(user | proxy | store | policy)
```

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$
$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$
$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$
$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

---

$$\Gamma \vdash \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$$

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$
$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$
$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$
$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\Gamma \vdash c : \text{Ch}(\text{Un})$$
$$\Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Un}$$

---

$$\Gamma \vdash \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$$

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\frac{\Gamma \vdash c : \text{Un} \quad \Gamma \vdash \text{Un} <: \text{Ch}(\text{Un})}{\Gamma \vdash c : \text{Ch}(\text{Un})}$$

$$\Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Un}$$

---

$$\Gamma \vdash \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$$

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\text{sign} : (T, \text{SigKey}(T)) \mapsto \text{Signed}(T)$$

$$\frac{\Gamma \vdash c : \text{Un} \quad \Gamma \vdash \text{Un} <: \text{Ch}(\text{Un}) \quad \Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Signed}(T) \quad \Gamma \vdash \text{Signed}(T) <: \text{Un}}{\Gamma \vdash c : \text{Ch}(\text{Un})}$$
$$\frac{}{\Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Un}}$$

$$\Gamma \vdash \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$$

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\text{sign} : (T, \text{SigKey}(T)) \mapsto \text{Signed}(T)$$

		$T <: \text{Un}$
$\Gamma \vdash c : \text{Un}$	$\Gamma \vdash \text{Un} <: \text{Ch}(\text{Un})$	$\Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Signed}(T)$
		$\Gamma \vdash \text{Signed}(T) <: \text{Un}$
<hr/>		<hr/>
$\Gamma \vdash c : \text{Ch}(\text{Un})$	$\Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Un}$	
<hr/>		
$\Gamma \vdash \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$		

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\text{sign} : (T, \text{SigKey}(T)) \mapsto \text{Signed}(T)$$

	$\frac{\Gamma \vdash \langle \text{emo} \rangle : T \quad \Gamma \vdash \text{ka} : \text{SigKey}(T)}{\Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Signed}(T)}$	$T <: \text{Un}$
$\Gamma \vdash c : \text{Un} \quad \Gamma \vdash \text{Un} <: \text{Ch}(\text{Un})$	$\Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Signed}(T)$	$\Gamma \vdash \text{Signed}(T) <: \text{Un}$
$\Gamma \vdash c : \text{Ch}(\text{Un})$	$\Gamma \vdash \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Un}$	
$\Gamma \vdash \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$		

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

	$\Gamma  - \text{emo} : T \quad \Gamma  - \text{ka} : \text{SigKey}(T)$	$T <: \text{Un}$ ✓
$\Gamma  - c : \text{Un} \quad \Gamma  - \text{Un} <: \text{Ch}(\text{Un})$	$\Gamma  - \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Signed}(T)$	$\Gamma  - \text{Signed}(T) <: \text{Un}$
$\Gamma  - c : \text{Ch}(\text{Un})$	$\Gamma  - \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Un}$	
$\Gamma  - \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$		

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$\Gamma  - \text{emo} : \text{Un}$	$\text{forms}(\Gamma)  = \text{Order}(\text{alice}, \text{emo})$	$\Gamma  - \langle \text{emo} \rangle : T$	$\Gamma  - \text{ka} : \text{SigKey}(T)$	$T <: \text{Un}$	✓
$\Gamma  - c : \text{Un}$	$\Gamma  - \text{Un} <: \text{Ch}(\text{Un})$	$\Gamma  - \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Signed}(T)$	$\Gamma  - \text{Signed}(T) <: \text{Un}$		
$\Gamma  - c : \text{Ch}(\text{Un})$		$\Gamma  - \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Un}$			
		$\Gamma  - \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$			

# Type-checking the example

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$

$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$

$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$

$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

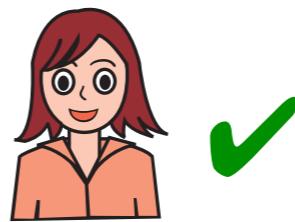
$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$\Gamma  - \text{emo} : \text{Un}$	$\text{forms}(\Gamma)  = \text{Order}(\text{alice}, \text{emo})$ ✓				
		$\Gamma  - \langle \text{emo} \rangle : T$	$\Gamma  - \text{ka} : \text{SigKey}(T)$		
			$T <: \text{Un}$ ✓		
$\Gamma  - c : \text{Un}$	$\Gamma  - \text{Un} <: \text{Ch}(\text{Un})$	$\Gamma  - \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Signed}(T)$	$\Gamma  - \text{Signed}(T) <: \text{Un}$		
		<hr/>			
$\Gamma  - c : \text{Ch}(\text{Un})$	$\Gamma  - \text{sign}(\langle \text{emo} \rangle, \text{ka}) : \text{Un}$				
	<hr/>				
			$\Gamma  - \text{out}(c, \text{sign}(\langle \text{emo} \rangle, \text{ka}))$		

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

```
let user = assume Order(alice, emo) |  
  out(c, sign(<emo>, ka)).
```



```
let proxy = assume Registered(alice) |  
  in(c,m);  
let <s> = check(m, vk(ka)) in  
  out(c, sign(<alice, s>, kp)).
```



```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka :  $\text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \})$ ;  
new kp :  $\text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$ ;  
(user | proxy | store | policy)
```

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$
$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$
$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$
$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

---

$$\Gamma \vdash \mathbf{in}(c, m); \mathbf{let} \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \mathbf{in} \mathbf{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$\Gamma \vdash c : \text{Ch}(\text{Un})$

---

$\Gamma \vdash \mathbf{in}(c, m); \mathbf{let} \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \mathbf{in} \mathbf{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\Gamma' = \Gamma, m : \text{Un}$$
$$\Gamma |- c : \text{Ch}(\text{Un})$$
$$\Gamma' |- \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

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$$\Gamma |- \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\Gamma' = \Gamma, m : \text{Un}$$
$$\text{check} : (\text{Signed}(T), \text{VerKey}(T)) \mapsto T$$
$$\Gamma \vdash c : \text{Ch}(\text{Un})$$
$$\Gamma' \vdash \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

---

$$\Gamma \vdash \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$

$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$

$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$

$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

check : (Signed( $T$ ), VerKey( $T$ ))  $\mapsto T$

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$$\Gamma' \vdash m : \text{Signed}(T) \quad \Gamma' \vdash \text{vk}(\text{ka}) : \text{VerKey}(T)$$


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$$\Gamma \vdash c : \text{Ch}(\text{Un}) \quad \Gamma' \vdash \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$


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$$\Gamma \vdash \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$

$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$

$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$

$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$vk : (\text{SigKey}(T)) \mapsto \text{VerKey}(T)$$

$$\frac{\Gamma' \vdash ka : \text{SigKey}(T)}{\Gamma' \vdash m : \text{Signed}(T) \quad \Gamma' \vdash vk(ka) : \text{VerKey}(T)}$$

$$\Gamma \vdash c : \text{Ch}(\text{Un}) \qquad \Gamma' \vdash \text{let } \langle s \rangle = \text{check}(m, vk(ka)) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp))$$

$$\Gamma \vdash \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, vk(ka)) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp)).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$$vk : (\text{SigKey}(T)) \mapsto \text{VerKey}(T)$$

$$\frac{\Gamma' \vdash ka : \text{SigKey}(T)}{\Gamma' \vdash m : \text{Signed}(T) \quad \Gamma' \vdash vk(ka) : \text{VerKey}(T)}$$

$$\Gamma \vdash c : \text{Ch}(\text{Un}) \quad \Gamma' \vdash \text{let } \langle s \rangle = \text{check}(m, vk(ka)) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp))$$

$$\Gamma \vdash \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, vk(ka)) \text{ in out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp)).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$$\Gamma'' = \Gamma', s : \text{Un}, \text{Order}(\text{alice}, s)$$

$$vk : (\text{SigKey}(T)) \mapsto \text{VerKey}(T)$$

	$\frac{\Gamma' \vdash ka : \text{SigKey}(T)}{\Gamma' \vdash m : \text{Signed}(T) \quad \Gamma' \vdash vk(ka) : \text{VerKey}(T)}$	$\Gamma'' \vdash \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp))$
$\Gamma \vdash c : \text{Ch}(\text{Un})$	$\Gamma' \vdash \text{let } \langle s \rangle = \text{check}(m, vk(ka)) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp))$	
$\Gamma \vdash \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, vk(ka)) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp)).$		

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$$\Gamma'' = \Gamma', s : \text{Un}, \text{Order}(\text{alice}, s)$$

	$\frac{\Gamma'  - ka : \text{SigKey}(T)}{\Gamma'  - m : \text{Signed}(T)}$	$\frac{\Gamma''  - \text{sign}(\langle \text{alice}, s \rangle, kp) : \text{Signed}(T') \quad T' <: \text{Un}}{\Gamma''  - \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp))}$
$\Gamma  - c : \text{Ch}(\text{Un})$	$\Gamma'  - \text{let } \langle s \rangle = \text{check}(m, \text{vk}(ka)) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp))$	
$\Gamma  - \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, \text{vk}(ka)) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp)).$		

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$$\Gamma'' = \Gamma', s : \text{Un}, \text{Order}(\text{alice}, s)$$

sign :  $(T', \text{SigKey}(T')) \mapsto \text{Signed}(T')$

$$\frac{\Gamma' |- ka : \text{SigKey}(T) \quad \Gamma'' |- \text{sign}(\langle \text{alice}, s \rangle, kp) : \text{Signed}(T') \quad T' <: \text{Un}}{\Gamma' |- m : \text{Signed}(T) \quad \Gamma' |- \text{vk}(ka) : \text{VerKey}(T)} \quad \frac{}{\Gamma'' |- \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp))}$$

$$\Gamma |- c : \text{Ch}(\text{Un})$$

$$\Gamma' |- \text{let } \langle s \rangle = \text{check}(m, \text{vk}(ka)) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp))$$

$$\Gamma |- \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, \text{vk}(ka)) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, kp)).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$$\Gamma'' = \Gamma', s : \text{Un}, \text{Order}(\text{alice}, s)$$

$$\text{sign} : (T', \text{SigKey}(T')) \mapsto \text{Signed}(T')$$

$$\frac{\Gamma'' \vdash \langle \text{alice}, s \rangle : T' \quad \Gamma'' \vdash \text{kp} : \text{SigKey}(T')}{\Gamma' \vdash \text{ka} : \text{SigKey}(T) \quad \Gamma'' \vdash \text{sign}(\langle \text{alice}, s \rangle, \text{kp}) : \text{Signed}(T') \quad T' <: \text{Un}}$$

$$\frac{\Gamma' \vdash m : \text{Signed}(T) \quad \Gamma' \vdash \text{vk}(\text{ka}) : \text{VerKey}(T)}{\Gamma'' \vdash \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))}$$

$$\Gamma \vdash c : \text{Ch}(\text{Un})$$

$$\Gamma' \vdash \text{let } s = \text{check}(m, \text{vk}(\text{ka})) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

$$\Gamma \vdash \text{in}(c, m); \text{let } s = \text{check}(m, \text{vk}(\text{ka})) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$$\Gamma'' = \Gamma', s : \text{Un}, \text{Order}(\text{alice}, s)$$

$$T' = \langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \\ \{ \text{Order}(\text{user}, \text{song}) \\ \wedge \text{Registered}(\text{user}) \}$$

$$\frac{}{\Gamma'' |- \langle \text{alice}, s \rangle : T' \quad \Gamma'' |- \text{kp} : \text{SigKey}(T')}$$

$$\frac{\Gamma' |- \text{ka} : \text{SigKey}(T)}{\Gamma' |- m : \text{Signed}(T)} \quad \frac{\Gamma'' |- \text{sign}(\langle \text{alice}, s \rangle, \text{kp}) : \text{Signed}(T') \quad T' <: \text{Un}}{\Gamma'' |- \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))}$$

$$\Gamma' |- m : \text{Signed}(T) \quad \Gamma' |- \text{vk}(\text{ka}) : \text{VerKey}(T)$$

$$\Gamma'' |- \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

$$\Gamma |- c : \text{Ch}(\text{Un})$$

$$\Gamma' |- \text{let } s = \text{check}(m, \text{vk}(\text{ka})) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

$$\Gamma |- \text{in}(c, m); \text{let } s = \text{check}(m, \text{vk}(\text{ka})) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$$\Gamma'' = \Gamma', s : \text{Un}, \text{Order}(\text{alice}, s)$$

$$T' = \langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \\ \{ \text{Order}(\text{user}, \text{song}) \\ \wedge \text{Registered}(\text{user}) \}$$

$$\Gamma'' |- \text{alice} : \text{Un} \quad \Gamma'' |- s : \text{Un} \quad \text{forms}(\Gamma'') |= \text{Order}(\text{alice}, s) \wedge \text{Registered}(\text{alice})$$

$$\Gamma'' |- \langle \text{alice}, s \rangle : T' \quad \Gamma'' |- \text{kp} : \text{SigKey}(T')$$

$$\Gamma' |- \text{ka} : \text{SigKey}(T) \quad \Gamma'' |- \text{sign}(\langle \text{alice}, s \rangle, \text{kp}) : \text{Signed}(T') \quad T' <: \text{Un}$$

$$\Gamma' |- m : \text{Signed}(T) \quad \Gamma' |- \text{vk}(\text{ka}) : \text{VerKey}(T) \quad \Gamma'' |- \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

$$\Gamma |- c : \text{Ch}(\text{Un}) \quad \Gamma' |- \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

$$\Gamma |- \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$$

# Type-checking the proxy

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, m : \text{Un}$$

$$T = \langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}$$

$$\Gamma'' = \Gamma', s : \text{Un}, \text{Order}(\text{alice}, s)$$

$$T' = \langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \\ \{ \text{Order}(\text{user}, \text{song}) \\ \wedge \text{Registered}(\text{user}) \}$$

$$\Gamma'' |- \text{alice} : \text{Un} \quad \Gamma'' |- s : \text{Un} \quad \text{forms}(\Gamma'') |= \text{Order}(\text{alice}, s) \wedge \text{Registered}(\text{alice}) \quad \checkmark$$

$$\Gamma'' |- \langle \text{alice}, s \rangle : T' \quad \Gamma'' |- \text{kp} : \text{SigKey}(T')$$

$$\Gamma' |- \text{ka} : \text{SigKey}(T) \quad \Gamma'' |- \text{sign}(\langle \text{alice}, s \rangle, \text{kp}) : \text{Signed}(T') \quad T' <: \text{Un}$$

$$\Gamma' |- m : \text{Signed}(T) \quad \Gamma' |- \text{vk}(\text{ka}) : \text{VerKey}(T) \quad \Gamma'' |- \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

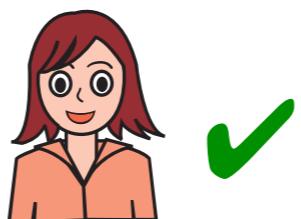
$$\Gamma |- c : \text{Ch}(\text{Un}) \quad \Gamma' |- \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp}))$$

$$\Gamma |- \text{in}(c, m); \text{let } \langle s \rangle = \text{check}(m, \text{vk}(\text{ka})) \text{ in } \text{out}(c, \text{sign}(\langle \text{alice}, s \rangle, \text{kp})).$$

# Type-checking the store

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

```
let user = assume Order(alice, emo) |  
    out(c, sign(<emo>, ka)).
```



```
let proxy = assume Registered(alice) |  
    in(c,m);  
let <s> = check(m, vk(ka)) in  
    out(c, sign(<alice, s>, kp)).
```



```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```



```
let policy = assume  $\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka :  $\text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \})$ ;  

 $\Gamma \vdash \text{new kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$ ;  

  (user | proxy | store | policy)
```

# Type-checking the store

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$
$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$
$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$
$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

---

$$\Gamma \vdash \mathbf{in}(c, x); \mathbf{let} \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \mathbf{in assert} \text{CanDownload}(\text{user}, \text{song})$$

# Type-checking the store

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$
$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$
$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$
$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\Gamma' = \Gamma, x : \text{Un}$$
$$\Gamma' \vdash \text{let } \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \text{ in assert } \text{CanDownload}(\text{user}, \text{song})$$

---

$$\Gamma \vdash \text{in}(c, x); \text{let } \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \text{ in assert } \text{CanDownload}(\text{user}, \text{song})$$

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$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$
$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\Gamma' = \Gamma, x : \text{Un}$$
$$\text{check} : (\text{Signed}(T), \text{VerKey}(T)) \mapsto T$$
$$\Gamma' \vdash \text{let } \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \text{ in assert } \text{CanDownload}(\text{user}, \text{song})$$
$$\Gamma \vdash \text{in}(c, x); \text{let } \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \text{ in assert } \text{CanDownload}(\text{user}, \text{song})$$

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$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$
$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$
$$\Gamma' = \Gamma, x : \text{Un}$$
$$\Gamma' \vdash x : \text{Signed}(\text{T}) \quad \Gamma' \vdash \text{vk}(\text{kp}) : \text{VerKey}(\text{T})$$

---

$$\Gamma' \vdash \mathbf{let} \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \mathbf{in} \mathbf{assert} \text{CanDownload}(\text{user}, \text{song})$$

---

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$$\Gamma' = \Gamma, x : \text{Un}$$
$$T = \langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \\ \{ \text{Order}(\text{user}, \text{song}) \\ \wedge \text{Registered}(\text{user}) \}$$
$$\Gamma' \vdash x : \text{Signed}(T) \quad \Gamma' \vdash \text{vk}(\text{kp}) : \text{VerKey}(T)$$

---

$$\Gamma' \vdash \mathbf{let} \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \mathbf{in} \mathbf{assert} \text{CanDownload}(\text{user}, \text{song})$$

---

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$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}),$$

$$\text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \})$$

$$\text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}),$$

$$\forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, x : \text{Un}$$

$$\Gamma'' = \Gamma', \text{user} : \text{Un}, \text{song} : \text{Un},$$

$$\text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user})$$

$$T = \langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle$$

$$\{ \text{Order}(\text{user}, \text{song})$$

$$\wedge \text{Registered}(\text{user}) \}$$


---


$$\Gamma' \vdash x : \text{Signed}(T) \quad \Gamma' \vdash \text{vk}(\text{kp}) : \text{VerKey}(T) \quad \Gamma'' \vdash \textbf{assert } \text{CanDownload}(\text{user}, \text{song})$$


---


$$\Gamma' \vdash \textbf{let } \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \textbf{ in assert } \text{CanDownload}(\text{user}, \text{song})$$


---


$$\Gamma \vdash \textbf{in}(c, x); \textbf{let } \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \textbf{ in assert } \text{CanDownload}(\text{user}, \text{song})$$

# Type-checking the store

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, x : \text{Un}$$

$$\Gamma'' = \Gamma', \text{user} : \text{Un}, \text{song} : \text{Un}, \\ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user})$$

$$T = \langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \\ \{ \text{Order}(\text{user}, \text{song}) \\ \wedge \text{Registered}(\text{user}) \}$$

$$\text{forms}(\Gamma'') \models \text{CanDownload}(\text{user}, \text{song})$$


---


$$\Gamma' \vdash x : \text{Signed}(T) \quad \Gamma' \vdash \text{vk}(\text{kp}) : \text{VerKey}(T) \quad \Gamma'' \vdash \textbf{assert} \text{CanDownload}(\text{user}, \text{song})$$


---


$$\Gamma' \vdash \textbf{let } \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \textbf{ in assert } \text{CanDownload}(\text{user}, \text{song})$$


---


$$\Gamma \vdash \textbf{in}(c, x); \textbf{let } \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \textbf{ in assert } \text{CanDownload}(\text{user}, \text{song})$$

# Type-checking the store

$$\Gamma = c : \text{Un}, \text{emo} : \text{Un}, \text{alice} : \text{Un}, \text{ka} : \text{SigKey}(\langle \text{song} : \text{Un} \rangle \{ \text{Order}(\text{alice}, \text{song}) \}), \\ \text{kp} : \text{SigKey}(\langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \{ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user}) \}) \\ \text{Order}(\text{alice}, \text{emo}), \text{Registered}(\text{alice}), \\ \forall u, s. (\text{Order}(u, s) \wedge \text{Registered}(u) \Rightarrow \text{CanDownload}(u, s))$$

$$\Gamma' = \Gamma, x : \text{Un}$$

$$\Gamma'' = \Gamma', \text{user} : \text{Un}, \text{song} : \text{Un}, \\ \text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user})$$

$$T = \langle \text{user} : \text{Un}, \text{song} : \text{Un} \rangle \\ \{ \text{Order}(\text{user}, \text{song}) \\ \wedge \text{Registered}(\text{user}) \}$$

forms( $\Gamma''$ ) |= CanDownload(user, song) ✓

---


$$\Gamma' |- x : \text{Signed}(T) \quad \Gamma' |- \text{vk}(\text{kp}) : \text{VerKey}(T) \quad \Gamma'' |- \textbf{assert} \text{CanDownload}(\text{user}, \text{song})$$


---

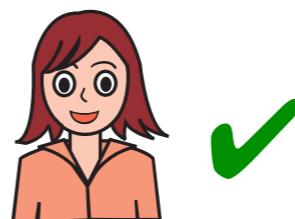

$$\Gamma' |- \textbf{let} \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \text{ in } \textbf{assert} \text{CanDownload}(\text{user}, \text{song})$$


---


$$\Gamma |- \textbf{in}(c, x); \textbf{let} \langle \text{user}, \text{song} \rangle = \text{check}(x, \text{vk}(\text{kp})) \text{ in } \textbf{assert} \text{CanDownload}(\text{user}, \text{song})$$

# Type-checking done ✓

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



```
let proxy = assume Registered(alice) |  
          in(c,m);  
let <s> = check(m, vk(ka)) in  
        out(c, sign(<alice, s>, kp)).
```



```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```



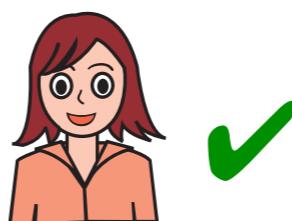
```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka : SigKey(<song:Un>{Order(alice, song)});  
new kp : SigKey(<user:Un, song:Un>{Order(user, song) ∧ Registered(user)});  
(user | proxy | store | policy)
```

# Type-checking done ✓

- The type system enforces robust safety (we will prove it soon)
- So successfully type-checking the protocol means it is **robustly safe**

```
let user = assume Order(alice, emo) |  
        out(c, sign(<emo>, ka)).
```



```
let proxy = assume Registered(alice) |  
          in(c,m);  
let <s> = check(m, vk(ka)) in  
        out(c, sign(<alice, s>, kp)).
```



```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```



```
let policy = assume ∀ u, s. (Order(u, s) ∧ Registered(u) ⇒ CanDownload(u, s)).
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```
process new ka : SigKey(<song:Un>{Order(alice, song)});  
new kp : SigKey(<user:Un, song:Un>{Order(user, song) ∧ Registered(user)});  
(user | proxy | store | policy)
```

# Illustrating the proof technique (on the board)

# Proving safety

- Theorem (Subject-reduction)
  1. If  $\Gamma \vdash P$  and  $P \equiv Q$ , then  $\Gamma \vdash Q$ ;
  2. If  $\Gamma \vdash P$  and  $P \rightarrow Q$  then  $\Gamma \vdash Q$ .
- Lemma (Correct Environment Extraction)

If  $Q \rightsquigarrow \Gamma_Q$  then  $\exists \mathcal{E} = \mathbf{new}~b_1 : U_1 \dots \mathbf{new}~b_m : U_m. [ ] \mid Q'$  s.t.  
 $\{b_1, \dots, b_m\} = \text{dom}(\Gamma_Q)$ ,  $Q \equiv \mathcal{E}[\mathbf{assume}~C_1 \mid \dots \mid \mathbf{assume}~C_n]$ ,  
and  $\text{forms}(\Gamma_Q) = \{C_1, \dots, C_n\}$ .
- Theorem (Safety)

If  $\vdash_{\mathsf{Un}} P$  then  $P$  is safe.

( $\vdash_{\mathsf{Un}} P$  denotes  $u_1 : \mathsf{Un}, \dots, u_n : \mathsf{Un} \vdash P$  for  $u_i \in \text{free}(P)$ )

# Proving robust safety

- **Theorem (Safety)**  
If  $\vdash_{U_n} P$  then  $P$  is safe.
- **Lemma (Universal Type)**  
If  $\Gamma \vdash \diamond$  then  $\Gamma \vdash U_n <:> T$  for all  
 $T \in \{Ch(U_n), \langle x_1 : U_n, \dots, x_n : U_n \rangle \{true\}, SigKey(U_n), VerKey(U_n),$   
 $Signed(U_n), PubKey(U_n), PrivKey(U_n), PubEnc(U_n), Hash(U_n)\}$
- **Lemma (Term Un-typability)**  $\forall M. \vdash_{U_n} M : U_n$
- **Lemma (Opponent Typability)**  
For all opponents  $O$  we have  $\vdash_{U_n} O$ .
- **Theorem (Robust Safety).**  
If  $\vdash_{U_n} P$  then  $P$  is robustly safe.

# Safety despite compromised participants

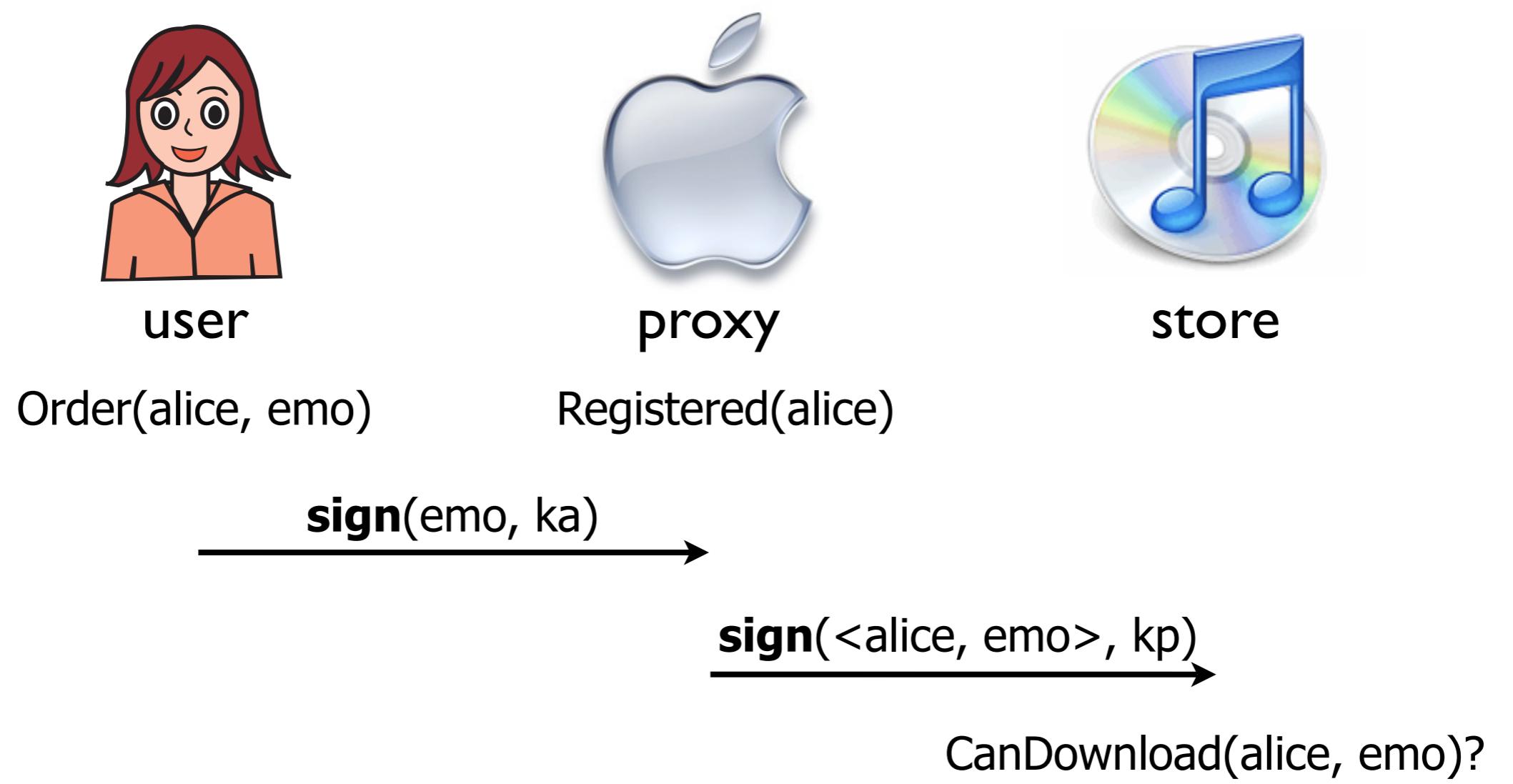
(slides I needed to skip)

# Informal principle

- An invalid authorization decision by an uncompromised participant should only arise if participants on which the decision logically depends are compromised.
- In general, we check conformance of code to a logical policy, so that we can focus security reviews on the policy
- Hence, the impact of partial compromise should be apparent from the policy, without study of the code

# Expressing protocol participants

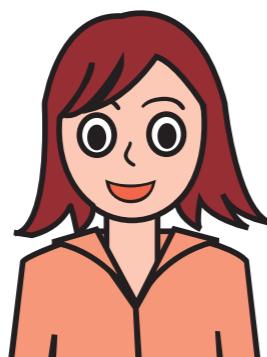
- Extend the authorization logic with **says** operator



policy =  $\forall \text{user}, \forall \text{song} (\text{Order}(\text{user}, \text{song}) \wedge \text{Registered}(\text{user})) \Rightarrow \text{CanDownload}(\text{user}, \text{song})$

# Expressing protocol participants

- Extend the authorization logic with **says** operator



user



proxy



store

alice **says** Order(emo)

proxy **says** Registered(alice)

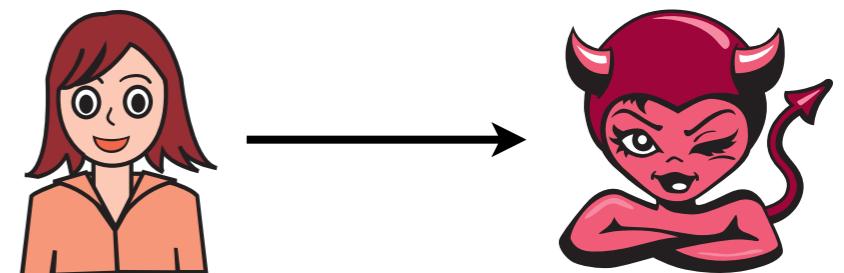
**sign**(emo, ka)

**sign**(<alice, emo>, kp)

CanDownload(alice, emo)?

policy =  $\forall \text{user}, \forall \text{song} (\text{user } \mathbf{says} \text{ Order(song)} \wedge \text{proxy } \mathbf{says} \text{ Registered(user)} \Rightarrow \text{CanDownload(user, song)})$

# Safety despite compromised participants



- Definition

Let  $\mathcal{E} = \mathbf{new} c_1 : T_1 \dots \mathbf{new} c_m : T_m. [ ] \mid R$

and  $P_{b_1}, \dots, P_{b_n}$  be  $n$  protocol participants

( $P_{b_i}$  only contains formulas of the form  $b_i$  says  $C$ ).

$\mathcal{E}[P_{b_1} \mid \dots \mid P_{b_n}]$  is safe despite  $\{b_1, \dots, b_n\}$  iff  $\exists Q', \sigma$

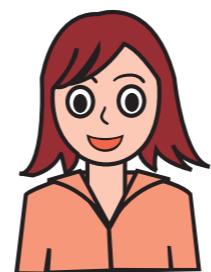
(1)  $P_{b_1} \mid \dots \mid P_{b_n} = Q' \sigma$  with  $fn(Q') \cap \{c_1, \dots, c_n\} = \emptyset$

(2) **assume**  $\wedge_{b \in \{b_1, \dots, b_n\}} b$  says **false**  $\mid \mathcal{E}[\prod_{x \in dom(\sigma)} \mathbf{out}(c, x\sigma)]$   
is robustly safe (where  $c \notin \{c_1, \dots, c_n\}$ )

- The type system can also enforce this property

- Basically all we need to check is that all variables in the domain of  $\sigma$  can be given type **Un**

# Compromising the user



user



proxy



store

alice **says** Order(alice, emo) proxy **says** Registered(alice)

sign(emo, ka)

```
let user = assume alice says Order(emo) |
    out(c, sign(<emo>, ka)).
```

sign(<alice, emo>, kp)

CanDownload(alice, emo)?

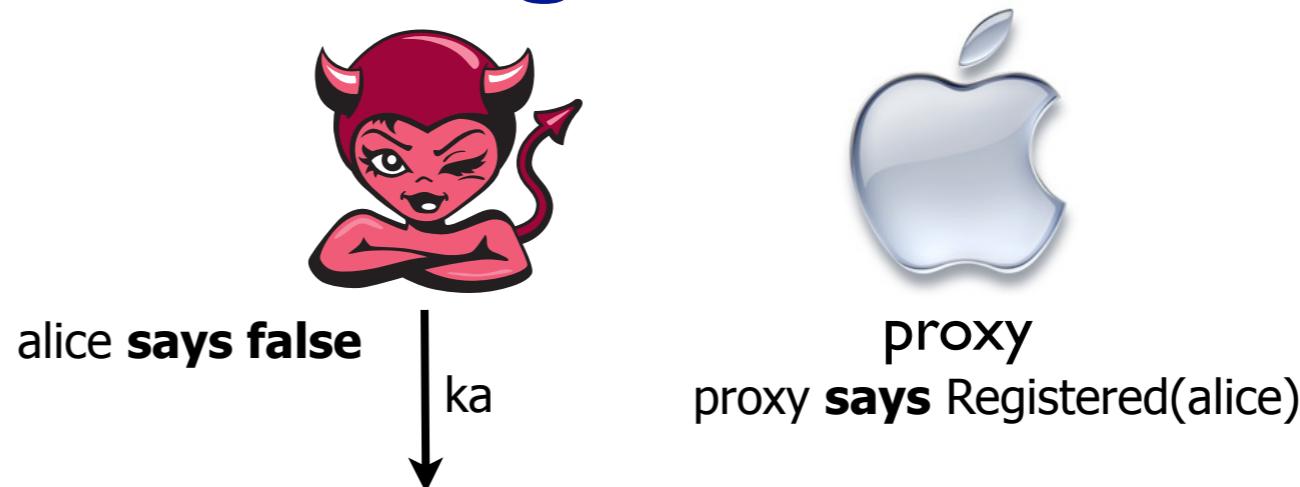
```
let proxy = assume proxy says Registered(alice) |
    in(c,m);
    let <s> = check(m, vk(ka)) in
    out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);
    let <user, song> = check(x, vk(kp)) in
    assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (u \text{ says } Order(s) \wedge proxy \text{ says } Registered(u) \Rightarrow CanDownload(u, s))$ .
```

```
process new ka; new kp;
    [user] | proxy | store | policy
```

# Compromising the user



```
let proxy = assume proxy says Registered(alice) |  
in(c,m);  
let <s> = check(m, vk(ka)) in  
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (u \text{ says Order}(s) \wedge \text{proxy says Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka; new kp;  
assume alice says false | [out(c, ku)] | proxy | store | policy
```

# Compromising the user



```
let proxy = assume proxy says Registered(alice) |
```

```
in(c,m);
```

```
let <s> = check(m, vk(ka)) in
out(c, sign(<alice, s>, kp)).
```

```
let store = in(c, x);
```

```
let <user, song> = check(x, vk(kp)) in
assert CanDownload(user, song).
```

```
let policy = assume ∀ u, s.(u says Order(s) ∧ proxy says Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;
```

```
assume alice says false | [out(c, ku)] | proxy | store | policy
```

# Compromising the user



```
let proxy = assume proxy says Registered(alice).
in(c,m);
```

```
let <s> = check(m, vk(ka)) in
out(c, sign(<alice, s>, kp)).
```

With the new definition the system is safe despite the compromise of the user

```
let store = in(c, x);
```

```
let <user, song> = check(x, vk(kp)) in
assert CanDownload(user, song).
```

```
let policy = assume ∀ u, s.(u says Order(s) ∧ proxy says Registered(u) ⇒ CanDownload(u, s)).
```

```
process new ka; new kp;
```

```
assume alice says false | [out(c, ku)] | proxy | store | policy
```

# Compromising the user



**let proxy = assume proxy says Registered(alice).** |

**in(c,m);**

**let <s> = check(m, vk(ka)) in**  
**out(c, sign(<alice, s>, kp)).**

**let store = in(c, x);**

**let <user, song> = check(x, vk(kp)) in**  
**assert CanDownload(user, song).**

**let policy = assume**  $\forall u, s. (u \text{ says Order}(s) \wedge \text{proxy says Registered}(u) \Rightarrow \text{CanDownload}(u, s)).$

**process new ka; new kp;**

**assume alice says false | [out(c, ku)] | proxy | store | policy**

- With the new definition the system is safe despite the compromise of the user
- All assertions only depending on alice saying something will hold (since alice says false)

# Compromising the user



**let proxy = assume proxy says Registered(alice).** |  
**in(c,m);**

**let <s> = check(m, vk(ka)) in**  
**out(c, sign(<alice, s>, kp)).**

**let store = in(c, x);**

**let <user, song> = check(x, vk(kp)) in**  
**assert CanDownload(user, song).**

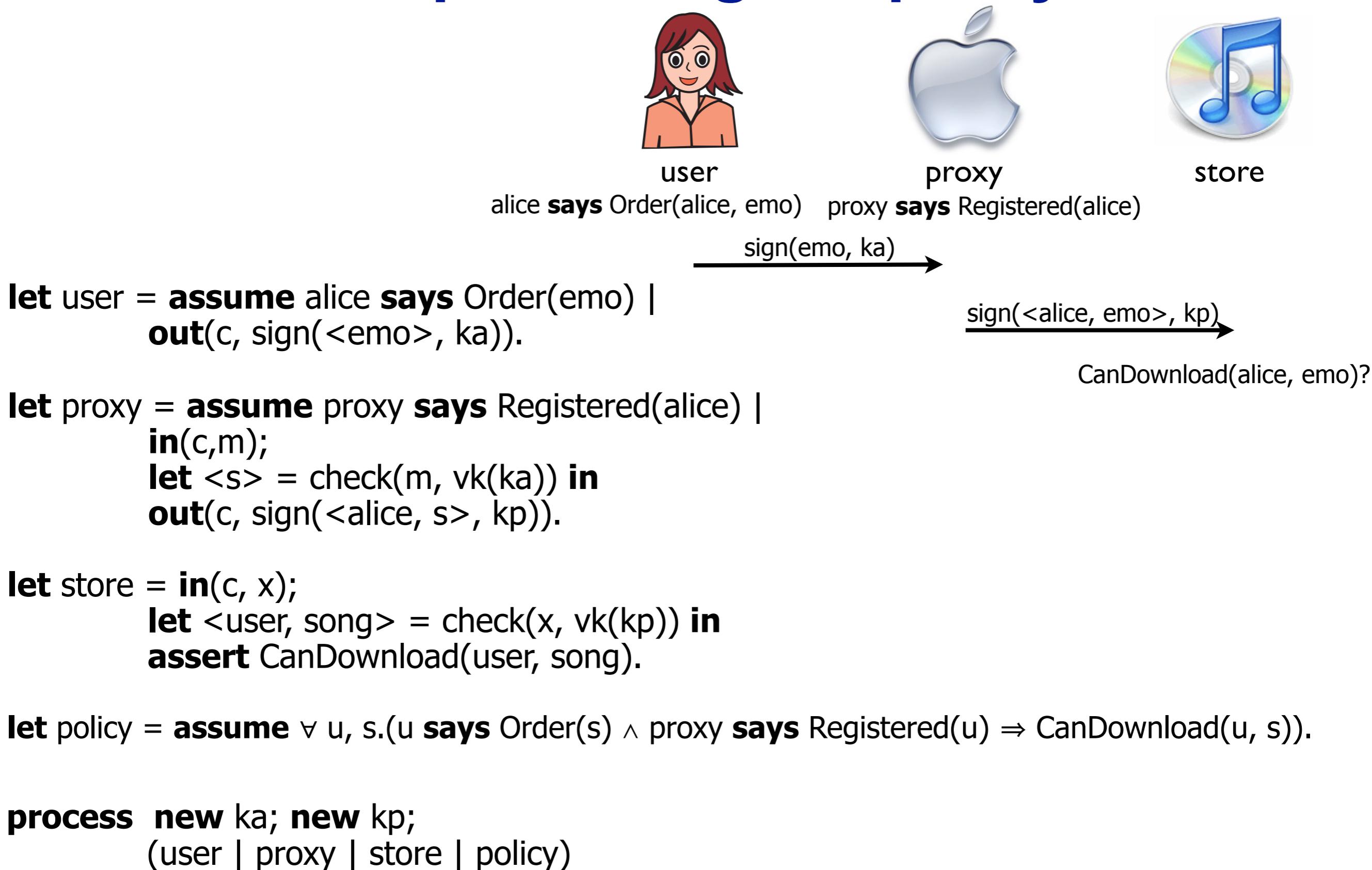
**let policy = assume**  $\forall u, s. (u \text{ says Order}(s) \wedge \text{proxy says Registered}(u) \Rightarrow \text{CanDownload}(u, s)).$

**process new ka; new kp;**

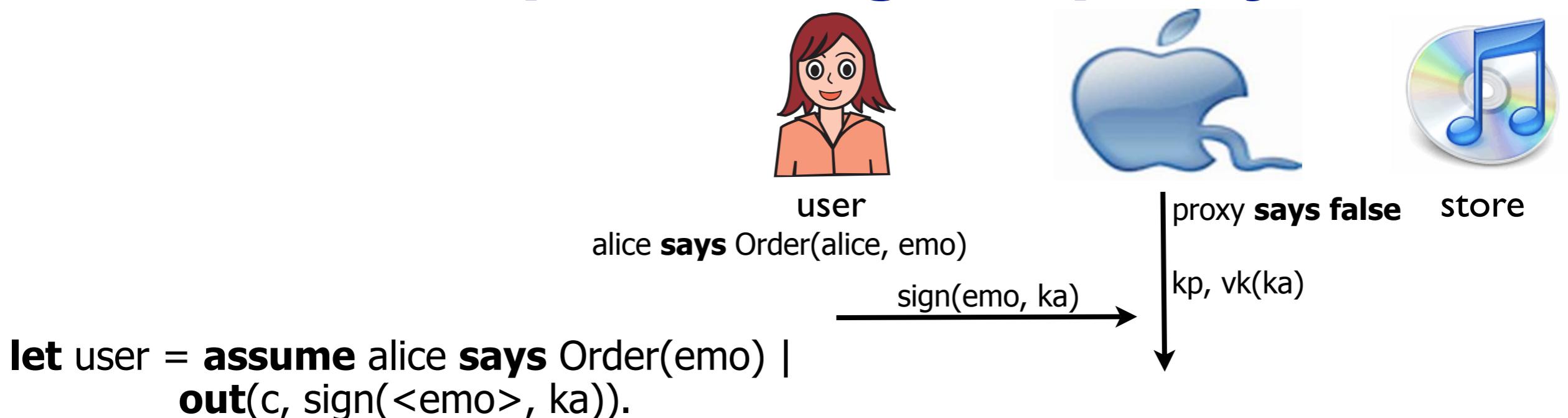
**assume alice says false | [out(c, ku)] | proxy | store | policy**

- With the new definition the system is safe despite the compromise of the user
- All assertions only depending on alice saying something will hold (since alice says false)
- This is **not** a problem

# Compromising the proxy



# Compromising the proxy



```
let user = assume alice says Order(emo) |  

out(c, sign(<emo>, ka)).
```

```
let store = in(c, x);  

let <user, song> = check(x, vk(kp)) in  

assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (u \text{ says Order}(s) \wedge \text{proxy says Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

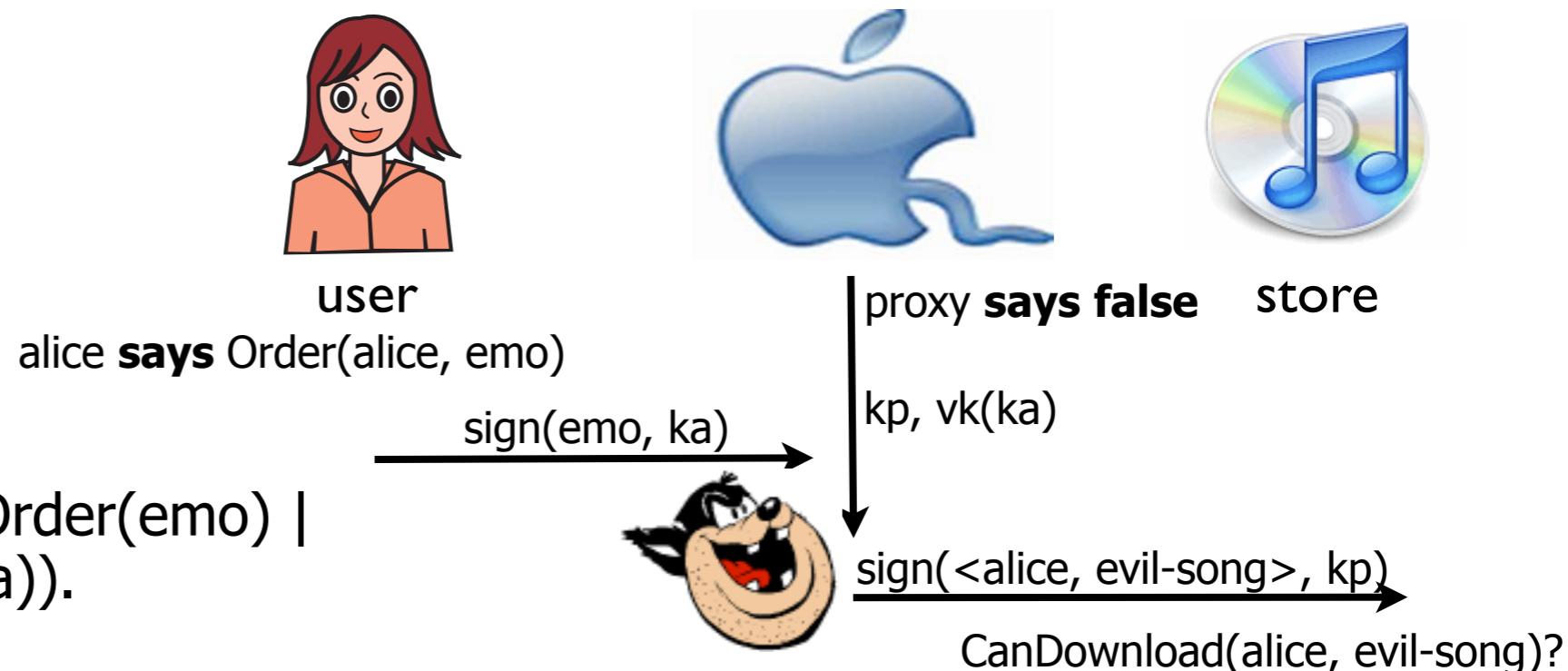
```
process new ka; new kp;  

  (user | [assume proxy says false | out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Compromising the proxy

```
let user = assume alice says Order(alice, emo) |  

out(c, sign(<emo>, ka)).
```



```
let store = in(c, x);  

let <user, song> = check(x, vk(kp)) in  

assert CanDownload(user, song).
```

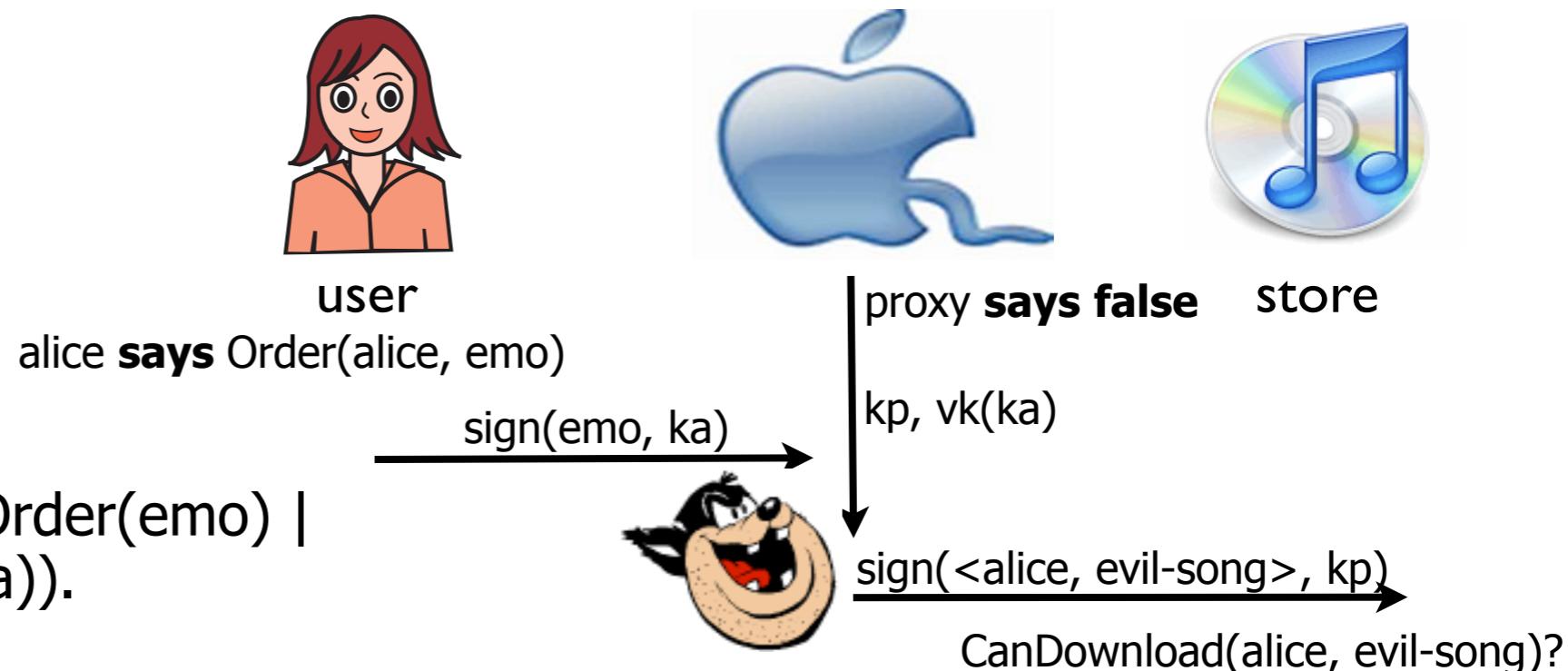
```
let policy = assume  $\forall u, s. (u \text{ says } Order(s) \wedge \text{proxy says Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka; new kp;  

(user | [assume proxy says false | out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Compromising the proxy

```
let user = assume alice says Order(emo) |  
out(c, sign(<emo>, ka)).
```



- The system is still **not** safe despite the compromise of the proxy

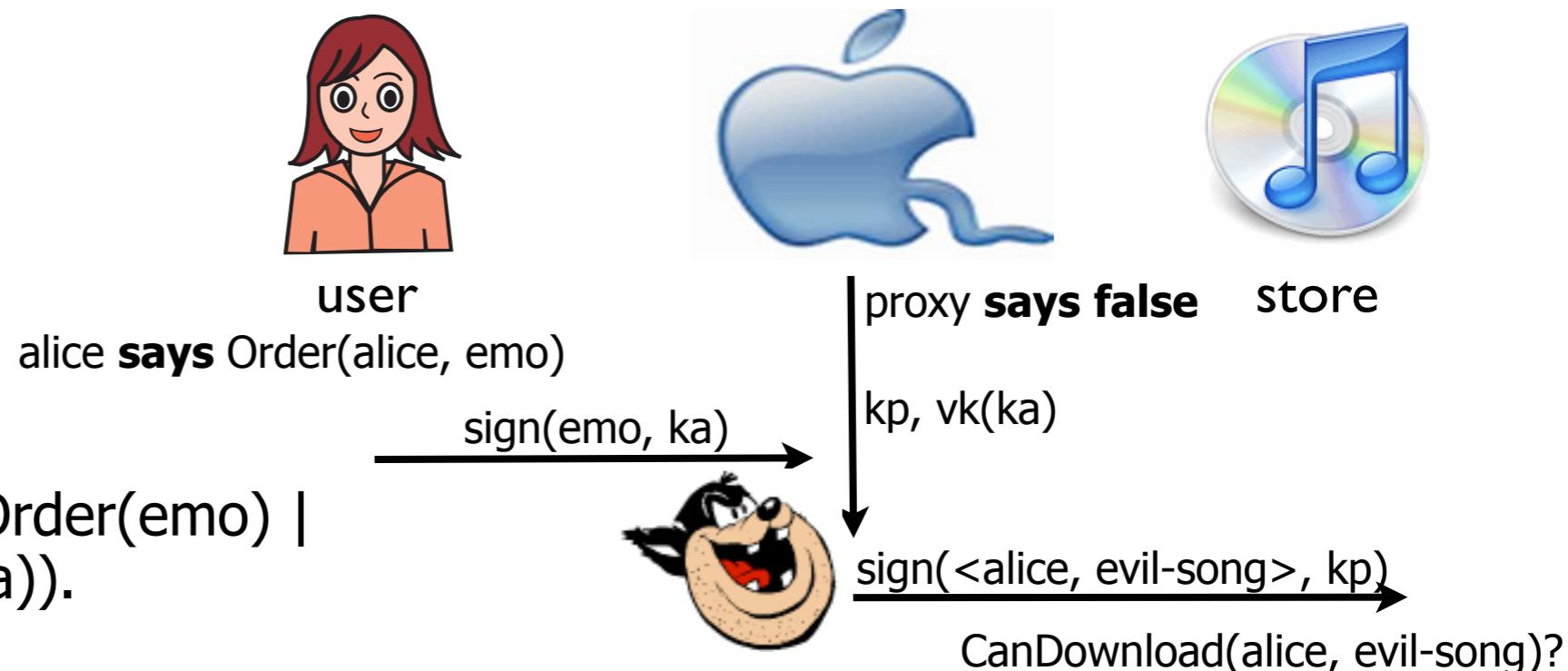
```
let store = in(c, x);  
let <user, song> = check(x, vk(kp)) in  
assert CanDownload(user, song).
```

```
let policy = assume  $\forall u, s. (u \text{ says } Order(s) \wedge \text{proxy says Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka; new kp;  
(user | [assume proxy says false | out(c, kp) | out(c, vk(ka))] | store | policy)
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# Compromising the proxy

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```

- The system is still **not** safe despite the compromise of the proxy
- These are the kind of bugs we are actually looking for

```
let policy = assume  $\forall u, s. (u \text{ says } Order(s) \wedge \text{proxy says Registered}(u) \Rightarrow \text{CanDownload}(u, s))$ .
```

```
process new ka; new kp;  
(user | [assume proxy says false | out(c, kp) | out(c, vk(ka))] | store | policy)
```

# Making the policy more explicit

- To fix the undocumented dependency on the proxy we can change the policy
  - $\text{policy} = \forall \text{user}, \forall \text{song}(\text{proxy } \mathbf{controls} (\text{user } \mathbf{says} \text{ Order}(\text{song})) \wedge \text{proxy } \mathbf{says} \text{ Registered}(\text{user}) \Rightarrow \text{CanDownload}(\text{user}, \text{song}))$   
where “a **controls** C” means “(a **says** C)  $\Rightarrow$  C”
- With respect to this policy the protocol is safe despite the compromise of the proxy
- But this doesn’t make the protocol any safer
  - it only documents all the implications of compromising the proxy

# Related and future work