Algoritmica Grafurilor Tema 2

Iordache Iustin-Ionut **Grupa B2**
Vascan Dumitru **Grupa B2**

7 Ianuarie 2014

Problema 1

Pentru a demonstra ca doar una dintre proprietati a sau b pot avea loc, vom demonstra ca $(a) \Rightarrow !(b)$ si $(b) \Rightarrow !(a)$

$(a) \Rightarrow !(b)$

$$l(e) \leq f(e) \leq u(e) \Rightarrow \begin{cases} l(e) \leq f(e) \Rightarrow \sum\limits_{e \epsilon \delta^-(X)} l(e) \leq \sum\limits_{e \epsilon \delta^-(X)} f(e) & (1) \\ f(e) \leq u(e) \Rightarrow \sum\limits_{e \epsilon \delta^+(X)} f(e) \leq \sum\limits_{e \epsilon \delta^+(X)} u(e) & (2) \end{cases}$$

$$\begin{cases} \sum\limits_{e \epsilon \delta^-(v)} f(e) = \sum\limits_{e \epsilon \delta^+(v)} f(e) \\ v \epsilon V si X \subseteq V \end{cases} \Rightarrow \sum\limits_{e \epsilon \delta^-(X)} f(e) = \sum\limits_{e \epsilon \delta^+(X)} f(e) \ (3)$$

Din (1), (2) si (3) $\Rightarrow \sum\limits_{e \epsilon \delta^-(X)} l(e) \leq \sum\limits_{e \epsilon \delta^+(X)} u(e)$

$(b) \Rightarrow !(a)$

Stim din ipoteza ca $l(e) \leq u(e)$ pentru orice arc $e \epsilon E \Rightarrow$

$$\Rightarrow \begin{cases} \sum\limits_{e \epsilon \delta^-(X)} l(e) \leq \sum\limits_{e \epsilon \delta^-(X)} u(e) & (4) \\ \sum\limits_{e \epsilon \delta^+(X)} l(e) \leq \sum\limits_{e \epsilon \delta^+(X)} u(e) & (5) \end{cases}$$

Din (4), (5) si (b) $\Rightarrow \sum\limits_{e \epsilon \delta^-(X)} l(e) \leq \sum\limits_{e \epsilon \delta^-(X)} u(e) < \sum\limits_{e \epsilon \delta^+(X)} l(e) \leq \sum\limits_{e \epsilon \delta^+(X)} u(e) \quad (6)$

Pentru ca (a) sa fie adevarat, trebuie sa avem $l(e) \leq f(e) \leq u(e) \Rightarrow$

$$\Rightarrow \begin{cases} \sum\limits_{e \epsilon \delta^-(X)} l(e) \leq \sum\limits_{e \epsilon \delta^-(X)} f(e) \leq \sum\limits_{e \epsilon \delta^-(X)} u(e) & (7) \\ \sum\limits_{e \epsilon \delta^+(X)} l(e) \leq \sum\limits_{e \epsilon \delta^+(X)} f(e) \leq \sum\limits_{e \epsilon \delta^+(X)} u(e) & (8) \end{cases}$$

Din (6), (7) si (8) $\Rightarrow \sum_{e \epsilon \delta^-(X)} f(e) < \sum_{e \epsilon \delta^+(X)} f(e)$, ceea ce contrazice ipoteza.

Problema 2

a) Notam cu V_i a i-ea componenta conexa a grafului V.

Cum componentele conexe sunt arbori, iar un arbore cu k varfuri are k-1 muchii, si cum arborele$V_i$este e-par daca are un numar par de muchii$\Rightarrow |V_i| \equiv 1(mod 2)$.

eh(V)=$|V_1|(mod2) + |V_2|(mod2) + ... + |V_n|(mod2)$ (1)

$|V| = |V_1| + |V_2| + ... + |V_n|$

$|V|(mod2) = (|V_1| + |V_2| + ... + |V_n|)(mod2) = (|V_1|(mod2) + |V_2|(mod2) + ... + |V_n|(mod2))(mod2)$ (2)

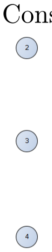
Din (1), (2) $\Rightarrow eh(V) \equiv V(mod)$

Problema 4

Consideram n=4, si D={3,3,3,3}.

Vom construi graful $M_D$astfel :
1.Construim subgrafurile C_i,$1 \leq i \leq n$ ca fiind induse de $R_i \bigcup S_i$
2.Unim componentele C_i,$1 \leq i \leq n$ si construim muchiile $\{r_{i,j}, r_{j,i}\}$pentru $i, j \epsilon \{1, ..., n\}$si $i \neq j$

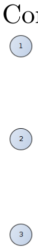
Construim C_1



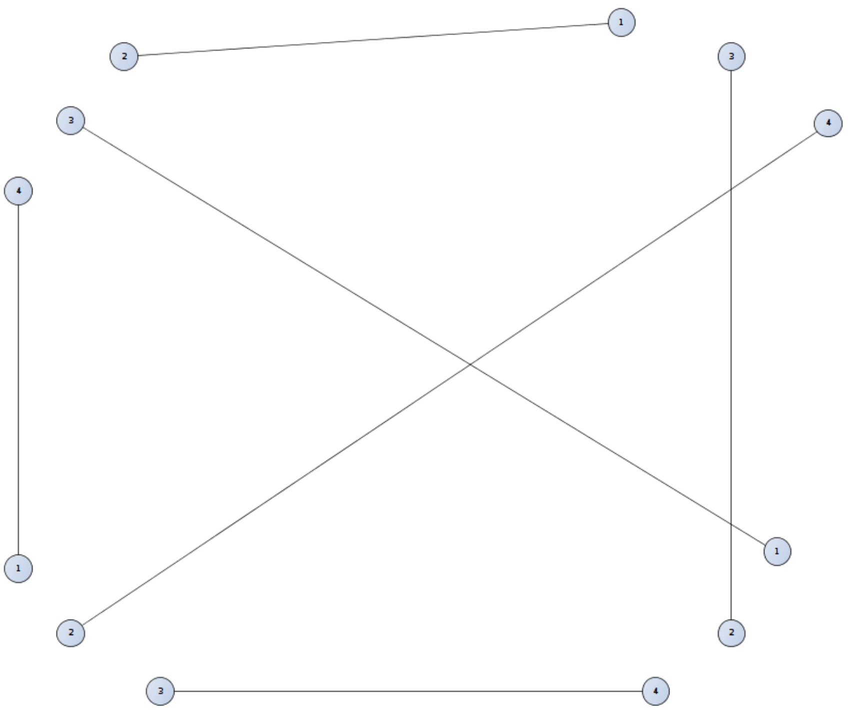
Construim C_2

Construim C_3

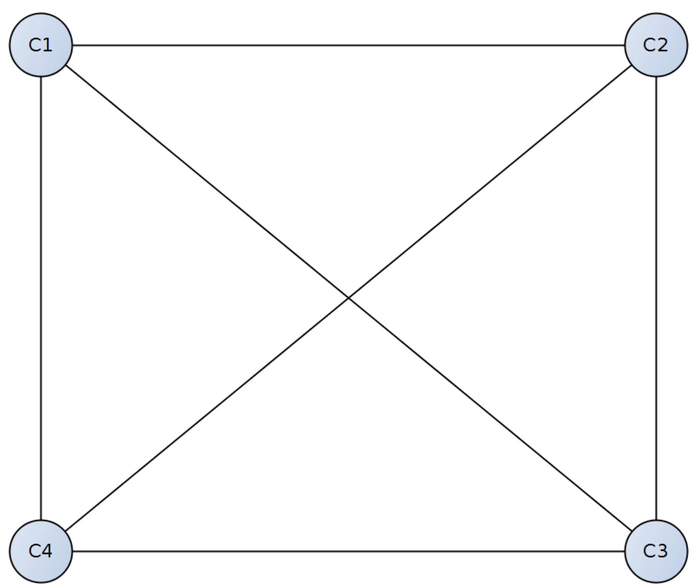
Construim C_4



Avand C_1, C_2, C_3, C_4vom construi graful $M_D$unind varfurile din $R_i$cu cele din $R_j$cum am scris la 2.



" $\Leftarrow$ "Se observa pe reprezentarea de mai sus ca graful M_D are un cuplaj perfect, iar gradele varfurilor sale sunt d_i



" $\Rightarrow$ "In reprezentarea de mai sus, fiecare din componentele C_1, C_2, C_3, C_4este un subgraf format dupa constructia enuntata, iar fiecare din varfurile acestori subgrafuri este unit cu cate un varf din alte subgrafuri, la fel urmand regula de constructie a muchiilor.