Calcul Numeric - Referat

Bucătaru Andreea A2

Interpolation by Inverse Distance Weighting

Interpolation is the process of using points with known values or sample points to estimate values at other unknown points.

Inverse distance weighting (IDW) is a type of deterministic method for multivariate interpolation with a known scattered set of points. The assigned values to unknown points are calculated with a weighted average of the values available at the known points.

Multivariate interpolation or spatial interpolation is interpolation on functions of more than one variable. The function to be interpolated is known at given points $(x_i, y_i, z_i, \dots)$ and the interpolation problem consist of yielding values at arbitrary points $(x, y, z, \dots)$.

Definition of the problem

Given $n+1$ distinct points, $x_0, x_1, ..., x_n$ $(x_i \in \mathbb{R} \forall i, x_i \neq x_j, i \neq j)$ and $n+1$ values of a function f, $y_0 = f(x_0), y_1 = f(x_1), ..., y_n = f(x_n)$, approximate $f(\overline{x})$ for a given $\overline{x}, \overline{x} \neq x_i$ $(i = 0, ..., n)$.

The expected result is a discrete assignment of the unknown function u in a study region: $u(x) : x \to \mathbb{R} \quad x \in \mathbf{D} \subset \mathbb{R}^n$, where $\mathbf{D}$ is the study region.

IDW interpolation explicitly implements the assumption that things that are close to one another are more alike than those that are farther apart. To predict a value for any unmeasured location, IDW will use the measured values surrounding the prediction location. Those measured values closest to the prediction location will have more influence on the predicted value than those farther away. Thus, IDW assumes that each measured point has a local influence that diminishes with distance.

Using the IDW method, the weight of any known point is set inversely proportional to its distance from the estimated point.

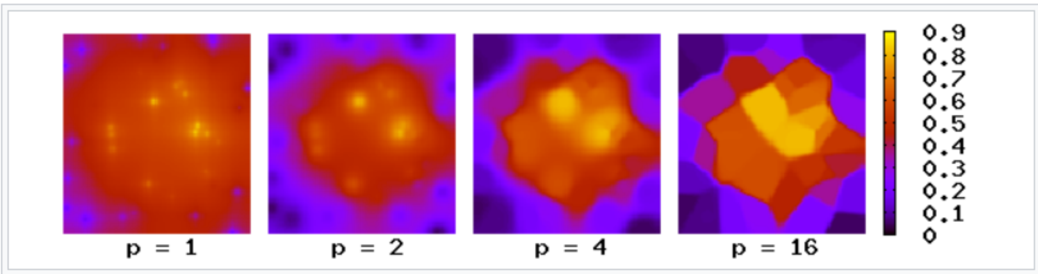
The simplest weighting function is inverse power: $\omega(d) = \dfrac{1}{d^p}$, with $p > 0$ (the most common choice is $p = 2$).

$$\hat{u} = \frac{\sum_{i=1}^{n} \frac{1}{d_i} u_i}{\sum_{i=1}^{n} \frac{1}{d_i}} = \frac{\sum_{i=1}^{n} \omega_i u_i}{\sum_{i=1}^{n} \omega_i}$$ (Inverse Distance Weighting IDW – basic formula)

where $\hat{u}$ is the value to be estimated, u_i is the known value and $d_i, \ldots, d_n$ are distances from the n data points to the point estimated n.

In most cases, the following variation can be found, in which the influence of the distance can be additionally controlled by an exponent:

$$\hat{u} = \frac{\sum_{i=1}^{n} \frac{1}{d_i^p} u_i}{\sum_{i=1}^{n} \frac{1}{d_i^p}} = \frac{\sum_{i=1}^{n} \omega_i^p u_i}{\sum_{i=1}^{n} \omega_i^p}$$ (Most common form of IDW formula with added distance weighting exponent)



Interpolation for different power parameters p, for function $z = \exp(-x^2 - y^2)$

As a result, as the distance increases, the weights decrease rapidly. How fast the weights decrease is dependent on the value for p.

The lower the exponent, the more uniformly all neighbors are incorporated into the calculation (regardless of their distance), and therefore, the "smoother" the estimated surface. The higher the exponent, the more accentuated and "unsettled" is the surface because only the weight of the nearest neighbors is integrated in the interpolation.

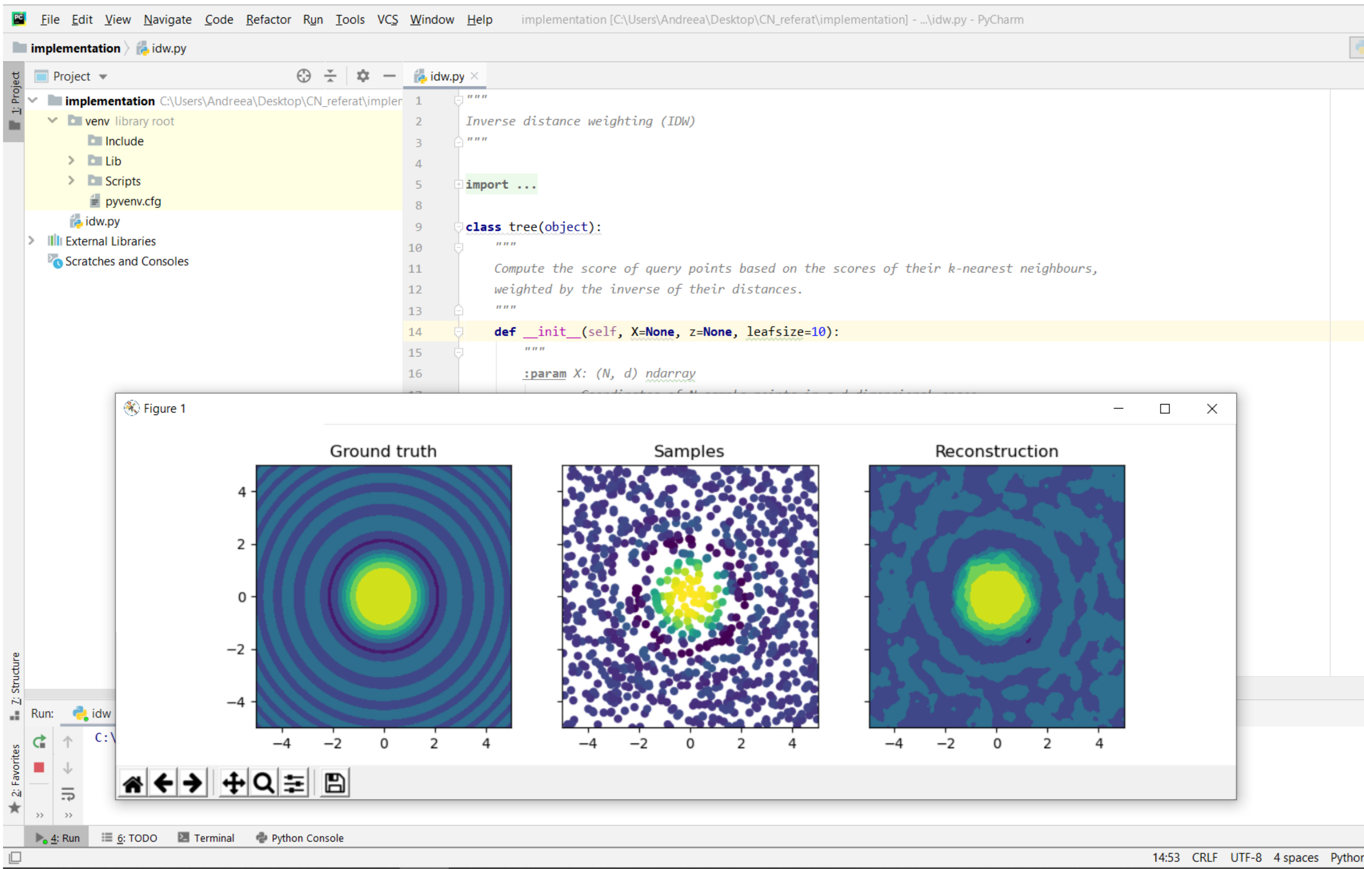
Advantages and use of the IDW interpolation:

- It allows for very fast calculations
- Different distances are integrated in the estimation
- The distance-weighting exponent is able to precisely control the influence of the distances

Multivariate interpolation is particularly important in geostatistics, where it is used to create a digital elevation model from a set of points on the Earth's surface (for example, spot heights in a topographic survey or depths in a hydrographic survey).

Implementation

As each query point is evaluated using the same number of data points, this method allows for strong gradient changes in regions of high sample density while imposing smoothness in data sparse regions.



Bibliography

- https://en.wikipedia.org/wiki/Inverse_distance_weighting
- https://en.wikipedia.org/wiki/Multivariate_interpolation
- http://www.gisresources.com/types-interpolation-methods_3/

- http://www.gitta.info/ContiSpatVar/en/html/Interpolatio_learningObject2.xhtml
- http://www.ncgia.ucsb.edu/pubs/spherekit/inverse.html