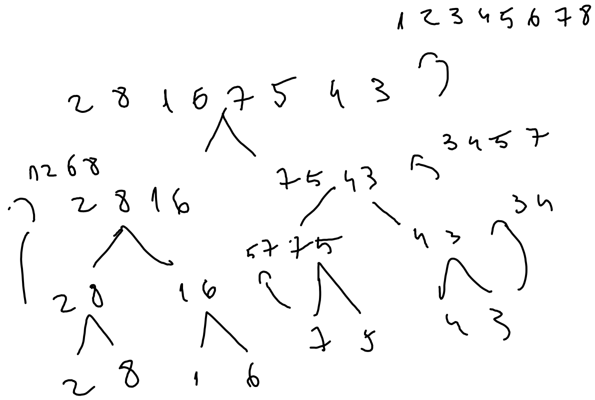
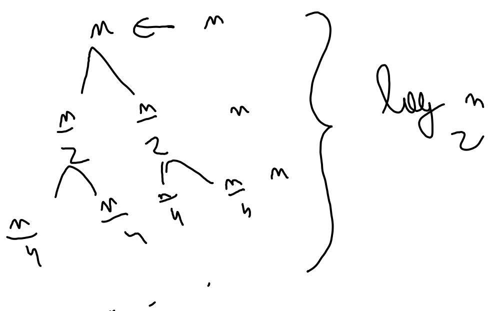
Recurențe Teorema Master

Merge Sort.



Arborele de recursie/recurență



$T(n) \in O(n \log_2 n)$

$T(n) = 2T(\frac{n}{2}) + n$

Metoda substituției

$T(n) < c n \log_2 n \;\; \forall\, c\, n \log_2 n) \;\; \forall n \geq 1$

$T(n-1) \Rightarrow T(n)$

$T(1), T(2), \dots, T(n-1), \Rightarrow T(n)$

$T(n) = 2\underbrace{T(\frac{n}{2})} + 1 \leq 2\frac{cn}{2}\log_2\frac{n}{2}$

$\leq c\frac{n}{2}\log_2\frac{n}{2}$

$= c \cdot n(\log_2 n - 1) + n -$ $\quad n-n$

$\leq c \cdot n \log_2 n - n(c-1)$

$\forall c > 1$

$\Rightarrow T(n) \in O(n \log_2 n)$

$T(n) = aT(\frac{n}{b}) + f(n)$, $a \geq 1$, $b > 1$

I) $\exists \varepsilon > 0$ a.î. $f(n) \in O(n^{\log_b a - \varepsilon}) \Rightarrow T(n) \in \Theta(n^{\log_b a})$

II) $f(n) \in \Theta(n^{\log_b a}) \Rightarrow T(n) \in \Theta(n^{\log_b a} \lg n)$

III) $\exists \varepsilon > 0, c < 1, n_0 \in \mathbb{N}$ a.î. $f(n) \in \Omega(n^{\log_b a + \varepsilon})$ și $a f(\frac{n}{b}) \leq c f(n), \forall n \geq n_0$

$\Rightarrow T(n) \in \Theta(f(n))$

$T(n) = T(n-1) + n \quad \forall n \geq 1$

$T(n) \leq cn^2, \forall n \geq 1$

$T(n) = T(n-1) + n$

$\leq c(n-1)^2 + n$

$cn^2 - 2cn + c + n$

$= cn^2 + c + n(1-2c) \leq cn^2$

$T(n) \leq cn^2 - c, \forall n \geq 1$

$T(n) = T(n-1) + n \leq c(n-1)^2 - c + n$

$cn^2 + n(1-2c) \leq cn^2 \quad \forall c \geq \frac{1}{2}$

$T(n) = 2T(\frac{n}{4}) + (1) \quad \sqrt{n} \quad n$

$T(n) \in \Theta(n^{\frac{1}{2}})$

$O(\sqrt{n})$

$T(n) = 4T(\frac{n}{2}) + n^2 \log n$

2 8 1 7 6 5 3

2 1 5 3 $\boxed{6}$ 8 7 → Quickselect

$K=4$

$T(n) = T(\frac{n}{2}) + n$

1) Nr de inversiuni

6 4 2 1 3 5

(i, j)

$i<j$ și $a_i > a_j$

$n \log_2 n$

123 456

6 4 2 1 3 5

246 135

642 135

24 35

6 42 1 35

4 2 3 5

+1
+1
+1
+3
+2
+1

$i = 1$

$m = \frac{n}{2}$

$j = \frac{n}{2} + 1$

```
while(i ≤ m && j ≤ n)
  if(a i < a j){
      c[i++k] = a i ,
        i++
  }else{
      c[i++k] = a j ;
      m+ = m - i + 1
  }
```

$|n| = |X| = |Y|$

$x = 123$

$y = 456$

$O(n^2)$

$O(n^{\log_2 3})$

$$x = x_L \, 10^{\frac{n}{2}} + x_R$$

$$y = y_L \cdot 10^{\frac{n}{2}} + y_R \;,\; x_L, x_R, y_L, y_R < 10^{\frac{n}{2}}$$

$$x \cdot y = x_L \cdot y_L \cdot 10^n + x_L \cdot y_R \, 10^{\frac{n}{2}} + x_R \cdot y_L \cdot 10^{\frac{n}{2}} + x_R \cdot y_R$$

$$= \underline{x_L \cdot y_R} \cdot 10^n + 10^{\frac{n}{2}} (x_L \cdot y_R + x_R \cdot y_L) + \underline{x_R y_R}$$

$$(x_L + x_R)(y_L + y_R) - x_L y_L - x_R y_R$$

$T(n) = 4T(\frac{n}{2}) + n$