Temă

V.a. discrete

1. a)

$$3X: \begin{pmatrix} 3\cdot 2 & 3\cdot 3 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$X^{-1}: \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$\cos\left(\frac{\pi}{2}\cdot X\right): \begin{pmatrix} \cos\left(\frac{\pi}{2}\cdot 2\right) & \cos\left(\frac{\pi}{2}\cdot 3\right) \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} \cos(\pi) & \cos\frac{3\pi}{2} \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$Y^2: \begin{pmatrix} (-3)^2 & (-2)^2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 9 & 4 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$$

$$Y+3: \begin{pmatrix} 3-3 & 3-2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$$

b)

$$X-1: \begin{pmatrix} 0-1 & 9-1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -1 & 8 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$X^{-2}: \begin{pmatrix} 0 & \frac{1}{9^2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{81} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\sin\left(\frac{\pi}{4}\cdot X\right): \begin{pmatrix} \sin\left(\frac{\pi}{4}\cdot 0\right) & \sin\left(\frac{\pi}{4}\cdot 9\right) \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \sin(0) & \sin\frac{9\pi}{4} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 & 0{,}71 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$Y\cdot 5: \begin{pmatrix} 5\cdot(-3) & 5\cdot 1 \\ \frac{1}{7} & \frac{6}{7} \end{pmatrix} = \begin{pmatrix} -15 & 5 \\ \frac{1}{7} & \frac{6}{7} \end{pmatrix}$$

$$e^Y: \begin{pmatrix} e^{-3} & e^1 \\ \frac{1}{7} & \frac{6}{7} \end{pmatrix} = \begin{pmatrix} \frac{1}{e^3} & e \\ \frac{1}{7} & \frac{6}{7} \end{pmatrix}$$

c) $2X:\begin{pmatrix}2\cdot5 & 2\cdot8\\ \frac{1}{3} & \frac{2}{3}\end{pmatrix}=\begin{pmatrix}10 & 16\\ \frac{1}{3} & \frac{2}{3}\end{pmatrix}$

$X^{-3}:\begin{pmatrix}5^{-3} & 8^{-3}\\ \frac{1}{3} & \frac{2}{3}\end{pmatrix}=\begin{pmatrix}\frac{1}{125} & \frac{1}{512}\\ \frac{1}{3} & \frac{2}{3}\end{pmatrix}$

$tg(\pi\cdot X):\begin{pmatrix}tg(5\pi) & tg(8\pi)\\ \frac{1}{3} & \frac{2}{3}\end{pmatrix}:\begin{pmatrix}0\\ 1\end{pmatrix}$ $\boxed{tg5\pi=tg8\pi}$

$Y-2:\begin{pmatrix}-1-2 & 1-2\\ \frac{1}{6} & \frac{5}{6}\end{pmatrix}=\begin{pmatrix}-3 & -1\\ \frac{1}{6} & \frac{5}{6}\end{pmatrix}$

$|Y|:\begin{pmatrix}1\\ \frac{1}{6}+\frac{5}{6}\end{pmatrix}=\begin{pmatrix}1\\ 1\end{pmatrix}$

d) $2-X:\begin{pmatrix}2-(-3) & 2-6\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}=\begin{pmatrix}5 & -4\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}$

$X^3:\begin{pmatrix}(-3)^3 & 6^3\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}=\begin{pmatrix}-27 & 216\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}$

$\cos(\frac{\pi}{6}\cdot X):\begin{pmatrix}\cos\frac{\pi}{{}_2\cancel{6}}\cdot(-\cancel{3}) & \cos(\frac{\pi}{\cancel{6}}\cdot\cancel{6})\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}=\begin{pmatrix}0 & -1\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}$

$Y^{-1}:\begin{pmatrix}e^{-1} & (e^3)^{-1}\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}=\begin{pmatrix}\frac{1}{e} & \frac{1}{e^3}\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}$

$\ln Y:\begin{pmatrix}\ln e & \ln e^3\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}=\begin{pmatrix}1 & 3\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}$

2) a) $2X: \begin{pmatrix} 4 & 6 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$ $3Y: \begin{pmatrix} -9 & -6 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$$2X + 3Y: \begin{pmatrix} 4-9 & 4-6 & 6-9 & 6-6 \\ \frac{1}{5} \cdot \frac{4}{5} & \frac{1}{5} \cdot \frac{1}{5} & \frac{4}{5} \cdot \frac{4}{5} & \frac{4}{5} \cdot \frac{1}{5} \end{pmatrix}$$

$$2X+3Y: \begin{pmatrix} -5 & -2 & -3 & 0 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix}$$

$3X: \begin{pmatrix} 6 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$ $Y: \begin{pmatrix} -3 & -2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$$3X - Y: \begin{pmatrix} 6+3 & 6+2 & 3+9 & 9+2 \\ \frac{1}{5} \cdot \frac{4}{5} & \frac{1}{5} \cdot \frac{1}{5} & \frac{4}{5} \cdot \frac{4}{5} & \frac{4}{5} \cdot \frac{1}{5} \end{pmatrix}$$

$$3X - Y: \begin{pmatrix} 9 & 8 & 12 & 11 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix}$$

$X^2: \begin{pmatrix} 4 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$ $Y^3: \begin{pmatrix} 27 & 8 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$$X^2 \cdot Y^3: \begin{pmatrix} 108 & 32 & 243 & 72 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix}$$

b.) $X-Y: \begin{pmatrix} 0-(-3) & 0-1 & 9-(-3) & 9-1 \\ \frac{1}{2}\cdot\frac{1}{7} & \frac{1}{2}\cdot\frac{6}{7} & \frac{1}{2}\cdot\frac{1}{7} & \frac{1}{2}\cdot\frac{6}{7} \end{pmatrix}$

$X-Y: \begin{pmatrix} 3 & -1 & 12 & 8 \\ \frac{1}{14} & \frac{6}{14} & \frac{1}{14} & \frac{6}{14} \end{pmatrix}$

$X-Y: \begin{pmatrix} -1 & 3 & 8 & 12 \\ \frac{6}{14} & \frac{1}{14} & \frac{6}{14} & \frac{1}{14} \end{pmatrix}$

$\cos(\pi\cdot X\cdot Y): \begin{pmatrix} \cos(0\cdot(-3)\cdot\pi) & \cos(0\cdot1\cdot\pi) & \cos(9\cdot(-3)\cdot\pi) & \cos(9\cdot1\cdot\pi) \\ \frac{1}{2}\cdot\frac{1}{7} & \frac{1}{2}\cdot\frac{6}{7} & \frac{1}{2}\cdot\frac{1}{7} & \frac{1}{2}\cdot\frac{6}{7} \end{pmatrix}$

$\cos(\pi X\cdot Y): \begin{pmatrix} 1 & 1 & -1 & -1 \\ \frac{1}{14} & \frac{6}{14} & \frac{1}{14} & \frac{6}{14} \end{pmatrix}$

$\downarrow$

$\cos(\pi X\cdot Y): \begin{pmatrix} -1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$X^2+3Y: \begin{pmatrix} 0^2+3\cdot(-3) & 0^2+3\cdot1 & 9^2+3\cdot(-3) & 9^2+3\cdot1 \\ \frac{1}{2}\cdot\frac{1}{7} & \frac{1}{2}\cdot\frac{6}{7} & \frac{1}{2}\cdot\frac{1}{7} & \frac{1}{2}\cdot\frac{6}{7} \end{pmatrix}$

$X^2+3Y: \begin{pmatrix} -9 & 3 & 72 & 84 \\ \frac{1}{14} & \frac{6}{14} & \frac{1}{14} & \frac{6}{14} \end{pmatrix}$

c.)

$$X+Y: \begin{pmatrix} 5-1 & 5+1 & 8-1 & 8+1 \\ \frac{1}{3}\cdot\frac{1}{6} & \frac{1}{3}\cdot\frac{5}{6} & \frac{2}{3}\cdot\frac{1}{6} & \frac{2}{3}\cdot\frac{5}{6} \end{pmatrix}$$

$$X+Y: \begin{pmatrix} 4 & 6 & 7 & 9 \\ \frac{1}{18} & \frac{5}{18} & \frac{2}{18} & \frac{10}{18} \end{pmatrix}$$

$$\sin\left(\frac{\pi}{2}\cdot X\cdot Y\right): \begin{pmatrix} \sin\left(\frac{\pi}{2}\cdot 5\cdot(-1)\right) & \sin\left(\frac{\pi}{2}\cdot 5\cdot 1\right) & \sin\left(\frac{\pi}{2}\cdot 8\cdot(-1)\right) & \sin\left(\frac{\pi}{2}\cdot 8\cdot 1\right) \\ \frac{1}{18} & \frac{5}{18} & \frac{2}{18} & \frac{10}{18} \end{pmatrix}$$

$$\sin\left(\frac{\pi}{2}\cdot X\cdot Y\right): \begin{pmatrix} \sin\left(-\frac{5\pi}{2}\right) = -1 & \sin\left(\frac{5\pi}{2}\right) = 1 & \overset{0}{\sin(-4\pi)} & \overset{0}{\sin(4\pi)} \\ \frac{1}{18} & \frac{5}{18} & \underline{\frac{2}{18}} & \underline{\frac{10}{18}} \end{pmatrix}$$

$$\sin\left(\frac{\pi}{2}\cdot X\cdot Y\right): \begin{pmatrix} -1 & 0 & 1 \\ \frac{1}{18} & \frac{2}{3} & \frac{5}{18} \end{pmatrix}$$

$$\frac{1}{X}+\frac{1}{Y}: \begin{pmatrix} \frac{1}{5}+(-1) & \frac{1}{5}+1 & \frac{1}{8}-1 & \frac{1}{8}+1 \\ \frac{1}{18} & \frac{5}{18} & \frac{2}{18} & \frac{10}{18} \end{pmatrix} = \begin{pmatrix} -\frac{4}{5} & \frac{6}{5} & -\frac{7}{8} & \frac{9}{8} \\ \frac{1}{18} & \frac{5}{18} & \frac{2}{18} & \frac{10}{18} \end{pmatrix}$$

$$\frac{1}{X}+\frac{1}{Y}: \begin{pmatrix} -\frac{7}{8} & -\frac{4}{5} & \frac{6}{5} & \frac{9}{8} \\ \frac{2}{18} & \frac{1}{18} & \frac{5}{18} & \frac{10}{18} \end{pmatrix}$$

d.)

$$X\cdot Y: \begin{pmatrix} -3\cdot e & -3\cdot e^3 & 6\cdot e & 6\cdot e^3 \\ \frac{1}{8}\cdot\frac{1}{4} & \frac{1}{8}\cdot\frac{3}{4} & \frac{7}{8}\cdot\frac{1}{4} & \frac{7}{8}\cdot\frac{3}{4} \end{pmatrix}$$

$$X\cdot Y: \begin{pmatrix} -3e^3 & -3e & 6e & 6\cdot e^3 \\ \frac{3}{32} & \frac{1}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$\frac{X}{Y} \overset{not}{=} X \cdot Y^{-1}$

$$X \cdot Y^{-1} : \begin{pmatrix} -3 \cdot \frac{1}{e} & -3 \cdot \frac{1}{e^3} & 6 \cdot \frac{1}{e} & 6 \cdot \frac{1}{e^3} \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$$X \cdot Y^{-1} : \begin{pmatrix} -\frac{3}{e^3} & -\frac{3}{e} & \frac{6}{e^3} & \frac{6}{e} \\ \frac{3}{32} & \frac{1}{32} & \frac{21}{32} & \frac{7}{32} \end{pmatrix}$$

$$|X - Y^2| : \begin{pmatrix} |-3-e^2| & |-3-e^6| & |6-e^2| & |6-e^6| \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$$|X - Y^2| : \begin{pmatrix} 3+e^2 & 3+e^6 & e^2-6 & e^6-6 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$$|X - Y^2| : \begin{pmatrix} e^2-6 & 3+e^2 & e^6-6 & 3+e^6 \\ \frac{7}{32} & \frac{1}{32} & \frac{21}{32} & \frac{3}{32} \end{pmatrix}$$

3.) a) $p, q - ?$

$X : \begin{pmatrix} 1 & 2 \\ p & q \end{pmatrix}$ $Y : \begin{pmatrix} 3 & 9 \\ 0,1 & \frac{p^2+0,02}{2} \end{pmatrix}$ v.a. bine def.

$$\begin{cases} p+q=1 \\ p \geq 0 \\ q \geq 0 \\ \frac{p^2+0,02}{2} = 1-0,1 \end{cases} \Leftrightarrow \begin{cases} p+q=1 \\ p \geq 0 \\ q \geq 0 \\ \frac{p^2+0,02}{2} = 0,9 \end{cases}$$

$$\begin{cases} p+q=1 \\ p \geq 0 \\ q \geq 0 \\ \dfrac{p^2+0{,}02}{2} = 0{,}09 \end{cases}$$

Rezolvăm ecuația:

$$\frac{p^2+0{,}02}{2} = 0{,}09 \Leftrightarrow$$

$$\Leftrightarrow \frac{p^2+\frac{1}{50}}{2} = 0{,}9 \Leftrightarrow \frac{\frac{50p^2+1}{50}}{2} = 0{,}9 \Leftrightarrow \frac{50p^2+1}{100} = 0{,}9$$

$$\Leftrightarrow 50p^2+1=90 \Leftrightarrow 50p^2 = 89 \Leftrightarrow p^2 = \frac{89}{50} \Rightarrow p = \begin{cases} -1{,}33 \\ 1{,}33 \end{cases}$$

deoarece $p \geq 0 \Rightarrow p = 1{,}33$

$q = 1 - p \Rightarrow q = -0{,}33.$

p/u X p și q Nu pot fi valorile probabilităților deși $p+q=1$, $p>1$, iar $q<0$. Prin urmare, $q \in \emptyset$.

b.) $X : \begin{pmatrix} 1 & 2 & 3 \\ \frac{1}{3} & p & q^2 \end{pmatrix}$ $X^2 : \begin{pmatrix} 1 & 4 & 9 \\ p & p & p^2 \end{pmatrix}$

$$\begin{cases} p \geq 0 \\ q \in \mathbb{R} \\ p = \frac{1}{3} \\ q^2 = p^2 \end{cases} \Rightarrow \begin{cases} p = \frac{1}{3} \\ q_1 = \frac{1}{3} \\ q_2 = -\frac{1}{3} \end{cases}$$

R|S: $p = \frac{1}{3}$ $q = \pm\frac{1}{3}$

c) $X: \begin{pmatrix} -1^2=1^2 & 0 & 1 \\ p & p^2 & q \end{pmatrix}$ $\quad X^4: \begin{pmatrix} 0 & 1 \\ \frac{9}{25} & \underbrace{\frac{16}{25}}_{p+q} \end{pmatrix}$

$$\begin{cases} q \geq 0 \\ p \geq 0 \\ p^2 = \frac{9}{25} \\ p + q = \frac{16}{25} \end{cases} \Rightarrow \begin{cases} p = \frac{3}{5} \\ q = \frac{1}{25} \end{cases}$$

R/s: $p = \frac{3}{5}$ $\quad q = \frac{1}{25}$

d) $X: \begin{pmatrix} -1 & 1 \\ 2p & q \end{pmatrix}$ $\quad Y: \begin{pmatrix} 0 & 1 \\ q & 7q \end{pmatrix}$

$$\begin{cases} p \geq 0 \\ q \geq 0 \\ 2p + q = 1 \\ q + 7q = 1 \end{cases} \Rightarrow \begin{cases} p, q \geq 0. \\ p = \frac{7}{16} \\ q = \frac{1}{8} \end{cases}$$

R/s: $p = \frac{7}{16}$ $\quad q = \frac{1}{8}$

4.) a) $\mathbb{P}(2X+3Y>1) =$

$$2X+3Y : \begin{pmatrix} -5 & -2 & -3 & 0 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix}$$

$$\mathbb{P}(2X+3Y>1) = \mathbb{P}(2X+3Y=0) = \frac{4}{25}$$

$$\mathbb{P}(2X+3Y>1 \mid X>0) = 0$$

$$\mathbb{P}(2X+3Y<3 \mid Y<-2) = \frac{\mathbb{P}(2X+3Y<3)\cdot\mathbb{P}(Y<-2)}{\mathbb{P}(Y<-2)} =$$

$$= \mathbb{P}(2X+3Y<3) = 1-\mathbb{P}(2X+3Y\geq 3) = 1$$

$$\mathbb{P}(X^2\cdot Y^3>3) = 1-\mathbb{P}(X^2\cdot Y^3\leq 3) = 1$$

$$\mathbb{P}(X^2\cdot Y^3\leq 3) = 0$$

$$\mathbb{P}(2X+3Y<3X-Y) = \mathbb{P}(X-4Y>0) = 1$$

$X-4Y>0$

$$X-4Y : \begin{pmatrix} 14 & 10 & 15 & 11 \\ \frac{4}{25} & \frac{1}{25} & \frac{16}{25} & \frac{4}{25} \end{pmatrix}$$

б) $\mathbb{P}(X-Y>0) = 1-\mathbb{P}(X-Y\leq 0) = 1-\frac{6}{14} = \frac{8}{14} = \frac{4}{7}$

$>0 \quad y<x$

$$\mathbb{P}(X-Y<0 \mid X>0) = \cancel{\frac{\mathbb{P}(X-Y<0)\,\mathbb{P}(X>0)}{\mathbb{P}(X>0)}} = \frac{6}{14}$$

$$\mathbb{P}(X-Y>0 \mid Y\leq 0) = \frac{4}{7}$$

$$\mathbb{P}(\cos(\pi XY)<\tfrac{1}{2}) = \mathbb{P}(\cos(\pi XY)=-1) = \frac{1}{2}$$

$$\mathbb{P}(X^2+3Y\geq 3) = 1-\mathbb{P}(X^2+3Y\leq 3) = 1-\mathbb{P}(X^2+3Y=-9) =$$

$$= 1-\frac{1}{14} = \frac{13}{14}$$

$\mathbb{P}(X - Y < X^2 + 3Y) = 1$

c). $\mathbb{P}(X + Y < 2) = 0$

$\mathbb{P}(X + Y \geq 2 \mid X > 5) = 1$

$\mathbb{P}(X + Y < 12 \mid Y < 0) = 1$

$\mathbb{P}(\sin(\frac{\pi}{2} \cdot XY) \leq \frac{1}{2}) = 1 - \mathbb{P}(\sin(\frac{\pi}{2} XY) > \frac{1}{2}) = 1 - \mathbb{P}(\sin(\frac{\pi}{2} XY) = 1)$

$= 1 - \frac{5}{18} = \frac{13}{18}$

$\mathbb{P}(\frac{1}{X} + \frac{1}{Y} < 1 \mid Y < 0) = \frac{2}{18} + \frac{1}{18} = \frac{3}{18}$

$\mathbb{P}(\frac{1}{X} + \frac{1}{Y} < X + Y) = 1$

d.) $\mathbb{P}(X \cdot Y \leq e^4) = 1$

$\mathbb{P}(X \cdot Y \geq 7 \mid X < 0) = \frac{7}{32} + \frac{21}{32} = \frac{28}{32} = \frac{7}{8}$

$\mathbb{P}(X \cdot Y < 9 \mid Y > 3) = \frac{3}{32} + \frac{1}{32} = \frac{4}{32} = \frac{1}{8}$

$\mathbb{P}(\frac{X}{Y} < 1) = 1 - \mathbb{P}(\frac{X}{Y} \geq 1) = 1 - \frac{7}{32} = \frac{25}{32}$

$\mathbb{P}(|X - Y^2| \geq 3) = 1 - \mathbb{P}(|X - Y^2| < 3) =$

$= 1 - \frac{7}{32} = \frac{25}{32}$

$\mathbb{P}(\frac{X}{Y} < |X - Y^2|) = 1$

Prob:

Un student are 2 teancuri cu foi. În fiecare dintre ele există amestecat foi albe și ciorne.

Studentul extrage câte o foaie la întâmplare din cele 2 teancuri. Știind că în I-l teanc 10% foi albe; II-lea - 50% foi albe. Construiți v.a. X asociată nr de foi albe extrase în total din cele 2 teancuri

$$X_1 \begin{pmatrix} 0 & 1 \\ 0,9 & 0,1 \end{pmatrix} \qquad X_2 : \begin{pmatrix} 0 & 1 \\ 0,5 & 0,5 \end{pmatrix}$$

X_1, X_2 - nr succese (f. albe) pot fi obținute la o extragere

$$X = X_1 + X_2$$

X_1, X_2 independente.

$$X : \begin{pmatrix} 0 & 1 & 2 \\ 0,45 & 0,5 & 0,05 \end{pmatrix}$$

Ipoteza: extragem foi până la obținerea unei foi albe.

$$A = \{ \ldots \}$$

$P(A)$ - ?

după Pascal: $P(A) = \underset{p\ succes}{p} \cdot \underbrace{q^{k-1}}_{p.\ eșec}.$