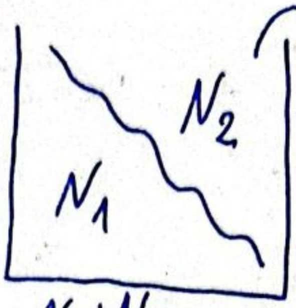
2) Schema cu bila NEREVENITĂ:




$n = n_1 + n_2$

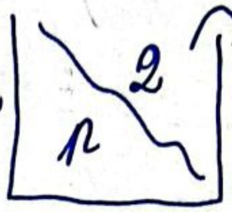
$N_1 + N_2 = n$

Pt. 2 culori: $\mathbb{P}(n, n_1, n_2) = \dfrac{C_{N_1}^{n_1} \cdot C_{N_2}^{n_2}}{C_N^n}$

Pt. K culori:

$$\mathbb{P}(n, n_1, \dots, n_k) = \frac{C_{N_1}^{n_1} \cdot \ldots \cdot C_{N_k}^{n_k}}{C_N^n}$$

3) Schema lui Pascal (geometrică):

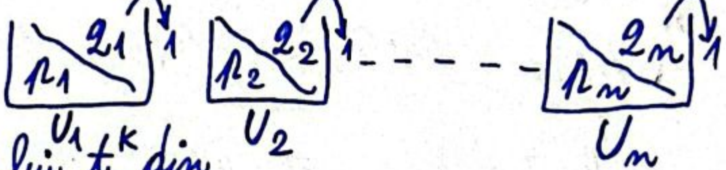



extrageri independente, mă opresc la primul succes.

A = ev. că primele K-1 extrageri sunt eșecuri și al K-ulea este succes.

$\mathbb{P}(A) = p \cdot q^{k-1}$

4) Schema lui Poisson:




$\mathbb{P}(n, K, n-K)$ = coef. lui $t^k$ din polinomul:

succese eșecuri

$$Q(t) = \prod_{i=1}^{n} (p_i \cdot t + q_i)$$

---

$\mathbb{E}[X] = \sum_{i=1}^{n} x_i \cdot p_i$ ; $Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

Proprietăți: Fie $a, b, c \in \mathbb{R}$

$X \sim \binom{c}{1}$

1) $\mathbb{E}[X] \in \mathbb{R}$
2) $\mathbb{E}[aX \pm bY] = a\mathbb{E}[X] \pm b\mathbb{E}[Y]$
3) $\mathbb{E}[X \cdot Y] = \mathbb{E}[X] \cdot \mathbb{E}[Y]$, iff. $X \perp\!\!\!\perp Y$
4) $\mathbb{E}(c) = c$

1) $Var(X) \geq 0$ ($= 0 \Rightarrow X$ constantă)
2) $Var(aX \pm bY) = a^2 Var(X) + b^2 Var(Y)$, iff. $X \perp\!\!\!\perp Y$.
3) $Var(X \pm c) = Var(X)$
4) $Var(a \cdot X) = a^2 Var(X)$
5) $Var(c) = 0$

Covarianța: $cov(X,Y) = \mathbb{E}[X \cdot Y] - \mathbb{E}[X] \cdot \mathbb{E}[Y]$

Coef. de corelație (Pearson): $\rho(X,Y) = \dfrac{cov(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}} \in [-1, 1]$

Obs!: Semnul lui $\rho$ ne spune tipul de legătură X-Y (+ → directă, − → inversă)

$\rho = 0 \Rightarrow X, Y$ = NECORELATE

$|\rho| \in (0, 0{,}25) \Rightarrow X, Y$ = SLAB CORELATE

$|\rho| \in [0{,}25, 0{,}75] \Rightarrow X, Y$ = CORELATE

$|\rho| \in (0{,}75, 1] \Rightarrow X, Y$ = PUTERNIC CORELATE

# V.A. DISCRETE

$\cdot f(x) \geq 0 \ (\forall x)$ și $\sum f(x) = 1$

$$f(x) = \begin{cases} p_i, & x = x_i, \ i = \overline{1,n} \\ 0, & \text{altfel} \end{cases}$$ — funcția de masă a v.a. X

$F(x) = \mathbb{P}(X \leq x)$ — funcția de repartiție a v.a. X

$$X \sim \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ p_1 & p_2 & \dots & p_n \end{pmatrix} \qquad Y \sim \begin{pmatrix} y_1 & y_2 & \dots & y_m \\ q_1 & q_2 & \dots & q_m \end{pmatrix}, \quad \boxed{X \perp\!\!\!\perp Y} \ !$$

$$\underset{(\forall c \in \mathbb{R})}{X \pm c} \sim \begin{pmatrix} x_1 \pm c & \dots & x_n \pm c \\ p_1 & \dots & p_n \end{pmatrix} \qquad X^c \sim \begin{pmatrix} x_1^c & x_2^c & \dots & x_n^c \\ p_1 & p_2 & \dots & p_n \end{pmatrix}$$

$$X \pm Y \sim \begin{pmatrix} x_1 \pm y_1 & x_1 \pm y_2 & \dots & x_1 \pm y_m & x_2 \pm y_1 & x_2 \pm y_2 & \dots & x_n \cdot y_m \\ p_1 \cdot q_1 & p_1 \cdot q_2 & \dots & p_1 \cdot q_m & p_2 \cdot q_1 & p_2 \cdot q_2 & \dots & p_n \cdot q_m \end{pmatrix}$$

---

$\mathbb{P}(A|B) = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$, pt. $\mathbb{P}(B) \neq 0$ → prob. condiționată

Dacă $A \perp\!\!\!\perp B \Rightarrow \mathbb{P}(A|B) = \mathbb{P}(A)$ și $\mathbb{P}(B|A) = \mathbb{P}(B)$

Formula lui Bayes: $\mathbb{P}(A|B) \cdot \mathbb{P}(B) = \mathbb{P}(B|A) \cdot \mathbb{P}(A)$

Dacă $A \perp\!\!\!\perp B \Rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$

Altfel $(A \not\perp\!\!\!\perp B) \Rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(B) \cdot \mathbb{P}(A|B)$

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

$\mathbb{P}(X \cap Y) = \mathbb{P}(X) \cdot \mathbb{P}(Y)$, $X \perp\!\!\!\perp Y$

$\mathbb{P}(A) + \mathbb{P}(B) = \mathbb{P}(A \cup B)$, A,B inc.

$$\mathbb{P}(A \cap B) + \mathbb{P}(\bar{A} \cap B) \ \overset{A, \bar{A} \text{ incomp.}}{\underset{\text{deci și } A \cap B, \ \bar{A} \cap B \text{ incomp.}}{=}} \ \mathbb{P}((A \cap B) \cup (\bar{A} \cap B)) = \mathbb{P}((A \cup \bar{A}) \cap B) = \mathbb{P}(B)$$

---

## 1) Schema cu bila REVENITĂ:

**Pentru 2 culori:**

$\mathbb{P}(n, n_1, n_2) = C_n^{n_1} \cdot p_1^{n_1} \cdot p_2^{n_2}$


$n = n_1 + n_2$ (nr. bune, nr. rele)

$p_1 + p_2 = 1$

**Pentru K culori:**

$$\mathbb{P}(n, n_1, n_2, \dots, n_K) = \frac{n!}{n_1! \cdot \ldots \cdot n_K!} \cdot p_1^{n_1} \cdot \ldots \cdot p_K^{n_K}$$

## V.A. DISCRETE:

$\mathbb{P}(a<X\leq b)=F(b)-F(a)$

$\mathbb{P}(a\leq X\leq b)=F(b)-F(a)+\mathbb{P}(X=a)$

$\mathbb{P}(a<X<b)=F(b)-F(a)-\mathbb{P}(X=b)$

$\mathbb{P}(a\leq X<b)=F(b)-F(a)+\mathbb{P}(X=a)-\mathbb{P}(X=b)$

$\pi_{ij}=\mathbb{P}(X=x_i, Y=y_j)$

$X|Y=y_j : \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ \frac{\pi_{1j}}{q_j} & \frac{\pi_{2j}}{q_j} & \cdots & \frac{\pi_{nj}}{q_j} \end{pmatrix}$

$q_j=$ prob. coresp. lui $y_j$

---

Atunci când $X \perp\!\!\!\perp Y$: $Var(X\pm Y)=Var(X)+Var(Y)\pm 2\,cov(X,Y)$

**Proprietățile covarianței:**

1) $cov(X,X)=Var(X)$
2) $cov(X,Y)=cov(Y,X)$
3) $cov(X,c)=0\ (\forall c\in\mathbb{R})$
4) $cov(aX+bY, cX+dY)=ac\cdot Var(X)+(ad+bc)\cdot cov(X,Y)+bd\cdot Var(Y)$

---

## V.A. CONTINUE:

$f(x)=$ densitate de prob. pt. v.a. $X \iff \begin{cases} f(x)\geq 0\ (\forall x\in\mathbb{R}) \\ \int_{-\infty}^{+\infty} f(x)\,dx=1 \end{cases}$

$F(x)=\int_{-\infty}^{x} f(t)\,dt \rightarrow$ funcție de repartiție $=\mathbb{P}(X\leq x)$

**Proprietăți ale funcției de repartiție:** (discret și continuu)

1) $Im\,F=[0,1]$
2) $\lim_{x\to-\infty}F(x)=0$, $\lim_{x\to\infty}F(x)=1$
3) $F$ este crescătoare ($x_1<x_2\Rightarrow F(x_1)\leq F(x_2)$)
4) $F$ e cont. la dr. ($\lim_{\substack{x\to x_0\\ x>x_0}}F(x)=F(x_0)$)

**În cazul CONTINUU:** $\mathbb{P}(a<X\leq b)=\mathbb{P}(a\leq X\leq b)=\mathbb{P}(a\leq X<b)=$
$=\mathbb{P}(a<X<b)=F(b)-F(a)=\int_a^b f(x)\,dx$

$X\sim Norm(m,\sigma^2)$

$f(x)=\frac{1}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-\frac{(x-m)^2}{2\sigma^2}}$

$F(x)=\int_{-\infty}^{x}f(t)\,dt$ (nu se poate evalua)

**Media și momentele unei v.a. continue:**

$\mathbb{E}[X]=\int_{-\infty}^{\infty}x\,f(x)\,dx$

$Var(X)=\mathbb{E}[X^2]-\mathbb{E}[X]^2=\int_{-\infty}^{\infty}(x-\mathbb{E}[X])^2\cdot f(x)\,dx$

---

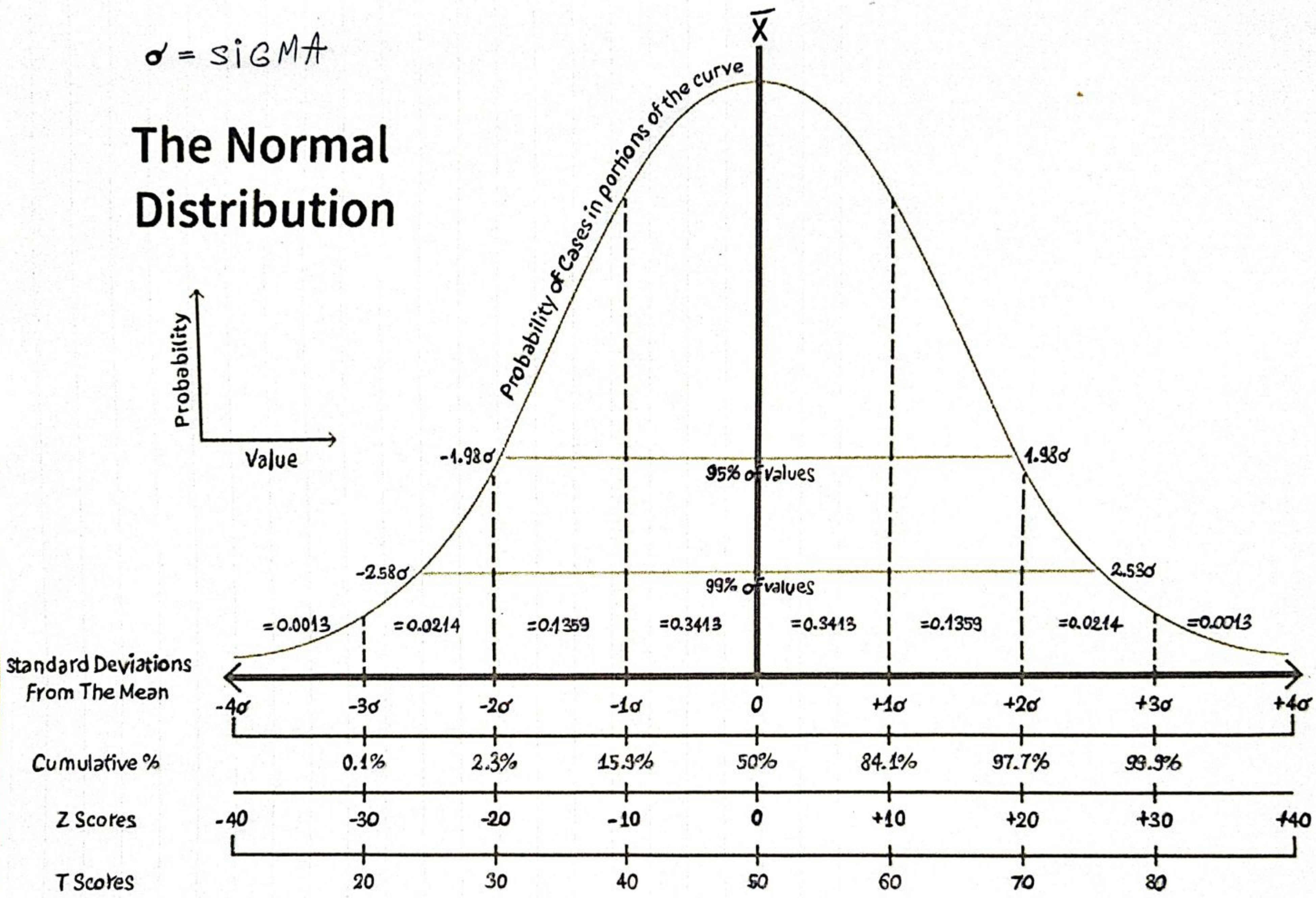
**Funcția Gauss-Laplace:** $\phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{t^2}{2}}\,dt \rightarrow$ fct. de repartiție pt. $N(0,1)$

**Procedeul de standardizare:** $X$ v.a. cont., $X\sim N(m,\sigma^2)$, cu $Z=\frac{X-m}{\sigma}\Rightarrow Z\sim N(0,1)$.

$\phi(-x)=1-\phi(x)$ | $\mathbb{P}(X\leq x)=\mathbb{P}\left(\frac{X-m}{\sigma}\leq\frac{x-m}{\sigma}\right)=\phi\left(\frac{x-m}{\sigma}\right)$

σ = SIGMA

# The Normal Distribution

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