Ex 1

$$X : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 3p & 4p & 2p & p & p \end{pmatrix} \qquad p \in \mathbb{R}$$

a) $3p + 4p + 2p + p + p = 11p$

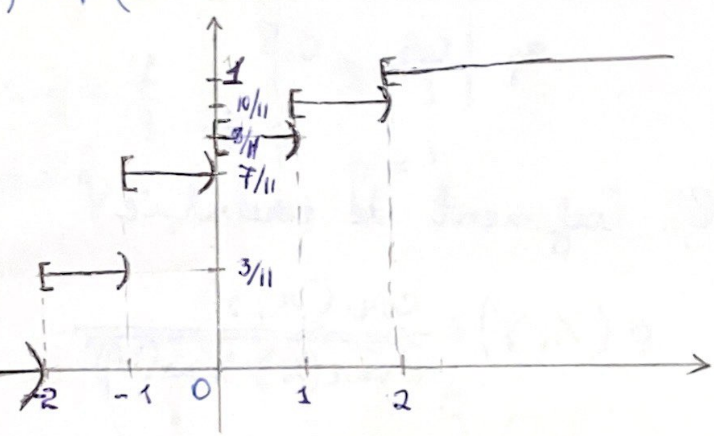
$11p = 1 \Rightarrow p = \frac{1}{11}$

$$X : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ \frac{3}{11} & \frac{4}{11} & \frac{2}{11} & \frac{1}{11} & \frac{1}{11} \end{pmatrix}$$

b) Pentru repartiția de mai sus avem următoarea funcție de repartiție:

$F : \mathbb{R} \to [0, 1] \qquad F(x) = \mathbb{P}(X \leq x)$

$$F x = \begin{cases} 0, & x < -2 \\ \frac{3}{11}, & -2 \leq x < -1 \\ \frac{7}{11}, & -1 \leq x < 0 \\ \frac{9}{11}, & 0 \leq x < 1 \\ \frac{10}{11}, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$



c) $\mathbb{E}[16X - 23]$ -? $\text{Var}(16X - 23)$ -? $\mathbb{E}[3X - 2]$ -? $\text{Var}(3X-2)$ -?

$$\mathbb{E}[16X - 23] = \overset{-165}{-55 \cdot \frac{3}{11}} + \overset{-156}{(-39) \cdot \frac{4}{11}} + \overset{-46}{(-23) \cdot \frac{2}{11}} + \overset{-7}{(-7) \cdot \frac{1}{11}} + \overset{9}{9 \cdot \frac{1}{11}} = -\frac{365}{11} \simeq \underline{-33,182}$$

$$\text{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$\mathbb{E}[\underbrace{(16X - 23)}_{y}{}^2] = \overset{9075}{3025 \cdot \frac{3}{11}} + \overset{6084}{1521 \cdot \frac{4}{11}} + \overset{1058}{529 \cdot \frac{2}{11}} + \overset{49}{49 \cdot \frac{1}{11}} + \overset{81}{\frac{81}{11}} = \frac{16347}{11} \simeq 1486,09$$

$$\mathbb{E}[16X - 23]^2 = 1101,03$$

$$\text{Var}(16X - 23) = 1486,09 - 1101,03 = \underline{385,06}$$

$$\mathbb{E}[3X - 2] = \overset{-24}{-8 \cdot \frac{3}{11}} + \overset{-20}{(-5) \cdot \frac{4}{11}} + \overset{-4}{(-2) \cdot \frac{2}{11}} + \overset{1}{1 \cdot \frac{1}{11}} + \overset{4}{4 \cdot \frac{1}{11}} = -\frac{43}{11} \simeq \underline{-3,91}$$

$$\mathbb{E}[(3X-2)^2] = 64 \cdot \frac{3}{11} + 25 \cdot \frac{4}{11} + 4 \cdot \frac{2}{11} + 1 \cdot \frac{1}{11} + 16 \cdot \frac{1}{11} = \frac{317}{11} \simeq 28,82$$

$\mathbb{E}[3X-2]^2 = (-3,91)^2 \simeq 15,288$

$Var(3X-2) = \mathbb{E}[(3X-2)^2] - \mathbb{E}[3X-2]^2 = 28,82 - 15,288 \simeq \underline{13,53}$

2. $X: \begin{pmatrix} 0 & 1 \\ 0.4 & 0.6 \end{pmatrix}$ $\quad Y: \begin{pmatrix} -1 & 1 \\ 0.5 & 0.5 \end{pmatrix}$

a)

X \ Y	-1	1	p_i
0	0.2	0.2	0.4
1	0.3	0.3	0.6
q_j	0.5	0.5	1

$p_i \cdot q_j$

→ Rep comună p/u X, Y.

0 0 -1 1
0,2 0,2 0,3 0,3
0,4 0,3 0,3

b). Coeficient de corelație:

$$\rho(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(X)\cdot Var(Y)}}$$

$cov(X,Y) = \mathbb{E}(X\cdot Y) - \mathbb{E}(X)\cdot\mathbb{E}(Y)$

$\mathbb{E}(X) = 0\cdot 0,4 + 1\cdot 0,6 = 0,6$ $\quad \mathbb{E}(X)^2 = 0,36$

$\mathbb{E}(X^2) = 0,6$

$Var(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = 0,6 - 0,36 = 0,24$

$\mathbb{E}(Y) = -0,5 + 0,5 = 0$ $\quad \mathbb{E}(Y)^2 = 0$

$\mathbb{E}(Y^2) = 1$

$Var(Y) = \mathbb{E}(Y^2) - \mathbb{E}(Y)^2 = 1 - 0 = 1$

$\mathbb{E}(X\cdot Y) = 0\cdot 0,4 + (-1)\cdot 0,3 + 1\cdot 0,3 = 0$

$cov(X,Y) = 0 - 0,6\cdot 0 = 0$

$\rho(X,Y) = 0 \Rightarrow X, Y$ necorelate.

c) din ce am calculat anterior $\Rightarrow X, Y$ necorelate. ③

deci $k \in \{0.2, 0.3\}$

$$\left.\begin{array}{l} 0.2 = 0,4 \cdot 0,5 \\ 0.3 = 0,5 \cdot 0,6 \end{array}\right\} \xRightarrow{\pi_{ij} = q_j \cdot p_i} X, Y \text{ independente.}$$

---

3. $X : \begin{pmatrix} a & 1 & 2 \\ \frac{1}{3} & p & q \end{pmatrix}$ $\qquad Y : \begin{pmatrix} a+1 & 1 & 2 \\ \frac{1}{3} & \frac{2}{3}-q & p \end{pmatrix}$

$\mathrm{Var}(X-Y) = \frac{4}{9}$ $\qquad \underline{a - ?}$

$$\begin{cases} \frac{1}{3} + p + q = 1 \\ \frac{1}{3} + \frac{2}{3} - q + p = 1 \end{cases} \Rightarrow \begin{cases} 2q = \frac{2}{3} \\ p = q \end{cases} \Rightarrow \begin{cases} q = \frac{1}{3} \\ p = \frac{1}{3} \end{cases}$$

Rescriu repartițiile:

$X : \begin{pmatrix} a & 1 & 2 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ $\qquad Y : \begin{pmatrix} a+1 & 1 & 2 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

$$\mathrm{Var}(X-Y) \xlongequal{X,Y \text{ indep}} \mathrm{Var}(X) + \mathrm{Var}(Y)$$

$$\mathrm{Var}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

$$\mathbb{E}(X^2) = a^2 \cdot \frac{1}{3} + \frac{1}{3} + \frac{4}{3} = \frac{1}{3}a^2 + \frac{5}{3} = \frac{a^2+5}{3}$$

$$\mathbb{E}(X) = a \cdot \frac{1}{3} + \frac{1}{3} + \frac{2}{3} = \frac{1a}{3} + \frac{3}{3} = \frac{a+3}{3} \qquad \frac{(a+3)^2}{9} = \frac{a^2+6a+9}{9}$$

$$\mathrm{Var}(X) = \frac{a^2+5}{3} - \frac{a^2+6a+9}{9} = \frac{3a^2+15-a^2-6a-9}{9} = \frac{2a^2-6a+6}{9}$$

$$\mathbb{E}(Y^2) = (a+1)^2 \cdot \frac{1}{3} + \frac{1}{3} + \frac{4}{3} = \frac{(a+1)^2+5}{3} = \frac{a^2+2a+6}{3}$$

$$\mathbb{E}(Y) = \frac{a+1}{3} + \frac{1}{3} + \frac{2}{3} = \frac{a+1+3}{3} = \frac{a+4}{3}$$

$$\left(\frac{a+4}{3}\right)^2 = \frac{(a+4)^2}{9} = \frac{a^2+8a+16}{9}$$

$$\text{var}(Y) = \frac{a^2+2a+6}{3} - \frac{a^2+8a+16}{9} = \frac{3a^2+6a+18-a^2-8a-16}{9} =$$

$$= \frac{2a^2-2a+2}{9}$$

$$\text{var}(X-Y) = \frac{2a^2-6a+6+2a^2-2a+2}{9} = \frac{4a^2-8a+8}{9}$$

$$\frac{4a^2-8a-8}{9} = \frac{4(a^2-2a+2)}{9}$$

$$\frac{4(a^2-2a+2)}{9} = \frac{4}{9} \Rightarrow a^2-2a+2=1$$

$$a^2-2a+1=0$$

$$\Delta = 0 \qquad a = -\frac{b}{2a} = 1$$

$$\underline{a=1}$$

X, Y- indep => a nu influențează valoarea lui ρ.
coeficientul de corelație pentru X, Y:

$$\rho(X,Y) = \frac{\text{cov}(X,Y)}{\sqrt{\text{var}(X)\cdot\text{var}(Y)}}$$

$$\text{cov}(X,Y) = \mathbb{E}(X\cdot Y) - \mathbb{E}(X)\cdot\mathbb{E}(Y) = \frac{20}{9} - \frac{4}{3}\cdot\frac{5}{3} = 0$$

$$X\cdot Y: \begin{pmatrix} 2 & 1 & 2 & 2 & 1 & 2 & 4 & 2 & 4 \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$$

$$X\cdot Y: \begin{pmatrix} 2 & 1 & 4 \\ \frac{5}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \qquad \mathbb{E}[X\cdot Y] = 2\cdot\frac{5}{9} + 1\cdot\frac{2}{9} + 4\cdot\frac{2}{9} = \frac{20}{9}$$

$$\mathbb{E}(X) = 1\cdot\frac{1}{3} + 1\cdot\frac{1}{3} + 2\cdot\frac{1}{3} = \frac{4}{3} \qquad \mathbb{E}(Y) = 2\cdot\frac{1}{3} + 1\cdot\frac{1}{3} + 2\cdot\frac{1}{3} = \frac{5}{3}$$

$cov(X, Y) = 0$, iar X, Y independente.

Deoarece X, Y independente valoarea lui $\underline{a}$ NU influențează coeficientul de corelație ρ. Acesta va fi $\underline{0}$, adică v.a. sunt necorelate.

4. $X: \begin{pmatrix} -2 & 3 & 4 & 6 \\ 6p & 2p & 9p & p \end{pmatrix}$, $p \in \mathbb{R}$

$Y = aX + b$ $\qquad \mathbb{E}(Y) = 57$

$\qquad \qquad Var(Y) = 75$

① $6p + 2p + 9p + p = 1$

$p = \frac{1}{18}$

② $X: \begin{pmatrix} -2 & 3 & 4 & 6 \\ \frac{6}{18} & \frac{2}{18} & \frac{9}{18} & \frac{1}{18} \end{pmatrix}$

$\mathbb{E}(Y) = \mathbb{E}(aX + b) = (-2a+b)\frac{1}{3} + (3a+b)\frac{1}{9} + (4a+b)\frac{1}{2} + (6a+b)\frac{1}{18} =$

$= 2a + b$

$\underline{2a + b = 57}$.

$Var(Y) = \mathbb{E}(Y^2) - \mathbb{E}(Y)^2$

$\mathbb{E}(Y^2) = (-2a+b)^2\frac{1}{3} + (3a+b)^2\frac{1}{9} + (4a+b)^2\frac{1}{2} + (6a+b)^2\frac{1}{18} =$

$= \frac{3b^2 + 12ab + 37a^2}{3}$

$\begin{cases} 2a + b = 57 \\ \frac{3b^2 + 12ab + 37a^2}{3} = 75 \end{cases} \Rightarrow \begin{cases} b = 57 - 2a \\ 3(57-2a)^2 + 12a(57-2a) + 37a^2 = 225 \end{cases}$

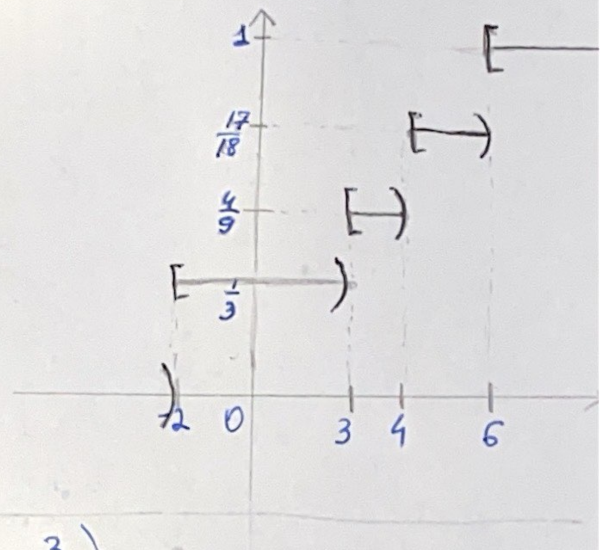
$\Rightarrow \begin{cases} b = 57 - 2a \\ a \notin \mathbb{R} \end{cases} \Rightarrow (a, b) \notin \mathbb{R}^2$

$$X: \begin{pmatrix} -2 & 3 & 4 & 6 \\ \frac{1}{3} & \frac{1}{9} & \frac{1}{2} & \frac{1}{18} \end{pmatrix}$$

Fct de repartiție pentru X

fie $F: \mathbb{R} \to [0,1]$ $\quad F(x) = \mathbb{P}(X \leq x)$

$$F(x) = \begin{cases} 0, & x < -2 \\ \frac{1}{3}, & -2 \leq x < 3 \\ \frac{4}{9}, & 3 \leq x < 4 \\ \frac{17}{18}, & 4 \leq x < 6 \\ 1, & x \geq 6 \end{cases}$$



5.

$$X: \begin{pmatrix} -2 & 1 \\ 0.4 & 0.6 \end{pmatrix} \qquad Y: \begin{pmatrix} -1 & 3 \\ 0.3 & 0.7 \end{pmatrix}$$

X\Y	-1	3	p_i
-2	0,12	0,28	0.4
1	0,18	0,42	0.6
q_j	0.3	0.7	1.

→ Rep comună p/u X, Y

Coef de corelație:

$$\rho(X, Y) = \frac{cov(X, Y)}{\sqrt{var(X) \cdot var(Y)}}$$

$cov(X, Y) = \mathbb{E}(X \cdot Y) - \mathbb{E}(X) \cdot \mathbb{E}(Y) = 0,36 + 0,2 \cdot 1,8 = 0,72$

$\mathbb{E}(X \cdot Y) = 2 \cdot 0,12 + (-2) \cdot 3 \cdot 0,28 - 1 \cdot 0,18 + 3 \cdot 0,42 = 0,36$

$\mathbb{E}(X) = -2 \cdot 0,4 + 1 \cdot 0,6 = -0,2$

$\mathbb{E}(Y) = -0,3 + 3 \cdot 0,7 = 1,8$

$Var(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = 4 \cdot 0{,}4 + 1 \cdot 0{,}6 - (-0{,}2)^2 = 2{,}16$

$Var(Y) = \mathbb{E}(Y^2) - \mathbb{E}(Y)^2 = 1 \cdot 0{,}3 + 9 \cdot 0{,}7 - (1{,}8)^2 = 3{,}36.$

$$\rho(X,Y) = \frac{0{,}72}{\sqrt{2{,}16 \cdot 3{,}36}} \simeq 0{,}267$$

$|\rho| \in [0{,}25, 0{,}75] \Rightarrow X, Y$ sunt corelate.

b) $k = P(X=-2, Y=3)$

$k = 0{,}28.$ din datele problemei.

Pentru ca X, Y să fie necorelate $k = 0$.

Ex 6.

$$X : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 3p & 4p & 2p & p & p \end{pmatrix}, \ p \in \mathbb{R}$$

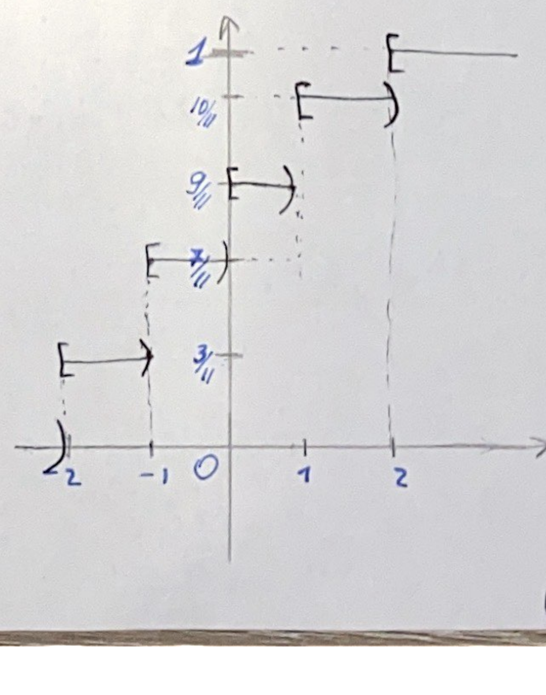
a) p - ? $\quad p \in \mathbb{R}$

$3p + 4p + 2p + p + p = 1 \Rightarrow p = \frac{1}{11}$

b) $$X : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ \frac{3}{11} & \frac{4}{11} & \frac{2}{11} & \frac{1}{11} & \frac{1}{11} \end{pmatrix}$$

fie $F : \mathbb{R} \to [0,1] \qquad F(x) = \mathbb{P}(X \leq x)$

$$F(x) = \begin{cases} 0, & x < -2 \\ \frac{3}{11}, & -2 \leq x < -1 \\ \frac{7}{11}, & -1 \leq x < 0 \\ \frac{9}{11}, & 0 \leq x < 1 \\ \frac{10}{11}, & 1 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$



c) $\mathbb{E}(3X-2) = (3\cdot(-2)-2)\cdot\frac{3}{11} + (3\cdot(-1)-2)\cdot\frac{4}{11} + (-2)\cdot\frac{2}{11} + \frac{1}{11} + 4\cdot\frac{1}{11} =$

$= -\frac{43}{11}$

$Var(6X-3) = (-12-3)\cdot\frac{3}{11} + (-6-3)\cdot\frac{4}{11} + (-3)\cdot\frac{2}{11} + \frac{3}{11} + (12-3)\cdot\frac{1}{11} =$

$= -\frac{75}{11}$

$\mathbb{E}(X+X^2) = \mathbb{E}(X) + \mathbb{E}(X^2) = -\frac{7}{11} + \frac{21}{11} = \frac{14}{11}$

$\mathbb{E}(X) = -2\cdot\frac{3}{11} + (-1)\cdot\frac{4}{11} + 0 + \frac{1}{11} + \frac{2}{11} = -\frac{7}{11}$

$\mathbb{E}(X^2) = 4\cdot\frac{3}{11} + 1\cdot\frac{4}{11} + 0 + \frac{1}{11} + \frac{4}{11} = \frac{21}{11}$

d) $P(\underbrace{|X|<\frac{1}{2}}_{=A} \mid \underbrace{-1{,}25 < X < 0{,}75}_{=B})$ — ?

$|X| < \frac{1}{2} \Rightarrow X \in (-\frac{1}{2}, \frac{1}{2})$

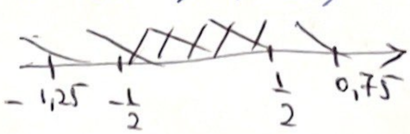
$P(A|B) = \frac{P(A\cap B)}{P(B)}$

$P(A) = \frac{2}{11}$

$P(B) = \frac{4}{11} + \frac{2}{11} = \frac{6}{11}$

$P(A\cap B)$ $\quad (-\frac{1}{2}; \frac{1}{2}) \cap (-1{,}25; 0{,}75) = (-\frac{1}{2}; \frac{1}{2})$

$P(A\cap B) = P(A) = \frac{2}{11}$



$P(|X| < \frac{1}{2} \mid -1{,}25 < X < 0{,}75) = P(A|B) = \frac{\frac{2}{11}}{\frac{6}{11}} = \frac{1}{3}$

Ex 7.

Se consideră v.a. (X, Y)

a)

$X \backslash Y$	-2	0	9	$P(X = X_i)$
-1	$\frac{1}{15}$	$\frac{2}{15}$	0	$\frac{1}{5}$
0	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$	$\frac{4}{5}$
$P(Y=Y_j)$	$\frac{4}{15}$	$\frac{6}{15}$	$\frac{1}{3}$	1

$b + 2b + 0 + 3b + 4b + 5b = 1 \qquad b \in \mathbb{R}$

$15b = 1 \Rightarrow \underline{b = \frac{1}{15}}$

b) independența variabilelor aleatoare X și Y

$P(X \cdot Y \neq 0)$ - ?

X și Y <u>nu sunt</u> independente deoarece:

$\frac{1}{15} \neq \frac{1}{5} \cdot \frac{4}{15}$

$P(X \cdot Y \neq 0) = \frac{1}{15}$ din valorile repart. comune.

c) $Var(3X - 2Y) = 9\,Var(X) + 4\,Var(Y) - 2 \cdot 3 \cdot 2 \cdot Cov(X, Y)$

$Cov(X, Y) = \mathbb{E}(X \cdot Y) - \mathbb{E}(X) \cdot \mathbb{E}(Y) = \frac{2}{15} + \frac{1}{5} \cdot \frac{7}{5} = \frac{31}{75} = 0{,}413$

$Var(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = \frac{1}{5} - \left(-\frac{1}{5}\right)^2 = \frac{1}{5} - \frac{1}{25} = \frac{4}{25} = 0{,}16$

$\mathbb{E}(X) = -1 \cdot \frac{1}{5} + 0 \cdot \frac{4}{5} = -\frac{1}{5}$

$\mathbb{E}(X^2) = 1 \cdot \frac{1}{5} + 0 = \frac{1}{5}$

$\mathbb{E}(Y) = -2 \cdot \frac{4}{5} + 0 + \cancel{9}^{3} \cdot \frac{1}{\cancel{3}} = -\frac{8}{5} + 3 = \frac{7}{5}$

$\mathbb{E}(Y^2) = 4 \cdot \frac{4}{5} + 0 + 81 \cdot \frac{1}{3} = \frac{151}{5}$

$Var(Y) = \mathbb{E}(Y^2) - \mathbb{E}(Y)^2 = \frac{151}{5} - \left(\frac{7}{5}\right)^2 = \frac{706}{25} = 28{,}24$

$\mathbb{E}(X \cdot Y) = 2 \cdot \frac{1}{15} + 0 + 0 + 0 + 0 + 0 = \frac{2}{15}$

$Var(3X-2Y) = 9 \cdot 0{,}16 + 4 \cdot 28{,}24 - 12 \cdot 0{,}413 = \underline{109{,}4}$

Ex 8.

! Rezolvat la laborator !

Ex 9.

$X \backslash Y$	-2	-1	0	1	2	p_i
-1	$\frac{1}{10}$	$\frac{1}{50}$	$\frac{3}{50}$	$\frac{1}{50}$	$\frac{1}{10}$	$\frac{3}{10}$
0	$\frac{1}{25}$	$\frac{3}{25}$	$\frac{1}{25}$	$\frac{3}{25}$	$\frac{1}{25}$	$\frac{9}{25}$
1	$\frac{2}{25}$	$\frac{1}{50}$	$\frac{7}{50}$	$\frac{1}{50}$	$\frac{2}{25}$	$\frac{17}{50}$
q_j	$\frac{11}{50}$	$\frac{4}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{11}{50}$	1

coef de corelație între X și Y:

b) $\rho(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}}$

$cov(X,Y) = \mathbb{E}(X \cdot Y) - \mathbb{E}(X) \cdot \mathbb{E}(Y)$

$\mathbb{E}(X \cdot Y) = 2 \cdot \frac{1}{10} + \frac{1}{50} + (-1) \cdot \frac{1}{50} + (-2) \cdot \frac{1}{10} + 0 + (-2) \cdot \frac{2}{25} + (-1) \cdot \frac{1}{50} + \frac{1}{50} + \frac{4}{25} = 0$

$\mathbb{E}(X) = (-1) \cdot \frac{3}{10} + 0 + 1 \cdot \frac{17}{50} = 0{,}04$

$\mathbb{E}(Y) = -2 \cdot \frac{11}{50} + (-1) \cdot \frac{4}{25} + 0 + \frac{4}{25} + 2 \cdot \frac{11}{50} = 0$

$cov(X,Y) = 0.$

$\rho(X,Y) = 0 \quad \Rightarrow \quad X, Y$ necorelate.

c) $X \mid Y=0 : \begin{pmatrix} -1 & 0 & 1 \\ \frac{1}{5} & \frac{1}{9} & \frac{7}{17} \end{pmatrix}$ $\qquad Y \mid X=1 : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ \frac{4}{11} & \frac{1}{8} & \frac{7}{12} & \frac{1}{8} & \frac{4}{11} \end{pmatrix}$

$\mathbb{E}[X \mid Y] = -\frac{1}{5} + \frac{7}{17} = \frac{18}{85}$ $\qquad \mathbb{E}[Y \mid X] = -\frac{8}{11} - \frac{1}{8} + \frac{1}{8} + \frac{8}{11} = \frac{14}{11}$

d) $\mathrm{Var}(3X+5) = 9\,\mathrm{Var}(X)$

$\mathrm{Var}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$

$\mathbb{E}(X^2) = 1\cdot\frac{3}{10} + 0 + 1\cdot\frac{17}{50} = \frac{15+17}{50} = \frac{32}{50} = 0{,}64$

$\mathbb{E}(X)^2 = 0{,}0016.$

$\mathrm{Var}(X) = 0{,}64 - 0{,}0016 = 0{,}638$

e) $P(X<1,\ Y>0) = \frac{1}{50} + \frac{5}{50} + \frac{6}{50} + \frac{2}{50} = \frac{14}{50}$

Ex 10.

X \ Y	1	2	3	4	p_i
0	$\frac{4}{40}$	$\frac{3}{40}$	$\frac{2}{40}$	$\frac{1}{40}$	$\frac{1}{4}$
1	$\frac{1}{40}$	$\frac{4}{40}$	$\frac{3}{40}$	$\frac{2}{40}$	$\frac{1}{4}$
2	$\frac{2}{40}$	$\frac{1}{40}$	$\frac{4}{40}$	$\frac{3}{40}$	$\frac{1}{4}$
3	$\frac{3}{40}$	$\frac{2}{40}$	$\frac{1}{40}$	$\frac{4}{40}$	$\frac{1}{4}$
q_j	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1

$\rho(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\cdot\mathrm{Var}(Y)}}$

$\mathrm{cov}(X,Y) = \mathbb{E}(X\cdot Y) - \mathbb{E}(X)\cdot\mathbb{E}(Y)$

$\mathbb{E}(X\cdot Y) = 0 + \frac{1}{40} + 2\cdot\frac{4}{40} + 3\cdot\frac{3}{40} + 4\cdot\frac{2}{40} + 2\cdot\frac{2}{40} + 2\cdot2\cdot\frac{1}{40} + 6\cdot\frac{4}{40} + 8\cdot\frac{3}{40} +$

$+ 3\cdot\frac{3}{40} + 6\cdot\frac{2}{40} + 9\cdot\frac{1}{40} + 12\cdot\frac{4}{40} = 4$

$\mathbb{E}(X) = 0 + 1\cdot\frac{1}{4} + 2\cdot\frac{1}{4} + 3\cdot\frac{1}{4} = \frac{3}{2}$

$\mathbb{E}(Y) = 1\cdot\frac{1}{4} + 2\cdot\frac{1}{4} + 3\cdot\frac{1}{4} + 4\cdot\frac{1}{4} = \frac{5}{2}$

$\mathrm{cov}(X,Y) = 4 - \frac{3}{2}\cdot\frac{5}{2} = \frac{1}{4}$

$\mathrm{Var}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = \frac{7}{2} - \frac{9}{4} = \frac{5}{4}$

$\mathbb{E}(X^2) = \frac{1}{4} + \frac{4}{4} + 9\cdot\frac{1}{4} = \frac{7}{2}$ $\qquad (\mathbb{E}(X))^2 = \frac{3^2}{2^2} = \frac{9}{4}$

$$\mathrm{var}(Y) = \mathbb{E}(Y^2) - \mathbb{E}(Y)^2 = \frac{15}{2} - \frac{25}{4} = \frac{5}{4}$$

$$\mathbb{E}(Y^2) = 1 \cdot \frac{1}{4} + 2^2 \cdot \frac{1}{4} + 3^2 \cdot \frac{1}{4} + 4^2 \cdot \frac{1}{4} = \frac{15}{2}$$

$$(\mathbb{E}(Y))^2 = \frac{5^2}{2^2} = \frac{25}{4}$$

$$\rho(X, Y) = \frac{\frac{1}{4}}{\sqrt{\frac{5}{4} \cdot \frac{5}{4}}} = \frac{1}{5}$$

$|\rho| \in (0, 0{,}25) \Rightarrow X, Y$ sunt slab corelate.

c) $X|Y=3 : \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{2}{10} & \frac{3}{10} & \frac{4}{10} & \frac{1}{10} \end{pmatrix}$

$$\mathbb{E}[X|Y=3] = \frac{3}{10} + \frac{8}{10} + \frac{3}{10} = \frac{14}{10} = \frac{7}{5}$$

$Y|X=1 : \begin{pmatrix} 1 & 2 & 3 & 4 \\ \frac{1}{10} & \frac{4}{10} & \frac{3}{10} & \frac{2}{10} \end{pmatrix}$

$$\mathbb{E}[Y|X=1] = \frac{1}{10} + \frac{8}{10} + \frac{9}{10} + \frac{8}{10} = \frac{13}{5}$$

d) $\mathrm{var}(-X+5) = \mathrm{var}(X) = \frac{5}{4}$

e) $P(X < 1, Y > 3) = \frac{1}{40}$