Coursework 1: Image filtering

In this coursework you will practice image filtering techniques, which are commonly used to smooth, sharpen or add certain effects to images. The coursework includes both coding questions and written questions. Please read both the text and code comment in this notebook to get an idea what you are expected to implement.

What to do?

- Complete and run the code using `jupyter-lab` or `jupyter-notebook` to get the results.
- Export (File | Export Notebook As...) or print (using the print function of your browser) the notebook as a pdf file, which contains your code, results and text answers, and upload the pdf file onto Cate.
- If Jupyter-lab does not work for you, you can also use Google Colab to write the code and export the pdf file.

Dependencies:

If you do not have Jupyter-Lab on your laptop, you can find information for installing Jupyter-Lab here.

There may be certain Python packages you may want to use for completing the coursework. We have provided examples below for importing libraries. If some packages are missing, you need to install them. In general, new packages (e.g. imageio etc) can be installed by running

```
`pip3 install [package_name]`
```

in the terminal. If you use Anaconda, you can also install new packages by running `conda install [package_name]` or using its graphic user interface.

In [1]:
```
# Import libaries (provided)
import imageio
import numpy as np
import matplotlib.pyplot as plt
import noise
import scipy
import scipy.signal
import math
import time
```

1. Moving average filter.

Read a specific input image and add noise to the image. Design a moving average filter of kernel size 3x3 and 11x11 respectively. Perform image filtering on the noisy image.

Design the kernel of the filter by yourself. Then perform 2D image filtering using the function `scipy.signal.convolve2d()` .

In [2]:
```
# Read the image (provided)
image = imageio.imread('hyde_park.jpg')
plt.imshow(image, cmap='gray')
plt.gcf().set_size_inches(10, 8)
```

In [3]:
```
# Corrupt the image with Gaussian noise (provided)
image_noisy = noise.add_noise(image, 'gaussian')
plt.imshow(image_noisy, cmap='gray')
plt.gcf().set_size_inches(10, 8)
```

Note: from now on, please use the noisy image as the input for the filters.

1.1 Filter the noisy image with a 3x3 moving average filter. Show the filtering results. (5 points)

In [4]:
```
# Design the filter h
h = np.ones((3, 3)) / 9

# Convolve the corrupted image with h using scipy.signal.convolve2d functior
image_filtered = scipy.signal.convolve2d(image_noisy, h)

# Print the filter (provided)
print('Filter h:')
print(h)

# Display the filtering result (provided)
plt.imshow(image_filtered, cmap='gray')
plt.gcf().set_size_inches(10, 8)
```

```
Filter h:
[[0.11111111 0.11111111 0.11111111]
 [0.11111111 0.11111111 0.11111111]
 [0.11111111 0.11111111 0.11111111]]
```

1.2 Filter the noisy image with a 11x11 moving average filter. (5 points)

In [5]:
```
# Design the filter h
h = np.ones((11, 11)) / 121

# Convolve the corrupted image with h using scipy.signal.convolve2d functior
image_filtered = scipy.signal.convolve2d(image_noisy, h)

# Print the filter (provided)
print('Filter h:')
print(h)

# Display the filtering result (provided)
plt.imshow(image_filtered, cmap='gray')
plt.gcf().set_size_inches(10, 8)
```

```
Filter h:
[[0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
 [0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
 [0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
 [0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
 [0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
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  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
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  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
 [0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
 [0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
 [0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]
 [0.00826446 0.00826446 0.00826446 0.00826446 0.00826446 0.00826446
  0.00826446 0.00826446 0.00826446 0.00826446 0.00826446]]
```

1.3 Comment on the filtering results. How do different kernel sizes influence the filtering results? (10 points)

For big kernel sizes, the image gets blurrier and the colours are blended together more, which leads to losing details. Smaller kernel sizes still blur the image, but not as much as the big kernel sizes, and keeps more details.

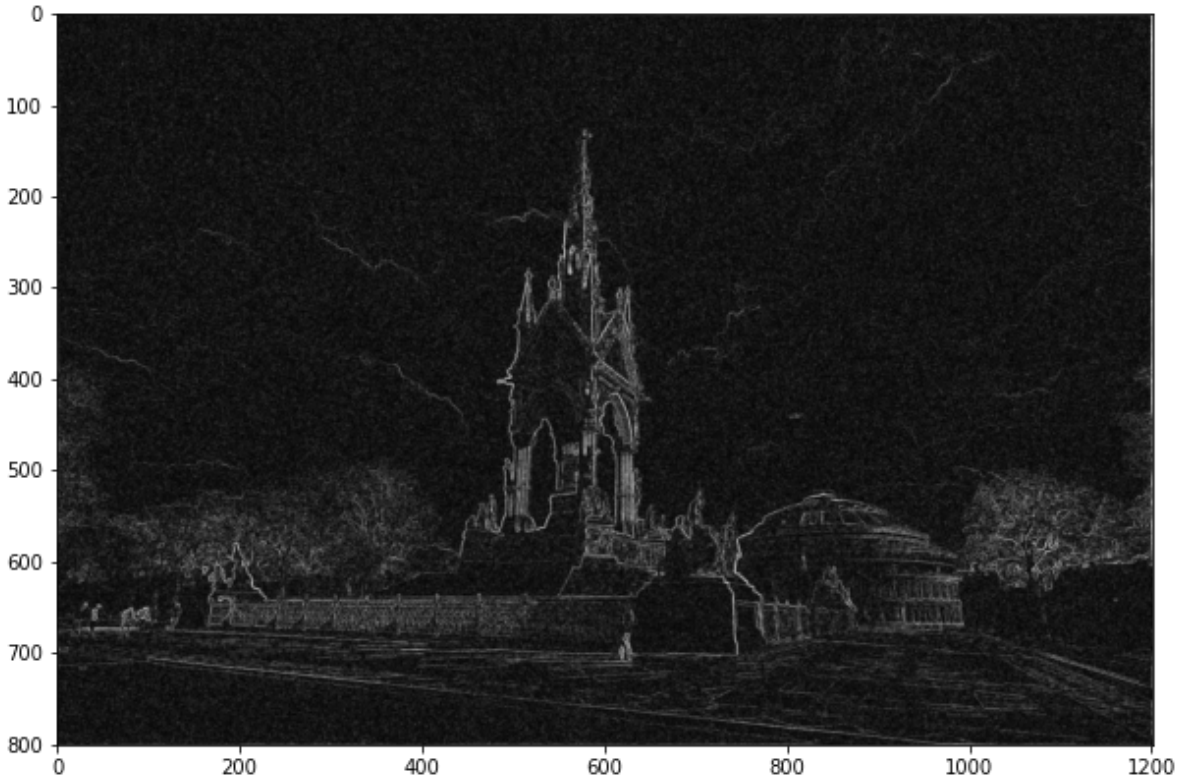
2. Edge detection.

Perform edge detection using Prewitt filtering, as well as Gaussian + Prewitt filtering.

2.1 Implement 3x3 Prewitt filters and convolve with the noisy image. (10 points)

In [6]:
```
# Design the Prewitt filters
prewitt_x = np.array([[1, 0, -1], [1, 0, -1], [1, 0, -1]])
prewitt_y = np.transpose(prewitt_x)

# Prewitt filtering
image_filtered_prewitt_x = scipy.signal.convolve2d(image_noisy, prewitt_x)
image_filtered_prewitt_y = scipy.signal.convolve2d(image_noisy, prewitt_y)

# Calculate the gradient magnitude
grad_mag = np.sqrt(np.square(image_filtered_prewitt_x) + np.square(image_fil

# Print the filters (provided)
print('prewitt_x:')
print(prewitt_x)
print('prewitt_y:')
print(prewitt_y)

# Display the gradient magnitude image (provided)
plt.imshow(grad_mag, cmap='gray')
plt.gcf().set_size_inches(10, 8)
```

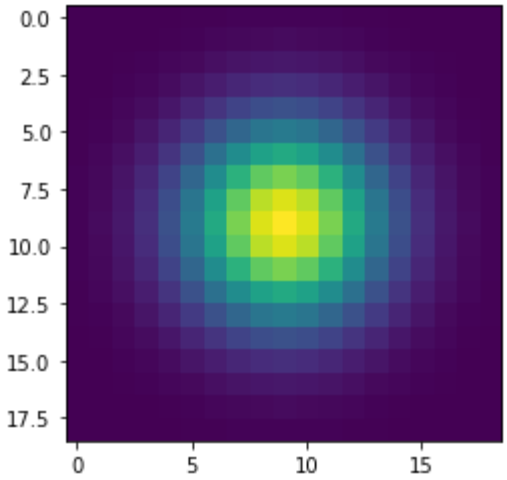
```
prewitt_x:
[[ 1  0 -1]
 [ 1  0 -1]
 [ 1  0 -1]]
prewitt_y:
[[ 1  1  1]
 [ 0  0  0]
 [-1 -1 -1]]
```



2.2 Implement a function that generates a 2D Gaussian filter given the parameter σ. (10 points)

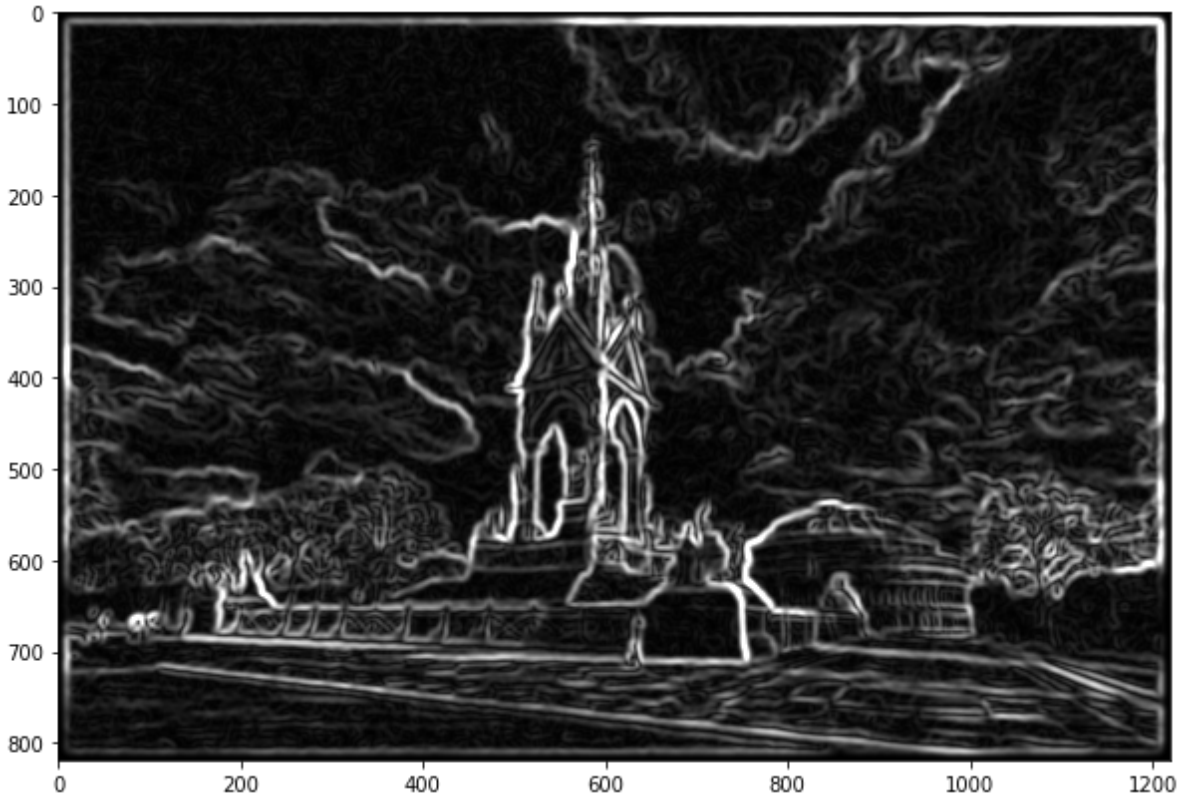
In [7]:
```
# Design the Gaussian filter
def gaussian_filter_2d(sigma):
    # sigma: the parameter sigma in the Gaussian kernel (unit: pixel)
    #
    # return: a 2D array for the Gaussian kernel

    k = 3
    var = sigma * k
    size = 2 * var + 1
    h = np.array([math.exp(-(i * i + j * j) / (2 * sigma * sigma)) / (2 * ma
                  for i in range(-var, var + 1) for j in range(-var, var + 1
    return h

# Visualise the Gaussian filter when sigma = 3 pixel (provided)
sigma = 3
h = gaussian_filter_2d(sigma)
plt.imshow(h)
```

Out[7]:
```
<matplotlib.image.AxesImage at 0x7f5852121160>
```



2.3 Perform Gaussian smoothing (σ = 3 pixels), followed by Prewitt filtering, show the gradient magnitude image. (5 points)

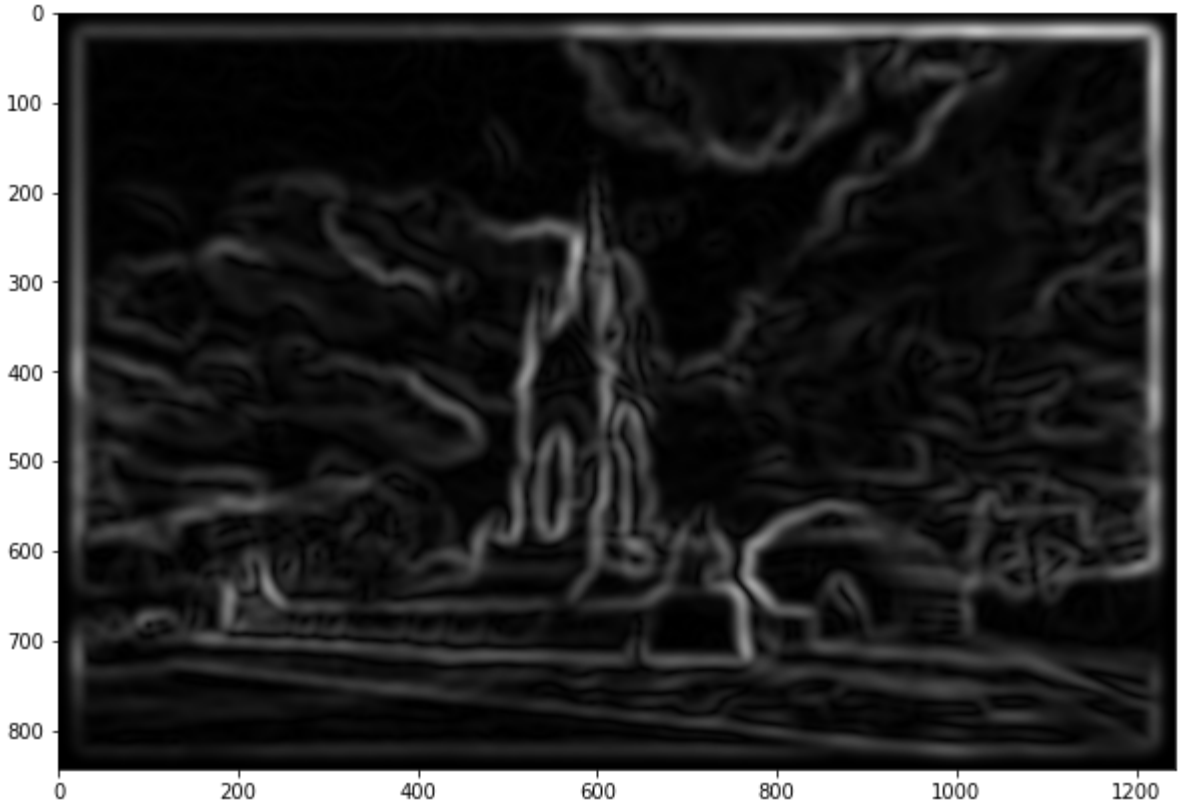
In [8]:
```
# Perform Gaussian smoothing before Prewitt filtering
image_gaussian = scipy.signal.convolve2d(image_noisy, h)

# Prewitt filtering
image_prewitt_x = scipy.signal.convolve2d(image_gaussian, prewitt_x)
image_prewitt_y = scipy.signal.convolve2d(image_gaussian, prewitt_y)

# Calculate the gradient magnitude
grad_mag = np.sqrt(np.square(image_prewitt_x) + np.square(image_prewitt_y))

# Display the gradient magnitude image (provided)
plt.imshow(grad_mag, cmap='gray', vmin=0, vmax=100)
plt.gcf().set_size_inches(10, 8)
```



2.4 Perform Gaussian smoothing (σ = 7 pixels) and evaluate the computational time for Gaussian smoothing. After that, perform Prewitt filtering. (7 points)

In [9]:
```
# Construct the Gaussian filter
h = gaussian_filter_2d(7)

# Perform Gaussian smoothing and count time
time_start = time.process_time()
image_gaussian = scipy.signal.convolve2d(image_noisy, h)
time_elapsed = (time.process_time() - time_start)
print("Computational time for Gaussian smoothing =", time_elapsed)

# Prewitt filtering
image_prewitt_x = scipy.signal.convolve2d(image_gaussian, prewitt_x)
image_prewitt_y = scipy.signal.convolve2d(image_gaussian, prewitt_y)

# Calculate the gradient magnitude
grad_mag = np.sqrt(np.square(image_prewitt_x) + np.square(image_prewitt_y))

# Display the gradient magnitude image (provided)
plt.imshow(grad_mag, cmap='gray', vmin=0, vmax=100)
plt.gcf().set_size_inches(10, 8)
```

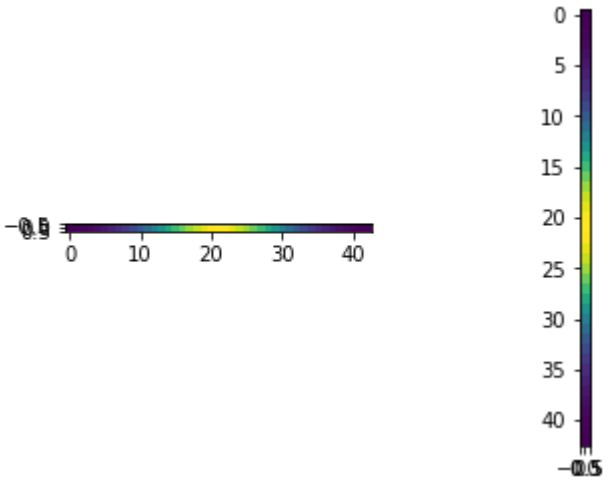
```
Computational time for Gaussian smoothing = 3.0132627329999995
```



2.5 Implement a function that generates a 1D Gaussian filter given the parameter σ. Generate 1D Gaussian filters along x-axis and y-axis respectively. (10 points)

In [10]:
```
# Design the Gaussian filter
def gaussian_filter_1d(sigma):
    # sigma: the parameter sigma in the Gaussian kernel (unit: pixel)
    #
    # return: a 1D array for the Gaussian kernel

    k = 3
    var = sigma * k
    size = 2 * var + 1
    h = np.array([math.exp(-(l * l) / (2 * sigma * sigma)) / (math.sqrt(2 *
                  for l in range(-var, var + 1)])
    return h

# sigma = 7 pixel (provided)
sigma = 7

gaussian = gaussian_filter_1d(sigma)
sz = gaussian.shape[0]

# The Gaussian filter along x-axis. Its shape is (1, sz).
h_x = gaussian.reshape((1, sz))

# The Gaussian filter along y-axis. Its shape is (sz, 1).
h_y = gaussian.reshape((sz, 1))

# Visualise the filters (provided)
plt.subplot(1, 2, 1)
plt.imshow(h_x)
plt.subplot(1, 2, 2)
plt.imshow(h_y)
```

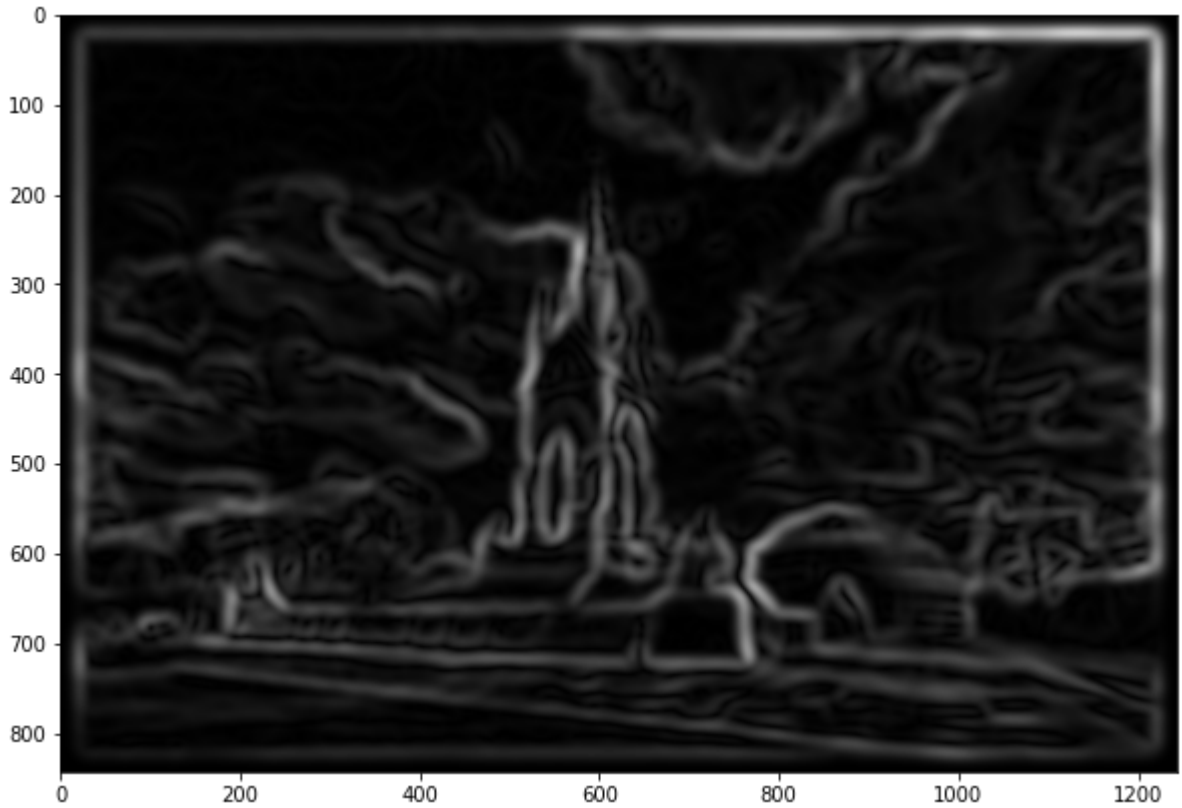
Out[10]:
```
<matplotlib.image.AxesImage at 0x7f5851fecc10>
```



2.6 Perform Gaussian smoothing (σ = 7 pixels) using two separable filters and evaluate the computational time for separable Gaussian filtering. After that, perform Prewitt filtering, show results and check whether the results are the same as the previous one without separable filtering. (9 points)

In [11]:
```
# Perform separable Gaussian smoothing and count time
time_start = time.process_time()
image_gaussian_x = scipy.signal.convolve2d(image_noisy, h_x)
image_gaussian = scipy.signal.convolve2d(image_gaussian_x, h_y)
time_elapsed = (time.process_time() - time_start)
print("Computational time for Gaussian smoothing =", time_elapsed)

# Prewitt filtering
image_prewitt_x = scipy.signal.convolve2d(image_gaussian, prewitt_x)
image_prewitt_y = scipy.signal.convolve2d(image_gaussian, prewitt_y)

# Calculate the gradient magnitude
grad_mag2 = np.sqrt(np.square(image_prewitt_x) + np.square(image_prewitt_y))

# Display the gradient magnitude image (provided)
plt.imshow(grad_mag2, cmap='gray', vmin=0, vmax=100)
plt.gcf().set_size_inches(10, 8)

# Check the difference between the current gradient magnitude map
# and the previous one produced without separable filtering. You
# can report the mean difference between the two.
mse = np.mean((grad_mag - grad_mag2) ** 2)
print("Mean squared error =", mse)
```

```
Computational time for Gaussian smoothing = 0.27624437100000065
Mean squared error = 1.597776592280473e-25
```



2.7 Comment on the Gaussian + Prewitt filtering results and the computational time. (9 points)

The Prewitt filtering alone generates an image where the fine lines are visible. The addition of the Gaussian smoothing leads to thicker and bolder lines.

When we're using Gaussian + Prewitt filtering we can notice that the small sigma (3) gives a result that clearly highlights the edges, whilst the bigger sigma (7) blends them with the background, since a greater sigma means that the current pixel is influenced by more neighbour-pixels.

The Gaussian + Prewitt filtering is significantly faster (10 times faster, from 2.895238699 to 0.27624437100000065) when we're using two 1D Gaussian filters instead of one 2D Gaussian filter. This result is expected since 2D Gaussian filtering has complexity O(N^2*M^2), whilst 1D Gaussian filtering has complexity O(N^2*M).

The very small MSE (1.597776592280473e-25) shows that the 2 options are very similar, almost the same. Since the MSE is so small and the computational time is significantly smaller for the 2nd approach, it is safe to say that the two 1D Gaussian filtering is much better.

3. Challenge: Implement a 2D Gaussian filter using Pytorch.

Pytorch is a machine learning framework that supports filtering and convolution.

The Conv2D operator takes an input array of dimension NxC1xXxY, applies the filter and outputs an array of dimension NxC2xXxY. Here, since we only have one image with one colour channel, we will set N=1, C1=1 and C2=1. You can read the documentation of Conv2D for more detail.

In [12]:
```
# Import libaries (provided)
import torch
```

3.1 Expand the dimension of the noisy image into 1x1xXxY and convert it to a Pytorch tensor. (7 points)

In [13]:
```
# Expand the dimension of the numpy array
x, y = image_noisy.shape
new_image_noisy = image_noisy.reshape((1, 1, x, y))

# Convert to a Pytorch tensor using torch.from_numpy
tensor_image_noisy = torch.from_numpy(new_image_noisy)
```

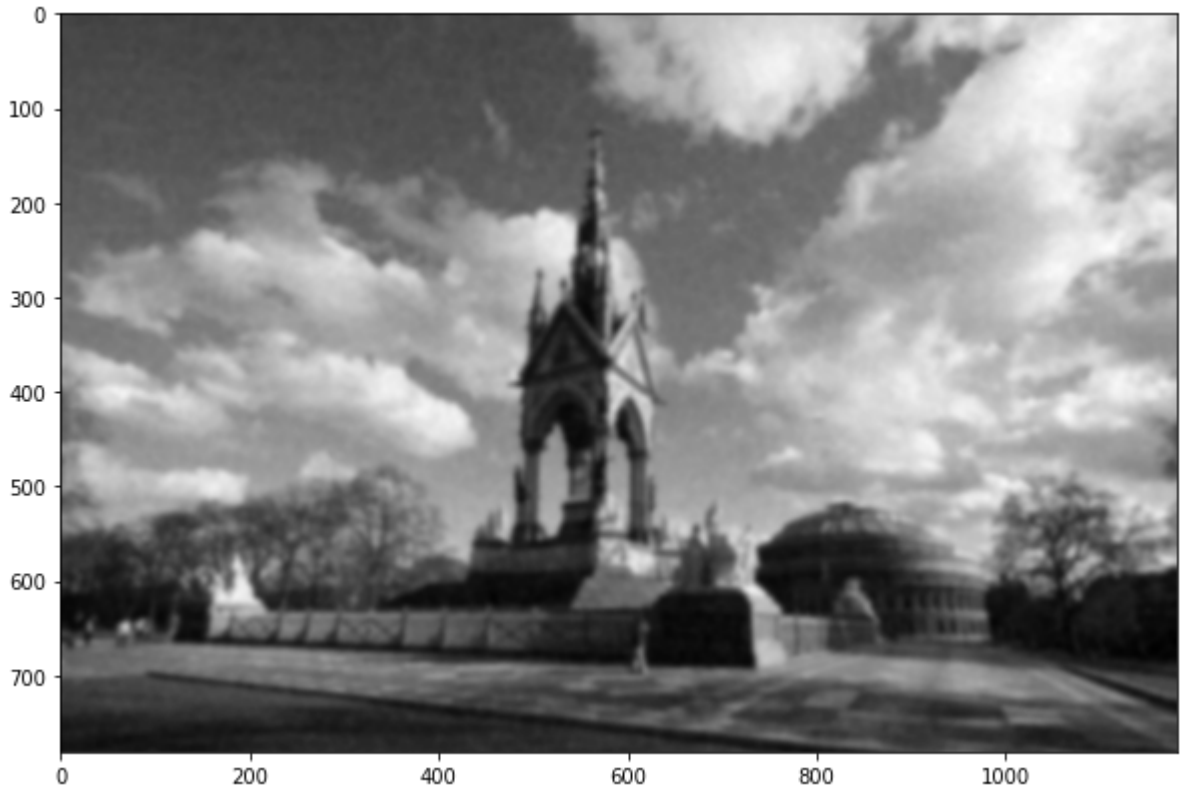
3.2 Create a Pytorch Conv2D filter, set its kernel to be a 2D Gaussian filter. (7 points)

In [14]:
```
# A 2D Gaussian filter when sigma = 3 pixel (provided)
sigma = 3
h = gaussian_filter_2d(sigma)
hx, hy = h.shape
h_expanded = h.reshape((1, 1, hx, hy))
h_torch = torch.from_numpy(h_expanded)
print(h.shape)

# Create the Conv2D filter (provided)
conv = torch.nn.Conv2d(in_channels = 1, out_channels = 1, kernel_size = h_ex

# Set the kernel weight
conv.weight = torch.nn.Parameter(h_torch)
```

```
(19, 19)
```

3.3 Apply the filter to the noisy image tensor and display the output image. (6 points)

In [15]:
```
# Filtering
image_filtered = conv(tensor_image_noisy).detach().numpy()[0, 0]

# Display the filtering result (provided)
plt.imshow(image_filtered, cmap='gray')
plt.gcf().set_size_inches(10, 8)
```



4. Survey: How long does it take you to complete the coursework?

In [16]:
```
2-3 hours
```

```
  Input In [16]
    2-3 hours
        ^
SyntaxError: invalid syntax
```