Dem pt mai:

$(R^0 \cup R^1)^{n+1} = (R^0 \cup R^1)^n \circ (R^0 \cup R^1) = (R^0 \cup R^1 \cup \dots \cup R^n) \circ (R^0 \cup R^1) =$

$= (R^0 \circ R^0) \cup (R^1 \circ R^0) \cup \dots \cup (R^n \circ R^0) \cup (R^0 \circ R^1) \cup (R^1 \circ R^1) \cup \dots \cup (R^n \circ R^1)$

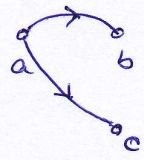
$= R^0 \cup R^1 \cup \dots \cup R^n \cup R^1 \cup R^2 \cup \dots \cup R^{n+1} =$

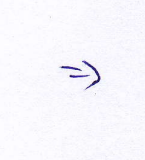
$= R^0 \cup R^1 \cup \dots \cup R^{n+1} \Rightarrow \mathcal{T}(R) = \bigcup_{n=0}^{\infty} (R^0 \cup R^1 \cup \dots \cup R^n) =$

$= \bigcup_{n=0}^{\infty} R^n = \mathcal{R}(\mathcal{T}(R))$

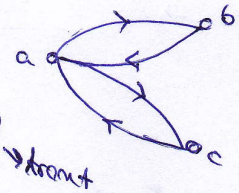
(3) Exemplul 1:

Fie $A = \{a, b, c\}$ $(|A| = 3)$ $R \subseteq A^2$

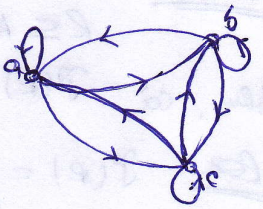
R: 

$\Rightarrow R^{-1}$: 

$\Rightarrow$

$\Rightarrow R \cup R^{-1}$: 

$\equiv \mathcal{R}(R \cup R^{-1})$ $\rightarrow$ tranz

$\Rightarrow \mathcal{T}(R \cup R^{-1})$: 

$\Rightarrow \mathcal{T}(R \cup R^{-1}) = A^2$ $\quad \mathcal{T}(\mathcal{T}(R))$

R tranzitivă $\Rightarrow \mathcal{T}(R) = R \Rightarrow \mathcal{T}(\mathcal{T}(R)) = \mathcal{T}(R) = R \cup R^{-1} \subseteq A^2 \Rightarrow$

$\Rightarrow \mathcal{T}(\mathcal{T}(R)) = A^2$

Exemplul 2: $A = \mathbb{N}$, $R = | = \{(n, an) \mid n, a \in \mathbb{N}\} \subseteq \mathbb{N}^2$

$\Rightarrow R^{-1} = |^{-1} = \vdots = \{(an, n) \mid n, a \in \mathbb{N}\}. \Rightarrow$

$\Rightarrow \mathcal{T}(R) = R \cup R^{-1} = |\vdots \Rightarrow \mathcal{T}(\mathcal{T}(R)) = \mathcal{T}(|\vdots) = \mathbb{N}^2$

Fie $x, y \in \mathbb{N} \Rightarrow \begin{cases} (x, x \cdot y) \in | \\ (x \cdot y, y) \in \vdots \end{cases} \Rightarrow (x, x \cdot y)(x \cdot y, y) \in |\vdots \subseteq$

$\subseteq \mathcal{T}(|\vdots) \rightarrow$ tranz $\Rightarrow$

$\Rightarrow (x, y) \in \mathcal{T}(|\vdots) \Rightarrow \mathbb{N}^2 \subseteq \mathcal{T}(|\vdots) \subseteq \mathbb{N}^2 \Rightarrow \mathcal{T}(|\vdots) = \mathbb{N}^2 \Rightarrow$

$\Rightarrow \mathcal{T}(\mathcal{T}(R)) = \mathbb{N}^2$