$a_2 \nleq a_1 \Rightarrow (a_2,b_1) \cancel{(\leq\times\sqsubseteq)} (a_1,b_2)$

$b_2 \sqsubseteq b_1 \Rightarrow (a_1,b_2) \cancel{(\leq\times\sqsubseteq)} (a_2,b_1)$

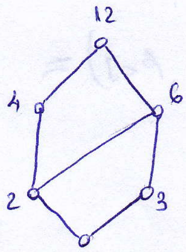
$\Rightarrow (A,\leq)\times(B,\sqsubseteq)$ nu e lanț

Exerc: Să se descompună în prod. direct de lanțuri laticea divizorilor nat. a unui nr. nat. nenul.

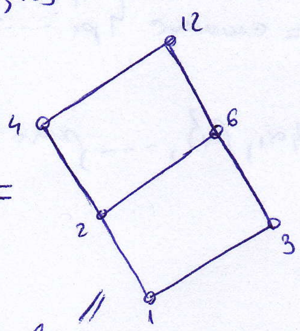
$(\forall n\in\mathbb{N}^*)$ $D_n := \{d\in\mathbb{N} \mid d|n\} \subseteq \mathbb{N}^*$, și

$\mathcal{D}_n := (D_n, \text{cmmmc}, \text{cmmdc}, |, 1, n) \to$ latice mărg.

$D_{12} = \{1,2,3,4,6,12\}$



$(\mathcal{D}_{12})$ =



$\parallel$

$\mathcal{L}_3\times\mathcal{L}_2$ $\quad (\mathcal{D}_{12})$

$\parallel \qquad \parallel$

$2+1 \qquad 1+1$

$\uparrow \qquad \uparrow$

$12 = 2^2\cdot 3 = 2^2\cdot 3^1$

Fie $n = p_1^{e_1}\cdots p_k^{e_k}$ desc. canonică a lui n, i.e. $k\in\mathbb{N}$, $p_1,\dots,p_k\in\mathbb{P}$ (nr nat prime), 2×2 distincte

$(\forall i\in\overline{1,k})$ $e_i = \max\{\alpha\in\mathbb{N} \mid p_i^{\alpha} | n\}$, $p_1|n, \dots, p_k|n$

$\Downarrow \qquad \Downarrow$

$e_1\in\mathbb{N}^* \ \dots \ e_k\in\mathbb{N}^*$

$\Downarrow$

$D_n = \{p_1^{\alpha_1}\cdots p_k^{\alpha_k} \mid \alpha_1\in\overline{0,e_1}, \dots, \alpha_k\in\overline{0,e_k}\}$

$(\mathcal{D}_2)$

$(\mathcal{D}_3)$

$(\mathcal{D}_{2^2})$

$(\forall i\in\overline{1,k})$ $L_{e_i+1} := \overline{0,e_i}$ și considerăm $\mathcal{L}_{e_i+1} = (L_{e_i+1}, \leq)$ → ordinea naturală

Fie $f : \prod_{i=1}^{k} \mathcal{L}_{e_i+1} \to D_n$, $(\forall \alpha_1\in\overline{0,e_1})\dots(\forall\alpha_k\in\overline{0,e_k})$

$\parallel$

$\prod_{i=1}^{k}\overline{0,e_i}$

$f(\alpha_1,\dots,\alpha_k) := p_1^{\alpha_1}\cdots p_k^{\alpha_k}\in D_n$

$D_n \cong \prod_{i=1}^{k}\mathcal{L}_{e_i+1}$ (izomorfism de latici mărg)

$\Rightarrow f\left(\prod_{i=1}^{k} L_{e_i+1}\right) = D_n \Rightarrow f \to$ surj