$\exists \max(\mathcal{F}(T)) = T \Rightarrow \exists \inf(\emptyset) = T = \bigcap_{X\in\emptyset} X$

$\exists \min(\mathcal{F}(T)) = \emptyset \Rightarrow \exists \sup(\emptyset) = \emptyset = \bigcup_{X\in\emptyset} X$

<u>Caz 2</u>: $A \neq \emptyset$ ($A \subseteq \mathcal{F}(T)$)

Fie $Y \in A$ ($Y \in \mathcal{F}(T)$) $\Rightarrow \bigcap_{X\in A} X \subseteq Y \quad (*)$

$\Rightarrow Y \subseteq \bigcup_{X\in A} X \quad (**)$

Fie $M, P \in \mathcal{F}(T)$, a.î. $\begin{cases} (\forall X\in A)\, M \subseteq X \Leftrightarrow M \subseteq \bigcap_{X\in A} X & (*\,*) \\ (\forall X\in A)\, X \subseteq P \Leftrightarrow P \supseteq \bigcup_{X\in A} X & (*\,*\,*\,*) \end{cases}$

$(*), (**) \Rightarrow \bigcap_{X\in A} X = \inf(A)$

$(**), (****) \Rightarrow \bigcup_{X\in A} X = \sup(A)$

<u>Exerc.</u>: $(L, \leq) \to$ poset $X \subseteq Y \subseteq L$. Dem că:

(1) dacă $\exists$ în $(L,\leq)$ $\inf(X)$ și $\inf(Y) \Rightarrow \inf(Y) \leq \inf(X)$

(2) dacă $\exists$ în $(L,\leq)$ $\sup(X)$, $\sup(Y) \Rightarrow \sup(X) \leq \sup(Y)$

<u>Rez</u>:

<u>Caz 1</u> $X = \emptyset$

(1) $\exists \inf(X) = \inf(\emptyset) = \max(L)$

$\exists \inf(Y) \in L \Rightarrow \inf(Y) \leq \max(L)$

$\Rightarrow \inf(Y) \leq \inf(X)$

(2) $\exists \sup(X) = \sup(\emptyset) = \min(L)$

$\exists \sup(Y) \in L \Rightarrow \min(L) \leq \sup(Y)$

$\Rightarrow \sup(X) \leq \sup(Y)$

<u>Caz 2</u>: $X \neq \emptyset$

(1) $\exists \inf(X), \inf(Y)$ $X \subseteq Y$

Fie $a \in X \Rightarrow a \in Y \Rightarrow \inf(Y) \leq a \Rightarrow \inf(Y) \leq \inf(X)$

(2) $\exists \sup(X)$, $\sup(Y)$

Fie $a \in X \overset{X\subseteq Y}{\Longrightarrow} a \in Y \Rightarrow a \leq \sup(Y) \Rightarrow \sup(X) \leq \sup(Y)$