Peste tot în cele ce urmează, vom avea aceste ipoteze:

$A, B, C, J \rightarrow$ mulțimi nevide;

$\rho \subseteq C \times A$, $\begin{cases} R \subseteq A \times B \\ S \subseteq A \times B \end{cases}$, $T \subseteq B \times C$,

$(R_i)_{i \in J} \subseteq \mathcal{P}(A \times B)$ (i.e. $(\forall i \in J)\ R_i \subseteq A \times B$).

$\Delta_A = \{(a,a) \mid a \in A\} \subseteq A^2 = A \times A$ $\quad \boxed{\Delta_A^{-1} = \Delta_A}$

$\Delta_A = id_A = "="$ pe A $\quad (\forall a, b \in A)(a \Delta_A b \Leftrightarrow a = b)$

$R^{-1} \subseteq B \times A$, $(\forall a \in A)(\forall b \in B)\ b R^{-1} a \Leftrightarrow a R b$

$T \circ R = \{(a,c) \mid a \in A, c \in C, (\exists b \in B)(a R b \text{ și } b T c)\}$

$\boxed{\begin{array}{l} R \circ \rho \subseteq C \times B \\ T \circ R \subseteq A \times C \end{array}}$

• $(R^{-1})^{-1} = R$ $\quad (R^{-1})^{-1} \subseteq A \times B$

$(\forall a \in A)(\forall b \in B)\ a (R^{-1})^{-1} b \Leftrightarrow b R^{-1} a \Leftrightarrow a R b \Rightarrow (R^{-1})^{-1} = R$

• $R \subseteq S \Leftrightarrow R^{-1} \subseteq S^{-1} \Rightarrow [R = S \Leftrightarrow R^{-1} = S^{-1}$

"$\Rightarrow$": $R \subseteq S$

Fie $a \in A$ și $b \in B$, $b R^{-1} a \Leftrightarrow \underset{\subseteq S}{a R b} \Rightarrow a S b \Leftrightarrow b S^{-1} a \Rightarrow$

$\Rightarrow R^{-1} \subseteq S^{-1}$

"$\Leftarrow$" $R^{-1} \subseteq S^{-1} \xrightarrow{"\Rightarrow"} \underbrace{(R^{-1})^{-1}}_{R} \subseteq \underbrace{(S^{-1})^{-1}}_{S} \Leftrightarrow R \subseteq S$

• $R \subseteq S \Rightarrow \begin{cases} R \circ \rho \subseteq S \circ \rho \\ T \circ R \subseteq T \circ S \end{cases}$

$\rho \subseteq C \times A$

$R, S \subseteq A \times B$

$T \subseteq B \times C$

$S \circ \rho, R \circ \rho \subseteq C \times B$

$T \circ R, T \circ S \subseteq A \times C$