Not: $(\forall x \in \mathbb{R})$ $\begin{cases} [x] := \max\{n \in \mathbb{Z} / n \le x\} \in \mathbb{Z} \\ \text{Not parte fracționară: } \{x\} := x - [x] \in [0;1) \ (\subset \mathbb{R}) \end{cases}$

Ex: $[-7] = -7$, $\{-7\} = 0$

$[8,3] = 8$ $\quad \{8,3\} = 0,3$

$[-9,2] = -10$ $\quad \{-9,2\} = 0,8$

Exerc: $\sim \subseteq \mathbb{R}^2$; $(\forall x, y \in \mathbb{R})$

$x \sim y \overset{def}{\iff} x - y \in \mathbb{Z}$

Dem că: (1) $(\forall x, y \in \mathbb{R})$ $x \sim y \iff \{x\} = \{y\}$;

(2) $\sim \in Echiv(\mathbb{R})$;

(3) $\mathbb{R}/\sim \ \simeq [0;1)$ $(\subset \mathbb{R})$ (Mulț. factor. a lui $\sim$ bij)

(1) Fie $x, y \in \mathbb{R}$, arb. fixate.

$x \sim y \iff x - y \in \mathbb{Z} \iff [x] - [y] + \{x\} - \{y\} \in \mathbb{Z} \iff$

($x = [x] + \{x\}$, $y = [y] + \{y\}$)

$\{x\} - \{y\} \in \mathbb{Z} \iff \{x\} - \{y\} \in \mathbb{Z} \cap (-1,1) = \{0\} \iff$

($\{x\} \in [0,1)$, $\{y\} \in (-1,0]$, $\{x\} - \{y\} \in (-1,1)$)

$\iff \{x\} - \{y\} = 0 \iff \{x\} = \{y\}$

(2) Fie $x, y, z \in \mathbb{R}$ arb. fixate

$\{x\} = \{x\} \overset{(1)}{\iff} x \sim x \Rightarrow \sim$ reflexivă

Dacă $x \sim y \overset{(1)}{\iff} \{x\} = \{y\} \Rightarrow \{y\} = \{x\} \overset{(1)}{\iff} y \sim x \Rightarrow \sim$ simetrică

Dacă $x \sim y$ și $y \sim z \overset{(1)}{\iff} \{x\} = \{y\}$ și $\{y\} = \{z\}$

$\Rightarrow \{x\} = \{z\} \overset{(1)}{\iff} x \sim z \Rightarrow \sim$ tranzitivă

$\Rightarrow \sim \in Echiv(\mathbb{R})$