(3) Fie $f : \mathbb{R}/_{\sim} \to [0,1)$, $(\forall x \in \mathbb{R})\ f(\hat{x}) := \mathrm{frac}\{x\} \in [0,1)$

$\underbrace{f \to \text{funcţie}}_{''}$, i.e $\underbrace{f \to \text{bine definită}}_{''}$, i.e $f \to$ $\underbrace{\text{indep. de reprezentanţii clasei}}_{''}$

Fie $x, y \in \mathbb{R}$, arb, fixate

$\hat{x} = \hat{y} \Leftrightarrow x \sim y \overset{(1)}{\Leftrightarrow} \mathrm{frac}\{x\} = \mathrm{frac}\{y\} \Leftrightarrow f(\hat{x}) = f(\hat{y})$

În acest şir de echivalenţe implicaţiile de la stg la dr "$\Rightarrow$" $\rightsquigarrow$ $f \to$ bine definită
arată că

"$\Leftarrow$" $\rightsquigarrow$ $f \to$ injectivă

Fie $\alpha \in [0,1)$, arb, fixat

$f(\hat{\alpha}) := \mathrm{frac}\{\alpha\} = \alpha - \overbrace{[\alpha]}^{0} = \alpha \Rightarrow f \to$ surjectivă $\to$

f bijectivă.

Exerc: $A \to$ mulţ, $R \subseteq A^2$. Arm că:

(a) $\mathrm{Pre}(R) = \bigcup_{n=0}^{\infty} R^n \ (= \mathcal{R}(\mathcal{T}(R)))$ → temă pt acasă

(b) $\underset{''}{E(R) =} \bigcup_{n=1}^{\infty} (\Delta_A \cup R \cup R^{-1})^n \ (= \mathcal{T}(\mathcal{R}(\mathcal{S}(R))))$.

includere tranz — includ. reflex. — includ. simetrică

REZ:

(b) Not $Q := \bigcup_{n=1}^{\infty} (\Delta_A \cup R \cup R^{-1})^n = \mathcal{T}(\mathcal{R}(\mathcal{S}(R))) \subseteq A^2$

$\underbrace{E(R) = Q}_{''}$, i.e $\underbrace{Q \in \mathrm{Echiv}(A)}_{''}$, $\underbrace{R \subseteq Q}_{''}$, şi

$(\forall S \subseteq A^2)(S \in \mathrm{Echiv}(A)$ şi $R \subseteq S \underset{''}{\Rightarrow} Q \subseteq S)$

$Q = \bigcup_{n=1}^{\infty} (\Delta_A \cup R \cup R^{-1})^n \supseteq (\Delta_A \cup R \cup R^{-1})^1 = \Delta_A \cup R \cup R^{-1} \supseteq R \ (*)$
$\supseteq \Delta_A \ (1)$

Fie $S \in \mathrm{Echiv}(A)$ cu $R \subseteq S$. $\left.\begin{matrix} \\ S \to \text{simetrică} \end{matrix}\right\}$ $\overset{\text{def includ simetr}}{\Longrightarrow}$ $\left.\begin{matrix} \mathcal{S}(R) \subseteq S \\ S \to \text{refl.} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \mathcal{R}(\mathcal{S}(R)) \\ \text{\rotatebox{90}{$\subseteq$}} \\ S \\ S \to \text{tranz} \end{matrix}\right\} \Rightarrow$

$\Rightarrow \mathcal{T}(\mathcal{R}(\mathcal{S}(R))) = Q \Leftrightarrow Q \subseteq S \ (*)$