Comutarea imag și preimag

$f: A \to B$; $C \subseteq A$; $\delta \subseteq B$

Imag $f(C) = \{ f(x) \mid x \in C \} \subseteq B$

Preimag $f^{-1}(\delta) = \{ x \in A \mid f(x) \in \delta \}$
$\subseteq A$

$f^{-1}(B) = A$

$f(A) = B \Leftrightarrow f$ surj

Temă: de dem.

" $f^{-1}(f(C)) \supseteq C$, cu "=" $\forall C \subseteq A \Leftrightarrow f$ inj

" $f(f^{-1}(\delta)) \subseteq \delta$, cu "=" $\forall \delta \subseteq B \Leftrightarrow f$ surj

$f(\underbrace{f^{-1}(B)}_{=A}) = f(A)$

Exerciții: $A, B \to$ mult, $A \neq \emptyset$, $B \neq \emptyset$,

$f: A \to B$

$I, J \to$ mult, $I \neq \emptyset$, $J \neq \emptyset$

$(A_i)_{i \in I} \subseteq \mathcal{P}(A)$ (i.e $(\forall i \in I)\ A_i \in \mathcal{P}(A)$)

$(B_j)_{j \in J} \subseteq \mathcal{P}(B)$ ($\Leftrightarrow (\forall j \in J)\ B_j \in \mathcal{P}(B)$)

Demonstrați:

(1) $f^{-1}\left(\bigcup_{j \in J} B_j\right) = \bigcup_{j \in J} f^{-1}(B_j)$

(2) $f\left(\bigcup_{i \in I} A_i\right) = \bigcup_{i \in I} f(A_i)$