① Max $12x_1 + 16x_2 + 3x_3$

$$\left.\begin{array}{l} 5x_1+2x_2+3x_3 \le 16 \to u_1 \\ x_1+3x_2-x_3 \le 13 \to u_2 \\ -x_1+x_2-3x_3 \le 1 \to u_3 \end{array}\right\} \text{primala} \to \begin{cases} 5x_1+2x_2+3x_3+w_1=16 \\ x_1+3x_2-x_3+w_2=13 \\ -x_1+x_2-3x_3+w_3=1 \\ w_1,w_2,w_3\ge 0 \end{cases}$$

↓ duala

Min $16u_1+13u_2+1u_3$

$$\left.\begin{cases} 5u_1+1u_2-1u_3 \ge 12 \\ 2u_1+3u_2+1u_3 \ge 16 \\ 3u_1-1u_2-3u_3 \ge 3 \\ u_1,u_2,u_3\ge 0 \end{cases}\right\} \text{duala} \Rightarrow \begin{cases} 5u_1+1u_2-1u_3-v_1=12 \\ 2u_1+3u_2+1u_3-v_2=16 \\ 3u_1-1u_2-3u_3-v_3=3 \end{cases}$$

ⓑ $(x_1^*, x_2^*, x_3^*) = (0,5,2)$.

Th. slaba a ecartului $\Rightarrow \begin{cases} x_j^*\cdot v_j^*=0 \\ u_i^*\cdot w_i^*=0 \end{cases}$

$$\begin{cases} x_1^*\cdot v_1^*=0 \Rightarrow x_1^*=0 \Rightarrow \boxed{v_1^*\ne 0} \\ x_2^*\cdot v_2^*=0 \Rightarrow 5\cdot v_2^*=0 \Rightarrow \boxed{v_2^*=0} \\ x_3^*\cdot v_3^*=0 \to 2\cdot v_3^*=0 \Rightarrow \boxed{v_3^*=0} \\ u_1^*\cdot w_1^*=0 \Rightarrow u_1^*\cdot 0=0 \Rightarrow \boxed{u_1^*\ne 0} \\ u_2^*\cdot w_2^*=0 \Rightarrow u_2^*\cdot 0=0 \Rightarrow \boxed{u_2^*\ne 0} \\ u_3^*\cdot w_3^*=0 \Rightarrow u_3^*\cdot 2=0 \Rightarrow \boxed{u_3^*=0} \end{cases}$$

$w_1^*=16-5x_1-2x_2-3x_3$

$w_1^*=16-5\cdot 0-2\cdot 5-3\cdot 2$

$w_1^*=16-0-10-6$

$w_1^*=0$

$w_2^*=13-x_1-3x_2+x_3$

$w_2^*=13-0-3\cdot 5+2$

$w_2^*=13-15+2$

$w_2^*=0$

$u_2=3\cdot\frac{25}{11}-3$

$u_2=\frac{75-33}{11}$

$\boxed{u_2=\frac{42}{11}}$

$w_3^*=1-x_1-x_2+3x_3$

$w_3^*=1-0-5+3\cdot 2$

$w_3^*=2$

$2u_1+3u_2+1u_3-v_2=16 \Rightarrow$

$\Rightarrow 2u_1+3u_2=16$

$3u_1-1u_2-3u_3-v_3=3$

$\Rightarrow 3u_1-1u_2=3$

$2u_1+3u_2=16$

$3u_1-1u_2=3 \to -u_2=3-3u_1$

$\boxed{u_2=3u_1-3}$

$2u_1+3(3u_1-3)=16.$

$2u_1+9u_1-9=16$

$11u_1=25$

$\boxed{u_1=\frac{25}{11}}$

$u^*=\left(\frac{25}{11}, \frac{42}{11}, 0\right).$

2. Max $x_1+x_2+(\beta+1)x_3$

$$(4+1)x_1+(2-1)x_2+\beta x_3=(6-1)+\alpha \Leftrightarrow \begin{cases}5x_1+1x_2+\beta x_3=5+\alpha\\ 2x_1+x_2+x_3+x_4=2+2\alpha\\ x_1,x_2\geq 0\end{cases}$$

$$(1+1)x_1+x_2+x_3+x_4=2+2\alpha$$

$A=\begin{pmatrix}5 & 1 & \beta & 0\\ 2 & 1 & 1 & 1\end{pmatrix}$ $b=\begin{pmatrix}5+\alpha\\ 2+2\alpha\end{pmatrix}$ $c=\begin{pmatrix}1\\1\\ \beta+1\\0\end{pmatrix}$ $B=\begin{pmatrix}5 & 1\\ 2 & 1\end{pmatrix}\rightarrow$ bază dată de x_1,x_2.

$B^{-1}=\frac{1}{\det B}\cdot B^*$ $\quad B^T=\begin{pmatrix}5 & 2\\ 1 & 1\end{pmatrix}$

$B^{-1}=\frac{1}{3}\cdot B^*$

$B^{-1}=\begin{pmatrix}\frac{1}{3} & -\frac{1}{3}\\ -\frac{2}{3} & \frac{5}{3}\end{pmatrix}$ $\quad B^*=\begin{pmatrix}(-1)^{1+1}\cdot 1 & (-1)^{1+2}\cdot 1\\ (-1)^{2+1}\cdot 2 & (-1)^{2+2}\cdot 5\end{pmatrix}$

$B^*=\begin{pmatrix}1 & -1\\ -2 & 5\end{pmatrix}$

B-primal admisibilă dacă $B^{-1}\cdot b\geq 0$

$$\begin{pmatrix}\frac{1}{3} & -\frac{1}{3}\\ -\frac{2}{3} & \frac{5}{3}\end{pmatrix}\begin{pmatrix}5+\alpha\\ 2+2\alpha\end{pmatrix}=\begin{pmatrix}\frac{1}{3}(5+\alpha)-\frac{1}{3}(2+2\alpha)\\ -\frac{2}{3}(5+\alpha)+\frac{5}{3}(2+2\alpha)\end{pmatrix}\geq 0.$$

$$\begin{matrix}\frac{1}{3}(5+\alpha)-\frac{1}{3}(2+2\alpha)\geq 0\\ -\frac{2}{3}(5+\alpha)+\frac{5}{3}(2+2\alpha)\geq 0\end{matrix}\quad \begin{cases}\frac{5}{3}+\frac{\alpha}{3}-\frac{2}{3}-\frac{2\alpha}{3}\geq 0\\ -\frac{10}{3}-\frac{2\alpha}{3}+\frac{10}{3}+\frac{10\alpha}{3}\geq 0\end{cases}\Leftrightarrow\begin{cases}\frac{3}{3}-\frac{\alpha}{3}\geq 0\Rightarrow -\frac{\alpha}{3}\geq -\frac{3}{3}\Rightarrow \frac{\alpha}{3}\leq 1\\ \frac{8\alpha}{3}\geq 0 \quad \frac{8}{3}\cdot\alpha\geq 0\Rightarrow \alpha\geq 0\end{cases}$$

$\rightarrow \alpha\in[0,3]$

$C_B^T\cdot y-c$

$$y=B^{-1}\cdot A=\begin{pmatrix}\frac{1}{3} & -\frac{1}{3}\\ -\frac{2}{3} & \frac{5}{3}\end{pmatrix}\begin{pmatrix}5 & 1 & \beta & 0\\ 2 & 1 & 1 & 1\end{pmatrix}=\begin{pmatrix}\frac{5}{3}-\frac{2}{3} & \frac{1}{3}-\frac{1}{3} & \frac{\beta}{3}-\frac{1}{3} & -\frac{1}{3}\\ -\frac{10}{3}+\frac{10}{3} & -\frac{2}{3}+\frac{5}{3} & -\frac{2\beta}{3}+\frac{5}{3} & \frac{5}{3}\end{pmatrix}$$

$$y=\begin{pmatrix}1 & 0 & \frac{\beta-1}{3} & -\frac{1}{3}\\ 0 & 1 & \frac{-2\beta+5}{3} & \frac{5}{3}\end{pmatrix}\quad C_B^T\cdot y-c^T=(1,1)\cdot\begin{pmatrix}1, & 0, & \frac{\beta-1}{3}, & -\frac{1}{3}\\ 0 & 1, & \frac{-2\beta+5}{3}, & \frac{5}{3}\end{pmatrix}-(1,1,\beta+1,0)$$

$$C_B^T\cdot y-c=\left(1,1,\frac{\beta-1}{3}-\frac{2\beta+5}{3},-\frac{1}{3}+\frac{5}{3}\right)-(1,1,\beta+1,0)$$

$$C_B^T\cdot y-c=\left(0,0,\frac{-\beta+4}{3}-(\beta+1),1\right)$$

$$C_B^T\cdot y-c=\left(0,0,\frac{-\beta+4}{3}-\frac{\beta-1}{1},1\right)\Rightarrow C_B^T\cdot y-c=\left(0,0,\frac{-4\beta+\cancel{4}}{3},1\right)$$

$\rightarrow$ Th1.

$-\frac{4\beta+1}{3}\leq 0$

$-4\beta+1\leq 0$

$-4\beta\leq -1 \mid\cdot(-1)$

$\beta\geq\frac{1}{4}$

$\boxed{\beta\in\left[\frac{1}{4};+\infty\right)}$

Sol mult $\Rightarrow \beta=\frac{1}{4}$

Max $x_1+x_2+4x_3$

$5x_1+x_2+3x_3=5$

$2x_1+x_2+x_3+x_4=2.$

$A=\begin{pmatrix}5&1&3&0\\2&1&1&1\end{pmatrix}$ $\quad b=\begin{pmatrix}5\\2\end{pmatrix}$ $\quad c=\begin{pmatrix}1\\1\\4\\0\end{pmatrix}$

Fie $B=\begin{pmatrix}1&0\\1&1\end{pmatrix}$ bază

$\downarrow c_2 \quad \downarrow c_4$

$B^{-1}=\begin{pmatrix}1&0\\-1&1\end{pmatrix}$

$\bar{x}^B=B^{-1}\cdot b=\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}5\\2\end{pmatrix}=\begin{pmatrix}5\\-3\end{pmatrix}\geq 0$ F

nu e bază, alegem $B=\begin{pmatrix}3&0\\1&1\end{pmatrix}$ (c_3, c_4) $\Rightarrow B^{-1}=\begin{pmatrix}\frac{1}{3}&0\\-\frac{1}{3}&1\end{pmatrix}\begin{pmatrix}5\\2\end{pmatrix}=\begin{pmatrix}\frac{5}{3}\\-\frac{5}{3}+2\end{pmatrix}=\begin{pmatrix}\frac{5}{3}\\\frac{1}{3}\end{pmatrix}\geq 0$

$y=B^{-1}\cdot A=\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}5&1&3&0\\2&1&1&1\end{pmatrix}$ $\quad y=\begin{pmatrix}\frac{5}{3}&\frac{1}{3}&1&0\\\frac{1}{3}&\frac{2}{3}&0&1\end{pmatrix}.$

$c_B^T\cdot y-c^T=(4,0)\begin{pmatrix}\frac{5}{3}&\frac{1}{3}&1&0\\\frac{1}{3}&\frac{2}{3}&0&1\end{pmatrix}-(1\ 1\ 4\ 0)=\begin{pmatrix}\frac{20}{3}&\frac{4}{3}&4&0\end{pmatrix}-(1\ 1\ 4\ 0)$

$=\begin{pmatrix}\frac{17}{3}&\frac{1}{3}&0&0\end{pmatrix}$

Th1

VB	VVB	y_1	y_2	y_3	y_4
x_3	$5/3$	$\frac{5}{3}$	$\frac{1}{3}$	1	0
x_4	$1/3$	$\frac{1}{3}$	$\frac{2}{3}$	0	1
		$\frac{17}{3}$	$\frac{1}{3}$	0	0

$\frac{17}{3}\leq 0$ F

$\frac{1}{3}\leq 0$ F $\to$ nu avem sol optimă.

Th2 $\frac{17}{3}>0$ $\quad\frac{1}{3}>0\Rightarrow\begin{pmatrix}\frac{5}{3}\\\frac{1}{3}\end{pmatrix}\leq 0$ F $\quad\begin{pmatrix}\frac{1}{3}\\\frac{2}{3}\end{pmatrix}\leq 0$ F. $\to$ nu avem opt inf

Th3 Max $\left(\frac{17}{3},\frac{1}{3}\right)=\frac{17}{3}=K=1.$ $\to$ x_1 intră în bază

min $\left(\frac{\frac{5}{3}}{\frac{5}{3}},\ \frac{\frac{1}{3}}{\frac{1}{3}}\right)=(1,1)$ $\to$ x_3 iese din bază

end of pas.