~~(1)~~ $(\forall A_1, A_2 \in \mathcal{P}(A))$

$A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

$(\forall B_1, B_2 \in \mathcal{P}(B))$

$B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

(4) $(\forall i \in J) \bigcap_{i \in J} A_i = \begin{cases} A_k, \text{ dacă } J = \{k\} \\ A_k \cap \left(\bigcap_{i \in J \setminus \{k\}} A_i\right) \subseteq A_k \text{ dacă } J \setminus \{k\} = \emptyset \end{cases}$

$\Rightarrow (\forall k \in J) \bigcap_{i \in J} A_i \subseteq A_k \Rightarrow (\forall k \in J) f\left(\bigcap_{i \in J} A_i\right) \subseteq f(A_k) \Leftrightarrow$

$\Leftrightarrow f\left(\bigcap_{i \in J} A_i\right) \subseteq \bigcap_{k \in J} f(A_k) = \bigcap_{i \in J} f(A_i)$

u $A = B = \mathbb{R}$; $f: \mathbb{R} \to \mathbb{R}$

$(\forall x \in \mathbb{R}) f(x) = x^2$

$J = \{1, 2\}$; $A_1 = [-1, 0]$; $A_2 = (0, 1)$

$A_1 \cap A_2 = \emptyset \Rightarrow f(A_1 \cap A_2) = f(\emptyset) = \emptyset$

$f(A_1) = [0, 1]$

$f(A_2) = (0, 1)$

$f(A_1) \cap f(A_2) = (0, 1) \subset \emptyset$

u $f \to inj$ $\Leftarrow$ u $(\forall J \neq \emptyset)(\forall (A_i)_{i \in J} \in \mathcal{P}(A)) f\left(\bigcap_{i \in J} A_i\right) = \bigcap_{i \in J} f(A_i)$

u $\subseteq$ u Fie $x, y \in A$ cu $f(x) = f(y)$. $J = \{1, 2\}$, $A_1 = \{x\}$, $A_2 = \{y\}$.

$f(\{x\} \cap \{y\}) = f(\{x\}) \cap f(\{y\}) = \{f(x)\} \cap \{f(y)\}$

$= \{f(x)\} \cap \{f(x)\} = \{f(x)\} \neq \emptyset \Rightarrow \{x\} \cap \{y\} \neq \emptyset$

$\subseteq \{x\}$ $\Rightarrow$