Exerc.

$T \to$ mulț., $T \neq \emptyset$, $A, B \in \mathcal{P}(T)$

$f : \mathcal{P}(T) \to \mathcal{P}(A) \times \mathcal{P}(B)$, $(\forall X \subseteq T)$ $f(X) := (X \cap A, X \cap B)$

$(\forall X \subseteq T)$ $\overline{X} := T \setminus X$

• Dem. că:

(a) $f \to$ inj. $\Leftrightarrow A \cup B = T$

(b) $f \to$ surj $\Leftrightarrow A \cap B = \emptyset$

(c) $f \to$ bij $\Leftrightarrow A = \overline{B} \Leftrightarrow B = \overline{A}$

Rez:

(a) "$\Leftarrow$": Fie $X, Y \in \mathcal{P}(T)$ a.î. $f(X) = f(Y) \Leftrightarrow$

$\Leftrightarrow (X \cap A, X \cap B) = (Y \cap A, Y \cap B)$

$\Leftrightarrow \begin{cases} X \cap A = Y \cap A \\ \text{și} \\ X \cap B = Y \cap B \end{cases} \Rightarrow (X \cap A) \cup (X \cap B) = (Y \cap A) \cup (Y \cap B) \Leftrightarrow$

$\Leftrightarrow X \cap (A \cup B) = Y \cap (A \cup B) \Leftrightarrow (A \cup B = T)$

$X \cap T = Y \cap T \Leftrightarrow X = Y \Rightarrow f \to$ inj.

"$\Rightarrow$" Pp absurd $\underbrace{A \cup B}_{\subseteq T} \neq T \Rightarrow A \cup B \subsetneq T \Leftrightarrow \exists \alpha \in T \setminus (A \cup B) \Leftrightarrow$

$A \cup B \neq T$

$\Leftrightarrow \alpha \in T$ și $\alpha \notin A \cup B \Leftrightarrow \alpha \in T$ și $\alpha \notin A$ și $\alpha \notin B$

$f(\{\alpha\}) = (\{\alpha\} \cap A, \{\alpha\} \cap B) = (\emptyset, \emptyset) = (\emptyset \cap A, \emptyset \cap B) = f(\emptyset)$

$\{\alpha\} \neq \emptyset$

abs cu $f \to$ inj.

(b) "$\Rightarrow$" Pp absurd $A \cap B \neq \emptyset \Leftrightarrow \exists \alpha \in A \cap B \Leftrightarrow \alpha \in A$

și $\alpha \in B$

$\{\alpha\} \in \mathcal{P}(A)$