

$$\Rightarrow \underbrace{\{x\} \cap \{y\} = \{x\}}_{\subseteq \{y\}} \Rightarrow \{x\} \subseteq \{y\} \Rightarrow x \in \{y\} \Rightarrow x = y$$

$$'\geq': \text{For } J \neq \emptyset, (A_i)_{i \in J} \in \mathcal{P}(A)$$

$$\text{Conform Definit (4)} \Rightarrow f\left(\bigcap_{i \in J} A_i\right) \subseteq \bigcap_{i \in J} f(A_i) \stackrel{(*)}{\neq}$$

$$\text{For } y \in \bigcap_{i \in J} f(A_i) \Leftrightarrow (\forall i \in J) y \in f(A_i) \Leftrightarrow (\forall i \in J) (\exists x_i \in A_i) y = f(x_i)$$

$$y = f(x_i)$$

$$\text{For } i_0 \in J \Rightarrow (\exists x_{i_0} \in A_{i_0}) y = f(x_{i_0})$$

$$\Rightarrow (\forall i \in J) f(x_i) = y = f(x_{i_0}) \xrightarrow{f \text{ surj}} (\forall i \in J) x_i \in A_i \Rightarrow x = x_{i_0} \Rightarrow$$

$$\Rightarrow (\exists i_0 \in J) x_{i_0} \in A_{i_0} \Rightarrow x_{i_0} \in \bigcap_{i \in J} A_i \Rightarrow f(x_{i_0}) \in f\left(\bigcap_{i \in J} A_i\right) \Leftrightarrow y \in f\left(\bigcap_{i \in J} A_i\right) \Rightarrow '\geq' \Rightarrow '='$$

$$\text{Lemma: } 1) \text{ T-multiplicative, } T \neq \emptyset; A, B \in \mathcal{P}(T),$$

$$\forall x \in \mathcal{P}(T) \quad \bar{x} := T \setminus x$$

$$f: \mathcal{P}(T) \rightarrow \mathcal{P}(A) \times \mathcal{P}(B), \quad \bar{f}(b) \in \mathcal{P}(B)$$

$$(\forall x \in \mathcal{P}(T)) f(x) = (\overbrace{x \cap A}^{\in \mathcal{P}(A)}, \overbrace{x \cap B}^{\in \mathcal{P}(B)}) \in \mathcal{P}(A) \times \mathcal{P}(B)$$

$$\text{from case: (a) } f \text{ surj} \text{ oblige } A \cup B = T$$

$$(b) f \text{ surj} \Leftrightarrow A \cap B = \emptyset$$

$$(c) f \text{ bij} \Leftrightarrow A = \bar{B} \Leftrightarrow B = \bar{A}$$