Fie $\alpha_1, \beta_1 \in \overline{0,e_1}, \dots, \alpha_k, \beta_k \in \overline{0,e_k}$

$$f(\alpha_1,\dots,\alpha_k) = f(\beta_1,\dots,\beta_k) \Leftrightarrow p_1^{\alpha_1}\cdots p_k^{\alpha_k} = p_1^{\beta_1}\cdots p_k^{\beta_k} \xrightarrow{\text{(dat unicității desc. canonice)}}$$

$$\Rightarrow \alpha_1 = \beta_1, \dots, \alpha_k = \beta_k \Leftrightarrow (\alpha_1,\dots,\alpha_k) = (\beta_1,\dots,\beta_k) \Rightarrow f \to \text{inj}$$

$$f(0,\dots,0) = p_1^0 \cdots p_k^0 = 1 \cdots 1 = 1 = \min(\Delta_n)$$

$$f(e_1,\dots,e_k) = p_1^{e_1} \cdots p_k^{e_k} = n = \max(\Delta_n)$$

Fie $\alpha_1, \beta_1 \in \overline{0,e_1}, \dots, \alpha_k, \beta_k \in \overline{0,e_k}$.

$$\text{cmmmc}\{f(\alpha_1,\dots,\alpha_k), f(\beta_1,\dots,\beta_k)\} = \text{cmmmc}\{p_1^{\alpha_1}\cdots p_k^{\alpha_k}, p_1^{\beta_1}\cdots p_k^{\beta_k}\} =$$

$$= p_1^{\max\{\alpha_1,\beta_1\}} \cdots p_k^{\max\{\alpha_k,\beta_k\}} = f(\max\{\alpha_1,\beta_1\},\dots,\max\{\alpha_k,\beta_k\}) =$$

$$= f((\alpha_1,\dots,\alpha_k) \underset{\substack{\downarrow \\ \text{disjuncția în } \prod_{i=1}^{k} L_{e_i+1}}}{\vee} (\beta_1,\dots,\beta_k))$$

Analog, $\text{cmmdc}\{f(\alpha_1,\dots,\alpha_k), f(\beta_1,\dots,\beta_k)\} = f(\min\{\alpha_1,\beta_1\},\dots,\min\{\alpha_k,\beta_k\}) =$

$$= f((\alpha_1,\dots,\alpha_k) \wedge (\beta_1,\dots,\beta_k))$$

$\Longrightarrow f \to$ izom. de lat. marg.