

$$\Rightarrow (\{\alpha\}, \emptyset) \in \mathcal{P}(A) \times \mathcal{P}(B) \xrightarrow{f \text{ surj}} (\exists x \in \mathcal{P}(T)) \underset{(x \cap A, x \cap B)}{\underset{\parallel}{f(x)}} = \{\{\alpha\}, \emptyset\} \Rightarrow$$

$$\Leftrightarrow \begin{cases} x \cap A = \{\alpha\} \Rightarrow \alpha \in x \cap A \subseteq x \Rightarrow \alpha \in x \\ x \cap B = \emptyset \end{cases} \not\Rightarrow \alpha \in x \cap B = \emptyset \Rightarrow \alpha \in \emptyset \Rightarrow$$

$$\Rightarrow A \cap B = \emptyset$$

$$\text{'suff'} \quad \forall A_1 \in \mathcal{P}(A), B_1 \in \mathcal{P}(B) \Rightarrow A_1 \cup B_1 \subseteq A \cup B \subseteq T \cup T = T$$

$$f(A_1 \cup B_1) = ((A_1 \cup B_1) \cap A, (A_1 \cup B_1) \cap B) = (A_1, B_1) \Rightarrow f \text{ surj}$$

$$(A_1 \cup B_1) \cap A = (\overset{\subseteq A}{A_1 \cap A}) \cup (\overset{\subseteq B \cap A = \emptyset}{B_1 \cap A}) = A_1 \cup \emptyset = A_1$$

$$(c) \quad f \rightarrow b_j' \Leftrightarrow \begin{cases} f \rightarrow m_j' \xrightarrow{(a)} \boxed{A \cup B = T} (*) \\ f \rightarrow surj' \xrightarrow{(b)} A \cap B = \emptyset \Leftrightarrow \boxed{A \subseteq \overline{B}} (***) \end{cases}$$

$$(*) \Rightarrow \underbrace{\overline{\overline{B}}}_{\subseteq T} = \overline{B} \cap T \overset{(**)}{=} \overline{B} \cap (A \cup B) = (A \cap \overline{B}) \cup (\overline{B} \cap \overline{B}) = \boxed{A \cap B}$$

$$\Rightarrow \overline{B} \subseteq A \xrightarrow{(***)} \boxed{A = \overline{B}} \Rightarrow \boxed{A \cup B = \overline{B} \cup B = A \cup B = T}$$

$$\boxed{f \rightarrow b_j'} \Leftrightarrow A \subseteq \overline{B} \wedge A = \overline{B} \Leftrightarrow \boxed{A = \overline{B}} \Leftrightarrow \overline{A} = \overline{\overline{B}} \Leftrightarrow \boxed{B = A}$$

Distributivität

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$\Downarrow \\ \forall n \in \mathbb{N} \quad A \cup (A_1 \cap A_2 \cap \dots \cap A_n) = (A \cup A_1) \cap \dots \cap (A \cup A_n)$$