$= \{(x_0, x_{n+1}) \mid x_0, x_{n+1} \in A, (\exists x_1, x_2, \dots, x_{n-1}, x_n \in A)\ (x_0,x_1), \dots, (x_{n-1}, x_n), (x_n, x_{n+1}) \in R\}$

Dem prin inducție matem. după $j \in N$, $j \ge k+1$, că $(\forall j \in N)$ $j \ge k+1 \Rightarrow R^j \subseteq \bigcup_{n=1}^{k} R^n$

$j = k+1$: Fie $x_0, x_{k+1} \in A$ a.î. $(x_0, x_{k+1}) \in R^{k+1} \Leftrightarrow$

$\Leftrightarrow (\exists x_1, \dots, x_k \in A)\ (x_0, x_1) \dots (x_k, x_{k+1}) \in R$

$$\left.\begin{array}{l}\{x_0, x_1, \dots, x_k\} \subseteq A \\ |A| = k;\ |\overline{0,k}| = k+1\end{array}\right\} \Rightarrow (\exists i_1, i_2 \in \overline{0,k})\ i_1 < i_2 \text{ și } x_{i_1} = x_{i_2}.$$

$\Rightarrow$ Drumul $(x_0, x_1), (x_1, x_2), \dots, (x_{i_1-1}, x_{i_1} = x_{i_2}), (x_{i_2} = x_{i_1}, x_{i_2+1}), (x_{i_2+1}, x_{i_2+2}), \dots, (x_k, x_{k+1})$ are lungimea $i_1 + k - i_2 + 1 =$

$$= \underbrace{k - \underbrace{(\overset{\le k}{i_2} - \overset{\ge 0}{i_1})}_{\ge 1} + 1}_{\ge k-k+1=1} \le k-1+1 = k$$

$\Rightarrow (x_0, x_{k+1}) \in \bigcup_{n=1}^{k} R^n \Rightarrow R^{k+1} \subseteq \bigcup_{n=1}^{k} R^n$

$j \ge k+1 \to j+1$: Fie $j \in N$, $j \ge k+1$

Pp. că $R^j \subseteq \bigcup_{n=1}^{k} R^n \ /\circ R \Rightarrow R^{j+1} \subseteq \left(\bigcup_{n=1}^{k} R^n\right) \circ R = \bigcup_{n=1}^{k} (R^n \circ R) =$

$$= \bigcup_{n=1}^{k} R^{n+1} = \bigcup_{n=2}^{k+1} R^n = \underbrace{\bigcup_{n=2}^{k} R^n}_{\subseteq \bigcup_{n=1}^{k} R^n} \cup \underbrace{R^{k+1}}_{\subseteq \bigcup_{n=1}^{k} R^n} \subseteq \bigcup_{n=1}^{k} R^n.$$

$$\Rightarrow \mathcal{T}(R) = \bigcup_{n=1}^{\infty} R^n = \left(\bigcup_{n=1}^{k} R^n\right) \supseteq \cup \underbrace{\left(\bigcup_{n=k+1}^{\infty} R^n\right)}_{\subseteq \bigcup_{n=1}^{k} R^n} = \bigcup_{n=1}^{k} R^n$$