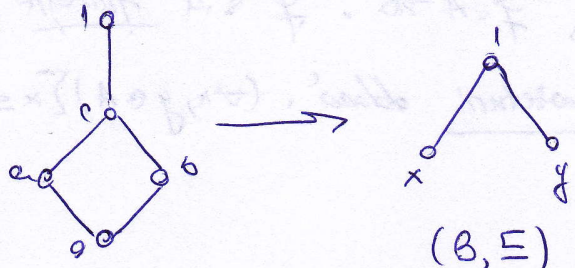
• Să se det toate func. izotone inj. și toate func izot. surj. între urm. poseturi:




$(A, \leq)$ $\quad$ $(B, \sqsubseteq)$

$|A| = 5 > 3 = |B| \Rightarrow$

$\Rightarrow \nexists$ fct injectivă $f: A \to B \Rightarrow$

$\Rightarrow \nexists$ fct izotonă inj $f: A \to B$

$0 = \min(A) \Leftrightarrow (\forall \alpha \in A)\ 0 \leq \alpha$

Fie $f: A \to B$, $f \nearrow$, surj $\Rightarrow (\forall \alpha \in A)\ f(0) \sqsubseteq f(\alpha)$ (*)

Caz 1: $f(0) = x \overset{(*)}{\Rightarrow} (\forall \alpha \in A)\ x \sqsubseteq f(\alpha)$, $x \neq y \Rightarrow (\nexists \alpha \in A)\ f(\alpha) = y \to$

$\to$ ✗ cu $f \to$ surj

Caz 2: $f(0) = y \overset{(*)}{\Rightarrow} (\forall \alpha \in A)\ y \sqsubseteq f(\alpha)$, $y \neq x \Rightarrow (\nexists \alpha \in A)\ f(\alpha) = x$ ✗ cu $f$ surj

Caz 3: $f(0) = 1 \overset{(*)}{\Rightarrow} (\forall \alpha \in A)\ 1 \sqsubseteq f(\alpha)$

$x \sqsubsetneq 1 \Leftrightarrow \begin{cases} x \neq 1 \to \text{✗} \Rightarrow 1 \not\sqsubseteq x \\ x \sqsubseteq 1 \overset{x \neq 1}{\Rightarrow} x = 1 \end{cases} \Rightarrow (\nexists \alpha \in A)\ f(\alpha) = x$ ✗ cu $f$ surj

Deci am avea $1 \sqsubseteq x$

$\to$ nu există fct $\nearrow$ surj $f: A \to B$

Fie: $(L, \leq) \to$ lant, $L \neq \emptyset$, $(M, \sqsubseteq) \to$ poset, $M \neq \emptyset$;

$f: L \to M$, $f \nearrow$, surj

Al. $\Rightarrow (M, \sqsubseteq) \Rightarrow$ lant

Fie $x, y \in M$, $f$ surj $\Rightarrow (\exists a, b \in L)\ f(a) = x$ și $f(b) = y$

$(L, \leq) \to$ lant $\Rightarrow a \leq b$ sau $b \leq a$, $f \nearrow \Rightarrow f(a) \sqsubseteq f(b)$ sau $f(b) \sqsubseteq f(a) \Rightarrow$

$\Rightarrow x \sqsubseteq y$ sau $y \sqsubseteq x \Leftrightarrow (M, \sqsubseteq) \to$ lant