<u>Obs</u>: $\mathcal{A} := (A, \vee, \wedge, \leq, \bar{\cdot}, 0, 1)$, $\mathcal{B} := (B, \vee, \wedge, \leq, \bar{\cdot}, 0, 1)$ } alg Boole

$f: A \to B \to$ morfism boolean

At. $\begin{cases} f^{-1}(\{0\}) \supseteq \{0\} \\ f^{-1}(\{1\}) \supseteq \{1\} \end{cases}$

<u>Dem</u>: $f(0) = 0 \Leftrightarrow 0 \in f^{-1}(\{0\}) \Leftrightarrow \{0\} \subseteq f^{-1}(\{0\})$

La fel pt. 1

<u>Exerc</u>: Fie $A, B \to$ alg Boole; $f: A \to B \to$ morfism boolean.

Să dem. că urm. afirmații sunt echivalente:

(1) $f \to$ injectiv

(2) $f^{-1}(\{0\}) = \{0\}$

(3) $f^{-1}(\{1\}) = \{1\}$

<u>Rez</u> $\underset{\text{"}}{(1) \Rightarrow (3)}$: $f \to$ injectiv, $f(1) = 1$

Fie $x \in f^{-1}(\{1\}) \Leftrightarrow f(x) = 1 \Rightarrow f(x) = f(1) \Rightarrow x = 1 \Leftrightarrow x \in \{1\}$

$\Rightarrow f^{-1}(\{1\}) \subseteq \{1\} \Leftrightarrow f^{-1}(\{1\}) = \{1\}$

$\underset{\text{"}}{(3) \Rightarrow (1)}$: Fie $x, y \in A$, cu $f(x) = f(y) \Leftrightarrow \underbrace{f(x) \leftrightarrow f(y)}_{= f(x \leftrightarrow y)} = 1$

$\Leftrightarrow f(x \leftrightarrow y) = 1 \Leftrightarrow x \leftrightarrow y \in \underbrace{f^{-1}(\{1\})}_{= \{1\}} \Leftrightarrow x \leftrightarrow y \in \{1\} \Leftrightarrow x \leftrightarrow y = 1 \Leftrightarrow$

$\Leftrightarrow x = y \Rightarrow f \to$ inj!

$\underset{\text{"}}{(1) \Leftrightarrow (2)}$: Rezultă din $[(1) \Leftrightarrow (3)]$ prin dualitate

<u>Altfel</u>: $\underset{\text{"}}{(2) \Leftrightarrow (3)}$

$f^{-1}(\{1\}) = \{1\} \Leftrightarrow (\forall x \in A)[x \in f^{-1}(\{1\}) \Leftrightarrow x \in \{1\}] \Leftrightarrow (\forall x \in A)[f(x) = 1 \Leftrightarrow$

$\Leftrightarrow x = 1]$

$f(x) = 1 \Leftrightarrow f(\bar{x}) = 0$