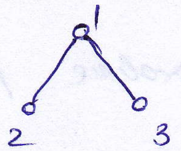
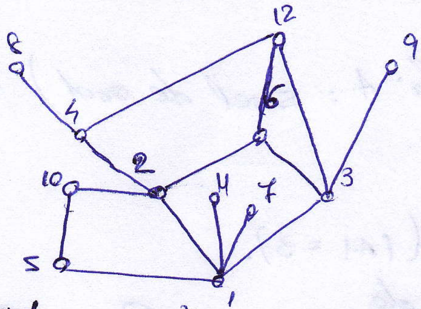
$(\{1,2,3\}, \vdots) \longrightarrow$ 

(ce divide)

$(\{\overline{1,12}\}, |)$ 

$(\forall A \to \text{mulţ})(\forall R \subseteq A^2)$

$\mathcal{R}(R) = \Delta_A \cup R$

$\mathcal{S}(R) = R \cup R^{-1}$

$\mathcal{T}(R) = \bigcup\limits_{n \geq 1}^{\infty} R^n$

<u>Exerc</u>: $A \to$ mulţ, $R \subseteq A^2$.

(1) $\mathcal{R}(\mathcal{S}(R)) \underset{?}{=} \mathcal{S}(\mathcal{R}(R))$

(2) $\mathcal{R}(\mathcal{T}(R)) \underset{?}{=} \mathcal{T}(\mathcal{R}(R))$

(3) $\mathcal{S}(\mathcal{T}(R))$ şi $\mathcal{T}(\mathcal{S}(R))$ nu sunt neapărat egale

(1) $\mathcal{R}(\mathcal{S}(R)) = \Delta_A \cup (\mathcal{S}(R)) = \Delta_A \cup R \cup R^{-1}$

$\mathcal{S}(\mathcal{R}(R)) = \mathcal{R}(R) \cup (\mathcal{R}(R))^{-1} = \Delta_A \cup R \cup (\Delta_A \cup R)^{-1} = \Delta_A \cup R \cup \overset{\Delta_A}{\overset{\shortparallel}{\Delta_A^{-1}}} \cup R^{-1} =$

$= \Delta_A \cup R \cup R^{-1} = \mathcal{R}(\mathcal{S}(R))$

(2) $\mathcal{R}(\mathcal{T}(R)) = \overset{R^0}{\overset{\shortparallel}{\Delta_A}} \cup (\mathcal{T}(R)) = \Delta_A \cup \bigcup\limits_{n \geq 1}^{\infty} R^n = \bigcup\limits_{n \geq 0}^{\infty} R^n$

$\mathcal{T}(\mathcal{R}(R)) = \bigcup\limits_{n \geq 1}^{\infty} (\mathcal{R}(R))^n = \bigcup\limits_{n \geq 1}^{\infty} (\Delta_A \cup R)^n = \bigcup\limits_{n \geq 1}^{\infty} (R^0 \cup R^1)^n$

Dem. prin inducţie după $n \in \mathbb{N}^*$ că $(\forall n \in \mathbb{N}^*)\ (R^0 \cup R^1)^n =$

$= R^0 \cup R^1 \cup R^2 \cup \ldots \cup R^n$

<u>E.I.</u>

$n = 1$ $(R^0 \cup R^1)^n = R \cup R^1$

<u>E.I.</u> $n \to n+1$ Fie $n \in \mathbb{N}$ arb, fixat

P.P. că $(R^0 \cup R^1)^n = R^0 \cup R^1 \cup \ldots \cup R^n$