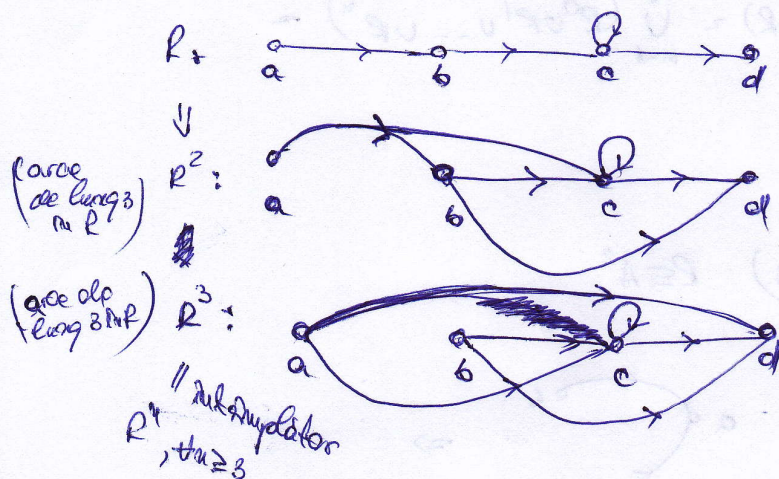


$$(\forall x, y, z \in \mathbb{N}) \quad x/y \wedge y/z \Rightarrow x/z \Rightarrow R \neq P \Rightarrow \text{transitiv} \Rightarrow P(R) = R = I \Rightarrow \\ \Rightarrow P(P(R)) = P(I) = \{ \emptyset \} \neq I \neq P(P(R)) \Rightarrow P(P(R)) \neq \mathbb{N}^2 = P(P(R))$$

$$A = \{a, b, c, d\} \quad (|A| = 4) \quad ; \quad R \subseteq A^2$$



Exerc. \$A \rightarrow mult, |A| = K \in \mathbb{N}^+\$;  
\$R \subseteq A^2\$

dem. ca \$P(R) = \bigcup\_{n=1}^{\infty} R^n\$

Rez: \$P(R) = \bigcup\_{n=1}^{\infty} R^n\$

dem. ca \$(\forall p \in \mathbb{N}) \quad p \geq K \Rightarrow R^p \subseteq \bigcup\_{n=1}^K R^n\$

dem. prin inductie matca alupa \$u \in \mathbb{N}^+\$ ca \$(\forall u \in \mathbb{N}^+)\$ este adevarata

\$P(u): R^u = \{ (x\_0, x\_u) \mid x\_0, x\_u \in A, (\exists x\_1, x\_2, \dots, x\_{u-1} \in A) \text{ ca } (x\_0, x\_1), (x\_1, x\_2), \dots, (x\_{u-1}, x\_u) \in R \}\$

\$u=1\$: \$R^1 = R = \{ (x\_0, x\_1) \mid x\_0, x\_1 \in A, (x\_0, x\_1) \in R \}\$

\$u \Rightarrow u+1\$: For \$u \in \mathbb{N}^+\$ arb, frat.

\$P\_p\$ \$P(u)\$ adev.

\$R^{u+1} = R^u \circ R = R \circ R^u = \{ (x\_0, x\_{u+1}) \mid x\_0, x\_{u+1} \in A, (\exists x\_u \in A) (x\_0, x\_u) \in R^u\$

\$\wedge (x\_u, x\_{u+1}) \in R \} \stackrel{(P(u))}{=} \{ (x\_0, x\_{u+1}) \mid x\_0, x\_{u+1} \in A, (\exists x\_u \in A) ( \exists x\_1, \dots, x\_{u-1} \in A) (x\_0, x\_1), \dots, (x\_{u-1}, x\_u) \in R \wedge (x\_u, x\_{u+1}) \in R =