(2) $x \le y \Leftrightarrow x \vee y = y \overset{(1)}{\Leftrightarrow} \overline{x \vee y} = \bar{y} \Leftrightarrow \bar{x} \wedge \bar{y} = \bar{y} \Leftrightarrow \bar{y} \le \bar{x}$

(3) $x \le y \overset{\text{lema}}{\Longrightarrow} \cancel{x \vee y = y}\ x \wedge \bar{y} \le y \wedge \bar{y} = 0 = \min(B) \Rightarrow x \wedge \bar{y} = 0 \Longrightarrow$

$$\left.\begin{array}{l} \Rightarrow \underbrace{(x \wedge \bar{y}) \vee y}_{\| \ (x \vee y) \wedge (\underbrace{\bar{y} \vee y}_{= 1})} = 0 \vee y = y \\ (x \vee y) \wedge (\bar{y} \vee y) = x \vee y \end{array}\right) \Rightarrow x \vee y = y \Leftrightarrow x \le y$$

Așadar $x \le y \Leftrightarrow x \wedge \bar{y} = 0 \overset{(1)}{\Leftrightarrow} \overline{x \wedge \bar{y}} = \bar{0} \Leftrightarrow \bar{x} \vee \bar{\bar{y}} = 1 \Leftrightarrow \bar{x} \vee y = 1$

(4) $x \le y \overset{(3)}{\Leftrightarrow} \underbrace{\bar{x} \vee y}_{\|\text{def} \ x \to y} = 1 \Leftrightarrow x \to y = 1$

(5) $x = y \Leftrightarrow \begin{cases} x \le y \\ \text{și} \\ y \le x \end{cases} \overset{(4)}{\Leftrightarrow} \begin{cases} x \to y = 1 \\ \text{și} \\ y \to x = 1 \end{cases} \Leftrightarrow \underbrace{(x \to y) \wedge (y \to x)}_{\|\text{def } 7 \ x \leftrightarrow y} = 1 \wedge 1 = 1 \Leftrightarrow$

$\Leftrightarrow x \leftrightarrow y = 1$

(6) $x \wedge y \le z \overset{\text{lema}}{\Rightarrow} (x \wedge y) \vee \bar{y} \le z \vee \bar{y} \Rightarrow \bar{y} \vee z = y \to z$

$(x \wedge y) \vee \bar{y} = (x \vee \bar{y}) \wedge (\underbrace{y \vee \bar{y}}_{=1}) = x \vee \bar{y} \ge x$

$\overset{\text{tranz.}}{\Longrightarrow} x \le y \to z$

$x \le \underbrace{y \to z}_{\| \ \bar{y} \vee z} \overset{\text{lema}}{\Longrightarrow} x \wedge y \le (\bar{y} \vee z) \wedge y = (\underbrace{\bar{y} \wedge y}_{=0}) \vee (z \wedge y) = z \wedge y \le z \overset{\text{tranz.}}{\Longrightarrow}$

$\Rightarrow x \wedge y \le z$

Exerc: $(B, \vee, \wedge, \le, \bar{\cdot}, 0, 1) \to$ alg Boole completă (i.e. alg Boole care este latice completă). $(\forall (a_i)_{i \in I} \subseteq B)$ $(\forall I \to$ mult$)$ not.:

$$\begin{cases} \bigvee_{i \in I} a_i := \sup\{a_i \mid i \in I\} \\ \bigwedge_{i \in I} a_i := \inf\{a_i \mid i \in I\} \end{cases}$$

At.: $(\forall I, J \to$ mult$)$ $(\forall (a_i)_{i \in I} \subseteq B)$ $(\forall (b_j)_{j \in J} \subseteq B)$

$$\left(\bigvee_{i \in I} a_i\right) \wedge \left(\bigvee_{j \in J} b_j\right) = \bigvee_{i \in I} \bigvee_{j \in J} (a_i \wedge b_j) = \bigvee_{j \in J} \bigvee_{i \in I} (a_i \wedge b_j) = \bigvee_{(i,j) \in I \times J} (a_i \wedge b_j) \quad (\circ)$$