5.2.4. Utilizând metoda tablelor semantice, verificați dacă are loc:

2. $\vdash (\exists x)(\forall y)p(x,y) \longleftrightarrow (\forall y)(\exists x)\,p(x,y)$

$(A \longleftrightarrow B)$ $\quad (A \to B) \wedge (B \to A)$

$\neg\Big(\big((\exists x)(\forall y)p(x,y) \to (\forall y)(\exists x)p(x,y)\big) \wedge \big((\forall y)(\exists x)p(x,y) \longrightarrow (\exists x)(\forall y)p(x,y)\big)\Big)\ (1)$

$\beta(1)$

$\neg\big((\exists x)(\forall y)p(x,y) \to (\forall y)(\exists x)p(x,y)\big)\,(2)$

$\neg\big((\forall y)(\exists x)p(x,y) \to (\exists x)(\forall y)p(x,y)\big)\,(3),$

$\alpha(3)$

$(\forall y)(\exists x)\,p(x,y)\ (4)$

$\neg\big((\exists x)(\forall y)p(x,y)\big)\ (5)$

$\gamma(4)$, a - const. implicit

$(\exists x)\,p(x,a)\ (6)$ ✓

$(\forall y)(\exists x)\,p(x,y)\ (4')$

$\delta(6)$, b - const. nouă

$p(b,a)$

$\gamma(5)$, a, b - const. existente

$\neg(\forall y)p(a,y)\ (7)$ ✓

$\neg(\forall y)\,p(b,y)\ (8)$ ✓

$\neg\big((\exists x)(\forall y)p(x,y)\big)\ (5')$

$\delta(7)$, c - const. nouă

$\neg p(a,c)$

$\delta(8)$, d - const. nouă

$\neg p(b,d)$

$\gamma(5')$, c, d - const. exist.

$\neg(\forall y)\,p(c,y)\ (9)$

$\neg(\forall y)p(d,y)\ (10)$ ✓

$\neg(\exists x)(\forall y)p(x,y)\ (5'')$

$\delta(9)$, e - const. nouă

$\neg p(c,e)$

$\delta(10)$, f - const. nouă

$\neg p(d,f)$

$\Rightarrow$ avem ciclu infinit și nu putem decide tipul formulei

5.2.8.2. Sunt unificabili atomii din perechile urm.? Dacă da, identificați cel mai general unificator

$A_1 = P(a, x, f(g(y)))$
$A_2 = P(y, f(z), f(z))$

P1: Același simbol pred? DA (P)
P2: Aceeași aritate DA (3)
P3: $\neq$
I chiar $\neq$ subst. (var)

$\lambda_1 = [x \leftarrow a]$

$\lambda_1(A_1) = P(a, x, f(g(a)))$

$\lambda_1(A_2) = P(a, f(z), f(z))$

$\lambda_2 = [x \leftarrow f(z)]$

$\lambda_2(\lambda_1(A_1)) = P(a, f(z), f(g(a)))$

$\lambda_2(\lambda_1(A_2)) = P(a, f(z), f(z))$

$\lambda_3 = [z \leftarrow g(a)]$

$\lambda_3(\lambda_2(\lambda_1(A_1))) = P(a, f(g(a)), f(g(a)))$

$||$

$\lambda_3(\lambda_2(\lambda_1(A_2))) = P(a, f(g(a)), f(g(a)))$

$\Downarrow$

A_1, A_2 unificabile,

$mgu(A_1, A_2) = [y \leftarrow a, x \leftarrow f(g(a)), z \leftarrow g(a)]$

$A_3 = P(x, g(f(a)), f(b))$
$A_4 = P(f(y), z, z)$

P_1: DA, P
P_2: DA, 3
P_3: $\theta_1 = [x \leftarrow f(y)]$
$\theta_1(A_3) = P(f(y), g(f(a)), f(b))$
$\theta_1(A_4) = P(f(y), z, z)$
$\theta_2 = [z \leftarrow g(f(a))]$
$\theta_2(\theta_1(A_3)) = P(f(y), g(f(a)), f(b))$
$\theta_2(\theta_1(A_4)) = P(f(y), g(f(a)), g(f(a)))$
$f \neq g$ – nu ∃ substituție între două simboluri de funcție
$\Rightarrow A_3, A_4$ – nu sunt unificabile

$A_5 = P(a, x, f(g(y)))$
$A_6 = P(z, h(z, u), f(b), z)$

P_1: DA (P)
P_2: $3 \neq 4 \Rightarrow A_5$ și A_6 nu sunt unificabile

9.2.16 Verificați dacă următoarele formule sunt teoreme utilizând
2. rezoluția generală.

$U = (\forall x)(\forall y)\,(P(x,y) \to P(y,x)) \to (\forall x)P(x,x)$

Nd. Res. e primară.

$\neg U \equiv \neg\left((\forall x)(\forall y)\,(P(x,y) \to P(y,x)) \to (\forall x)P(x,x)\right)$

P1 $\not\to$

$\neg U \equiv \neg\left((\forall x)(\forall y)(\neg P(x,y) \vee P(y,x)) \vee \underline{(\forall x)P(x,x)}\right)$

P2 DM $\neg U \equiv (\forall x)(\forall y)(\neg P(x,y) \vee P(y,x)) \wedge (\exists x)\neg P(x,x)$

P3 Reden. var. legate a. î. să fie ≠

$\neg U \equiv (\forall x)(\forall y)(\neg P(x,y) \vee P(y,x)) \wedge (\exists z)\neg P(z,z)$

P4 $\exists\forall$

$\neg U \equiv (\neg U)^P \equiv (\exists z)(\forall x)(\forall y)\left((\neg P(x,y) \vee P(y,x)) \wedge \neg P(z,z)\right)$

P5 $\cancel{\exists}$ $z \leftarrow a$

$(\neg U)^S \equiv (\forall x)(\forall y)\left((\neg P(x,y) \vee P(y,x)) \wedge \neg P(a,a)\right)$

P6 $\cancel{\forall}$

$(\neg U)^{SZ} \equiv (\neg P(x,y) \vee P(y,x)) \wedge \neg P(a,a) = (\neg U)^C$

P7 —

$\Rightarrow S = \{\underbrace{\neg P(x,y) \vee P(y,x)}_{C_1},\ \underbrace{\neg P(a,a)}_{C_2}\}$

$C_1 = \neg P(x,y) \vee \underline{P}(y,x)$

$C_2 = \underline{\neg P}(a,a)$

$\xrightarrow{\text{Res},\ [y \leftarrow a,\ x \leftarrow a]} C_3 = \neg P(a,a) = C_2 \Rightarrow S =$ consistentă

$\Rightarrow U$ nu este teoremă