Partea II, pt. prob. intro II

I. m Dom. ∞ pt. $\mathcal{U} \stackrel{not.}{=} (\exists x) P(x) \vee (\exists x) Q(x) \rightarrow (\exists x)(P(x) \vee Q(x))$

$$\mathfrak{I}_2 = \langle D_2, m_2 \rangle$$

$$D_2 = \mathbb{R}$$

$m_2(P) : \mathbb{R} \rightarrow \{T, F\}$, $m_2(P)(x) =$ "$x > 0$"

$m_2(Q) : \mathbb{R} \rightarrow \{T, F\}$, $m_2(Q)(x) =$ "$[x] \% 2 == 0$"

$$V^{\mathfrak{I}_2}(\mathcal{U}) = V^{\mathfrak{I}_2}\big((\exists x) P(x) \vee (\exists x) Q(x) \rightarrow (\exists x)(P(x) \vee Q(x))\big) =$$

$$= V^{\mathfrak{I}_2}\big((\exists x) P(x) \vee (\exists x) Q(x)\big) \rightarrow V^{\mathfrak{I}_2}\big((\exists x)(P(x) \vee Q(x))\big) =$$

$$= V^{\mathfrak{I}_2}\big((\exists x) P(x)\big) \vee V^{\mathfrak{I}_2}\big((\exists x) Q(x)\big) \rightarrow V^{\mathfrak{I}_2}\big((\exists x)(P(x) \vee Q(x))\big) =$$

$=$ "$\exists x \in \mathbb{R}, x > 0$" $\vee$ "$\exists x \in \mathbb{R}, [x] \% 2 == 0$" $\rightarrow$

"$\exists x \in \mathbb{R}$ a.î. $x > 0$ sau $[x] \% 2 == 0$" $=$

$= T \vee T \rightarrow T = T \rightarrow T = T \Rightarrow \mathfrak{I}_2$ este model (și $\mathcal{U}$ este consistentă)

9.2 7.nb2

Aduceți la o formă normală prenexă și la o formă normală clauzală

2) $(\exists x)(\forall y)((\exists z)P(z) \wedge (\exists w)(Q(x,u) \longrightarrow (\exists z)Q(y,z)))$

Pas 1. $\rightarrow \Rightarrow \vee$

$(\exists x)(\forall y)((\exists z)P(z) \wedge (\exists u)(\neg Q(x,u) \vee \underline{(\exists z)} Q(y,\underline{z})))$

Pas 2. Legile lui De Morgan: Nu e cazul

Pas 3. Redenumirea variabilelor legate astfel să fie distincte

$(\exists x)(\forall y)(\exists z)P(z) \wedge (\exists u)(\neg Q(x,u) \vee (\exists w)Q(y,w)))$

Pas 4. Extragerea cuantificatorilor în fața formulei:

$(\exists x)(\forall y)(\exists z)(\exists u)(\exists w)(P(z) \wedge (\neg Q(x,u) \vee Q(y,w)))$

O Formă prenexă

Pas 5. Skolemizare Eliminare cuant. $\exists$ prin substit. var legate $\exists$ cu fct. de var legate universal anterioare

$x \leftarrow c$

$z \leftarrow f(y)$

$u \leftarrow g(y)$

$w \leftarrow h(y)$

$(\forall y)(P(f(y)) \wedge (\neg Q(c,g(y)) \vee Q(y,h(y))))$

Forma Skolem

Pas 6. Eliminăm cuant. universali

$P(f(y)) \wedge (\neg Q(c,g(y)) \vee Q(y,h(y)))$

Pas 7. Aducerea la forma clauzală

Nu e cazu

9.2.6.2. Construiți toate formele normale Prenexe, Skolem și Clauzale:

$$\mathcal{U} \stackrel{\text{not}}{=} (\exists x)\left((\exists y)P(y) \to \neg(\forall y)(Q(y) \to R(x))\right)$$

Pas 1: $\not\to$ $(A \to B \equiv \neg A \vee B)$

$$\mathcal{U} \equiv (\exists x)(\neg(\exists y)P(y) \vee \neg(\forall y)(\neg Q(y) \vee R(x)))$$

Pas 2: De Morgan

$$\mathcal{U} \equiv (\exists x)((\forall y)\neg P(y) \vee (\exists y)(Q(y) \wedge \neg R(x)))$$

Pas 3: Redenumirea var. legate a.î. să fie distincte

$$\mathcal{U} \equiv (\exists x)((\forall y)\neg P(y) \vee (\exists z)(Q(z) \wedge \neg R(x)))$$

Pas 4: Extragerea cuantif. în fața formulei

$$\mathcal{U} \equiv \mathcal{U}^{P_1} = (\exists x)(\forall y)(\exists z)(\neg P(y) \vee (Q(z) \wedge \neg R(x)))$$

$$\mathcal{U} \equiv \mathcal{U}^{P_2} = (\exists x)(\exists z)(\forall y)(\neg P(y) \vee (Q(z) \wedge \neg R(x)))$$

Pas 5: $\not\exists$ var. legate $\exists \longleftarrow$ fct (var leg $\forall$ anterioare)

$x \leftarrow a,\ z \leftarrow f(y)$ $\quad \mathcal{U} \not\equiv \mathcal{U}^{S_1} = (\forall y)(\neg P(y) \vee (Q(f(y)) \wedge \neg R(a)))$

$x \leftarrow b,\ z \leftarrow c$ $\quad \mathcal{U} \not\equiv \mathcal{U}^{S_2} = (\forall y)(\neg P(y) \vee (Q(c) \wedge \neg R(b)))$

Pas 6: $\not\forall$ ([illegible])

$$\mathcal{U} \not\equiv \mathcal{U}^{S_{21}} = \neg P(y) \vee (Q(f(y)) \wedge \neg R(a))$$

$$\mathcal{U} \not\equiv \mathcal{U}^{S_{22}} = \neg P(y) \vee (Q(c) \wedge \neg R(b))$$

Pas 7: aducerea la forma clauzală (asoc., distr. $\vee$ față de $\wedge$, ...)

$$\mathcal{U} \not\equiv \mathcal{U}^{C_1} = (\neg P(y) \vee Q(f(y))) \wedge (\neg P(y) \vee \neg R(a))$$

$$\mathcal{U} \not\equiv \mathcal{U}^{C_2} = (\neg P(y) \vee Q(c)) \wedge (\neg P(y) \vee \neg R(b))$$

9.2 13/2

Utilizând metoda tabelelor semantice, dem. distributivitatea "$\forall$" față de "$\wedge$"

$$\vdash (\forall x)P(x)\wedge(\forall x)Q(x) \leftrightarrow (\forall x)(P(x)\wedge Q(x)) \stackrel{not}{=} U$$

$$\neg\Big(\big((\forall x)P(x)\wedge(\forall x)Q(x)\to(\forall x)(P(x)\wedge Q(x))\big)\wedge\big((\forall x)(P(x)\wedge Q(x))\to(\forall x)P(x)\wedge(\forall x)Q(x)\big)\Big)\ (1)\ \checkmark$$

β pt (1)

Ramura stângă:

- $\neg\big((\forall x)P(x)\wedge(\forall x)Q(x)\to(\forall x)(P(x)\wedge Q(x))\big)$ (2) ✓
- α pt (2)
- $(\forall x)P(x)\wedge(\forall x)Q(x)$ (4) ✓
- $\neg\big((\forall x)(P(x)\wedge Q(x))\big)$ (5) ✓
- α pt (4)
- $(\forall x)P(x)$ (6) ✓
- $(\forall x)Q(x)$ (7) ✓
- δ pt 5, a const. nouă
- $\neg(P(a)\wedge Q(a))$ (8) ✓
- γ pt (6), a const. existentă
- $P(a)$
- $(\forall x)P(x)$ (6')
- γ pt (7), a const. existentă
- $Q(a)$
- $(\forall x)Q(x)$ (7')
- β pt (8)
- $\neg P(a)$ ⊗ $\quad|\quad$ $\neg Q(a)$ ⊗

Ramura dreaptă:

- $\neg\big((\forall x)(P(x)\wedge Q(x))\to(\forall x)P(x)\wedge(\forall x)Q(x)\big)$ (3) ✓
- α pt (3)
- $(\forall x)(P(x)\wedge Q(x))$ (9) ✓
- $\neg\big((\forall x)P(x)\wedge(\forall x)Q(x)\big)$ (10) ✓
- β pt (10)

$\neg((\forall x)P(x))$ (11) ✓	$\neg((\forall x)Q(x))$ (12) ✓
δ (11) b const. nouă	δ (12) c const. nouă
$\neg P(b)$	$\neg Q(c)$
γ (9) b const. existentă	γ (9) c const. ex.
$P(b)\wedge Q(b)$ (13) ✓	$P(c)\wedge Q(c)$ (14) ✓
$(\forall x)(P(x)\wedge Q(x))$ (9')	$(\forall x)(P(x)\wedge Q(x))$ (9'')
α pt (13)	α pt (14)
$P(b)$	$P(c)$
$Q(b)$	$Q(c)$
⊗	⊗

$\Rightarrow$ tabela sem. închisă $\xRightarrow{TCC}$ U tautologie

$\Rightarrow$ teoremă