

Large Minimum Separation

May 10, 2019

1 Assumptions

1. Orthogonal signs, equal to standard basis vectors ($\mathbf{V} = \mathbf{I}$)
2. $m = s$
3. $\Delta \approx (1 + C)\lambda_c$.

2 Setup

The solution is:

$$(\mathbf{K}_r)_{ij} = K^{(r)}(t_i - t_j) \quad (1)$$

$$(\mathbf{k}_r(t))_i = K^{(r)}(t - t_i) \quad (2)$$

$$\beta = \frac{\text{diag}(\mathbf{K}_0^{-1} \mathbf{K}_1)}{\text{diag}(\mathbf{K}_2) - \text{diag}(\mathbf{K}_1 \mathbf{K}_0^{-1} \mathbf{K}_1)}, \quad (3)$$

$$\mathbf{q}(t) = \mathbf{K}_0^{-1} \mathbf{k}_0(t) + \mathbf{D}_\beta \left[\mathbf{k}_1(t) - \mathbf{K}_1 \mathbf{K}_0^{-1} \mathbf{k}_0(t) \right] \quad (4)$$

The norm is described by:

$$N(t) = \|\mathbf{q}(t)\|_2^2, \quad (5)$$

$$N'(t) = 2\text{Re}(\langle \mathbf{q}(t), \mathbf{q}'(t) \rangle), \quad (6)$$

$$N''(t) = 2 \left(\text{Re}(\langle \mathbf{q}(t), \mathbf{q}''(t) \rangle) + \|\mathbf{q}'(t)\|_2^2 \right). \quad (7)$$

3 t Near t_i , Negative Term of $N''(t)$

Main contribution:

$$\langle \mathbf{K}_0^{-1} \mathbf{k}_0(t), \mathbf{K}_0^{-1} \mathbf{k}_2(t) \rangle = \mathbf{k}_0(t)^\top \mathbf{K}_0^{-2} \mathbf{k}_2(t).$$

4 t Near t_i , Positive Term $N''(t)$

$$\begin{aligned}\|\mathbf{q}'(t)\|_2^2 &= \left\| \mathbf{K}_0^{-1} \mathbf{k}_1(t) + \mathbf{D}_\beta \left[\mathbf{k}_2(t) - \mathbf{K}_1 \mathbf{K}_0^{-1} \mathbf{k}_1(t) \right] \right\|_2^2 \\ &\leq \left(\|\mathbf{K}_0^{-1}\| \cdot \|\mathbf{k}_1(t)\|_2 + \|\mathbf{D}_\beta\| \cdot \|\mathbf{k}_2(t)\|_2 + \|\mathbf{D}_\beta\| \cdot \|\mathbf{K}_1\| \cdot \|\mathbf{K}_0^{-1}\| \cdot \|\mathbf{k}_1(t)\|_2 \right)^2\end{aligned}$$

(Writing $\|\mathbf{X}\|$ for operator norm.) Will control operator norms by:

$$\begin{aligned}\|\mathbf{D}_\beta\| &= \max_{i \in [s]} |\beta_i| \\ \|\mathbf{X}\| &\leq \|\mathbf{X}\|_\infty = \max_{i \in [s]} \sum_{j=1}^s |X_{ij}| \\ \|\mathbf{X}^{-1}\|_\infty &\leq \frac{1}{1 - \|\mathbf{I} - \mathbf{X}\|_\infty}\end{aligned}$$

5 t Away from Support

$$\begin{aligned}\|\mathbf{q}(t)\|_2^2 &= \left\| \mathbf{K}_0^{-1} \mathbf{k}_0(t) + \mathbf{D}_\beta \left[\mathbf{k}_1(t) - \mathbf{K}_1 \mathbf{K}_0^{-1} \mathbf{k}_0(t) \right] \right\|_2^2 \\ &\leq \left(\|\mathbf{K}_0^{-1}\| \cdot \|\mathbf{k}_0(t)\|_2 + \|\mathbf{D}_\beta\| \cdot \|\mathbf{k}_1(t)\|_2 + \|\mathbf{D}_\beta\| \cdot \|\mathbf{K}_1\| \cdot \|\mathbf{K}_0^{-1}\| \cdot \|\mathbf{k}_0(t)\|_2 \right)^2\end{aligned}$$