Large Minimum Separation

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1 Assumptions

1. Orthogonal signs, equal to standard basis vectors ($\boldsymbol{V} = \boldsymbol{I}$)
2. $m = s$
3. $\Delta \approx (1 + C)\lambda_c$.

2 Setup

The solution is:

$$(\boldsymbol{K}_r)_{ij} = K^{(r)}(t_i - t_j) \tag{1}$$
$$(\boldsymbol{k}_r(t))_i = K^{(r)}(t - t_i) \tag{2}$$
$$\boldsymbol{\beta} = \frac{\mathsf{diag}(\boldsymbol{K}_0^{-1}\boldsymbol{K}_1)}{\mathsf{diag}(\boldsymbol{K}_2) - \mathsf{diag}(\boldsymbol{K}_1\boldsymbol{K}_0^{-1}\boldsymbol{K}_1)}, \tag{3}$$
$$\boldsymbol{q}(t) = \boldsymbol{K}_0^{-1}\boldsymbol{k}_0(t) + \boldsymbol{D}_{\boldsymbol{\beta}}\Big[\boldsymbol{k}_1(t) - \boldsymbol{K}_1\boldsymbol{K}_0^{-1}\boldsymbol{k}_0(t)\Big] \tag{4}$$

The norm is described by:

$$N(t) = \|\boldsymbol{q}(t)\|_2^2, \tag{5}$$
$$N'(t) = 2\mathrm{Re}(\langle \boldsymbol{q}(t), \boldsymbol{q}'(t)\rangle), \tag{6}$$
$$N''(t) = 2\left(\mathrm{Re}(\langle \boldsymbol{q}(t), \boldsymbol{q}''(t)\rangle) + \|\boldsymbol{q}'(t)\|_2^2\right). \tag{7}$$

3 t Near t_i, Derivative Term

$$\begin{aligned}
\|\boldsymbol{q}'(t)\|_2^2 &= \left\|\boldsymbol{K}_0^{-1}\boldsymbol{k}_1(t) + \boldsymbol{D}_{\boldsymbol{\beta}}\Big[\boldsymbol{k}_2(t) - \boldsymbol{K}_1\boldsymbol{K}_0^{-1}\boldsymbol{k}_1(t)\Big]\right\|_2^2 \\
&\leq \Big(\|\boldsymbol{K}_0^{-1}\| \cdot \|\boldsymbol{k}_1(t)\|_2 + \|\boldsymbol{D}_{\boldsymbol{\beta}}\| \cdot \|\boldsymbol{k}_2(t)\|_2 + \|\boldsymbol{D}_{\boldsymbol{\beta}}\| \cdot \|\boldsymbol{K}_1\| \cdot \|\boldsymbol{K}_0^{-1}\| \cdot \|\boldsymbol{k}_1(t)\|_2\Big)^2
\end{aligned}$$

(Writing $\|\boldsymbol{X}\|$ for operator norm.) Will control operator norms by:

$$\|\boldsymbol{D}_{\boldsymbol{\beta}}\| = \max_{i\in[s]} |\beta_i|$$

$$\|\boldsymbol{X}\| \leq \|\boldsymbol{X}\|_\infty = \max_{i\in[s]} \sum_{j=1}^{s} |X_{ij}|$$

4 t Away from Support

$$\begin{aligned}\|\boldsymbol{q}(t)\|_2^2 &= \left\|\boldsymbol{K}_0^{-1}\boldsymbol{k}_0(t) + \boldsymbol{D}_{\boldsymbol{\beta}}\left[\boldsymbol{k}_1(t) - \boldsymbol{K}_1\boldsymbol{K}_0^{-1}\boldsymbol{k}_0(t)\right]\right\|_2^2 \\ &\leq \left(\|\boldsymbol{K}_0^{-1}\| \cdot \|\boldsymbol{k}_0(t)\|_2 + \|\boldsymbol{D}_{\boldsymbol{\beta}}\| \cdot \|\boldsymbol{k}_1(t)\|_2 + \|\boldsymbol{D}_{\boldsymbol{\beta}}\| \cdot \|\boldsymbol{K}_1\| \cdot \|\boldsymbol{K}_0^{-1}\| \cdot \|\boldsymbol{k}_0(t)\|_2\right)^2\end{aligned}$$