Newton's law of cooling

1) Governing Differential Equation

$$\frac{dT}{dt} - k(T_{amb} - T) = 0$$

2) Initial and Boundary conditions

$T_{amb} = 27°C$

$T(t=0) = T_0 = 250°C$

$k = 0.45\ s^{-1}$

3) Residual (or $Loss_{PDE}$)

① r or $Loss_{PDE} = \frac{dT}{dt} - k(T_{amb} - T)$

② $Loss_{IC} = (T_0^{pred} - T_0)^2$

$\therefore$ total loss = $Loss_{PDE} + Loss_{IC}$

additional) Analytical Solution

$$T(t) = T_{amb} + (T_0 - T_{amb})e^{-kT}$$

from Governing eq.,

$\frac{dT}{dt} - k(T_{amb} - T) = 0 \rightarrow \frac{dT}{dt} = -k(T - T_{amb})$

· $Y(t) = T - T_{amb} \rightarrow \frac{dY}{dt} = \frac{dT}{dt}$

· $\frac{dY}{dt} = -kY \rightarrow \frac{1}{Y}\frac{dY}{dt} = -k$

· $\frac{1}{Y}dY = -k\,dt \rightarrow \int \frac{1}{Y}dY = \int -k\,dt$

$\ln|Y| = -kt + C \rightarrow Y = e^{-kt+C} = e^C \cdot e^{-kt}$

$e^C \approx C_1$ (constant = C)

$Y = C_1 \cdot e^{-kt}$ ← apply IC to this. → $T(0) = T_0$

$Y = C_1 \cdot e^0 = C_1$

→ $T_0 - T_{amb} = C_1$

$Y = (T_0 - T_{amb})^{-kt} \rightarrow T - T_{amb} = (T_0 - T_{amb})^{-kt}$

$\therefore T = T_{amb} + (T_0 - T_{amb})^{-kt}$

$T_0 = 250°C$

- Approaches for constraining IC/BC

① warm-start

② changing loss weights (PDE vs IC/BC)

③ Output transform to enforce IC/BC exactly

④ combine pre-training on data (analytic solution) with PINN fine-tuning (2-stage)