Exercise 1.1: let K be a compact of $C([0,1];\mathbb{R})$; show that K has empty interior. Deduce that $C_0(C([0,1];\mathbb{R})) = \{0\}$, and in particular the conclusion of Reisz' Theorem does not apply to $C([0,1];\mathbb{R})$.

By Ascoli's theorem, K is uniformly bounded, meaning that there exists $M \in \mathbb{R}$ such that $\sup_{f\in K} |f| \leq M$. K is equicontinuous, meaning for all $\epsilon > 0$, there exists $\delta > 0$ such that whenever $|x-y| \leq \delta$

$$\sup_{f\in K} |f(x) - f(y)| \leq \epsilon.$$

Let $u \in K$. Fix $\epsilon > 0$.

Exercise 1.2: show that $L^\infty((0,1)), C_b(\mathbb{R}^n)$ are not separable, but that $C_0(\mathbb{R}^n)$ is.

For $L^\infty((0,1))$, consider the set of characteristic functions $X = \{\chi_{B_r(1/2)}\}_{0<r<1/2} \subset L^\infty((0,1))$. For each $f, g \in X$, $\|f-g\|_{L^\infty} = 1$ provided that $f \neq g$. Hence, if D were a countable dense subset of $L^\infty((0,1))$, then for all $\epsilon > 0$, there is $f \in D$ such that for some $g \in X$, $\|f-g\|_{L^\infty} < \epsilon$. However, this would directly imply that X is countable as each pair of functions in X differ in norm by exactly 1, and hence a contradiction.
Similarly, for $C_b(\mathbb{R}^n)$, the same class of functions over $\mathbb{R}^n$ can be produced with $X = \{\chi_{B_r(0)}\}_{r>0}$. Then, clearly for $f, g \in X$ with $f \neq g$, $\|f-g\|_{\sup} = 1$.
For $C_0(\mathbb{R}^n)$