Computation 1: Find a number z so that $z^2 = i$. Write it in the form $z = x + iy$.

Solution: We wish to find a number z such that

$$(x+iy)^2 = i.$$

So $x^2 - y^2 = 0$ and $2xy = 1$. Thus we have $x = y = \pm\frac{\sqrt{2}}{2}$ and hence

$$z = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, \quad z = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}.$$

Exercise 1: Stein Shakarchi 3,4,5,7,23,24

3. With $\omega = se^{i\phi}$, where $s \geq 0$ and $\phi \in \mathbb{R}$, solve the equation $z^n = \omega$ in $\mathbb{C}$ where n is a natural number. How many solutions are there?

   Solution: Write z as $re^{i\theta}$ where $r \geq 0$ and $\theta \in \mathbb{R}$. Now,

   $$r^n e^{ni\theta} = se^{i\phi}$$

   so we have that $r = s^{1/n}$ and $\theta = \phi/n$. *Uhh, what's the number of soln here.*

4. Show that it is impossible to define a total ordering on $\mathbb{C}$.

   Solution: Assume such a relation, $\succ$, exists. Now, consider that $i \succ 0$. Furthermore, multiplying by i, we see that $-1 \succ 0$. Now, using (iii) again, multiplying by -1 gives $(-1)(-1) = 1 \succ 0$. This is a contradiction to (i) given that both $1 \succ 0$ and $-1 \succ 0$ hold.

   Solution: Assume such a relation, $\succ$, exists with $i \succ 0$. Using condition (iii), observe that

   $$\begin{aligned} i \cdot i \succ 0 \cdot i &\implies -1 \succ 0 \\ -1 \cdot i \succ 0 \cdot i &\implies -i \succ 0 \\ i \cdot (-i) \succ 0 \cdot (-i) &\implies 1 \succ 0. \end{aligned}$$

   This breaks condition (i), so we've arrived at a contradiction, and there is no such relation.

5. Prove that an open set Ω is pathwise connected if and only if Ω is connected.

   (a) Suppose first that Ω is open and pathwise connected, and that it can be written as $\Omega = \Omega_1 \cup \Omega_2$ where Ω_1 and Ω_2 are disjoint non-empty open sets. Choose two points $\omega_1 \in \Omega_1$ and $\omega_2 \in \Omega_2$ and let γ denote a curve in Ω joining ω_1 to ω_2. Consider a parametrization $z : [0,1] \to \Omega$ of this curve with $z(0) = \omega_1$ and $z(1) = \omega_2$, and let

   $$t^\star = \sup_{0 \leq t \leq 1} \{t : z(s) \in \Omega_1 \text{ for all } 0 \leq s < t\}.$$

   Arrive at a contradiction by considering the point $z(t^\star)$.

   Solution: 0 is contained in the set $T_1 = \{t : z(s) \in \Omega_1 \text{ for all } 0 \leq s < t\}$. Furthermore, when $t^\star = 1$, $z(t^\star) \in \Omega_2$ so the set $T_2 = \{t : z(s) \in \Omega_2 \text{ for all } t \leq s \leq 1\}$ is non-empty. Now, $T_1 \cup T_2 = [0,1]$, but intervals in $\mathbb{R}$ are connected