Exercise 17: Let U be a bounded domain in $\mathbb{R}^n$ and $\alpha \in (0,1)$. Suppose that $u \in C^2(U) \cap C(\overline{U})$ solves

$$\begin{cases} -\Delta u = |u|^\alpha & \text{in } U \\ u(x) = 0 & \text{on } \partial U. \end{cases}$$

Show that

$$\sup_U |u| \leq C$$

where C depends only on n, α, and $\operatorname{diam}(U)$.

Solution. By Evans 2.6,

$$\max_{\bar{U}} |u| \leq \frac{\max_{\overline{U}} |x|^2}{2n} \max_{\overline{U}} |u|^\alpha$$

□

Exercise 21:

Solution. Because v is a subsolution, $-\Delta v \leq 0$ in $B(0,1) \setminus \{0\}$ and $v \leq 0$ on $\partial B(0,1)$. □