Ec onda: $\frac{\partial^2 u}{\partial t^2} = \alpha^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$

$$\frac{u_{i,j}^{l+1} - 2u_{i,j}^{l} + u_{ij}^{l-1}}{(\Delta t)^2} = \alpha^2 \frac{u_{i+1,j}^{l} - 2u_{i,j}^{l} + u_{i-1,j}^{l}}{(\Delta x)^2} + \alpha^2 \frac{u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l}}{(\Delta y)^2}$$

- $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial x} + \frac{\partial u}{\partial \phi}\frac{\partial \phi}{\partial x}$ • $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial y} + \frac{\partial u}{\partial \phi}\frac{\partial \phi}{\partial y}$

- $\frac{\partial \rho}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2)^{1/2} = \frac{x}{\rho}$ • $\frac{\partial \rho}{\partial y} = \frac{y}{\rho}$ • $\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\arctan\left(\frac{y}{x}\right) = \frac{1}{1+(y/x)^2} \cdot y(-x^2) = -\frac{y}{r^2}$

- $\frac{\partial \phi}{\partial y} = \frac{x}{\rho^2}$

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \left(\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial u}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \phi^2} \right)$$

De las notas de clase tenemos.

- $\frac{\partial u}{\partial \rho} = \frac{u_{i,j}^{l} - u_{i-1,j}^{l}}{\Delta \rho}$ • $\frac{\partial^2 u}{\partial \rho^2} = \frac{u_{i+1,j}^{l} - 2u_{i,j}^{l} + u_{i-1,j}^{l}}{(\Delta \rho)^2}$ • $\frac{\partial^2 u}{\partial \phi^2} = \frac{u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l}}{(\Delta \phi)^2}$ • $\frac{1}{\rho} = \frac{1}{\rho(i)}$

$$\frac{u_{ij}^{l+1} - 2u_{i,j}^{l} + u_{ij}^{l-1}}{(\Delta t)^2} = \alpha^2 \left(\frac{1}{\rho(i)} \frac{\partial}{\partial \rho} \left(\rho(i) \left(\frac{u_{ij}^{l} - u_{i-1,j}^{l}}{\Delta \rho} \right) + \frac{1}{\rho(i)^2} \left(\frac{u_{i,j+1}^{l} - 2u_{ij}^{l} + u_{ij-1}^{l}}{(\Delta \phi)^2} \right) \right) \right)$$

$$= \alpha^2 \left(\frac{1}{\rho(i)} \left[\frac{u_{ij}^{l} - u_{i-1,j}^{l}}{\Delta \rho} + \rho(i) \left(\frac{u_{i+1,j}^{l} - 2u_{ij}^{l} + u_{i-1,j}^{l}}{(\Delta \rho)^2} \right) \right] + \frac{1}{\rho(i)^2} \left(\frac{u_{i,j+1}^{l} - 2u_{ij}^{l} + u_{i,j-1}^{l}}{(\Delta \phi)^2} \right) \right)$$

$$= \frac{\alpha^2}{\rho(i)} \left[\frac{u_{ij} - u_{i-1,j}}{\Delta \rho} \right] + \frac{\alpha^2}{(\Delta \rho)^2} \left(u_{i+1,j} - 2u_{ij}^{l} + u_{i-1,j} \right) + \frac{\alpha^2}{\rho(i)^2 (\Delta \phi)^2} \left[u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l} \right.$$

$$u_{i,j}^{l+1} = \frac{\alpha^2 \Delta t^2}{\rho(i)} [u_{ij} - u_{i-1,j}] + \frac{\alpha^2 \Delta t^2}{(\Delta \rho^2)} \left(u_{i+1,j}^{l} - 2u_{ij}^{l} + u_{i-1,j} \right) + \frac{\alpha^2 \Delta t^2}{\rho(i)^2 \Delta \phi^2} \left[u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j+1}^{l} \right] + 2u_{ij}^{l} - u_{i,j}^{l-1}$$

Definiendo $\lambda := \frac{\Delta \rho}{\Delta \phi}$ y $v := \frac{\alpha \Delta t}{\Delta \rho}$

$$\frac{v^2 \cdot \Delta \rho}{\rho(i)} \left(u_{ij}^{l} - u_{i-1,j}^{l} \right) + v^2 \left(u_{i+1,j}^{l} - 2u_{ij}^{l} + u_{i-1,j} \right) + \frac{v^2}{\rho(i)^2} \lambda^2 \left(u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1} \right) + 2u_{ij}^{l} - u_{i,j}^{l-1}$$

$$u_{ij}^{l+1} = v^2 \left[u_{i+1,j}^{l} - 2u_{i,j}^{l} + u_{i-1,j}^{l} + \frac{\Delta \rho}{\rho(i)} \left(u_{i,j}^{l} - u_{i-1,j}^{l} \right) + \frac{\lambda^2}{\rho(i)^2} \left(u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l} \right) \right]$$

$$+ 2u_{i,j}^{l} - u_{ij}^{l-1}$$