Punto 6

a) $y''(x) = -g(x)y(x) + s(x)$

expandimos en Taylor

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0)$$
$$+ \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y''''(x_0) + \frac{(x-x_0)^5}{5!}y'''''(x_0)$$

$h = x - x_0$

$$y(x) = y(x_0) + h\,y'(x_0) + \frac{h^2}{2!}y''(x_0) + \frac{h^3}{3!}y'''(x_0)$$
$$+ \frac{h^4}{4!}y''''(x_0) + \frac{h^5}{5!}y'''''(x_0) \quad O(h^6)$$

discretizando el espacio, $h = x_{n+1} - x_n$

$$y_{n+1} = y_n + h\,y'(x_n) + \frac{h^2}{2!}y''(x_n) + \frac{h^3}{3!}y'''(x_n) + \frac{h^4}{4!}y''''(x_n) + \frac{h^5}{5!}y'''''(x_n)$$
$$y_{n-1} = y_n - h\,y'(x_n) + \frac{h^2}{2!}y''(x_n) - \frac{h^3}{3!}y'''(x_n) + \frac{h^4}{4!}y''''(x_n) + \frac{h^5}{5!}y'''''(x_n)$$

sumando ambas:

$$y_{n+1} + y_{n-1} = 2y_n + h^2 y_n'' + \frac{h^4}{12}y_n''''$$

$$y_n'' = -g_n y_n + s_n$$

$$y_n'''' = \frac{d^2}{dx^2}(-g_n y_n + s_n)$$

$$y_n'''' = \frac{-g_{n+1} y_{n+1} + s_{n+1} + 2g_n y_n - 2s_n - g_{n-1} y_{n-1} + s_{n-1}}{h^2}$$

sustituimos.

$$y_{n+1} - 2y_n + y_{n-1} = h^2(-g_n y_n + s_n) + \frac{h^2}{12}(-g_{n+1} y_{n+1} + s_{n+1}$$

$$+ 2g_n y_n - 2s_n - g_{n-1} y_{n-1} + s_{n-1})$$

reorganizando

$$y_{n+1}\left(1 + \frac{h^2}{12} g_{n+1}\right) - 2y_n\left(1 - \frac{5h^2}{12} g_n\right) + y_{n-1}\left(1 + \frac{h^2}{12} g_{n-1}\right)$$

$$=$$

$$\frac{h^2}{12}(s_{n+1} + 10 s_n + s_{n-1})$$