$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{u_i^{t+1} - 2u_i^t + u_i^{t-1}}{\Delta t^2} = \frac{\alpha^2}{\Delta x} u_{i+1}^t - 2u_i^t + u_{i-1}^t$$

$$u_i^{t+1} - 2u_i^t + u_i^{t-1} = \lambda^2 (u_{i+1}^t - 2u_i^t + u_{i-1}^t)$$

$$2u_i^t \cos(k\omega t) - 2u_i^t = \lambda^2 (2\cos(k\Delta x) - 2u_i^t)$$

$$u_i^t \cos(k\Delta t) - u_i^t = \lambda^2 (\cos(k\Delta x) u_i^t - u_i^t)$$

$$u_i^t (\cos(k\Delta t) - 1 - \lambda^2 \cos(k\Delta x) + \lambda^2) = 0$$

$$\cos(k\Delta t) - 1 - \lambda^2 \cos(k\Delta x) + \lambda^2 = 0$$

$$\cos(k\Delta t) - \lambda^2 (\cos(k\Delta x) + 1) = 1$$

$$-\lambda^2 (\cos(k\Delta x) + 1) = 1 - \cos(k\Delta t)$$

$$= -2$$

$$-2\lambda^2 = -2$$

$$\lambda = 1$$