3.

$x^3y' = x^4y^2 - 2x^2y - 1$ $\qquad y_1 = x^{-2}$

$y' = xy^2 - \frac{2y}{x} - \frac{1}{x^3}$

$y = x^{-2} + \frac{1}{v(x)}$

$y' = -2x^{-3} - \frac{1}{v^2}v'(x)$

$-2x^{-3} - \frac{1}{v^2}v'(x) = -x^{-3} - 2x^{-1}\left(x^{-2} + \frac{1}{v(x)}\right) + x\left(x^{-2} + \frac{1}{v(x)}\right)^2$

$\frac{v'(x)}{v^2} = -x^{-3} + 2x^{-3} + \frac{2x^{-1}}{v(x)} + x\left(\frac{1}{x^2} + \frac{1}{v(x)}\right)^2$

$\frac{v'(x)}{v^2} = \frac{1}{x^3} + \frac{2}{xv(x)} + \frac{1}{x^3} - \frac{2}{xv(x)} - \frac{x}{v^2(x)}$

$\frac{v'(x)}{v^2} = \frac{1}{x^3} - \frac{x}{v^2}$

$v'(x) = -x$

$v(x) = -\frac{x^2}{2} + \tilde{c}$

$y = \frac{1}{x^2} + \frac{1}{-\frac{x^2}{2} + \tilde{c}} = \frac{1}{x^2} - \frac{2}{x^2 + c}$

$y(\sqrt{2}) = 0 = \frac{1}{2} - \frac{2}{2+c} \Rightarrow \frac{2}{2+c} = \frac{1}{2}$

$2 + c = 4 \qquad c = 2$

$y = \frac{1}{x^2} - \frac{2}{x^2+2}$