Métodos computacionales 2

1. $f'(x) = \dfrac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

$$f'(x^2) = \lim_{h\to 0} \frac{-x^2 - 4xh - 4h^2 + 4x^2 + 8xh + 4h^2 - 3x^2}{2h}$$

$$= \lim_{h\to 0} \frac{4xh}{2h} = 2x$$

$$f'(\sin x) = \lim_{h\to 0} \frac{-\sin(x+2h) + 4\sin(x+h) - 3\sin x}{2h}$$

$$= \lim_{h\to 0} -\frac{\sin(x)\cos(2h) - \cos(x)\sin(2h) + 4\sin(x)\cos(h)}{2h}$$

$$+ \frac{4\cos(x)\sin(h) - 3\sin(x)}{2h}$$

$$= \sin(x) \lim_{h\to 0} \left(\frac{-3 + 4\cos(h) - \cos(2h)}{2h}\right)$$

$$+ \cos(x) \lim_{h\to 0} \frac{-\sin 2h + 4\sin h}{2h}$$

$$= \sin(x) \lim_{h\to 0} \frac{-4\sin(h) + 2\sin(2h)}{2} + \cos(x) \lim_{h\to 0} \frac{-2\cos 2h + 4\cos h}{2}$$

$$= \sin(x) \cdot 0 + \cos(x)(2-1) = \cos(x)$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$f''(x^2) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2x^2 + x^2 - 2xh + h^2}{h^2}$$

$$= \lim_{h \to 0} \frac{2h^2}{h^2} = 2$$

$$f''(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - 2\sin x + \sin(x-h)}{h^2}$$

$$= \lim_{h \to 0} \sin(x)\cos(h) + \cos(x)\sin(h) - 2\sin(x) + \sin(x)\cos(h) - \cos(x)\sin(h)$$

$$= \sin(x) \lim_{h \to 0} \frac{2\cos(h) - 2}{h^2} = \sin(x) \lim_{h \to 0} - \frac{2\sin(h)}{2h}$$

$$= \sin(x) \lim_{h \to 0} -\cos(h) = -\sin(x)$$