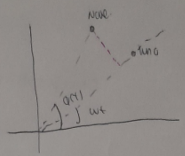
c) $r_L(r, \phi, t) = ?$



$r_N = \sqrt{x^2+y^2} = r(t)$

$r_{TL} = \sqrt{x_L^2+y_L^2} = d$

La distancia está dada por la suma magnitud de ambos vectores menos la proyección de r(t) sobre r_{TL}, es decir

$$r_L = \sqrt{(r(t)-d))^2} = \sqrt{r^2(t) - 2\,r(t)\cdot d + d^2} = \sqrt{r^2(t) - 2r(t)\cdot d\cos(\varphi) + d^2}$$

producto punto

ángulo entre ellos

Como la nave se encuentra a Φ y la luna a ωt definimos $\varphi = \phi - \omega t$ entonces

$$r_L = \sqrt{r^2(t) - 2r(t)\, d\cos(\phi - \omega t)}$$

d) $H = p_r \dot{r} + p_\phi \dot{\Phi} - L$

$x = r\cos\phi \qquad \dot{x} = \dot{r}\cos\phi - r\,\mathrm{sen}\phi$

$y = r\,\mathrm{sen}\,\phi \qquad \dot{y} = \dot{r}\,\mathrm{sen}\phi + r\cos\phi$

$T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x}^2+\dot{y}^2)$

$= \dot{r}^2\cos^2\phi - 2\dot{r}r\,\mathrm{sen}\phi\cos\phi + r^2\mathrm{sen}^2\phi$

$+ \dot{r}^2\mathrm{sen}^2\phi + 2\dot{r}r\,\mathrm{sen}\phi\cos\phi + r^2\cos\phi$

$= (\dot{r}^2 + r^2)\frac{1}{2}m$

$= p_r\dot{r} + p_\phi\dot{\phi}^2 - (\dot{r}^2+r^2)\frac{1}{2}m + \frac{Gm_Tm}{r}$

$- \frac{Gmm_L}{r_L(r,\phi,t)}$

$= \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - \frac{Gmm_T}{r} - G\frac{mm_L}{r_L(r,\phi,t)}$

$U = -\frac{Gm_Tm}{r} - \frac{Gm_Lm}{r_L}$

$L = T - U$

$r = \sqrt{x^2+y^2} \Rightarrow \dot{r} = \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2x\dot{x} + 2y\dot{y}$

~~$\langle\Psi|\Phi\rangle\langle\Psi|\Phi\rangle^*$~~

e)

$\dot{r} = \frac{\partial H}{\partial p_r} = \frac{2p_r}{2m} = \frac{p_r}{m}$

$\dot{\phi} = \frac{\partial H}{\partial p_\phi} = \frac{2p_\phi}{2mr^2} = \frac{p_\phi}{mr^2}$

$\dot{p}_r = -\frac{\partial H}{\partial r} = -\left(\frac{+Gmm_T}{r^2} - \frac{p_\phi^2}{mr^3}\right) - \left(\frac{1}{2}\frac{Gmm_L}{r_L(\phi,t)^3} \cdot (2r - 2d\cos(\phi - \omega t))\right)$

$\{r(t)^2 + d - 2r(t)\,d\cos(\phi - \omega t)\}^{-3/2}$

$= -\frac{Gmm_T}{r^2} + \frac{p_\phi^2}{mr^3} - \frac{Gmm_L}{r_L(r,\phi,t)^3}(r - d\cos(\phi - \omega t))$

$\dot{p}_\phi = \frac{-\partial H}{\partial \phi} = -\left[\frac{+1}{2}\frac{Gmm_L}{r_L(r,\phi,t)^3}\cdot(+2r(t)\,d\,\mathrm{sen}(\phi - \omega t))\right]$

$= -\frac{Gmm_L}{r_L(r,\phi,t)^3}(r(t)\,d\,\mathrm{sen}(\phi - \omega t))$

f) ~~$\dot{\tilde{r}} = \frac{p_r}{m} = \tilde{p}_r$~~

- $\dot{\tilde{r}} = \frac{d}{dt}\left(\frac{r}{d}\right) = \frac{1}{d}\frac{dr}{dt} = \frac{1}{d}\dot{r} = \frac{1}{d}\frac{p_r}{m} = \tilde{p}_r$
- $\dot{\phi} = \frac{p_\phi}{mr^2} = \tilde{p}_\phi d^2 \cdot \frac{1}{\tilde{r}^2 d^2} = \frac{\tilde{p}_\phi}{\tilde{r}^2}$
- ~~$\dot{\tilde{p}}_r = -\frac{Gm_T}{\tilde{r}\,d^2} + \frac{\tilde{p}_\phi^2 d^4}{\tilde{r}^3 d^3} - \frac{Gm_L}{r_L(r,\phi,t)^3}(\tilde{r}d - d\cos(\phi - \omega t))$~~

$\dot{\tilde{p}}_r = \frac{d}{dt}(\tilde{p}_r) = \frac{d}{dt}\left(\frac{p_r}{md}\right) = \frac{1}{md}\dot{p}_r = \frac{1}{md}\left(\frac{p_\phi^2}{mr^3} - \frac{Gmm_T}{r^2} - \frac{Gmm_L}{r_L(r,\phi,t)^3}[r - d\cos(\phi - \omega t)]\right)$

$= \frac{p_\phi^2}{m^2 d\, r^3} - \frac{Gm_T}{r^2 d} - \frac{Gm_L}{d\, r_L(r,\phi,t)^3}[r - d\cos(\phi - \omega t)]$

$= \frac{\tilde{p}_\phi^2 d^4}{d\,r^3} - \frac{Gm_T}{d^3}\left(\frac{d^2}{\ldots}\right) + \frac{d^2\,Gm_L}{\ldots}[r - d\cos(\phi - \omega t)]$

$$= \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{\mu d^2}{r_2(r,\phi,t)}\left[\tilde{r}d - d\cos(\phi-\omega t)\right]\right]$$

$$= \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{\mu d}{r_2(r,\phi,t)^3}\left[\tilde{r} - \cos(\phi-\omega t)\right]\right].$$

$$\rightarrow (r^2+d^2-2rd\cos(\phi-\omega t))^{3/2}$$

$$(\tilde{r}^2 d^2 + d^2 - 2\tilde{r}d^2\cos(\phi-\omega t))^{3/2}$$

$$d(\tilde{r} + \ -2\tilde{r}\cos(\phi-\omega t))^{3/2}$$

$$= \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}\left[\tilde{r} - \cos(\phi-\omega t)\right]\right].$$

$$\circ\ \mathring{\tilde{p}}_\phi = \frac{d}{dt}(\tilde{p}_\phi) = \frac{d}{dt}\left(\frac{p_\phi}{md^2}\right) = \frac{1}{md^2}\dot{p}_\phi = \frac{-G m_2}{d\, r_2(r,\phi,t)^3} r\,\mathrm{sen}(\phi-\omega t).$$

$$= \frac{-\Delta\mu d}{d^2 r'^3}\tilde{r}d\,\mathrm{sen}(\phi-\omega t). = \frac{-\Delta\mu\tilde{r}}{\tilde{r}'^3}\mathrm{sen}(\phi-\omega t).$$

$$\mathring{\tilde{p}}_r = \left(\frac{\Delta\mu\tilde{r}^0\,\mathrm{sen}(\phi)}{(\underbrace{1+\tilde{r}^{0\,2}-2\tilde{r}^0\cos(\phi)}_{Q})^{3/2}}\right)^2 \cdot \frac{1}{\tilde{r}^{0\,3}} - \Delta\left\{\frac{1}{\tilde{r}_0^2} + \frac{\mu}{Q}(\tilde{r}^0 - \cos\phi)\right]$$

$$= \frac{\mathring{p}_r}{md} = \frac{1}{m}\frac{m\mathring{r}}{d} = \frac{1}{d}\frac{dr}{dt} = \frac{1}{d}\left(\frac{d}{dt}(\sqrt{x^2+y^2})\right) = \frac{1}{d}\frac{1}{2}(x^2+y^2)^{-1/2} \circ 2x\dot{x} + 2y\dot{y}$$

$$= \frac{1}{rd}\ x\dot{x}+y\dot{y} = \frac{v_0}{d}\cos(\theta-\phi) = \tilde{v}_0\cos(\theta-\phi).$$

$$\mathring{\tilde{p}}_\phi = \frac{\mathring{p}_\phi}{md^2} = \frac{r^2}{md^2}\cdot m\frac{d\theta}{dt} = \frac{r^2}{d^2}\frac{d}{dt}\arctan\left(\frac{y}{x}\right) = \frac{\tilde{r}^2}{1+y^2/x^2}\frac{d}{dt}\left(\frac{y}{x}\right).$$

$$= \frac{\tilde{r}^2x^2}{x^2+y^2}\frac{d}{dt}\left(\frac{y}{x}\right) = \frac{\tilde{r}^2x^2}{r^2}\frac{d}{dt}\left(\frac{y}{x}\right) = \frac{\tilde{r}^2x^2}{r^2}\left(\frac{\dot{y}x - y\dot{x}}{x^2}\right).$$

$$= \frac{\tilde{r}^2}{r^2}(\dot{y}x - y\dot{x}) = \frac{r_0}{d}\frac{v_0}{d}\mathrm{sen}(\theta-\phi) = \tilde{r}_0\tilde{v}_0\,\mathrm{sen}(\theta-\phi).$$