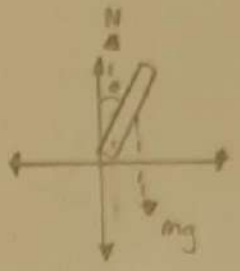
Falling Rod

a) $\Sigma_i F_y = N - mg \Rightarrow m\ddot{y} = N - mg$



b) $T = F \cdot d = F d \sin\theta \quad \tau = I\dot{\alpha} = I\ddot{\theta}$

$\tau = \Sigma_i m d \alpha_i = N \cdot \frac{L}{2} + mg \cdot 0 = N \frac{L}{2}$

$F d' \sin\theta = I\ddot{\theta}$

$N \frac{L}{2} \sin\theta = I\ddot{\theta}$

d: distancia al centro de masa

c) $x = L\cos\theta \qquad \dot{x} = -L\dot{\theta}\sin\theta$

$y = L\sin\theta \qquad \dot{y} =$

$x = L\cos\theta$

$y = L\sin\theta$

$x^2 + y^2 = L$

$\dot{x} = -L\sin\theta\,\dot{\theta} \qquad \ddot{x} = -L\cos\theta\,\dot{\theta} - L\sin\theta\,\ddot{\theta}$

$\dot{y} = L\cos\theta\,\dot{\theta} \qquad \ddot{y} = -L\sin\theta\,\dot{\theta} + L\cos\theta\,\ddot{\theta}$

$T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m(L^2\cos^2\theta\,\dot{\theta}^2 + L\sin\theta\,\ddot{\theta})^2 + (-L\sin\theta\,\dot{\theta}^2 + L\cos\theta\,\ddot{\theta})^2$

$V = mgy = L\sin\theta$

$T = \frac{1}{2}m\left(L^2\sin^2\theta\,\dot{\theta}^2 + L^2\cos^2\theta\,\dot{\theta}^2\right) = \frac{1}{2}mL^2\dot{\theta}^2(\sin^2\theta + \cos^2\theta) = \frac{1}{2}mL^2\dot{\theta}^2$

$L = \frac{1}{2}mL^2\dot{\theta}^2 - L\sin\theta$

$-L\cos\theta = mL^2\ddot{\theta}$

$\frac{\partial L}{\partial \theta} = -L\cos\theta$

$\ddot{\theta} = -\frac{L\cos\theta}{mL^2} = -\frac{\cos\theta}{mL}$

$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2}mL^2\,2\dot{\theta} = mL^2\dot{\theta}$