Circuit Theory and Electronics Fundamentals

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Circuit Analysis Methods

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1 Introduction

The main objective of this laboratory assignment is to study the following circuit. For that we will be using two methods that we learnt in TCFE class: the Mesh Method and the Node Voltage Method. Then, we want to see if the values that we obtain in this two methods are the same and if they are equal to the ones of Ngspice. We decided to separate this work in other three sections. In the first one,Analysis, we will explain the theoretical analysis of this circuit, the methods that we used to analyse it and the theoretical values of current and voltage, for that we use the math tools of Octave. Using Ngspice tools, in the second section Simulation, we will present a simulation of our circuit and compare it with the previous theoretical results. Lastly, Conclusion, we will summarise the contents of the report and discuss the achieved results.

2 Theoretical Analysis

First of all, we need to analyse the circuit in general. It has thirteen meshes, of which four are elementar, nine nodes and eleven branches (seven resistors, one independent voltage source, V_A, one independent current source, I_D, one voltage controlled current source I_B and one current controlled voltage source V_C. In order to discover the currents and the voltages in nodes we could use Kirchhoff's Current Law (KCL) and the Kirchhoff's Voltage Law (KVL), but

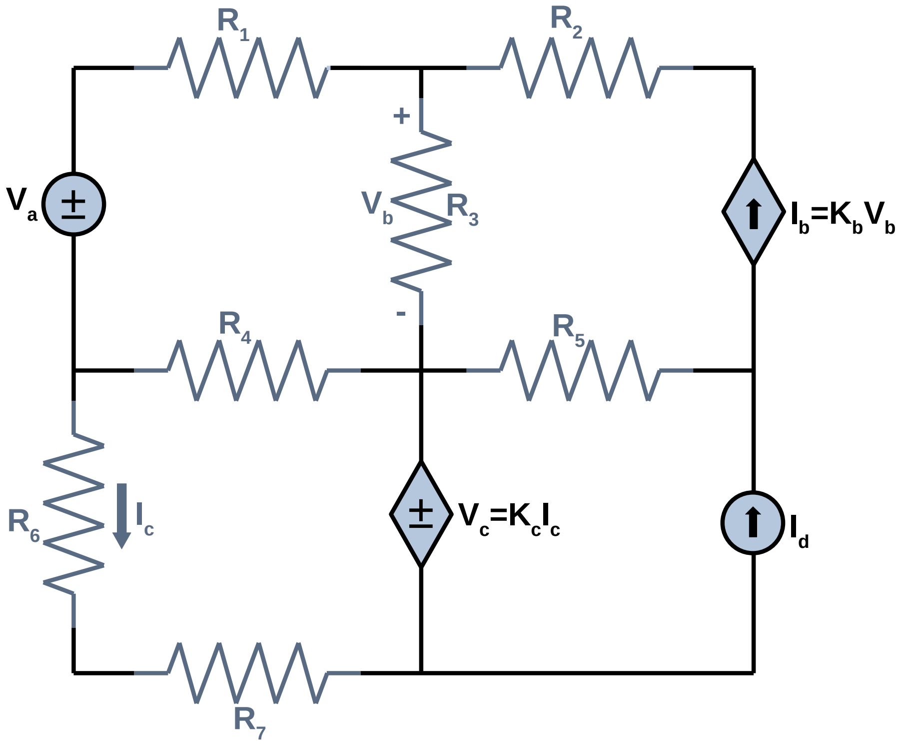

Figure 1: Circuit

we used two methods (that are based on these two laws), which are explained in the following sections.

2.1 Mesh method

To use this method we need to identify the loops, as we can see in the image above. Each one of the loops have one associated current, that we call I_A, I_B, I_C and I_D. To discover the value of this currents we need to analyse each mesh by itself, but we have to beware that components that make part of more than one loop have a current that is influenced by the currents of those loops. For example for the first loop, we have to sum the voltage of each component. In the voltage source we know that the voltage is V_A, for the resistor, R_1, we use the Ohm's Law and we get that $V_1 = R_1$. However for the others two resistor, we will use the Ohm's Law again and we will notice that they make part of two loops. The voltage that goes through the resistor is multiplication the value of the resistor and the current. Looking at the branches in between two nodes we can see that both of the loop currents go through the branch but in opposite directions. The current with the same direction as the branch current (+ $\rightarrow$ -) will have a positive sign and the loop current going through the opposite direction will have a negative sign. We do the same for the resistor R_4. For the others two loops we did the same way and we got 3 equations in total:

$$I_1(R_1 + R_4 + R_3) - I_2R_3 - I_3R_4 = -V_A \tag{1}$$

Name	Value [A or V]
I1	0.00022339172
I2	0.00023403434
I3	-0.00099456437

$$I_1(-R_3K_b) + I_2(K_bR_3 - 1) = 0 \tag{2}$$

$$I_1(-R_4) + I_3(-K_c - R_6 - R_7 + R_4) = 0 \tag{3}$$

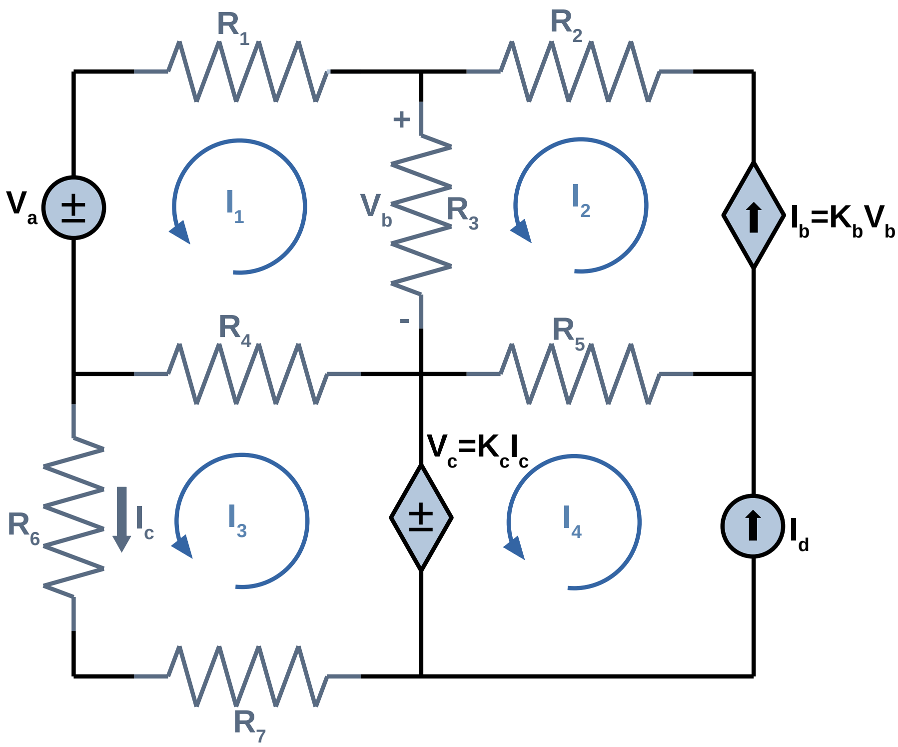

Figure 2: Mesh method

To solve this system of linear equations we used Octave and we get the following values:
The last calculations are presented in the table below

Name	Value [A or V]
Vc	-7.9685230178
I2	0.03289187116

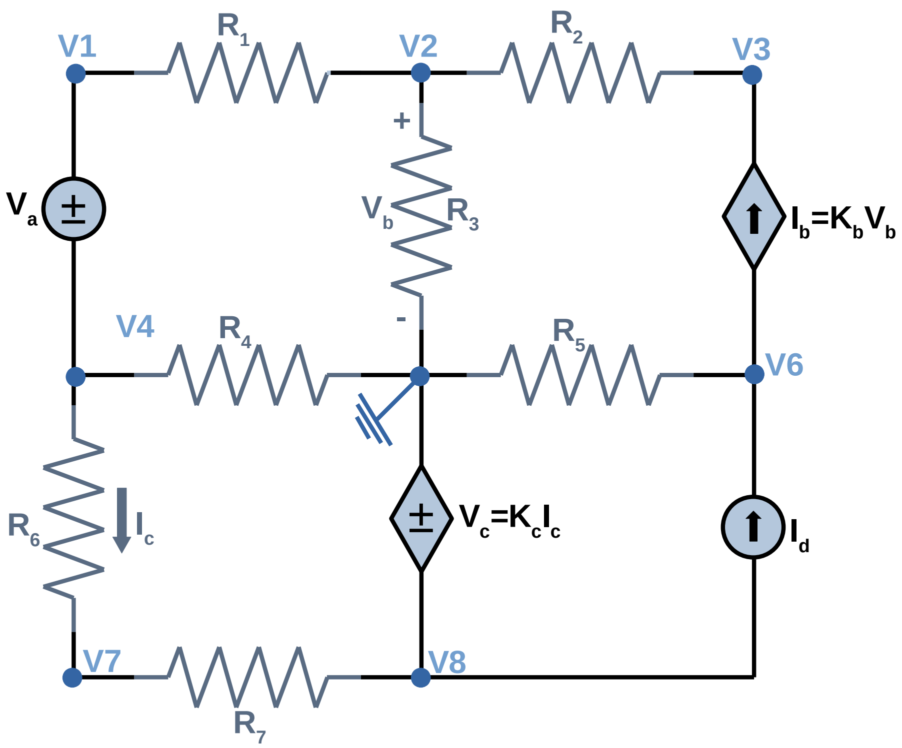

Figure 3: Node method: labeling of the nodes

2.2 Node Voltage Method

A really powerful way of analyzing circuits is called the Node Voltage Method, which is based on Kirchhoff's Current Law (KCL).

To begin with, we should start by explaining what node voltage means. When we use the term node voltage, we are referring to the potential difference between two nodes of a circuit. This is still a voltage, it's not anything too strange. We start by labelling the nodes with numbers and assigning each with its own variable, V_1, V_2, etc. Then, we select one of the nodes in the circuit to be the reference node and call this reference node the ground node. It gets a ground symbol in the schematic. The potential of the ground node is defined to be $0V$. The choice of the reference node is somewhat arbitrary, but it's always best to choose it based on its connections to voltage sources, because by this selection we already know the node voltages of other nodes, i.e. the ones that the reference node is connected to them by voltage sources. We chose for the ground node node 5, since it is connected to dependent voltage source V_c and three different resistors. Therefore, $V_5 = 0V$. Choosing node 5 as the ground node also allows us to state that $V_2 = V_b$, for instance. After that is done, we apply KCL for each node. Taking all of this into account, we end up with the following equations

$$V_1 - V_4 = V_a \tag{4}$$

$$V_1(-G_1) + V_2(G_1 + G_2 + G_3) + V_3(-G_2) = 0 \tag{5}$$

Name	Value [A or V]
V1	0.1950637950
V2	-0.03289187116
V3	-0.50193690692
V4	-4.87868080023
V6	2.44434955329
V7	-6.93173304431
V8	-7.96852301779

Name	Value [A or V]
Ic	-0.0039547
Vc	-31.6853746
Ib	-0.0035714

$$V_2(-K_b - G_2) + V_3(G_2) = 0 \tag{6}$$

$$V_2(K_b) + V_6(G_5) = I_d \tag{7}$$

$$V_4(G_6) + V_7(-G_6 - G_7) + V_8(G_7) = 0 \tag{8}$$

$$V_1(-G_1) + V_2(G_1) + V_4(-G_4 - G_6) + V_7(G_6) = 0 \tag{9}$$

$$V_4(K_c G_6) + V_7(-K_c G_6) + V_8 = 0 \tag{10}$$

where G is the conductance of the resistors, given by the relation $G = \frac{1}{R}$. Solving these equations in order of V_n, n = {1,2,3,4,5,6,7,8}, allows us to then simply calculate the currents and voltages in play in this circuit, using Ohm's Law, among others. Octave was used to compute the solution of this 7 equation system, giving the following solutions

Values for I_b, I_c and V_c were obtained using the following expressions

$$I_b = K_b V_b = K_b V_2 \tag{11}$$

$$I_c = \frac{V_7 - V_4}{R_6} \tag{12}$$

$$V_c = K_c I_c \tag{13}$$

3 Simulation Analysis

Comparing to the theoretical analysis results, one notices that the values obtained for each node voltage are the same. However, for the mesh method, I_1 and I_2 are the same as the ones obtained in the simulation, but I_3 is off by 2 orders of magnitude. This huge difference is most likely related to the equations used to solve the system, since the values of I_1 and I_2 are correct.Consequently, we cannot rule out the precision of the mesh method since in this case the inaccuracy of the values obtained is apparently down to the equations used to achieve them and not the method itself. This means that the Node Voltage Method is an accurate and exact method and can be confidently used to solve complex and extensive electric circuits. From our observations we can't say with conviction that the Mesh Method is as exact and accurate as the the other method used, due to the reasons stated before.

Name	**Value [A or V]**
i(hvc)	-4.58297e-05
i(va)	-2.23392e-04
@gib[i]	-2.34034e-04
@id[current]	1.040394e-03
@r1[i]	2.233917e-04
@r2[i]	-2.34034e-04
@r3[i]	-1.06426e-05
@r4[i]	-1.21796e-03
@r5[i]	1.274428e-03
@r6[i]	9.945644e-04
@r7[i]	9.945644e-04
v(1)	1.950638e-01
v(2)	-3.28919e-02
v(3)	-5.01937e-01
v(4)	-4.87868e+00
v(6)	3.863224e+00
v(7)	-6.93173e+00
v(8)	-7.96852e+00
v(9)	-4.87868e+00

4 Conclusion

In this laboratory assignment, the objective of analysing a complex electric circuit using different circuit analysis methods mesh method and node voltage method has been achieved. The circuit was theoretically analysed combining each of the circuit analysis methods and the Octave maths tool and by running a circuit simulation using the Ngspice tool. The simulation results matched the results obtained with the Node Voltage Method precisely, but were not totally satisfactory regarding the Mesh Method. The reason for this mismatch using the Mesh Method were the equations used to solve the problem. When dealing with circuits with so many components it gets difficult to evaluate each of the components' signs, which way the current is flowing, etc, and a mixed sign can greatly affect the outcome. Opposed to this, the reason for the perfect match obtained using the Mesh Method is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot