Circuit Theory and Electronics Fundamentals

Lab 5: Bandpass filter using OP-AMP

Technological Physics Engineering

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Contents

1 Introduction **2**

2 Simulation Analysis **2**

3 Theoretical Analysis **2**

3.1 Input and Output Impedances 3

3.2 Gain 3

3.3 Low Cut-Off Frequency, High Cut-Off Frequency, and Central Frequency 4

3.4 Theoretical results and comparison 4

4 Conclusion **5**

1 Introduction

The main objective of this laboratory assignment is to study the circuit depicted in 1. We decided to separate this work in three different sections.

In the first one,Theoretical Analysis 3, we will analyze the circuit in 3 different time intervals, $t < 0$, $t = 0$ and $t > 0$. For the first 2 time intervals we will obtain linear equations using methods learnt in the TCFE class, which can be solved with $Octave$. These equation allow to find the voltage in each node and the current in each branch. With $t \to \infty$, a first order linear equation is obtained, which can be solved to obtain and plot the total solution of the circuit.

Using Ngspice tools, in the second section Simulation 2, we will present a simulation of the circuit and compare it with the previous theoretical results.

Lastly, Conclusion 4, the contents of the report will be summarised and the achieved results discussed.

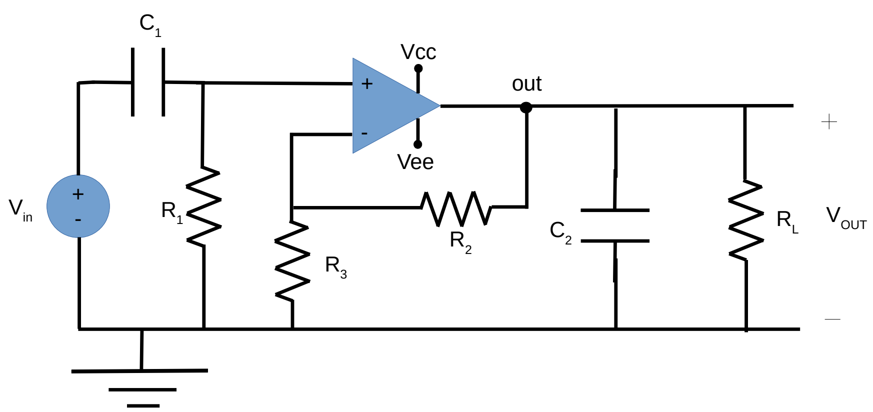


Figure 1: Complet circuit

2 Simulation Analysis

Using ngspice we are able to produce this graphs and obtain the values of the gain, central frequency and input and output impedances. This results will be showed later on. We were also able to optain the value of merith which is showed in the table below.

Name	Value [A or V]
mer	8.230015e-07

3 Theoretical Analysis

In this section we are going to theorically analyse the following circuit that was already showed.

The circuit is divided in 3 parts: the High Pass Filter Stage, the Amplifier Stage and the Low Pass Filter Stage, each with a different purpose. Their goal is to block out low frequencies, amplify the voltage signal and block out high frequencies, respectivelly. Moreover, the OP AMP is implemented in the circuit in a non-inverting configuration. This way, the circuit acts as an Active Non-inverting OP AMP Bandpass Filter.

Firstly, we proceed to compute the input and output impedances, as well as the gain of the circuit.

3.1 Input and Output Impedances

The input impedance is computed as:

$$Z_I = \left.\frac{V_i}{I_i}\right|_{Z_L=\infty} \tag{1}$$

where Z_L is the load impedance and I_i is the current that passes through the voltage source V_i. Since no current enters an OP AMP (in the $+$ and $-$ terminals, since it's input impedance is ∞), this leaves us with a capacitor in series with a resistor. Therefore, the input impedance is:

$$Z_I = \frac{1}{jwC_1} + R_1 \tag{2}$$

The output impedance is computed as:

$$Z_O = \left.\frac{V_o}{I_o}\right|_{V_i=0} \tag{3}$$

Once again, since no current enters an OP AMP and $V_i = 0$, then $V_+ = V_- = 0$ and we are left with capacitor C_2 in series with a parallel of resistor R_4 and capacitor C_3. Thus, the output impedance is given by:

$$Z_O = \frac{1}{\frac{1}{R_4} + jwC_2} \tag{4}$$

3.2 Gain

To compute the gain, we can divide the problem in 3 parts. Starting with the High Pass Filter Stage and using a Voltage Divider, we get:

$$V_+ = \frac{R_1}{R_1 + \frac{1}{jwC_1}} V_i \tag{5}$$

With this expression for V_+ we can already see the purpose of this part of the circuit. As the name suggests, for high frequencies $V_+ \to V_i$ and for low frequencies $V_+ \to 0$. Hence, the High Pass Filter Stage blocks the signals with low frequencies.

Now, moving on to the amplifier stage, by inspection we see that $V_A = V_- + R_2 I_2$, where $I_2 = I_3 = (V_- - 0)/R_3$. So, we get:

$$V_A = V_- + R_2\frac{V_-}{R_3} \equiv V_A = (1 + \frac{R_2}{R_3})V_- \tag{6}$$

This expression unfolds the amplifying properties of this part. The greater R_2 is compared to R_3, the greater V_A will be.

Lastly, we move onto the Low Pass Filter Stage. Using voltage dividir:

$$V_o = \frac{\frac{1}{jwC_2}}{\frac{1}{jwC_2} + R_4} V_a \tag{7}$$

This expression explain the name Low Pass Filter Stage. For low frequencies $V_o \to V_a$, but for high frequencies $V_o \to 0$.

Now, using equations (5) , (6) and (7) and recalling that $V_+ = V_-$, the gain is given by:

$$\frac{V_o(\omega)}{V_i(\omega)} = \left(\frac{R_1}{R_1 + \frac{1}{j\omega C_1}}\right)\left(1 + \frac{R_2}{R_3}\right)\left(\frac{\frac{1}{jwC_2}}{\frac{1}{jwC_2} + R_4}\right) \tag{8}$$

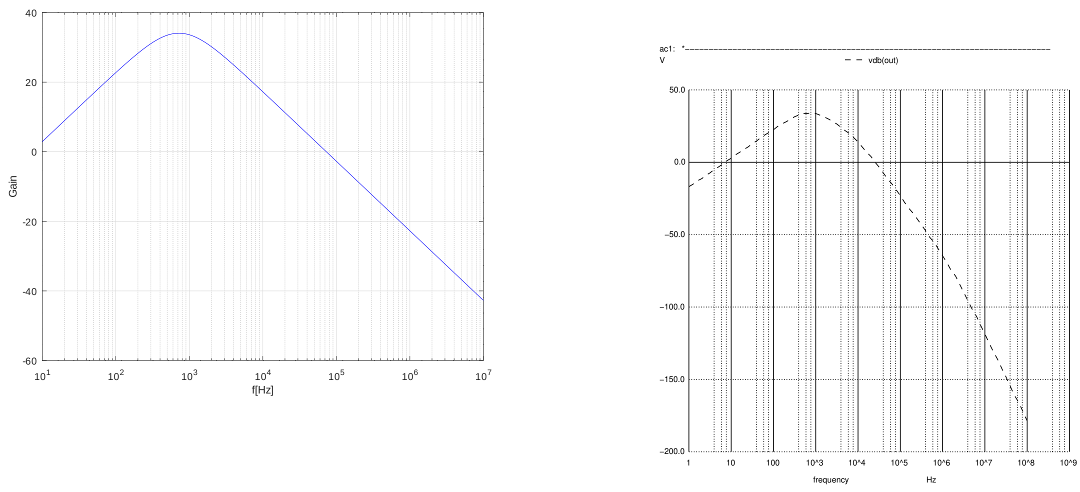
3.3 Low Cut-Off Frequency, High Cut-Off Frequency, and Central Frequency

In this section we analyse the methods used to determine the Low Cut-Off Frequency (LCF), the High Cut-Off Frequency (HCF), and the Central Frequency.
By definition the LCF and HCF are defined by the values of frequency from and to which, respectively, the gain at said frequency is greater or equal than the maximum gain divided by $\sqrt{2}$, which is equivalent to a gain 3dB lower than the maximum gain. The central frequency can be obtained by applying the geometric mean to the LCF and HCF:

$$CF = \sqrt{HCF \times LCF} \tag{9}$$

3.4 Theoretical results and comparison

With the help of *Octave* we plotted the gain as a function of the frequency $f = w/(2\pi)$. The following figure shows the frequency response from the theoretical analysis compared with the one we got from the simulation:



(b) *Ngspice*'s gain frequency response

(a) *Octave*'s gain frequency response

Figure 2: Side-by-side comparison of *Ngspice*'s and *Octave*'s frequency response

The table below presents the key results from the theoretical analysis alongside their simulation counterparts and their respective errors assuming ngspice's results as the exact results.

As we can see the results were very similar in both the simulation and the theoretical analysis.

Table 1: Theoretical Analysis

Name	**Value [A or V]**
$f_{central}(Hz)$	7.237456e+02
$Gain$	3.406583e+01
$Z_{in}(\Omega)$	1.000000e+03
$Z_{out}(\Omega)$	3.435534e+02

Table 2: Simulation Analysis

Name	**Value [A or V]**
centralfq	1.013979e+03
gain	3.354093e+01
zi	1.362822e+03
zo	7.205190e+14

4 Conclusion

In this laboratory assignment the objective of develop a audio amplifier circuit has been achieved. The simulation results have matched with the theoretical results almost perfectly, means that the central frequency, the gain, the input and output impedances are similar in the two parts of this work.