Circuit Theory and Electronics Fundamentals

Lab 2: RC Circuit Analysis

Technological Physics Engineering

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1 Introduction

The main objective of this laboratory assignment is to study the circuit depicted in 1. We decided to separate this work in three different sections.

In the first one,Theoretical Analysis 2, we will analyze the circuit in 3 different time intervals, $t < 0$, $t = 0$ and $t > 0$. For the first 2 time intervals we will obtain linear equations using methods learnt in the TCFE class, which can be solved with $Octave$. These equation allow to find the voltage in each node and the current in each branch. With $t \to \infty$, a first order linear equation is obtained, which can be solved to obtain and plot the total solution of the circuit.

Using Ngspice tools, in the second section Simulation 3, we will present a simulation of the circuit and compare it with the previous theoretical results.

Lastly, Conclusion 4, the contents of the report will be summarised and the achieved results discussed.

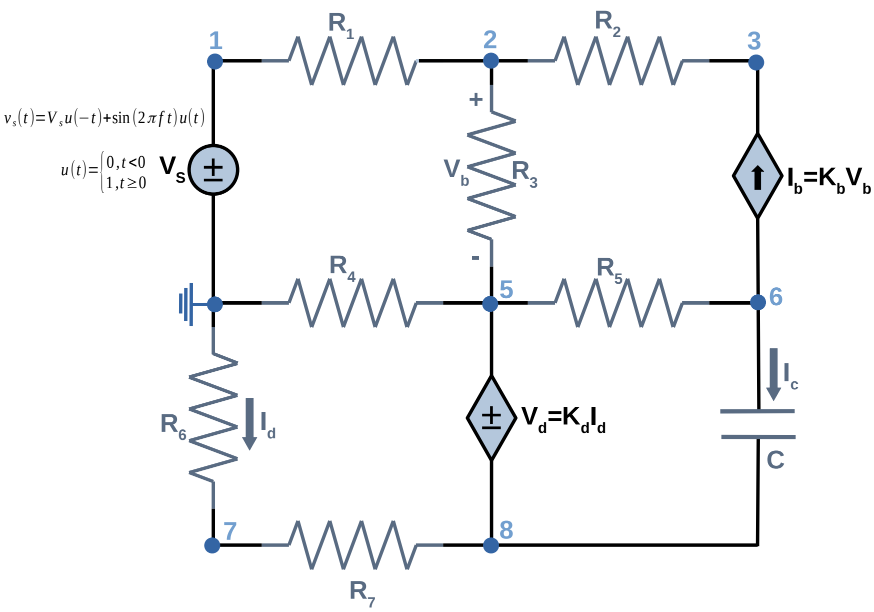

Figure 1: Circuit

2 Theoretical Analysis

First of all, we need to analyse the circuit in general. It has thirteen meshes, of which four are elementar, nine nodes and eleven branches (seven resistors, one independent voltage source V_s, one voltage controlled current source I_b, one current controlled voltage source V_d and one capacitor with charge C.

2.1 The voltages in all nodes and the currents in all branch for $t < 0$

According to information provided in the circuit, when $t < 0$, $u(t) = 0$. Therefore, $V_s(t) = V_s$, so that means $V_s(t)$ is constant. We also assume that the capacitor already has a static value, so the voltage in nodes 6 and 8 is the same, so there is no current passing through this branch and we have what is called an open circuit. In order to determine the currents and voltages in the nodes we use the Node Voltage Method. When we use the term node voltage, we are referring to the potential difference between two nodes of a circuit. This is still a voltage, it's not anything too strange. We start by labelling the nodes with numbers and assigning each with its own variable, V_1, V_2, etc. Then, we select one of the nodes in the circuit to be the reference node and call this reference node the ground node. The potential of the ground node is defined to be $0V$. The choice of the reference node is somewhat arbitrary, but it's always best to choose it based on its connections to voltage sources, because by this selection we already know the node voltages of other nodes, i.e. the ones that the reference node is connected to them by voltage sources. In our case we already have the ground in the circuit, so we use that. With this information we are able to obtain the following equations:

$$V_s = V_1$$
$$V_b = V_2 - V_5$$

We use the previous equations to facilitate the equations referring to the Node Voltage Method and we get these equations:

$$V_s = V_1$$
$$V_1(-G_1) + V_2(G_1 + G_2 + G_3) + V_3(-G_2) + V_5(-G_3) = 0$$
$$V_2(-G_2 - K_b) + V_3G_2 + V_5K_b = 0$$
$$V_2(-G_3) + V_5(G_3 + G_5 + G_4 + \frac{1}{K_d}) + V_6(-G_5) + V_8(\frac{-1}{K_d}) = 0$$
$$V_2K_b + V_5(-K_b + -G_5) + V_6(G_5) = 0$$
$$V_7(G_6 - G_7) + V_8G_7 = 0$$
$$V_5 + V_7(K_dG_6) - V_8 = 0$$

where G is the conductance of the resistors, given by the relation $G = \frac{1}{R}$. Solving these equations in order of V_n, n = {1,2,3,5,6,7,8}, allows to then simply calculate the currents and voltages in play in this circuit, using Ohm's Law, among others. Octave was used to compute the solution of this 7 equation system, giving the following solutions:

2.2 The equivalent resistance R_{eq}

To learn the equivalent resistance R_{eq} as seen from the capacitor terminals, we make $V_s = 0V$ and replace the capacitor with a voltage source $V_x = V_6 - V_8$ (V_6 and V_8 are the voltages in the nodes 6 and 8 obtained in 2.2). This way, we apply again the Node Voltage Method to determine I_x and V_x, in order to calculate the equivalent resistance $R_{eq} = \frac{V_x}{I_x}$ and then the time constante τ. Written below are the equations for the nodes:

Name	Value [A or V]
Ic	0.0
Ib	-0.00139222950
I1	-0.00132891839
I2	-0.00139222950
I3	-0.00006331111
I4	0.00097696107
I5	0.00139222950
I6	0.00035195732
I7	0.00035195732
V1	5.0737445952
V2	3.71767606204
V3	0.92740888235
V5	3.91334407167
V6	8.13366324395
V7	0.72653594426
V8	1.09343609721
Id	0.00035195732

Table 1: Current and voltage in each node

$$
\begin{gathered}
V_1 = V_0 \\
V_1G_1 + V_2(-G_1 - G_2 - G_3) + V_3G_2 + V_5G_3 = 0 \\
V_2(G_2 + K_B) - V_3G_2 - V_5K_b = 0 \\
-V_1G_1 + V_2G_1 + V_5G_4 + V_7G_6 = 0 \\
V_5 + V_7K_dG_6 - V_8 = 0 \\
V_6 - V_8 = V_x \\
V_7(-G_6 - G_7) + V_8G_7 = 0
\end{gathered}
$$

Solving this system of linear equations in $Octave$ we get:

Using the values obtained, we can find out the expression for voltages in the terminal of the capacitor. The solution has two components, the natural solution and the forced solution. Doing this procedure allows to easily find out the natural solution of the circuit.

2.3 The Natural solution

The natural solution depends on the initial voltage, equivalent resistance and capacitance of the capacitor. By solving the first order differential equation given by analyzing the circuit, we get the following natural solution:

$$v_{6n}(t) = V_x e^{\frac{-t}{R_{eq}C}}$$

Plotting the expression in a graph using the range of $[0, 20]ms$ we get:

Name	Value [A or V]
Ib	-0.00139222950
I1	0.00000000000
I2	0.00000000000
I3	0.00000000000
I4	0.00000000000
I5	-0.00227796363
I6	0.00000000000
Id	-0.00000000000
V1	0.0000000000
V2	0.00000000000
V3	0.00000000000
V5	0.00000000000
V6	7.04022714674
V7	0.00000000000
V8	0.00000000000
R_{eq}	3031.33870093000

Table 2: Table with the values of current and voltage

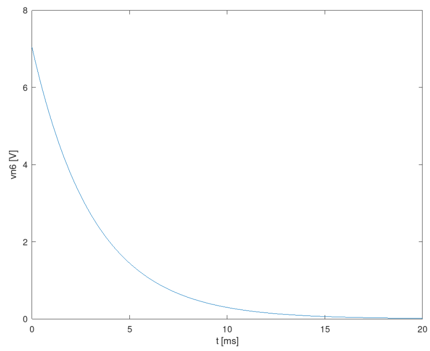

Figure 2: Representation of the Natural Solution in $[0, 20]ms$

2.4 The Forced solution

Now that we have the expression for the natural solution we need to find the expression of the forced solution in order to obtain the total solution of the circuit. This solution depends on the voltage source, equivalent resistance and the capacitance of capacitor.

$$v_{6f}(t) = sin(2\pi ft)$$

To obtain this expression for $f = 1kHz$,we start by using a phasor voltage source $V_s = 1$, replacing C with its impedance Z_c and using Node Analysis once again to determine the phasor voltages in each node. We get the following expressions:

$$\widetilde{V_s} = e^{-j\frac{\pi}{2}} = V_1$$
$$-V_1G_1 + V_2G_1 + V_5G_4 + V_7G_6 = 0$$
$$V_1G_1 + V_2(-G_1 - G_2 - G_3) + V_3G_2 + V_5G_3 = 0$$
$$V_2(G_2 + K_b) - V_3G_2 - V_5K_b = 0$$
$$-V_2K_B + V_5(K_b + G_5) + V_6(-G_5 - jwC) + V_8(jwC) = 0$$
$$V_7(-G_6 - G_7) + V_8G_7 = 0$$
$$V_5 + V_7(K_dG_6) - V_8 = 0$$

Name	**Amplitude**	**Angle [rad]**
$\tilde{V_1}$	1.0000000	-1.5707963
$\tilde{V_2}$	0.9550715	-1.5707963
$\tilde{V_3}$	0.8626260	-1.5707963
$\tilde{V_4}$	0.0	0.0
$\tilde{V_5}$	0.9615543	-1.5707963
$\tilde{V_6}$	0.6107405	1.7122338
$\tilde{V_7}$	0.4046424	1.5707963
$\tilde{V_8}$	0.6089866	1.5707963

Table 3: Complex amplitudes and angles in each node for the forced solution

2.5 The Final Total Solution

The Final Total Solution is the sum of the Natural Solution and the Forced Solution, obtained in 2.3 and 2.4 respectively:

$$v_6(t) = V_x e^{\frac{-t}{R_{eq}C}} + sin(2\pi ft)$$

Plotting the expression in a graph we get:

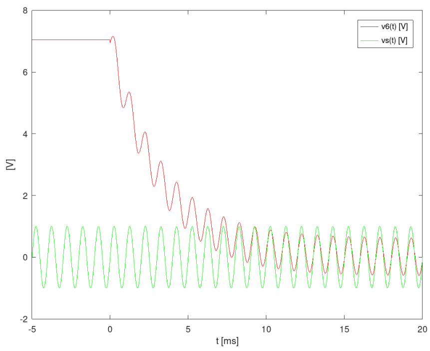

Figure 3: Representation of the Final Total Solution in $[-5, 20]ms$

2.6 The Final Total Solution for frequency range $0.1Hz$ to $1MHz$

Now we plot the graph of V_s, V_6 and $V_C = V_6 - V_8$ (db) in function of the frequency (using a logarithmic scale) and in function of the phase (degrees). We obtained the following graphics:

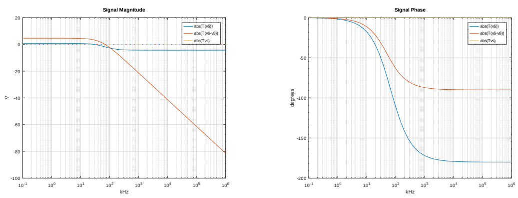

Figure 4: V(f) with frequency range $0.1Hz$ to $1MHz$ (left) and V(p) (right).

Comparing the $V(f)$ graphs, they are very distinct from each other. The magnitude of $V_s(f)$ is always ≈ 0. It never changes because it's an imposed voltage from the voltage source. $V_c(f)$ has a steady positive value for a while and then rapidly decreases permanently. $V_6(f)$ starts off constant and positive then softly decreases and stabilizes at a negative constant value. With big values of frequency, of the voltage $|V_c|$ in the capacitor also increases. Consequently, the voltage $|V_6|$ will have a bigger value. However, this value will be constant due to the exponential contribution $\exp(\frac{-t}{R_{eq}C})$.

Now looking at the $V(p)$ graphs, they are also quite different. While $V_s(p)$ is always ≈ 0 , both $V_C(p)$ and $V_6(p)$ start off at ≈ 0 but then rapidly decrease to a constant negative value ($\approx -180o$ and $\approx -85o$ respectively)

3 Simulation Analysis

3.1 Node voltage and branch current for $t < 0$

Comparing (4) to the theoretical analysis results (1), one notices that the values obtained for each node voltage are the same up to the 7th decimal place and for the branch current are the same up to the 6th decimal place. As we concluded in the last project, the exactness of the values obtained simulating the circuit vs theoretically analyzing it just proves and powerful and useful the nodal voltage method is.

3.2 Simulation of the operating point for $v_s(0) = 0$

As in 3.1, comparing (6)to the theoretical analysis results (2), one notices that the values obtained for each node voltage are the same up to the 7th decimal place and for the branch current are the same up to the 6th decimal place, proving once again the efficacy of the nodal voltage method. The capacitor is replaced with a voltage source $V_x = V_6 - V_8$, where V_6 and V_8 are the voltages in nodes 6 and 8 as obtained in order to preserve the voltage in the terminal of the capacitor. This allows to obtain the boundary conditions of the system for $t = 0$ and therefore simulate the natural response of the circuit in 3.3.

Table 4: Simulation Analysis

Name	Value [A or V]
@cc[i]	0.000000e+00
@gib[i]	-1.39223e-03
@r1[i]	1.328918e-03
@r2[i]	-1.39223e-03
@r3[i]	-6.33111e-05
@r4[i]	9.769611e-04
@r5[i]	-1.39223e-03
@r6[i]	-3.51957e-04
@r7[i]	-3.51957e-04
v(1)	5.073745e+00
v(2)	3.717676e+00
v(3)	9.274089e-01
v(5)	3.913344e+00
v(6)	8.133663e+00
v(7)	7.265359e-01
v(8)	1.093436e+00
v(9)	0.000000e+00
i(hvd)	3.519573e-04

Table 5: Theoretical Analysis

Name	Value [A or V]
Ic	0.0
Ib	-0.00139222950
I1	-0.00132891839
I2	-0.00139222950
I3	-0.00006331111
I4	0.00097696107
I5	0.00139222950
I6	0.00035195732
I7	0.00035195732
V1	5.0737445952
V2	3.71767606204
V3	0.92740888235
V5	3.91334407167
V6	8.13366324395
V7	0.72653594426
V8	1.09343609721
Id	0.00035195732

Table 6: Simulation Analysis

Name	Value [A or V]
@gib[i]	-3.81380e-18
@r1[i]	3.640368e-18
@r2[i]	-3.81380e-18
@r3[i]	-1.73431e-19
@r4[i]	-7.93568e-19
@r5[i]	-2.32248e-03
@r6[i]	2.858886e-19
@r7[i]	2.858886e-19
v(1)	0.000000e+00
v(2)	-3.71474e-15
v(3)	-1.13582e-14
v(5)	-3.17874e-15
v(6)	7.040227e+00
v(7)	-5.90152e-16
v(8)	-8.88178e-16
v(9)	0.000000e+00
i(hvd)	2.322481e-03

Table 7: Theoretical Analysis

Name	Value [A or V]
Ib	-0.00139222950
I1	0.00000000000
I2	0.00000000000
I3	0.00000000000
I4	0.00000000000
I5	-0.00227796363
I6	0.00000000000
Id	-0.00000000000
V1	0.0000000000
V2	0.00000000000
V3	0.00000000000
V5	0.00000000000
V6	7.04022714674
V7	0.00000000000
V8	0.00000000000
R_{eq}	3031.33870093000

3.3 Natural response of the circuit

In this simulation, we set $V_s = 0$ since we want the natural solution.

Looking at the graphs, the exactness is unquestionably clear. Therefore we can conclude that using Ngpice's transient analysis is equivalent to solving the first order differential equation given by theoretically analyzing the circuit. That equation leads to the natural solution of v_6, as in 2.3.

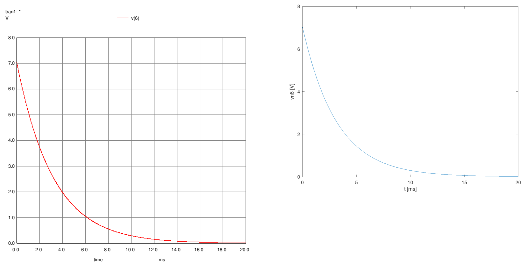

Figure 5: Graphs of the natural solution of $v_6(t)$. Simulation on the left, theoretical analysis on the right.

3.4 The Final Total Solution

Simulating the total solution (sum of the natural and forced solutions) we simulate the transient solution and obtain the graphic bellow:

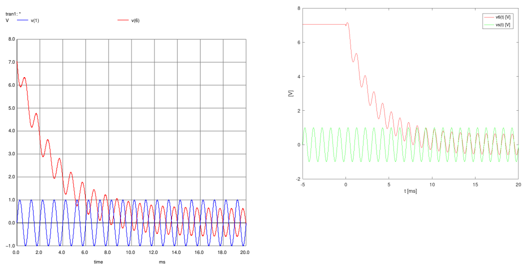

Figure 6: Graphs of the total transient solution of $v_6(t)$ and $v_s(t)$. Simulation on the left, theoretical analysis on the right.

As in 3.3, both graphs are exactly the same from $t = 0$ up until the end. Consequently, converting the phasors to real time functions for f=1KHz, and superimposing the natural and forced solutions is equivalent to repeating 3.3 with $V_s(t)$ as given in 1 and f=1kHz.

3.5 The Final Total Solution for the frequency range $0.1Hz$ to $1MHz$

Now we simulate both the frequency response of the voltage in node 6, of V_c and of V_S and the phase response of this voltages.

It is very noticeable that the graphs are the same in both the theorical analysis and the simulation.

The phase reponse graphs are also the same.

We can conclude that both analysis methods are exact and can be confidently used in future projects.

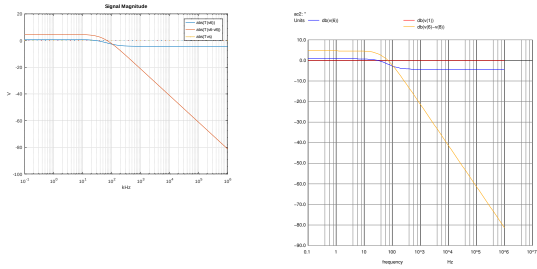

Figure 7: Representation of Total Solution in $[0, 20]ms$

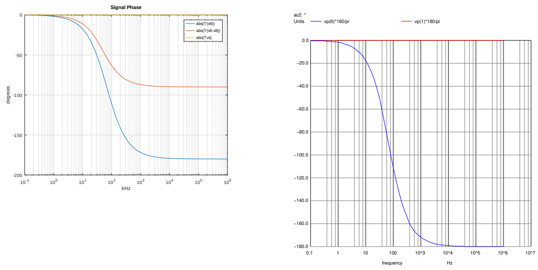

Figure 8: Representation of Total Solution in $[0, 20]ms$

4 Conclusion

In this laboratory assignment, the objective of analysing a complex electric circuit using different circuit analysis methods mesh method and node voltage method has been achieved. The circuit was theoretically analysed combining each of the circuit analysis methods and the Octave maths tool and by running a circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for the perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ.