Stock Reduction Analysis using catch at length data

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Abstract

Last thing to be written

1 Introduction

Modern stock assessments typically attempt to fit population dynamics models to catch at age and catch at length data, in hopes of extracting information from these data about age/size vulnerability and fishing mortality patterns (Methot and Wetzel, 2013; Hilborn and Walters, 1992). In cases where age data are lacking, models like MULTIFAN-CL attempt to obtain estimates of vulnerability and fishing mortality only from size distribution data (Fournier et al., 1998). Combined with a few assumptions regarding the structure and variability in length at age, this procedure can even be used to attempt to recover information about changes in body growth patterns if there is a strong age-class signal in the length frequency data (Fournier et al., 1998). Some assessment methods attempt to put aside the length frequency data, by converting these data to age compositions using age-from-length tables, perhaps using iterative methods to estimate proportions of fish at age for each length interval (Kimura and Chikuni, 1987). It is typical for assessment results from length-based assessment models to show substantial deviations between predicted and observed length distributions of catches, reflecting both sampling variation in the length composition data and incorrect assumptions about stability of growth and vulnerability patterns (Hilborn and Walters, 1992).

The vulnerability process is the combination of two processes: selectivity of the fishing gear and availability of the fished population in the area being fished (Beverton and Holt, 1957). Both processes can vary over time and therefore modify the resulting selectivity. Although selectivity process can often be directly measured through gear experiments, availability is generally harder to measure as it depends on the distribution of the exploited population. Fish movement, changes in fish distribution, and changes in fleet distribution, can all affect availability and consequently lead to vulnerability changes. Changes in vulnerability are not uncommon (Sampson and Scott, 2012) but are usually difficult to track over time. This difficulty is associated with an inability to distinguish between changes in fishing mortality and changes in vulnerability in most age and length based stock assessment methods. For this reason, many assessment methods rely on ad hoc parametric vulnerability models that may or may not include changes over time (Maunder et al., 2014). If misspecified, such models might lead to severe bias in fishing mortality estimates, which could result in misleading management advice (Martell and Stewart, 2014).

Here we suggest an alternative approach to assessment modeling that begins by assuming that the assessment model should exactly reproduce the observed catch at length distribution. This approach follows the dynamics of an age structred stock reduction analysis (SRA) (Walters et al., 2006) which follows a "conditioned on catch" format, subtracting observed catches at age from modeled numbers at age in estimation of numbers at age over time. This assumption is analogous to the classical assumption in virtual population analysis that reconstructed numbers at age should exactly match observed catch at age data (Hilborn and Walters, 1992). The suggested approach may have two key advantages over statistical catch at age and/or catch at length models: (1) it does not require estimation of age or size vulnerability schedules, and (2) catch at length data are commonly available for every year, even when age composition sampling has not been conducted.

We this approach a Length-SRA assessment model. Here we present the model formulation, the demonstrate its performance with a simulation-evaluation analysis and apply it to real fisheries data from the Peruvian jack mackerel (*Trachurus murphyi*) and Pacific Hake (*Merluccius productus*) fisheries.

2 Methods

In this section we describe the stock reduction analysis with catch at length data (Length-SRA), describe the simulation analysis and scenarios used to test the model and provide a description of the real data used to illustrate the model applicability.

2.1 Stock reduction analysis with catch at length data - length-SRA

The stock reduction analysis (SRA) described here proceeds through the following steps: (1) Compute numbers at age (based on recruitment estimates and mortality in the previous year); (2) Convert numbers at age into numbers at length based on the proportions of individuals at length given each age class; (3) Calculate the exploitation rate at length based on numbers at length and observed catch at length; (3) Convert the exploitation rate at length to exploitation rate at age; (4) Compute numbers in the following year using the exploitation rate at age, natural mortality and recruitment estimates.

A crucial component of the length-SRA is the calculation of the proportions of individual at length given each age class ($P_{l|a}$ - eqs. T3.1-T3.5). The calculation of such proportions (eq. T3.1) relies on four main assumptions regarding the distribution of length at age: (1) The mean length at age follows a von Bertalanffy growth curve (eq.T3.4), (2) The length at age is normally distributed (eqs. T3.1 -T3.3), (3) The standard deviation of the length at age is defined (e.g. eq.T3.5) and (4) $P_{L|a}$ is constant for all lengths equal or greater than L (eq.T3.3).

The proportions of length at age are used to convert the length-based quantities into age based quantities which are used to propagate the age structured population dynamics forward (Table 4 - Population dynamics). We assume that recruitment follows a Beverton-Holt type recruitment curve (eq. T3.6), that harvesting occurs over a short, discrete season in each time step (year or shorter time period) and that natural survival rate is known and stable over time (eqs. T3.6-T3.10). We used the same model structure to simulate data and as an assessment model. The computation of numbers at age in the initial year (i.e first year in which data is reported) is different from that in the remaining years (eq. T3.11). When the model is used as an stock assessment model, the recruitment in the initial year is given by the parameter R_{init}. This feature frees the model from the assumption that the population is at unfished equilibrium at the beginning of the data recording time series.

When the same model dynamics was used as an operating model, we initialized the model at unfished conditions (eq. T4.1) but only started reporting data for the simulation evaluation procedure after the *init* year. The period between the first time step and init functions is used as a burn-in period. On both the simulation and assessment models we used equilibrium yield per recruit quantities to calculate MSY and U_{MSY} (eqs.T4.6 to T4.12). These quantites depend on the vulnerability curves calculated for each year (eq. T4.8).

The stock assessment model estimates three main parameters: average unexploited recruitment R_0, recruitment compensation ratio κ and recruitment in the initial year R_{init} (we assume that the data collection for the fisheries does not start when the population is at an unfished equilibrium state). In addition, the recruitment deviations w_t are estimated for all cohorts observed in the model, that is, the number of recruitment deviations is equal to the number of years in the time series plus the number of age classes greater that recruitment age.

The objective function (eq. T5.12) is composed of a negative log likelihood component, three penalties and a prior component for the recruitment compensation ratio κ. The negative log likelihood component minimizes the differences between the predicted and observed index of abundance (eq. T5.1). We assume that such differences are lognormally distributed (eqs. T5.3-T5.4) and use the conditional maximum likelihood

Table 1: Indexes, variable definition and values used in simulation-evaluation

Symbol	Value	Description
l	$\{l_o,\ldots,L\}$	Central point of length bin, $L=15$
a	$\{a_o,\ldots,A\}$	Age-class, $A=10$
t	$\{1,\ldots,T\}$	Annual time step, $T=50$
a_o	1	First age or age of recruitment
l_o	1	Central point of first length bin
$init$	21	Annual time step in which data starts to be reported
Distribution of length given age		
L_{inf}	10	Maximum average length
k	0.3	Rate of approach to L_{inf}
t_o	-0.1	Theoretical time in which length of individuals is zero
cv_l	0.08	Coefficient of variation for length at age curve
$P_{l\|a}$		Matrix of proportions of length at age
Φ		Standard normal distribution
$z1_{a,l}$		Normalized z score for lower limit length bins
$z2_{a,l}$		Normalized z score for upper limit length bins
$b1_l$		Lower limit of length bins
$b2_l$		Upper limit of length bins
$\bar{L}_a$		Mean length at age
σ_{L_a}		Standard deviation of length at age
Population dynamics		
R_o	100	Average unfished recruitment
κ	10	Goodyear recruitment compensation ratio
S	0.7	Natural survival
σ_{rec}	0.4	standard deviation for recruitment deviations
w_t	$\mathcal{N}(0,\sigma_R)$	Recruitment deviations for years {init-A+1,...,T}
$N_{a,t}$		Numbers of fish at age and time
SB_t		Spawning biomass at time
mat_a		Proportion of mature individuals at age
a_{rec}, b_{rec}		Beverton & Holt stock recruitment parameters
R_{init}		Recruitment in the year data starts to be reported
$U_{a,init}$		Exploitation rate at age before data starts being reported.
$U_{a,t}$		Exploitation rate at age and time
$U_{l,t}$		Exploitation rate at length and time
$C_{l,t}$		Catch at length and time
$N_{l,t}$		Numbers at length and time
lx_a		Unfished survivorship at age
Φ_e		Unfished average spawning biomass per recruit

Table 2: Indexes, variable definition for operating model and MSY quantities

Symbol	Value	Description
Operating model		
$sel_{l,t}$		Fishing selectivity at length and time
g,a,b		Parameters for selectivity function
U_t		Annual maximum exploitation rate
$C_{l,t}$		Catch at length and time
$N_{l,t}$		Numbers at length and time
I_t		Index of abundance at time
VB_t		Biomass that is vulnerable to the survey at time
q	1.0	catchability coefficient
MSY quantities		
lz_a		
U_z	seq(0.0,1.0,by=0.001)	Hypothetical exploitation rates to calculate MSY
Φ_z		Unfished average spawning biomass per recruit
Φ_{eq}		Fished under U_z average spawning biomass per recruit
sel_a		Selectivity at age
R_{eq}		Average fished recruitment under U_z
$Yield_z$		Equilibrium yield under U_z
MSY		Maximum sustainable yield based on optimum spawner per recruit
U_{MSY}		Exploitation rate that leads to maximum sustainable yield

estimator described by Walters and Ludwig (1994) to estimate the catchability coefficient q (eq. T5.2). A lognormal penalty is added to the negative log-likelihood function to constrain annual recruitment residuals so that the estimates have mean of zero and fixed standard deviation σ_R (eq. T5.5). the second penalty, $P_{U_{max}}$ was implemented in order to prevent estimated values of exploitation rate at length greater than one ($U_{l,t} > 1$) (eq. T5.9) and the third penalty term P_U was added in order to limit variability in vulnerability at length across the years, and therefore limit the influence of observation error on vulnerability estimates. Lastly, a normal prior for $log(\kappa)$ was included in the objective function (eq. T5.11).

2.2 Simulation evaluation

In order to perform a simulation evaluation of the Length-SRA under various scenarios we used the same model dynamics described in Table 4 - Population dynamics, as an operating model. However we modified the model to control annual exploitation rate (eq. T4.2), time varying selectivity (eq. T4.4), and observation and process errors. Selectivity in the operating model was computed with the three parameter selectivity equation described by Thompson (1994) (eq. T4.4). The observation error in the operating model included lognormal

Table 3: population dynamics for Length-SRA and operating model

	Distribution of length given age
(T3.1)	$P_{l\|a} = \int_{z1_{a,l}}^{z2_{a,l}} \Phi(z)dz$
(T3.2)	$z1_{a,l} = \frac{b1_l - \bar{L}_a}{\sigma_{L_a}}$
(T3.3)	$z2_{a,l} = \begin{cases} \frac{b2_l - \bar{L}_a}{\sigma_{L_a}} & l < L \\ 1.0 & l = L \end{cases}$
(T3.4)	$\bar{L}_a = L_{inf} \cdot (1 - \exp^{(-k\cdot(a-t_o))})$
(T3.5)	$\sigma_{L_a} = \bar{L}_a \cdot cv_l$
	Population dynamics
(T3.6)	$N_{a,t>init} = \begin{cases} \frac{a_{rec}\cdot SB_{t-1}}{1+b_{rec}\cdot SB_{t-1}} \cdot e^{w_t}, & a = a_o \\ N_{a-1,t-1} \cdot S \cdot (1 - U_{a-1,t-1}), & a_o < a < A \\ \frac{N_{a-1,t-1}\cdot S\cdot(1-U_{a-1,t-1})}{1-S\cdot(1-Ua,t)}, & a = A \end{cases}$
(T3.7)	$U_{a,t} = \sum_a (P_{l\|a} \cdot U_{l,t})$
(T3.8)	$U_{l,t} = \frac{C_{l,t}}{N_{l,t}}$
(T3.9)	$N_{l,t} = \sum_a (P_{l\|a} \cdot N_{a,t})$
(T3.10)	$SB_t = \sum_a (mat_a \cdot w_a \cdot N_{a,t})$
	Initial year and incidence functions
(T3.11)	$N_{a,init} = \begin{cases} R_{init} \cdot e^{w_{init}} & a = a_o \\ N_{a-1,init} \cdot S \cdot (1 - U_{a-1,init}) \cdot e^{w_{init-a+1}}, & a_o < a < A \\ \frac{N_{a-1,init}\cdot S\cdot(1-U_{a-1,init})}{1-S\cdot(1-Ua,init)} \cdot e^{w_{init-a+1}}, & a = A \end{cases}$
(T3.12)	$a_{rec} = \frac{\kappa}{\phi_e}$
(T3.13)	$b_{rec} = \frac{\kappa - 1}{R_o \cdot \phi_e}$
(T3.14)	$\Phi_e = \sum_a lx_a \cdot mat_a \cdot w_a$
(T3.15)	$lx_a = \begin{cases} 1, & a = 1 \\ lx_{a-1} \cdot S, & 1 < a < A \\ \frac{lx_{a-1}\cdot S}{1-S}, & a = A \end{cases}$

Table 4: MSY quantities and operating model

	Operating model
(T4.1)	$N_{a,t=1} = lx_a * R_o$
(T4.2)	$U_{l,t} = U_t \cdot sel_{l,t}$
(T4.3)	$C_{l,t} = N_{l,t} \cdot U_{l,t} \cdot P_{l\|a}$
(T4.4)	$sel_{l,t} = \frac{1}{1-g} \cdot \left(\frac{1-g}{g}\right)^g \cdot \frac{e^{a \cdot g \cdot (b-l)}}{1+e^{a \cdot (b-l)}}$
(T4.5)	$I_t = q \cdot VB_t \cdot e^{(\mathcal{N}(0,\sigma_{I_t})}$
	MSY quantities
(T4.6)	$lz_a = \begin{cases} lz_a = 1 & a = a_o \\ lz_{a-1} \cdot S \cdot (1-U_z) & a_o < a < A \\ \frac{lz_{a-1} \cdot S \cdot (1-U_z)}{1-S*(1-U_z)} & a = A \end{cases}$
(T4.7)	$\Phi_z = \sum_a lz_a \cdot mat_a \cdot w_a$
(T4.8)	$\Phi_{eq} = \sum_a lz_a \cdot sel_a \cdot w_a$
(T4.9)	$R_{eq} = R_o \cdot \frac{\kappa - \Phi_e/\Phi_z}{\kappa - 1}$
(T4.10)	$Yield_z = U_z * R_{eq} * \Phi_{eq}$
(T4.11)	$MSY = max(Yield_z)$
(T4.12)	$U_{MSY} = U_z \rightarrow max(Yield_z)$

Table 5: Likelihood functions and penalties

Conditional Likelihood	
(T5.1)	$Z_t = log(I_t) - log(VB_t)$
(T5.2)	$q = e^{\bar{Z}}$
(T5.3)	$Zstat = Z - \bar{Z}$
(T5.4)	$LL_1 \sim \mathcal{N}(Zstat \mid \mu = 0, \sigma = \sigma_{I_t})$
Penalties	
(T5.5)	$P_{wt} \sim \mathcal{N}(wt \mid \mu = 0, \sigma = \sigma_R)$
(T5.6)	$\mu_{U_{t-2:t}} = \left(\sum_{t-2}^{t} \frac{U_{l,t}}{\bar{U}_t} \right) / 3$
(T5.7)	$U_t^{pen} = \left(\frac{U_{l,t}}{\bar{U}_t} - \mu_{U_{t-2:t}} \right)^2$
(T5.8)	$SS_{vul} = \sum_{t=3}^{t=T} U_t^{pen}$
(T5.9)	$P_U = \frac{SS_{vul}}{\sigma_{vul}}$
(T5.10)	$P_{U_{max}} = \sum^{t} \sum^{l} U_{t,l}{}^{10}$
Priors	
(T5.11)	$prior(log(\kappa)) \sim \mathcal{N}(log(true\kappa), 0.9)$
Objective function	
(T5.12)	$Obj = -log(LL_1) + -log(P_{wt}) + P_U + P_{U_{max}} + prior(log(\kappa))$

error in the index of abundance and logistic multivariate error in the catch numbers at length. Recruitment deviations were assumed to be lognormally distributed.

We considered a total of six different scenarios in the simulation evaluation runs. Three different historical exploitation rate trajectories were used: contrast, one way trip and U ramp. In the contrast scenario the exploitation rate (U_t) starts low and increases beyond U_{msy} and then decreases until $U_t = U_{msy}$. In the one way trip scenario U increased through time until $U = 2 \cdot U_{msy}$. In the U ramp scenario, U_t increases steadily until $U_t = U_{msy}$ and remains constant thereafter. In addition to the exploitation rate scenarios, we considered two selectivity scenarios: constant and time varying selectivity. In the constant selectivity scenario, selectivity was assumed to follow a sigmoid shape. In the time varying selectivity scenario, the selectivity curve was assumed to vary every year, progressively changing form a dome shaped curve to sigmoid and back to dome shaped. The complete list of scenarios and the acronym used for them is presented in Table 6.

All simulation runs had 30 years of data and we used 100 simulation runs for each scenario. We evaluated the distribution of the % relative error ($\frac{esimated - simulated}{simulated}$) for the main parameter estimates (R_0, R_{init} and κ) and for four derived quantities (Depletion: $\frac{B_t}{B_0}$, MSY, U_{MSY} and q).

Table 6: Simulation-estimation scenarios

Scenario Code	Selectivity	U trajectory
CC	constant	contrast
CO	constant	one way trip
CR	constant	U ramp
VC	time-varying	contrast
VO	time-varying	one way trip
VR	time-varying	U ramp

2.3 Real data examples

Two species were chosen to illustrate the application of the Length-SRA to real datasets: Chilean jack mackerel and Pacific hake. Both species are believed to be subject to time varying selectivity.

The Pacific hake fishery is believed to exhibit time varying selectivity due to cohort targeting and annual changes fleet spatial distribution. The population is know to have spasmodic recruitment, with high recruitment events occurring once or twice every decade (Ressler et al., 2007). Pacific hake tends to segregate by size during their annual migration(Ressler et al., 2007), allowing the fishing fleet to target the strong cohorts by changing the spatial distribution of fishing effort as the cohort ages. Hake catch at length data was available for the period between 1975 and 2013. The survey index of abundance was available itermitently from 1995 to 2013.

The movement pattern of jack mackerel is not as well known, although fish appear to move between spawning and feeding areas (Gerlotto et al., 2012). Variability in selectivity patterns for the jack mackerel fishery are believed to be associated both with evolution of fleet capacity and gear utilization and with compression and expansion of the species range associated with abundance changes (Gerlotto et al., 2012). Jack mackerel catch at length data was available from 1980 to 2013 and the survey index was avaialble for the years between 1986 and 2013, with the exception of the year of 2010.

3 Results

3.1 Simulation-evaluation

Even though the parameter estimates were not entirely unbiased, the amount of bias was relatively low, especially for the leading parameters R_0 and κ (Figure 1). The parameter R_0 was slightly accurately estimated for the contrast and one-way-trip scenarios, with median % errors between -1.8% and 2.5%. In the U ramp scenarios, R_0 was underestimated with with median % errors of -5.9% (CR) and -10.6% (VR). The parameter R_{init} was underestimated for all scenarios with relatively high median % errors varying between -14.4% and -33.9%. Once again the U ramp scenarios produced the highest bias (Figure 1). The κ parameter was the most accurately and precisely estimated with median % errors varying between 0.5% and 2.4%.

In relation to the derived quantities (Figure 2), the Length-SRA model tended to underestimate depletion with median % errors ranging between -17.0% and -1.3%. MSY was also underestimated for all scenarios, with median % errors ranging between - 15.8% and -4.6%. The estimates for U_{MSY} showed very low (<3%) median % errors for all scenarios, and were underestimated for the time varying selectivity scenarios and overestimated for the constant selectivity scenarios. The estimates of q were positively biased, particularly for the the U ramp scenarios (CR and VR), this is likely to be associated with the underestimations of both R_0 and R_{init} for those scenarios.

The simulation-evaluation exercise showed that the Length-SRA model is able to track selectivity changes over time (Figure 3). However, the estimates of selectivity are less accurate in the initial year of data (year 21) for all scenarios and particulatly unnacurate for the Uramp scenarios. Precision and accuracy of selectivity estimates were better for the contrast and one-way trip scenarios for both the constant and time-varying selectivity.

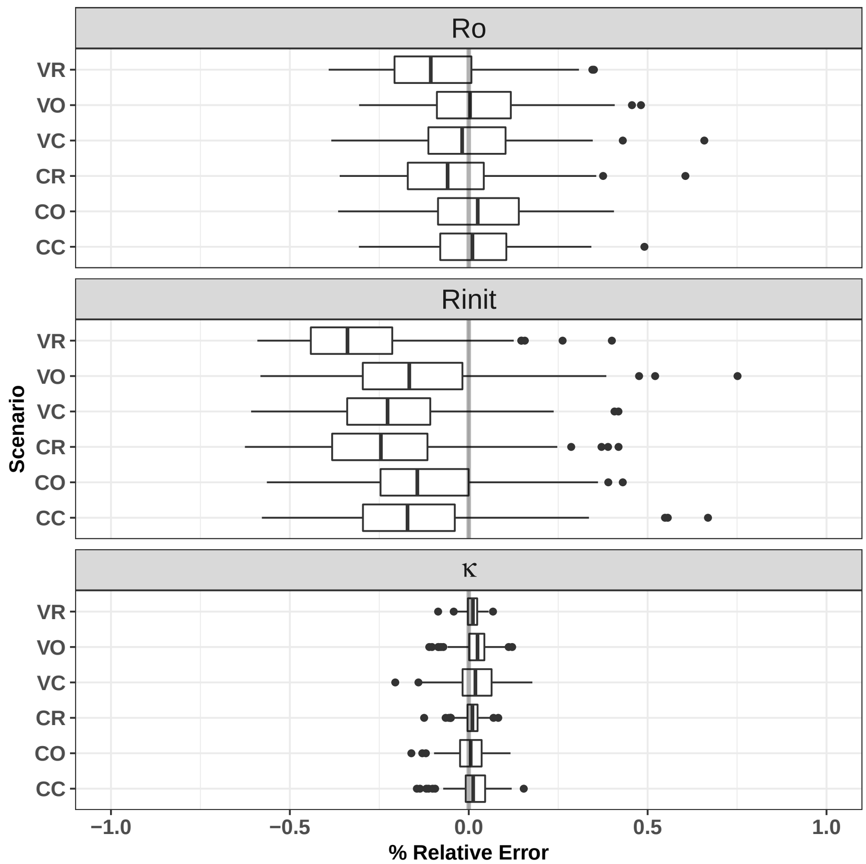

Figure 1: Relative error (%) for main parameters for all scenarios considered in the simulation-evaluation.

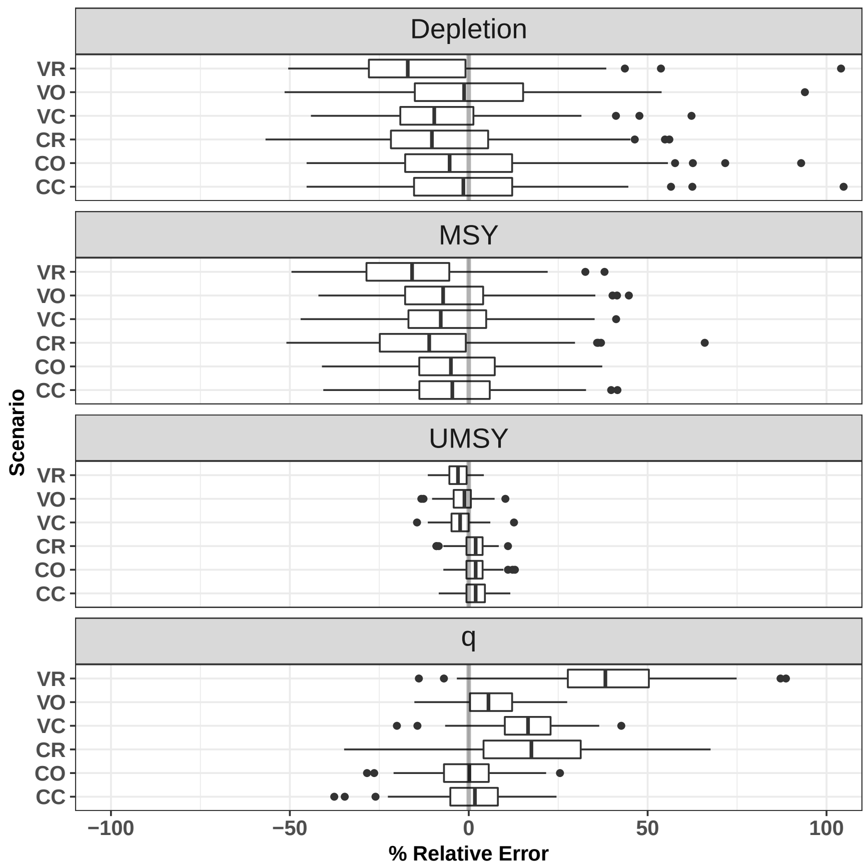

Figure 2: Relative error (%) for main parameters for all scenarios considered in the simulation-evaluation.

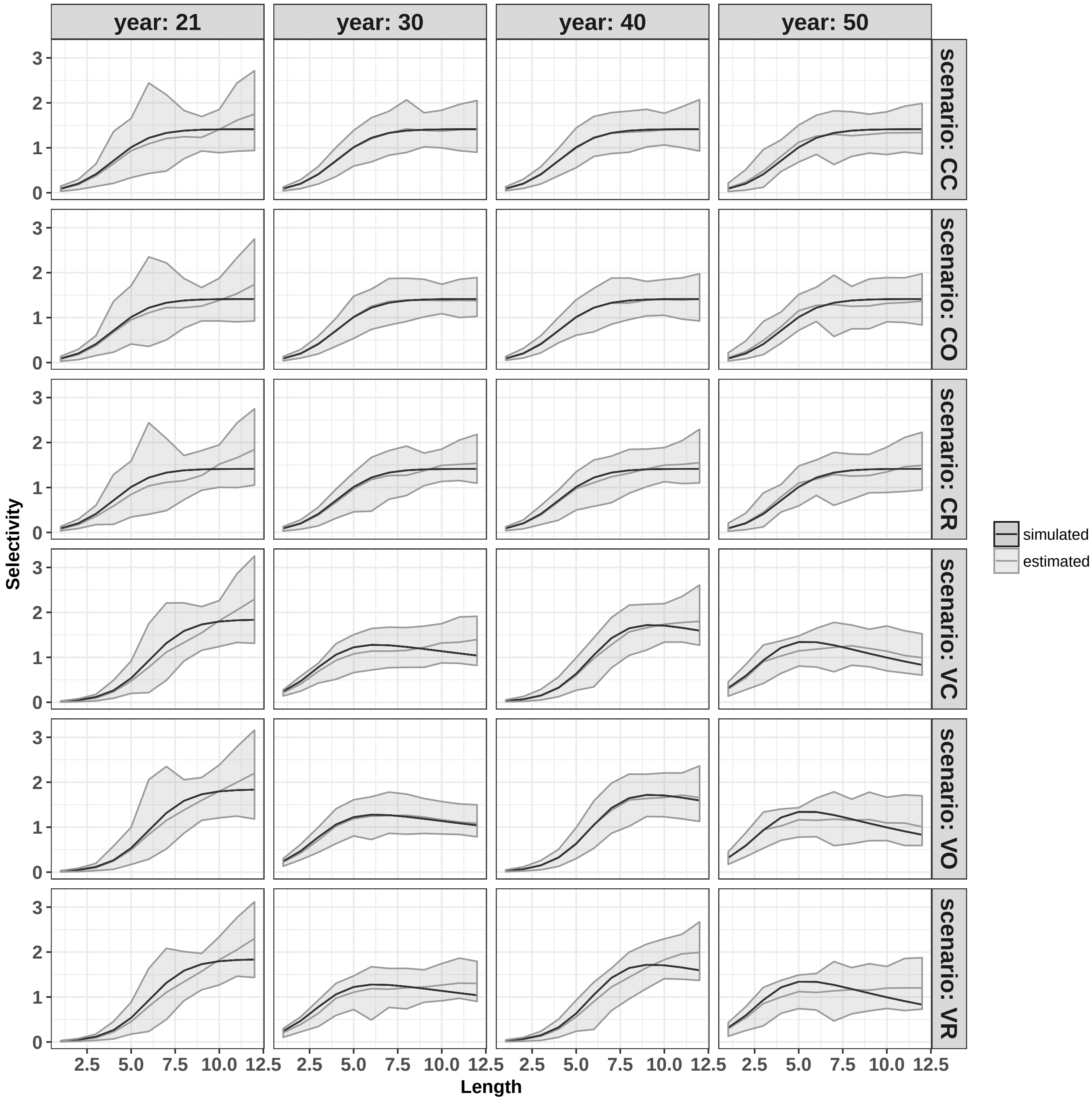

Figure 3: Simulated and realized selectivity estimates for a set of years within simulation-evaluation time series.

3.2 Real data examples

The model fit to the Pacific hake and jack mackerel indexes of abundance relatively well (Figure 4). The model fit for both species resulted in time varying selectivities that lead to variability in the MSY and U_{MSY} estimates.

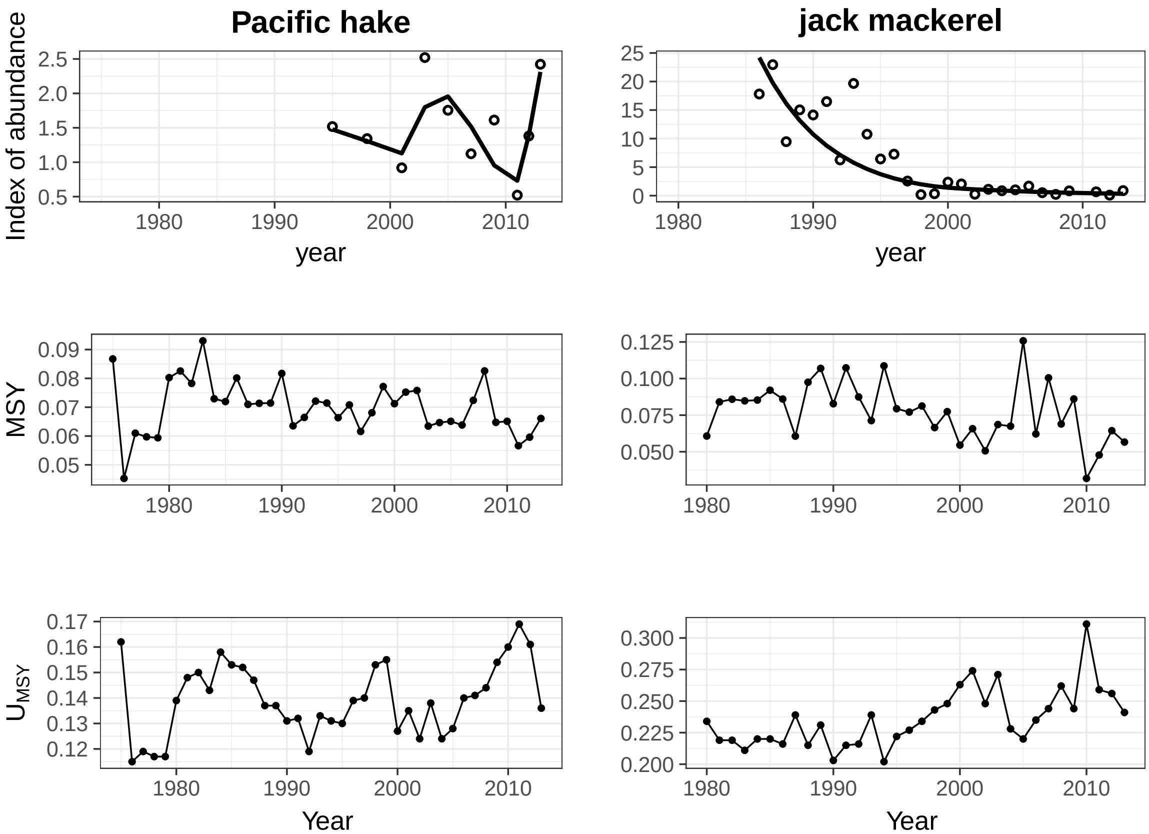

Figure 4: Fit to index of abundance for Pacific hake and jack mackerel.

4 Discussion

We present a length-based stock reduction analysis (SRA) that allows monitoring of time varying selectivity. In this model catch at length is assumed to be known without error and exploitation rate at length is derived directly from the estimates of numbers at length. This fact is important because it allows the mode to bypass the requirement for the estimation of a selectivity ogive, as is required in other length based models (Sullivan et al., 1990; Fournier et al., 1998).

Selectivity parameters can be particularly hard to estimate, especially if it changes over space or time (Martell and Stewart, 2014). Accurate estimates of selectivity are particularly important if the fishery is managed based on yield per recruit reference points. The yield per recruit of a fishery depends on the selectivity curve (Vasilakopoulos et al., 2016) and for this reason, changes in selectivity over time will directly affect values reference points.

The approach used in the length-based SRA described here is analogous to that used in virtual population analysis (VPA). Here we assume that catch at length in numbers is assumed to known without error. Therefore selectivity estimates are the result of the estimated exploitation rate and observation and sampling error. Because of this assumption, it is important that the catch at length sampling is representative of the total removals from the population. Biased sampling will lead to biased estimates of selectivity and result in bias in the fishery reference points.

Main talking points:

- The model is not that good, main parameters are biased and selectivity estimates are not particularly accurate or precise
- Why is this failing?
- Is this still useful?
- Depletion and MSY are underestimated - the model tends to produce conservative benchmarks.
- U ramp scenario yield very bad results - lack of information in the time series.
- Time- varying growth might result in even worse results. Suggest estimating cohort-specific growth curves or implementing the density dependent functions shown in multifan-CL?
- Further testing of this model in a closed-loop simulation set up would provide more insight on the model performance on achieving management outcomes

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