

# Stock Reduction Analysis using catch at length data

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## Abstract

Last thing to be written

## 1 Introduction

Modern stock assessments typically attempt to fit population dynamics models to catch at age and catch at length data, in hopes of extracting information from these data about age/size vulnerability and fishing mortality patterns (Methot and Wetzel, 2013; Hilborn and Walters, 1992). In cases where age data are lacking, models like MULTIFAN-CL attempt to obtain estimates of vulnerability and fishing mortality only from size distribution data (Fournier et al., 1998). Combined with a few assumptions regarding the structure and variability in length at age, this procedure can even be used to attempt to recover information about changes in body growth patterns if there is a strong age-class signal in the length frequency data (Fournier et al., 1998). Some assessment methods attempt to put aside the length frequency data, by converting these data to age compositions using age-from-length tables, perhaps using iterative methods to estimate proportions of fish at age for each length interval (Kimura and Chikuni, 1987). It is typical for assessment results from length-based assessment models to show substantial deviations between predicted and observed length distributions of catches, reflecting both sampling variation in the length composition data and incorrect assumptions about stability of growth and vulnerability patterns (reference).

The vulnerability process is the combination of two processes: selectivity of the fishing gear and availability of the fished population in the area being fished (Beverton and Holt, 1957). Both processes can vary over time and therefore modify the resulting selectivity. Although selectivity process can often be directly measured through gear experiments, availability is generally harder to measure as it depends on the distribution of the exploited population. Changes in availability occur when changes in the spatial distribution of the fleet in

25 relation to the fished population happen. Availability changes can be caused by fish movement, changes in  
26 fish distribution, and by changes in fleet distribution. Changes in vulnerability are not uncommon (Sampson  
27 and Scott, 2012) but are usually difficult to track overtime. This difficulty is associated with the inability to  
28 distinguish between changes in fishing mortality and changes in vulnerability in most age and length based  
29 stock assessment methods. For this reason, many assessment methods rely on ad hoc parametric vulnerability  
30 models that may or may not include changes over time (Maunder et al., 2014). If misspecified, such models  
31 might lead to severe bias in fishing mortality estimates, which could result in misleading management advice  
32 (Martell and Stewart, 2014).

33 Here we suggest an alternative approach to assessment modeling that begins by assuming that the assess-  
34 ment model should exactly reproduce the observed catch at length distribution. This is similar to the classical  
35 assumption in virtual population analysis that reconstructed numbers at age should exactly match observed  
36 catch at age data (Hilborn and Walters, 1992). This assumption is also analogous to the suggestion by Schnute  
37 (1994) that statistical catch at age models might best be run in a “conditioned on catch” format by subtracting  
38 observed catches at age from modeled numbers at age in estimation of numbers at age over time. The sug-  
39 gested approach may have two key advantages over statistical catch at age and/or catch at length models: (1)  
40 it does not require estimation of age or size vulnerability schedules, and (2) catch at length data are commonly  
41 available for every year, even when age composition sampling has not been conducted.

42 We demonstrate the performance of this model with a simulation-evaluation analysis and apply it to real  
43 fisheries data from the Chilean jack mackerel and Pacific Hake fisheries.

## 44 **2 Methods**

45 In this section we describe the stock reduction analysis with catch at length data (Length-SRA), describe  
46 the simulation analysis and scenarios used to test the model and provide a description of the real data used to  
47 illustrate the model applicability.

### 48 **2.1 Stock reduction analysis with catch at length data**

49 The stock reduction analysis described here starts by calculating , the proportions of individual at length  
50 for each age class (Table 1. The calculation of such proportions relies on three main assumptions regarding the  
51 distribution of length at age: (1) The mean length at age follows a von Bertalanffy growth curve (eq.T1.5), (2)

**Table 1: Age at Length**

Variable definition	
$P_{l a}$ = Matrix of proportions of length at age	
$z1_{a,l}$ = Normalized Z score for lower limit length bins	
$z2_{a,l}$ = Normalized Z score for upper limit length bins	
$b1_l$ = Lower limit of length bins	
$b2_l$ = upper limit of length bins	
$\bar{L}_a$ = Mean length at age	
$\sigma_{L_a}$ = Standard deviation of length at age	
$L_{inf}$ = Maximum average length	
$k$ = rate of approach to $L_{inf}$	
$t_o$ = Theoretical time in which length of individuals is zero	
$cvL$ = Coefficient of variation for length curve	(T1.1)
<hr/>	
$P_{l a} = \int_{z1_{a,l}}^{z2_{a,l}} \mathcal{N}(0, 1)$	(T1.2)
$z1_{a,l} = \frac{b1_l - \bar{L}_a}{\sigma_{L_a}}$	(T1.3)
$z2_{a,l} = \frac{b2_l - \bar{L}_a}{\sigma_{L_a}}$	(T1.4)
$\bar{L}_a = L_{inf} \cdot (1 - \exp^{-k \cdot (a - t_o)})$	(T1.5)
$\sigma_{L_a} = \bar{L}_a \cdot cvL$	(T1.6)
<hr/>	

52 The length at age is normally distributed (eqs. T1.2 -T1.4) and (3) The standard deviations of the length at age  
53 distributions is given by the product of the mean length at age and a constant CV (eq.T1.6).

54 The proportions of length at age is used to convert the length based quantities into age based quantities  
55 which are used to propagate the age structured population dynamics forward (Table 2). We assume that recruit-  
56 ment follows a Beverton & Holt type recruitment curve (eq. T2.1), that harvesting occurs over a short, discrete  
57 season in each time step (year or shorter time period) and that natural survival rate is stable over time (eqs.  
58 T2.1-T2.5). Differences in the population dynamics equations in the initial year as well as incidence functions  
59 are shown in Table 2.

**Table 2:** Population dynamics

Variable definition	
$N_{a,t}$	Numbers of fish at age and time
$SB_t$	Spawning biomass at time t
$a_{rec}, b_{rec}$	Beverton & Holt stock recruitment parameters
$\kappa$	Goodyear recruitment compensation ratio
$wt$	Normally distributed recruitment deviations
$S_a$	Survival rate at age
$U_{a,t}$	Exploitation rate at age and time
$U_{l,t}$	Exploitation rate at length and time
$C_{l,t}$	Catch at length and time
$N_{l,t}$	Numbers at length and time
$syr$	Initial year of data
$a_o$	Age of recruitment
$\phi_e$	Unfished average spawning biomass per recruit

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$$N_{a,t > syr} = \begin{cases} \frac{a_{rec} \cdot SB_{t-1}}{1 + b_{rec} \cdot SB_{t-1}} \cdot e^{wt}, & a = a_o \\ N_{a-1,t-1} \cdot S_{a-1} \cdot (1 - U_{a-1,t-1}), & 1 < a < A \\ \frac{N_{a-1,t-1} \cdot S_{a-1} \cdot (1 - U_{a-1,t-1})}{1 - S_A \cdot 1 - U_{a,t}}, & a = A \end{cases} \quad (T2.1)$$

$$U_{a,t} = \sum_a (P_{l|a} \cdot U_{l,t}) \quad (T2.2)$$

$$U_{l,t} = \frac{C_{l,t}}{N_{l,t}} \quad (T2.3)$$

$$N_{l,t} = \sum_a (P_{l|a} \cdot N_{a,t}) \quad (T2.4)$$

$$SB_t = \sum_a (fec_a \cdot w_a \cdot N_{a,t}) \quad (T2.5)$$


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Initial year

$$N_{a_o, syr} = R_{init} * e^{wt} \quad (T2.6)$$

$$a_{rec} = \frac{\kappa}{\phi_e} \quad (T2.7)$$

$$b_{rec} = \frac{\kappa - 1}{R_o \cdot \phi_e} \quad (T2.8)$$

$$\phi_e = \sum_a lx_a \quad (T2.9)$$

$$lx_a = \begin{cases} 1, & a = a_o \\ lx_{a-1} \cdot S_{a-1}, & 1 < a < A \\ \frac{lx_{a-1} \cdot S_{a-1}}{1 - S_A}, & a = A \end{cases} \quad (T2.10)$$


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**Table 3: Operating model dynamics**

Variable definition	
$U_{l,t}$ = Exploitation rate at length and time	
$sel_{l,t}$ = fishing selectivity at length and time	
$U_t$ = annual maximum exploitation rate	
$C_{l,t}$ = Catch at length and time	
$N_{l,t}$ = Numbers at length and time	
$I_t$ = Index of abundance at time t	
$VB_t$ = Biomass that is vulnerable to the survey at time t	
$q$ = catchability coefficient	
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$U_{l,t} = U_t \cdot sel_{l,t}$	(T3.1)
$C_{l,t} = N_{l,t} \cdot U_{l,t} \cdot P_{l a}$	(T3.2)
$sel_{l,t} = \frac{1}{1-g} \cdot \left( \frac{1-g}{g} \right)^g \cdot \frac{e^{a \cdot g \cdot (b-l)}}{1 + e^{a \cdot (b-l)}}$	(T3.3)
$I_t = q \cdot VB_t$	(T3.4)

60 The model estimates three main parameters: the average unexploited recruitment  $R_0$ , the recruitment com-  
61 pensation ratio  $\kappa$  and the recruitment in the initial year  $R_{init}$ . In addition, the recruitment deviations are esti-  
62 mated for all cohorts observed in the model, that is, the number of recruitment deviations is equal to the number  
63 of years in the time series plus the number of age classes greater than recruitment age. The parameters of the  
64 model are estimated with two likelihood components: Index of abundance and Recruitment deviations, both  
65 are assumed to be lognormally distributed with fixed variances.

## 66 2.2 Simulation evaluation

67 In order to perform a simulation evaluation of the Length-SRA under various scenarios we used the same  
68 model dynamics described in Table 2 as an operating model. However we modified the model population  
69 dynamics to include controlled time varying selectivity, a maximum annual exploitation rate (eq. T3.1) and  
70 observation and process errors. Selectivity in the operating model was computed with the three parameter  
71 selectivity equation described by Thompson (1994) (eq. T3.3). The observation error in the operating model  
72 included lognormal error in the index of abundance and logistic multivariate error in the catch numbers at  
73 length. Recruitment deviations were assumed to be lognormally distributed.

We considered a total of six different scenarios in the simulation evaluation runs. Three different historical exploitation rate trajectories were used: contrast, one way trip and  $U$  ramp. In the contrast scenario the exploitation rate( $U$ ) starts low and increases up to  $U > U_{msy}$  and then decreases until  $U = U_{msy}$ . In the one way trip scenario  $U$  increased through time until  $U = 2 \cdot U_{msy}$ . In the  $U$  ramp scenario,  $U$  increases steadily until  $U = U_{msy}$  and remains constant thereafter. In addition to the exploitation rate scenarios, we considered two selectivity scenarios: constant and time varying selectivity. In the constant selectivity scenario, selectivity was assumed to follow a sigmoid shape. In the time varying selectivity scenario, the selectivity curve was assumed to vary every year, progressively changing from a dome shaped curve to sigmoid and back to dome shaped. The complete list of scenarios and the code used for them is presented in Table 4.

All simulation runs had 30 years of data and we used 200 simulation runs for each scenario. We evaluated the distribution of the % relative error ( $\frac{estimated-simulated}{simulated} \cdot 100$ ) for the main parameter estimates ( $R_0$ ,  $R_{init}$  and  $\kappa$ ) and for four derived quantities (Depletion:  $\frac{B_t}{B_0}$ ,  $MSY$ ,  $U_{MSY}$  and  $q$ ).

**Table 4:** Simulation-estimation scenarios

Scenario Code	Selectivity	$U$ trajectory
CC	constant	contrast
CO	constant	one way trip
CR	constant	$U$ ramp
VC	time-varying	contrast
VO	time-varying	one way trip
VR	time-varying	$U$ ramp

## 2.3 Real data examples

Two species were chosen to illustrate the application of the Length-SRA to real datasets: Chilean jack mackerel and Pacific hake. Both species are believed to be subject to time varying selectivity. In the case of Pacific hake, time varying selectivity is believed to be associated with cohort targeting and fleet spatial distribution. The population is known to have spasmodic recruitment, with high recruitment events occurring once or twice every decade (Ressler et al., 2008). Pacific hake tends to segregate by size during their annual migration (Ressler et al., 2008), allowing the fishing fleet to target the strong cohorts by changing the spatial distribution of fishing effort as the cohort ages. In the case of jack mackerel, the movement pattern is not as well known. But fish do appear to move between spawning and feeding areas (Gerlotto et al., 2012). Variability in selectivity patterns for the jack mackerel fishery are believed to be associated both with evolution of fleet

96 capacity and with compression and expansion of the species range associated with abundance changes (Gerlotto  
97 et al., 2012).

## 98 **3 Results**

### 99 **3.1 Simulation-evaluation**

100 The simulation-evaluation showed that the parameters  $R_0$  and  $R_{init}$  tend to be underestimated but with very  
101 low bias. The median % errors for those parameters were within the  $\pm 10\%$  interval, with the exception of  
102 the  $U$  ramp scenarios (CR and VR) in which the median % error for  $R_0$  and  $R_{init}$  was as high as 25% (Figure  
103 1). The parameter  $\kappa$  was underestimated in all scenarios with higher median % error varying between 9% and  
104 40%. Once again, the  $U$  ramp scenarios (CR and VR) resulted in the highest bias (Figure 1).

105 In relation to the derived quantities, the Length-SRA tended to underestimate both depletion and MSY  
106 estimates with median % errors ranging between -26% and -2% . Once again the  $U$  ramp scenarios (CR and  
107 VR) yielded the worst bias (Figure 2). The estimates for  $U_{MSY}$  showed very low (<5%) median % errors for all  
108 scenarios except for the CC scenario (Contrast and constant selectivity) which showed a 13% median % error  
109 (Figure 2). The estimates of  $q$  tended to be underestimated for the  $U$  ramp scenarios and overestimated for the  
110 remaining scenarios, with median % error ranging between -3% and 10% (Figure 2).

111 The simulation-evaluation exercise showed that the Length-SRA model is able to track selectivity changes  
112 over time relatively well. There is a tendency to underestimate selectivity for lower lengths and overestimate it  
113 for higher lengths across all scenarios, However, this pattern is particularly prominent for the  $U$  ramp scenarios  
114 (Figure 3).

### 115 **3.2 Real data examples**

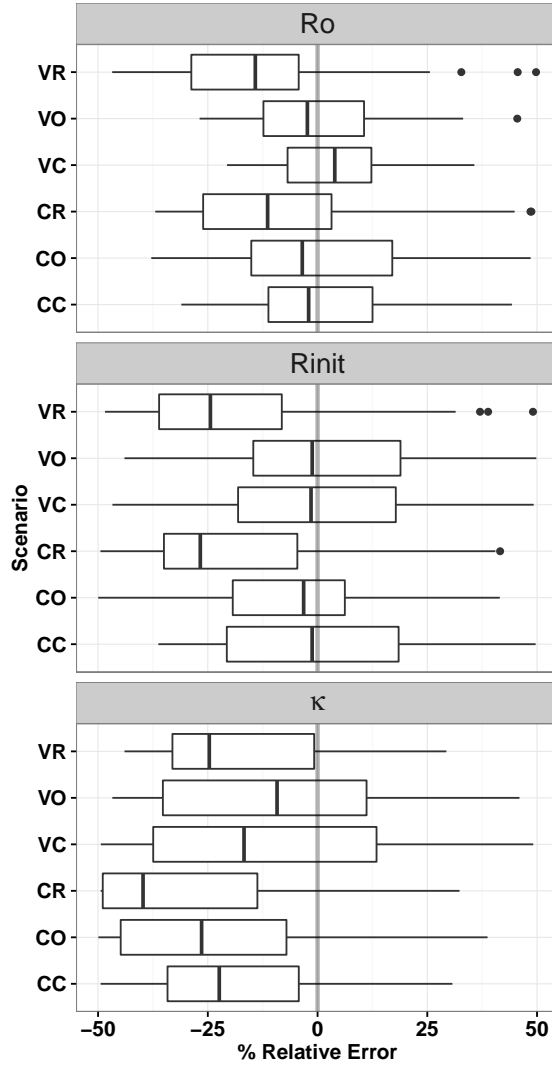
116 show figures with real

## 117 **4 Discussion**

118 main conclusions: Length-SRA underestimates kappa

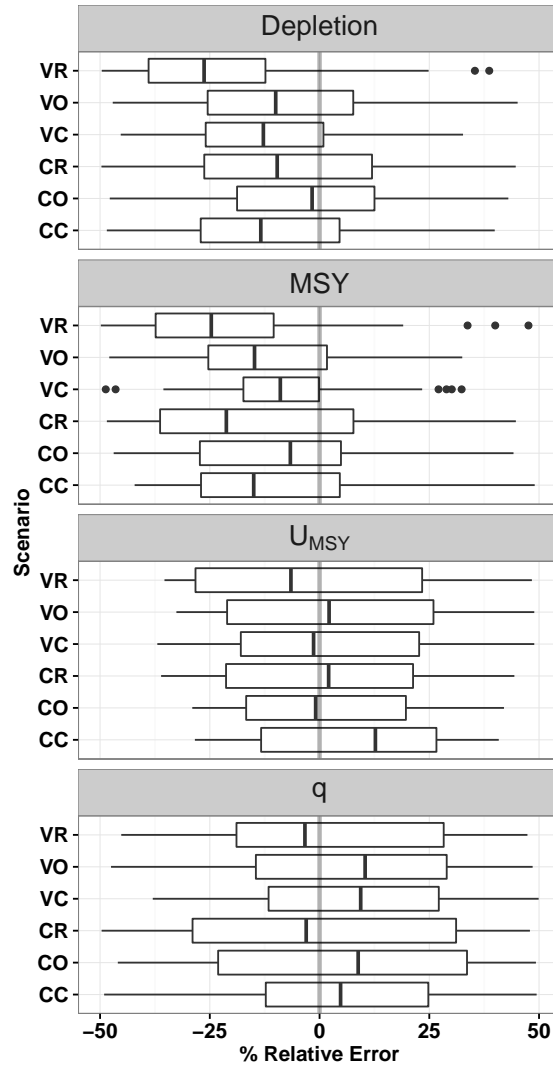
119 Management quantities Depletion and MSY are underestimated - the model tends to produce conservative  
120 benchmarks.

121 UMSY estimates had low bias- good?

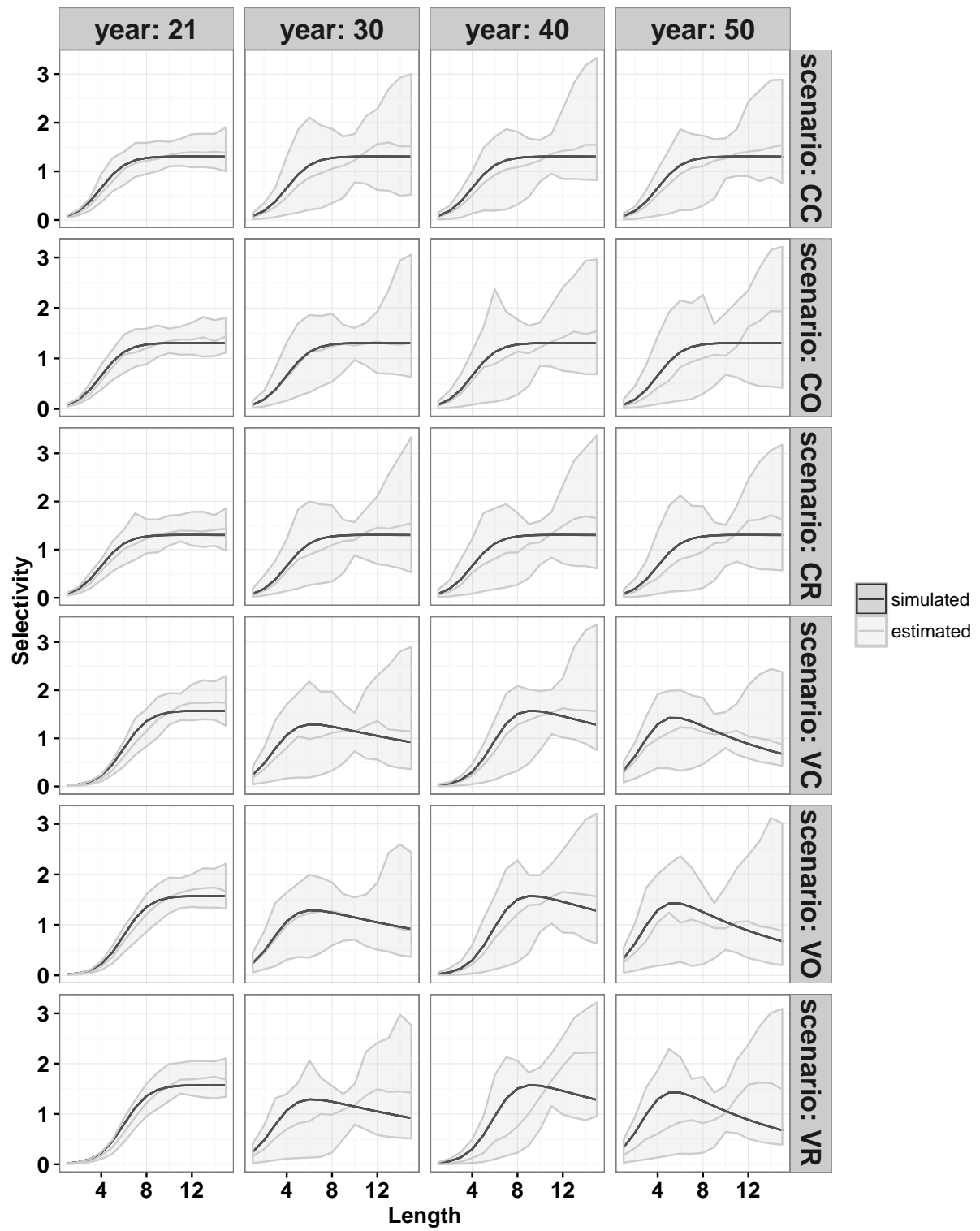


**Figure 1:** Relative error (%) for main parameters for all scenarios considered in the simulation-evaluation.





**Figure 2:** Relative error (%) for main parameters for all scenarios considered in the simulation-evaluation.



**Figure 3:** Simulated and realized selectivity estimates for a set of years within simulation-evaluation time series.

122 U ramp scenario yield very bad results - lack of information in the time series.  
123 Model is able to track selectivity over time, - good  
124 Does the model work? Could it produce useful management advice?  
125 assumption regarding  $U_{msy}$  -  $i$  changes with selectivity -  $i$   
126 Time- varying growth might render the model less useful. Suggest estimating cohort-specific growth curves  
127 or implementing the density dependent functions shown in multifan-CL.

## 128 **5 Acknowledgments**

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130 provision of Jack mackerel data.

## References

- Beverton, R. J. H. and Holt, S. J. (1957). *On the Dynamics of Exploited Fish Populations*, volume 19 of *Investment series*. U.K. Ministry of Agriculture and Fisheries, London. Google-Books-ID: BqbnCAAQBAJ.
- Fournier, D. A., Hampton, J., and Sibert, J. R. (1998). MULTIFAN-CL: A length-based, age-structured model for fisheries stock assessment, with application to South Pacific albacore, *Thunnus alalunga*. *Canadian Journal of Fisheries and Aquatic Sciences*, 55(9):2105–2116.
- Gerlotto, F., Gutiérrez, M., and Bertrand, A. (2012). Insight on population structure of the Chilean jack mackerel (*Trachurus murphyi*). *Aquatic Living Resources*, 25(4):341–355.
- Hilborn, R. and Walters, C. J. (1992). *Quantitative Fisheries Stock Assessment: Choice, Dynamics and Uncertainty/Book and Disk*. Springer Science & Business Media.
- Kimura, D. K. and Chikuni, S. (1987). Mixtures of Empirical Distributions: An Iterative Application of the Age- Length Key. *Biometrics*, 43(1):23–35.
- Martell, S. and Stewart, I. (2014). Towards defining good practices for modeling time-varying selectivity. *Fisheries Research*, 158:84–95.
- Maunder, M. N., Crone, P. R., Valero, J. L., and Semmens, B. X. (2014). Selectivity: Theory, estimation, and application in fishery stock assessment models. *Fisheries Research*, 158:1–4.
- Methot, R. D. and Wetzel, C. R. (2013). Stock synthesis: A biological and statistical framework for fish stock assessment and fishery management. *Fisheries Research*, 142:86–99.
- Ressler, P. H., Holmes, J. A., Fleischer, G. W., Thomas, R. E., and Cooke, K. C. (2007(2008) 2008). Pacific hake, *Merluccius productus*, autecology: A timely review. *U S National Marine Fisheries Service Marine Fisheries Review*, 69(1-4). ZOOREC:ZOOR14601004724.
- Sampson, D. B. and Scott, R. D. (2012). An exploration of the shapes and stability of population–selection curves. *Fish and Fisheries*, 13(1):89–104.
- Schnute, J. T. (1994). A General Framework for Developing Sequential Fisheries Models. *Canadian Journal of Fisheries and Aquatic Sciences*, 51(8):1676–1688.

156 Thompson, G. G. (1994). Confounding of gear selectivity and the natural mortality rate in cases where the  
157 former is a nonmonotone function of age. *Canadian Journal of Fisheries and Aquatic Sciences*, 51(12):2654–  
158 2664.