

Stock Reduction Analysis using catch at length data: Length-SRA

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Abstract

Many modern stock assessments convert length frequency data from catches to age compositions to then conduct age-structured assessment. These assessments must estimate age- or length-based vulnerability to fishing, often using simple parametric functions describing asymptotic or dome-shaped vulnerability. We present a length-based stock reduction analysis (Length-SRA), which bypasses the requirement of estimating a vulnerability by estimating exploitation rate at length directly from observed catch at length data. We test the performance of Length-SRA with a simulation-evaluation framework under three exploitation rate trajectories and under fixed and time-varying vulnerability scenarios. The model yields relatively low bias in parameter estimates and conservative management benchmarks. Length-SRA is relatively accurate when tracking changes in selectivity through time. We further use Length-SRA to assess two species, Pacific hake and Peruvian jack mackerel, to show that vulnerability is quite variable in both species over time, leading to time-varying management reference points. Length-SRA provides assessment results with accuracy comparable to other methods, such as Virtual Population Analysis and Statistical Catch at Age. However, Length-SRA has the additional advantage of providing accurate estimates of vulnerability over time.

1 Introduction

Modern stock assessments typically attempt to fit population dynamics models to catch at age and catch at length data, in hopes of extracting information from these data about age/size vulnerability, cohort strength and fishing mortality patterns (Methot and Wetzel, 2013; Hilborn and Walters, 1992). Some assessment methods attempt to put aside the length frequency data, by converting these data to age compositions using age-from-length tables, perhaps using iterative methods to estimate proportions of fish at age for each length interval

25 (Kimura and Chikuni, 1987). In cases where age data are lacking, models like MULTIFAN-CL attempt to
26 obtain estimates of vulnerability, fishing mortality and population dynamics parameters only from size distri-
27 bution data (Fournier et al., 1998). Combined with a few assumptions regarding the structure and variability in
28 length at age, this procedure can even be used to attempt to recover information about changes in body growth
29 patterns if there is a strong age-class signal in the length frequency data (Fournier et al., 1998). It is typical for
30 assessment results from length-based assessment models to show substantial deviations between predicted and
31 observed length distributions of catches, reflecting both sampling variation in the length composition data and
32 incorrect assumptions about stability of growth and vulnerability patterns (Hilborn and Walters, 1992).

33 Vulnerability to fishing is the combination of two processes: selectivity of the fishing gear and availability
34 of the fished population in the area being fished (Beverton and Holt, 1957). Both processes can vary over
35 time and therefore modify the resulting selectivity. Although selectivity process can often be directly measured
36 through gear experiments, availability is generally harder to measure as it depends on the size-based distribution
37 of the exploited population and the spatial distribution of the fishing fleet. Fish movement, size-structured
38 changes in fish distribution, and changes in fleet distribution, can all affect availability and consequently lead to
39 vulnerability changes. Changes in vulnerability are not uncommon (Sampson and Scott, 2012) but are usually
40 difficult to track over time. This difficulty is associated with an inability to distinguish between changes in
41 fishing mortality and changes in vulnerability in most age- and length-based stock assessment methods. For
42 this reason, many assessment methods rely on ad hoc parametric vulnerability models that may or may not
43 include changes over time (Maunder et al., 2014). If misspecified, such models might lead to severe bias in
44 fishing mortality estimates and other model parameters, which could result in misleading management advice
45 (Martell and Stewart, 2014).

46 Here we suggest an alternative approach to assessment modeling that begins by assuming that the as-
47 sessment model should exactly reproduce the observed catch at length composition. This approach follows
48 the dynamics of an age structured stock reduction analysis (SRA) (Walters et al., 2006; Kimura et al., 1984;
49 Kimura and Tagart, 1982) which follows a “conditioned on catch” format, subtracting observed catches at age
50 from modeled numbers at age in estimation of numbers at age over time. This assumption is analogous to
51 the classical assumption in virtual population analysis that reconstructed numbers at age should exactly match
52 observed catch at age data (Hilborn and Walters, 1992). The suggested approach may have two key advantages
53 over statistical catch at age and/or catch at length models: (1) it does not require estimation of age or size
54 vulnerability schedules, and (2) catch at length data are commonly available for every year, even when age
55 composition sampling has not been conducted.

We named this approach a Length-SRA assessment model. Here we present the model formulation, demonstrate its performance with a simulation-evaluation analysis and apply it to real fisheries data from the Peruvian jack mackerel (*Trachurus murphyi*) and Pacific hake (*Merluccius productus*) fisheries.

2 Methods

2.1 Stock reduction analysis with catch at length data - length-SRA

The stock reduction analysis (SRA) described here proceeds through the following steps: (1) compute numbers at age (based on recruitment estimates and mortality in the previous year); (2) convert numbers at age into numbers at length using the proportions of individuals at length given each age class; (3) calculate the exploitation rate at length using numbers at length and observed catch at length; (3) convert the exploitation rate at length to exploitation rate at age; (4) compute numbers in the following year using the exploitation rate at age, natural mortality, and recruitment estimates.

A crucial component of the length-SRA is the calculation of proportions of individual at length given each age class ($P_{l|a}$ - eqs. T3.1-T3.5). The calculation of such proportions (eq. T3.1) relies on four main assumptions regarding the distribution of length at age: (1) The mean length at age follows a von Bertalanffy growth curve (eq.T3.4), (2) The length at age is normally distributed (eqs. T3.1 -T3.3), (3) The standard deviation of the length at age is defined (e.g. eq.T3.5), and (4) $P_{L|a}$ is constant for all lengths equal or greater than a maximum length L (eq.T3.3).

The proportions of length at age are used to convert the length-based quantities into age-based quantities which are used to propagate the age structured population dynamics forward (Table 3). We assume that recruitment follows a Beverton-Holt type recruitment curve (eq. T3.6), that harvesting occurs over a short, discrete season in each time step (year or shorter time period), and that natural survival rate is known and stable over time (eqs. T3.6-T3.10). The computation of numbers at age in the initial year (i.e., first year in which data is reported - t_{init}) is different from that in the remaining years (eq. T3.12). Recruitment in the initial year is given by the parameter R_{init} , which allows the model to reflect that the population has been fished prior to the beginning of the time series. If the population is initially unfished, this parameter can be set to equal to R_0 . We used equilibrium yield per recruit quantities to calculate MSY and U_{MSY} (Table 4 - eqs.T4.6 to T4.12). As in all yield per recruit calculations, our MSY and U_{MSY} estimates depend on the vulnerability curves calculated for each year (eq. T4.8).

In order to assess how well the model tracked changes in selectivity over time, we calculated the resulting selectivity estimates by normalizing the yearly vectors of exploitation rate at length ($U_{l,t}$) by the yearly average exploitation rate at length (\bar{U}_l) (eq. T3.11). The selectivity vectors were normalized by the average yearly exploitation rate because the average (\bar{U}_l) is more stable than the maximum yearly exploitation rate ($\max U_l$). This happens because observation errors tend to average out over the length classes, diminishing the variability of \bar{U}_l in relation to $\max U_l$.

The Length-SRA model estimates three main parameters: average unexploited recruitment R_0 , recruitment compensation ratio κ , and recruitment in the initial year R_{init} . In addition, the annual recruitment deviations w_t are estimated for all cohorts observed in the model. That is, the number of recruitment deviations is equal to the number of years in the time series plus the number of age classes greater than recruitment age.

The objective function (eq. T5.13) is composed of a negative log-likelihood component, three penalties, and a prior component for the recruitment compensation ratio κ . The negative log-likelihood component minimizes the differences between the predicted and observed index of abundance (eq. T5.1). We assume that such differences are lognormally distributed (eqs. T5.3-T5.4) and use the conditional maximum likelihood estimator described by Walters and Ludwig (1994) to estimate the survey catchability coefficient q (eq. T5.2). A lognormal penalty is added to the negative log-likelihood function to constrain annual recruitment residuals so that the estimates have mean of zero and fixed standard deviation σ_R (Maunder and Deriso, 2003) (eq. T5.5). A second penalty, $P_{U_{max}}$ was implemented in order to prevent estimated values of exploitation rate at length greater than one ($U_{l,t} > 1$) (eq. T5.9) and a third penalty P_U was added to limit variability in vulnerability at length across years, and therefore limit the influence of observation error on vulnerability estimates. Lastly, an informative normal prior for $\log(\kappa)$ was included in the objective function (eq. T5.12).

2.2 Simulation-evaluation

Model performance was evaluated using a simulation-evaluation populated with the biological parameters of an hypothetical fish species. We used the same model structure described in Table 3 for both the simulation and estimation models. However, the operating model was modified to control annual exploitation rate (eq. T4.2), time varying selectivity (eq. T4.4), and observation and process errors.

The simulation model was initialized at unfished conditions (eq. T4.1) but only started reporting data for the simulation-evaluation procedure after the t_{init} year. The period between the first time step and t_{init} is used as a burn-in period. Selectivity in the operating model was computed with the three parameter selectivity function described by Thompson (1994) (eq. T4.4). We chose to use this three parameter selectivity curve

because of its flexibility, which allowed us to switch between logistic and dome-shaped selectivity curves in the scenarios in which time varying selectivity was considered. The observation error in the operating model included lognormal error in the index of abundance and logistic multivariate error (Schnute and Richards, 1995) in the catch numbers at length (Table 2). Recruitment deviations were assumed to be lognormally distributed with constant σ_R (Table 1).

We considered a total of six different scenarios in simulation-evaluation trials, including three historical exploitation rate trajectories (contrast, one-way trip and U -ramp) and two selectivity patterns (constant and time-varying). In the contrast scenario the exploitation rate (U_t) starts low and increases beyond U_{MSY} and then decreases until $U_t = U_{MSY}$. In the one-way trip scenario U increased through time until $U = 2 \cdot U_{MSY}$. In the U -ramp scenario, U_t increases steadily until $U_t = U_{MSY}$ and remains constant thereafter. In the constant selectivity scenario, selectivity was assumed to follow a sigmoid shape. In the time varying selectivity scenario, the selectivity curve was assumed to vary every year, progressively changing from a dome shaped curve to sigmoid and back to dome shaped. The complete list of scenarios and the acronym used for them is presented in Table 6.

All simulations had 30 years of data and 200 simulation trials were performed for each scenario. We evaluated the distribution of the relative proportional error ($\frac{estimated-simulated}{simulated}$) for the main parameter estimates (R_0 , R_{init} , and κ) and for four derived quantities (Depletion: $\frac{SB_t}{SB_0}$, MSY , U_{MSY} , and q).

2.3 Real data examples

Two case studies were chosen to illustrate the application of the Length-SRA to real datasets: Pacific hake and Peruvian jack mackerel. Both species are believed to be subject to time varying selectivity.

The Pacific hake fishery is believed to exhibit time varying selectivity due to cohort targeting and annual changes fleet spatial distribution (Ruttan, 2003). The population is known to have spasmodic recruitment, with high recruitment events occurring once or twice every decade (Ressler et al., 2007). Pacific hake tends to segregate by size during their annual migration (Ressler et al., 2007), allowing the fishing fleet to target strong cohorts by changing the spatial distribution of fishing effort as the cohort ages. Hake catch at length data was available for the period between 1975 and 2013. The survey index of abundance was available intermittently from 1995 to 2013.

The movement pattern of jack mackerel is not as well known, although fish appear to move between spawning and feeding areas (Gerlotto et al., 2012). Variability in selectivity patterns for the jack mackerel fishery are believed to be associated both with evolution of fleet capacity and gear utilization and with compression and

expansion of the species range associated with abundance changes (Gerlotto et al., 2012). Jack mackerel catch at length data was available from 1980 to 2013 and the survey index was available for the years between 1986 and 2013, with the exception of the year of 2010.

3 Results

3.1 Simulation-evaluation

We evaluated the performance of the model in relation to the main parameters and derived management quantities with boxplots of the relative proportional error, throughout the result we use the terms positive and negative median bias to indicate that the median relative proportional error is above or below zero. The relative proportional error sign indicate if a parameter has been underestimated or overestimated, the majority of the time.

Simulation-evaluation of Length-SRA model resulted in a small positive median bias for the κ parameter across all scenarios (Figure 1). The R_0 relative error medians were positively biased for the contrast and one-way-trip scenarios (CC, CO, VC and VO) and negatively biased for the U -ramp scenarios (Figure 1). The R_{init} median relative error was negatively biased for all scenarios with higher absolute biases associated with the contrast scenario with time-varying selectivity (VC) and U -ramp scenarios (CR and VR) (Figure 1). Overall, The contrast with constant selectivity and one-way trip scenarios yielded the best performance.

The depletion (SB_t/SB_o) estimates resulted in negative median relative error for all scenarios (Figure 2). MSY was also underestimated for most scenarios, except for the Contrast scenario with constant vulnerability, for which the median relative proportional error was zero. The estimates for U_{MSY} showed very low (<0.071) absolute median proportional errors for all scenarios (Figure 2). The estimates of q were more variable, with particularly high positive bias for the the contrast and time-varying selectivity scenario (VC) and U -ramp scenarios (CR and VR); this is likely to be associated with the more severe underestimations of and R_{init} for those scenarios.

The simulation-evaluation exercise showed that the Length-SRA model is able to track vulnerability for the CC, CO and VO scenarios (Figure 3). The capability of the model to track vulnerability through time was diminished for the CR, VC and VR scenarios. The decreased accuracy for the CR, VC and VR scenarios is likely associated with the lack of information in the data, this was an expected outcome for the U -ramp scenarios (CR and VR) but it was somewhat unexpected for the contrast scenario with time varying vulnerability (VC). However, upon closer inspection of the biomass trends in the VC scenario (results not shown), we realized

173 that the interaction between time-varying vulnerability and changes in exploitation rate resulted in a somewhat
174 stable biomass trend over time resulting in lack of contrast in the time series.

175 3.2 Real data examples

176 The model fit the Pacific hake and jack mackerel indexes of abundance relatively well (Figure 4), despite
177 some limitation in the available data. The Pacific hake index of abundance time-series is quite short and
178 intermittent (survey happens every three or two years). The index of abundance time series for jack mackerel
179 was longer but it indicates a downward trend in abundance with low contrast in the last ten years of data.

180 The model fit for both species resulted in time varying selectivities that lead to variation in MSY and conse-
181 quent changes in U_{MSY} . This is because changes in selectivity result in changes to the vulnerable biomass even
182 if total biomass is constant. The time-varying selectivity calculated for each species (Figure 5) results in up
183 to two-fold change in MSY throughout the time series for both species (Figure 4). The selectivity curves esti-
184 mated for Pacific hake and jack mackerel are quite variable. It is important to note that this observed variability
185 might indicate real changes in selectivity (e.g. cohort targeting) or might also be caused by misspecification
186 of the growth parameters (see Appendix). At this point it impossible to determine what are the causes for the
187 resulting patterns in vulnerability observed with the Length-SRA fit. Further investigation would be needed if
188 this model is to be used for management purposes.

189 4 Discussion

190 We present a length-based stock reduction analysis (Length-SRA) that allows monitoring of time varying
191 vulnerability. In the Length-SRA model, catch at length is assumed to be known without error and exploita-
192 tion rate at length is calculated directly from estimates of numbers at length. In turn, numbers at length are
193 produced based on numbers at age and on probabilities derived from growth curve parameters and coefficient
194 of variation. This fact is important because it allows the model to bypass the requirement for the estimation
195 of a vulnerability ogive, as is required in more traditional age- and length-based models (Sullivan et al., 1990;
196 Mesnil and Shepherd, 1990, e.g.) and in more recent length based state-space modelling approaches (White
197 et al., 2016). Estimation of vulnerability ogives can be very difficult, especially if vulnerability is believed to
198 vary over time unpredictably (Martell and Stewart, 2014; Linton and Bence, 2011).

199 The accuracy in the estimates of vulnerability obtained with the Length-SRA are comparable with those
200 presented by Nielsen and Berg (2014), especially for the one-way trip scenarios (scenarios with the most

informative data) . Nielsen and Berg (2014) account for time varying selectivity by treating fishing mortality at age as stochastic processes that are correlated over age and time. Their model seems to perform extremely well, however they only considered one exploitation rate trajectory, with significant contrast in the data. In addition, the changes in vulnerability considered in their study are more subtle than the ones considered in this study.

Another attractive approach to model catch at length data is the one described by White et al. (2016). Their state space model estimates annual recruitment, harvest rate, and error terms from catch at length data and seems to obtain good precision and accuracy in their estimates. However, it is important to note that their model relies on two assumptions that are likely violated in real data examples. First, they assume that vulnerability is constant over time and follows a cumulative normal distribution parametric form. Second, they assume that the growth component of the transition matrix $P(y,x)$ to be constant and given by the growth curve. This is incorrect because fishing causes changes in the age composition of fish of size x , i.e. under fishing, proportion of younger fish (with higher $y|x$) is expected to increase.

An important advancement of Length-SRA over conventional stock-assessment models is a calculation of time-varying vulnerability. This information alone will go a long way in helping to characterize the complexity of the fishery system. Length-SRA on its own is reasonably accurate in deriving important management-oriented parameters (depletion and U_{MSY}), however another option may be to combine findings from this model with another assessment model, such as a statistical catch at age (SCA) model. In this framework, Length-SRA can be used to calculate annual vulnerability, which are reasonably accurate of there is enough contrast in the biomass time series. These vulnerability estimates can then become an input into an SCA to calculate other important variables and produce management advice. This combination of models has been used in the past (Walters and Punt, 1994); we suggest Length-SRA may be an extremely useful tool in this context.

Accurate estimates of vulnerability are particularly important if the fishery management is based on yield per recruit reference points. The yield per recruit of a fishery depends on the vulnerability curve (Beverton and Holt, 1957) and for this reason, changes in vulnerability over time will directly affect reference points (Beverton and Holt, 1957; Hilborn and Walters, 1992). We observed vulnerability changes for both Pacific hake and jack mackerel and show how this variability can lead to a two-fold difference between the maximum and minimum estimates of MSY calculated along the time series. We believe that tracking these changes is important not only to ensure appropriate management recommendations, but also to illustrate the relationship between vulnerability patterns and MSY (Vasilakopoulos et al., 2016).

231 One potential point of concern that should be considered when using the Length-SRA is that it assumes
232 that the biological parameters used in the growth curve and catch at age relationship are known without error
233 and constant over time. We have tested the Length-SRA under misspecification of the von Bertalanffy growth
234 parameters (see Appendix) and we observed additional bias in the estimates of parameter and management
235 quantities as well as strong distortions in the resulting selectivity parameters. Similarly, Minte-Vera et al. (2017)
236 showed that misspecification in biological parameters, especially in asymptotic length, can have a significant
237 impact in assessment results. Other length models, e.g. MULTIFAN-CL (Fournier et al., 1998), overcome the
238 assumption of known growth parameters by estimating the von Bertalanffy parameters alongside the assess-
239 ment parameters. The estimation of the growth parameters is made possible by assuming that vulnerability
240 follows a parametric function (usually logistic). Once a simple vulnerability curve is assumed, all deviations
241 in observed catch at length are explained by adjusting the growth parameters. This assumption can also lead to
242 bias in parameter estimates, as other studies show that variability in selectivity and non-asymptotic patterns are
243 common (Waterhouse et al., 2014). In reality, in most cases it is difficult to know if patterns observed in catch at
244 length are caused by fisheries targeting (i.e. vulnerability) or if they would be more appropriately explained by
245 adjusting the growth parameters. Therefore, we recommend that, when using the Length-SRA, the user should
246 perform extensive sensitivity analyses over the possible range of values for the growth parameters, particularly
247 if the predicted vulnerability patterns are highly variable.

248 The approach used in the Length-SRA is analogous to that used in virtual population analysis in that the
249 length composition data is assumed to be known without error. For this reason, the vulnerability estimates
250 include extra variability due to observation and sampling error. We attempted to minimize this effect by adding
251 a time varying vulnerability penalty to the model's objective function, however this penalty is not capable of
252 completely removing the observation error effect from the vulnerability estimates. Because of the assumption
253 of known catch at length, it is important that the catch sampling is representative of the total removals from
254 the population (Pope, 1972). As in any other fisheries model, biased sampling and/or low sampling effort will
255 result in bias in parameter and fishery reference point estimates (Coggins and Quinn, 1998; Bunch et al., 2013).

256 Some management parameters are underestimated (namely depletion and MSY in most scenarios) and the
257 relative error indicated that the estimates are not very precise which may be cause for concern. However,
258 it is important to note that both parameters are underestimated for the majority of scenarios and simulation
259 runs, which leads to conservative management recommendations. Although this may lead to losses in catch
260 in the short-term, it will result in healthier stocks and higher catches over the longer-term (once the stock
261 is recovered). The magnitude of bias is also dependent on the trajectory of the data: time-series with more

contrast will lead to more accurate estimates. The magnitude and direction of the bias in the estimates of *MSY* and depletion observed in this study are comparable to the results obtained by Martell and Stewart (2014) in a simulation study on the impacts of time varying vulnerability on the estimates generated by a statistical catch at age model. Other studies show even higher biases in face of time-varying vulnerabilities (e.g. Linton and Bence, 2011; Henríquez et al., 2016). Overall, estimates derived parameters are generally within the range of many other stock assessment models.

The simple nature of the Length-SRA model makes it a good candidate model for inclusion on closed-loop simulation studies. Further testing of this model in a closed-loop simulation set up would provide more insight on the model performance on achieving management outcomes (Punt et al., 2014). As mentioned before, we expect that the length-SRA, when used as an assessment tool, will yield slightly conservative management benchmarks which might lead to higher average catch in the long term if the stock is currently overfished. However, it is also possible that these conservative benchmarks result in forgone yield in the medium and long term. We recommend, therefore that this approach be tested in a simulation set up before it is considered as part of a management strategy.

In conclusion, we believe that the Length-SRA approach presented in this study is a useful approach to fisheries stock assessment. We believe that this is particularly true when time varying vulnerability is believed to occur, especially if the variability is not easily predictable from historical changes in gear use/fleet composition. We foresee the application of this model as an investigative tool to evaluate potential time-varying vulnerability patterns, as a stock assessment tool and as part of closed loop simulation studies.

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A Appendix

One important feature of the Length-SRA is that it assumes that growth follows a von Bertalanffy curve and that the growth parameters are known and constant over time. If this assumption is violated, the model outcomes will be impacted as the model will try to explain the deviations from the true growth curve with changes in the vulnerability pattern. Here we illustrate how the model outcomes are impacted by the misspecification of the growth parameters by purposefully misreporting the values of L_∞ (Table 7). We assumed a simple logistic selectivity curve for this exercise and therefore expect the model to produce logistic patterns in the exploitation rate at length $U_{l,t}$.

We found that misspecification of L_∞ has severe implications in the capability of the model to estimate exploitation rate at length $U_{l,t}$ (Figure 6). If the value of L_∞ was reported to be lower than true, the estimates

365 of $U_{l,t}$ were lower than true for most length and extremely high for high lengths (approaching the true L_∞). In
366 the scenario where L_∞ was reported to be higher than true, $U_{l,t}$ was estimated to follow a dome shaped pattern,
367 with very low exploitation rates for the higher lengths. This patterns occur because the model is trying to adjust
368 the mismatch between proportions of catch at length and the $P_{l|a}$ matrix by changing the predicted vulnerability
369 pattern. As a result, failure to adequately specify L_∞ leads to erroneous estimation of vulnerability patterns and,
370 consequently, to failure in estimating management quantities.

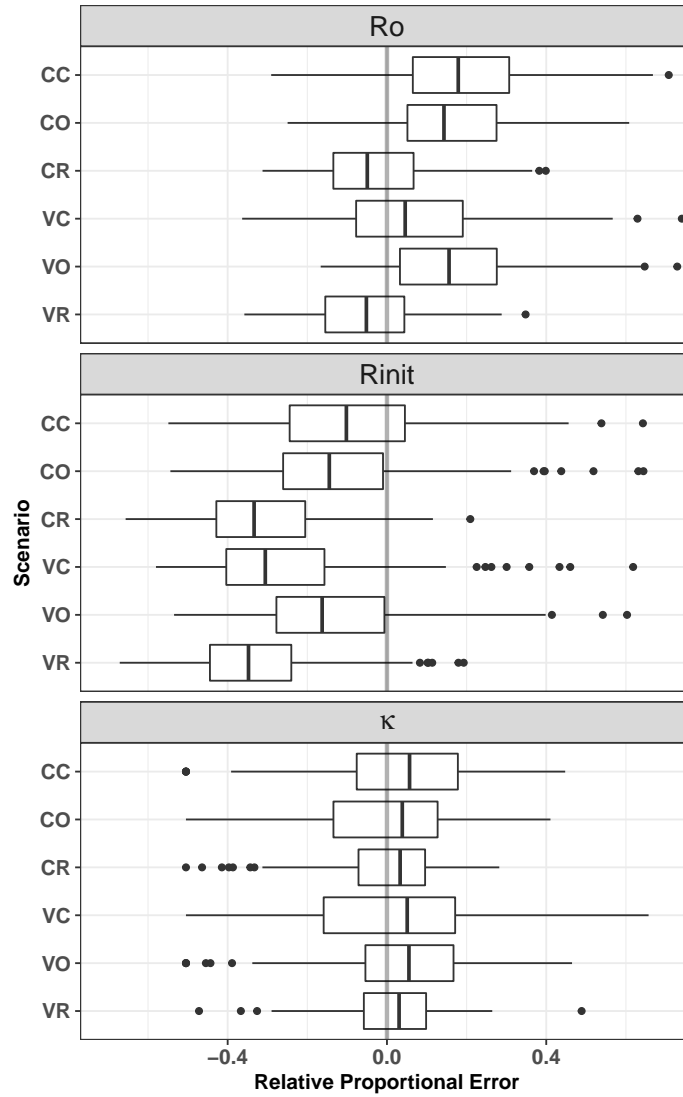


Figure 1: Relative proportional error for main parameters for all scenarios considered in the simulation-evaluation. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value $\pm 1.5 \times$ inter-quartile range (distance between the first and third quartiles).

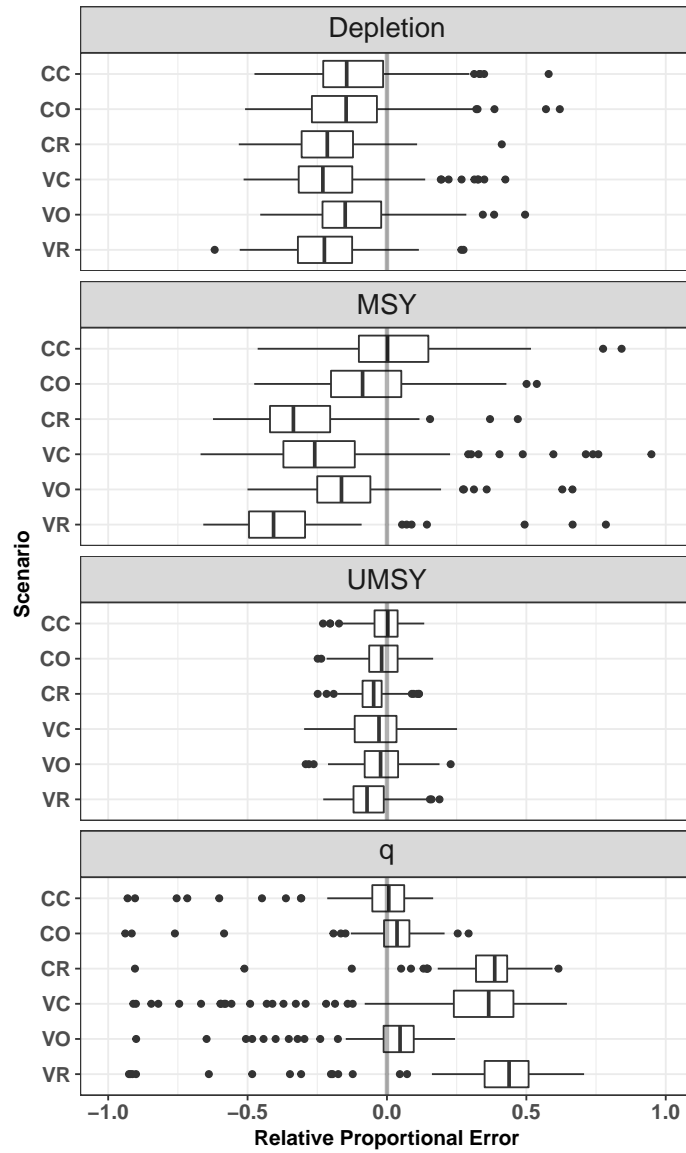


Figure 2: Relative proportional error for main parameters for all scenarios considered in the simulation-evaluation. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value $\pm 1.5 \times$ inter-quartile range (distance between the first and third quartiles).

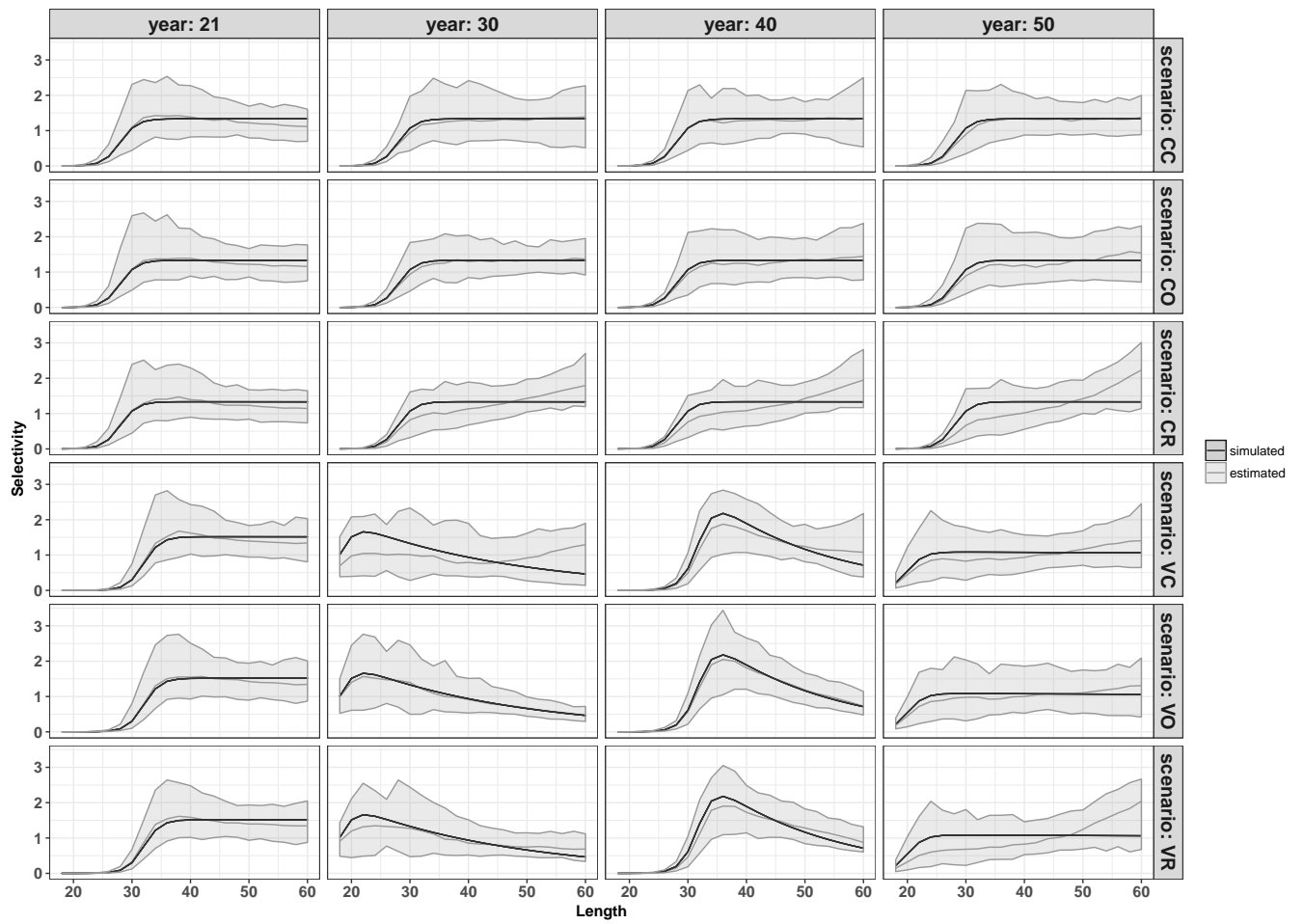


Figure 3: Simulated and realized selectivity estimates for a set of years within simulation-evaluation time series.

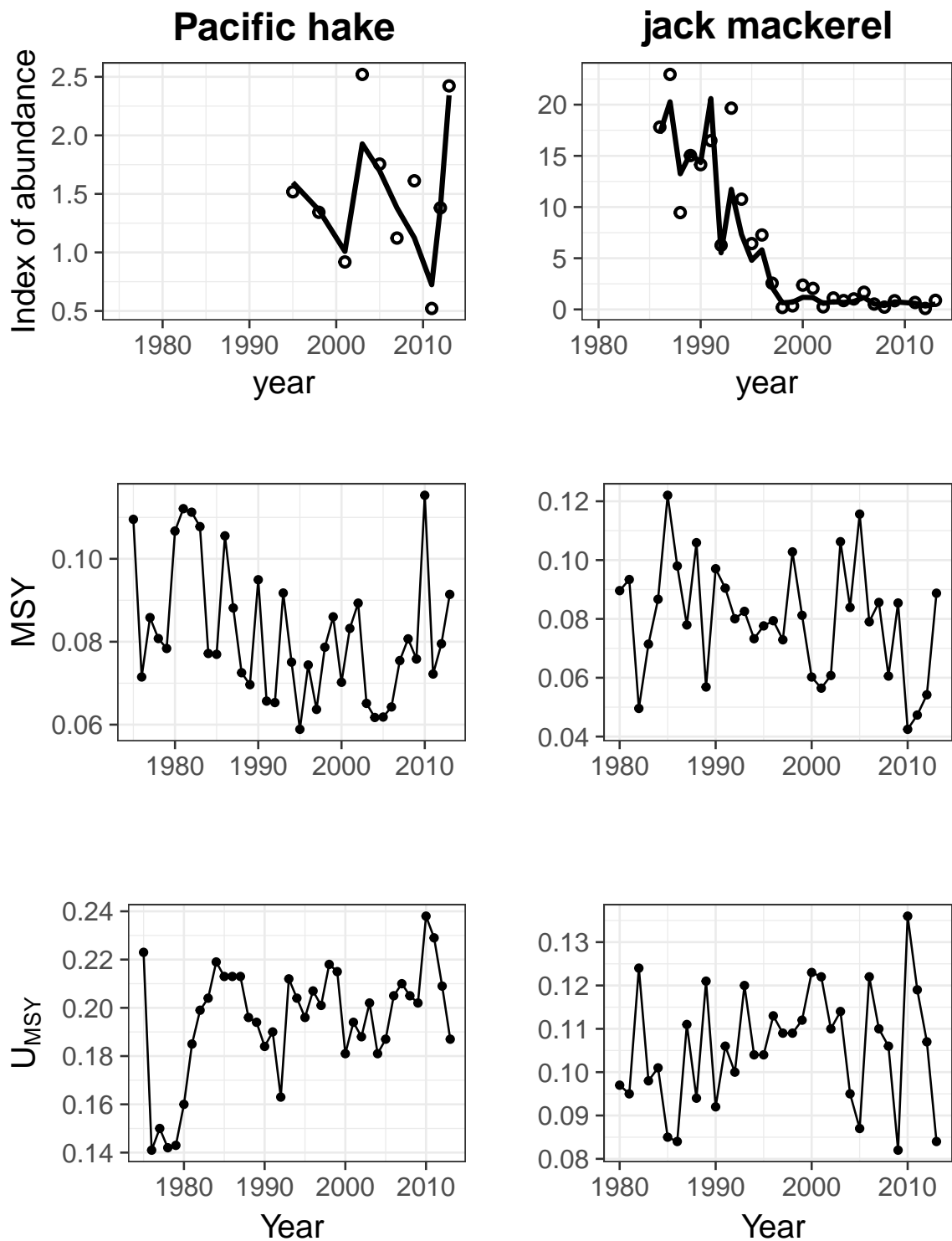


Figure 4: Fit to index of abundance for Pacific hake and jack mackerel.

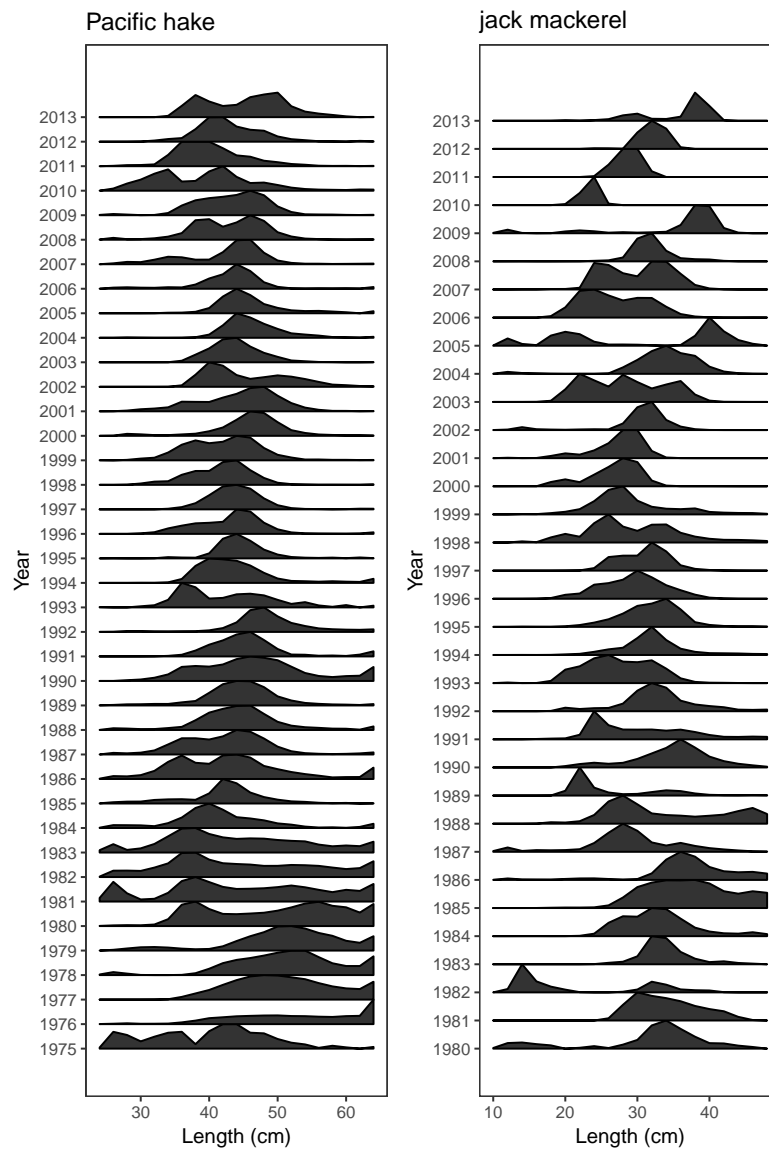


Figure 5: Realized selectivity patterns across years for Pacific hake and jack mackerel.

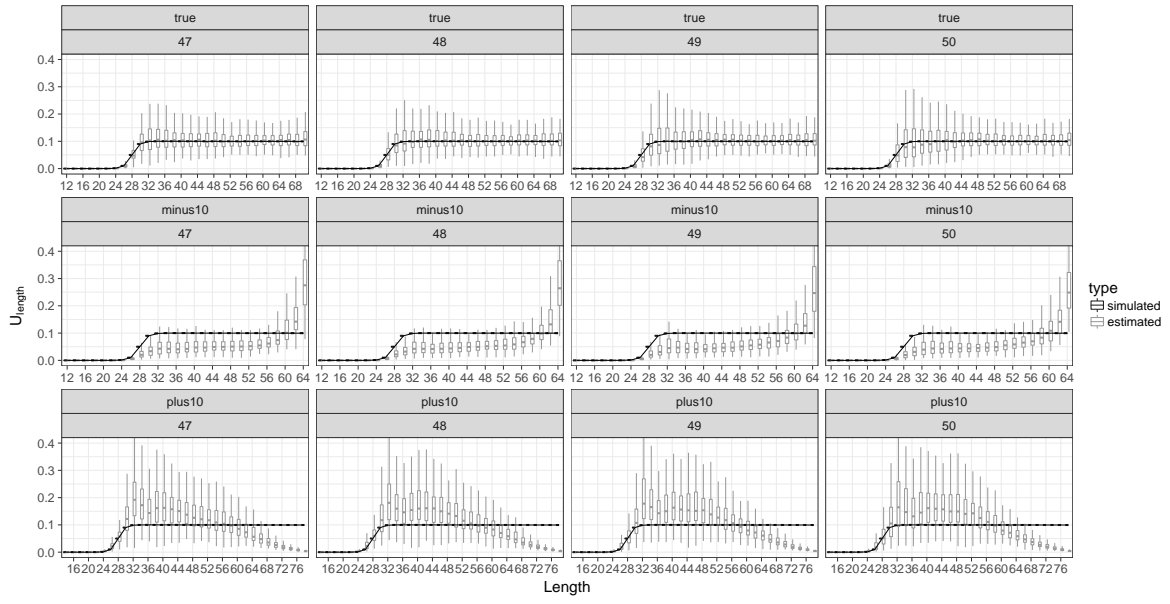


Figure 6: Simulated and realized exploitation rate at length $U_{l,t}$ for a set of years within simulation-evaluation time series.

Table 1: Indexes, stock assessment variable definition, and values used in simulation-evaluation

Symbol	Value	Description
l	$\{l_o, \dots, L\}$	Central point of length bin, $L = 50$ cma
a	$\{a_o, \dots, A\}$	Age-class, $A = 10$ years
t	$\{1, \dots, T\}$	Annual time step, $T = 50$ years
a_o	1	First age or age of recruitment
l_{bin}	2 cm	size of length bin
l_o	8 cm	Central point of first length bin
$init$	21	Annual time step in which data starts to be reported
Distribution of length given age		
L_∞	50 cm	Maximum average length
K	0.15	Rate of approach to L_∞
t_o	-0.1	Theoretical time in which length of individuals is zero
cv_l	0.08	Coefficient of variation for length at age curve
$P_{l a}$		Matrix of proportions of length at age
Φ		Standard normal distribution
$z1_{a,l}$		Normalized z score for lower limit length bins
$z2_{a,l}$		Normalized z score for upper limit length bins
$b1_l$		Lower limit of length bins
$b2_l$		Upper limit of length bins
\bar{L}_a		Mean length at age
σ_L		Standard deviation of length at age
Population dynamics		
R_o	100	Average unfished recruitment
κ	10	Goodyear recruitment compensation ratio
S	0.7	Natural annual survival
σ_R	0.6	standard deviation for recruitment deviations
w_t	$\mathcal{N}(0, \sigma_R)$	Recruitment deviations for years $\{init-A+1, \dots, T\}$
$N_{a,t}$		Numbers of fish at age and time
SB_t		Spawning biomass at time
mat_a		Proportion of mature individuals at age
a_{rec}, b_{rec}		Beverton & Holt stock recruitment parameters
R_{init}		Recruitment in the year data starts to be reported
$U_{a,init}$		Exploitation rate at age before data starts being reported.
$U_{a,t}$		Exploitation rate at age and time
$U_{l,t}$		Exploitation rate at length and time
$C_{l,t}$		Catch at length and time
$N_{l,t}$		Numbers at length and time
lx_a		Unfished survivorship at age
Φ_e		Unfished average spawning biomass per recruit
$sel_{l,t}^{result}$		Resulting selectivity estimates at length and time

Table 2: Indexes, variable definition for operating model, MSY quantities, and values used in simulation-evaluation

Symbol	Value	Description
Operating model		
$sel_{l,t}$		Fishing selectivity at length and time
g, d, b	vary by scenario	Parameters for selectivity function
U_t	vary by scenario	Annual maximum exploitation rate
$C_{l,t}^{Sim}$		Simulated catch at length and time
$N_{l,t}^{Sim}$		Simulated numbers at length and time
I_t		Index of abundance at time
σ_{I_t}	0.1	standard deviation for index of abundance deviates
VB_t		Biomass that is vulnerable to the survey at time
q	1.0	Catchability coefficient
MSY quantities		
l_{z_a}		Fished survivorship at age
U_z	seq(0.0,1.0,by=0.001)	Hypothetical exploitation rates to calculate MSY
Φ_z		Average spawning biomass per recruit
Φ_{eq}		Average exploited biomass per recruit under U_z and sel_a
sel_a		Selectivity at age
R_{eq}		Average equilibrium recruitment under U_z
$Yield_z$		Equilibrium yield under U_z
MSY		Maximum sustainable yield based on optimum spawner per recruit
U_{MSY}		Exploitation rate that leads to maximum sustainable yield

Table 3: population dynamics for Length-SRA and operating model

Distribution of length given age	
(T3.1)	$P_{l a} = \int_{z1_{a,l}}^{z2_{a,l}} \Phi(z) dz$
(T3.2)	$z1_{a,l} = \frac{b1_l - \bar{L}_a}{\sigma_{L_a}}$
(T3.3)	$z2_{a,l} = \begin{cases} \frac{b2_l - \bar{L}_a}{\sigma_{L_a}} & l < L \\ 1.0 & l = L \end{cases}$
(T3.4)	$\bar{L}_a = L_\infty \cdot (1 - e^{(-K \cdot (a - t_o))})$
(T3.5)	$\sigma_{L_a} = \bar{L}_a \cdot cv_l$
Population dynamics	
(T3.6)	$N_{a,t>init} = \begin{cases} \frac{a_{rec} \cdot SB_{t-1}}{1 + b_{rec} \cdot SB_{t-1}} \cdot e^{w_t}, & a = a_o \\ N_{a-1,t-1} \cdot S \cdot (1 - U_{a-1,t-1}), & a_o < a < A \\ \frac{N_{a-1,t-1} \cdot S \cdot (1 - U_{a-1,t-1})}{1 - S \cdot (1 - U_{a,t})}, & a = A \end{cases}$
(T3.7)	$U_{a,t} = \sum_l (P_{l a} \cdot U_{l,t})$
(T3.8)	$U_{l,t} = \frac{C_{l,t}}{N_{l,t}}$
(T3.9)	$N_{l,t} = \sum_a (P_{l a} \cdot N_{a,t})$
(T3.10)	$SB_t = \sum_a (mat_a \cdot w_a \cdot N_{a,t})$
(T3.11)	$sel_{l,t}^{result} = \frac{U_{l,t}}{\bar{U}_t}$
Initial year and incidence functions	
(T3.12)	$N_{a,init} = \begin{cases} R_{init} \cdot e^{w_{init}} & a = a_o \\ N_{a-1,init} \cdot S \cdot (1 - U_{a-1,init}) \cdot e^{w_{init}-a+1}, & a_o < a < A \\ \frac{N_{a-1,init} \cdot S \cdot (1 - U_{a-1,init})}{1 - S \cdot (1 - U_{a,init})} \cdot e^{w_{init}-a+1}, & a = A \end{cases}$
(T3.13)	$a_{rec} = \frac{\kappa}{\Phi_e}$
(T3.14)	$b_{rec} = \frac{\kappa - 1}{R_o \cdot \Phi_e}$
(T3.15)	$\Phi_e = \sum_a lx_a \cdot mat_a \cdot w_a$
(T3.16)	$lx_a = \begin{cases} 1, & a = 1 \\ lx_{a-1} \cdot S, & 1 < a < A \\ \frac{lx_{a-1} \cdot S}{1 - S}, & a = A \end{cases}$

Table 4: MSY quantities and operating model

Operating model	
(T4.1)	$N_{a,t=1}^{Sim} = lx_a \cdot R_o$
(T4.2)	$U_{l,t} = U_l \cdot sel_{l,t}$
(T4.3)	$C_{l,t}^{Sim} = N_{l,t}^{Sim} \cdot U_{l,t} \cdot P_{l a}$
(T4.4)	$sel_{l,t} = \frac{1}{1-g} \cdot \left(\frac{1-g}{g} \right)^g \cdot \frac{e^{d \cdot g \cdot (b-l)}}{1 + e^{d \cdot (b-l)}}$
(T4.5)	$I_t = q \cdot VB_t \cdot e^{(\mathcal{N}(0, \sigma_{I_t}))}$
MSY quantities	
(T4.6)	$lz_a = \begin{cases} lz_a = 1 & a = a_o \\ lz_{a-1} \cdot S \cdot (1 - U_z) & a_o < a < A \\ \frac{lz_{a-1} \cdot S \cdot (1 - U_z)}{1 - S \cdot (1 - U_z)} & a = A \end{cases}$
(T4.7)	$\Phi_z = \sum_a lz_a \cdot mat_a \cdot w_a$
(T4.8)	$\Phi_{eq} = \sum_a lz_a \cdot sel_a \cdot w_a$
(T4.9)	$R_{eq} = R_o \cdot \frac{\kappa - \Phi_e / \Phi_z}{\kappa - 1}$
(T4.10)	$Yield_z = U_z \cdot R_{eq} \cdot \Phi_{eq}$
(T4.11)	$MSY = \max(Yield_z)$
(T4.12)	$U_{MSY} = U_z \rightarrow \max(Yield_z)$

Table 5: Likelihood functions and penalties

Conditional Likelihood	
(T5.1)	$Z_t = \log(I_t) - \log(VB_t)$
(T5.2)	$q = e^{\bar{Z}}$
(T5.3)	$Zstat_t = Z_t - \bar{Z}$
(T5.4)	$LL_1 \sim \mathcal{N}(Zstat \mu = 0, \sigma = \sigma_{I_t})$
Penalties	
(T5.5)	$P_{wt} \sim \mathcal{N}(wt \mu = 0, \sigma = \sigma_R)$
(T5.6)	$\mu_{U_{t-2:t}} = \left(\sum_{t=2}^t \frac{U_{l,t}}{\bar{U}_t} \right) / 3$
(T5.7)	$U_t^{pen} = \left(\frac{U_{l,t}}{\bar{U}_t} - \mu_{U_{t-2:t}} \right)^2$
(T5.8)	$SS_{vul} = \sum_{t=3}^{t=T} U_t^{pen}$
(T5.9)	$P_U = \frac{SS_{vul}}{\sigma_{vul}}$
(T5.10)	$\sigma_{vul} = 200$
(T5.11)	$P_{U_{max}} = \sum_t^t \sum_l^l U_{t,l}^{10}$
Priors	
(T5.12)	$prior(\log(\kappa)) \sim \mathcal{N}(\log(\kappa), \sigma = 0.9)$
Objective function	
(T5.13)	$Obj = -\log(LL_1) + -\log(P_{wt}) + P_U + P_{U_{max}} + prior(\log(\kappa))$

Table 6: Simulation-estimation scenarios

Scenario Code	Selectivity	U trajectory
CC	constant	contrast
CO	constant	one-way trip
CR	constant	U -ramp
VC	time-varying	contrast
VO	time-varying	one-way trip
VR	time-varying	U -ramp

Table 7: Scenarios for testing misspecification of L_∞

Scenario name	version	L_∞ value
true	true	68
minus10	10% underestimated	61.2
plus10	10% overestimated	74.8