Stock Reduction Analysis using catch at length data: Length-SRA

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Abstract

Many modern stock assessments include age- or length-based selectivity in their list of estimable parameters, often using simple parametric functions describing asymptotic or dome-shaped selectivity. We present a length-based stock reduction analysis (Length-SRA), which bypasses the requirement of estimating selectivity by calculating exploitation rate at length directly from observed catch at length data. We test the performance of Length-SRA with a simulation-evaluation framework under three exploitation rate trajectories and under fixed and time-varying selectivity scenarios. We also explore the impacts of misspecification of growth parameters. The model yields low bias in parameter estimates and management benchmarks and is relatively accurate when tracking changes in selectivity through time. We use Length-SRA to assess two species, Pacific hake and Peruvian jack mackerel, showing that selectivity is quite variable in both species over time, leading to time-varying management reference points. Length-SRA provides assessment results with accuracy comparable to other methods, such as Virtual Population Analysis and Statistical Catch at Age with the additional advantage of providing estimates of selectivity over time.

Keywords: stock assessment, time-varying selectivity, length composition.

1 Introduction

Modern stock assessments typically attempt to fit population dynamics models to catch at age and catch at length data, in hopes of extracting information from these data about age/size selectivity, cohort strength and fishing mortality patterns (Methot and Wetzel 2013; Hilborn and Walters 1992). Some assessment methods attempt to put aside the length frequency data, by converting these data to age compositions using age-from-length tables, perhaps using iterative methods to estimate proportions of fish at age for each length interval (Kimura and Chikuni 1987). In cases where age data are lacking, models like MULTIFAN-CL attempt to obtain estimates of selectivity, fishing mortality and population dynamics parameters only from size distribution data (Fournier et al. 1998). Combined with a few assumptions regarding the structure and variability in length at age, this procedure can even be used to attempt to recover information about changes in body growth patterns if there is a strong age-class signal in the length frequency data (Fournier et al. 1998). It is typical for assessment results from length-based assessment models to show substantial deviations between predicted and observed length distributions of catches, reflecting both sampling variation in the length composition data and incorrect assumptions about stability of growth and selectivity patterns (Hilborn and Walters 1992).

Selectivity to fishing is the combination of two processes: vulnerability to the fishing gear and availability of the fished population in the area being fished (Beverton and Holt 1957). Both processes can vary over time and therefore modify the resulting selectivity. Although selectivity process can often be directly measured through gear experiments, availability is generally harder to measure as it depends on the size-based distribution of the exploited population and the spatial distribution of the fishing fleet. Fish movement, size-structured changes in fish distribution, and changes in fleet distribution, can all affect availability and consequently lead to selectivity changes. Changes in selectivity are not uncommon (Sampson and Scott 2012) but are usually difficult to track over time. This difficulty is associated with an inability to distinguish between changes in fishing mortality and changes in selectivity in most age- and length-based stock assessment methods. For this reason, many assessment methods rely on ad hoc parametric selectivity models that may or may not include changes over time (Maunder et al. 2014). If misspecified, such models might lead to severe bias in fishing mortality estimates and other model parameters, which could result in misleading management advice (Martell and Stewart 2014).

Here we suggest an alternative approach to assessment modeling that begins by assuming that the assessment model should exactly reproduce the observed catch at length composition. This approach follows the dynamics of an age structured stock reduction analysis (SRA) (Walters et al. 2006; Kimura et al. 1984; Kimura and Tagart 1982) which follows a "conditioned on catch" format, in which catch composition is assumed to be known without error. The observed catches at age are then subtracted from modeled numbers at age to project numbers at age over time. A good review of SRA-type models is provided in Thorson and Cope (2015). The assumption of known catch composition is analogous to the classical assumption in virtual population analysis that reconstructed numbers at age should exactly match observed catch at age data (Hilborn and Walters 1992). The suggested approach may have two key advantages over statistical catch at age and/or catch at length models: (1) it does not require estimation of age or size selectivity schedules, and (2) catch at length data are commonly available for every year, even when age composition sampling has not been conducted.

We named this approach a Length-SRA assessment model. Here we present the model formulation, demonstrate its performance with a simulation-evaluation analysis and apply it to real fisheries data from the Peruvian jack mackerel (*Trachurus murphyi*) and Pacific hake (*Merluccius productus*) fisheries.

2 Methods

2.1 Stock reduction analysis with catch at length data - length-SRA

The stock reduction analysis (SRA) described here proceeds through the following steps: (1) compute numbers at age (based on recruitment estimates and mortality in the previous year); (2) convert numbers at age into numbers at length using the proportions of individuals at length given each age class; (3) calculate the exploitation rate at length using numbers at length and observed catch at length; (4) convert the exploitation rate at length to exploitation rate at age; (5) compute numbers in the following year using the exploitation rate at age, natural mortality, and recruitment estimates.

The model requires data on length composition of catch in numbers (used in step 3), a prior distribution for the recruitment compensation ratio, and a survey index of abundance that is used to tune the model parameters to the most likely stock abundance trajectory. The model also requires good estimates of growth parameters, variability around mean lengh at age, and natural mortality. The stock assessment and simulation routines were written in ADMB (Fournier et al. 2012) and are available on `github.com/catarinawor/length_SRA`.

A crucial component of the length-SRA is the calculation of proportions of individuals at length given each age class ($P_{l|a}$ - eqs. T3.1-T3.5). The calculation of such proportions (eq. T3.1) relies on four main assumptions regarding the distribution of length at age: (1) The mean length at age follows a von Bertalanffy growth curve (eq.T3.4), (2) The length at age is normally distributed (eqs. T3.1 -T3.3), (3) The standard deviation of the length at age is defined (e.g. eq.T3.5), and (4) $P_{L|a}$ is constant for all lengths equal or greater than a maximum length L (eq.T3.3).

The proportions of length at age are used to convert the length-based quantities into age-based quantities which are used to propagate the age structured population dynamics forward (Table 3). We assume that recruitment follows a Beverton-Holt type recruitment curve (eq. T3.6), that harvesting occurs over a short, discrete season in each time step (year or shorter), and that natural survival rate is known and constant over time (eqs. T3.6-T3.10). The computation of numbers at age in the initial year (i.e., first year in which data is reported - $t = init$) is different from that in the remaining years (eq. T3.13). Recruitment in the initial year is given by the parameter R_{init} which is used to indicate that the population was not at equilibrium at the start of the time series.

We used equilibrium spawner per recruit (SPR) quantities to calculate management targets, for illustration purposes we use 40% as a SPR target and use $Yield_{SPR=40\%}$ and $U_{SPR=40\%}$ as target management benchmarks (Table 4 - eqs.T4.6 to T4.14). As in all spawner per recruit calculations, our $Yield_{target}$ and U_{target} estimates depend on the selectivity curves calculated for each year (eq. T4.9).

To assess how well the model tracked changes in selectivity over time, we calculated the resulting selectivity estimates by normalizing the yearly vectors of exploitation rate at length ($U_{l,t}$) by the yearly average exploitation rate at length ($\bar{U}_l$) (eq. T3.11), which is more stable than the maximum yearly exploitation rate ($maxU_l$). This happens because observation errors tend to average out over the length classes, diminishing variability of $\bar{U}_l$ in relation to $maxU_l$. When calculating the management targets, we used the same method to calculate the mean selectivity at age (eq. T4.7), however we also averaged the selectivity at age curves over the past two years (eq. T4.7) in order to further smooth the curves.

The Length-SRA model estimates three main parameters: average unexploited recruitment R_0, recruitment compensation ratio κ, and recruitment in the initial year R_{init}. In addition, the annual recruitment deviations w_t are estimated for all cohorts observed in the model. That is, the number of recruitment deviations is equal to the number of years in the time series plus the number of age classes greater than recruitment age.

The objective function (eq. T5.7) is composed of a negative log-likelihood component, one penalty, and a prior component for the recruitment compensation ratio κ. The negative log-likelihood component minimizes the differences between the predicted and observed index of abundance (eq. T5.1). We assume that such differences are lognormally distributed (eqs. T5.3-T5.4) and use the conditional maximum likelihood estimator described by Walters and Ludwig (1994) to estimate the survey catchability coefficient q (eq. T5.2). A lognormal penalty is added to the negative log-likelihood function to constrain annual recruitment residuals so estimates have mean of zero and fixed standard deviation σ_R (Maunder and Deriso 2003) (eq. T5.5). Lastly, an informative normal prior for $log(\kappa)$ was included in the objective function (eq. T5.6). In earlier versions of the model (results not shown), we found that it was difficult for the length-SRA to determine the difference between a large stock with low productivity and a small stock with high productivity. The inclusion of an informative prior on the κ helps to solve this problem.

2.2 Simulation-evaluation

Model performance was evaluated using a simulation-evaluation with the biological parameters of an hypothetical fish species. We used the same model structure described in Table 3 for both the simulation and estimation models.

Table 1 – Indexes, variable definition, and values used in simulation-evaluation

Symbol	Value	Description
l	$\{l_o, ..., L\}$	Central point of length bin, $L = 50$ cm
a	$\{a_o, ..., A\}$	Age-class, $A = 20$ years
t	$\{1, ..., T\}$	Annual time step, $T = 50$ years
a_o	1	First age or age of recruitment
l_{bin}	2 cm	Size of length bin
l_o	8 cm	Central point of first length bin
$init$	21	Annual time step in which data starts to be reported
Distribution of length given age		
L_∞	50 cm	Maximum average length
K	0.3	Rate of approach to L_∞
t_o	-0.1	Theoretical time in which length of individuals is zero
cv_l	0.08	Coefficient of variation for length at age curve
$P_{l\|a}$		Matrix of proportions of length at age
Φ		Standard normal distribution
$zl_{a,l}$		Normalized z score for lower limit length bins
$zu_{a,l}$		Normalized z score for upper limit length bins
bl_l		Lower limit of length bins
bu_l		Upper limit of length bins
$\bar{L}_a$		Mean length at age
σ_L		Standard deviation of length at age
Population dynamics		
R_o	100	Average unfished recruitment
κ	10	Goodyear recruitment compensation ratio
S	0.7	Natural annual survival
σ_R	0.6	standard deviation for recruitment deviations
w_t	$\mathcal{N}(0, \sigma_R)$	Recruitment deviations for years {init-A-a_o,...,T}
$N_{a,t}$		Numbers of fish at age and time
SB_t		Spawning biomass at time
mat_a		Proportion of mature individuals at age
a_{rec}, b_{rec}		Beverton & Holt stock recruitment parameters
VB_t		Biomass that is vulnerable to the survey at time t
v_a	{0,0.5,1,...,1}	Survey vulnerability at age
R_{init}		Recruitment in the year data starts to be reported
$U_{a,t}$		Exploitation rate at age and time
$U_{l,t}$		Exploitation rate at length and time
$C_{l,t}$		Catch at length and time
$N_{l,t}$		Numbers at length and time
lx_a		Unfished survivorship at age
ϕ_e		Unfished average spawning biomass per recruit
$\widehat{sel_{l,t}}$		Selectivity estimates at length and time

Table 2 – Indexes, variable definition for operating model, MSY quantities, and values used in simulation-evaluation

Symbol	Value	Description
Operating model		
$sel_{l,t}$		Fishing selectivity at length and time
g,d,k	vary by scenario	Parameters for selectivity function
U_t	vary by scenario	Annual maximum exploitation rate
I_t		Index of abundance at time
σ_{I_t}	0.1	standard deviation for index of abundance deviates
q	1.0	Catchability coefficient
τ		multivariate logistic error term with $\sigma_\tau = 0.1$
Management quantities		
lz_a		Fished survivorship at age
F_z	seq(0.0,1.0,by=0.001)	Hypothetical average fishing mortality to calculate management targets
ϕ_z		Average spawning biomass per recruit
ϕ_{eq}		Average exploited biomass per recruit under U_z
$\widehat{sel_{a,t}}$		Selectivity at age and time t
R_{eq}		Average equilibrium recruitment under U_z
$Yield_z$		Equilibrium yield under U_z
$Yield_{target}$		Yield that would reduce spawner per recruit to 40% of unfished levels
U_{target}		Exploitation rate that reduce spawner per recruit to 40% of unfished levels

However, the operating model was modified to control annual exploitation rate (eq. T4.2), time varying selectivity (eq. T4.4), and observation and process errors.

The simulation model was initialized at unfished conditions (eq. T4.1) but only started reporting data for the simulation-evaluation procedure after the t_{init} year. Selectivity in the operating model was computed with the three parameter selectivity function described by Thompson (1994) (eq. T4.4). We chose to use this three parameter selectivity curve because of its flexibility, which allowed us to switch between logistic and dome-shaped selectivity curves in the scenarios in which time varying selectivity was considered. The observation error in the operating model included lognormal error in the index of abundance and logistic multivariate error (Schnute and Richards 1995) in the catch numbers at length (Table 2). Recruitment deviations were assumed to be lognormally distributed with constant σ_R (Table 1).

We considered a total of six different scenarios in simulation-evaluation trials, including three historical exploitation rate trajectories (contrast, one-way trip and U-ramp) and two selectivity patterns (constant and time-varying). In the contrast scenario the exploitation rate (U_t) starts low and increases beyond U_{target} and then decreases until $U_t \approx U_{target}$. In the one-way trip scenario U increased through time until $U \approx 2 \cdot U_{target}$. In the U-ramp scenario, U_t increases steadily until $U_t \approx U_{target}$ and remains constant thereafter. In the constant selectivity scenario, selectivity was assumed to follow a sigmoid shape. In the time varying selectivity scenario, the selectivity curve was assumed to vary

Table 3 – population dynamics for Length-SRA and operating model

Distribution of length given age		
	$$P_{l\|a} = \int_{zl_{a,l}}^{zu_{a,l}} \Phi(z)dz$$	(T3.1)
	$$zl_{a,l} = \frac{bl_l - \bar{L}_a}{\sigma_{L_a}}$$	(T3.2)
	$$zu_{a,l} = \begin{cases} \frac{bu_l - \bar{L}_a}{\sigma_{L_a}} & l < L \\ 1.0 & l = L \end{cases}$$	(T3.3)
	$$\bar{L}_a = L_\infty \cdot (1 - e^{(-K\cdot(a-t_o))})$$	(T3.4)
	$$\sigma_{L_a} = \bar{L}_a \cdot cv_l$$	(T3.5)
Population dynamics		
	$$N_{a,t>init} = \begin{cases} \frac{a_{rec}\cdot SB_{t-1}}{1+b_{rec}\cdot SB_{t-1}} \cdot e^{w_t}, & a = a_o \\ N_{a-1,t-1} \cdot S \cdot (1 - U_{a-1,t-1}), & a_o < a < A \\ \frac{N_{a-1,t-1}\cdot S\cdot(1-U_{a-1,t-1})}{1-S\cdot(1-Ua,t)}, & a = A \end{cases}$$	(T3.6)
	$$U_{a,t} = \sum_l (P_{l\|a} \cdot U_{l,t})$$	(T3.7)
	$$U_{l,t} = \frac{C_{l,t}}{N_{l,t}}$$	(T3.8)
	$$N_{l,t} = \sum_a (P_{l\|a} \cdot N_{a,t})$$	(T3.9)
	$$SB_t = \sum_a (mat_a \cdot w_a \cdot N_{a,t})$$	(T3.10)
	$$\widehat{sel_{l,t}} = \frac{U_{l,t}}{\bar{U}_t}$$	(T3.11)
	$$VB_t = \sum^a N_{a,t} \cdot w_a$$	(T3.12)
Initial year and incidence functions		
	$$N_{a,t=init} = lx_a \cdot R_{init} \cdot e^{(w_{t=init} \ldots w_{t=init-A+a_o})}$$	(T3.13)
	$$a_{rec} = \frac{\kappa}{\phi_e}$$	(T3.14)
	$$b_{rec} = \frac{\kappa - 1}{R_o \cdot \phi_e}$$	(T3.15)
	$$\phi_e = \sum_a lx_a \cdot mat_a \cdot w_a$$	(T3.16)
	$$lx_a = \begin{cases} 1, & a = 1 \\ lx_{a-1} \cdot S, & 1 < a < A \\ \frac{lx_{a-1}\cdot S}{1-S}, & a = A \end{cases}$$	(T3.17)

Table 4 – Management quantities and operating model

Operating model	
$N_{a,t=1} = lx_a \cdot R_o$	(T4.1)
$U_{l,t} = U_t \cdot sel_{l,t}^{OM}$	(T4.2)
$C_{l,t} = N_{l,t} \cdot U_{l,t} \cdot P_{l\|a} \cdot \tau$	(T4.3)
$sel_{l,t} = \frac{1}{1-g} \cdot \left(\frac{1-g}{g}\right)^g \cdot \frac{e^{d \cdot g \cdot (k-l)}}{1+e^{d \cdot (k-l)}}$	(T4.4)
$I_t = q \cdot VB_t \cdot e^{(\mathcal{N}(0,\sigma_{I_t}))}$	(T4.5)
Management quantities	
$lz_a = \begin{cases} lz_a = 1 & a = a_o \\ lz_{a-1} \cdot S \cdot exp(-F_z \cdot \widehat{sel_{a-1,t}}) & a_o < a < A \\ \frac{lz_{a-1} \cdot S \cdot exp(-F_z \cdot \widehat{sel_{a-1,t}})}{1 - S \cdot exp(-F_z \cdot \widehat{sel_{A,t}})} & a = A \end{cases}$	(T4.6)
$\widehat{sel_{a,t}} = \frac{\frac{U_{a,t-1}}{U_{t-1}} + \frac{U_{a,t}}{U_t}}{2}$	(T4.7)
$\phi_z = \sum_a lz_a \cdot mat_a \cdot w_a$	(T4.8)
$Target_\phi = \|\frac{\phi_z}{\phi_e} - 0.4\|$	(T4.9)
$\phi_{eq} = \sum_a lz_a \cdot (1 - exp(-F_z * \widehat{sel_{a,t}})) \cdot w_a$	(T4.10)
$R_{eq} = R_o \cdot \frac{\kappa - \phi_e/\phi_z}{\kappa - 1}$	(T4.11)
$Yield_z = R_{eq} \cdot \phi_{eq}$	(T4.12)
$Yield_{target} = Yield_z \to min(Target_\phi)$	(T4.13)
$U_{target} = 1 - exp(-F_z) \to min(Target_\phi)$	(T4.14)

Table 5 – Likelihood functions and penalties

Conditional Likelihood	
$Z_t = log(I_t) - log(VB_t)$	(T5.1)
$q = e^{\bar{Z}}$	(T5.2)
$Zstat_t = Z_t - \bar{Z}$	(T5.3)
$LL_1 \sim \mathcal{N}(Zstat\|\mu = 0, \sigma = \sigma_{I_t})$	(T5.4)
Penalties	
$P_{wt} \sim \begin{cases} \mathcal{N}(wt\|\mu = 0, \sigma = \sigma_R) & phase < lastphase \\ \mathcal{N}(wt\|\mu = 0, \sigma = \sigma_R \cdot 2) & phase = lastphase \end{cases}$	(T5.5)
Priors	
$prior(log(\kappa)) \sim \mathcal{N}(log(\kappa), \sigma = 0.5)$	(T5.6)
Objective function	
$Obj = -log(LL_1) + (-log(P_{wt})) + prior(log(\kappa))$	(T5.7)

every year, progressively changing from a dome shaped curve to sigmoid and back to dome shaped. The complete list of scenarios and the acronyms used are presented in Table 6.

All simulations had 30 years of data and 200 simulation trials were performed for each scenario. We evaluated the distribution of the relative proportional error ($\frac{esimated-simulated}{simulated}$) for the main parameter estimates (R_0, R_{init}, and κ) and for four derived quantities (Depletion: $\frac{SB_t}{SB_0}$, $Yield_{target}$, U_{target}, and q).

Table 6 – Simulation-estimation scenarios

Scenario Code	Selectivity	U trajectory
CC	constant	contrast
CO	constant	one-way trip
CR	constant	U-ramp
VC	time-varying	contrast
VO	time-varying	one-way trip
VR	time-varying	U-ramp

2.3 Misspecification of growth parameters

One important feature of the Length-SRA is that it assumes that growth follows a von Bertalanffy curve and that the growth parameters are known and constant over time. If this assumption is violated, the model outcomes will be impacted as the model will try to explain the deviations from the true growth curve with changes in the selectivity pattern. Here we illustrate how the model outcomes are impacted by the misspecification of the growth parameters by purposefully misreporting the values of L_∞ (Table 7). We assumed a simple logistic selectivity curve for this exercise and therefore expect the model to produce logistic patterns in the exploitation rate at length $U_{l,t}$.

Table 7 – Scenarios for testing misspecification of L_∞

Scenario name	version	L_∞ value
true	true	68
plus10	10% overestimated	74.8
minus10	10% underestimated	61.2

2.4 Real data examples

Two case studies were chosen to illustrate the application of the Length-SRA to real datasets: Pacific hake and Peruvian jack mackerel. Both species are believed to be subject to time varying selectivity.

The Pacific hake fishery is believed to exhibit time varying selectivity due to cohort targeting and annual changes fleet spatial distribution (Ruttan 2003). The population is know to have spasmodic recruitment, with high recruitment events occurring once or twice every decade (Ressler et al. 2007). Pacific hake tends to segregate by size during their annual migration (Ressler et al. 2007), allowing the fishing fleet to target strong cohorts by changing the spatial distribution of fishing effort as the cohort ages. Hake catch at length data was available for the period between 1975 and 2013. The survey index of abundance was available intermittently from 1995 to 2013.

The movement pattern of jack mackerel is not as well known, although fish appear to move between spawning and feeding areas (Gerlotto et al. 2012). Variability in selectivity patterns for the jack mackerel fishery are believed to be associated both with evolution of fleet capacity and gear utilization and with compression and expansion of the species range associated with abundance changes (Gerlotto et al. 2012). Jack mackerel catch at length data was available from 1980 to 2013 and the survey index was available between 1986 and 2013, with the exception of 2010.

3 Results

3.1 Simulation-evaluation

We evaluated the performance of the model in relation to the main parameters and derived management quantities with boxplots of the relative proportional error. Throughout we use the terms positive and negative median bias to indicate that the median relative proportional error is above or below zero. The median relative proportional error sign indicate if a parameter has been underestimated or overestimated the majority of the time.

Simulation-evaluation of Length-SRA model resulted in positive median bias for all three of the main parameters (Figure 1). The R_0 relative error estimates had similar median bias and spread across all scenarios, with slightly higher absolute median bias for the fixed selectivity scenario (Figure 1 - top panel). The R_{init} median relative error was more prominently positively biased for all scenarios when compared to other parameters. The absolute median relative error was higher for the CC, CO and VO scenarios. However the spread of the relative error estimates were similar across all scenarios (Figure 1 - second panel). The relative error estimates for κ indicate that this parameter was nearly unbiased, an effect of the informative prior considered for that parameter (Figure 1 - bottom panel).

The depletion (SB_t/SB_o) estimates resulted in negative median relative error for all scenarios except for VO (Figure 2 - top panel). $Yield_{target}$ median relative error was overestimated for all scenarios, but the absolute median relative error was relatively low ($< 7\%$) (Figure 2 - second panel). The U_{target} relative error estimates were the most variable, with median relative errors ranging from negative (CC scenario) to unbiased (VC and VO scenarios) to positive (CO, CR and VR). In addition, the spread in the estimates of U_{target} seem to be associated with the exploitation history; the contrast scenarios had the most accurate estimates and the one-way trip the most variable (albeit least biased) estimates (Figure 2 - third panel). The estimates of q were the most precise of the derived quantities. However, marked positive biases for the contrast and time-varying selectivity scenario (VC) and U-ramp scenarios (CR and VR) (Figure 2 - bottom panel) .

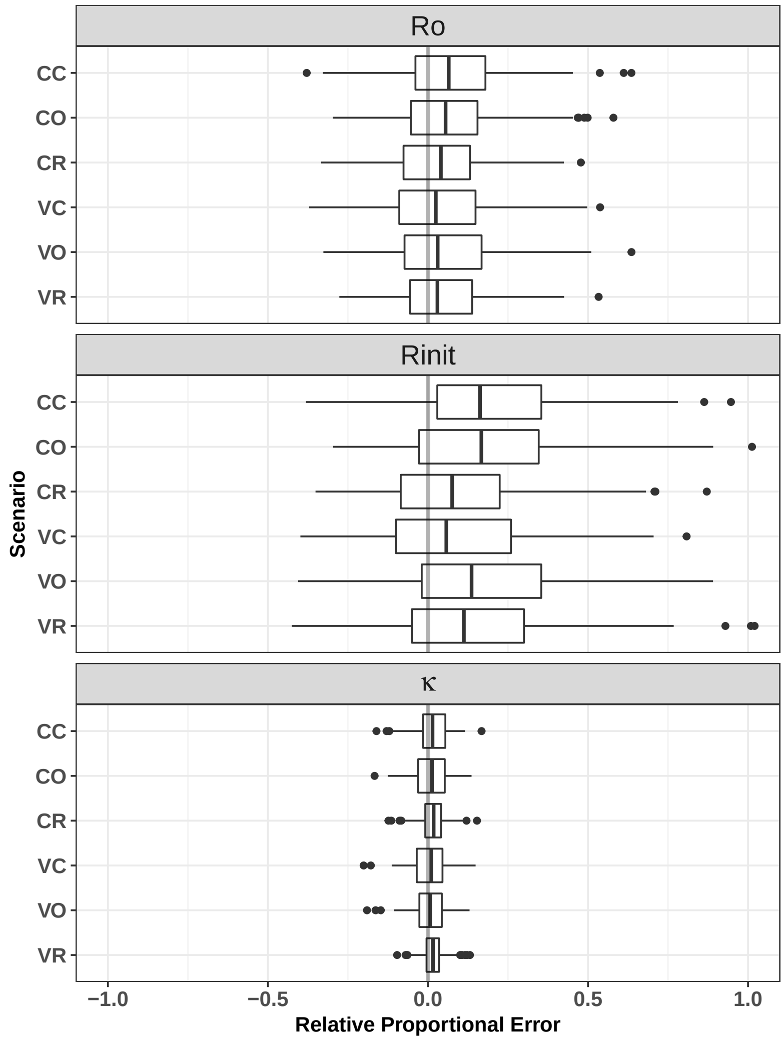

Figure 1 – Relative proportional error for main parameters for all scenarios considered in the simulation-evaluation. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value +/- 1.5 · inter-quartile range (distance between the first and third quartiles).

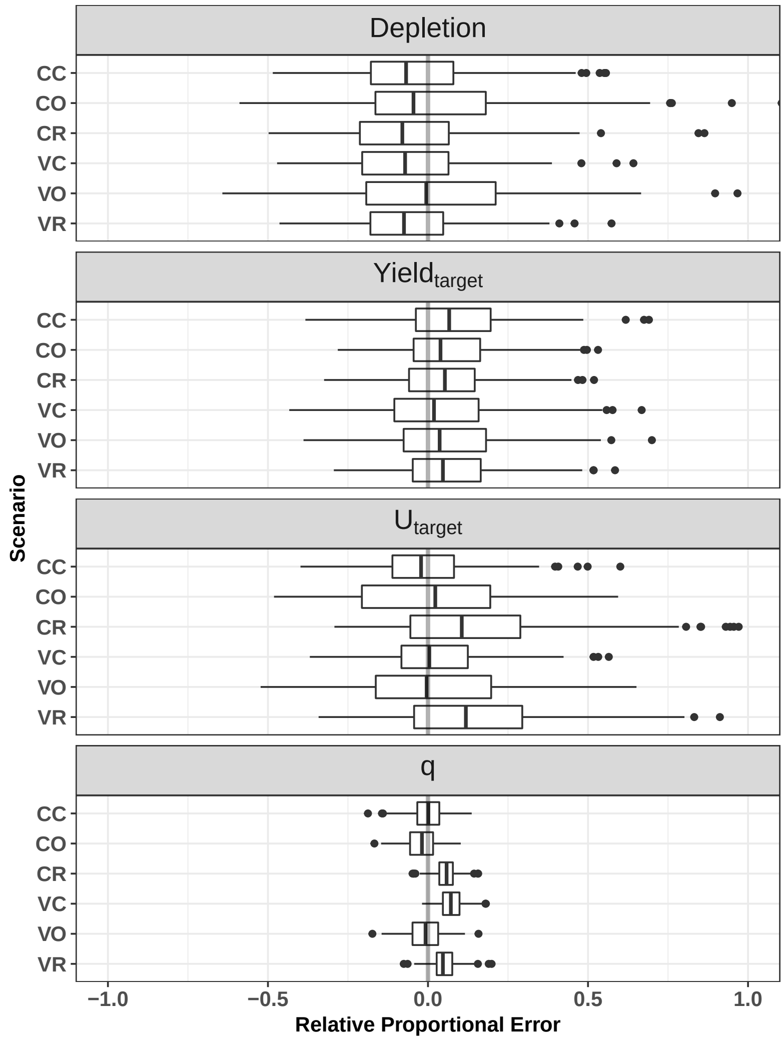

Figure 2 – Relative proportional error for main parameters for all scenarios considered in the simulation-evaluation. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value +/- 1.5 ·inter-quartile range (distance between the first and third quartiles).

The simulation-evaluation exercise showed that the Length-SRA model is able to track selectivity changes through time relatively well (Figure 3). However the selectivity estimates are quite variable, which is likely due to the observation error in the catch at length composition.

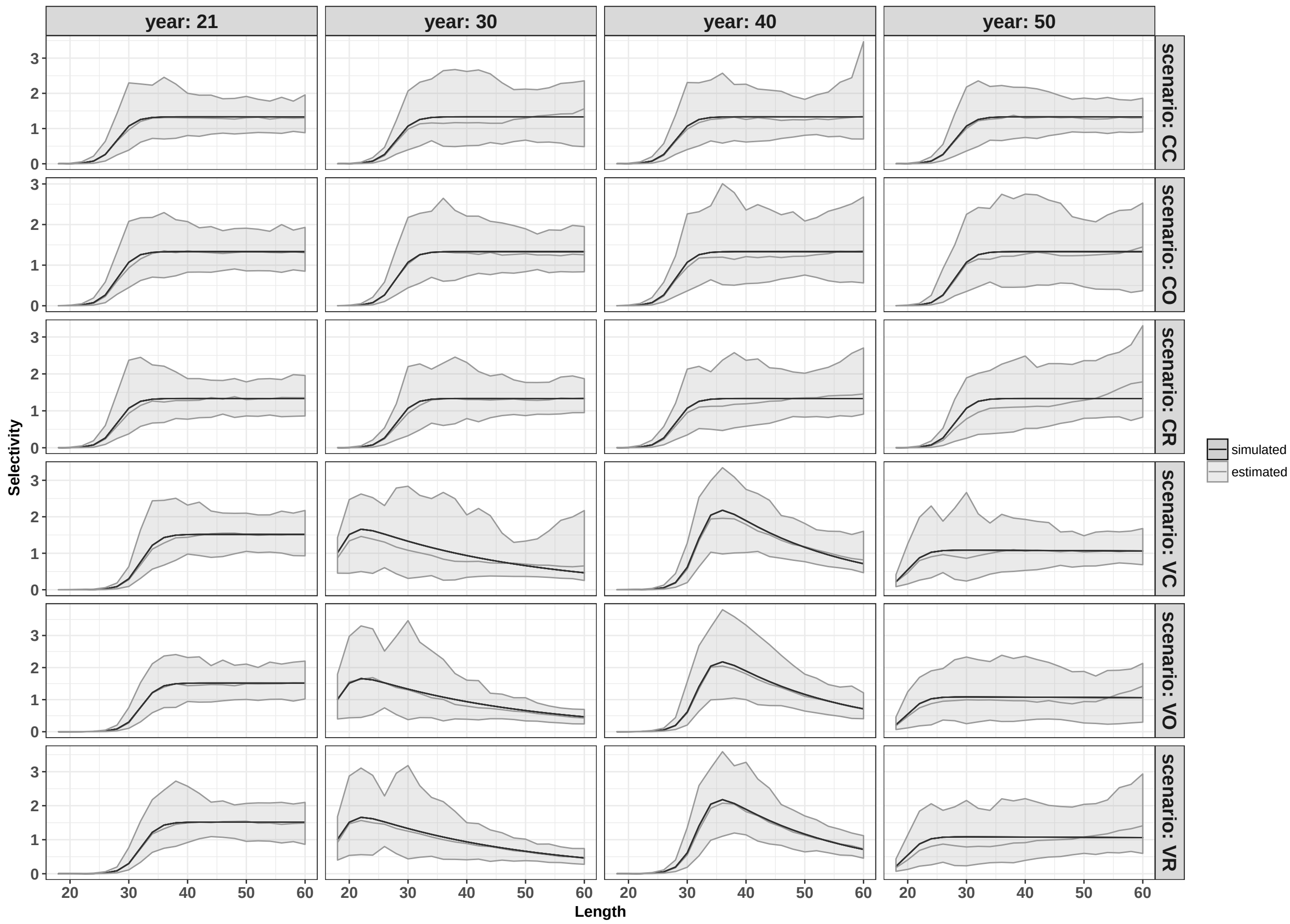

Figure 3 – Simulated and realized selectivity estimates for a set of years within simulation-evaluation time series. The estimated solid lines indicate median, 2.5% and 97.5% quantiles for the derived selectivities.

3.2 Misspecification of growth parameters

We found that misspecification of L_∞ has severe implications in the capability of the model to estimate exploitation rate at length $U_{l,t}$(Figure 4). If the value of L_∞ was specified to be lower than true, the estimates of $U_{l,t}$ were lower than true for most length and extremely high for high lengths (approaching the true L_∞). In the scenario where L_∞ was reported to be higher than true, $U_{l,t}$ was estimated to follow a dome shaped pattern, with very low exploitation rates for the higher lengths. These patterns occur because the model is trying to adjust the mismatch between proportions of catch at length and the $P_{l|a}$ matrix by changing the predicted selectivity pattern. As a result, failure to adequately specify L_∞ leads to erroneous estimation of selectivity patterns and, consequently, failure in estimating management quantities.

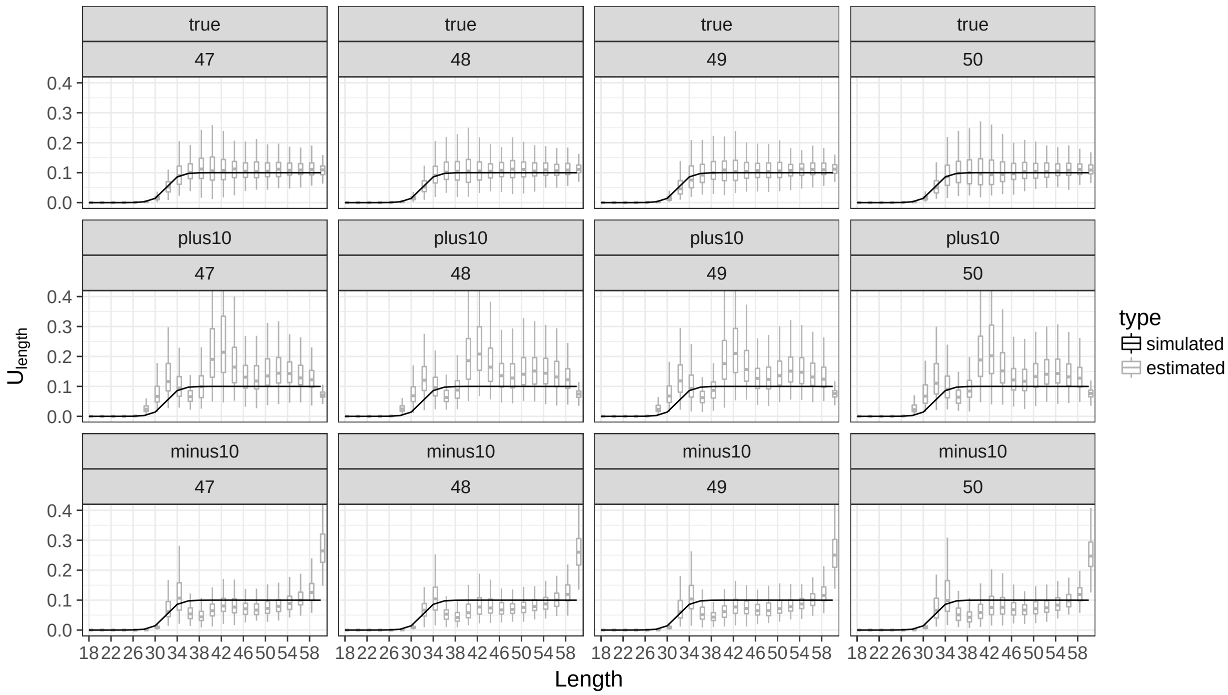

Figure 4 – Simulated and realized exploitation rate at length $U_{l,t}$ when L_∞ is misspecified. Results shown for the last four years of simulation-evaluation time series. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value +/- 1.5 · inter-quartile range (distance between the first and third quartiles).

3.3 Real data examples

The model fit the Pacific hake and jack mackerel indexes of abundance relatively well (Figure 5), despite some limitation in the available data. The Pacific hake index of abundance time-series is relatively short and intermittent (survey happens every two or three years). The index of abundance time series for jack mackerel was longer but it indicates a downward trend in abundance with low contrast in the last ten years of data.

The model fit for both species resulted in time varying selectivities that lead to variation in $Yield_{target}$ and consequent changes in U_{target} (Figure 5). This is because changes in selectivity result in changes to the vulnerable biomass even if total biomass is constant. A sharp peak in both $Yield_{target}$ and U_{target} is shown for both Pacific hake and jack mackerel. We believe these peaks are likely unrealistic and are associated with difficulties in estimating recruitment deviations in the early years.

The selectivity curves estimated for Pacific hake and jack mackerel are quite variable and mostly estimated to be dome shaped (Figure 6). It is important to note that this observed variability might indicate real changes in selectivity (e.g. cohort targeting) or might also be caused by misspecification of the growth parameters (see Figure 4). At this point it impossible to determine what are the causes for the resulting patterns in selectivity observed with the Length-SRA fit. Further investigation would be needed if this model is to be used for management purposes.

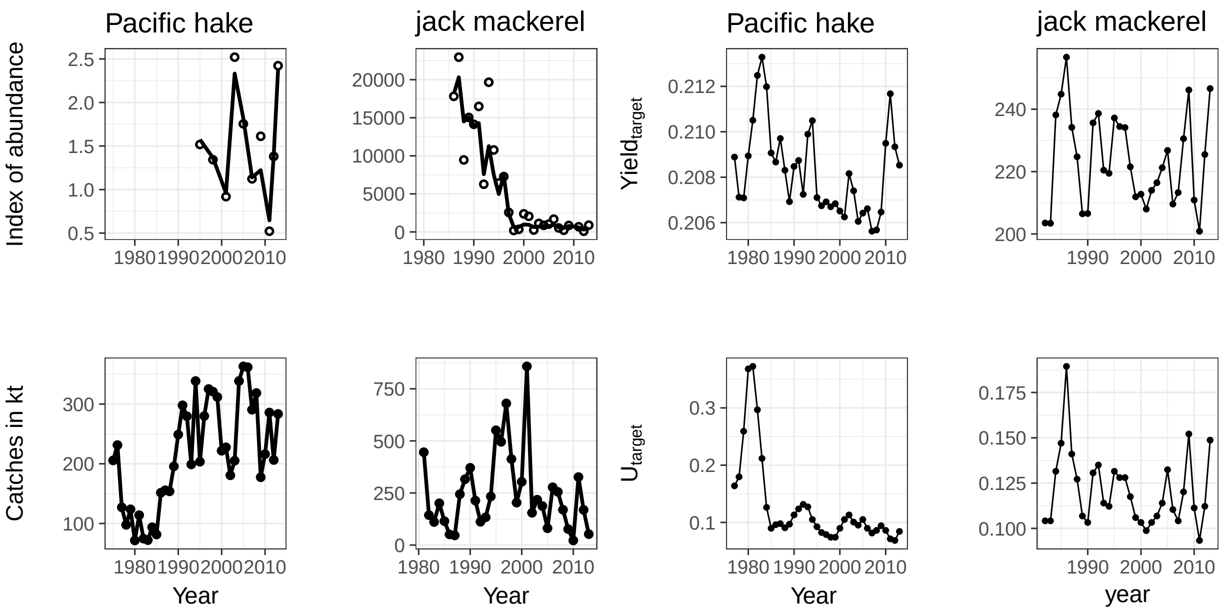

Figure 5 – Fit to index of abundance, historical catches, and $Yield_{target}$ and U_{target} estimates for Pacific hake and jack mackerel.

4 Discussion

We present a length-based stock reduction analysis (Length-SRA) that allows monitoring of time varying selectivity. In the Length-SRA model, catch at length is assumed to be known without error and exploitation rate at length is calculated directly from estimates of numbers at length. In turn, numbers at length are produced based on numbers at age and on probabilities derived from growth curve parameters and the assumed variability (standard deviation) around mean length at age. This fact is important because it allows the model to bypass the requirement for the estimation of a selectivity ogive, as is required in more traditional age- and length-based models (Sullivan et al. 1990; Mesnil and Shepherd 1990, e.g.) and in more recent length based state-space modelling approaches (White et al. 2016). Estimation of selectivity ogives can be very difficult, especially if selectivity is believed to vary over time unpredictably (Martell and Stewart 2014; Linton and Bence 2011).

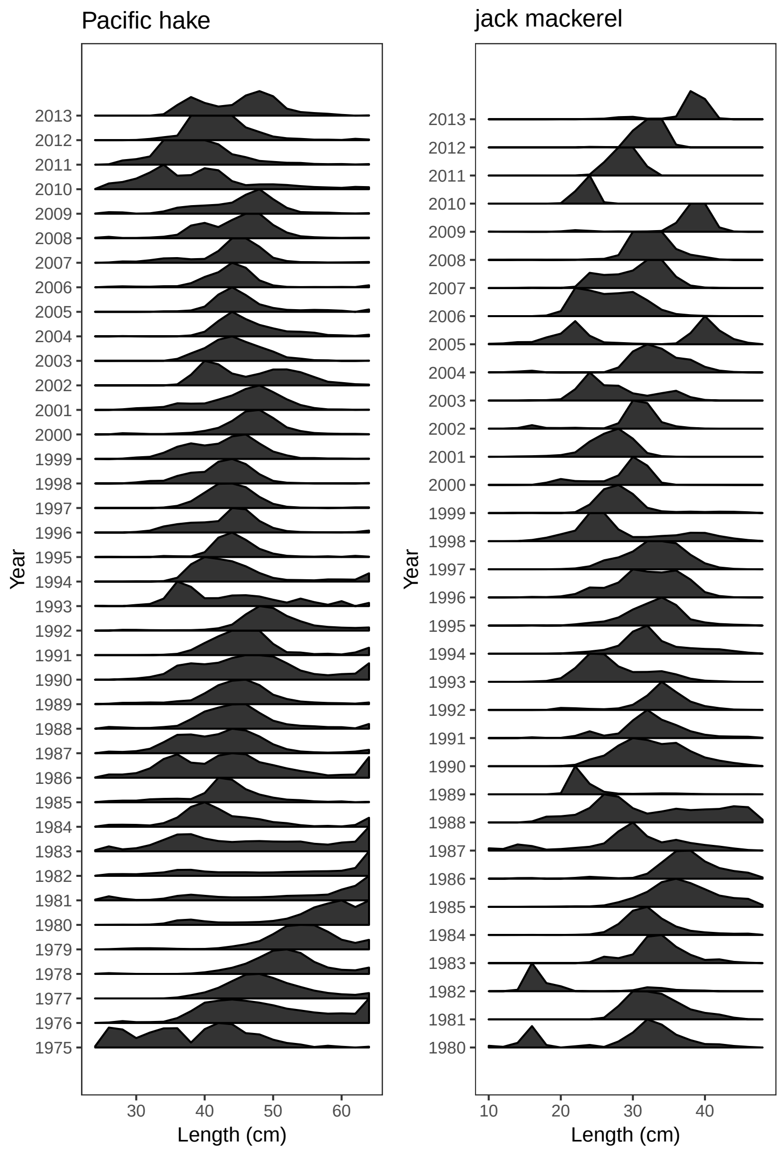

Figure 6 – Estimated selectivity patterns across years for Pacific hake and jack mackerel.

The accuracy in the estimates of selectivity obtained with the Length-SRA are comparable with those presented by Nielsen and Berg (2014), especially for the one-way trip scenarios. The Length-SRA selectivity estimates are less precise than those shown by Nielsen and Berg (2014), likely because the Length-SRA estimates incorporate observation error. The Nielsen and Berg (2014) approach accounts for time varying selectivity by treating fishing mortality at age as stochastic processes that are correlated over age and time. Their model seems to perform extremely well, however they only considered one exploitation rate trajectory, with significant contrast in the data. In addition, the changes in selectivity considered in their study are more subtle than the ones considered here.

Another attractive approach to model catch at length data is the one described by White et al. (2016). Their state space model estimates annual recruitment, harvest rate, and error terms from catch at length data and seems to obtain good precision and accuracy in their estimates. However, it is important to note that their model relies on two assumptions that are likely violated in real data examples. First, they assume that selectivity is constant over time and follows a cumulative normal distribution parametric form. Second, they assume that the growth component of the transition matrix (probabilities of age given length) to be constant and given by the growth curve. This is incorrect because fishing causes changes in the age composition of fish of size x, i.e. under fishing, proportion of younger fish in a given length/size class is expected to increase.

An important advancement of Length-SRA over conventional stock-assessment models is the indirect calculation of time-varying selectivity. This information alone can be used to characterize the complexity of the fishery system. Length-SRA on its own is reasonably accurate in deriving important management-oriented parameters (depletion and $Yield_{target}$). however another option may be to combine findings from this model with another assessment model, such as a statistical catch at age (SCA) model. In this framework, Length-SRA can be used to calculate annual selectivity patterns and provide an indication of possible changes over time. These selectivity estimates can then become an input into an SCA to calculate other important variables and produce management advice. This combination of models has been used in the past (Walters and Punt 1994); we suggest Length-SRA may be a useful tool in this context.

Accurate estimates of selectivity are particularly important if the fishery management is based on yield per recruit reference points. Fishery yield per recruit depends on the selectivity curve (Beverton and Holt 1957) and for this reason, changes in selectivity over time will directly affect reference points (Beverton and Holt 1957; Hilborn and Walters 1992). We observed selectivity changes for both Pacific hake and jack mackerel and show how this variability can lead to a difference between the maximum and minimum estimates of $Yield_{target}$ and U_{target} calculated along the time series. We believe that tracking these changes is important not only to ensure appropriate management recommendations, but also to illustrate the relationship between selectivity patterns and management targets (Vasilakopoulos et al. 2016).

One potential point of concern that should be considered when using the Length-SRA is that it assumes that the biological parameters used in the growth curve and catch at age relationship are know without error and constant over time. We have tested the Length-SRA under misspecification of the von Bertalanffy growth parameters and we observed additional bias in the estimates of parameter and management quantities as well as strong distortions in the resulting selectivity parameters. Similarly, Minte-Vera et al. (2017) showed that misspecification in biological parameters, especially in asymptotic length, can have a significant impact in assessment results. Other length models, e.g. MULTIFAN-CL (Fournier et al. 1998), overcome the assumption of known growth parameters by estimating the von Bertalanffy parameters alongside the assessment parameters. The estimation of the growth parameters is made possible by assuming that selectivity follows a parametric function (usually logistic). Once a simple selectivity curve is assumed, all deviations in observed catch at length are explained by adjusting the growth parameters. This assumption can also lead to bias in parameter estimates, as other studies show that variability in selectivity and non-asymptotic patterns are common (Waterhouse et al. 2014). In reality, in most cases it is difficult to know if patterns observed in catch at length are caused by fisheries targeting (i.e. selectivity) or if they would be more appropriately explained by adjusting the growth parameters. Therefore, we recommend that, when using the Length-SRA, the user should perform extensive sensitivity analyses over the possible range of values for the growth parameters, particularly if the predicted selectivity patterns are highly variable.

As mentioned previously, the model and simulation exercise presented here assumes that the growth parameters are known and constant through time. Consequently, time variability in growth patterns could also impacts the results produced by the model. We would not recommend attempting to estimate time varying growth parameters within the Length-SRA because growth and exploitation rates at length are confounded. However, if estimates of time-varying growth are available, preferably from fishery independent data, those could be used as an input to the Length-SRA model.

The approach used in the Length-SRA is analogous to that used in virtual population analysis in that the length composition data is assumed to be known without error. For this reason, the selectivity estimates include extra variability due to observation and sampling error. We attempted to minimize this effect by smoothing the predicted selectivity over two years, however this method is not capable of completely removing the observation error effect from the selectivity estimates. Because of the assumption of known catch at length, it is important that the catch sampling is representative of the total removals from the population (Pope 1972). As in any other fisheries model, biased sampling and/or low sampling effort will result in bias in parameter and fishery reference point estimates (Coggins and Quinn 1998; Bunch et al. 2013).

Some management parameters are consistently overestimated ($Yield_{target}$) and underestimated (depletion) which may be cause for concern. However, it is important to note that both parameters have low absolute median relative error (<7%). The magnitude of the bias in the estimates of $Yield_{target}$ and U_{target} observed in this study are comparable (in magnitude) to the results obtained by Martell and Stewart (2014) for MSY and F_{MSY} in a simulation study on the impacts of time varying selectivity on the estimates generated by a statistical catch at age model. Other studies show even higher biases in face of time-varying selectivity (e.g. Linton and Bence 2011; HenrÃ■quez et al. 2016). The estimates of depletion are also comparable to those produced with other SRA type assessments evaluated by Thorson and Cope (2015). Overall, parameter and derived parameters estimates are generally within the range of many other stock assessment models.

The Length-SRA approach presented in this study can be a useful tool for fisheries stock assessment. We believe that this is particularly true when time varying selectivity is thought to occur, especially if the variability is not easily predictable from historical changes in gear use/fleet composition. However, we would like to acknoledge that the selectivity estimates will only be reliable if the growth parameters for the population being assessed are known. In addition, the simple nature of the Length-SRA model makes it a good candidate model for inclusion on closed-loop simulation studies. Further testing of this model in a closed-loop simulation set up would provide more insight on the model performance on achieving management outcomes (Punt et al. 2014). We foresee the application of this model as an investigative tool to evaluate potential time-varying selectivity patterns, as a stock assessment tool and as part of closed loop simulation studies.

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