Stock Reduction Analysis using catch at length data:

Length-SRA

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Abstract

Many modern stock assessments convert length frequency data from catches to age compositions to then conduct age-structured assessment. These assessments must estimate age- or length-based vulnerability to fishing, often using simple parametric functions describing asymptotic or dome-shaped vulnerability. We present a length-based stock reduction analysis (Length-SRA), which bypasses the requirement of estimating a vulnerability by estimating exploitation rate at length directly from observed catch at length data. We test the performance of Length-SRA with a simulation-evaluation framework under three exploitation rate trajectories and under fixed and time-varying vulnerability scenarios. The model yields relatively low bias in parameter estimates and management benchmarks. Length-SRA is relatively accurate when tracking changes in vulnerability through time. We further use Length-SRA to assess two species, Pacific hake and Peruvian jack mackerel, to show that vulnerability is quite variable in both species over time, leading to time-varying management reference points. Length-SRA provides assessment results with accuracy comparable to other

methods, such as Virtual Population Analysis and Statistical Catch at Age. However, Length-SRA has the additional advantage of providing accurate estimates of vulnerability over time.

1   Introduction

Modern stock assessments typically attempt to fit population dynamics models to catch at age and catch at length data, in hopes of extracting information from these data about age/size vulnerability, cohort strength and fishing mortality patterns (Methot and Wetzel 2013; Hilborn and Walters 1992). Some assessment methods attempt to put aside the length frequency data, by converting these data to age compositions using age-from-length tables, perhaps using iterative methods to estimate proportions of fish at age for each length interval (Kimura and Chikuni 1987). In cases where age data are lacking, models like MULTIFAN-CL attempt to obtain estimates of vulnerability, fishing mortality and population dynamics parameters only from size distribution data (Fournier et al. 1998). Combined with a few assumptions regarding the structure and variability in length at age, this procedure can even be used to attempt to recover information about changes in body growth patterns if there is a strong age-class signal in the length frequency data (Fournier et al. 1998). It is typical for assessment results from length-based assessment models to show substantial deviations between predicted and observed length distributions of catches, reflecting both sampling variation in the length composition data and incorrect assumptions about stability of growth and vulnerability patterns (Hilborn and Walters 1992).

Vulnerability to fishing is the combination of two processes: selectivity of the fishing gear and availability of the fished population in the area being fished (Beverton and Holt 1957). Both processes can vary over time and therefore modify the resulting selectivity. Although selectivity process can often be directly measured through gear experiments, availability is generally harder to measure as it depends on the size-based distribution of the exploited population and the spatial distribution of the fishing fleet. Fish movement, size-structured changes in fish distribution, and changes in fleet distribution, can all affect availability and consequently lead to vulnerability changes. Changes in vulnerability are not uncommon (Sampson and Scott 2012) but are usually difficult to track over time. This difficulty is associated with an inability to distinguish between changes in fishing mortality and changes in vulnerability in most age- and length-based stock assessment methods. For this reason, many assessment methods rely on ad hoc parametric vulnerability models that may or may not include changes over time (Maunder et al. 2014). If misspecified, such models might lead to severe bias in fishing mortality estimates and other model parameters, which could result in misleading management advice (Martell and Stewart 2014).

Here we suggest an alternative approach to assessment modeling that begins by assuming that the assessment model should exactly reproduce the observed catch at length composition. This approach follows the dynamics of an age structured stock reduction analysis (SRA) (Walters et al. 2006; Kimura et al. 1984; Kimura and Tagart 1982) which follows a "conditioned on catch" format, subtracting observed catches at age from modeled numbers at age in estimation of numbers at age over time. This assumption is analogous to the classical assumption in virtual population analysis that reconstructed numbers at age should exactly match observed catch at age data (Hilborn and Walters 1992). The suggested approach may have two key advantages over statistical catch at age and/or catch at length models: (1) it does not require estimation of age or size vulnerability schedules, and (2) catch at length data are commonly available for every year, even when age composition sampling has not been conducted.

We named this approach a Length-SRA assessment model. Here we present the model formulation, demonstrate its performance with a simulation-evaluation analysis and apply it to real fisheries data from the Peruvian jack mackerel (*Trachurus murphyi*) and Pacific hake (*Merluccius productus*) fisheries.

2 Methods

2.1 Stock reduction analysis with catch at length data - length-SRA

The stock reduction analysis (SRA) described here proceeds through the following steps: (1) compute numbers at age (based on recruitment estimates and mortality in the previous year); (2) convert numbers at age into numbers at length using the proportions of individuals at length given each age class; (3) calculate the exploitation rate at length using numbers at length and observed catch at length; (3) convert the exploitation rate at length to exploitation rate at age; (4) compute numbers in the following year using the exploitation rate at age, natural mortality, and recruitment estimates.

A crucial component of the length-SRA is the calculation of proportions of individual at length given each age class ($P_{l|a}$ - eqs. T3.1-T3.5). The calculation of such proportions (eq. T3.1) relies on four main assumptions regarding the distribution of length at age: (1) The mean length at age follows a von Bertalanffy growth curve (eq.T3.4), (2) The length at age is normally distributed (eqs. T3.1 -T3.3), (3) The standard deviation of the length at age is defined (e.g. eq.T3.5), and (4) $P_{L|a}$ is constant for all lengths equal or greater than a maximum length L (eq.T3.3).

The proportions of length at age are used to convert the length-based quantities into age-based quantities which are used to propagate the age structured population dynamics forward (Table 3). We assume that recruit-

ment follows a Beverton-Holt type recruitment curve (eq. T3.6), that harvesting occurs over a short, discrete season in each time step (year or shorter time period), and that natural survival rate is known and stable over time (eqs. T3.6-T3.10). The computation of numbers at age in the initial year (i.e., first year in which data is reported - t_{init}) is different from that in the remaining years (eq. T3.12). Recruitment in the initial year is given by the parameter R_{init} which is used to indicate that the population was not at equilibrium at the start of the time series.

We used equilibrium yield per recruit quantities to calculate management targets, for illustration purposes we use $Yield_{SPR=40\%}$ and $U_{SPR=40\%}$ as target management benchmarks (Table 4 - eqs.T4.6 to T4.14). As in all yield per recruit calculations, our $Yield_{target}$ and U_{target} estimates depend on the vulnerability curves calculated for each year (eq. T4.9).

To assess how well the model tracked changes in vulnerability over time, we calculated the resulting vulnerability estimates by normalizing the yearly vectors of exploitation rate at length ($U_{l,t}$) by the yearly average exploitation rate at length ($\bar{U}_l$) (eq. T3.11), which is more stable than the maximum yearly exploitation rate ($maxU_l$). This happens because observation errors tend to average out over the length classes, diminishing variability of $\bar{U}_l$ in relation to $maxU_l$. When using the vulnerability estimates to calculate management targets we also averaged the selectivity curves over the past two years (eq. T4.7) in order to further smooth the curves.

The Length-SRA model estimates three main parameters: average unexploited recruitment R_0, recruitment compensation ratio κ, and recruitment in the initial year R_{init}. In addition, the annual recruitment deviations w_t are estimated for all cohorts observed in the model. That is, the number of recruitment deviations is equal to the number of years in the time series plus the number of age classes greater than recruitment age.

The objective function (eq. T5.7) is composed of a negative log-likelihood component, one penalty, and a prior component for the recruitment compensation ratio κ. The negative log-likelihood component minimizes the differences between the predicted and observed index of abundance (eq. T5.1). We assume that such differences are lognormally distributed (eqs. T5.3-T5.4) and use the conditional maximum likelihood estimator described by Walters and Ludwig (1994) to estimate the survey catchability coefficient q (eq. T5.2). A lognormal penalty is added to the negative log-likelihood function to constrain annual recruitment residuals so estimates have mean of zero and fixed standard deviation σ_R (Maunder and Deriso 2003) (eq. T5.5). Lastly, an informative normal prior for $log(\kappa)$ was included in the objective function (eq. T5.6).

2.2 Simulation-evaluation

Model performance was evaluated using a simulation-evaluation populated with the biological parameters of an hypothetical fish species. We used the same model structure described in Table 3 for both the simulation and estimation models. However, the operating model was modified to control annual exploitation rate (eq. T4.2), time varying selectivity (eq. T4.4), and observation and process errors.

The simulation model was initialized at unfished conditions (eq. T4.1) but only started reporting data for the simulation-evaluation procedure after the t_{init} year. Selectivity in the operating model was computed with the three parameter selectivity function described by Thompson (1994) (eq. T4.4). We chose to use this three parameter selectivity curve because of its flexibility, which allowed us to switch between logistic and dome-shaped selectivity curves in the scenarios in which time varying selectivity was considered. The observation error in the operating model included lognormal error in the index of abundance and logistic multivariate error (Schnute and Richards 1995) in the catch numbers at length (Table 2). Recruitment deviations were assumed to be lognormally distributed with constant σ_R (Table 1).

We considered a total of six different scenarios in simulation-evaluation trials, including three historical exploitation rate trajectories (contrast, one-way trip and U-ramp) and two selectivity patterns (constant and time-varying). In the contrast scenario the exploitation rate (U_t) starts low and increases beyond U_{MSY} and then decreases until $U_t = U_{MSY}$. In the one-way trip scenario U increased through time until $U = 2 \cdot U_{MSY}$. In the U-ramp scenario, U_t increases steadily until $U_t = U_{MSY}$ and remains constant thereafter. In the constant selectivity scenario, selectivity was assumed to follow a sigmoid shape. In the time varying selectivity scenario, the selectivity curve was assumed to vary every year, progressively changing from a dome shaped curve to sigmoid and back to dome shaped. The complete list of scenarios and the acronym used for them is presented in Table 6.

All simulations had 30 years of data and 200 simulation trials were performed for each scenario. We evaluated the distribution of the relative proportional error ($\frac{esimated - simulated}{simulated}$) for the main parameter estimates (R_0, R_{init}, and κ) and for four derived quantities (Depletion: $\frac{SB_t}{SB_0}$, $Yield_{target}$, U_{target}, and q).

2.3 Real data examples

Two case studies were chosen to illustrate the application of the Length-SRA to real datasets: Pacific hake and Peruvian jack mackerel. Both species are believed to be subject to time varying selectivity.

The Pacific hake fishery is believed to exhibit time varying selectivity due to cohort targeting and annual changes fleet spatial distribution (Ruttan 2003). The population is know to have spasmodic recruitment, with high recruitment events occurring once or twice every decade (Ressler et al. 2007). Pacific hake tends to segregate by size during their annual migration (Ressler et al. 2007), allowing the fishing fleet to target strong cohorts by changing the spatial distribution of fishing effort as the cohort ages. Hake catch at length data was available for the period between 1975 and 2013. The survey index of abundance was available intermittently from 1995 to 2013.

The movement pattern of jack mackerel is not as well known, although fish appear to move between spawning and feeding areas (Gerlotto et al. 2012). Variability in selectivity patterns for the jack mackerel fishery are believed to be associated both with evolution of fleet capacity and gear utilization and with compression and expansion of the species range associated with abundance changes (Gerlotto et al. 2012). Jack mackerel catch at length data was available from 1980 to 2013 and the survey index was available between 1986 and 2013, with the exception of 2010.

3 Results

3.1 Simulation-evaluation

We evaluated the performance of the model in relation to the main parameters and derived management quantities with boxplots of the relative proportional error. Throughout we use the terms positive and negative median bias to indicate that the median relative proportional error is above or below zero. The median relative proportional error sign indicate if a parameter has been underestimated or overestimated the majority of the time.

Simulation-evaluation of Length-SRA model resulted in a small positive median bias for the κ parameter across all scenarios (Figure 1). The R_0 relative error medians were positively biased for the contrast and one-way-trip scenarios (CC, CO, VC and VO) and negatively biased for the U-ramp scenarios (Figure 1). The R_{init} median relative error was negatively biased for all scenarios with higher absolute biases associated with the contrast scenario with time-varying selectivity (VC) and U-ramp scenarios (CR and VR) (Figure 1). Overall, The contrast with constant selectivity and one-way trip scenarios yielded the best performance.

The depletion (SB_t/SB_o) estimates resulted in negative median relative error for all scenarios (Figure 2). *MSY* was also underestimated for most senarios, except for the contrast scenario with constant vulnerability, for which the median relative proportional error was zero. The estimates for U_{MSY} showed very low (<0.071) absolute median proportional errors for all scenarios (Figure 2). The estimates of q were more variable, with

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particularly high positive bias for the the contrast and time-varying selectivity scenario (VC) and U-ramp scenarios (CR and VR). The overestimation in q is likely associated with the more severe underestimations of and R_{init} and MSY for those scenarios as both R_{init} and MSY are negatively correlated with q.

The simulation-evaluation exercise showed that the Length-SRA model is able to track vulnerability for the CC, CO and VO scenarios (Figure 3). The capability of the model to track vulnerability through time was diminished for the CR, VC and VR scenarios. The decreased accuracy for the CR, VC and VR scenarios is likely associated with the lack of information in the data, this was an expected outcome for the U-ramp scenarios (CR and VR) but it was somewhat unexpected for the contrast scenario with time varying vulnerability (VC). However, upon closer inspection of the biomass trends in the VC scenario (results not shown), we realized that the interaction between time-varying vulnerability and changes in exploitation rate resulted in a somewhat stable biomass trend over time resulting in lack of contrast in the time series.

3.2   Real data examples

The model fit the Pacific hake and jack mackerel indexes of abundance relatively well (Figure 4), despite some limitation in the available data. The Pacific hake index of abundance time-series is quite short and intermittent (survey happens every two or three years). The index of abundance time series for jack mackerel was longer but it indicates a downward trend in abundance with low contrast in the last ten years of data.

The model fit for both species resulted in time varying selectivities that lead to variation in MSY and consequent changes in U_{MSY}. This is because changes in selectivity result in changes to the vulnerable biomass even if total biomass is constant. The time-varying selectivity calculated for each species (Figure 5) results in up to two-fold change in MSY throughout the time series for both species (Figure 4). The selectivity curves estimated for Pacific hake and jack mackerel are quite variable. It is important to note that this observed variability might indicate real changes in selectivity (e.g. cohort targeting) or might also be caused by misspecification of the growth parameters (see Appendix). At this point it impossible to determine what are the causes for the resulting patterns in vulnerability observed with the Length-SRA fit. Further investigation would be needed if this model is to be used for management purposes.

4   Discussion

We present a length-based stock reduction analysis (Length-SRA) that allows monitoring of time varying vulnerability. In the Length-SRA model, catch at length is assumed to be known without error and exploitation

rate at length is calculated directly from estimates of numbers at length. In turn, numbers at length are produced based on numbers at age and on probabilities derived from growth curve parameters and coefficient of variation. This fact is important because it allows the model to bypass the requirement for the estimation of a vulnerability ogive, as is required in more traditional age- and length-based models (Sullivan et al. 1990; Mesnil and Shepherd 1990, e.g.) and in more recent length based state-space modelling approaches (White et al. 2016). Estimation of vulnerability ogives can be very difficult, especially if vulnerability is believed to vary over time unpredictably (Martell and Stewart 2014; Linton and Bence 2011).

The accuracy in the estimates of vulnerability obtained with the Length-SRA are comparable with those presented by Nielsen and Berg (2014), especially for the one-way trip scenarios (scenarios with the most informative data). The Length-SRA selectivity estimates are less precise than those shown by Nielsen and Berg (2014), likely because the Length-SRA estimates incorporate observation error. The Nielsen and Berg (2014) approach account for time varying selectivity by treating fishing mortality at age as stochastic processes that are correlated over age and time. Their model seems to perform extremely well, however they only considered one exploitation rate trajectory, with significant contrast in the data. In addition, the changes in vulnerability considered in their study are more subtle than the ones considered here.

Another attractive approach to model catch at length data is the one described by White et al. (2016). Their state space model estimates annual recruitment, harvest rate, and error terms from catch at length data and seems to obtain good precision and accuracy in their estimates. However, it is important to note that their model relies on two assumptions that are likely violated in real data examples. First, they assume that vulnerability is constant over time and follows a cumulative normal distribution parametric form. Second, they assume that the growth component of the transition matrix P(y,x) to be constant and given by the growth curve. This is incorrect because fishing causes changes in the age composition of fish of size x, i.e. under fishing, proportion of younger fish (with higher y|x) is expected to increase.

An important advancement of Length-SRA over conventional stock-assessment models is a calculation of time-varying vulnerability. This information alone will go a long way in helping to characterize the complexity of the fishery system. Length-SRA on its own is reasonably accurate in deriving important management-oriented parameters (depletion and U_{MSY}), however another option may be to combine findings from this model with another assessment model, such as a statistical catch at age (SCA) model. In this framework, Length-SRA can be used to calculate annual vulnerability, which are reasonably accurate of there is enough contrast in the biomass time series. These vulnerability estimates can then become an input into an SCA to calculate other

important variables and produce management advice. This combination of models has been used in the past (Walters and Punt 1994); we suggest Length-SRA may be a useful tool in this context.

Accurate estimates of vulnerability are particularly important if the fishery management is based on yield per recruit reference points. Fishery yield per recruit depends on the vulnerability curve (Beverton and Holt 1957) and for this reason, changes in vulnerability over time will directly affect reference points (Beverton and Holt 1957; Hilborn and Walters 1992). We observed vulnerability changes for both Pacific hake and jack mackerel and show how this variability can lead to a two-fold difference between the maximum and minimum estimates of *MSY* calculated along the time series. We believe that tracking these changes is important not only to ensure appropriate management recommendations, but also to illustrate the relationship between vulnerability patterns and *MSY* (Vasilakopoulos et al. 2016).

One potential point of concern that should be considered when using the Length-SRA is that it assumes that the biological parameters used in the growth curve and catch at age relationship are know without error and constant over time. We have tested the Length-SRA under misspecification of the von Bertalanffy growth parameters (see Appendix) and we observed additional bias in the estimates of parameter and management quantities as well as stong distortions in the resulting selectivity parameters. Similarly, Minte-Vera et al. (2017) showed that misspecification in biological parameters, especially in asymptotic length, can have a significant impact in assessment results. Other length models, e.g. MULTIFAN-CL (Fournier et al. 1998), overcome the assumption of known growth parameters by estimating the von Bertalanffy parameters alongside the assessment parameters. The estimation of the growth parameters is made possible by assuming that vulnerability follows a parametric function (usually logistic). Once a simple vulnerability curve is assumed, all deviations in observed catch at length are explained by adjusting the growth parameters. This assumption can also lead to bias in parameter estimates, as other studies show that variability in selectivity and non-asymptotic patterns are common (Waterhouse et al. 2014). In reality, in most cases it is difficult to know if patterns observed in catch at length are caused by fisheries targeting (i.e. vulnerability) or if they would be more appropriately explained by adjusting the growth parameters. Therefore, we recommend that, when using the Length-SRA, the user should perform extensive sensitivity analyses over the possible range of values for the growth parameters, particularly if the predicted vulnerability patterns are highly variable.

The approach used in the Length-SRA is analogous to that used in virtual population analysis in that the length composition data is assumed to be known without error. For this reason, the vulnerability estimates include extra variability due to observation and sampling error. We attempted to minimize this effect by adding a the time varying vulnerability penalty to the objective function, however this penalty is not capable of com-

pletely removing the observation error effect from the vulnerability estimates. Because of the assumption of known catch at length, it is important that the catch sampling is representative of the total removals from the population (Pope 1972). As in any other fisheries model, biased sampling and/or low sampling effort will result in bias in parameter and fishery reference point estimates (Coggins and Quinn 1998; Bunch et al. 2013).

Some management parameters are underestimated (namely depletion and *MSY* in most scenarios) and the relative error indicated that the estimates are not very precise which may be cause for concern. However, it is important to note that both parameters are underestimated for the majority of scenarios and simulation runs, which leads to conservative management recommendations. Although this may lead to losses in catch in the short-term, it will result in healthier stocks and higher catches over the longer-term (once the stock is recovered). The magnitude of bias is also dependent on the trajectory of the data: time-series with more contrast will lead to more accurate estimates. The magnitude and direction of the bias in the estimates of *MSY* and depletion observed in this study are comparable to the results obtained by Martell and Stewart (2014) in a simulation study on the impacts of time varying vulnerability on the estimates generated by a statistical catch at age model. Other studies show even higher biases in face of time-varying vulnerabilities (e.g. Linton and Bence 2011; Henríquez et al. 2016). Overall, parameter and derived parameters estimates are generally within the range of many other stock assessment models.

The simple nature of the Length-SRA model makes it a good candidate model for inclusion on closed-loop simulation studies. Further testing of this model in a closed-loop simulation set up would provide more insight on the model performance on achieving management outcomes (Punt et al. 2014). As mentioned before, we expect that the length-SRA, when used as an assessment tool, will yield slightly conservative management benchmarks which might lead to higher average catch in the long term if the stock is currently overfished. However, it is also possible that these conservative benchmarks result in forgone yield in the medium and long term. We recommend, therefore that this approach be tested in a simulation set up before it is considered as part of a management strategy.

In conclusion, we believe that the Length-SRA approach presented in this study is a useful approach to fisheries stock assessment. We believe that this is particularly true when time varying vulnerability is believed to occur, especially if the variability is not easily predictable from historical changes in gear use/fleet composition. We foresee the application of this model as an investigative tool to evaluate potential time-varying vulnerability patterns, as a stock assessment tool and as part of closed loop simulation studies.

5   Acknowledgments

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A Appendix

One important feature of the Length-SRA is that it assumes that growth follows a von Bertalanffy curve and that the growth parameters are known and constant over time. If this assumption is violated, the model outcomes will be impacted as the model will try to explain the deviations from the true growth curve with changes in the vulnerability pattern. Here we illustrate how the model outcomes are impacted by the misspecification of the growth parameters by purposefully misreporting the values of L_∞ (Table 7). We assumed a simple logistic selectivity curve for this exercise and therefore expect the model to produce logistic patterns in the exploitation rate at length $U_{l,t}$.

We found that misspecification of L_∞ has severe implications in the capability of the model to estimate exploitation rate at length $U_{l,t}$ (Figure 6). If the value of L_∞ was reported to be lower than true, the estimates

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of $U_{l,t}$ were lower than true for most length and extremely high for high lengths (approaching the true L_∞). In the scenario where L_∞ was reported to be higher than true, $U_{l,t}$ was estimated to follow a dome shaped pattern, with very low exploitation rates for the higher lengths. This patterns occur because the model is trying to adjust the mismatch between proportions of catch at length and the $P_{l|a}$ matrix by changing the predicted vulnerability pattern. As a result, failure to adequately specify L_∞ leads to erroneous estimation of vulnerability patterns and, consequently, to failure in estimating management quantities.
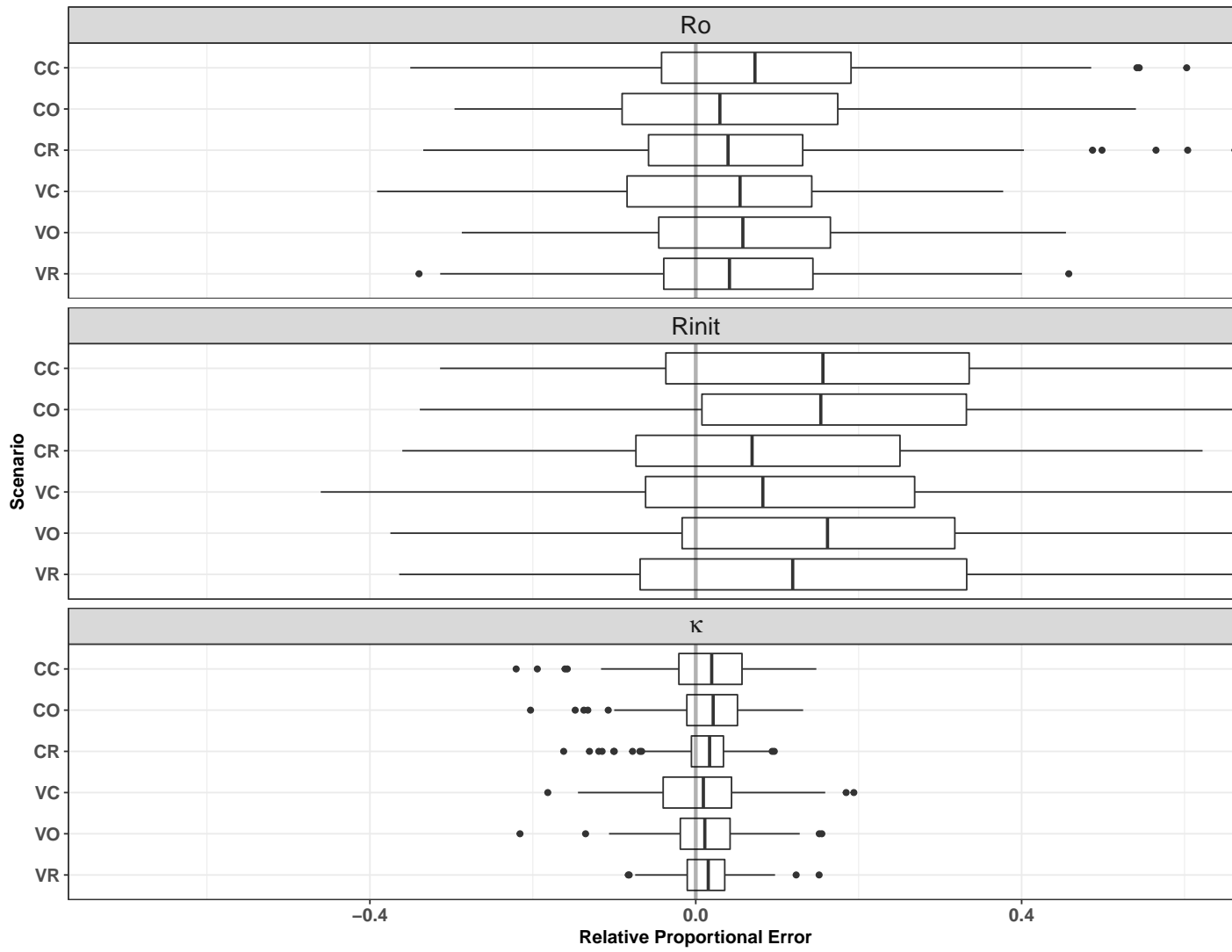
Figure 1: Relative proportional error for main parameters for all scenarios considered in the simulation-evaluation. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value +/- 1.5 * inter-quartile range (distance between the first and third quartiles).
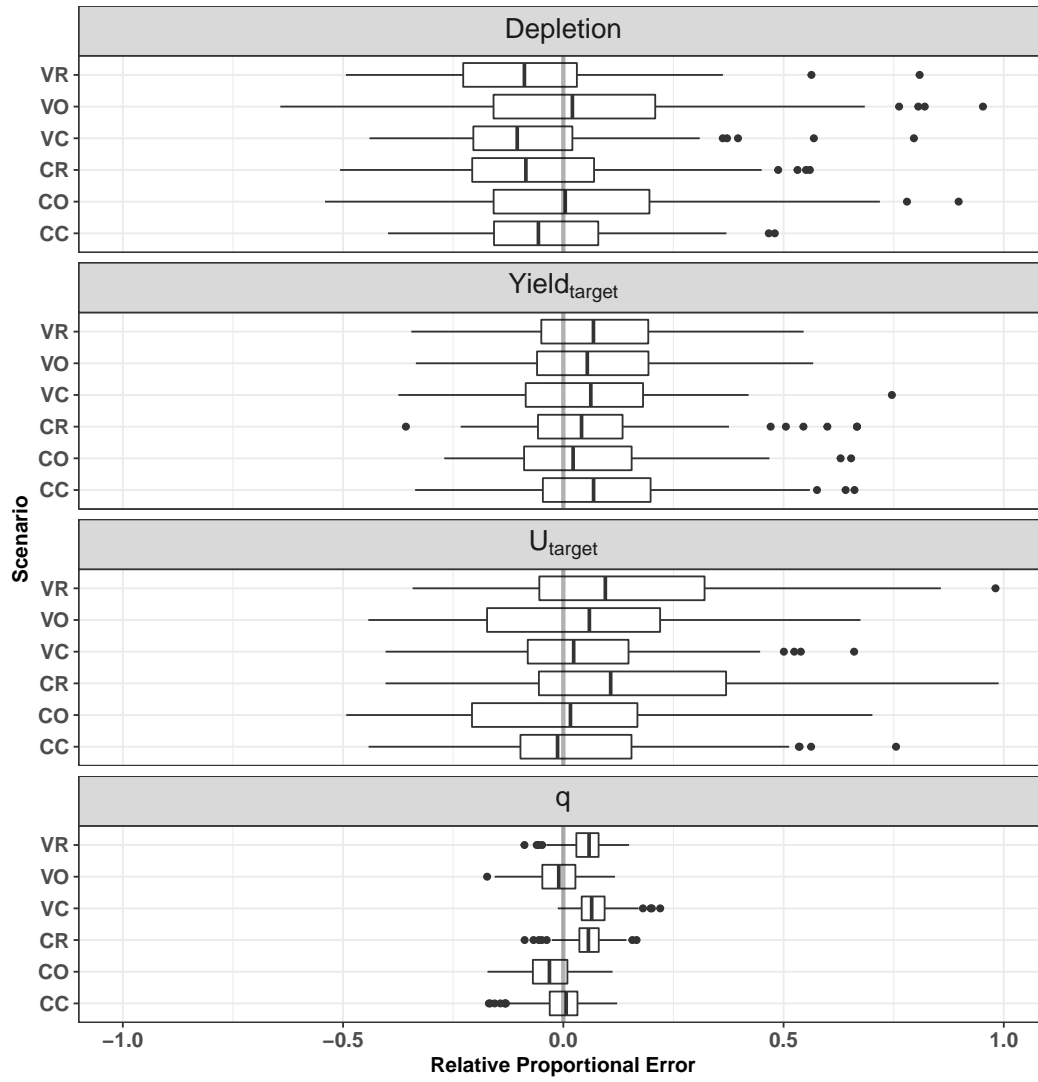
Figure 2: Relative proportional error for main parameters for all scenarios considered in the simulation-evaluation. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value +/- 1.5 * inter-quartile range (distance between the first and third quartiles).

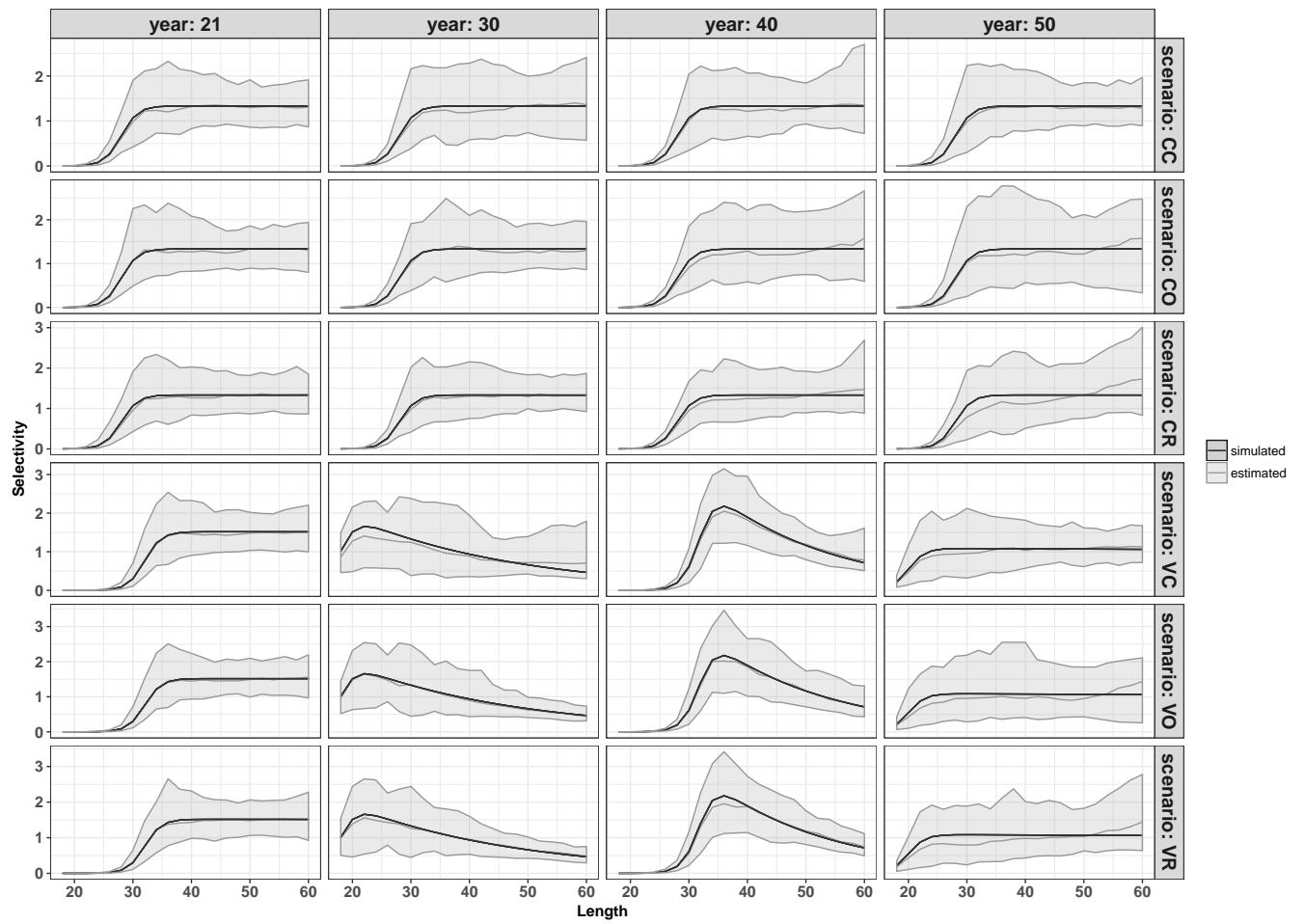Figure 3: Simulated and realized selectivity estimates for a set of years within simulation-evaluation time series.
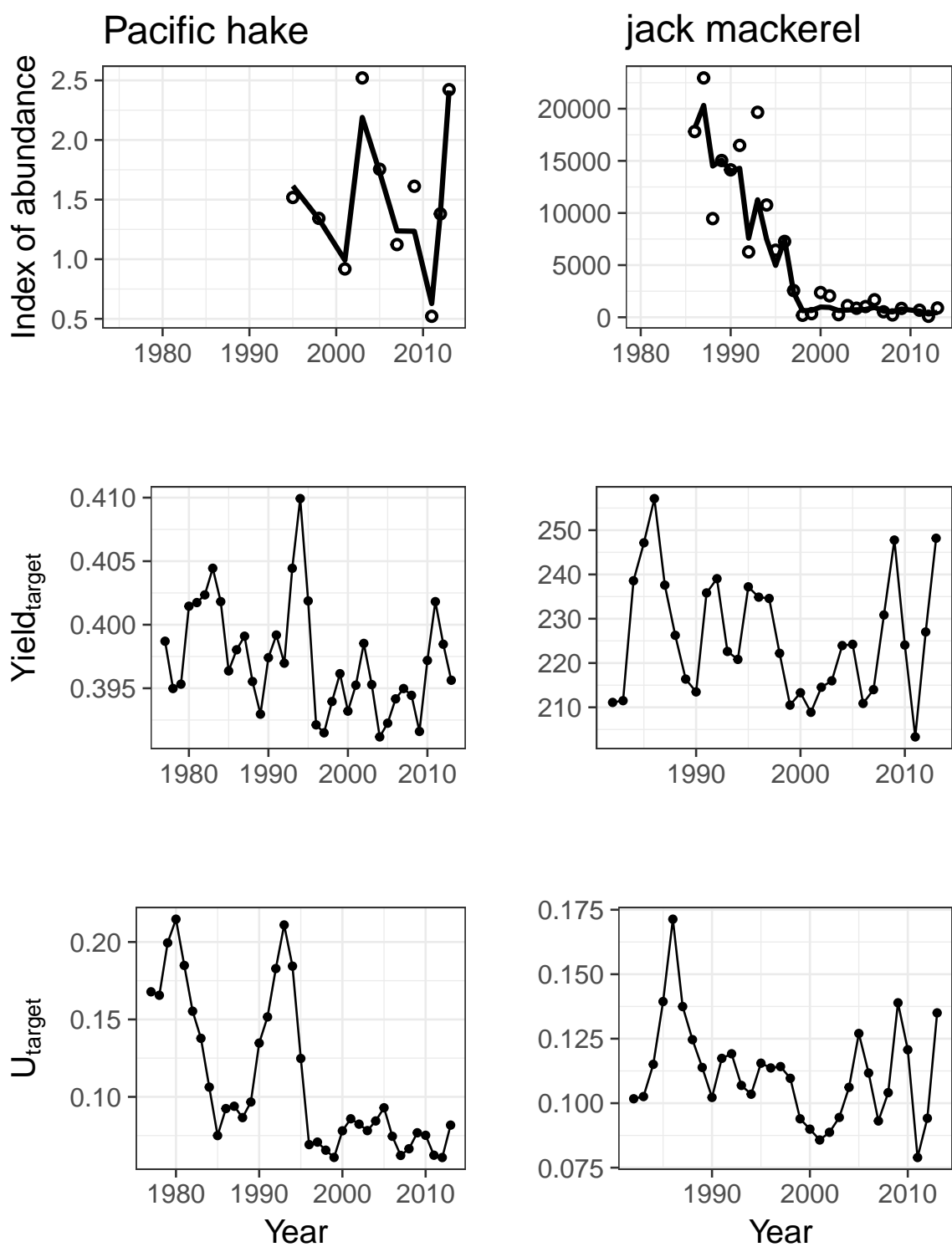
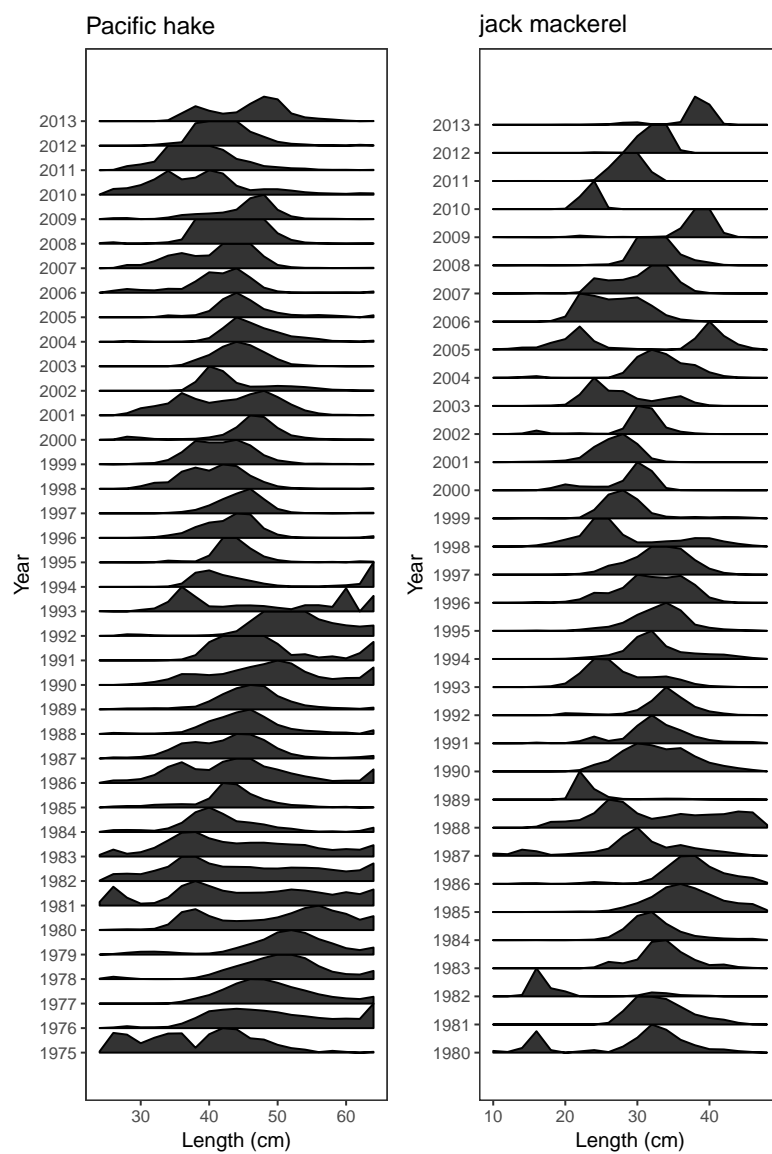Figure 4: Fit to index of abundance for Pacific hake and jack mackerel.

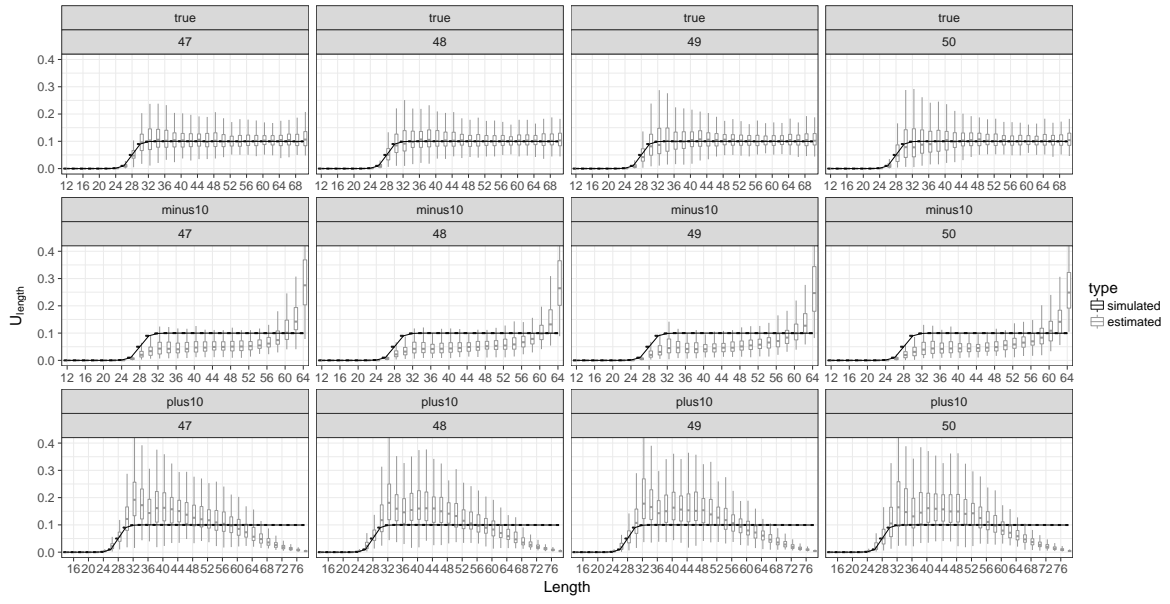Figure 5: Realized selectivity patterns across years for Pacific hake and jack mackerel.

Figure 6: Simulated and realized exploitation rate at length $U_{l,t}$ for a set of years within simulation-evaluation time series.

Table 1: Indexes, stock assessment variable definition, and values used in simulation-evaluation

Symbol	Value	Description
l	$\{l_o,...,L\}$	Central point of length bin, $L = 50$ cma
a	$\{a_o,...,A\}$	Age-class, $A = 10 years$
t	$\{1,...,T\}$	Annual time step, $T = 50 years$
a_o	1	First age or age of recruitment
$l_b in$	2 cm	size of length bin
l_o	8 cm	Central point of first length bin
$init$	21	Annual time step in which data starts to be reported
Distribution of length given age		
L_∞	50 cm	Maximum average length
K	0.15	Rate of approach to L_∞
t_o	-0.1	Theoretical time in which length of individuals is zero
cv_l	0.08	Coefficient of variation for length at age curve
$P_{l|a}$		Matrix of proportions of length at age
Φ		Standard normal distribution
$z1_{a,l}$		Normalized z score for lower limit length bins
$z2_{a,l}$		Normalized z score for upper limit length bins
$b1_l$		Lower limit of length bins
$b2_l$		Upper limit of length bins
$\bar{L}_a$		Mean length at age
σ_L		Standard deviation of length at age
Population dynamics		
R_o	100	Average unfished recruitment
κ	10	Goodyear recruitment compensation ratio
S	0.7	Natural annual survival
σ_R	0.6	standard deviation for recruitment deviations
w_t	$\mathcal{N}(0,\sigma_R)$	Recruitment deviations for years {init-A+1,...,T}
$N_{a,t}$		Numbers of fish at age and time
SB_t		Spawning biomass at time
mat_a		Proportion of mature individuals at age
a_{rec}, b_{rec}		Beverton & Holt stock recruitment parameters
R_{init}		Recruitment in the year data starts to be reported
$U_{a,init}$		Exploitation rate at age before data starts being reported.
$U_{a,t}$		Exploitation rate at age and time
$U_{l,t}$		Exploitation rate at length and time
$C_{l,t}$		Catch at length and time
$N_{l,t}$		Numbers at length and time
lx_a		Unfished survivorship at age
Φ_e		Unfished average spawning biomass per recruit
$vul_{l,t}^{result}$		Resulting vulnerability estimates at length and time

Table 2: Indexes, variable definition for operating model, MSY quantities, and values used in simulation-evaluation

Symbol	Value	Description
Operating model		
$vul^{OM}_{l,t}$		Fishing vulnerability at length and time
g,d,b	vary by scenario	Parameters for selectivity function
U_t	vary by scenario	Annual maximum exploitation rate
$C^{Sim}_{l,t}$		Simulated catch at length and time
$N^{Sim}_{l,t}$		Simulated numbers at length and time
I_t		Index of abundance at time
σ_{I_t}	0.1	standard deviation for index of abundance deviates
VB_t		Biomass that is vulnerable to the survey at time
q	1.0	Catchability coefficient
Managemen quantities		
lz_a		Fished survivorship at age
F_z	seq(0.0,1.0,by=0.001)	Hypothetical average fishing mortality to calculate management targets
Φ_z		Average spawning biomass per recruit
Φ_{eq}		Average exploited biomass per recruit under U_z
$vul^{result}_{a,t}$		Vulnerability at age and time t
R_{eq}		Average equilibrium recruitment under U_z
$Yield_z$		Equilibrium yield under U_z
$Yield_{target}$		Yield that would reduce spawner per recruit to 40% of unfished levels
U_{target}		Exploitation rate that reduce spawner per recruit to 40% of unfished levels

Table 3: population dynamics for Length-SRA and operating model

Distribution of length given age	

$$P_{l|a} = \int_{z1_{a,l}}^{z2_{a,l}} \Phi(z)dz \tag{T3.1}$$

$$z1_{a,l} = \frac{b1_l - \bar{L}_a}{\sigma_{L_a}} \tag{T3.2}$$

$$z2_{a,l} = \begin{cases} \frac{b2_l - \bar{L}_a}{\sigma_{L_a}} & l < L \\ 1.0 & l = L \end{cases} \tag{T3.3}$$

$$\bar{L}_a = L_\infty \cdot \left(1 - e^{(-K \cdot (a - t_o))}\right) \tag{T3.4}$$

$$\sigma_{L_a} = \bar{L}_a \cdot cv_l \tag{T3.5}$$

Population dynamics

$$N_{a,t>init} = \begin{cases} \frac{a_{rec} \cdot SB_{t-1}}{1 + b_{rec} \cdot SB_{t-1}} \cdot e^{w_t}, & a = a_o \\ N_{a-1,t-1} \cdot S \cdot (1 - U_{a-1,t-1}), & a_o < a < A \\ \frac{N_{a-1,t-1} \cdot S \cdot (1 - U_{a-1,t-1})}{1 - S \cdot (1 - U_{a,t})}, & a = A \end{cases} \tag{T3.6}$$

$$U_{a,t} = \sum_l \left(P_{l|a} \cdot U_{l,t}\right) \tag{T3.7}$$

$$U_{l,t} = \frac{C_{l,t}}{N_{l,t}} \tag{T3.8}$$

$$N_{l,t} = \sum_a \left(P_{l|a} \cdot N_{a,t}\right) \tag{T3.9}$$

$$SB_t = \sum_a (mat_a \cdot w_a \cdot N_{a,t}) \tag{T3.10}$$

$$vul_{l,t}^{result} = \frac{U_{l,t}}{\bar{U}_t} \tag{T3.11}$$

Initial year and incidence functions

$$N_{a,init} = lx_a \cdot R_{init} \cdot e^{w_{init}} \tag{T3.12}$$

$$a_{rec} = \frac{\kappa}{\Phi_e} \tag{T3.13}$$

$$b_{rec} = \frac{\kappa - 1}{R_o \cdot \Phi_e} \tag{T3.14}$$

$$\Phi_e = \sum_a lx_a \cdot mat_a \cdot w_a \tag{T3.15}$$

$$lx_a = \begin{cases} 1, & a = 1 \\ lx_{a-1} \cdot S, & 1 < a < A \\ \frac{lx_{a-1} \cdot S}{1 - S}, & a = A \end{cases} \tag{T3.16}$$

Table 4: Management quantities and operating model

Operating model	
$$N_{a,t=1}^{Sim} = lx_a \cdot R_o$$	(T4.1)
$$U_{l,t} = U_t \cdot vul_{l,t}^{OM}$$	(T4.2)
$$C_{l,t}^{Sim} = N_{l,t}^{Sim} \cdot U_{l,t} \cdot P_{l|a}$$	(T4.3)
$$sel_{l,t} = \frac{1}{1-g} \cdot \left(\frac{1-g}{g}\right)^g \cdot \frac{e^{d \cdot g \cdot (b-l)}}{1 + e^{d \cdot (b-l)}}$$	(T4.4)
$$I_t = q \cdot VB_t \cdot e^{(\mathcal{N}(0,\sigma_{I_t}))}$$	(T4.5)

Management quantities	
$$lz_a = \begin{cases} lz_a = 1 & a = a_o \\ lz_{a-1} \cdot S \cdot exp(-F_z \cdot vul_{a-1,t}^{result}) & a_o < a < A \\ \frac{lz_{a-1} \cdot S \cdot exp(-F_z \cdot vul_{a-1,t}^{result})}{1 - S \cdot exp(-F_z \cdot vul_{A,t}^{result})} & a = A \end{cases}$$	(T4.6)
$$vul_{a,t}^{result} = \frac{\frac{U_{a,t-1}}{U_{t-1}} + \frac{U_{a,t}}{U_t}}{2}$$	(T4.7)
$$\Phi_z = \sum_a lz_a \cdot mat_a \cdot w_a$$	(T4.8)
$$Target_{\Phi} = \left|\frac{\Phi_z}{\Phi_e} - 0.4\right|$$	(T4.9)
$$\Phi_{eq} = \sum_a lz_a \cdot (1 - exp(-F_z * vul_{a,t}^{result})) \cdot w_a$$	(T4.10)
$$R_{eq} = R_o \cdot \frac{\kappa - \Phi_e/\Phi_z}{\kappa - 1}$$	(T4.11)
$$Yield_z = R_{eq} \cdot \Phi_{eq}$$	(T4.12)
$$Yield_{target} = Yield_z \rightarrow min(Target_{\Phi})$$	(T4.13)
$$U_{target} = 1 - exp(-F_z) \rightarrow min(Target_{\Phi})$$	(T4.14)

Table 5: Likelihood functions and penalties

Conditional Likelihood	
$Z_t = log(I_t) - log(VB_t)$	(T5.1)
$q = e^{\bar{Z}}$	(T5.2)
$Zstat_t = Z_t - \bar{Z}$	(T5.3)
$LL_1 \sim \mathscr{N}(Zstat \| \mu = 0, \sigma = \sigma_{I_t})$	(T5.4)
Penalties	
$P_{wt} \sim \mathscr{N}(wt \| \mu = 0, \sigma = \sigma_R)$	(T5.5)
Priors	
$prior(log(\kappa)) \sim \mathscr{N}(log(\kappa), \sigma = 0.6)$	(T5.6)
Objective function	
$Obj = -log(LL_1) + (-log(P_{wt})) + prior(log(\kappa))$	(T5.7)

Table 6: Simulation-estimation scenarios

Scenario Code	Selectivity	U trajectory
CC	constant	contrast
CO	constant	one-way trip
CR	constant	U-ramp
VC	time-varying	contrast
VO	time-varying	one-way trip
VR	time-varying	U-ramp

Table 7: Scenarios for testing misspecification of L_∞

Scenario name	version	L_∞ value
true	true	68
minus10	10% underestimated	61.2
plus10	10% overestimated	74.8