1 Differential Equations: Questions   21 Jun 2023

Euler Method $y_{n+1} = y_n + hf(x_n, y_n)$

Write the Euler algorithm for this pair of equation. What do these equations signify?

$$\theta' = \omega \quad \omega' = -\frac{g}{L}\theta$$

Hint: $\theta_k = \theta_{k-1} + \omega_{k-1}dt$

What differential equation is this code solving?

```python
    for i in range(N):
        y = y + h*f_prime(x, y)   #y_n+1 = y_n + h * f
        x = x + h
        x_out.append(x) y_out.append(y)
    return x_out, y_out
def f_prime(x, y):
  return -0.5*np.exp(x/2)*np.sin(5*x)+5*np.exp(x/2)*np.cos(5*x)+y
```

Runge Kutta 4 $y_{n+1} = y_n + \frac{h}{6}(k_1 + k_2 + 2k_3 + k_4)$

Complete the code for this equation

```python
 for i in range(N):
        k1 = h*f_prime(x,y)
        k2 = h*f_prime(x+0.5*h,y+0.5*k1)
```

Adam-Bash Moulton $y_{n+1} = y_n + \frac{3}{2}hf(t_{n+1}, y_{n+1}) - \frac{1}{2}hf(t_n, y_n)$ $\quad y_{n+1} = y_n + \frac{1}{2}(hf(t_{n+1}, y_{n+1}) + \frac{1}{2}hf(t_n, y_n)$

Complete this FORTRAN code

```fortran
    do i=2,N
       y(i+1) = y(i) + (h/2)*(3*f_prime(x(i),y(i))-f_prime(x(i-1),y(i-1)))
       x(i+1) = x(i) + h
```

Explicit and Implicit

Write the explicit and implicit scheme to solve this equation.

$$y' = -50(y - cos(x))$$

Hint: $y(i + 1) = y(i) + 50[cos(t(i + 1)) - y(i + 1)]dt$

Stellar Structure

Construct an algorithm to solve the equations governing the structure of a main-sequence star

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad \frac{dm}{dr} = 4\pi r^2\rho \quad \frac{dL}{dr} = 4\pi r^2\epsilon \quad \frac{dT}{dr} = -\frac{3\kappa\rho L}{64\pi r^2\sigma T^3}$$

Partial Differential Equation: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$

Complete the code for 5-point stencil to solve the 2D Laplace equation

```python
        for i in range(1, lenX-1, delta):
            for j in range(1, lenY-1, delta):
                T[i, j] = 0.25 * (T[i+1][j] + T[i-1][j] + T[i][j+1] + T[i][j-1])
```