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Part I:  (1 point)
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Formally prove that 2^n + 1  is in  O(4^n - 16).

HINT:  You may assert without proof that, for all n >=1, 2^n >= 1.
(You may also assert without proof that 4^n and 2^n are monotonically
increasing, if you find it useful.)
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Solution:
Let T(n) = 2^n + 1, and f(n) = 4^n - 16
Choose c = 1, and N= 3.
Then we know T(n) is in O(f(n)) if we can prove or equivalently, T(n) <= c f(n), for all n >= N
we also know that 2^n - 4 >= 4 > 1, and 2^n + 1 < 2^n + 4, for all n >=3
so T(n) = 2^n + 1 < = (2^n + 4) (2^n - 4) = c f(n) for all n>=3

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Part II:  (1 point)
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Formally prove that if f(n) is in O(g(n)), and g(n) is in O(h(n)), then
f(n) is in O(h(n)).

NOTE:  The values of c and N used to prove that f(n) is in O(g(n)) are
not necessarily the same as the values used to prove that g(n) is in O(h(n)).
Hence, assume that there are positive c', N', c'', and N'' such that

    f(n) <= c' g(n)         for all n >= N', and
    g(n) <= c'' h(n)        for all n >= N''.
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Solution:
f(n) is in O(g(n)) means f(n) <= c1 * g(n)
g(n) is in O(h(n)) means g(n) <= c2 * h(n)
Let c1*c2 = c3, therefore,
f(n) <= c1 * g(n) = c1 * c2 * h(n) = c3 * h(n)
f(n) <= c3*h(n) means f(n)=O(h(n))

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Part III:  (2 points)
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Formally prove that 0.01 n^2 - 1  is NOT in  O(n).

We need to show that, no matter how large we choose c and N, we will never
obtain the desired inequality.  We cannot prove this by picking a specific
value of c and N.  Instead, we must study how the two functions behave as
n approaches infinity.

Let T(n) = 0.01 n^2 - 1, and let f(n) = n.  Prove that

                    c f(n)
          lim       ------ = 0,
      n->infinity    T(n)

no matter how large we choose c to be.  You will need to scale both the
numerator and the denominator by a well-chosen multiplier to get the result.

Use this result to show that there are no values c, N such that T(n) <= c f(n)
for all n >= N.
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Solution:
lim (n->inf) (c n) / (0.01 n^2 -1) = lim (n->inf) (100c) / (n -(100/n)) = lim (n->inf) (100c) / n = 0