EE5471: Least Squares Analysis of Data

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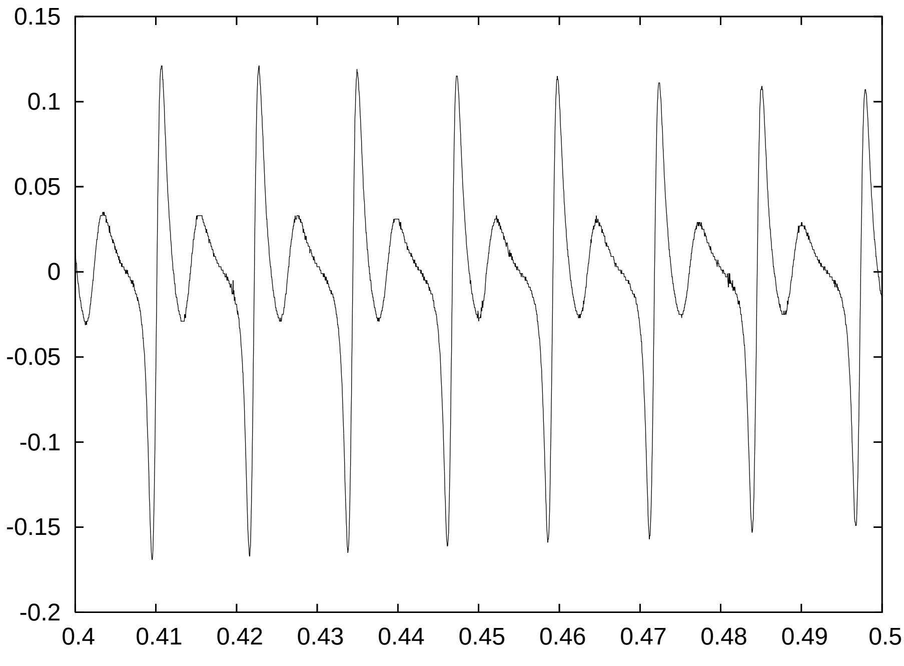
1 Reading Assignment

Sections 15.1 to 15.4 of Numerical Recipes.

2 The given function to estimate

The site contains a file tek00013.dat.gz. This is actual data acquired on a digital scope. A small portion of the data is shown below in the following figure:



The signal is nearly periodic, and has a rich spectrum. Your problem is to obtain the local period and spectrum as a function of position. The technique should be automated, i.e., you feed it the file and it figures out where to start and generates the fit.

- Obtain and plot the Fourier Transform of the signal (both magnitude and phase). Understand why the spectrum is broad band rather than a noisy line spectrum.
- Window the data (N samples, where N is varied from 64 to 2048) and study the spectrum. Are lines present? Do you see a $1/f$ fall off in the signal? That is due to the non-periodicity of the function in the window.
- Apply a windowing function - Hamming, Hanning, Kaiser. Look at the spectrum. Are the lines clearer? Is the $1/f$ noise suppressed? What is the effect of varying N?
- Find the times corresponding to zero-crossings of the function. From it attempt to estimate the frequency of the signal. Note that the number of zero-crossings per period will abruptly jump from 2 to 4 and vice versa. How will you handle this? Plot the frequency plot obtained.
- Instead attempt to locate positive peaks. Why are the peak locations not reliably obtained by finding the location of the maximum value of the signal? Instead find the times corresponding to $1/\sqrt{2}$ of the peak value and average those to get a good estimate of the peak position in time. Obtain a frequency estimate as a function of time. Note that the same problem of having one or two peaks per period will arise. Plot it.
- Filter the signal through a low pass filter (FIR) with a cut off frequency of 120 KHz and a very rapid transition region. The second and higher harmonics should be greatly suppressed. How will you design your filter?
- Apply your zero crossing and peak detection algorithms to the filtered signal. Plot the frequency vs time in each case.
- The power spectrum is the average $|F|^2(\omega)$ of the signal. Obtain the power spectrum by shifting the window by half a window and obtaining M samples and averaging them. What combination of N and M gives best results? Why?
- The autocorrelation of the signal contains average correlation between shifted locations of the signal. Its fourier transform also gives the power spectrum. Choose a window of size N and compute M autocorrelation values on it. $M \ll N$ is required since the signal is shifted and multiplied before being summed. So too large a shift leaves a very small sample that is nonzero. Take the fourier transform - which will be pure real since the autocorrelation is real and symmetric - and find the power spectrum. How does this power spectrum compare to the other one? What are optimal choices of M and N?
- Shift the windows and track the fundamental of the power spectrum to obtain $\omega(t)$. Plot it and compare with the zero-crossings approach.

None of these methods is quite satisfactory. But all of them do a crude job. They are simple techniques to keep in your arsenal of computer methods.