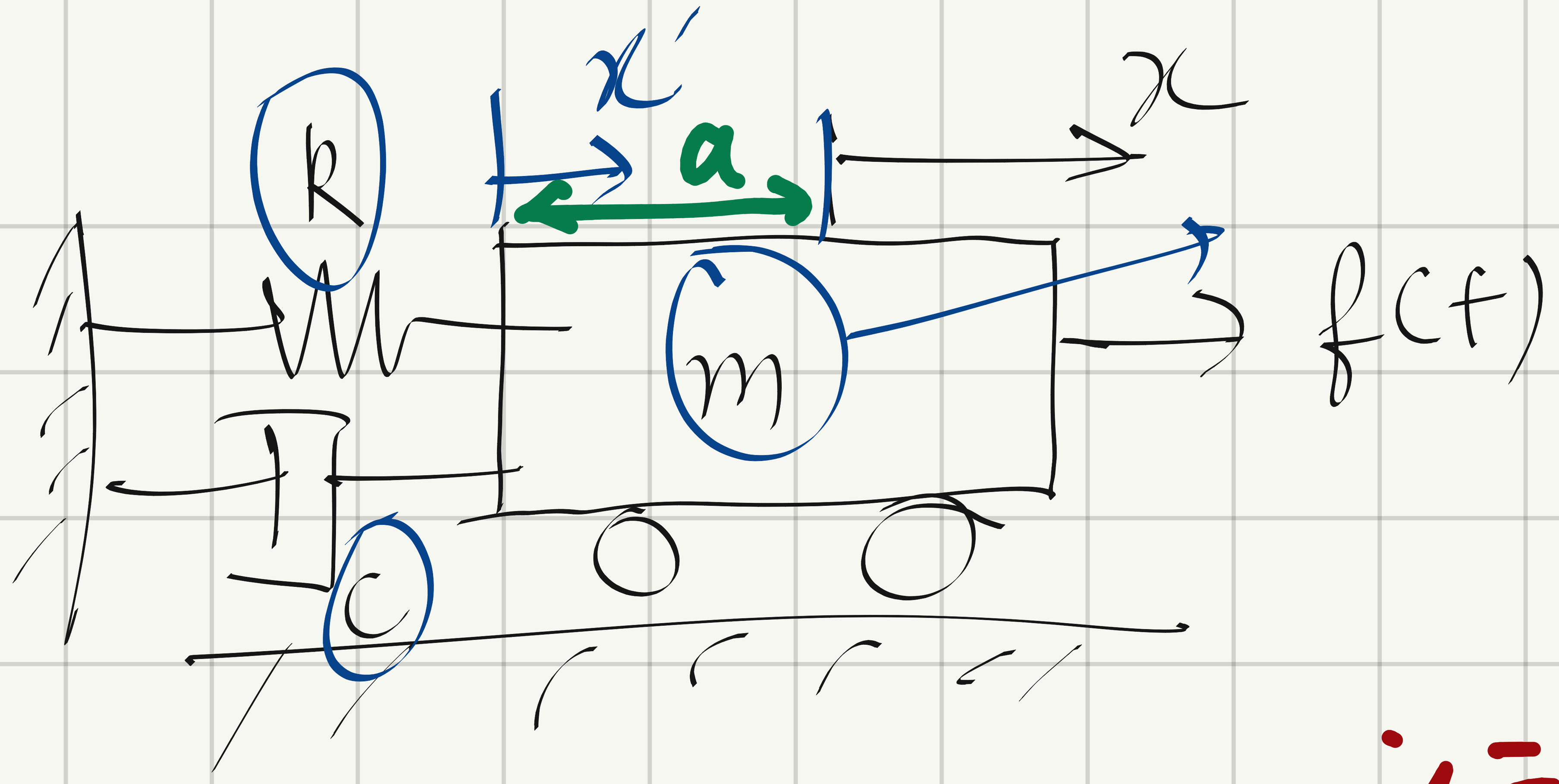
Laplace transforms

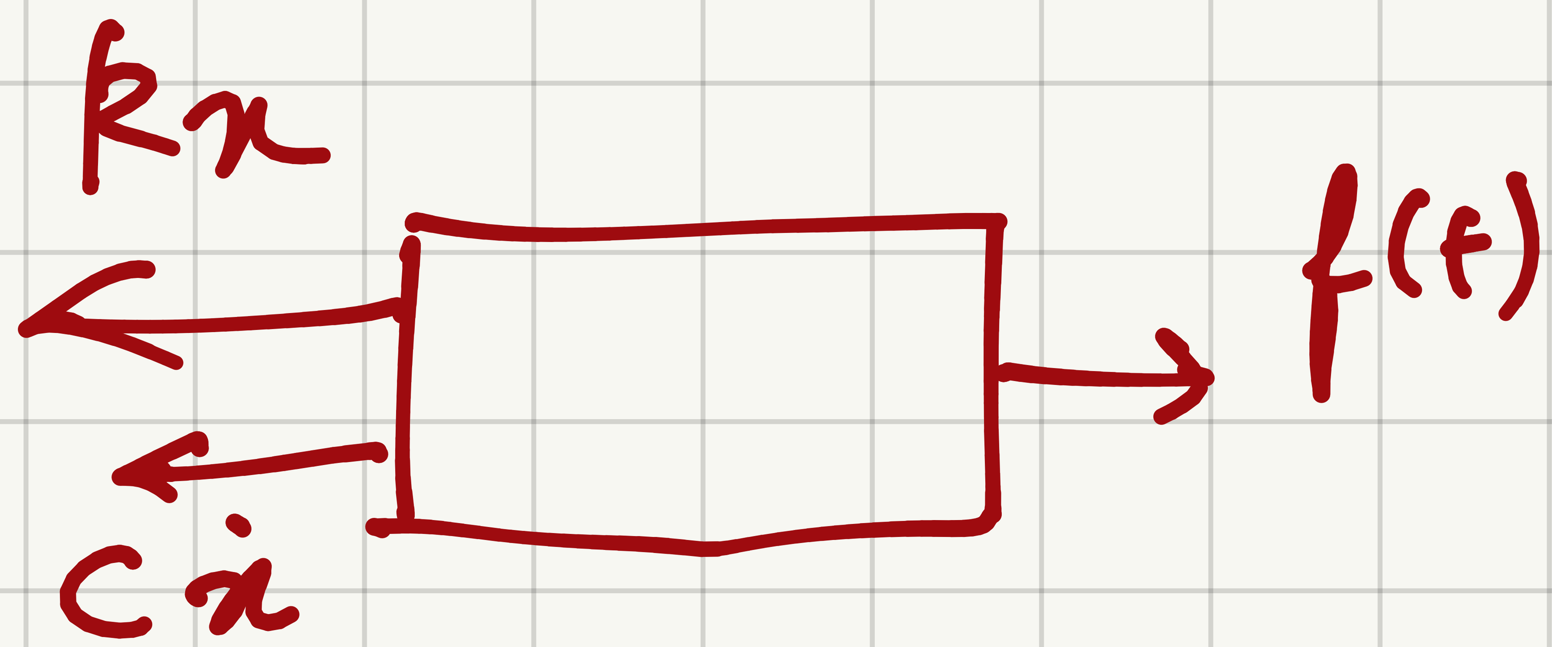
Modeling in frequency domain



$$\dot{x} = \frac{dx}{dt}$$

$$\ddot{x} = \frac{d^2x}{dt^2}$$

$$F = ma$$



$$f(t) - kx - c\dot{x} = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

Solving for $x(t)$

- 2nd order
- Non-Homogeneous
- ODE
- Time invariant
- Linear

Time invariant and Linear $\rightarrow$ LTI Systems.

$$x = (x' - a)$$

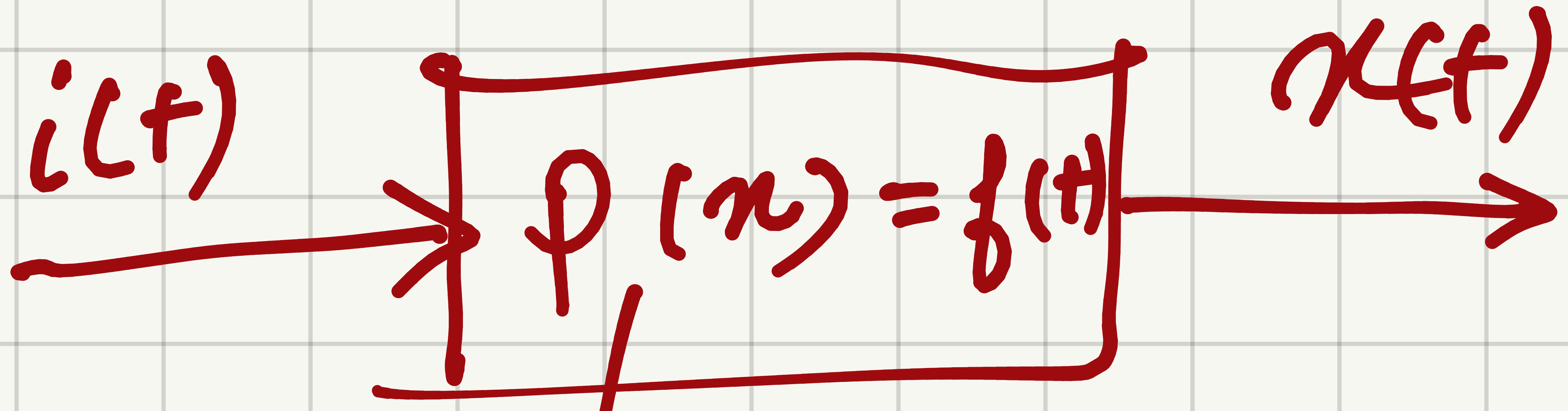
$$\dot{x} = \dot{x}'$$

$$\ddot{x} = \ddot{x}'$$

$$\cancel{\alpha}\ x = \alpha(x' - a)$$

$$f(x+a) = f(x) + f(a)$$

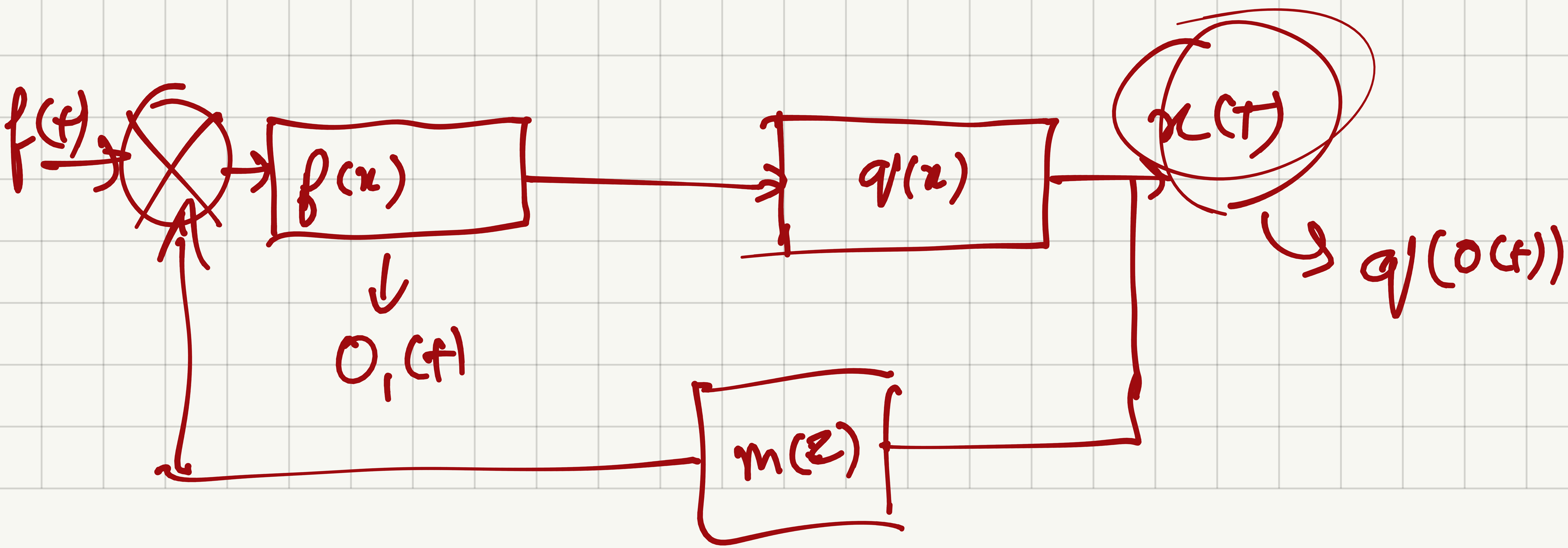
given initial conditions, system is solvable.



$x(t)$ = sol^n of $p(x) = f(t)$

Differential eq^n.

· for a system



Laplace Transform

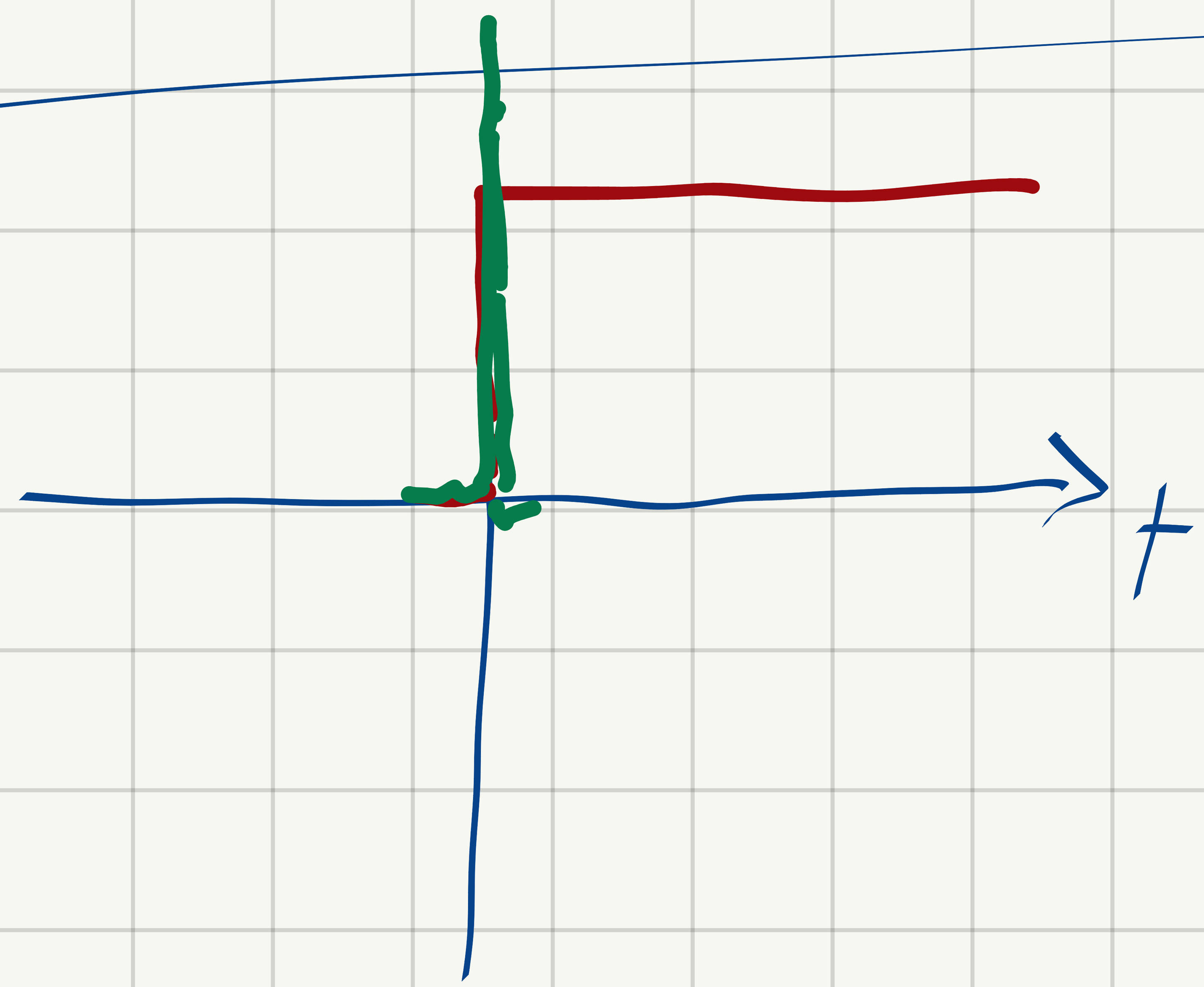
transform a differential eqn into algebraic equation

$$\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)\, e^{-st}\, dt$$

time domain

s- domain

$s = \sigma + j\omega$

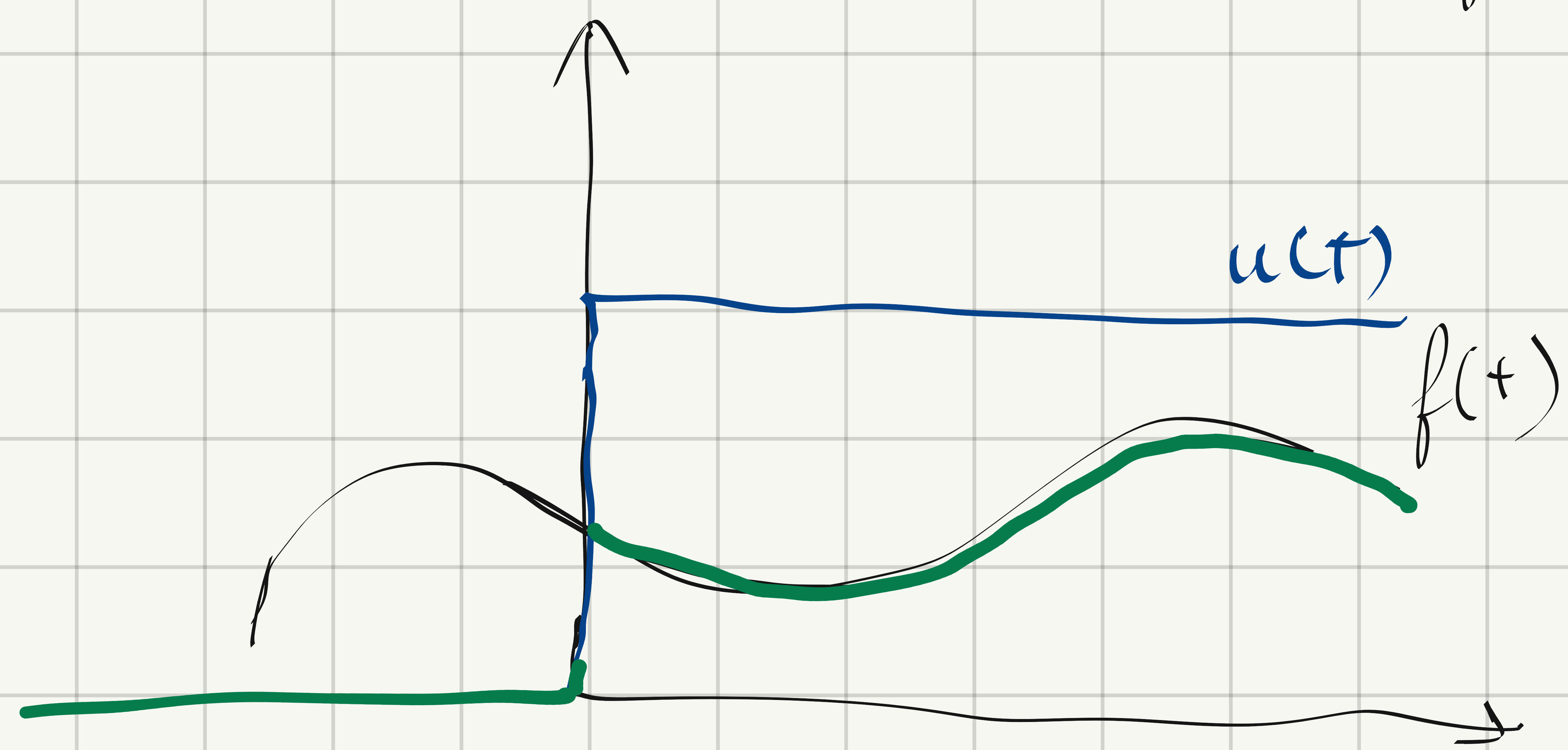


If there is a discontinuity at $t=0$

$\mathcal{L}$ exists as long as $F(s)$ converges.

Inverse Laplace

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma - j\omega}^{\sigma + j\omega} F(s)\, e^{st}\, ds = f(t)\underbrace{u(t)}_{\text{unit step function}}$$



Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

$$\int_0^\infty t e^{-st} dt = \frac{1}{s^2}$$

$$\boxed{e^{-at} f(t)}$$

$$\rightarrow \mathcal{L}[f(t)] = F(s)$$

$$\mathcal{L}[e^{-at} f(t)] = F(s+a)$$

$$\mathcal{L}[e^{at} t^n u(t)] = \frac{n!}{(s-a)^{n+1}}$$

$$\mathcal{L}[f(t) e^{-at}] = \int f(t) e^{-(s+a)t} dt$$

Properties of Laplace Transform

$$\mathcal{L}[f(t)] = F(s)$$

$$\mathcal{L}[k f(t)] = \int k f(t) e^{-st} dt = \boxed{k F(s)}$$

$$\mathcal{L}[f_1(t) + f_2(t)] = \int (f_1 + f_2) e^{-st} dt = \int f_1 e^{-st} dt + \int f_2 e^{-st} dt$$

$$= \boxed{F_1(s) + F_2(s)}$$

Linearity theorem

$$\mathcal{L}[e^{-at} f(t)] = F(s+a)$$

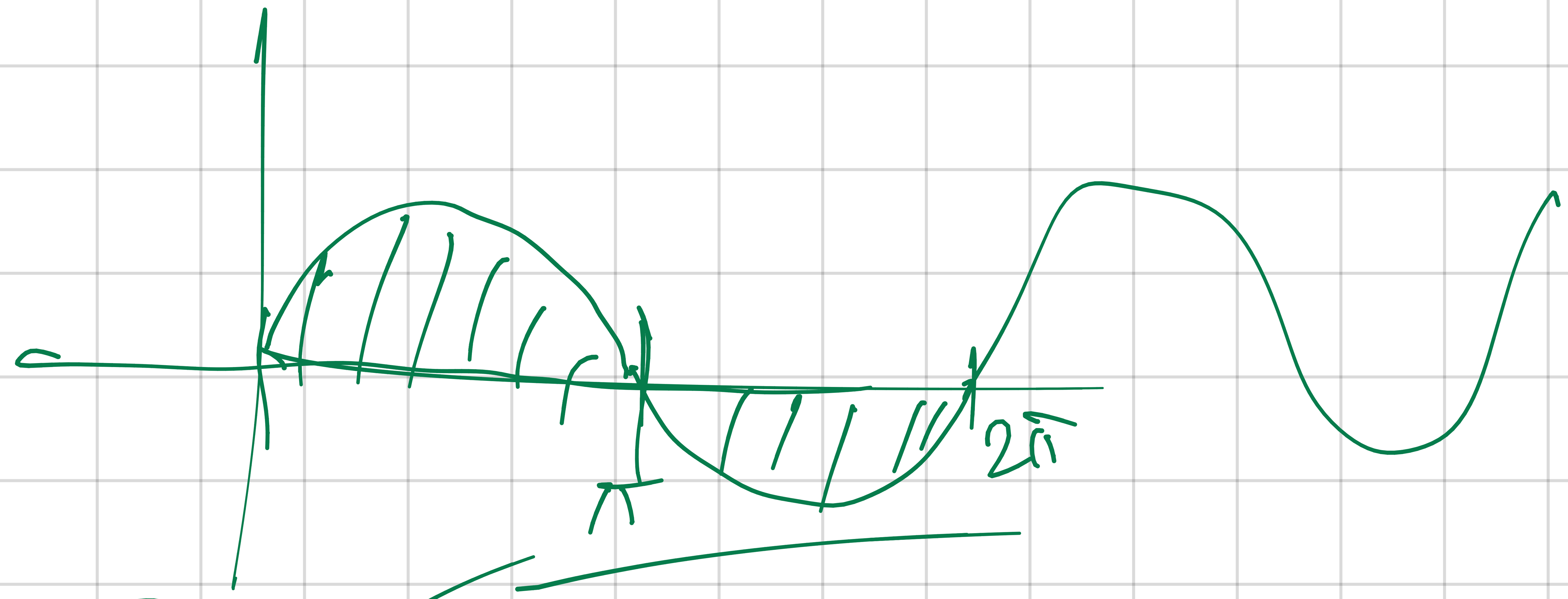
→ Frequency shift theorem.

$$\mathcal{L}[f(t-T)] = \int_0^\infty f(t-T) e^{-st} dt = \int_{0+T}^\infty f(\tau) e^{-s(\tau+T)} d\tau = \boxed{e^{-sT} F(s)}$$

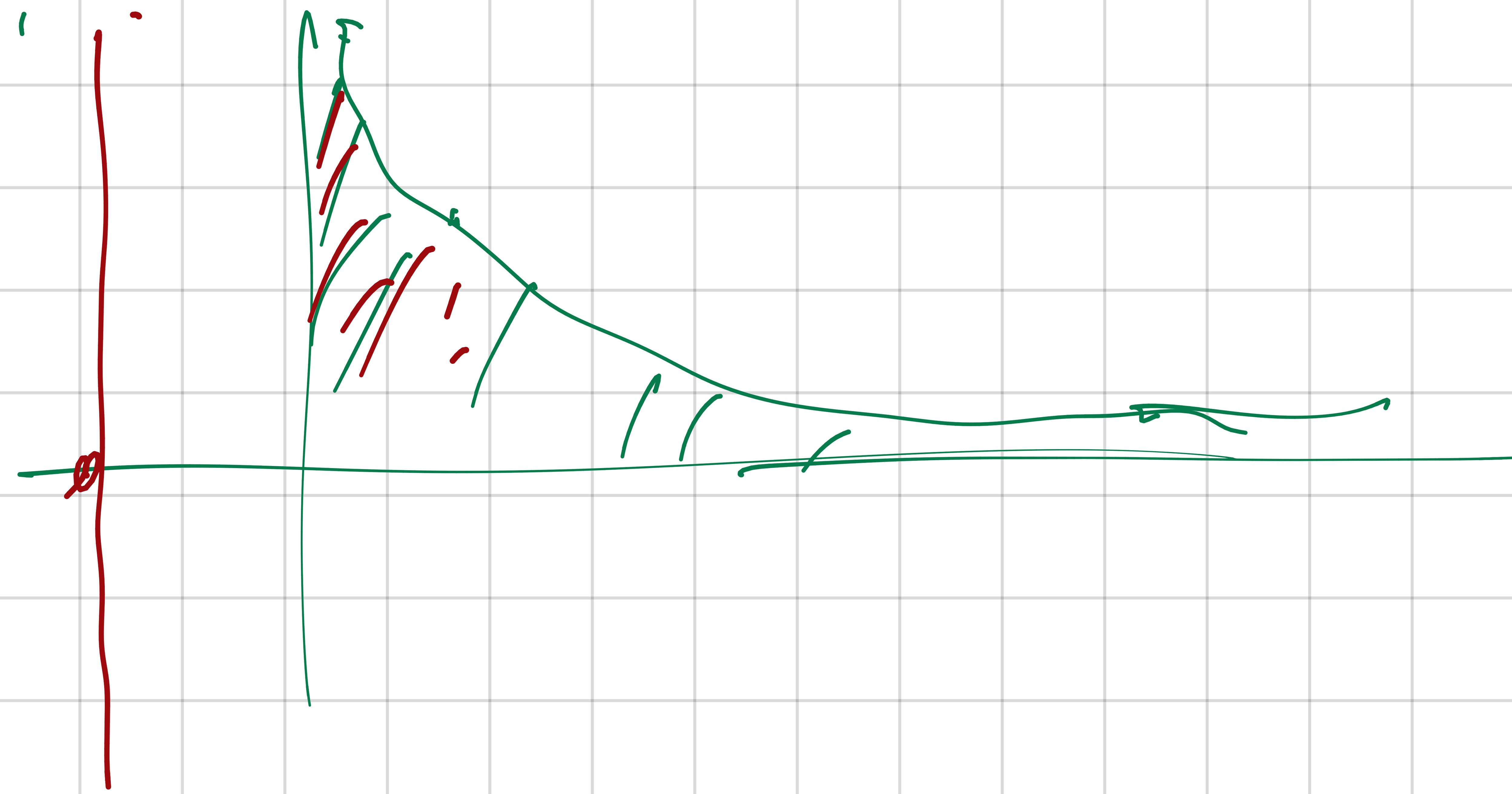
Replace $t - T = \tau$

Time shift theorem

$$\int_0^{\pi} \sin x \, dx = 1$$



$$\int_{0^-}^{\infty} f(t) e^{-st} dt = \int_{-T}^{\infty} f(\tau) e^{-s\tau} d\tau$$



$$\mathcal{L}[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$$ Scaling theorem.

$$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - \underbrace{f(0_-)}$$ → Initial value of function.

$$\mathcal{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - s\underbrace{f(0_-)} - \underbrace{f'(0_-)}$$ → initial value of first derivative.

$$\underbrace{\mathcal{L}\left[\frac{d^nf}{dt^n}\right]} = s^nF(s) - \sum_{k=1}^{n} s^{n-k} f^{k-1}(0_-)$$ → Sum of initial values upto $(n-1)$ derivative.

→ Differentiation theorem.

$$\mathcal{L}\left[\int_{0^-}^{t} f(\tau)\,d\tau\right] = \frac{F(s)}{s}$$ Integration theorem.

$f(\infty)$

Final value

$$= \lim_{s\to 0} sF(s)$$ Final value theorem

$f(0)$

Initial Value

$$= \lim_{s\to\infty} sF(s)$$

* For this to yield correct result roots of denominator of $F(s)$ must be $(-)$ve with no more than 1 root at origin.

$$F(s) = \frac{1}{s(s+1)(s+2)} \qquad f(\infty) = \frac{1}{2}$$

$$\frac{1}{s^2(s+1)(s+2)} \qquad f(\infty) = \infty \quad \text{NON}$$

Solution of a differential equation

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t)$$

given all I.C.s are 0.

$$\mathcal{L}[LHS] = \mathcal{L}[RHS]$$

$$s^2Y(s) - \cancel{sy(0)}^{\,0} - \cancel{y'(0)}^{\,0} + 12sY(s) - \cancel{y(0)}^{\,0} + 32Y(s) = \frac{32}{s}$$

$$Y(s)\left[s^2 + 12s + 32\right] = \frac{32}{s}$$

$$Y(s) = \frac{32}{s(s^2+12s+32)}$$

$$y(t) = ?$$

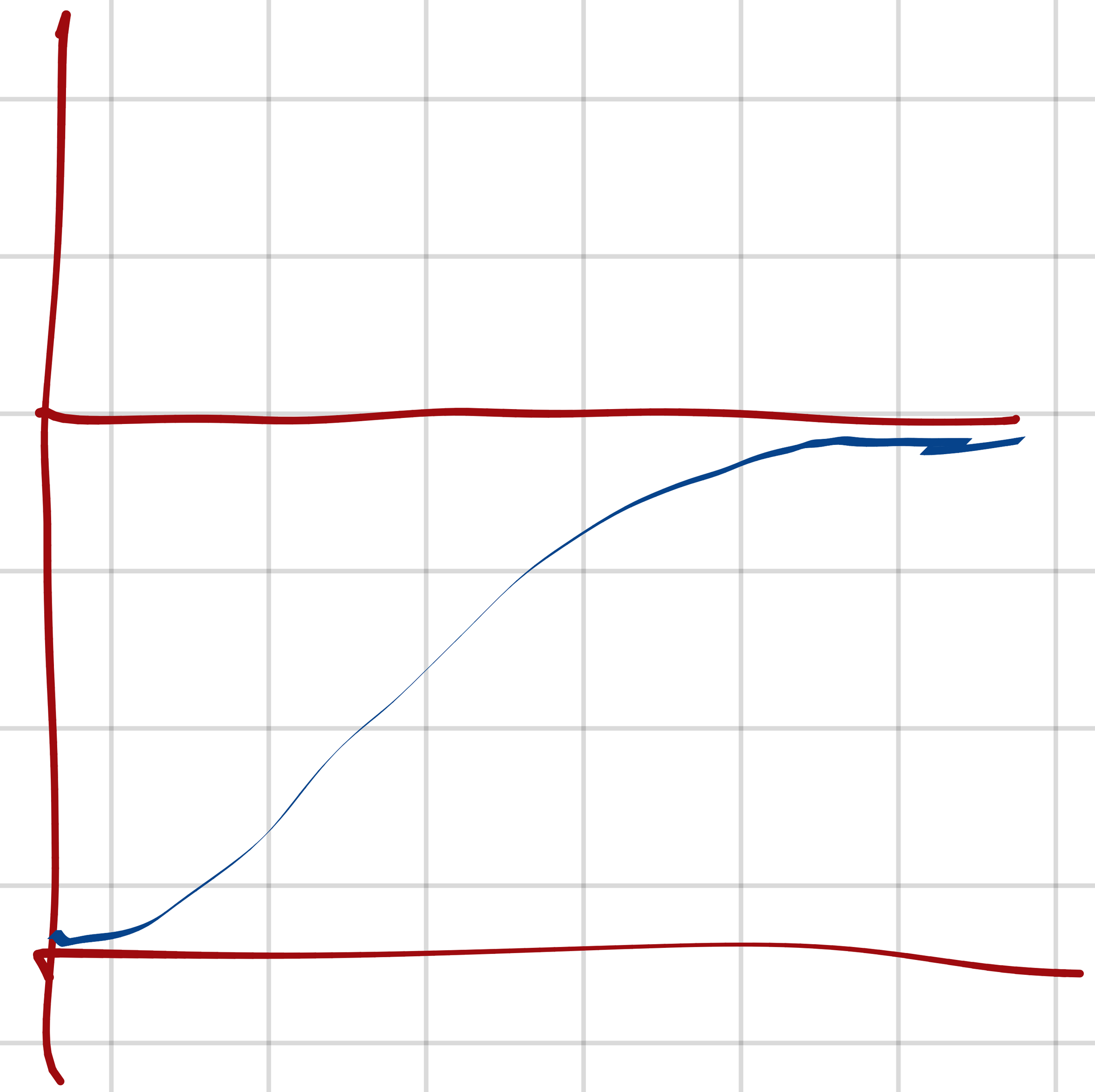
$$\frac{32}{s(s+4)(s+8)}$$

$$= \frac{K_1}{s} + \frac{K_2}{(s+4)} + \frac{K_3}{(s+8)}$$

$K_1 = 1$

$K_2 = -2$

$K_3 = 1$



$$\mathcal{L}^{-1}[Y(s)] = \boxed{u(t) - 2e^{-4t}u(t) + e^{-8t}u(t)}$$

decay

decay.

What if roots are complex?

$s = 0, -1 \pm 2i$

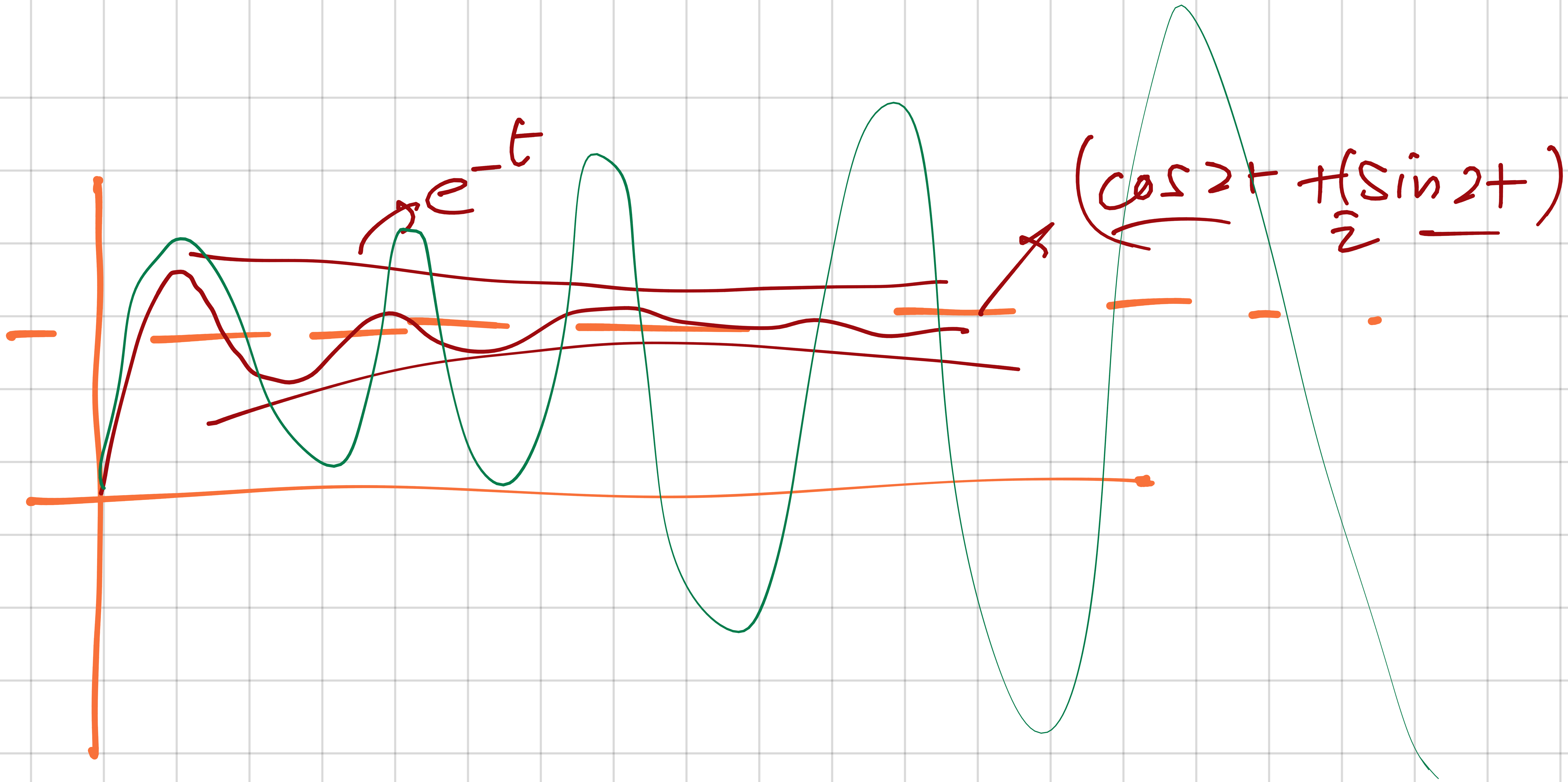
eg. $F(s) = \dfrac{3}{s(s^2+2s+5)} = \dfrac{3}{5}\dfrac{1}{s} - \dfrac{3}{5}\left(\dfrac{s+2}{s^2+2s+5}\right)$

$$= \frac{3}{5s} - \frac{3}{5}\left[\frac{s+1}{(s+1)^2+2^2} + \frac{1}{2}\times\frac{2}{(s+1)^2+2^2}\right]$$

$\dfrac{s}{s^2+\omega^2}$

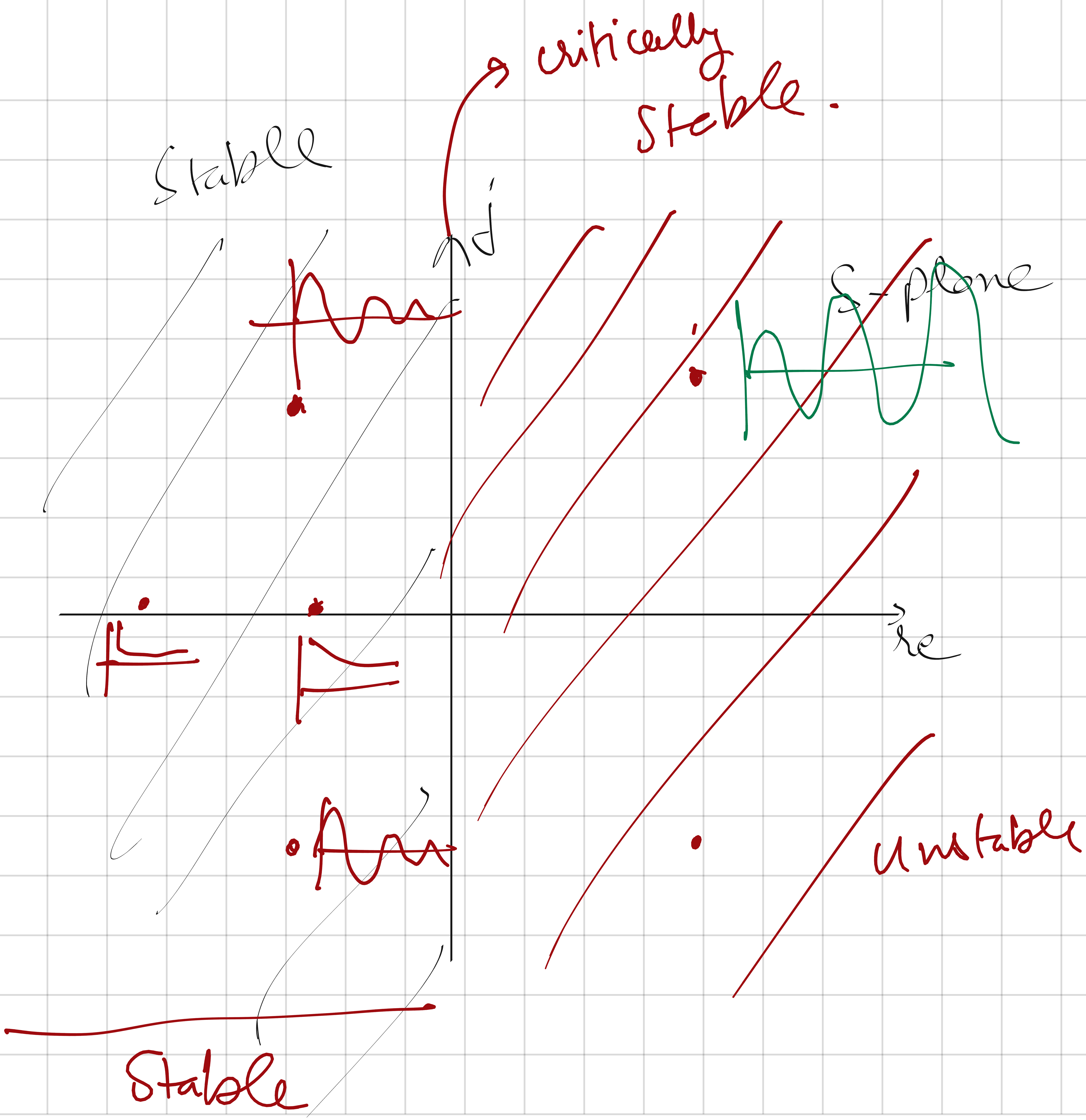
$\mathcal{L}\left[e^{-at}f(t)\right] = F(s+a)$

$$\mathcal{L}^{-1}\left[F(s)\right] = \frac{3}{5}u(t) - \frac{3}{5}e^{-t}\left[\cos 2t + \frac{1}{2}\sin 2t\right]$$



For a system to have a stable response, roots of denominator must lie in the

Left Half of s-plane



Roots of denominator determine stability of system

TRANSFER FUNCTION

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

$c(t) \rightarrow$ output.
$r(t) \rightarrow$ input.

$C(t) = F \cdot r(t)$ → Desired relationship to find the response

Take laplace on both sides :

$a_n s^n C(s) + a_{n-1} s^{n-1} C(s) + \ldots + a_0 C(s)$ + initial condition terms involving $c(t)$ & its derivatives.

$= b_m s^m R(s) + b_{m-1} s^{m-1} R(s) + \ldots + b_0 R(s)$ + I.C. terms of $r(t)$ & derivatives.

if all I.C. are 0

$$\frac{C(s)}{R(s)} = G(s) = \boxed{\frac{(b_m s^m + b_{m-1} s^{m-1} + \dots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \dots + a_0)}}$$

Transfer function

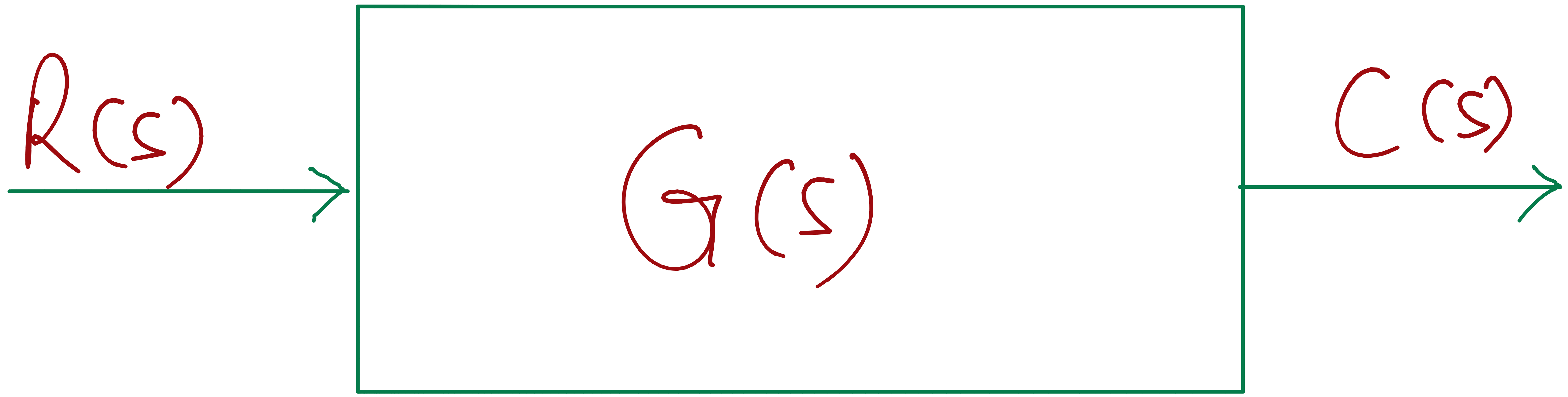
$$c(t) = r(t)\, g(t)$$

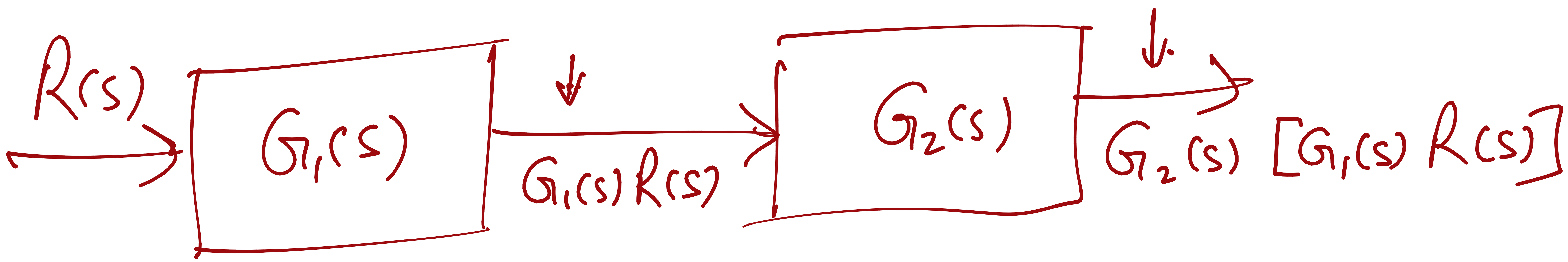
$$\mathcal{L}^{-1}[G(s) \cdot F(s)] \neq g(t)\, f(t)$$

$$c(t) = f(r(t))$$

$$\mathcal{L}^{-1}[C(s)] = \mathcal{L}^{-1}[G(s) \cdot R(s)]$$

Transfer function.





System response from transfer function

$$m\ddot{x} + c\dot{x} + kx = r(t)$$

Transfer function

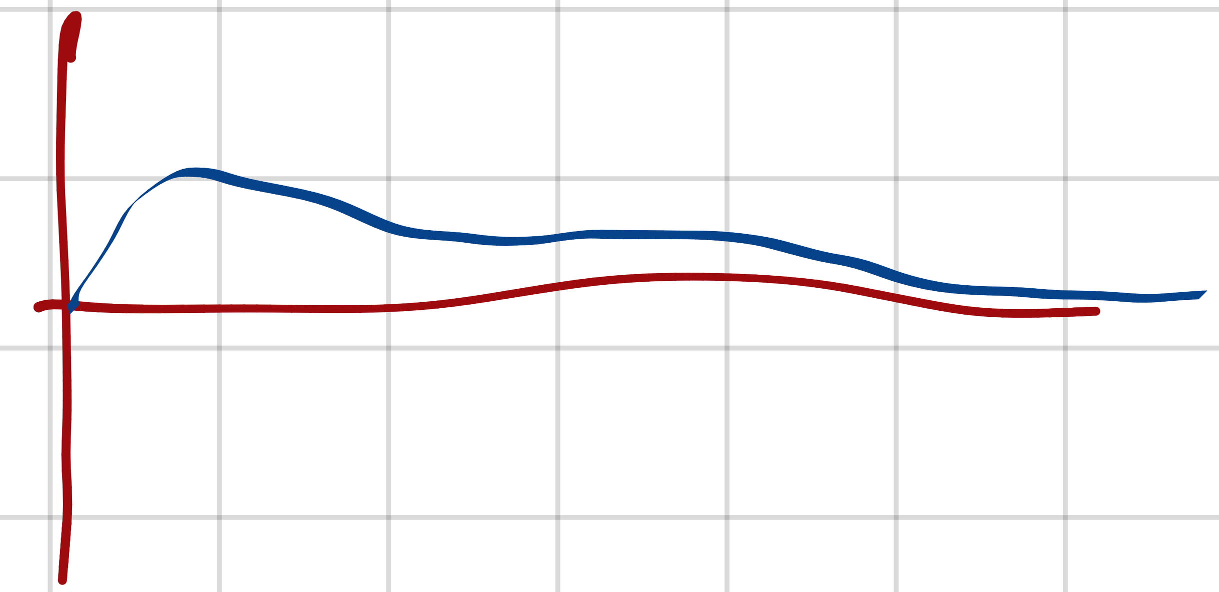
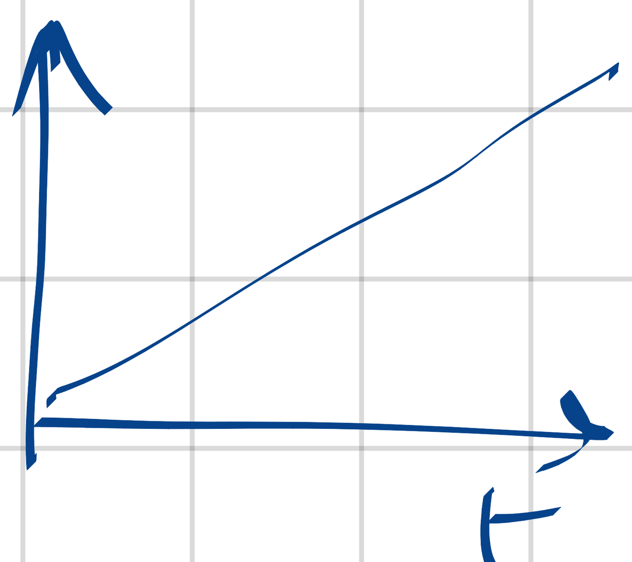
$$G(s) \rightarrow \frac{X(s)}{R(s)} = ?$$

$$G(s) = \frac{1}{ms^2 + cs + k}$$

$$X(s) = R(s) \cdot \frac{1}{ms^2 + cs + k}$$

$m = 1, \quad c = 3, \quad k = 2$ (say)

$$(s^2 + 3s + 2) = (s+2)(s+1)$$

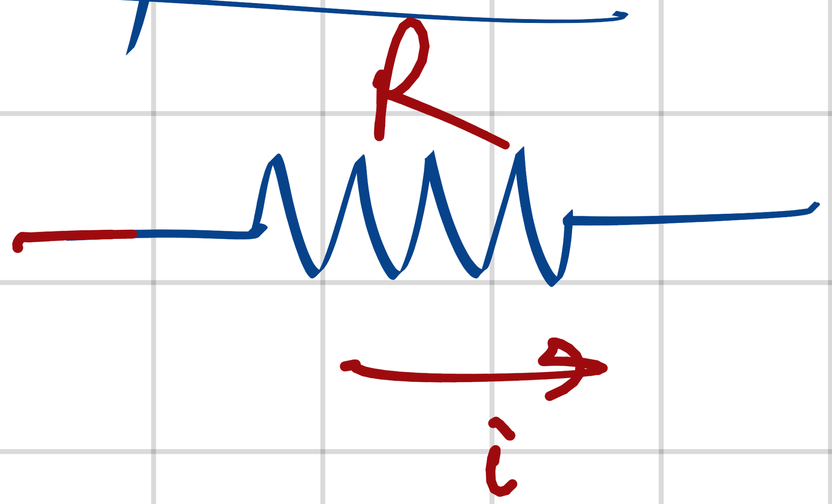
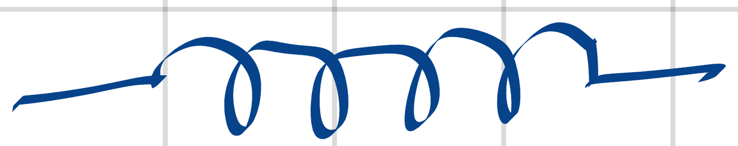
Input	$R(s)$	$X(s)$	$x(t)$
Impulse	1	$\frac{1}{(s+1)(s+2)}$	$[e^{-t} - e^{-2t}]\,u(t)$ 
Ramp I/P.  $t\,u(t)$	$\frac{1}{s^2}$	$\frac{1}{s^2}\,\frac{1}{(s+1)(s+2)}$	

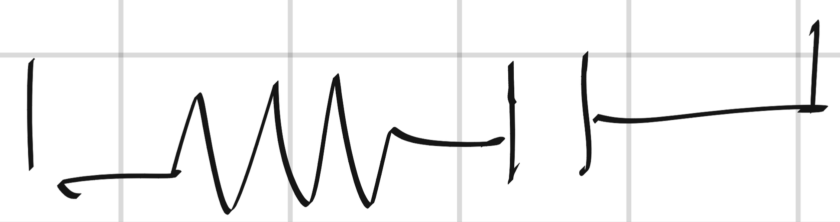
Real

Systems

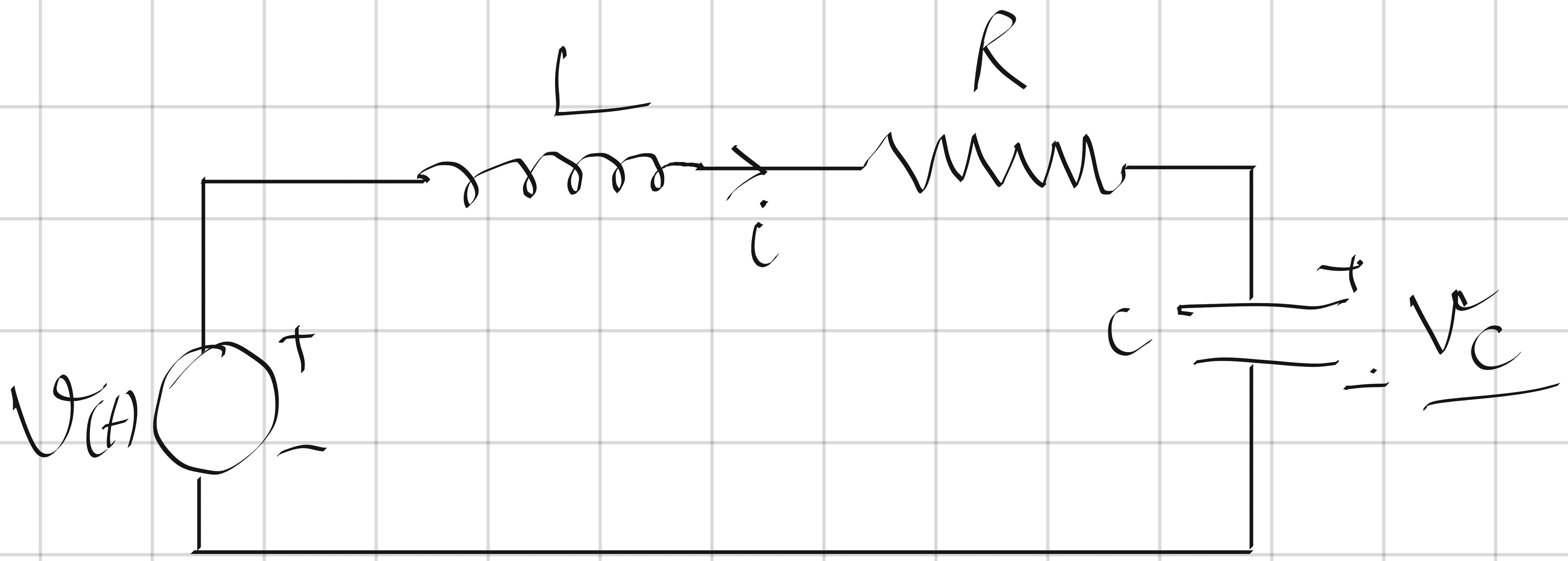
Electrical Systems

Passive Components

		$V-I$	Impedance $Z(s)$ [Transfer function $V(s)/I(s)$]	$V-q$	Admittance $I(s)/V(s)$
Resistor		$v(t) = R\, i(t)$	R	$v = R\dfrac{dq}{dt}$	$\dfrac{1}{R} = G$
Capacitor		$v(t) = \dfrac{1}{C}\int i\, dt$	$\dfrac{1}{Cs}$	$v = \dfrac{q}{C}$	Cs
Inductor		$v(t) = L\dfrac{di}{dt}$	Ls	$v = L\dfrac{d^2q}{dt^2}$	$\dfrac{1}{Ls}$



eg- Single loop



$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i \cdot d\tau = v$$

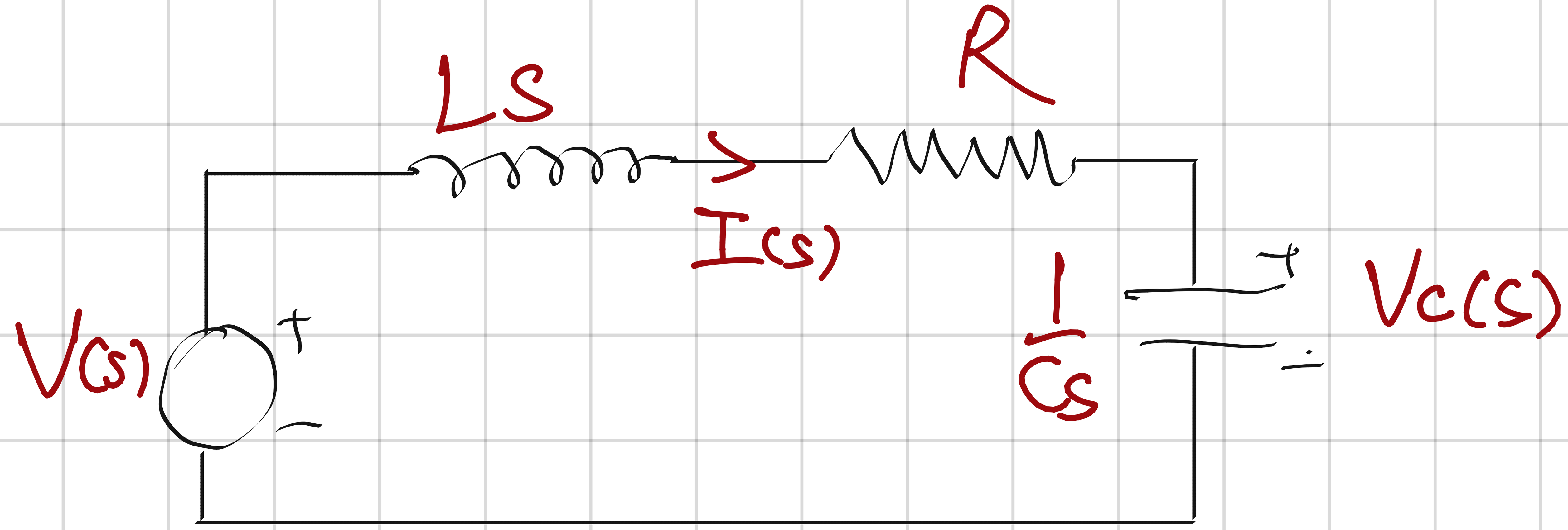
Substitute $i = \frac{dq}{dt}$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = v$$

$$q = Cv_C$$

$$\boxed{LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = V}$$

$$\left(Ls + R + \frac{1}{Cs}\right) I(s) = V(s)$$



$$\frac{V_c(s)}{I(s)} = \frac{1}{Cs}$$

$$I(s) = Cs\, V_c$$

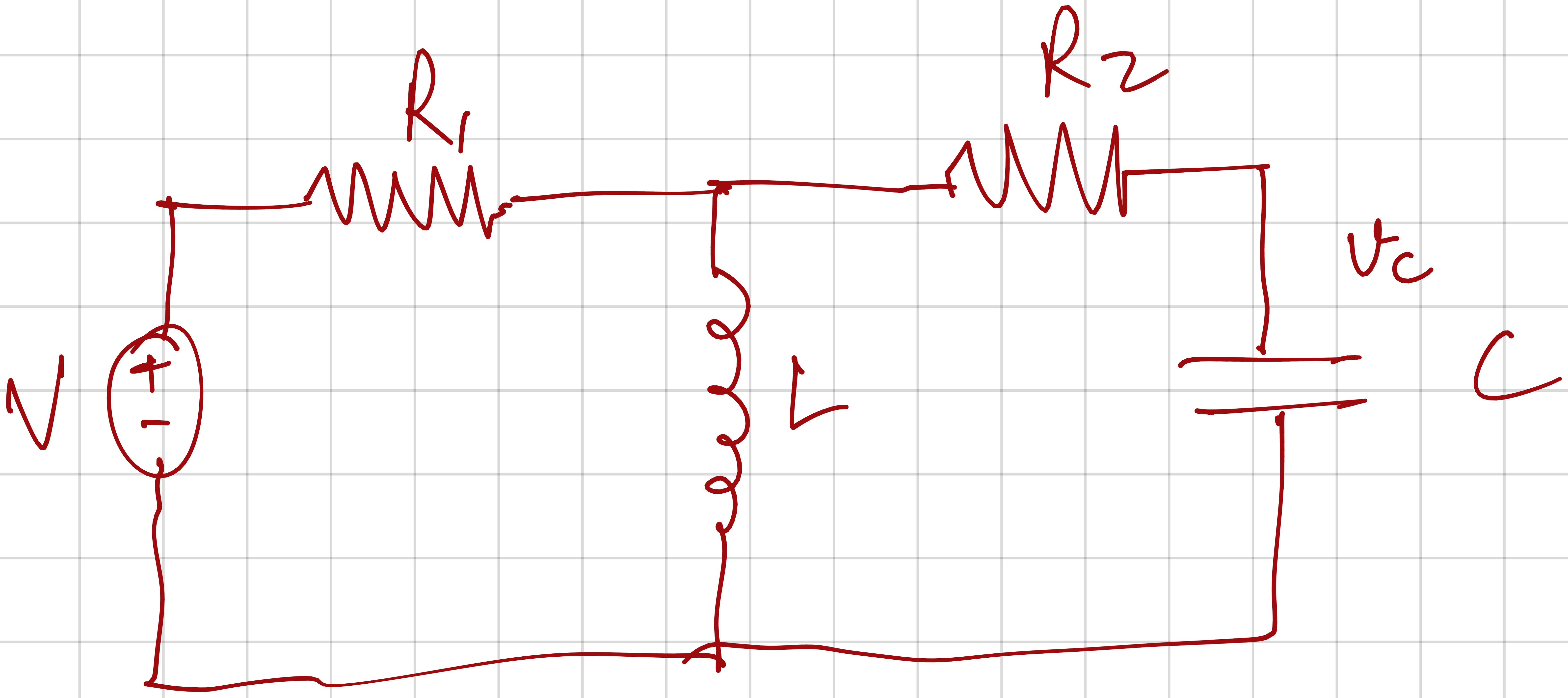
$$\left(Ls + R + \frac{1}{Cs}\right) Cs\, V_c = V$$

$$\boxed{\left(LCs^2 + RCs + 1\right) V_c = V} \equiv \boxed{LC\frac{d^2 v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = V}$$

s

t

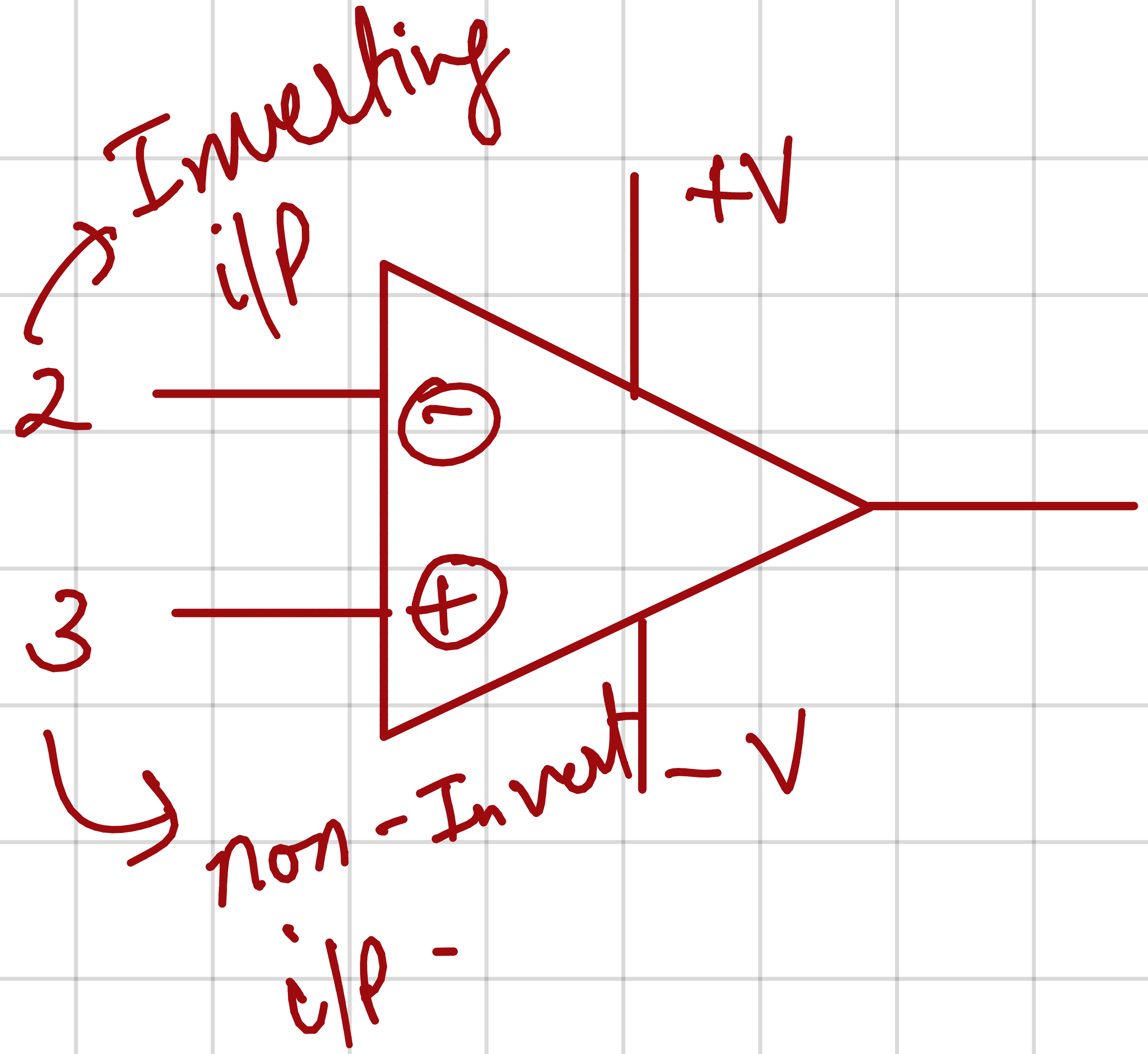
given I.Cs = 0



R1
R2
V
+
−
L
vc
C

Opamps

Ideal opamp assumptions:-

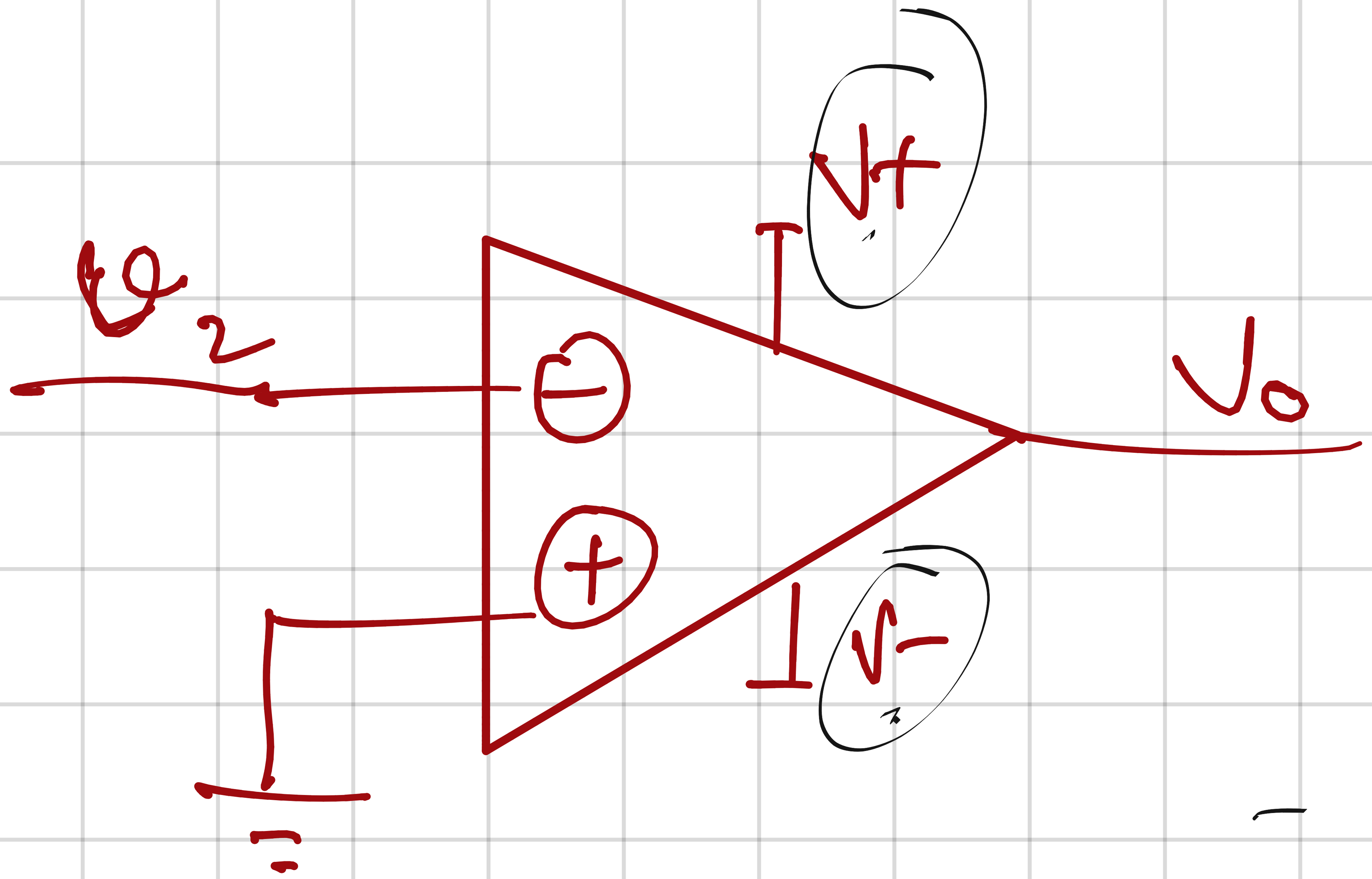


- Infinite resistance b/w 2 & 3

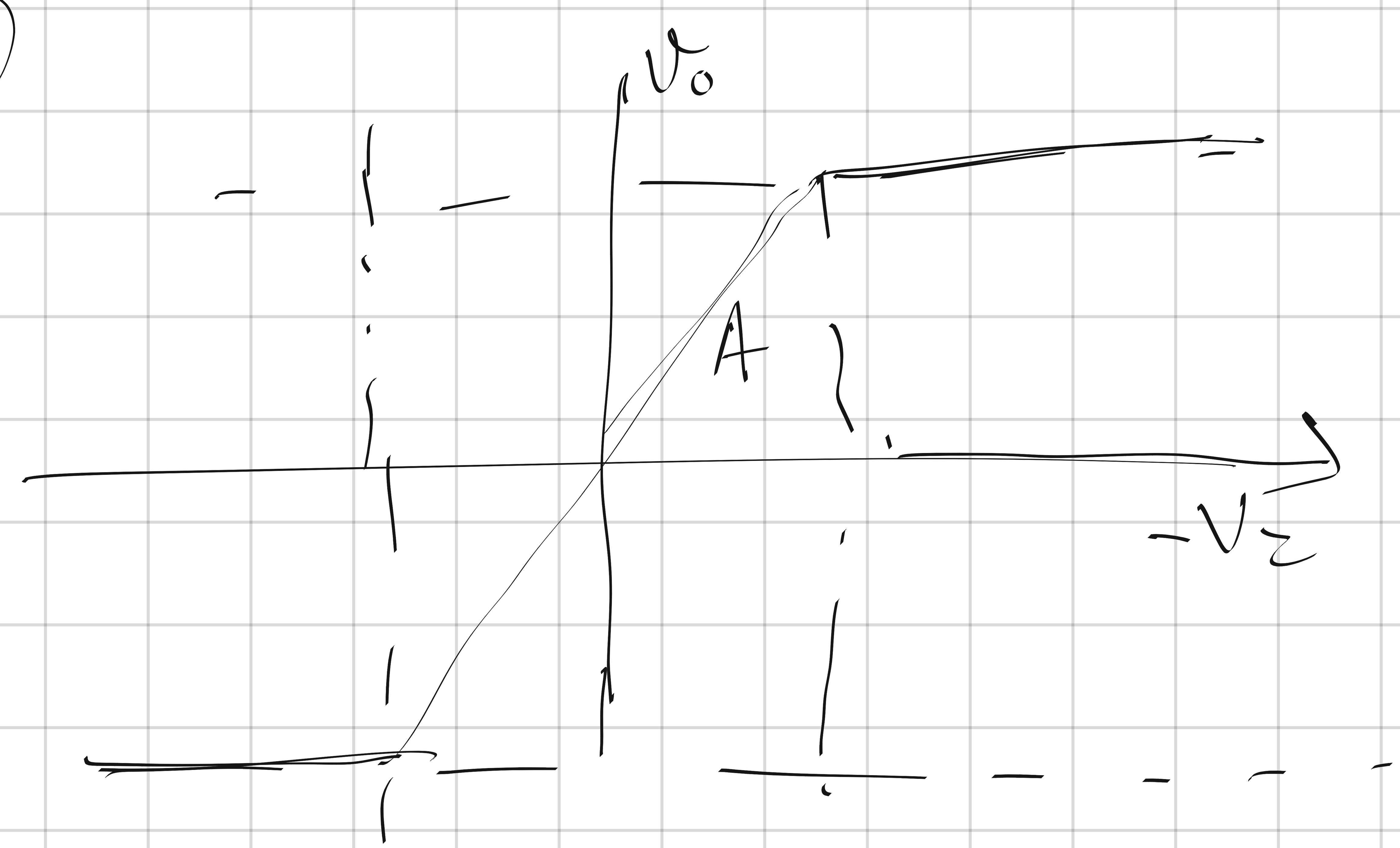
- Low o/p impedance = 0

- Infinite I/P impedance $Z_2 = Z_3 = \infty$ i.e. no current flows from source to opamp.

Inverting opamp



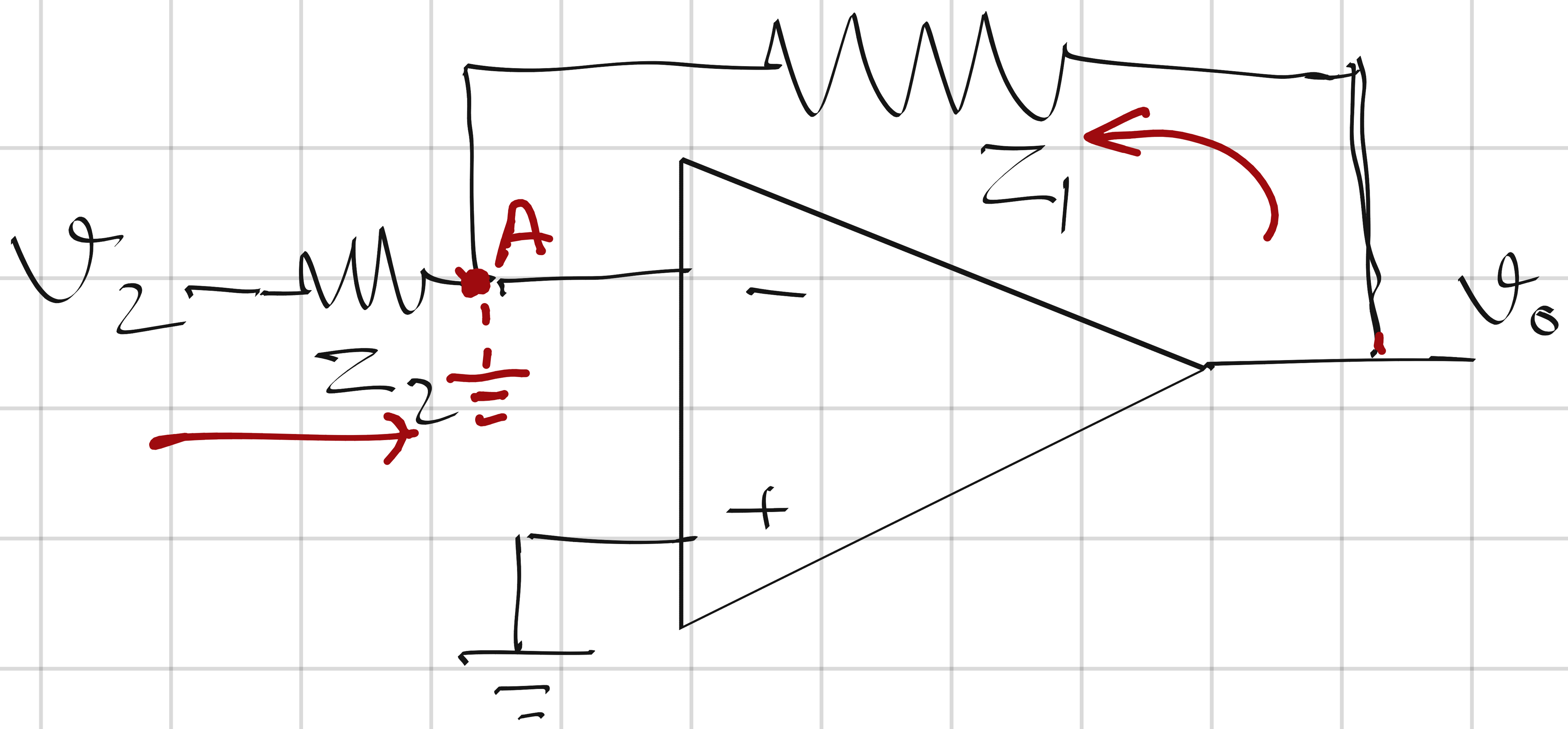
open loop gain $= A$



$$V_0 = -A\, v_2$$

Ideally $A \to \infty$

$$= -v_-$$



A → virtual ground.

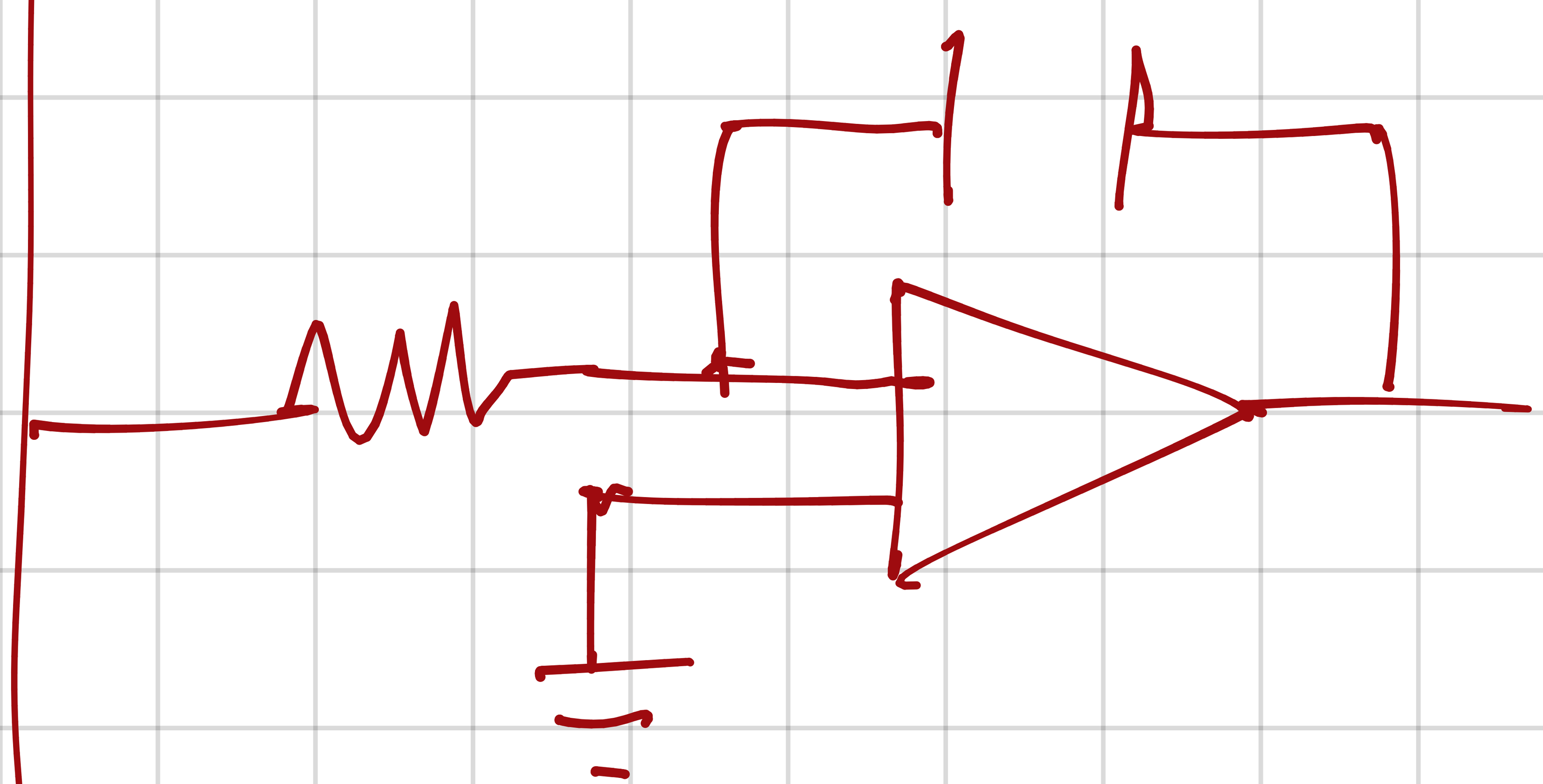
$$-\frac{v_0}{Z_1} \qquad \frac{v_2}{Z_2}$$

$$v_0 = -v_2 \cdot \frac{Z_1}{Z_2}$$

eg- write Z_1 & Z_2 in laplace

$Z_1 = R_1, Z_2 = R_2 \Rightarrow v_0 = -v_2 \frac{R_1}{R_2}$

$Z_1 = \frac{1}{CS} \quad Z_2 = R_2 \Rightarrow v_0 = -v_2 \frac{1}{R_2 CS}$



Time Response of Systems

$$G(s) = \frac{(S+a)}{(S+b)(S+c)} \times 1 = \frac{K_1}{(S+b)} + \frac{K_2}{(S+c)}$$

$$= K_1 e^{-bt} + K_2 e^{-ct}$$

$$a\dot{x} + bx + c\int_0^2 x\,dz$$

$$as + b + \frac{c}{s} = 1$$

$$G = \frac{(s+a)}{(s+b)(s+c)}$$

Zeros:

$G \to 0$
$s \to (-a)$

$a \to$ is a zero of the system

Poles:

$s \to -b, -c \quad G \to \infty$

$\hookrightarrow$ Poles of the system

Any poles that are common with zeros & vice versa are also poles & zeros

Step response of system

$$G(s) \cdot \frac{1}{s} \to \frac{1}{s} \cdot \frac{(s+a)}{(s+b)(s+c)} \longrightarrow \frac{K_1}{s} + \frac{K_2}{s+b} + \frac{K_3}{s+c} \quad — \quad (1)$$

$$\mathcal{L}^{-1}[(1)] = \left[K_1 u(t) + \left[K_2 e^{-bt} + K_3 e^{-ct}\right] u(t)\right) \qquad t \to \infty$$

$$O(t) \to K_1$$

dynamics → $Ae^{-\omega t}$

Dynamics of the system i.e. transient response primarily governs the system response.

$$\boxed{\frac{(s+3)}{(s+7)}}$$

$$\frac{s+1}{(s+1)(s+11)} = \frac{1}{s+1}$$

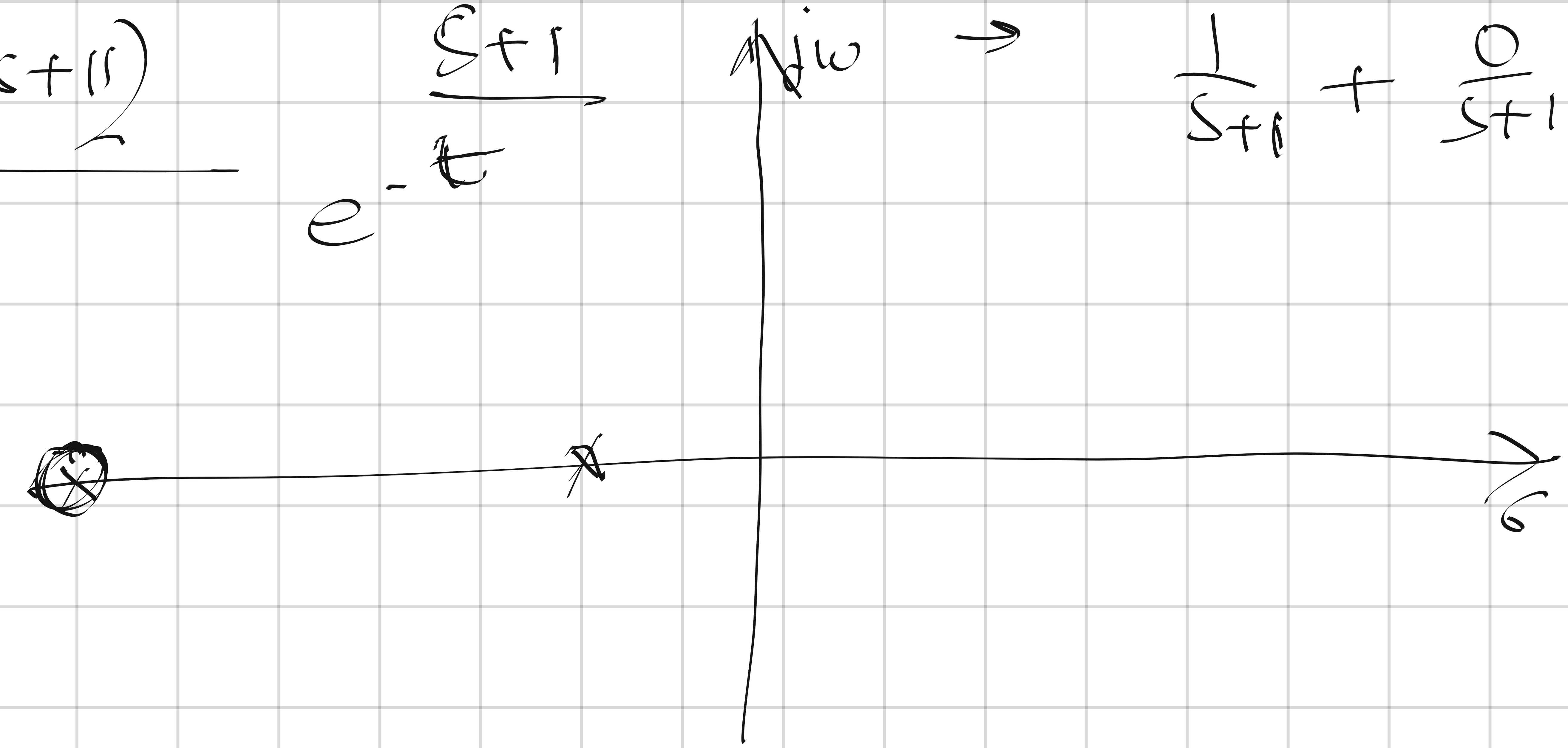
e^{-t}

$s = \sigma + j\omega$

$\rightarrow \frac{1}{s+1} + \frac{0}{s+11}$

Poles $\rightarrow$ X

Zeros $\rightarrow$ 0



$G(s) = \frac{s+3}{(s+7)}$ 1st order system

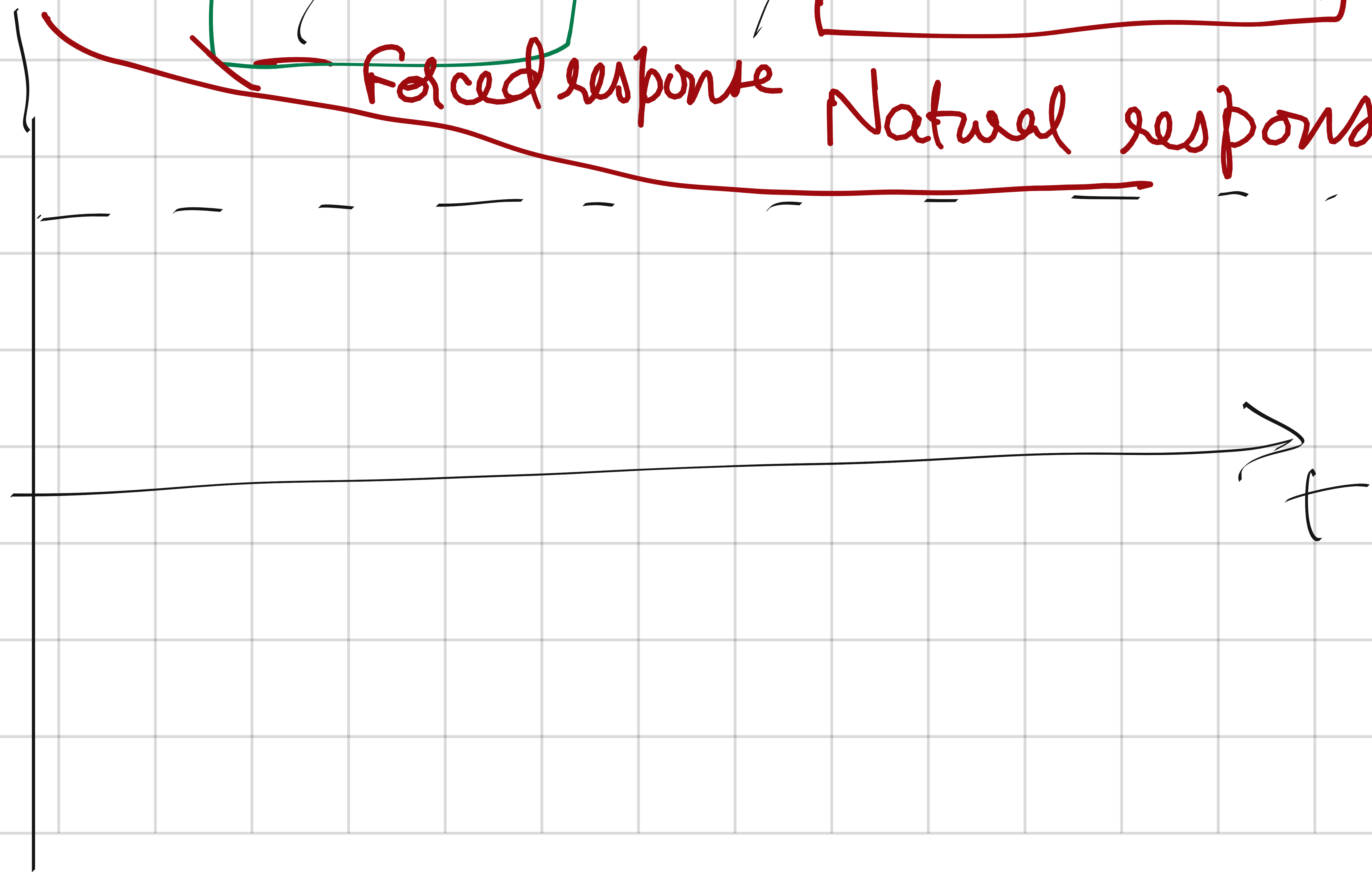
$O(s) = I(s) \cdot G(s)$

→ step response $1, \frac{1}{s}, \frac{1}{s^2}$

$$\frac{1}{s}\boxed{\frac{s+3}{(s+7)}} = \frac{3}{7}\frac{1}{s} + \frac{4}{7}\frac{1}{(s+7)} \quad - (2)$$

$$\mathcal{L}^{-1}[(2)] \rightarrow \boxed{\frac{3}{7}u(t)} + \frac{4}{7}e^{-7t}u(t)$$

Forced response Natural response



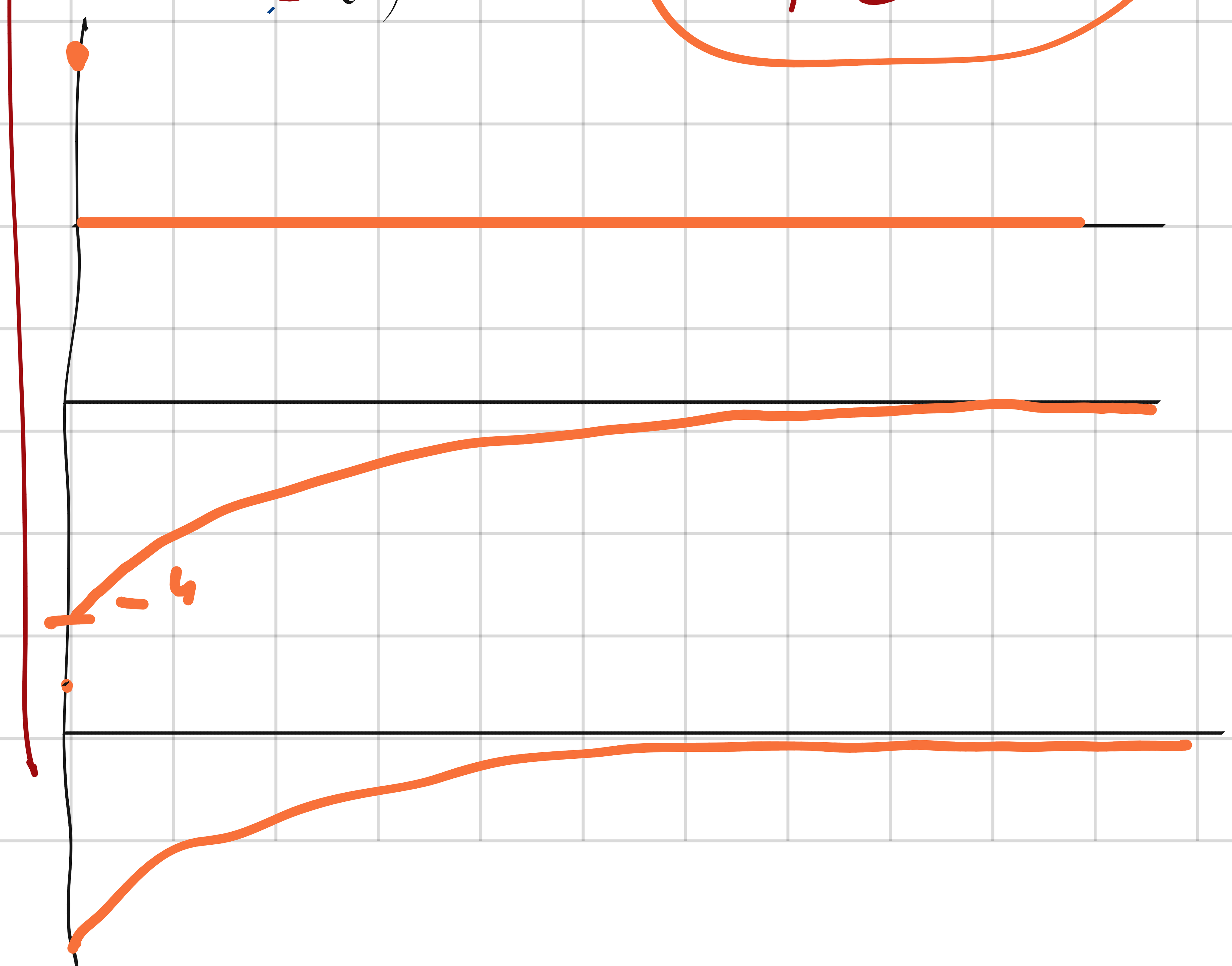
$\frac{3}{7}$

Static gain.

→ Impulse response?

$$\mathcal{L}^{-1}\left[\frac{(s+3)}{(s+7)}\right] = \mathcal{L}^{-1}\left[1 - \frac{4}{s+7}\right]$$

$$= \delta(t)u(t) - 4e^{-7t}$$



First Order Systems: Response and Performance Characteristics

$$G(s) = \frac{a}{(s+a)}$$

$a \rightarrow$ only parameter required to describe the sys.

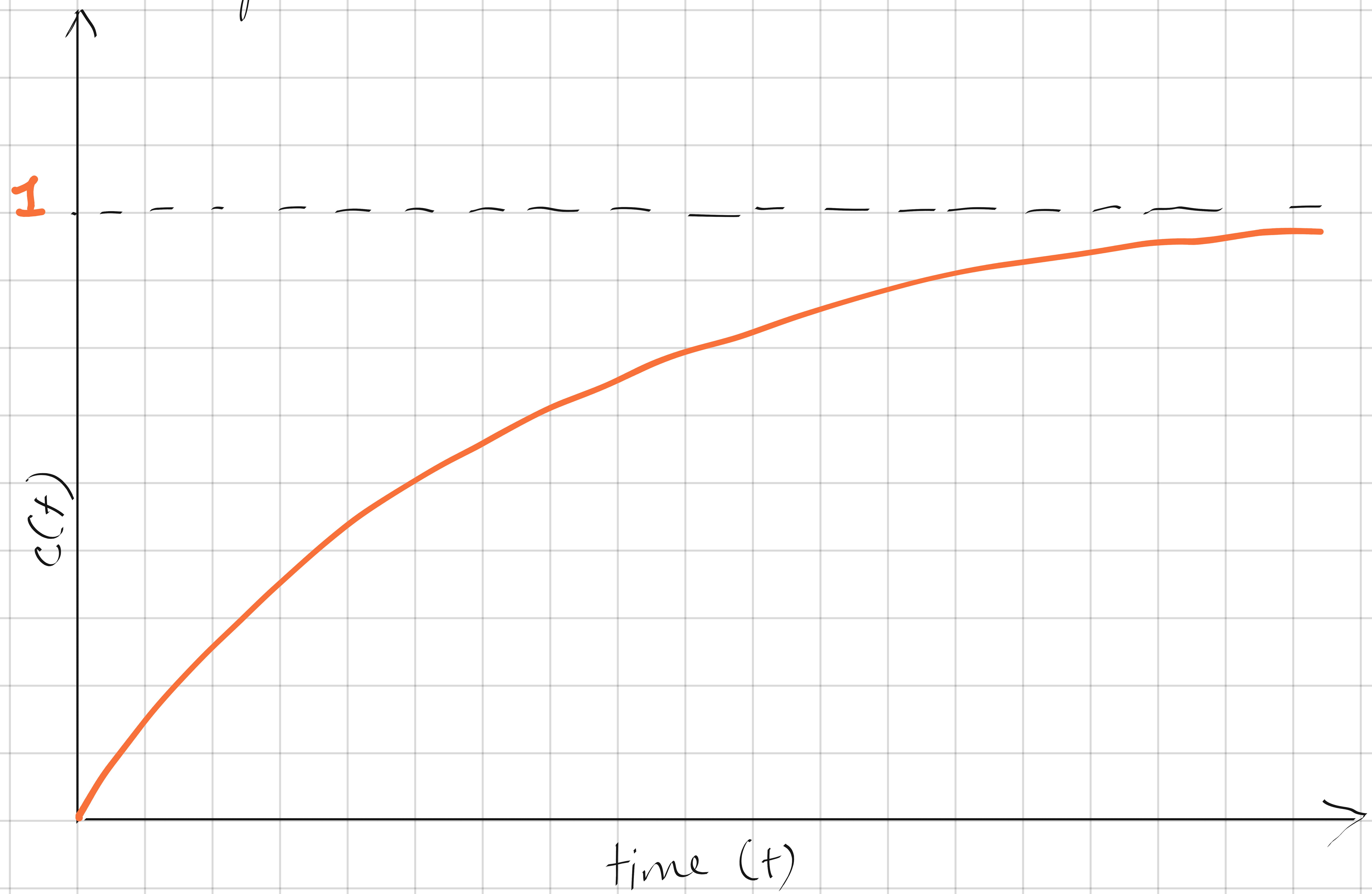
sys response to step i/p -

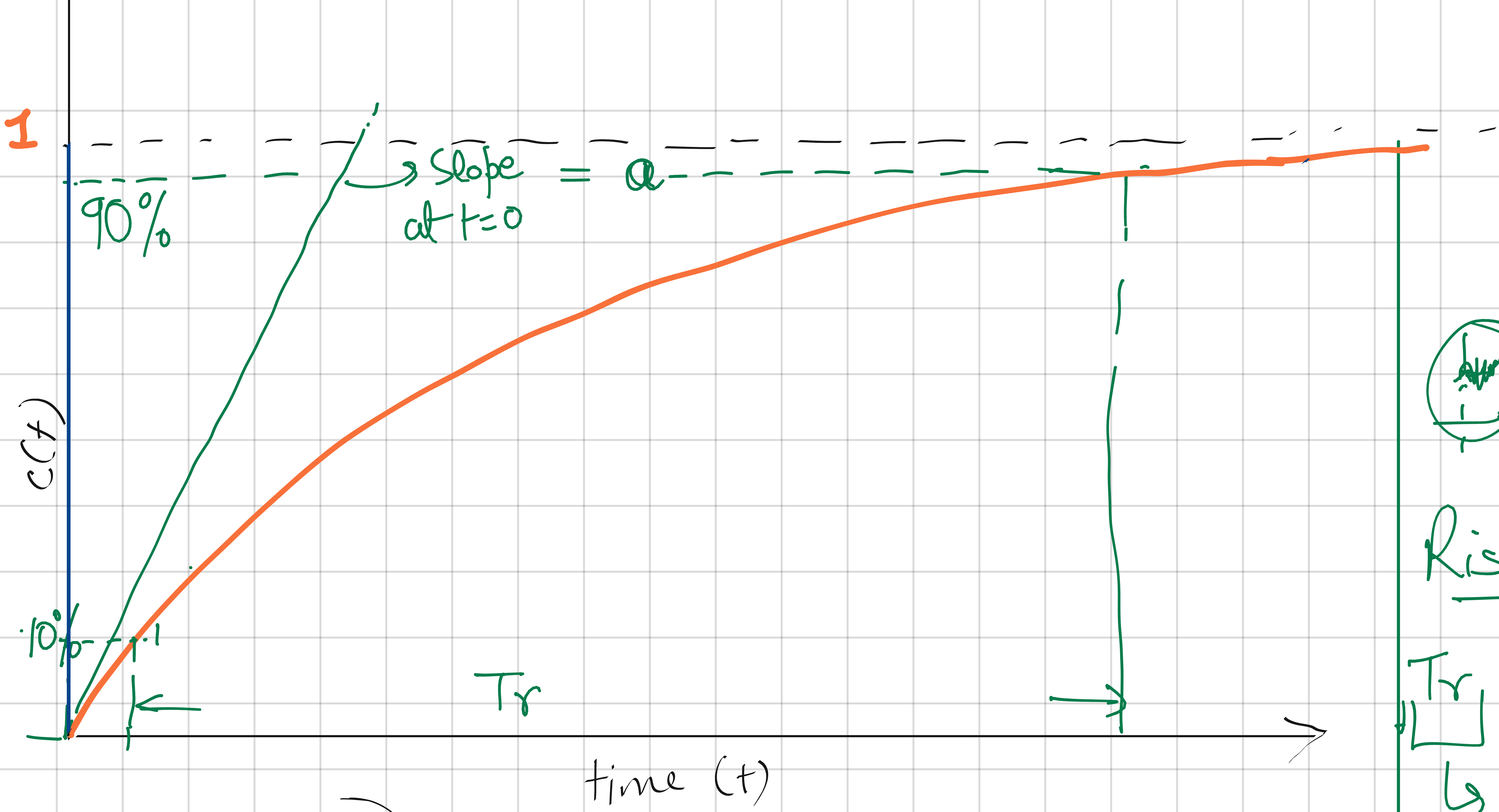
$$R(s) \rightarrow \boxed{G(s)} \rightarrow C(s)$$

$$C(s) = G(s)R(s) = \frac{a}{s(s+a)}$$

$$= \frac{1}{s} - \frac{1}{s+a}$$

$$\mathcal{L}^{-1}[C(s)] = [1 - e^{-at}]\,u(t)$$

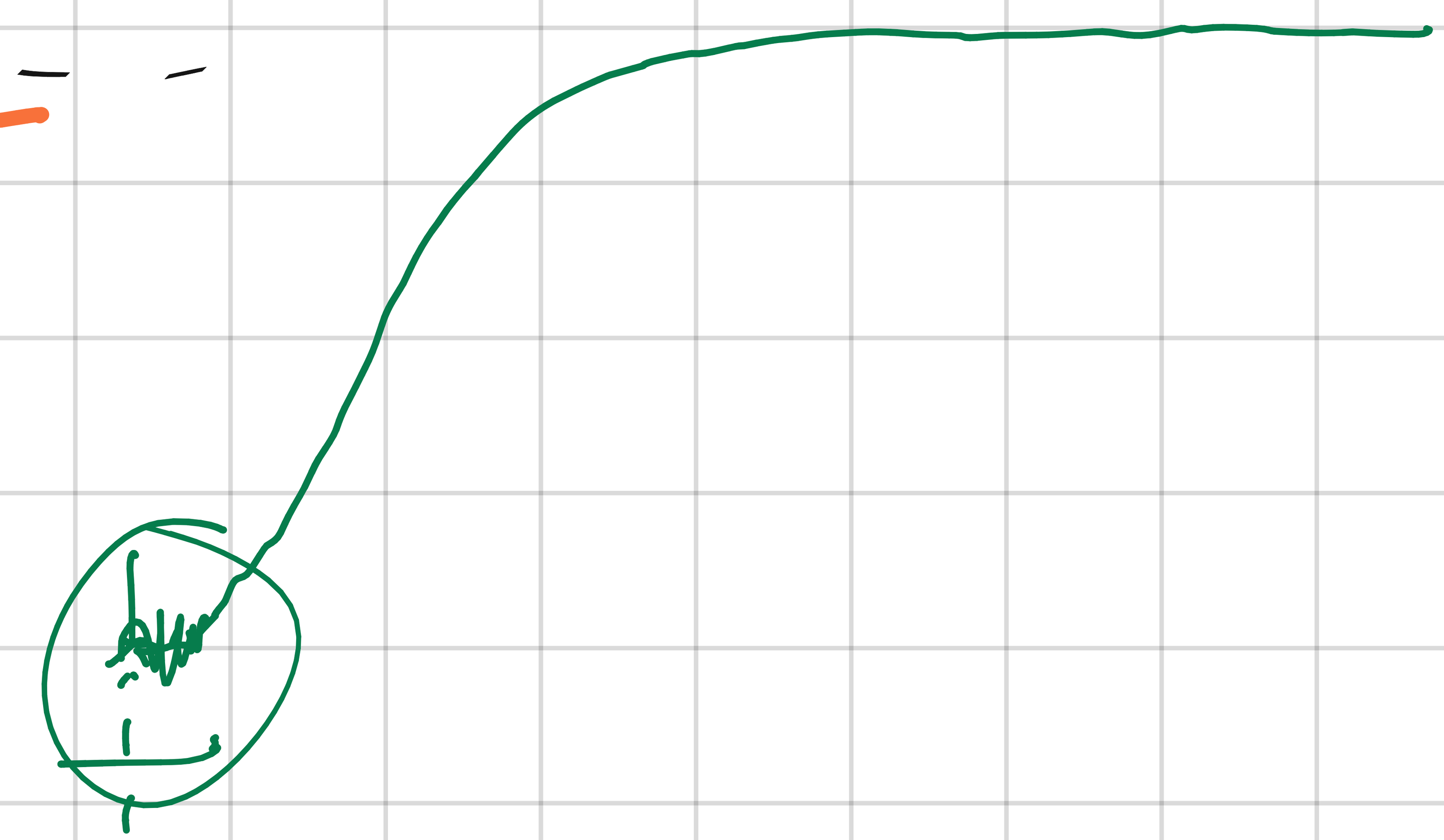




at $t = \frac{1}{a} = \tau \rightarrow$ Time constant
i.e. time at which response ≅ 63% of s.s.

$$c(t) = 1 - e^{-1} = 0.63$$

→ Useful for system I.D.



Rise Time

$T_r = t_{90\%} - t_{10\%}$

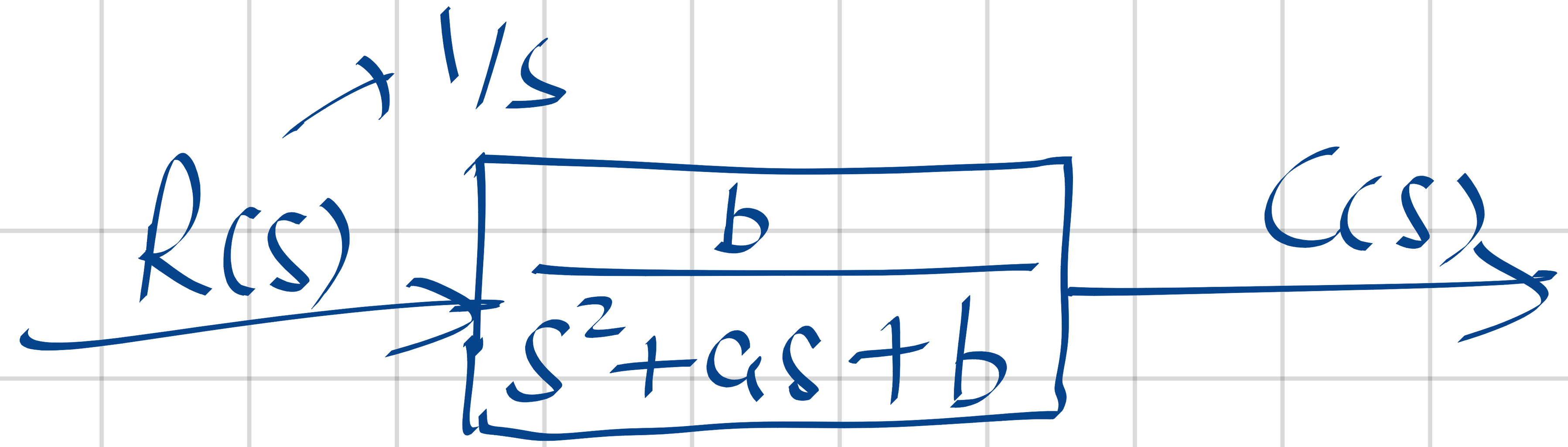
$$\boxed{T_r} = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$$

↳ measure experimentally.

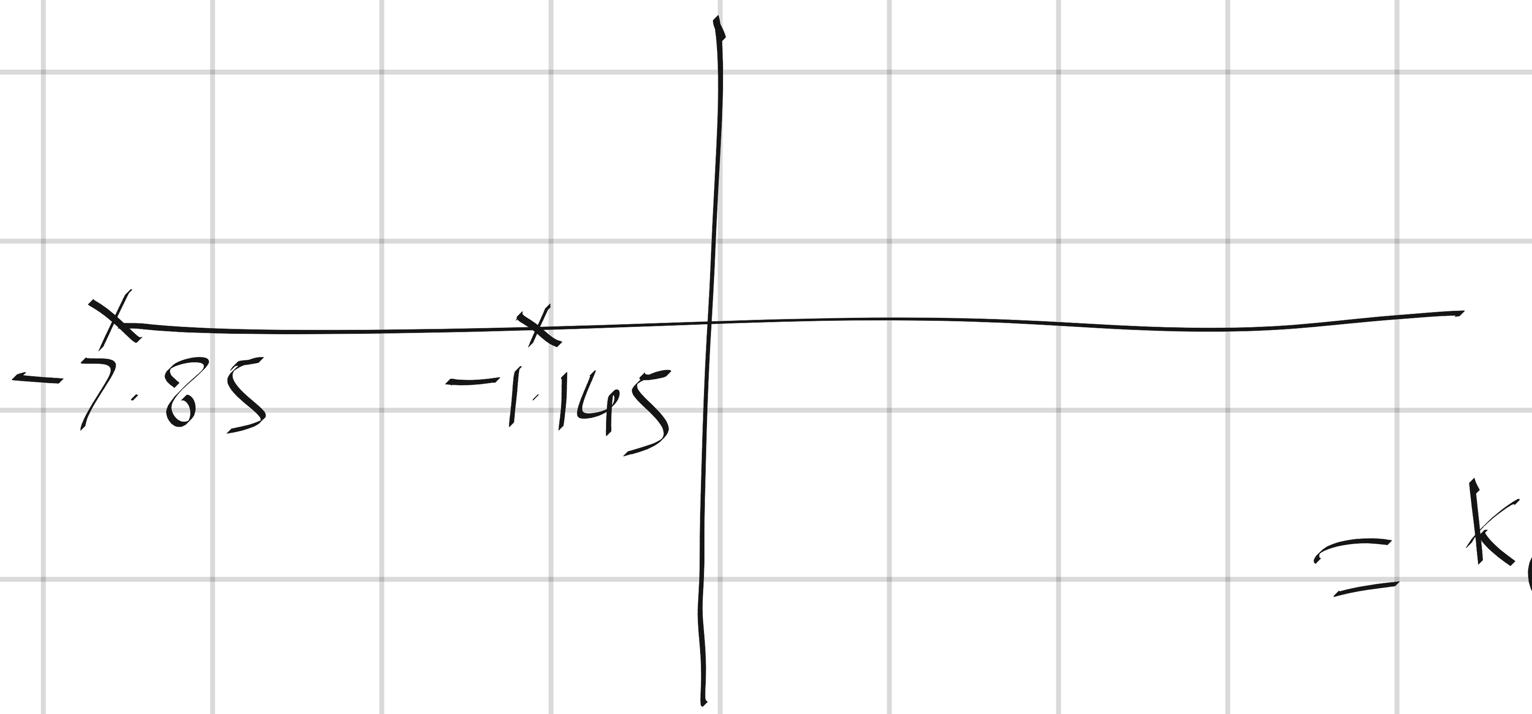
Settling Time

$$T_s = t_{98\%} = \frac{4}{a}$$

Second order Systems

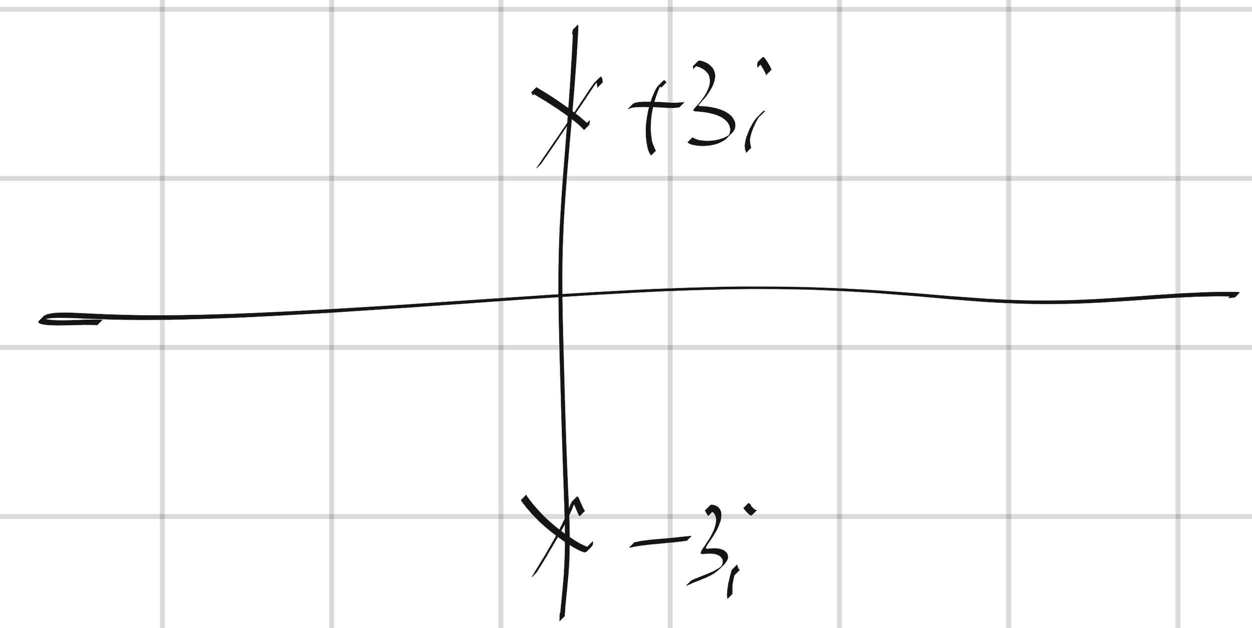


$$\frac{9}{s^2+9s+9} \to \mathcal{L}^{-1}\left[\frac{1}{s(s+7.85)(s+1.45)}\right]$$



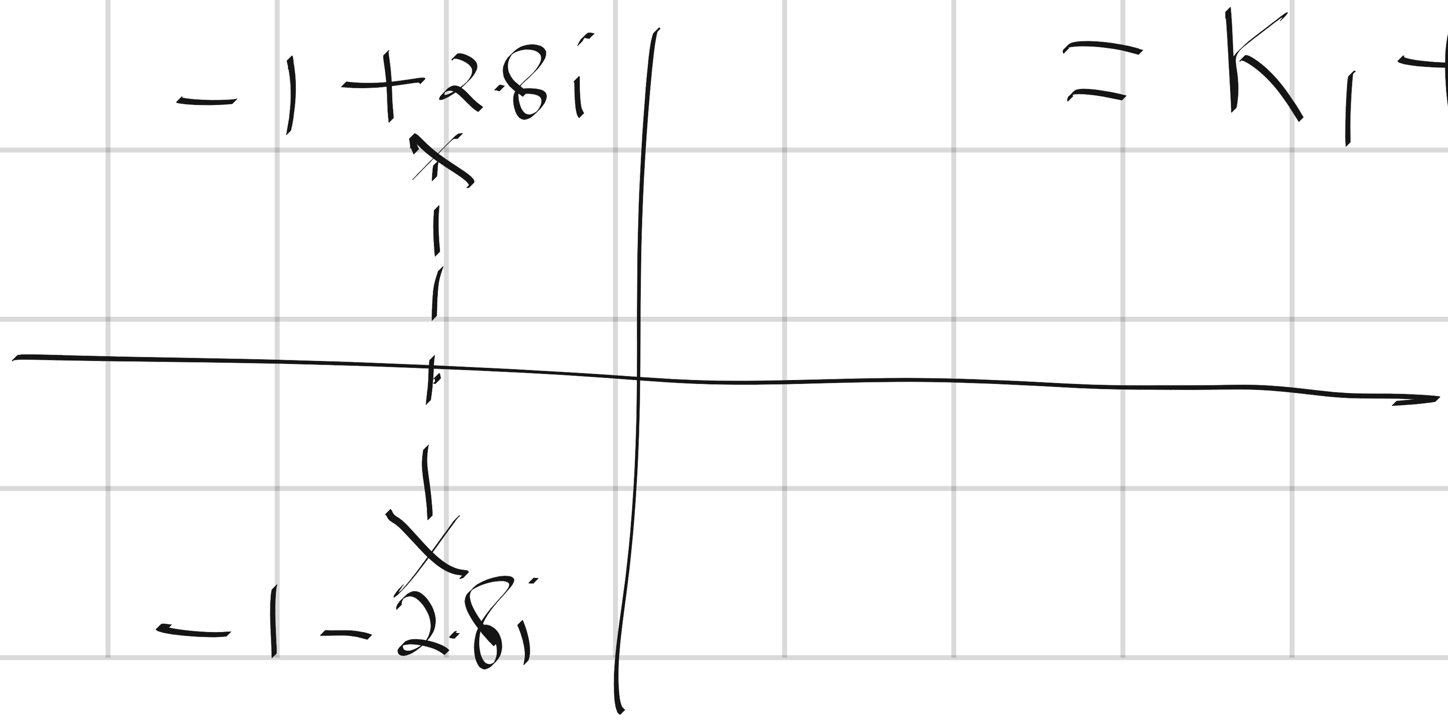
$$= K_1 + K_2 e^{-7.85t} + K_3 e^{-1.45t}$$

$$\frac{9}{s^2+9}$$



$$c(t) = 1 - \cos 3t$$

$$\frac{9}{s^2+2s+9}$$



$$= K_1 + K_2 e^{-t} \sin(2.8t + \phi)$$

$$\frac{1}{s(s+3)^2} = \frac{K_1}{s} + \frac{K_2}{s+3} + \frac{K_3}{(s+3)^2}$$

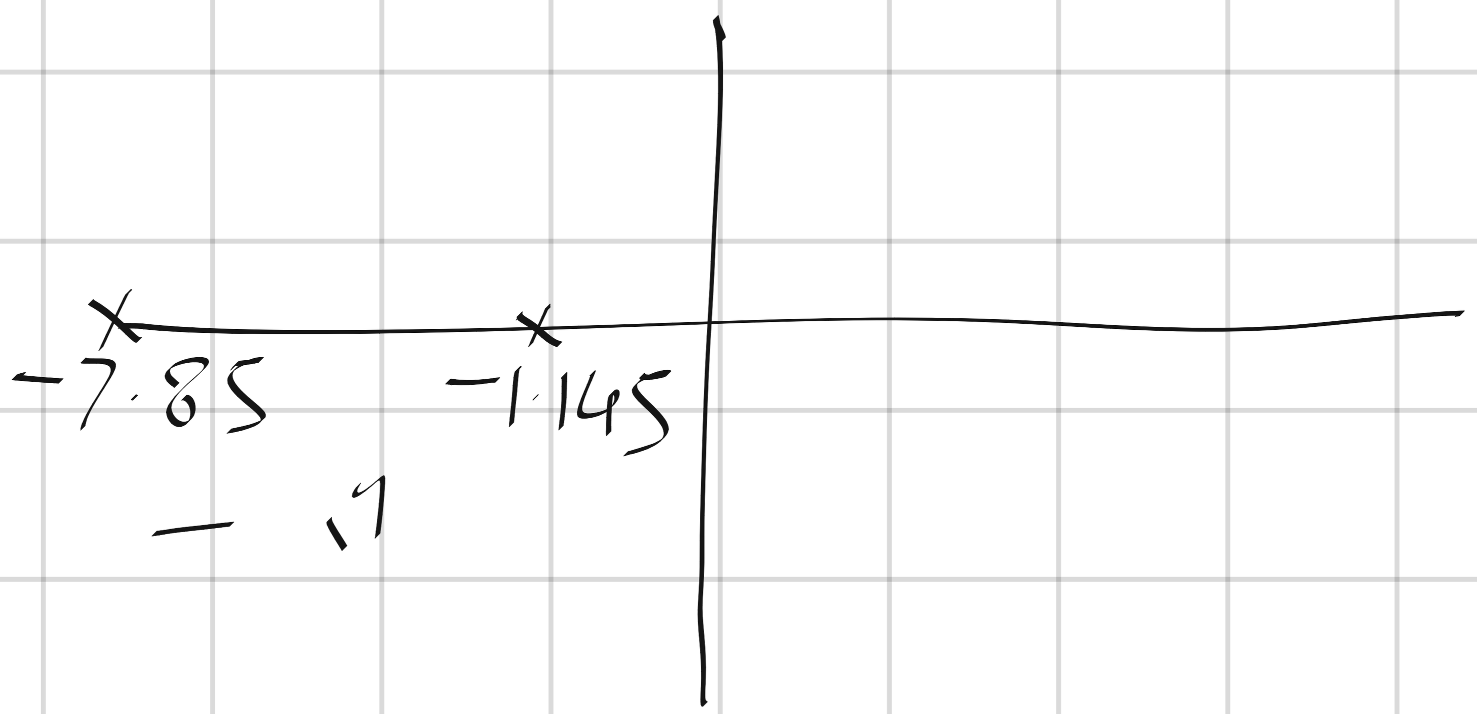
$$\frac{9}{s^2+6s+9}$$

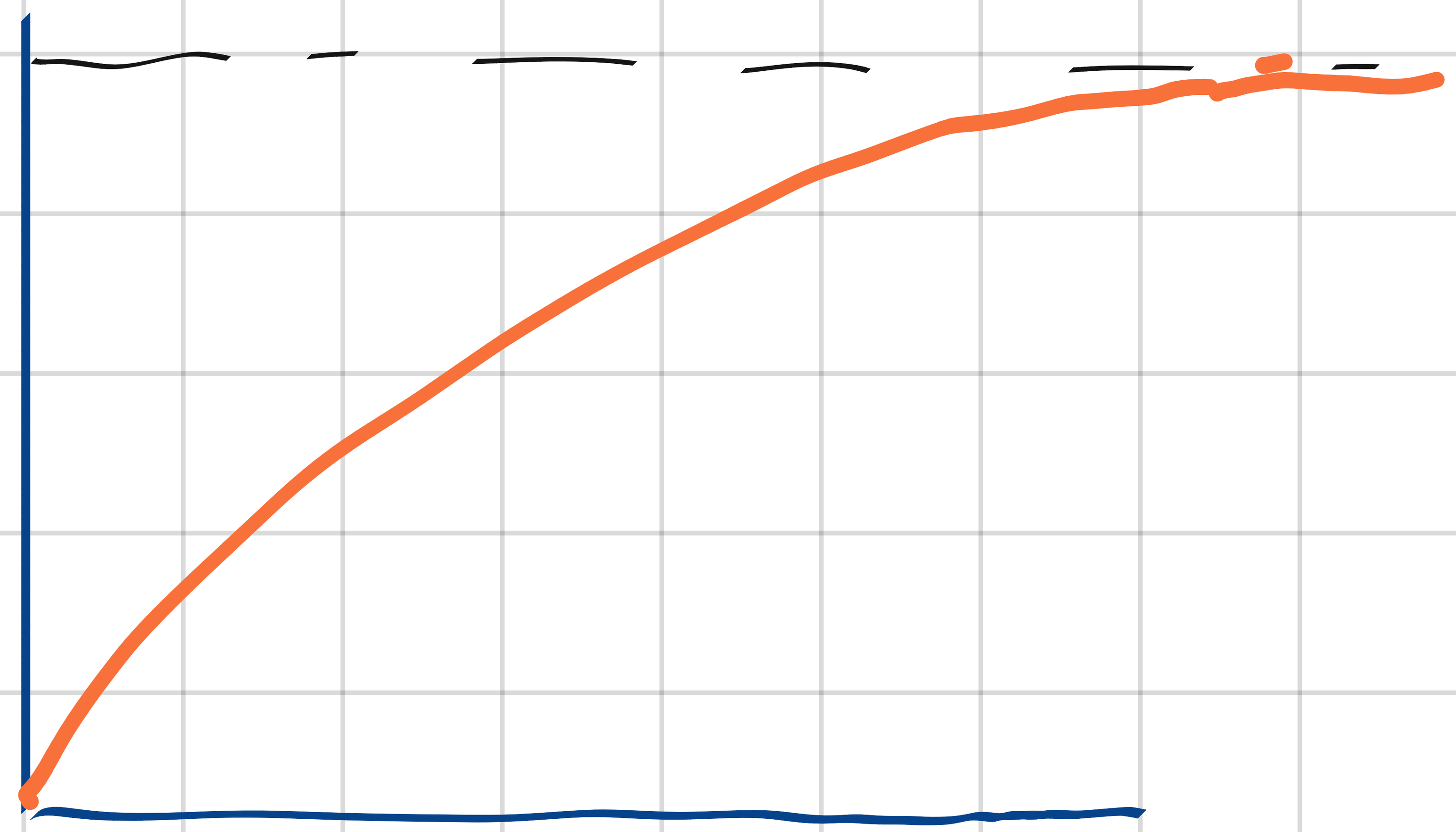
$$c(t) = 1 - 3te^{-3t} - e^{-3t}$$

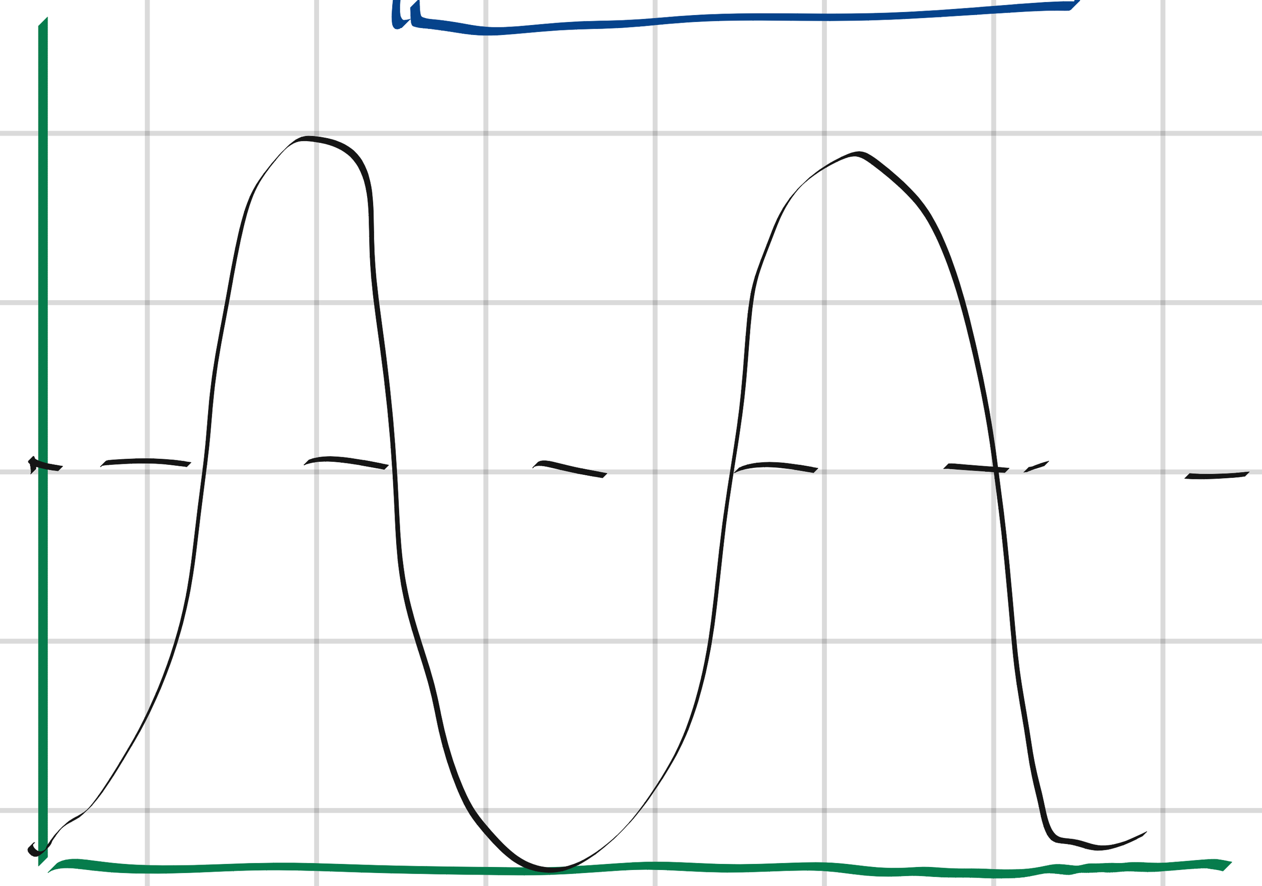
Second order Systems

$$R(s) \xrightarrow{1/s} \boxed{\frac{b}{s^2+as+b}} \rightarrow C(s)$$

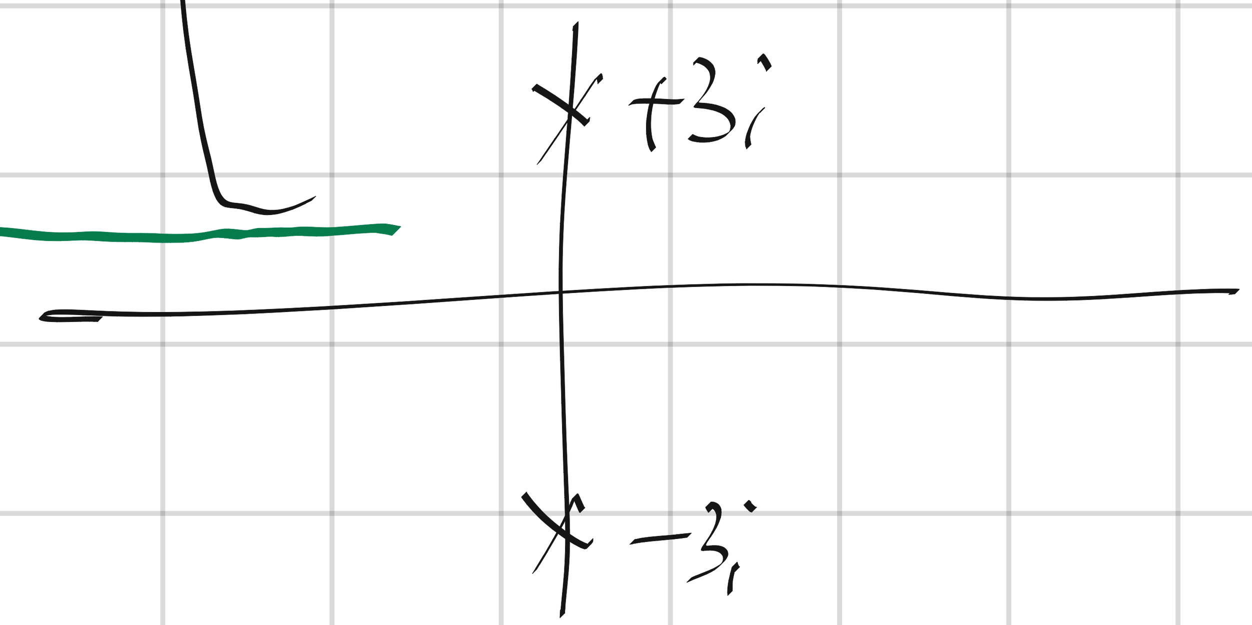
$$\frac{9}{s^2+9s+9} \rightarrow$$



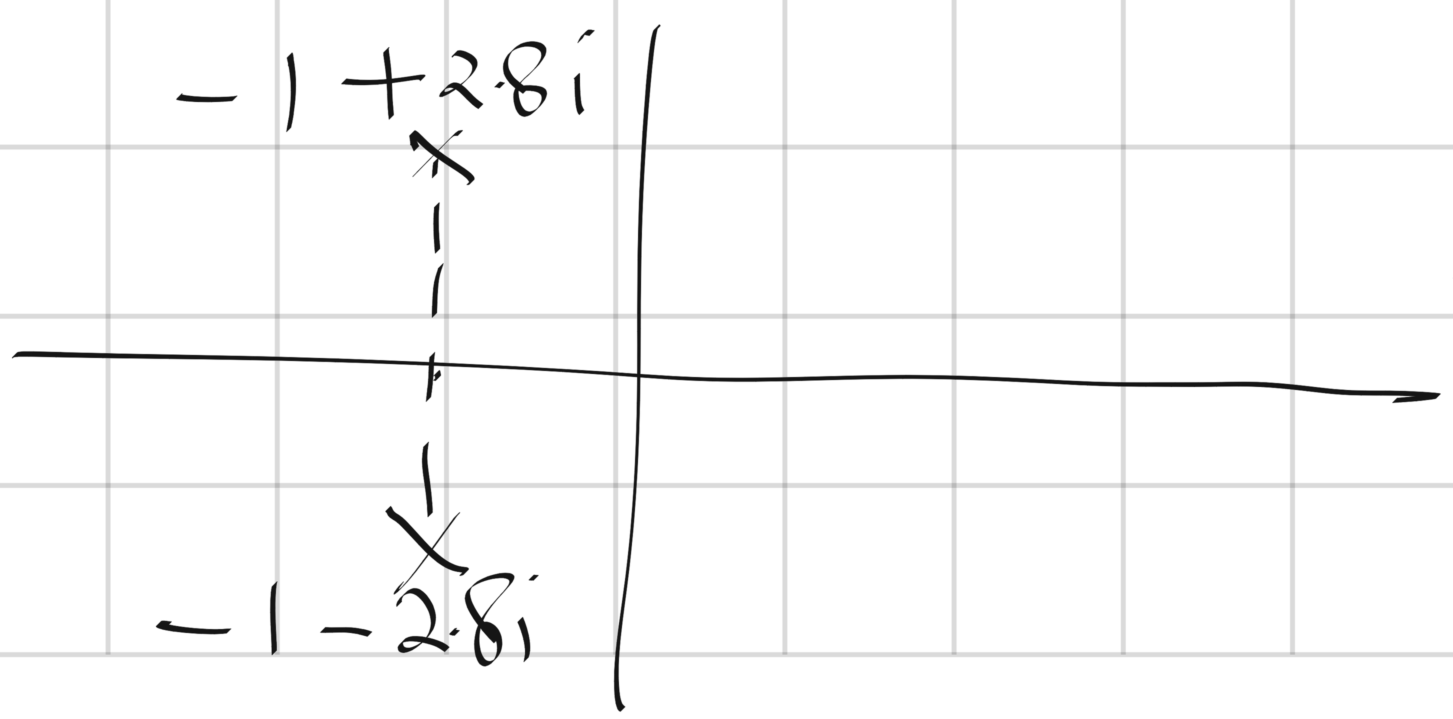


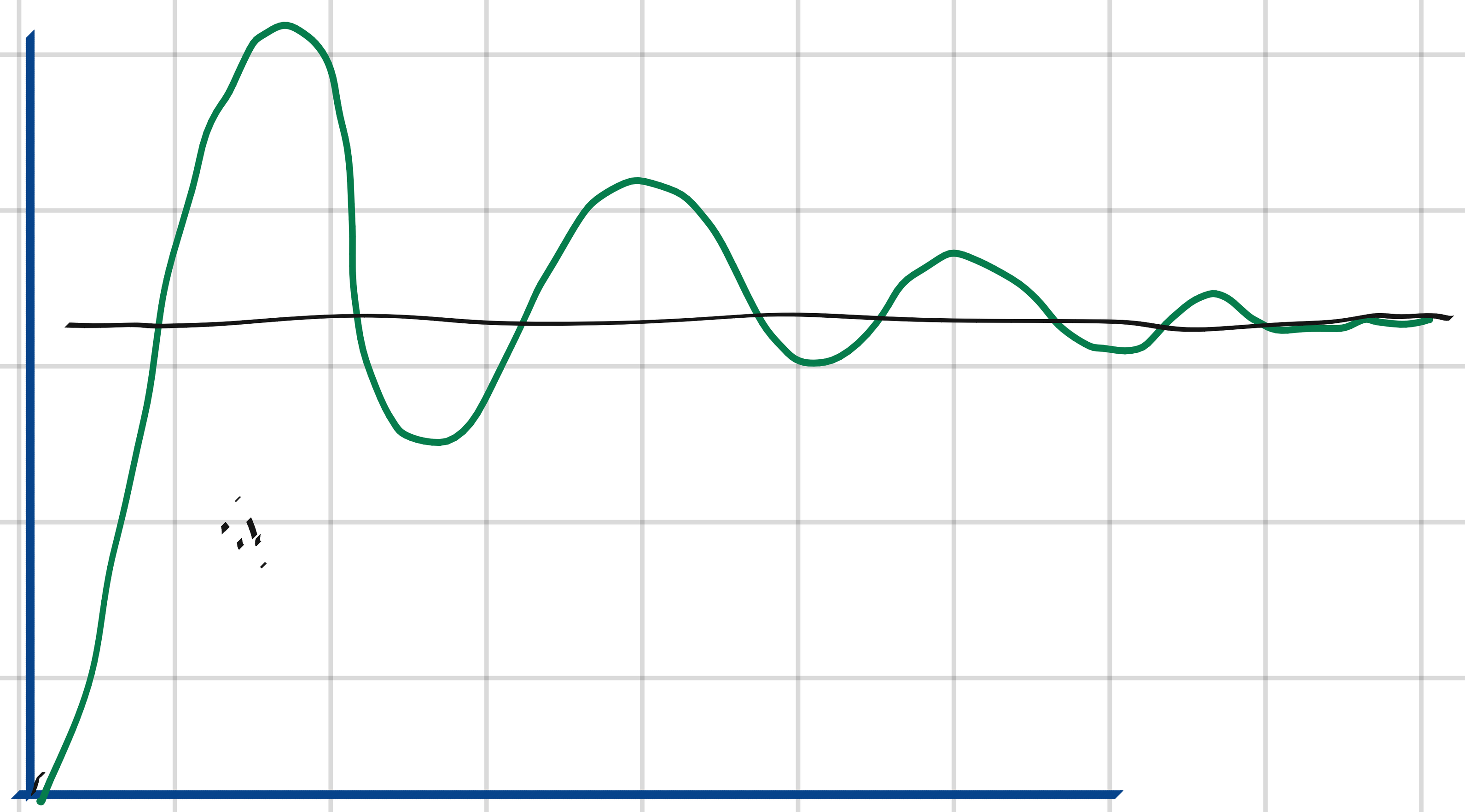


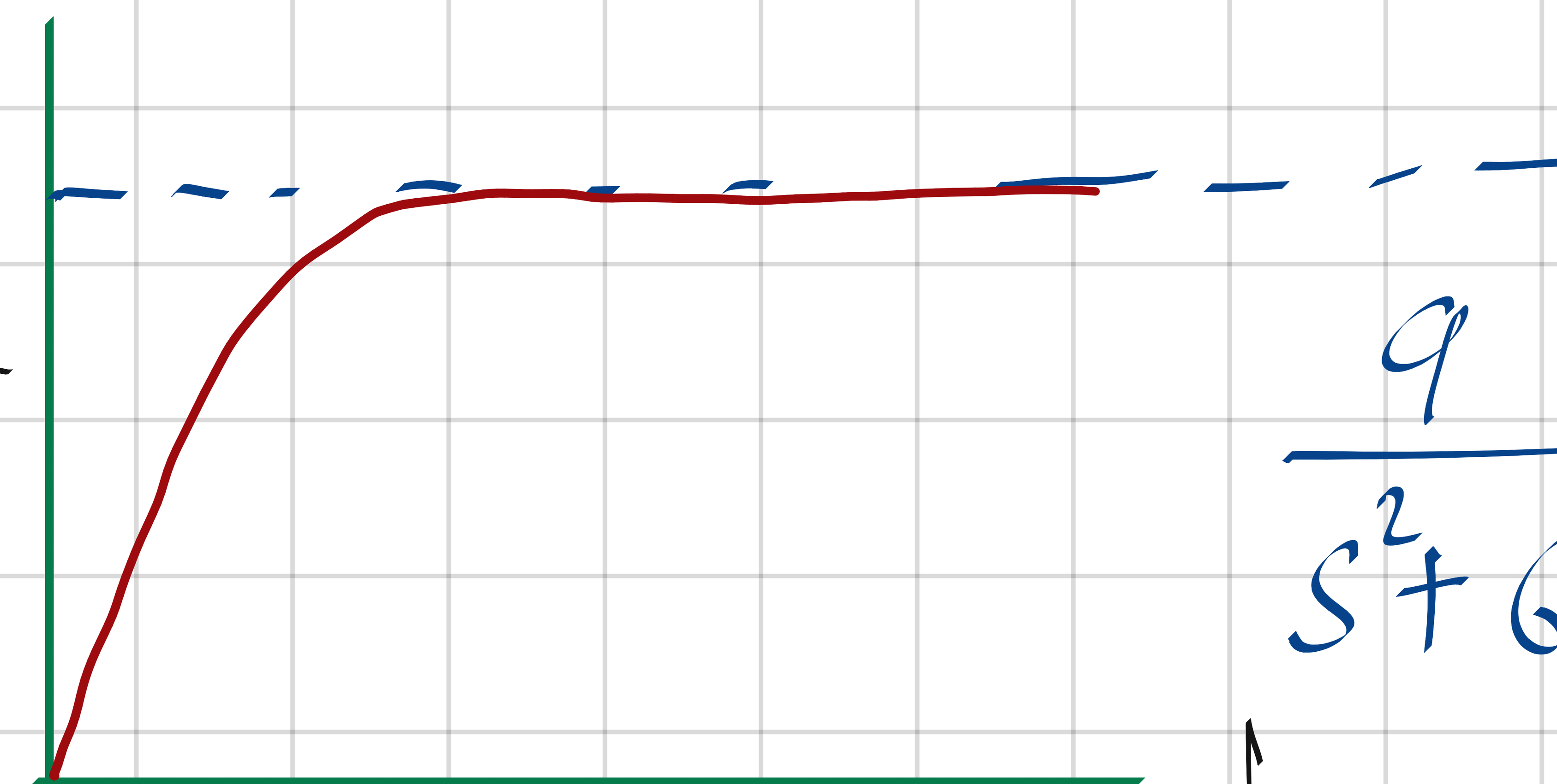
$$\frac{9}{s^2+9}$$



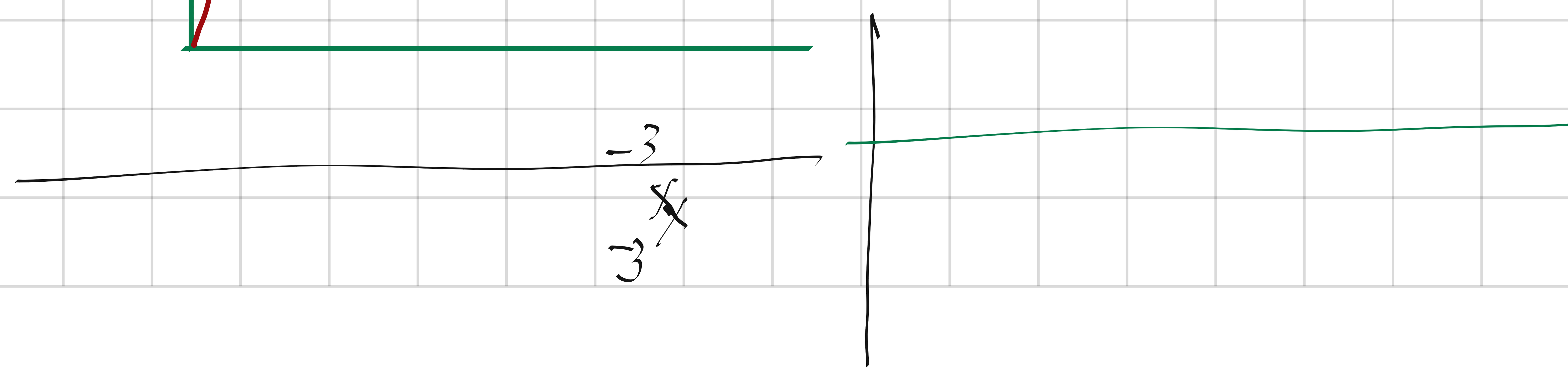
$$\frac{9}{s^2+2s+9}$$

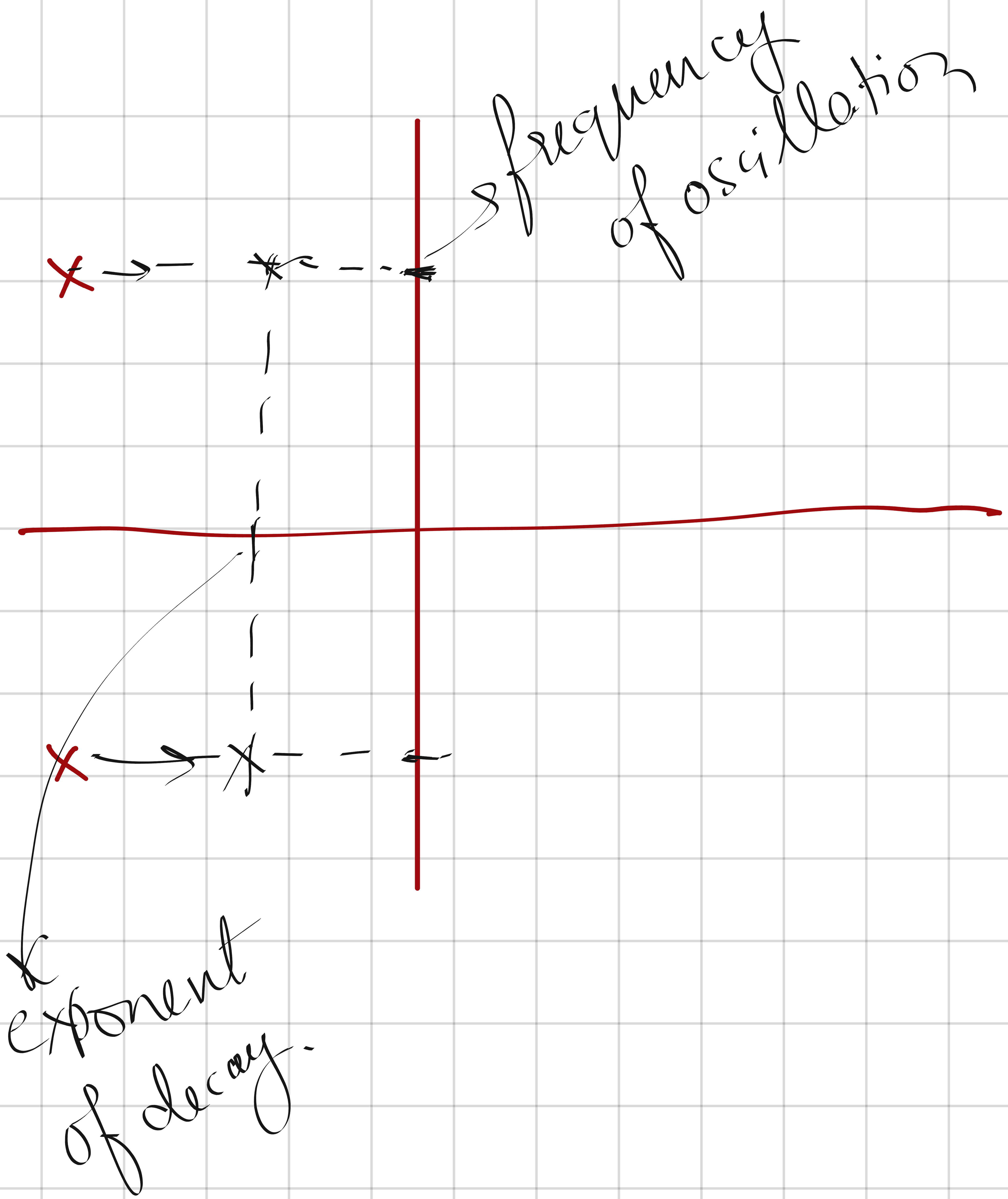


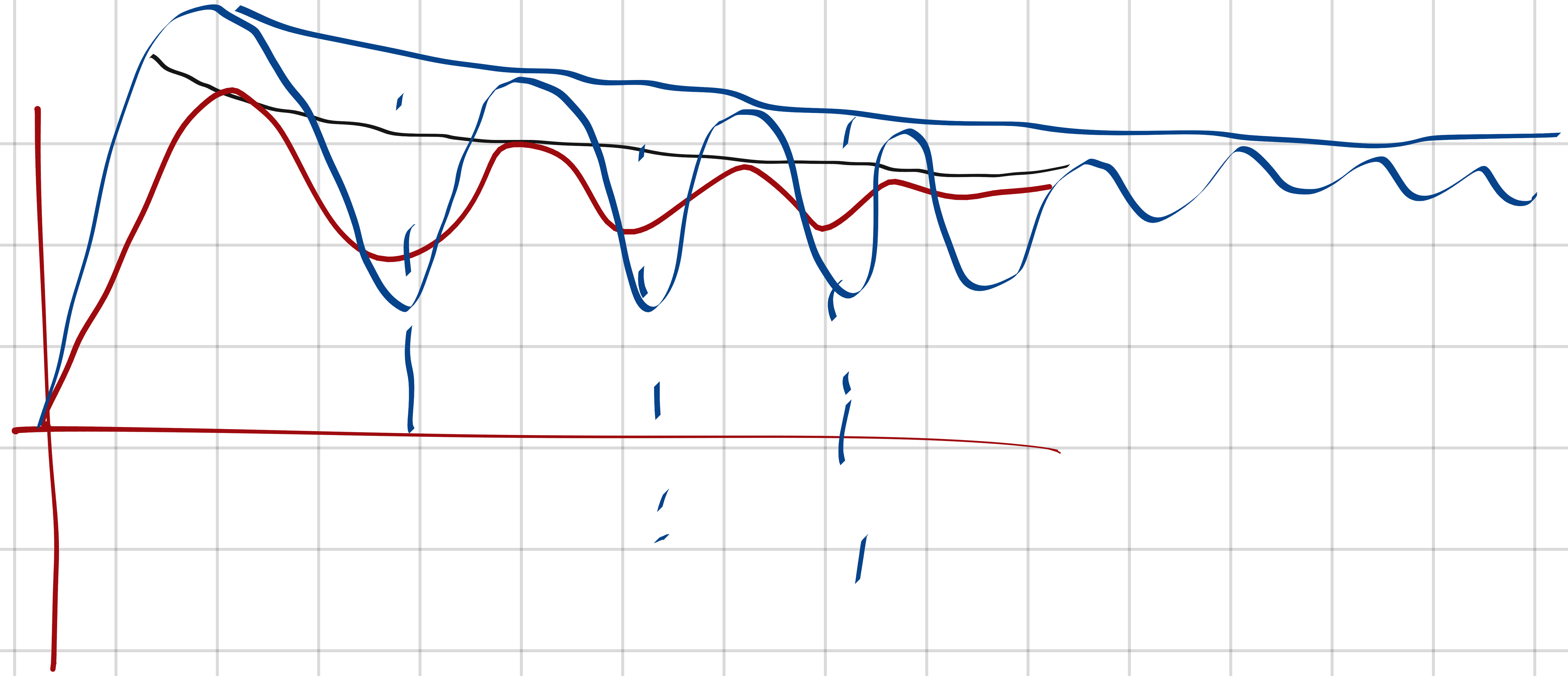




$$\frac{9}{s^2+6s+9}$$







$\omega_n \rightarrow$ natural frequency of system

$\xi \rightarrow$ damping ratio

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \rightarrow \text{Generalized } 2^{nd} \text{ order representation}$$

$\omega_n = 3$

$\xi =$

1.5 >1 → overdamped	0 → undamped.
0.33 → underdamped	$\boxed{1}$ → Critically damped system.

$\xi > 1$ No oscillation, Asymptotic convergence.

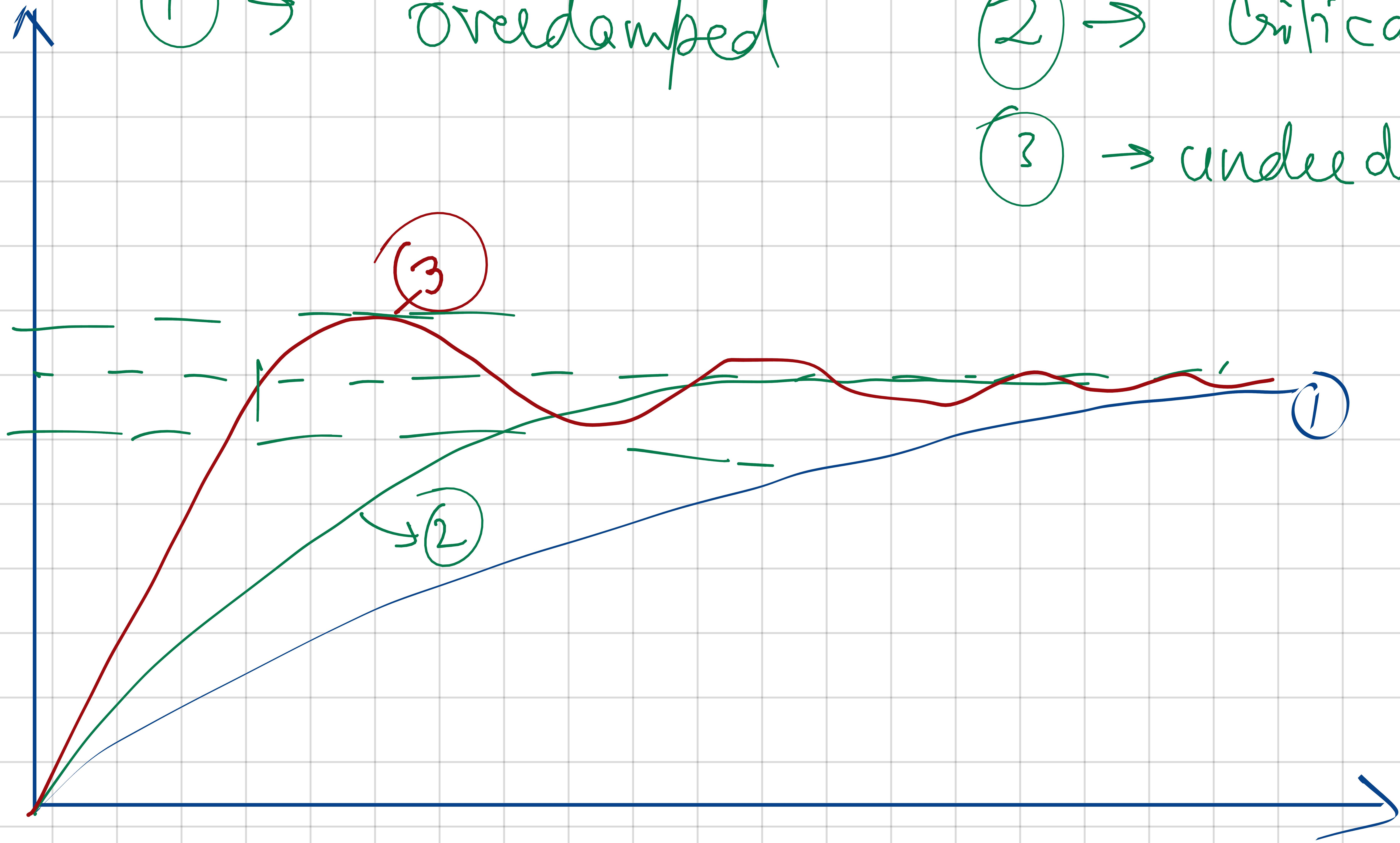
$\xi = 1$ No oscillation. Full convergence.

$\xi < 1$ Oscillations oscillating convergence.

1 → overdamped

2 → Critically

3 → underdamped.

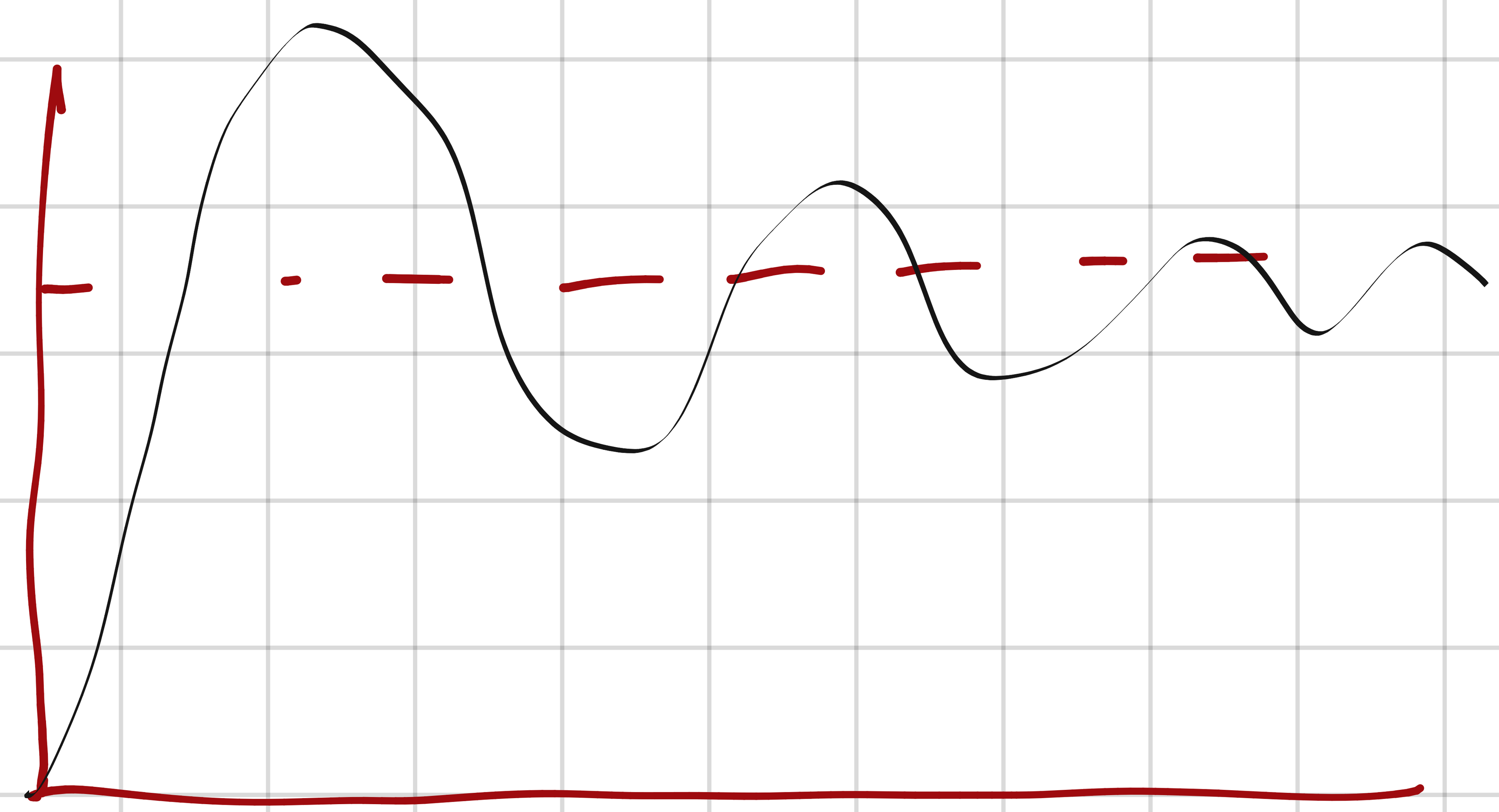


Underdamped response using poles:

By inspection, write the form of response of the following system to step input.

$$\frac{9}{(s^2+2s+9)}$$

$$s = -1 \pm 2.828i \ [\text{poles}]$$



$$c(t) = 1 + k_1 e^{-t}\left[k_2 \sin 2.828t + k_3 \cos 2.828t\right]$$

$$= 1 + k_4 e^{-t}\left[\cos(2.828t - \phi)\right]$$

$$\phi = \tan^{-1} k_2/k_3$$

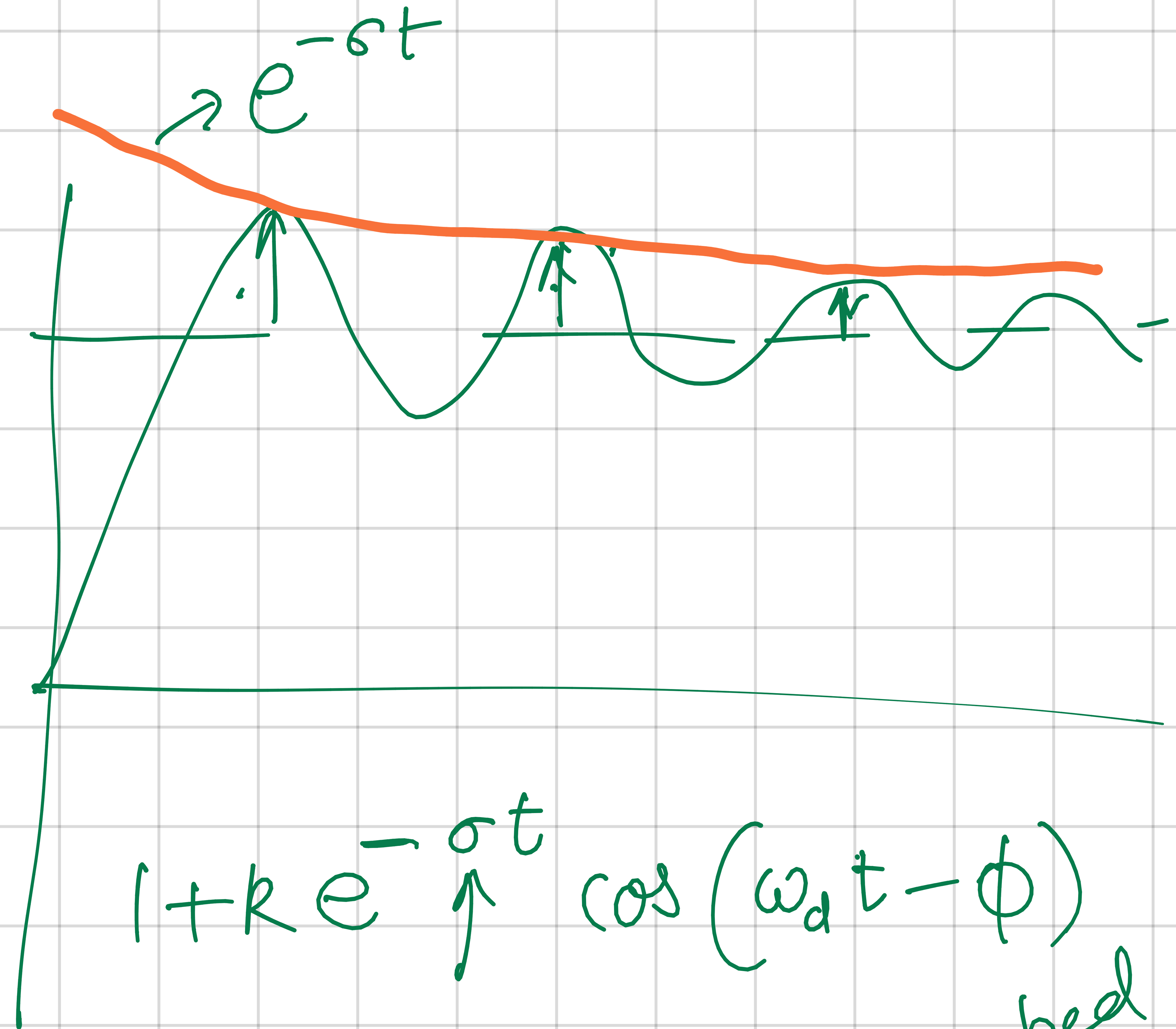
$$k_4 = k_1 \times \sqrt{k_2^2 + k_3^2}$$

ω_n = undamped frequency of oscillations

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ξ = $\dfrac{\text{exponential decay frequency}}{\text{natural frequency}}$

$= \dfrac{1}{2\pi} \dfrac{\text{time period of undamp oscillation}}{\text{Exponential time constant}}$



$1 + k e^{-\sigma t} \cos(\omega_d t - \phi)$

$s = -\sigma \pm i\,\omega_d$ → damped frequency of oscillation

1st order sys. → $\dfrac{a}{s+a} = \dfrac{1}{\frac{s}{\tau}+1}$

$1 - e^{-t/\tau}$

$= 1 - e^{-at}$ → τ → time constant of decay.

$\frac{1}{\tau} = a$ = exponential decay freq.

Underdamped Second Order System

Step response:

$\xi < 1$

$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\boxed{c(t) = 1 - \frac{1}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t} \cos\left(\omega_n \sqrt{1-\xi^2}\, t - \phi\right)}$$

$$1 - e^{-\xi\omega_n t}\left[\cos\left(\omega_n\sqrt{1-\xi^2}\, t\right) + \frac{\xi}{\sqrt{1-\xi^2}} \sin(\omega_d t)\right]$$

$$\phi = \tan^{-1} \frac{\xi}{\sqrt{1-\xi^2}}$$

Metrics:

Peak Time (T_p)

$$\dot{c}(t) = 0$$

$$\underbrace{\boxed{\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}}}_{\neq 0} \sin \omega_n \sqrt{1-\xi^2}\, t$$

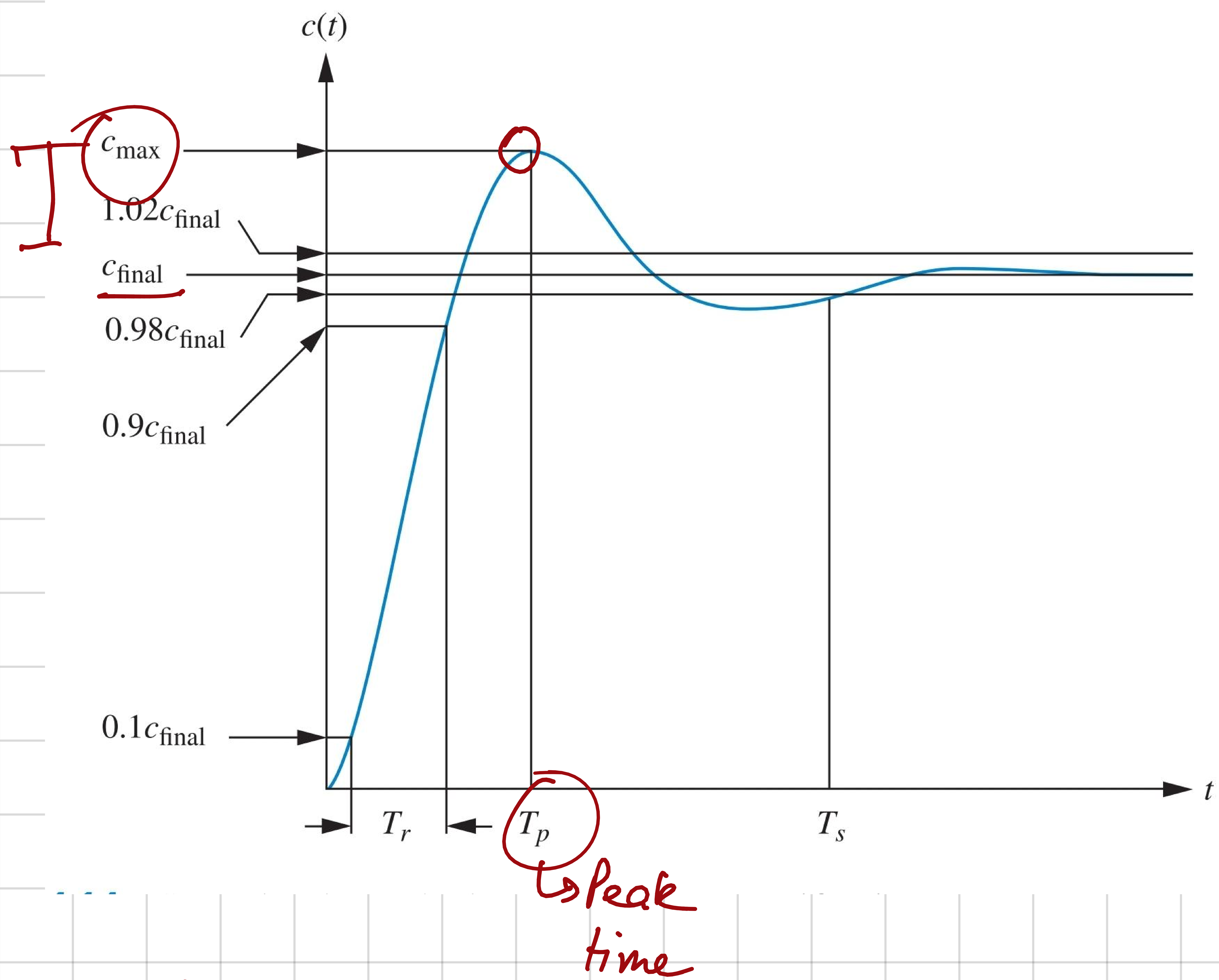
$$\Rightarrow \sin\left(\omega_n \sqrt{1-\xi^2}\, t\right) = 0$$

$$\omega_n \sqrt{1-\xi^2}\, t = n\pi$$

$$t = \frac{n\pi}{\omega_n \sqrt{1-\xi^2}}$$

t_p @ $n = 1$

i.e $t_p = \dfrac{\pi}{\omega_n \sqrt{1-\xi^2}} = \dfrac{\pi}{\omega_d} = f(\omega_n, \xi)$



Metrics:

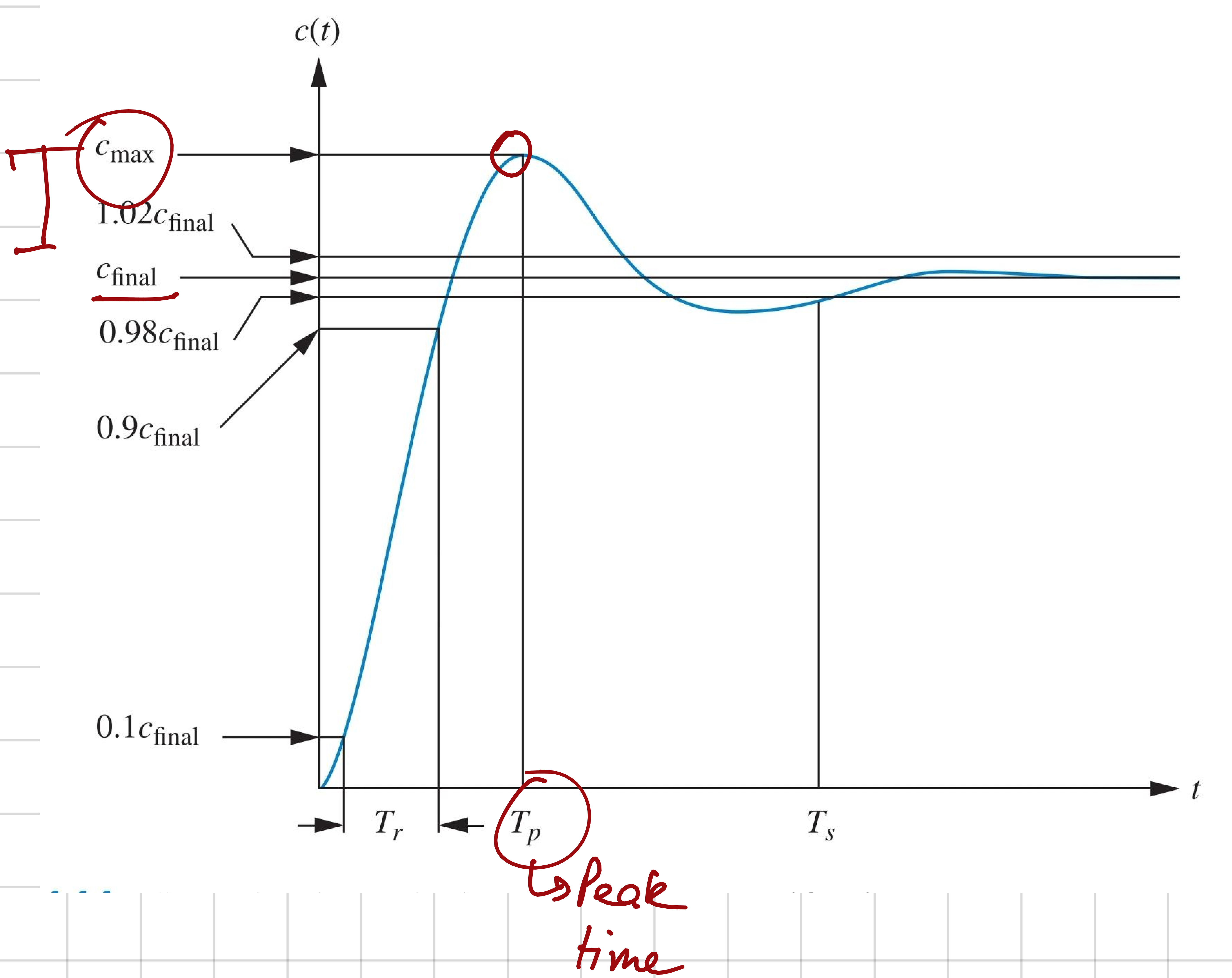
To find overshoot →

$$c(t_p) = e^{-\xi\pi/\sqrt{1-\xi^2}} + 1$$

$$\%OS = \frac{c(t_p) - c_{final}}{c_{final}} \times 100$$

$$\boxed{\%OS = 100 \cdot e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

↳ $f(\xi)$



$$\xi = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

→ To find the desired ξ for a specified OS

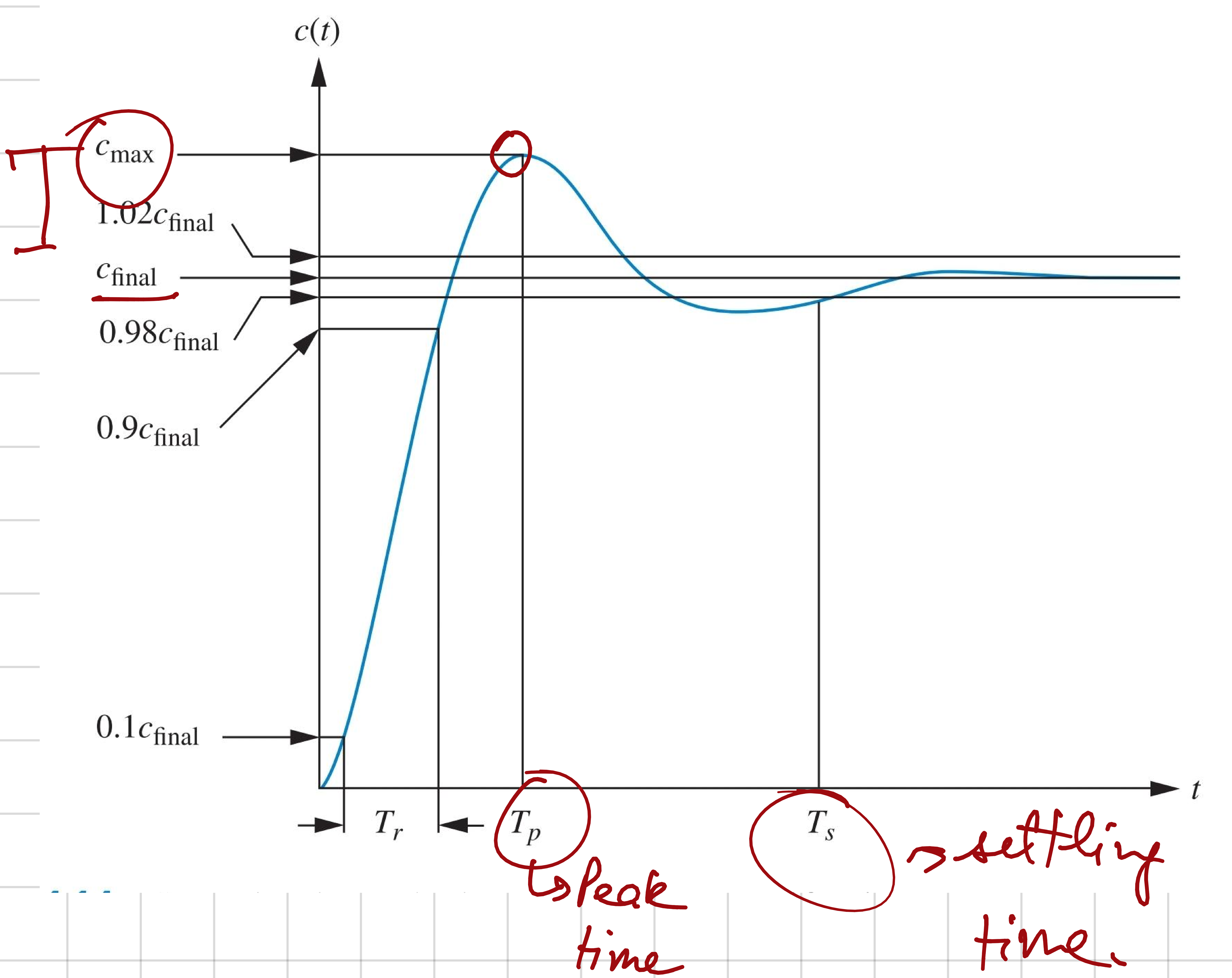
Metrics:

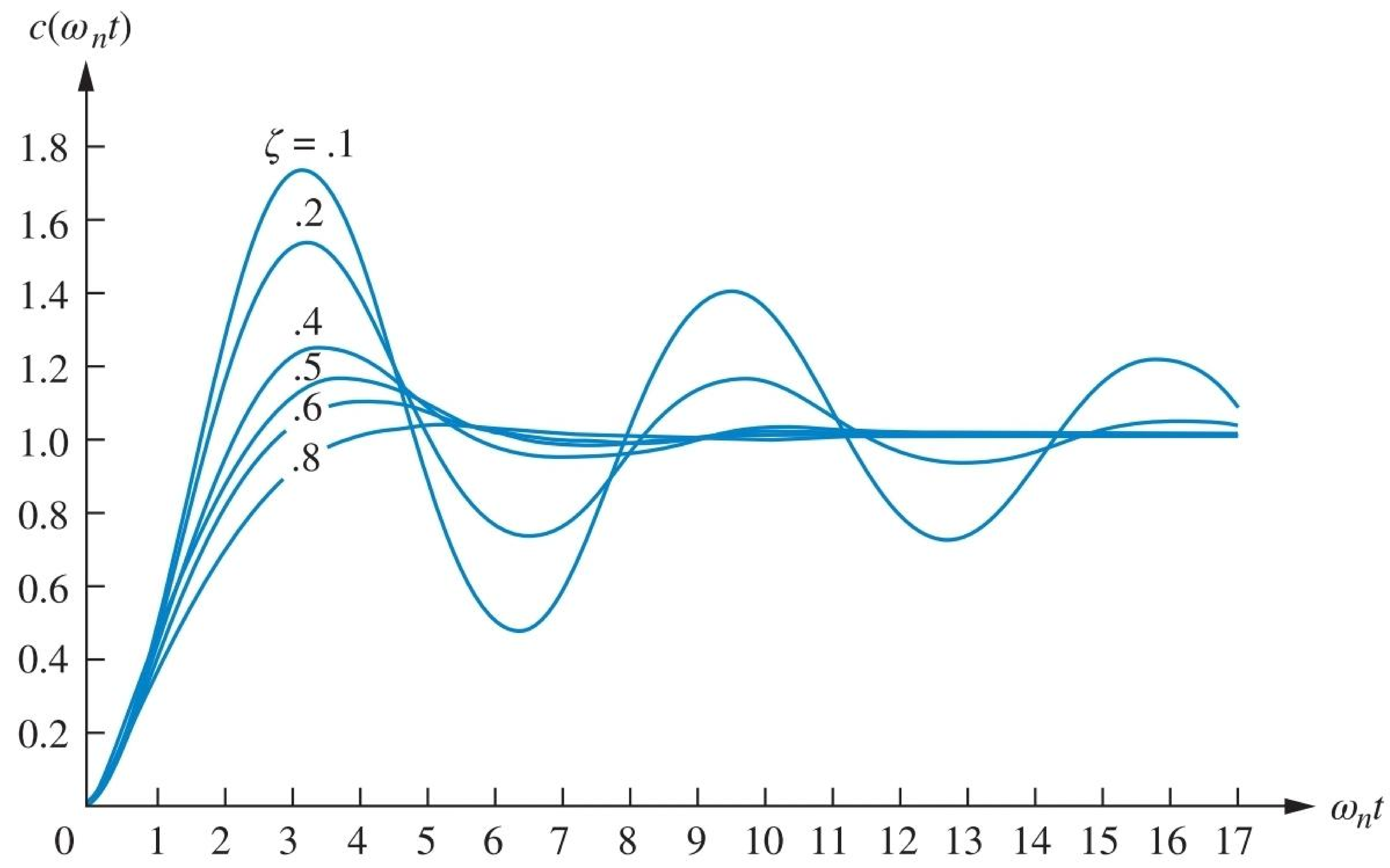
$$\boxed{\frac{e^{-\xi\omega_n t_s}}{\sqrt{1-\xi^2}}} = 0.02$$

approximation assuming

$$\cos\left(\omega_n\sqrt{1-\xi^2}\,t - \phi\right) \sim 1$$

$$T_s = \frac{4}{\xi\omega_n}$$

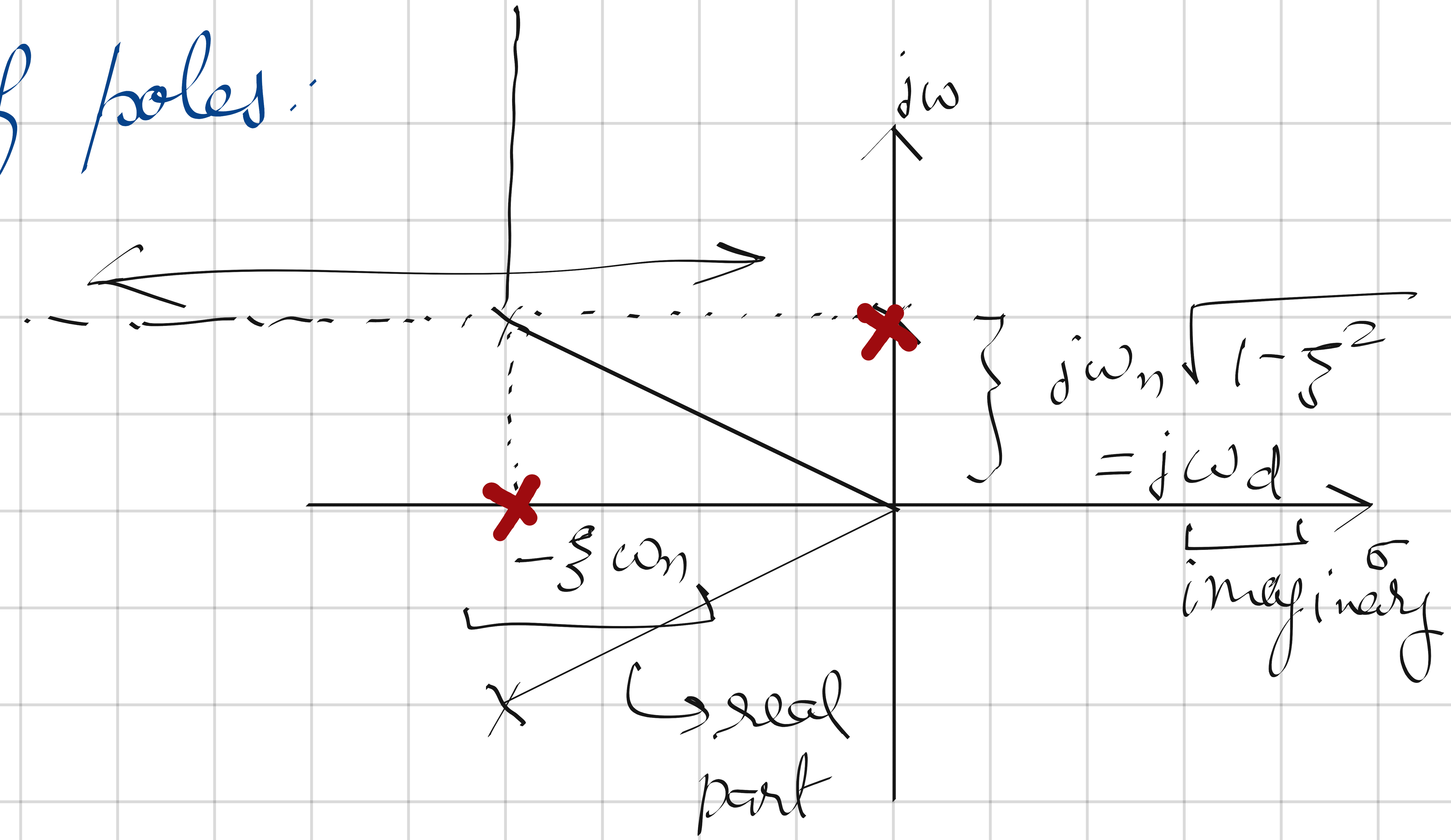




c(ωnt)
ζ = .1
.2
.4
.5
.6
.8
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
ωnt

Graphical Understanding of effect of poles:

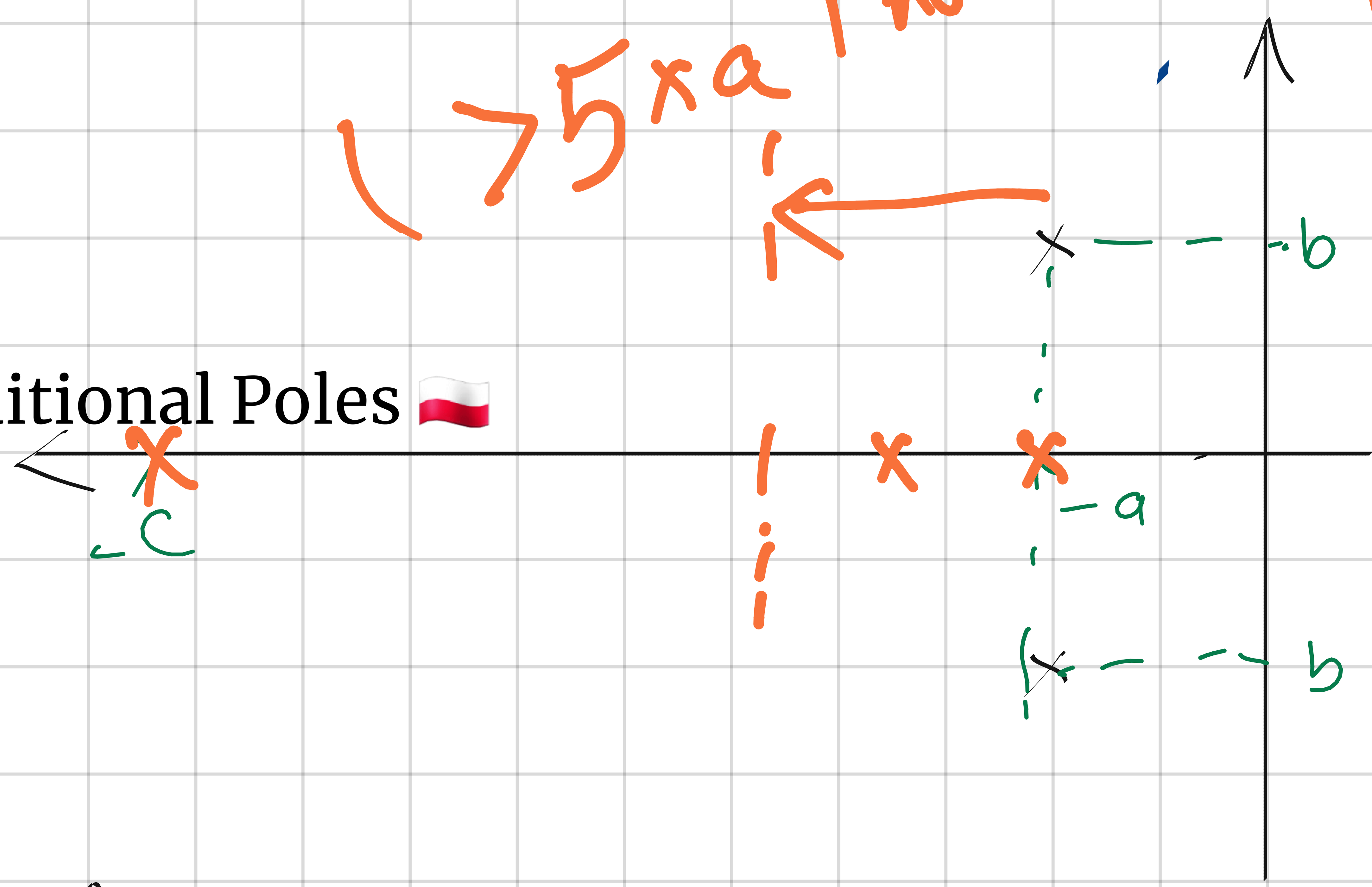
$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



II
line of const. ξ i.e. OS
const slope
Line of const. decay.
jω
line of const. frequency.
θ
σ

System response with additional Poles 🇵🇱

$> 5 \times a$ | non dominant pole.

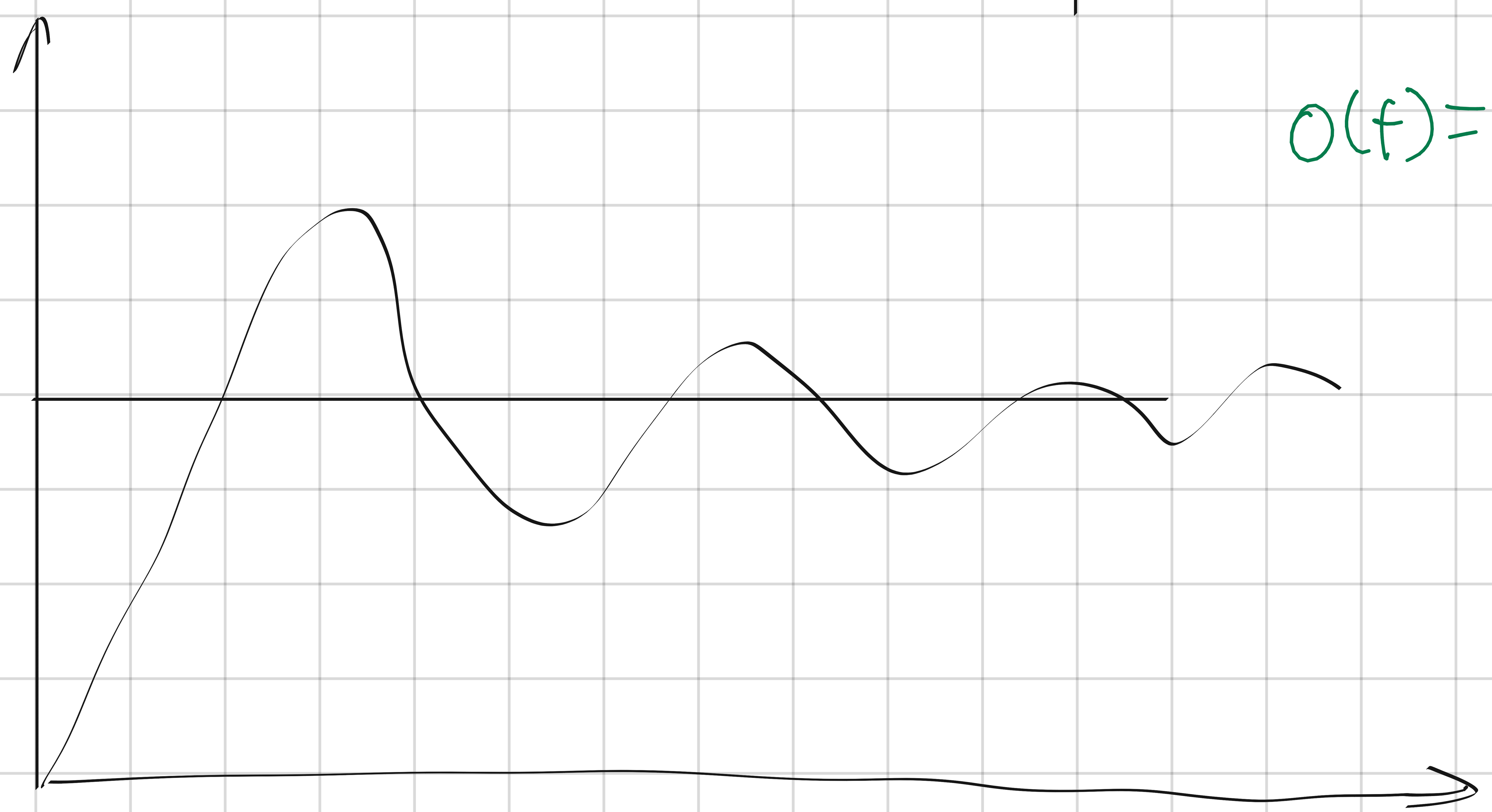


System response with additional Poles 🇵🇱

$$\frac{K_1}{(s-(-a+jb))(s-(-a-jb))}$$

$$\frac{K_2}{(s+c)(s-(-a+jb))(s-(-a-jb))}$$

$$c(t) = 1 - \beta e^{-ct} - e^{-R_2 t}\cos(\omega_d t - \phi)$$



System response with zeros

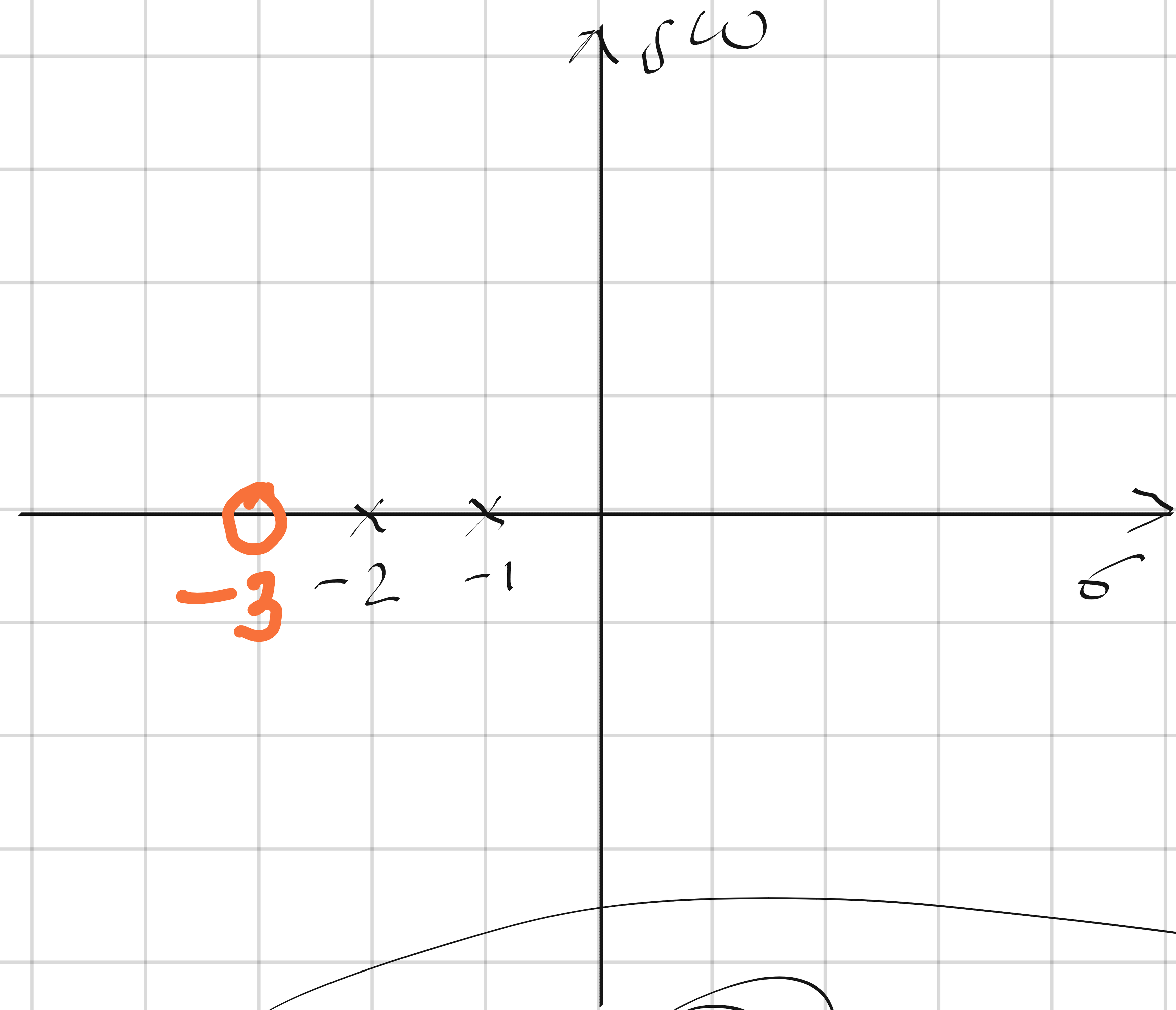
$$G(s) = \frac{1}{(s+1)(s+2)}$$

$$\frac{1}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

unit step response →

$$c(t) = 1 - k_1 e^{-t} - k_2 e^{-2t}$$

$$= 0.5 - e^{-t} + \frac{1}{2} e^{-2t}$$



New system

$$G_2(s) = \frac{s+3}{(s+1)(s+2)}$$

response to unit step I/p. →

$$1.5 - 2e^{-t} + 0.5e^{-2t}$$

$$G_2(s) = 3\left[\frac{1}{s+1} - \frac{1}{2}\frac{1}{(s+2)}\right]$$

$$G(s) = \frac{9}{s^2 + 2s + 9}$$

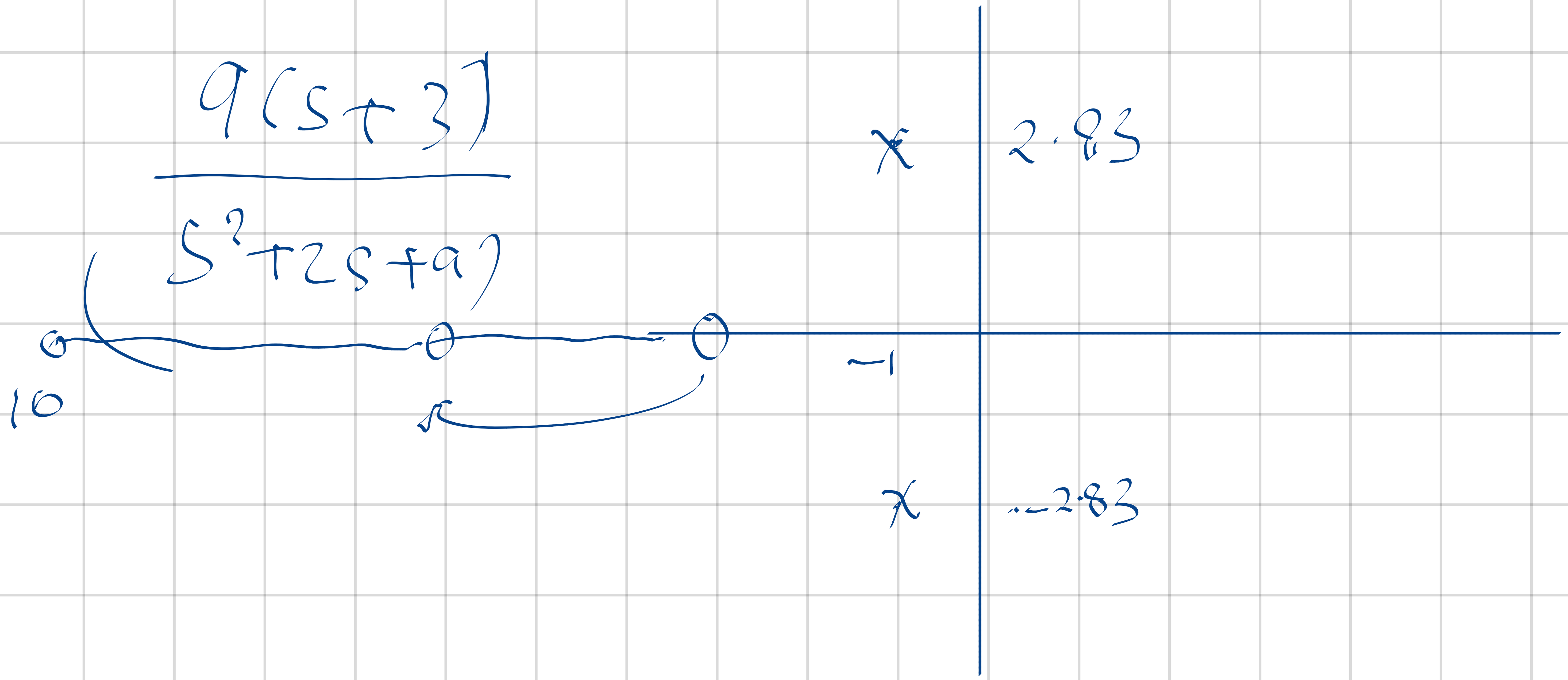
$$(s+a)$$

$$(s+a)\,G(s) = G_2(s) = \frac{9(s+a)}{(s^2+2s+9)} = s\,G(s) + a\,G(s)$$

$s\,G(s)$ → 1st order derivative

$a\,G(s)$ → Gain x G(s) → maintains original dynamics (scaled)

$$\frac{9(s+3)}{(s^2+2s+9)}$$

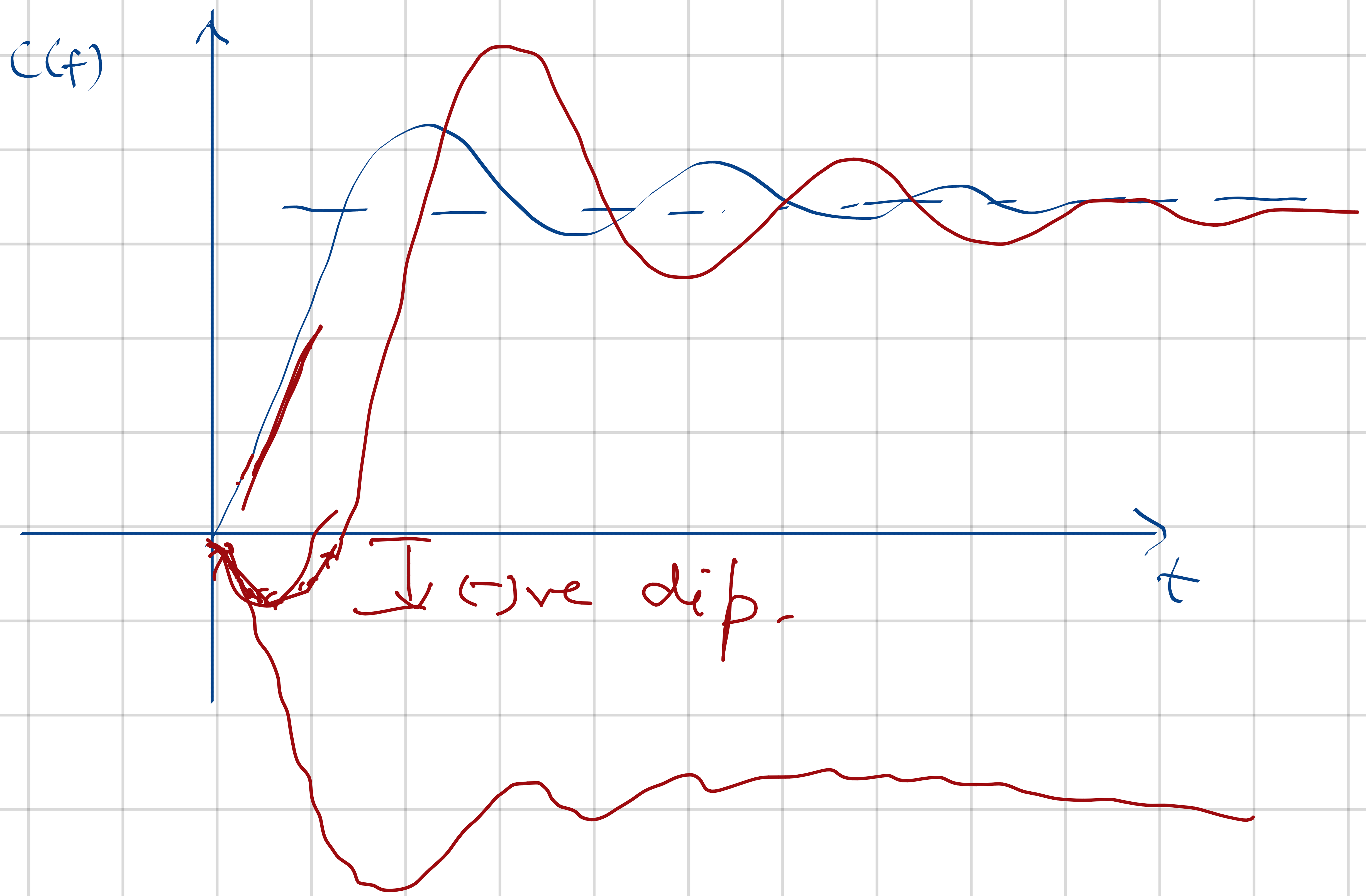


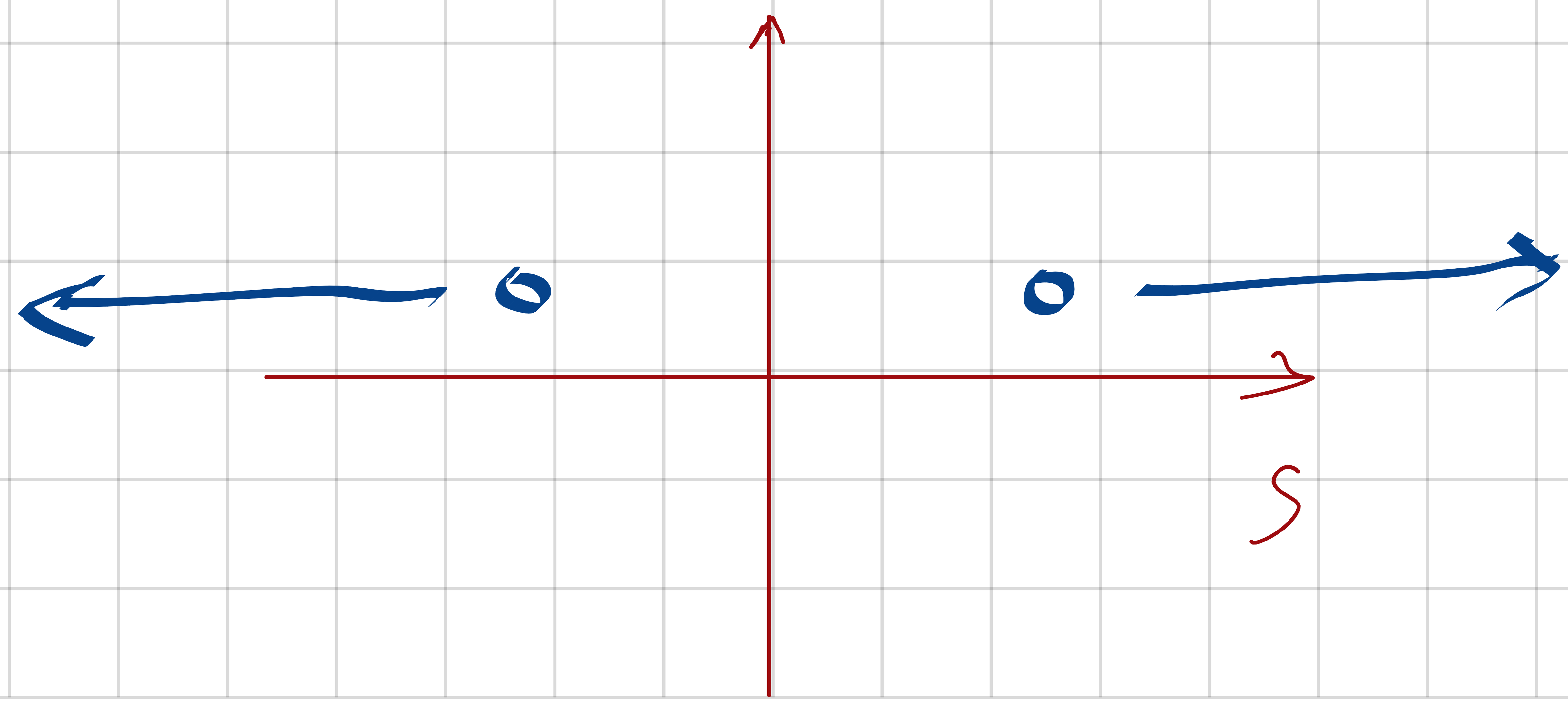
$sG(s) = a\,G(s)$

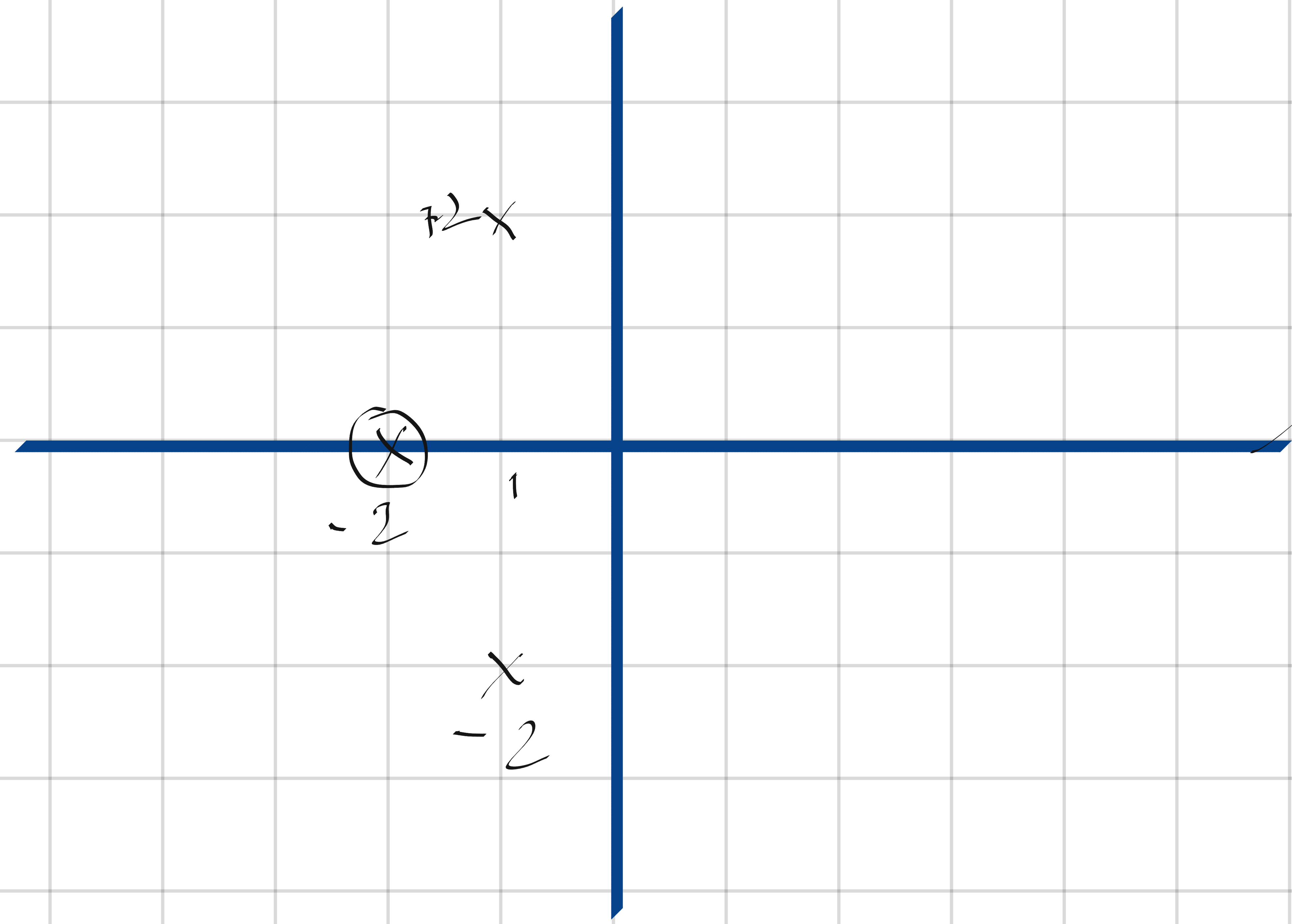
a → the number
Zero is in RHP.

$sG(s) + b\,G(s)$

4 -3





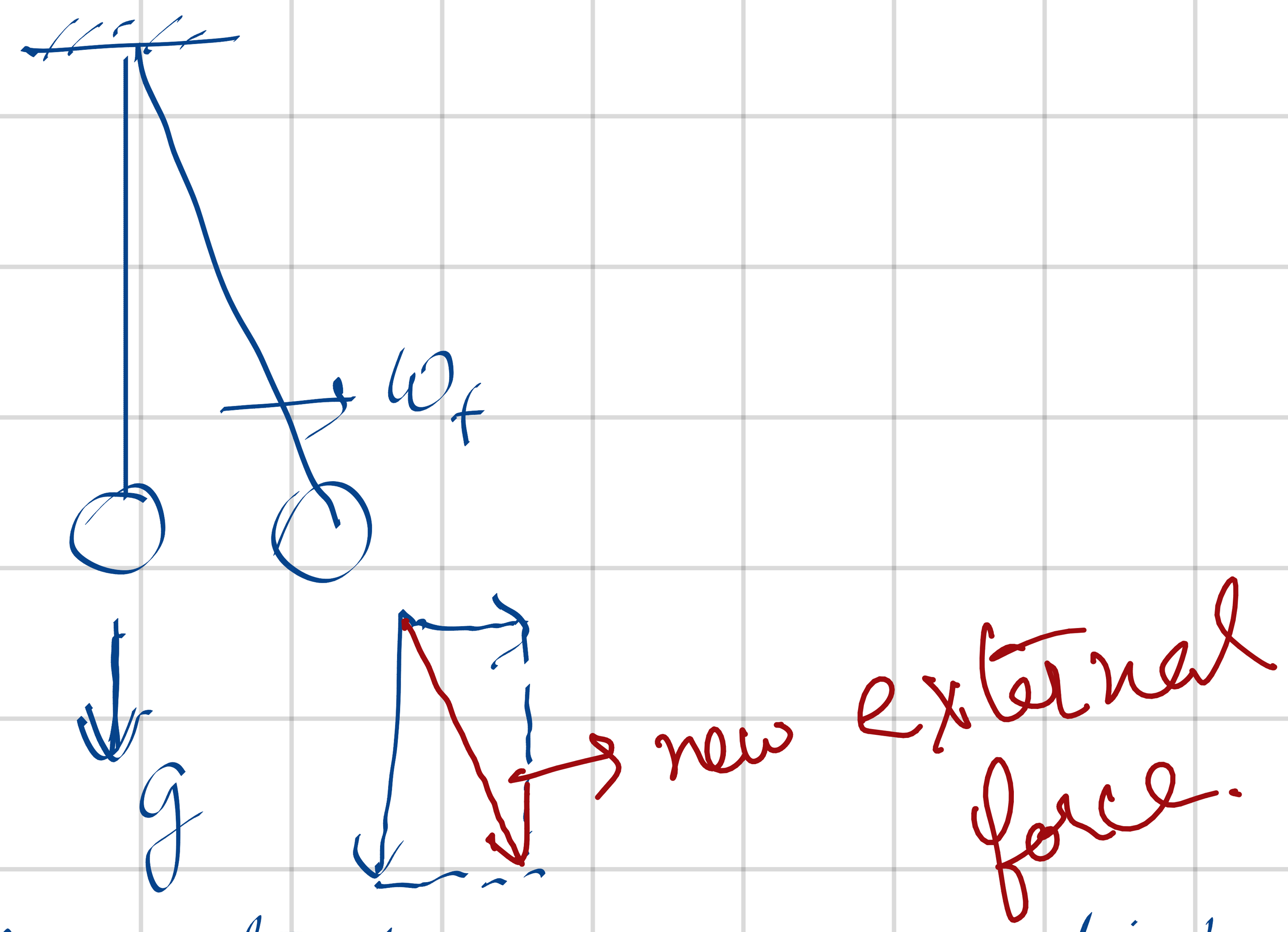


$$\frac{(s+1.9)}{(s+2)(s^2+2s+5)}$$

1

Stability

Criterions for stability.



System tends to a finite value.

System returns to its stable position when subjected to small disturbance.

Error wrt target $\longrightarrow 0$ as $t \to \infty$

$\longrightarrow$ finite value

denominator $a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} \dots\dots + a_0$

Routh-Hurwitz Criterion

$$\frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

Routh table to identify how many poles of the sys' are in the RHP without calculating the roots.

$a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$

$$
\begin{array}{llll}
s^4 & a_4 & a_2 & a_0 \\
s^3 & a_3 & a_1 & 0 \\
s^2 & \dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1 & \dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2 & \dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0 \\
s^1 & \dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1 & \dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0 & \\
s^0 & \dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1 & &
\end{array}
$$

$$
\begin{array}{ll}
a_3 & a_1 \\
b_1 & b_2
\end{array}
$$

$$
\frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1}
$$

Column 1

a_4

a_3

b_1

c_1

d_1

poles in RHP

= # sign changes in column 1

eg.

1

3

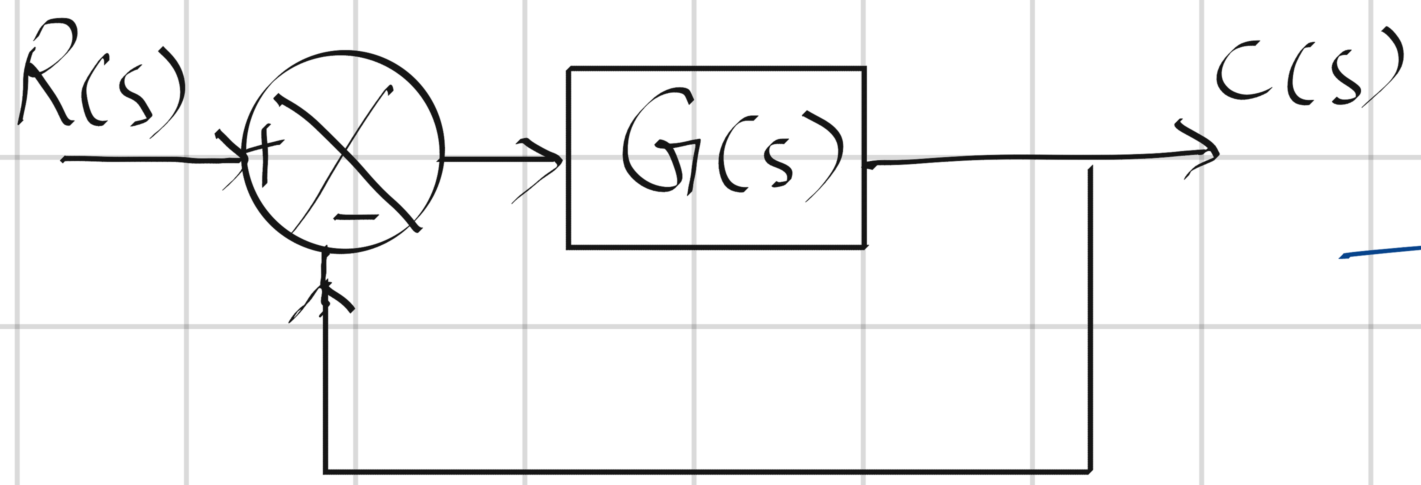
2

1

2

2 poles in RHP

$$G(s) = \frac{1000}{(s+2)(s+3)(s+5)}$$



$$\rightarrow \frac{G(s)}{1+G(s)}$$

$$\frac{1000}{(s+2)(s+3)(s+5)+1000}$$

$$= \frac{1000}{s^3+10s^2+31s+1030}$$

Roots $\rightarrow$ -13.413

$1.7068 \pm 8.595i$

$a_3 = 1$

$a_2 = 10$

$a_1 = 31$

$a_0 = 1030$

s^3	1	31	0
s^2	10	+1030	0
s^1	−72	−0	0
s^0	1030	+0	0

$$s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

s^5	1	3	5	0
s^4	2	6	3	0
s^3	$\cancel{0}\ \varepsilon$	3.5	0	0
s^2	$\dfrac{6\varepsilon - 7}{\varepsilon}$	3	0	0
s^1	$\dfrac{42\varepsilon - 49 - 6\varepsilon^2}{12\varepsilon - 14}$			
s^0	3			

$\varepsilon \to$ infinitesimally small number

	$\varepsilon \to (+)$ve	$\varepsilon \to (-)$ve
s^5	+	+
s^4	+	+
s^3	+	−
s^2	−	+
s^1	+	+
s^0	+	+

← 2 sign changes →