$$A\left[\begin{array}{ccc:ccc} \mathbf{v}_1 & \cdots & \mathbf{v}_r & \mathbf{v}_{r+1} & \cdots & \mathbf{v}_n \end{array}\right] = \left[\begin{array}{ccc:ccc} \mathbf{u}_1 & \cdots & \mathbf{u}_r & \mathbf{u}_{r+1} & \cdots & \mathbf{u}_m \end{array}\right]\left[\begin{array}{ccc:c} \sigma_1 & & & \\ & \ddots & & \mathbf{0} \\ & & \sigma_r & \\ \hdashline & \mathbf{0} & & \mathbf{0} \end{array}\right]$$