Interferometer for small-angle measurement based on total internal reflection

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We describe a new method for angle measurement based on the internal-reflection effect and heterodyne interferometry. A novel prism assembly is designed that can always parallel retroreflect the incoming light beams so the optical configuration is compact. As a differential common-path optical configuration is integrated into the design, the linearity of the method is greatly improved. Details of theoretical analysis of the method and experimental verification of the principle are presented. The resolution can be better than 0.3 arc sec. The experimental results and further improvements of the proposed method are also addressed. © 1998 Optical Society of America

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1. Introduction

There is great demand for small-angle measurements in the alignment, assembly, and calibration of machine tools and for the measurements of straightness, flatness, squareness, and parallelism. Many devices have been developed for measuring small angles.[1–16] Among them, angle interferometers, autocollimators, and levels are most widely used. With its high accuracy, ability to measure large distances, and ease of use, the angle interferometer has become a useful accessory of laser interferometers and is utilized widely today.[10,11] However, angle interferometers have some drawbacks. First, the angle interferometer is based on the sine principle, so there is a sinusoidal error in the measurement, especially when the angle to be measured is large. Second, because the resolution of an angle interferometer depends on the length of the sine arm, it is difficult to reduce the size of the system to achieve satisfactory resolution. Third, the accuracy of the angle interferometer is determined by the calibration and stability of the sine arm. Finally, as more cube prisms are used in angle interferometer accessories, the system becomes more expensive and difficult to manufacture.

In recent years an angle-measurement method that uses total internal reflection has attracted attention. Huang *et al.*[12] and Huang and Ni[13,14] proposed an angle-measurement method based on internal reflection. In this method a light beam is divided into two equal-intensity beams. The two beams are incident upon two separate prisms and are reflected in the vicinity of the critical angle. One can then estimate the small rotation angle of the two prisms by measuring the reflectance difference between the beams reflected by the two prisms. Multireflection within prisms can be used to increase the sensitivity for the angle measured. The method is compact in size, high in resolution, and low in cost, but it is also highly sensitive to variations in the intensity of the light source and to stray light because the angle measurement is directly proportional to the intensity of the reflected light. The measurement range is rather small ($\sim$1.6°). Chiu and Su proposed using heterodyne interferometry.[15,16] The rotary angle in this case is measured based on the phase difference between the parallel and the perpendicular polarization states of beams reflected from prisms at total internal reflection. The method is independent of the variation in light intensity and has better resistence to turbulence. However, in Chiu's optical configuration the incident and exit beams from the right-angle prism are perpendicular to each other, which makes the measurement system bulky. In addition, the direction of reflected beams from the right-angle prism is not constant during the measurement. When the right-angle prism rotates an angle of θ, the rotation angle of the exit beams is 2θ, which affects phase measurement and makes the re-

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ception of light beams difficult, especially if the measured angle is large. Furthermore, because the initial phase difference between the parallel and the perpendicular polarization states of the beams is unknown, the angle cannot be measured directly. No detail analyses were provided in Refs. 15 and 16.

We have designed a novel prism assembly that can always return the incoming light beams parallel so the optical configuration is a differential common path. This makes the measurement equipment compact. As the differential optical configuration is integrated into the design, the linearity is greatly improved and angle measurement becomes insensitive to variations in the environment. Thus, by utilizing the proposed prism assembly, we overcame the problems mentioned above and made the heterodyne interferometry method for angle measurement feasible and easy to use. Both theoretical analysis and experimental verification were carried out for this method. The results are presented in Section 3. The resolution of angle measurement can be better than 0.3 arc sec. The advantages of the method are compactness, high resolution, large measurement range, low cost, and ease of integration with commercially available or specially designed interferometers.

2. Principle of Operation

When a ray of light propagates from an optically denser medium into one that is optically less dense, and the incident angle is greater than the critical angle, all incident light will be reflected back into the first medium. This is called the total-reflection effect. Fresnel's principle[17] indicates that the phase difference of total-reflection light between parallel and perpendicular polarization states is a function of the incident angle. The total-reflection effect can be used to measure rotation angle. However, the direction of the reflected light is not constant during measurement. This makes the reception of light difficult, especially if the measurement angle is large. If there is a retroreflector that can return incident light parallel and does not affect the relationship between the phase difference and the incident angle, it is possible to measure the rotation angle based on the total-reflection effect. This fact was the motivation for the research reported in this paper.

To achieve this goal we designed a novel prism assembly that can always retroreflect the incoming light beams parallel. The principle of the prism assembly is shown schematically in Fig. 1. The prism assembly comprises a $\lambda/2$ plate and two right-angle prisms, which are parallel to each other; the $\lambda/2$ plate is placed between them. The fast or slow axis direction of the $\lambda/2$ plate is adjusted at 45° with respect to the polarization direction of the incident light.

We assume that two orthogonally linearly polarized laser beams from a stabilized laser are incident normally upon one surface of the first right-angle prism. Frequency f_1 is the parallel polarization component p, and frequency f_2 is the perpendicular polarization component s. The beams will be incident at $\theta_{i1} = \pi/4$ upon the hypotenuse and will be totally reflected by it and emerge from the first prism on another side. The beams pass through the $\lambda/2$ plate, and the polarization state is then changed. Frequency f_1 becomes the perpendicular polarization component s and frequency f_2 becomes the parallel polarization component p. They are incident normally on one surface of the second right-angle prism, too. The beams are incident at $\theta_{i2} = \pi/4$ upon the hypotenuse and emerge from the second prism from another side. The beams that exit from the prism assembly are now parallel with the incoming beams. Therefore the optical configuration becomes compact and can be measured. The prism assembly is placed upon a rotary table under test. When the table rotates an angle of θ, the angles at which the beams are incident upon the hypotenuses of the two right-angle prisms are

$$\theta_{i1} = \pi/4 + \sin^{-1}\left(\frac{\sin\theta}{n}\right), \tag{1}$$

$$\theta_{i2} = \pi/4 - \sin^{-1}\left(\frac{\sin\theta}{n}\right), \tag{2}$$

where n is the refractive index of the right-angle prisms, θ_{i1} is the incident angle upon the hypotenuse of the first right-angle prism, and θ_{i2} is the incident angle upon the hypotenuse of the second right-angle prism.

According to Fresnel's equation,[17] after the laser beams pass through the prism assembly the phase difference between frequency components f_1 and f_2 can be expressed as

$$\begin{aligned}\varphi = \varphi_0 &+ 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i1} - \frac{1}{n^2}\right)^{1/2}}{\frac{1}{n^2}\cos\theta_{i1}}\right] \\ &- 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i1} - \frac{1}{n^2}\right)^{1/2}}{\cos\theta_{i1}}\right] \\ &- 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i2} - \frac{1}{n^2}\right)^{1/2}}{\frac{1}{n^2}\cos\theta_{i2}}\right] \\ &+ 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i2} - \frac{1}{n^2}\right)^{1/2}}{\cos\theta_{i2}}\right] + \Delta\varphi,\end{aligned} \tag{3}$$

where φ_0 is the initial phase difference between f_1 and f_2, the magnitude of φ_0 depends on the distance that the beams travel before they arrive at the receiver and on the frequency difference between f_1 and f_2, and $\Delta\varphi$ is the phase change caused by the rotation of the prism assembly. We can estimate $\Delta\varphi$ by considering the rotation of the equivalent glass plate of

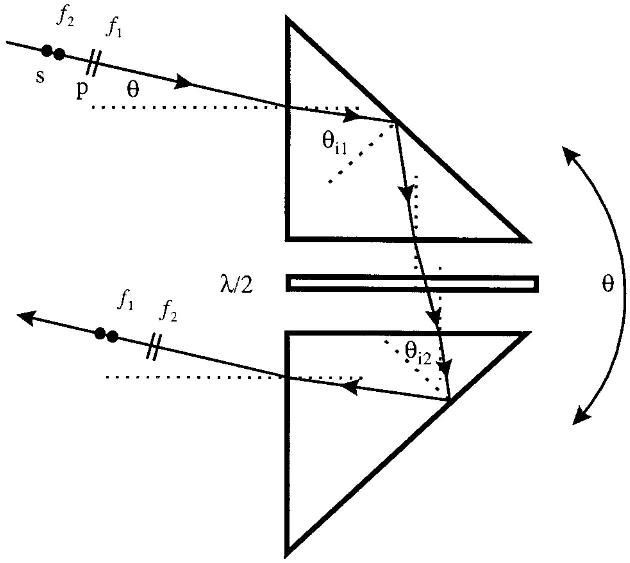

Fig. 1. Schematic of the prism assembly.

the prism assembly. The optical path difference (OPD) introduced by the rotation of a glass plate is a function of the rotation angle, as shown in Fig. 2. Accordingly, we have

$$\mathrm{OPD} = n\overline{AC} - n\overline{AB} - \overline{BE}, \tag{4}$$

where

$$\overline{AB} = t,$$
$$\overline{AC} = t\Big/\left(1 - \frac{1}{n^2}\sin^2\theta\right)^{1/2},$$
$$\overline{BE} = \overline{AC}\cos(\theta - \theta') - t,$$
$$\sin\theta = n\sin\theta'.$$

Substituting the above equations into Eq. (4), we have

$$\mathrm{OPD}(\theta) = \frac{n^2 t - t\cos\theta(n^2 - \sin^2\theta)^{1/2} - t\sin^2\theta}{(n^2 - \sin^2\theta)^{1/2}} - (n-1)t, \tag{5}$$

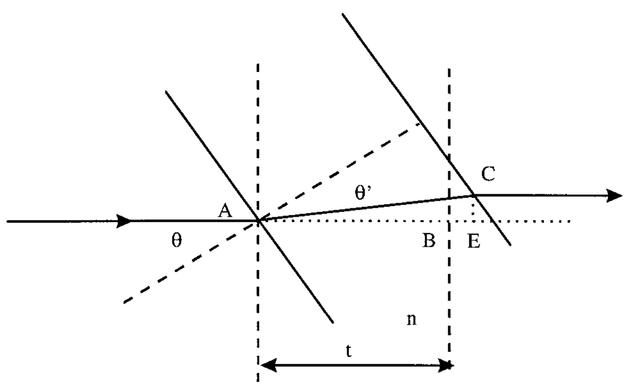

Fig. 2. Optical path difference introduced by the rotation of a glass plate.

where t is the thickness of the equivalent glass plate of the prism assembly and θ is the incident angle upon the equivalent glass plate, which is equal to the rotation angle of the prism assembly. Therefore the phase change introduced by the rotation of the prism assembly can be given by

$$\begin{aligned}\Delta\varphi &= \frac{2\pi}{\lambda}\frac{\Delta f}{f}\mathrm{OPD}(\theta)\\ &= \left[\frac{n^2 t - t\cos\theta(n^2 - \sin^2\theta)^{1/2} - t\sin^2\theta}{(n^2 - \sin^2\theta)^{1/2}} - (n-1)t\right]\frac{\Delta f}{f}\frac{2\pi}{\lambda},\end{aligned} \tag{6}$$

where f is the mean frequency of the laser, $f = (f_1 + f_2)/2$, Δf is the frequency difference between f_1 and f_2, and λ is the mean wave length of laser, $\lambda = c/f$. From Eq. (6) we see that Δφ is proportional to the frequency difference Δf and the thickness of the equivalent glass plate of the prism assembly t. Substituting Eq. (6) into Eq. (3), we get

$$\begin{aligned}\varphi = \varphi_0 &+ 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i1} - \frac{1}{n^2}\right)^{1/2}}{\frac{1}{n^2}\cos\theta_{i1}}\right]\\ &- 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i1} - \frac{1}{n^2}\right)^{1/2}}{\cos\theta_{i1}}\right]\\ &- 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i2} - \frac{1}{n^2}\right)^{1/2}}{\frac{1}{n^2}\cos\theta_{i2}}\right]\\ &+ 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i2} - \frac{1}{n^2}\right)^{1/2}}{\cos\theta_{i2}}\right]\\ &+ \left[\frac{n^2 t - t\cos\theta(n^2 - \sin^2\theta)^{1/2} - t\sin^2\theta}{(n^2 - \sin^2\theta)^{1/2}} - (n-1)t\right]\frac{\Delta f}{f}\frac{2\pi}{\lambda}.\end{aligned} \tag{7}$$

In Eq. (7) the first term, φ_0, is the initial phase difference; it cannot be determined exactly unless the frequency difference between f_1 and f_2 is zero. The other terms are the functions of θ and n. If the frequency difference between f_1 and f_2 is zero, the first and last terms are zero; thus one can directly determine the rotation angle by measuring the phase difference. In this case an absolute angle measurement can be performed. If the frequency difference between f_1 and f_2 is not zero, a measurement equation with satisfactory linearity is required for relative

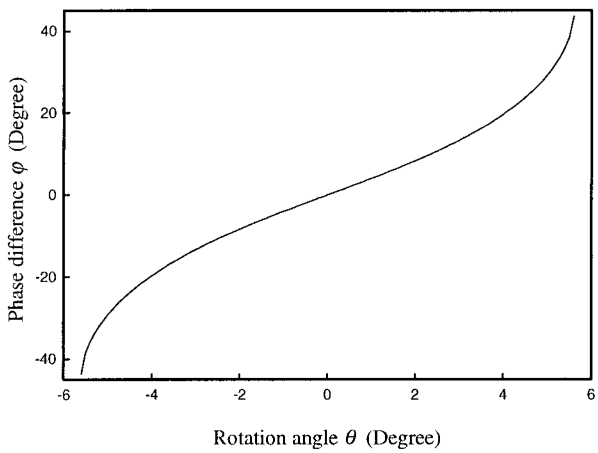

Fig. 3. Theoretical curve of φ versus θ.

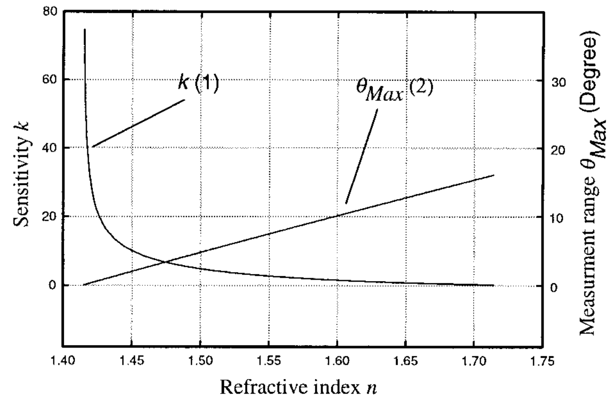

Fig. 4. Curves of sensitivity k and measurement range θ_{Max} versus refractive index n.

measurement of the angle. Therefore angle measurements based on Eq. (7) can be divided into two categories: absolute and relative measurements. The details are presented below.

A. Absolute Measurement

As we mentioned above, when the frequency difference between f_1 and f_2 is zero the first and last terms in Eq. (7) are zero. Therefore Eq. (7) becomes

$$\varphi = 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i1} - \frac{1}{n^2}\right)^{1/2}}{\frac{1}{n^2}\cos\theta_{i1}}\right] - 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i1} - \frac{1}{n^2}\right)^{1/2}}{\cos\theta_{i1}}\right] - 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i2} - \frac{1}{n^2}\right)^{1/2}}{\frac{1}{n^2}\cos\theta_{i2}}\right] + 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i2} - \frac{1}{n^2}\right)^{1/2}}{\cos\theta_{i2}}\right]. \tag{8}$$

It can be seen from Eq. (8) that the relationship between phase difference φ and rotation angle θ depends only on refractive index n. Figure 3 shows a typical curve of φ versus θ with a refractive index of 1.51509. Thus the precise rotation angle is known when the phase difference is measured. The angle sensitivity varies with the angle of rotation. When θ is small, the curve is approximately linear and the sensitivity is relatively low. The sensitivity is greatly enhanced as θ increases. If a phasemeter with a resolution of 0.01° is used, the angle resolution is ~0.3 arc sec at $\theta = \pm 5.6°$, and it is better than 9 arc sec over the measurement range of $|\theta| \le 5.6°$. As the refractive index n decreases, the sensitivity rises greatly.

The measurement range is determined by the critical angle of the right-angle prisms. When the prism assembly is being rotated during measurement, the incident angle at the hypotenuse of one prism increases while that of the other prism decreases. Therefore the measurement range should be $|\theta| \le \theta_{\text{Max}}$. According to refraction and total-reflection principles, we have

$$\theta_{\text{Max}} = \sin^{-1}\left\{n \sin\left[\frac{\pi}{4} - \sin^{-1}\left(\frac{1}{n}\right)\right]\right\}. \tag{9}$$

The measurement range is also determined only by n. The curve of measurement range θ_{Max} versus refractive index n is shown by the straight line in Fig. 4. When n increases, the sensitivity grows linearly. The measurement range can be as much as $\pm 20°$ when $n = 1.8$.

The absolute measurement of the rotation angle can be made without any major error and is easy to perform. A single-frequency laser is required as the light source. However, it is not easy to measure the phase difference of single-frequency laser interferometers dynamically.

B. Relative Measurement

If f_1 is not equal to f_2, the initial phase difference φ_0 and the phase change $\Delta\varphi$ caused by the rotation of the prism assembly will not be zero. Thus the measured phase difference at the output of the system is related not only to the rotation angle but also to the initial distance that the beams travel and to the size of the prism assembly.

Although the phase change $\Delta\varphi$ caused by the rotation of the prism assembly can be corrected according to Eq. (6), the initial phase difference φ_0 cannot be exactly known. To permit the angle to be measured, Eq. (7) should be linearized and simplified.

When θ is small, Eq. (6) can be simplified, by use of the Taylor series, as

$$\Delta\varphi = \frac{\Delta f}{f}\frac{2\pi}{\lambda} t \frac{n-1}{n}\frac{\theta^2}{2}. \tag{10}$$

It is clear that $\Delta\varphi$ is a quadratic function of θ. If t = 50 mm and Δf = 1 MHz, then $\Delta\varphi$ = 0.7 arc sec as $\theta = \pm 5.6°$. Therefore $\Delta\varphi$ can be neglected when Δf and t are selected to be comparatively smaller. Thus the phase difference between frequency components f_1 and f_2 at the output of the system can be given by

$$\begin{aligned}\varphi = \varphi_0 &+ 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i1} - \frac{1}{n^2}\right)^{1/2}}{\frac{1}{n^2}\cos\theta_{i1}}\right] \\ &- 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i1} - \frac{1}{n^2}\right)^{1/2}}{\cos\theta_{i1}}\right] \\ &- 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i2} - \frac{1}{n^2}\right)^{1/2}}{\frac{1}{n^2}\cos\theta_{i2}}\right] \\ &+ 2\tan^{-1}\left[-\frac{\left(\sin^2\theta_{i2} - \frac{1}{n^2}\right)^{1/2}}{\cos\theta_{i2}}\right].\end{aligned} \tag{11}$$

From Fig. 3 we can see that when θ is small, i.e., $|\theta| < 2°$, the relationship between φ and θ is approximately linear. Suppose that θ is small enough; the Taylor series expansion of $\varphi(\theta)$ is

$$\begin{aligned}\varphi(\theta) = \varphi_0 + \varphi(0) + \varphi'(0)\theta + (1/2!)\varphi''(0)\theta^2 \\ + (1/3!)\varphi'''(0)\theta^3 + \ldots,\end{aligned} \tag{12}$$

where

$$\varphi(0) = 0,$$
$$\varphi'(0) = \frac{4(3-n^2)}{(n^2-1)\sqrt{n^2-2}}.$$

When θ is small enough, the relation between φ and θ is approximately expressed as

$$\begin{aligned}\varphi(\theta) &\approx \varphi_0 + \varphi(0) + \varphi'(0)\theta \\ &= \varphi_0 + \frac{4(3-n^2)}{(n^2-1)\sqrt{n^2-2}}\theta \\ &= \varphi_0 + k\theta,\end{aligned} \tag{13}$$

where k is the sensitivity:

$$k = \frac{4(3-n^2)}{(n^2-1)\sqrt{n^2-2}}, \tag{14}$$

and depends only on refractive index n. The relationship between k and n is represented by the curve in Fig. 4. We can see from Fig. 4 that, when n decreases, the sensitivity grows greatly while the measurement range increases linearly. The sensitivity approaches infinity as n is close to $\sqrt{2}$. There is a trade-off between the measurement range and the sensitivity. For example, when the right-angle prism is made of BK7 glass with a refractive index of 1.51509, the measurement range is $\pm 5.6°$ and the sensitivity is ~ 4. If a phasemeter with a resolution of 0.01° is used, an angle resolution of ~ 9 arc sec will be achieved. If n = 1.415, the resolution is better than 0.4 arc sec.

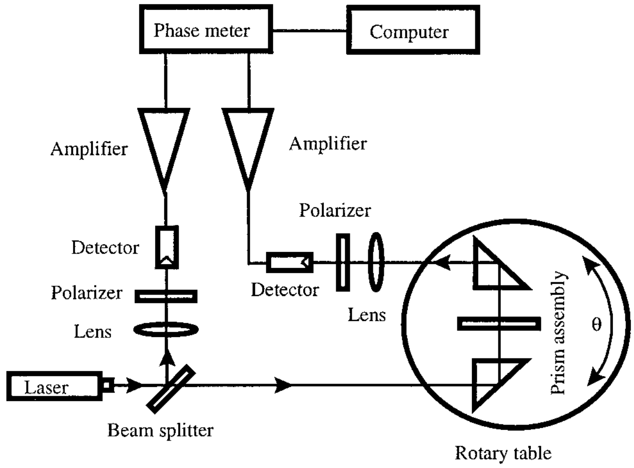

Fig. 5. Schematic diagram of the experiment setup.

3. Experiment and Discussions

To verify the theoretical analysis above we conducted experimental tests. Figure 5 is a schematic diagram of our experimental setup. Figure 6 is a photograph of the experimental setup. The details of the prism assembly can be seen in Fig. 7. An HP 5517 laser head was used as the light source. It emits two orthogonal linearly polarized light beams of frequencies f_1 and f_2. The frequency difference, $\Delta f = f_1 - f_2$, is ~ 2 MHz. The beams from the laser head are divided into two parts by a beam splitter. The reflected part is received by a photodiode after it passes through a lens and a polarizer. The photodiode outputs a reference signal with frequency Δf. The transmitted part is incident upon the prism assembly and retroreflected parallel. The reflected beams are

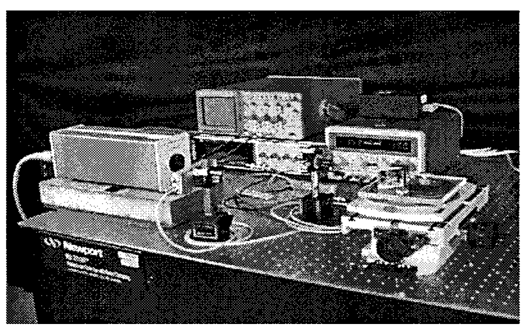

Fig. 6. Experimental setup for angle measurement.

Fig. 7. Details of the prism assembly.

received by another photodiode. The second photodiode gives the measurement signal. HP 10780C/F receivers were used in the experiment. The reference and measurement signals are transmitted to a phasemeter, which measures their phase difference. In our experiment an HP 3575 phasemeter with a resolution of 0.1° was used. The right-angle prism was made of BK7 glass with a refractive index of 1.51509. Therefore the resolution of experiment system is ~0.025°. The experimental results of phase difference φ versus rotation angle θ are shown in Fig. 8, which indicates that the curve obtained from the experiment is in good accordance with the theoretical curve shown in Fig. 3.

Because the objective of our experiment is just to demonstrate the feasibility of the proposed method, the experimental setup is relatively simple. To achieve higher resolution and greater accuracy, we should make some improvements. First, a laser with a lower frequency difference should be selected. The lower the frequency difference is, the smaller the additional phase change caused by the rotation of the equivalent glass plate of the prism assembly $\Delta\varphi$ will be. If possible, frequency difference Δf should be zero. When $\Delta f = 0$, $\varphi_0 = 0$ and $\Delta\varphi = 0$, which means that the conditions of Eq. (8) are completely satisfied, and the absolute measurement of angle can be performed without any major error. Although it will not be easy to measure the phase difference of single-frequency laser interferometers dynamically, the proposed method does promise to measure the rotation angle more accurately and effectively. Second, compared with multiple-order wave plates, zero-order wave plates are less sensitive to retardation change with rotation about the fast or the slow axis. This means that light that is not parallel to the optical axis will suffer a smaller change in retardation with zero-order wave plates than with multiple wave plates. Therefore a zero-order half-wave plate should be used to reduce the influence of the rotation of the wave plate during measurement. Finally, the accuracy and the resolution of angle measurement depend on those of the phasemeter, so a high-accuracy and high-resolution phasemeter should be adopted.

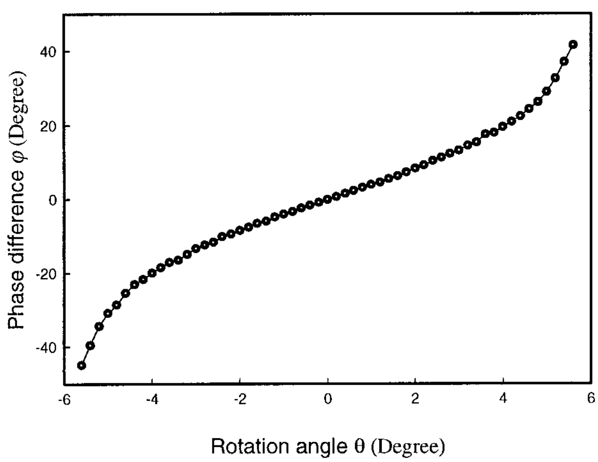

Fig. 8. Experimental curve of φ versus θ.

4. Conclusion

An angle-measurement method based on internal reflection and heterodyne interferometry has been presented. By utilizing the novel prism assembly proposed in this paper, we achieved a differential common path. As a result, the equipment can be compact and practical. The differential optical configuration is naturally designed into the system. Therefore the linearity of the system is greatly improved, and angle measurement is insensitive to variations in the environment. The theoretical analysis of the measurement principle indicates that the sensitivity and the measurement range of the method depend only on the refractive index of the right-angle prisms. When the refractive index decreases, the sensitivity increases significantly while the measurement range decreases linearly. Therefore there is a trade-off between the measurement range and the sensitivity. The resolution is better than 0.3 arc sec, depending on the refractive index selected. Our analysis indicates that the angle measurement can be further divided into absolute and relative measurements according to different laser sources. A two-frequency laser with a lower frequency difference is suggested to reduce the effect of phase change $\Delta\varphi$ caused by the rotation of the prism assembly and to increase the resolution and accuracy of phase measurement. An absolute measurement of the angle can be made if a single-frequency laser is used. The experiment results agree well with our theoretical analysis. The advantages of the method are compactness, high resolution, large measurement range, low cost, and ease of integration with commercially available or specially designed interferometers.

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