



# Roll angle measurement system based on differential plane mirror interferometer

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**Abstract:** A precision roll angle measurement system based on differential plane mirror interferometer (DPMI) is proposed and the measurement principle has been analyzed theoretically in detail. This system uses DPMI with wedgy angle prism and wedgy angle reflector to produce diagonal symmetry of the two frequency beams. The residual of nanopositioning stage and roll angle system is less than 1  $\mu$ rad, which has verified the correctness of the measuring principle. As result, the stability of the roll angle measurement system is satisfactory, and the average deviation of measurement experiments is less than 5  $\mu$ rad.

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**OCIS codes:** (120.3180) Interferometry; (120.0120) Instrumentation, measurement, and metrology; (120.3940) Metrology.

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## 1. Introduction

Precision motion system is a very important unit for any precision machining and precision measuring, so it will determine the quality of high-precision equipment. For machinery and equipment such as precision machine tool [1], lithography machine [2], CNC machine tools [3,4] etc., effective ways must be found to increase the accuracy of the motion system and reduce the geometric error [5,6].

By using commercial plane mirror interferometer(DPMI) for measuring multi-degree of freedom error [7–11], except for roll angle error, the positioning error, two straightness errors, pitch angle error and yaw angle error [12–14] can be easily measured. The reason is that the change of roll angle is not easy to represent as optical path difference in existing interference system. If roll angle can be measured by DPMI system, it means that we can achieve the full measurement of axial motion in six-degree-of-freedom [15–18].

In our previous works [19–21], we proposed a roll angle measurement system based on laser heterodyne interferometer. In the system, a wedge mirror is used to replace the reference mirror and a wedge prism is used for the roll angle sensing component. When the wedge prism is fixed on the translation stage, the optical path difference of roll angle error is changed due to the refractive index difference between the prism and air. But the designed interferometer is non-coaxial optical system, which need a complicated method for optical path adjustment. The special polarizing Koster prism is a key component in the system which is not easy to be manufactured. For another weakness, it's not easy to realize other five freedom measurement using the same system.

In this paper, we report a new use of commercial DPMI for roll angle measurement. With a wedgy prism and a wedgy reflector, the existing DPMI would be used for roll angle measuring compactly. Compared with our previous work, the main different is that it provides a coaxial optical system which is much easy to fabricate and adjust. This work extends the commercial DPMI functionality for roll angle measurement which can realize 6 freedom errors measurements of axial motion one by one. The detail of measurement principle is presented. And the calibration experiment, verification experiment and measurement experiment are given, which is a complete and systematic process. Lastly, the stability of this roll angle measurement system is reliable, and the average deviation of roll angle measuring results about linear translation stage is less than 5  $\mu\text{rad}$ . These prove the rationality of the proposed roll angle measurement system.

## 2. Principle

The schematic of roll angle measurement system is shown in Fig. 1. The system consists of dual-frequency laser head, DPMI, wedgy angle prism, wedgy angle reflector, photoelectric receiver, and phase meter. The DPMI is made by Zygo Corporation.

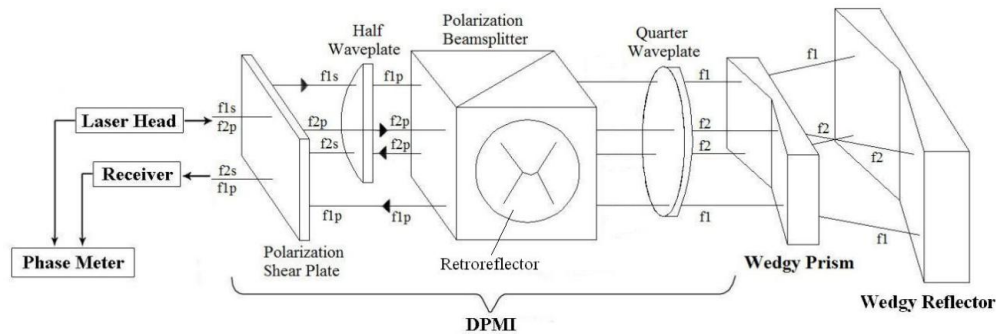
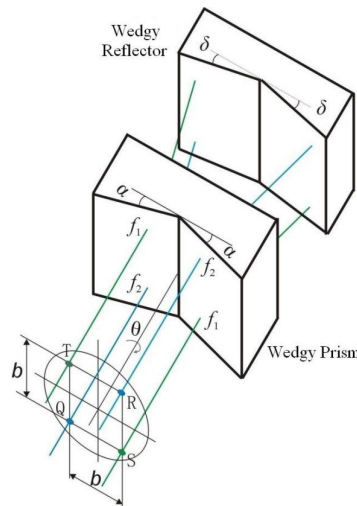


Fig. 1. Schematic of roll-angle measurement system.

DPMI is one of the heterodyne interferometer. It uses two orthogonal polarized laser beams with different frequency  $f1$  and  $f2$  to produce interference effect. One beam is horizontal polarization and the other is vertical polarization beam. In the process of measurement, the two laser beams will separately pass through one of the difference interference arms and carry different information of optical phases. When two laser beams meet, a beat frequency measuring signal will appear. If we compare it to a reference signal, we will get the linear relationship between the optical path difference  $\Delta l$  and the phase difference  $\Delta\phi$ , as shown in following Eq [21]:

$$\Delta l = \frac{\lambda}{2\pi} \Delta\phi. \quad (1)$$

Obviously, if the roll angle can be reflected in the optical path difference of the two beams of different frequency laser, we can easily get the roll angle. For DPMI system, it has a complete symmetrical configuration for the two frequency components, which have four symmetrical light paths in space. Figure 2 shows the light paths after the DPMI.

Fig. 2. Schematic of the light paths after the DPMI.  $\theta$  is the roll angle value;  $b$  is the distance of 4 symmetric light spots;  $\alpha$  is the wedge angle of wedgy angle prism;  $\delta$  is the wedge angle of wedgy angle reflector.

When the light beam of frequency  $f1$  passes through the incident point T, it will be reflected by the wedgy angle reflector after passing through the wedgy angle prism and returns back along the same route. After being switched by DPMI, the light beam of

frequency  $f_1$  will pass through the incident point S, and reflected by the wedgy angle reflector again after passing through the wedgy angle prism and return back. Similarly, the light beam of frequency  $f_2$  will pass through the point R, and be reflected by the wedgy angle reflector after passing through the wedgy angle prism and return back along the same route. After being switched by DPML, the light beam passes through incident point Q and return back.

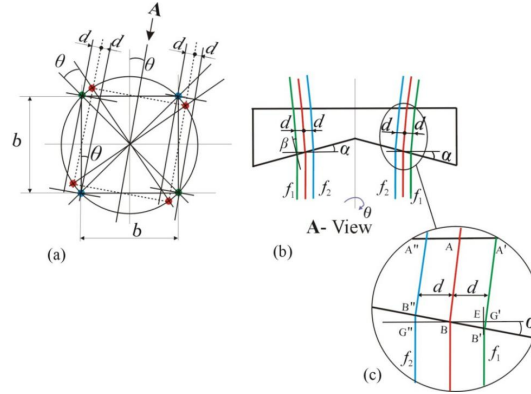


Fig. 3. Ichnography of four symmetrical optical paths system.

No matter the wedgy prism rolls or not, the position of 4 light beams will not change in geometric space. In the cross-section view of perpendicular to the light path (see Fig. 2), there are four intersections T, Q, R, S which compose a square with side length of  $b$ . The wedgy prism assembly is fixed on the moving stage to be measured. When the wedgy prism assembly rotates an angle  $\theta$  along with the roll of moving stage, the four light beams are still at the original spatial position, however the four former light sport positions on wedgy prism rotate an angle  $\theta$ , as shown in the Fig. 3(a) in red marked. If an observation is made from a direction perpendicular to the transversal cross section of the wedgy angle prism, e.g. from the direction of arrow A shown in Fig. 3(b), the two light beams of  $f_1$  (green marked T, S) will move outward along the radial direction relative to the red lines (i.e. the light beams before rolled). As the same way, the two light beams of  $f_2$  (blue marked T, S) will move inward along the radial direction relative to red lines (i.e. the light beams before rolled). See Fig. 3(b), the absolute values of the respective deviations are equal, set to  $d$ . Figure 3(a) shows that, in:

$$\sin \theta = \frac{2d}{b}, \quad (2)$$

$b$  is the distance of two beams, and the angle  $\theta$  can be calculated if  $d$  is measured.

The wedgy angle prism is fixed on the measured unit as the roll angle sensing device, others optical parts are fixed on the particular position. Because the wedgy angle prism is made of glass, and if the outside medium of prism is air (if it is a vacuum or other medium, the theory is also correct), the refractive index of the two medium are not equal. Therefore, the two light beams  $f_1$  and  $f_2$  through the total geometric path remain unchanged, but the optical path changed. The two light beams of  $f_1$  (green marked) move outward by AB change to the A'B', as shown in Fig. 3(c), the distance through the glass increases, that is, the optical path increases. Similarly, the two light beams of  $f_2$  (blue marked) move inward by AB change to the A''B'', as shown in Fig. 3(c), the distance through the glass decreases; that is, the optical path decreases. Because four light beams are centrosymmetric, the absolute values of the change of the geometric distance are equal. We set this distance as  $l$ , see Fig. 3(c), it will be given by as follows:

$$l = \overline{B'G'}. \quad (3)$$

Moreover, from Fig. 3(c), we have:

$$d = \overline{BE} + \overline{EG'}. \quad (4)$$

So,  $d$  is obtained by geometric computing:

$$d = l \frac{\cos \beta}{\sin \alpha}, \quad (5)$$

where  $\beta$  is the refraction angle entering the wedgy angle prism, when the size of prism and material of glass are selected, the  $\cos \beta$  is constant, of approximately  $\cos \beta = 1$ . Here  $l$  is directly related to the optical path difference  $\Delta l$ .

This is because each frequency light beam passes through the wedgy prism 4 times. In addition, as one change in the optical path of the two frequency light beams is positive and the other is negative, so the relationship between the total optical path difference  $\Delta l$  and  $l$  is:

$$\Delta l = 8l(n_g - n_{air}), \quad (6)$$

where  $n_g$  and  $n_{air}$  are the refractivity of glass and air. Substituting Eq. (5) and Eq. (6) into Eq. (2), Eq. (7) can be obtained as below:

$$\sin \theta = \frac{1}{4(n_g - n_{air})b \sin \alpha} \Delta l. \quad (7)$$

Substituting Eq. (1) into Eq. (7), Eq. (8) can be obtained as shown below:

$$\sin \theta = \frac{\lambda}{8(n_g - n_{air})\pi b \sin \alpha} \Delta \phi. \quad (8)$$

For conciseness, the refractivity of the wedge prism  $n_g$  is approximately 1.5, refractivity of air  $n_{air}$  is approximately 1. Their difference is about 0.5, so the Eq. (7) and Eq. (8) can be simplified as:

$$\sin \theta = \frac{1}{2b \sin \alpha} \Delta l \quad (9)$$

$$\sin \theta = \frac{\lambda}{4\pi b \sin \alpha} \Delta \phi \quad (10)$$

Because  $\alpha$ ,  $b$ , and  $\lambda$  are known, when phase change  $\Delta \phi$  of optical path difference between measurement and reference signal can be measured by phase meter, the roll angle  $\theta$  can be determined by Eq. (10). When  $\alpha = 1^\circ$ ,  $b = 9\text{mm}$ ,  $\lambda = 632.99\text{nm}$  and the resolution of phase meter is  $0.7^\circ$  (512 division), so resolution of roll angle measurement system is  $3.95 \mu\text{rad}$  ( $0.8143''$ ).

The maximum measuring distance (the allowable linear travel distance of the measured object) is theoretically unlimited. It depends on the width of the wedgy reflector. Assuming that the width of the wedgy reflector is 150 mm, the maximum measuring distance of the measured object is about 8 m. The maximum measuring speed is mainly limited by the frequency difference of the dual-frequency laser source. For the frequency difference of the laser source is 1.9-2.4 MHz, so the measurement speed of this system can reach up to 0.364 m/s. This system is not suitable for larger roll angle measurement and is designed for measuring a small roll angle error of a high-precision positioning device. Through comparative measurement in the  $2^\circ$  range of roll angle system and high-precision rotary table the two values are well-matched. In actual high-precision positioning systems, the roll angle error is very small, usually in the range of minute or second, so the measuring range of  $\pm 2^\circ$  is large enough.

For the centrosymmetrical configuration of this system, the two frequency beams are diagonal symmetry which have the same optical path change caused by positioning, two straightness and pitch angle errors. So these errors would not cause error crosstalk. For precision machine tools or translation stage, if its yaw angle error is controlled within  $300\ \mu\text{rad}$ , the roll angle does not change by more than  $2\ \mu\text{rad}$ . The installation error of wedgy prism does not affect the measurement accuracy for the same centrosymmetrical system. For the installation error of wedgy reflector, the misalignment of the rotation centers will not affect the direction of reflection beam. But the tilt installation of wedgy reflector will change the direction of reflection beam. The result is that the beam cannot be closed and we will avoid this happening in adjusting the light path. The testing principle of nonlinear error in our previous paper [22] was used to measure the overall nonlinearity of the roll angle system. The maximum value of nonlinear error of the system is about  $6.54\ \mu\text{rad}$ . High quality laser, wave plate and spectroscopic prism are used to reduce elliptical polarization of incident laser. Correct installation of optical components is needed. On this basis, the nonlinear error can be further reduced by various optical compensation, electronic compensation, hardware and software processing, etc.

### 3. Verification system

It is necessary to prove if the measuring principle is correct or not through the verification system. As there is no mature roll angle measurement instruments, so the main ideal of verification system is to use the nanopositioning stage to produce the precise rotations around the axes. In order to get the more precise benchmark of the verification system, we need to use an angular interferometer to calibrate the nanopositioning stage firstly.

For calibration experiment, the P-557.TCD(PI) nanopositioning stage is used as the benchmark which can produce precise rotation angle around axis x or y. So the target mirror of Zygo angular interferometer is fixed on the nanopositioning stage as the sensitive device to measure the rotation angle of P-557.TCD. The tilt angle measurement range of nanopositioning stage is  $200\ \mu\text{rad}$ , and stepping distance is  $50\ \mu\text{rad}$ . At the same time, 20 groups of data are collected at each angle by the angle interferometer, the curve of the measurement results is shown in Fig. 4.

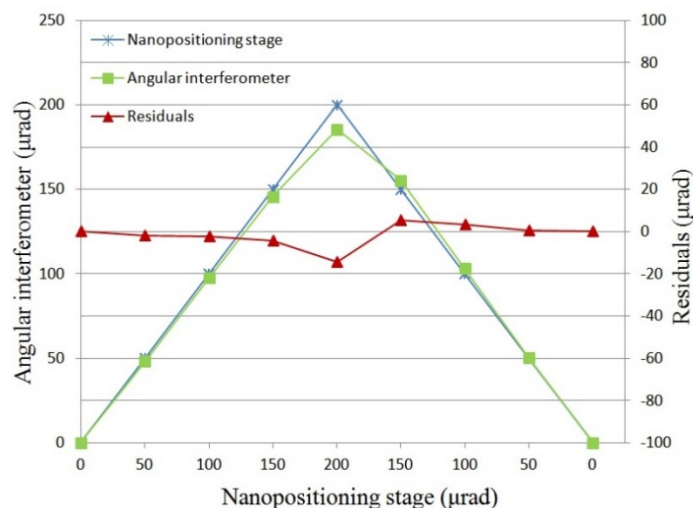


Fig. 4. Measurement results of calibration experiment.

As shown in Fig. 4, the rotation range of nanopositioning stage is identical with the measuring result of angular interferometer within  $50\ \mu\text{rad}$ . It is because of nanopositioning



stage has a big error in the range more than  $50\ \mu\text{rad}$ . So the precise rotation range ( $\leq 50\ \mu\text{rad}$ ) of the nanopositioning stage is credible and will be used in the following verification experiment.

For verification experiment, the wedgy angle prism is fixed on the center position of P-557.TCD. The DPMI module and the wedgy reflector are fixed at opposite ends of the travel axis. The rotation angle range of nanopositioning stage is 0 to  $50\ \mu\text{rad}$  and  $50$  to  $0\ \mu\text{rad}$ , and stepping distance is  $10\ \mu\text{rad}$ . At the same time, 50 sampling points at each angle are collected by phase meter (take 10 second) as shown in Fig. 5. Data of verification experiment are shown in Table 1.

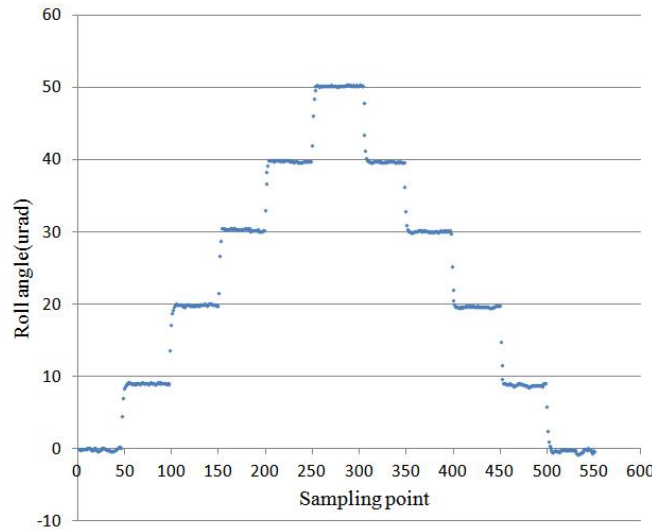


Fig. 5. Measurement results of verification experiment.

Table 1. Data of the verification experiment

Nanopositioning stage ( $\mu\text{rad}$ )	Average value of roll angle ( $\mu\text{rad}$ )	Residuals ( $\mu\text{rad}$ )
0	0.06	0.06
10	9.19	-0.81
20	20.01	0.01
30	30.49	0.49
40	39.96	-0.04
50	50.38	0.38
40	39.90	-0.10
30	30.24	0.24
20	19.81	-0.19
10	9.06	-0.94
0	-0.12	-0.12

As shown in Table 1, the residual of nanopositioning stage and roll angle system is less than  $1\ \mu\text{rad}$  which has verified the correctness of measuring principle. On the other hand, this result also indicates that the resolution of the system is less than  $10\ \mu\text{rad}$ .

#### 4. Measurement experiment

Experimental environment: Temperature ( $22 \pm 0.3^\circ\text{C}$ ), Humidity ( $55 \pm 3\%$ ), Pneumatic optical table.

For measurement experiment, the wedgy angle prism is fixed on the objective table of M-403.42S linear translation stage. The measurement experimental setup is shown in the Fig. 6. For the stability of the measurement system, the measuring results are shown in the Fig. 7

after 20 minutes continuous sampling. Variation range of measuring value is 6.37 to  $-7.64$   $\mu\text{rad}$ , average deviation is  $1.85$   $\mu\text{rad}$ .

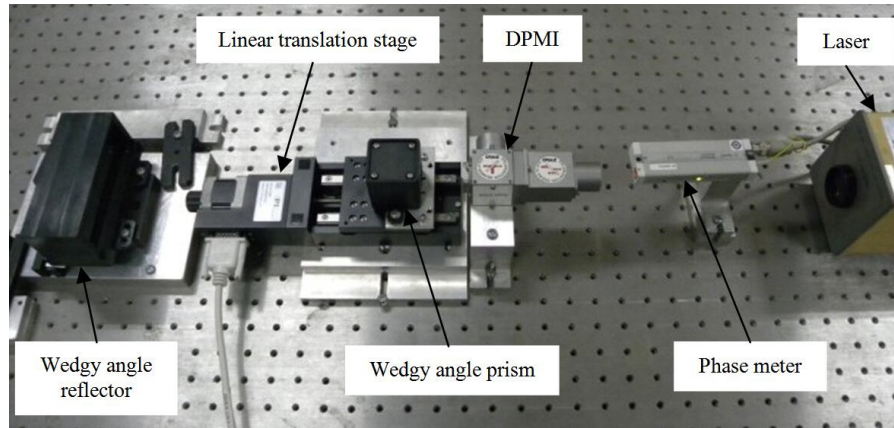


Fig. 6. The photo of the measurement experimental setup.

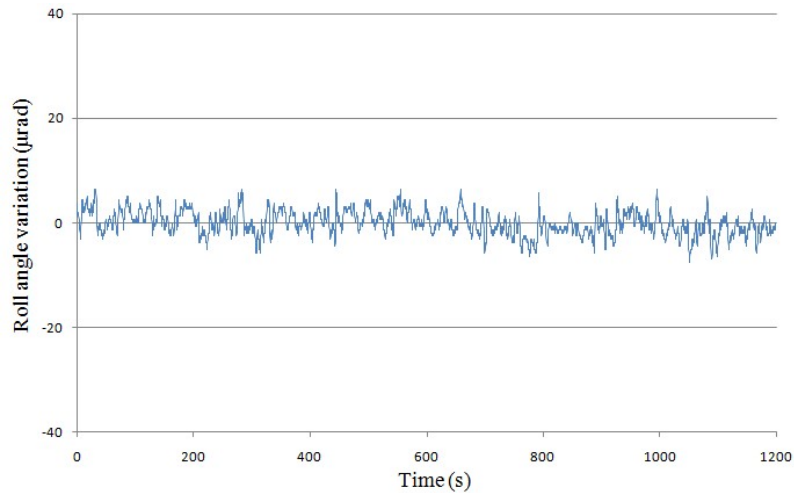


Fig. 7. Stability of the measurement experiment.

The travel range of M-403.42S linear translation stage is 60 mm, and stepping distance is 10 mm. For the same translation stage, three repeatability experiments have been done in the same situation. In each experiment, 10 groups of data is collected at each position by phase meter, average experimental data are shown in Table 2, the curve of measurement experiment is shown in Fig. 8.

Table 2. Data of measurement experiment

Position (mm)	Experiment 1 ( $\mu\text{rad}$ )	Experiment 2 ( $\mu\text{rad}$ )	Experiment 3 ( $\mu\text{rad}$ )	Average deviation ( $\mu\text{rad}$ )
0	0	0.637	0	0.28
10	57.07	64.08	63.70	3.03
20	107.43	101.51	105.61	2.23
30	144.27	134.98	141.36	3.48
40	189.69	193.43	186.54	2.36
50	252.31	260.40	265.62	4.76
60	299.39	307.14	298.75	3.59



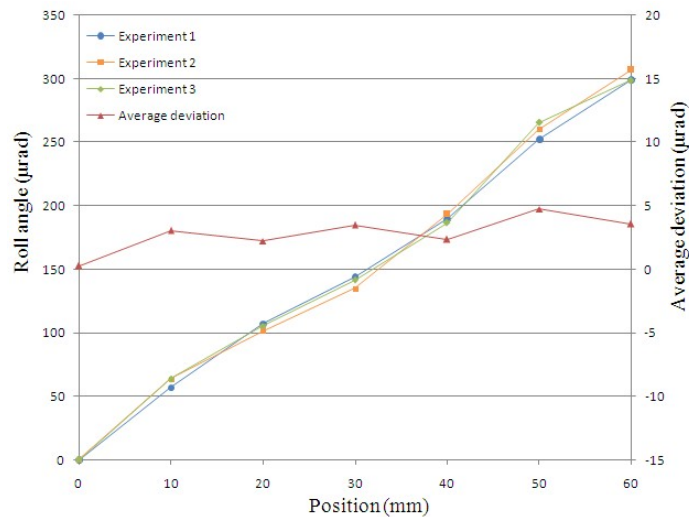


Fig. 8. Results of measurement experiment.

From the results, we get the conclusion that the stability of this roll angle measurement system is satisfactory, and the average deviation of three experiments is less than 5  $\mu\text{rad}$ . The roll angle value of M-403.42S linear translation stage is not given in its data table. So the experiment result of roll angle is very important and significant. This result can fill up the manufacturer's data blank of roll angle error.

## 5. Conclusion

Firstly, the calibration measurement between Zygo angular interferometer and PI nanopositioning stage has been done. After comparing of the results, the precise rotation range of nanopositioning stage is 50  $\mu\text{rad}$ . Through the verification experiment of roll angle measurement system, the experimental results have verified the availability of measurement system based on the measuring principle. Secondly, the measurement experiments of linear translation stage have been carried out by using same instruments and in the same environment situation. The average deviation of three experiments is less than 5  $\mu\text{rad}$ . It shows the reliability of this roll angle measurement system.

## Funding

National Natural Science Foundation of China (NSFC) (61378050); Key Laboratory Scientific Research Program of Shaanxi Provincial Department of Education(16JS042); Dean Fund Program from School of Optoelectronic Engineering, Xi'an Technological University (2017GDYJY02); President fund of Xi'an Technological University (XAGDXJJ16006).