

GRAVITATIONAL WAVE OBSERVATION FROM  
SPACE: OPTICAL MEASUREMENT TECHNIQUES  
FOR LISA AND LISA PATHFINDER

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## ABSTRACT

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The Laser Interferometer Space Antenna (LISA) is a joint ESA-NASA mission designed as the first space-based gravitational wave observatory and will operate in the frequency range between 0.1 mHz to 100 mHz. LISA will complement the ground-based observatories, as these low frequencies are inaccessible to detectors on Earth due to seismic noise predominance at frequencies lower than 10 Hz. LISA is a constellation of three identical spacecraft separated by 5 million kilometers, flying free-falling test masses. Relative changes in the separation between test masses located in different satellites reveal the presence of gravitational waves. LISA requires a measurement accuracy of better than  $40 \text{ pm}/\sqrt{\text{Hz}}$ , which is achieved by means of precision laser interferometry.

Due to the challenges LISA represents, ESA plans to launch the technology demonstration mission LISA Pathfinder in order to test LISA core technologies in the frequency range from 3 – 30 mHz. A high precision laser interferometer with picometer accuracy has been included to measure the displacement and attitude of the free-falling test masses and produces input signals for the test mass drag-free and spacecraft control.

This thesis describes three experiments related to LISA and LISA Pathfinder: an optical cavity, a phase-modulated homodyne interferometer, and a heterodyne interferometer were set up and characterized for test mass position and attitude measurements. During investigations on the LISA Pathfinder (LPF) interferometry, two testbeds were further developed: a laboratory test setup and a test facility for engineering models of subunits of the optical metrology system. The two setups and the results obtained are compared and described in detail.

Hardware simulations of the expected in-orbit cross-talk between test mass angular and displacement degrees of freedom have been conducted. Noise subtraction algorithms have been developed to correct for sensitivity limiting effects like the coupling of test mass jitter into displacement readout, and fluctuations of the laser frequency and the non-linear optical pathlength difference. A previously developed real-time wavefront detector has been used to help the adequate beam preparation, the manufacture of quasi-monolithic fiber injectors for the LPF optical bench, and the characterization of the LPF optical window.

Keywords: gravitational wave detection in space, laser interferometry, noise subtraction, data analysis

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## KURZZUSAMMENFASSUNG

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Die gemeinsame ESA-NASA-Mission Laser Interferometer Space Antenna (LISA) wird als das erste weltraumgestützte Gravitationswellenobservatorium im Frequenzbereich von 0.1 mHz bis 100 mHz konzipiert. LISA ergänzt hiermit erdgebundene Gravitationswellendetektoren, deren Empfindlichkeit durch seismisches Rauschen der Erde unterhalb 10 Hz begrenzt ist. LISA besteht aus drei identischen Satelliten in einem Abstand von 5 Millionen km mit frei fliegenden Testmassen. Von Gravitationswellen hervorgerufene relative Abstandsänderungen zwischen zwei Testmassen werden mit einer Genauigkeit besser als  $40 \text{ pm}/\sqrt{\text{Hz}}$  mittels hochempfindlicher Laserinterferometrie gemessen. Aufgrund der technologischen Herausforderungen von LISA beschloss die ESA, vor LISA den Technologiedemonstrator LISA Pathfinder (LPF) zu starten, der LISA Kerntechnologien im Frequenzbereich 3 – 30 mHz erprobt. Ein Laserinterferometer mit picometer-Genauigkeit wird zur Abstands- und Winkelmessung der Testmassen verwendet und generiert Signale zur Testmassen und Satellitenansteuerung. In dieser Arbeit werden drei Experimente für LISA und LISA Pathfinder behandelt: ein optischer Resonator, ein phasenmoduliertes Homodyninterferometer und ein Heterodyninterferometer wurden aufgebaut und charakterisiert, um Testmassenpositionen und Winkel zu messen.

Im Rahmen der LISA Pathfinder Interferometrie wurden zwei Testaufbauten weiterentwickelt: ein Labortestaufbau und ein Testaufbau für die Prototypen der Flugmodelle (Engineering Models). Die Aufbauten und damit erzielten Ergebnisse werden beschrieben und verglichen.

Laborhardwaresimulationen der im Weltraumflug zu erwartenden Kreuzkopplung zwischen Winkel- und Abstandsfreiheitsgraden der Testmassen wurden durchgeführt. Rauschsubtraktionsalgorithmen wurden zur Korrektur von Faktoren, die die Empfindlichkeit begrenzen, entwickelt, wie z.B. Testmassenrestwinkelrauschen, Laserfrequenz- und nicht-lineare optische Weglängenfluktuationen. Ein zuvor entwickelter Wellenfrontdetektor wurde charakterisiert und bei der Herstellung quasimonolithischer Faserauskoppler für LPF und bei der Charakterisierung des optischen Fensters in LPF eingesetzt.

Schlagworte: Gravitationswellendetektion im Weltraum, Laserinterferometrie, Rauschsubtraction, Datenanalyse

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## INTRODUCTION

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In his General Theory of Relativity presented in 1915<sup>1</sup> [1], Albert Einstein introduced a model for the geometric description of Gravitation as the spacetime curvature produced by masses and radiation present in it (see Figure 1 [2]).

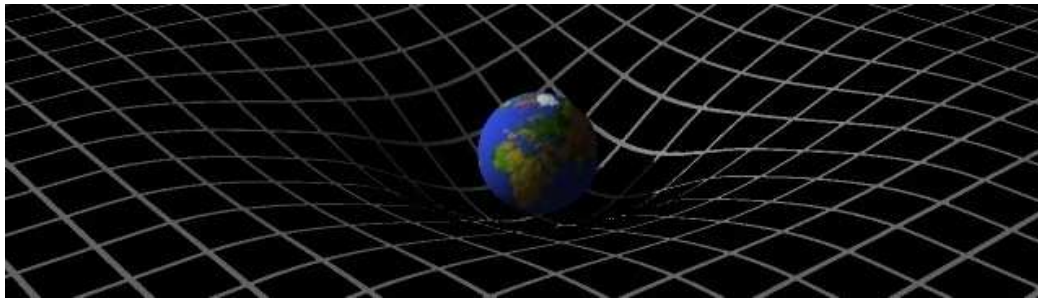


Figure 1: Illustration of Gravitation as the geometric curvature of spacetime.

Accelerated masses excite oscillations<sup>2</sup> of the spacetime itself which propagate at the speed of light and are called gravitational waves. Their theoretically predicted interaction with other physical phenomena is very small, which helps to preserve high quality information on the cosmological events emitting them while the waves travel through the universe. However, this fact also make gravitational waves extremely challenging to detect.

Current astronomy is mainly based on the detection of electromagnetic radiation emitted by celestial bodies and their interactions. Due to the fundamental difference in the physical process of their emission, the detection of gravitational radiation opens a completely novel field in astronomy that enables to reach a more extensive understanding of the universe and would help probe essential questions about its very origin and evolution.

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<sup>1</sup> Lecture given at the Preußische Akademie der Wissenschaften.

<sup>2</sup> "Similarly" to a ball moving on a rubber elastic bandage.

## GRAVITATIONAL WAVE OBSERVATION

The functional principle of laser interferometric gravitational wave detectors is based on measuring the distance between test masses – suspended mirrors of the interferometer – that changes at the pass of a gravitational wave.

The scientific community has been investing a considerable effort in the setup of a worldwide network consisting of five ground-based laser interferometric gravitational wave observatories.

Variations in the gravitational field of the Earth restrict the measurement bandwidth of these detectors to frequencies above a few hertz.

Therefore, a space-based gravitational wave observatory is currently under development in order to complement the ground-based network observation bandwidth in the millihertz range.

## LISA: LASER INTERFEROMETER SPACE ANTENNA

The Laser Interferometer Space Antenna (LISA) is the first space-based laser interferometric gravitational wave observatory, designed to operate in the frequency band between  $10^{-4}$  and  $10^{-1}$  Hz. Astrophysical sources such as super massive black hole binaries, coalescences and mergers, as well as a cosmological gravitational wave background are expected to be detectable in this frequency range.

LISA is a joint mission between the European Space Agency (ESA) and NASA, and consists of three identical spacecraft in an equilateral triangle formation, separated by 5 million kilometers. The LISA triangular constellation follows the Earth at a distance of 50 million km ( $20^\circ$ ) in a heliocentric orbit at 1 AU from the Sun. The plane of this assembly is tilted by  $60^\circ$  with respect to the ecliptic as shown in Figure 2.

LISA flies a total of six drag-free test masses, two in each spacecraft, and measures variations in their separation originating from gravitational waves with an accuracy of  $40 \text{ pm}/\sqrt{\text{Hz}}$  over their nominal separation of 5 million km, which corresponds to a relative length measurement precision of the order of  $\frac{\delta L}{L} = 8 \times 10^{-21} / \sqrt{\text{Hz}}$ . In order to achieve this displacement sensitivity for the gravitational wave measurement, effects of non-gravitational forces have to be



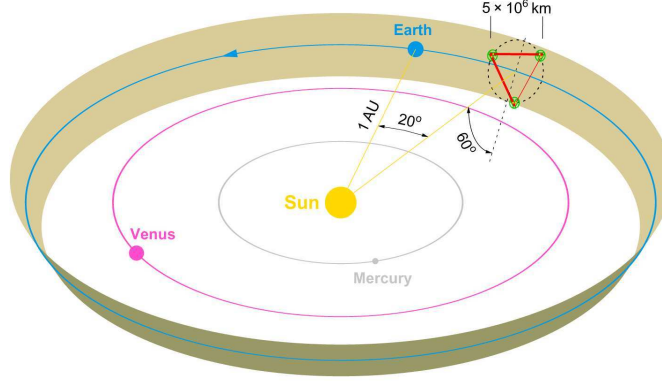


Figure 2: Heliocentric orbit of the spacecraft constituting LISA

suppressed below the level of  $3 \times 10^{-15} \text{ N}/\sqrt{\text{Hz}}$  at 3 mHz (see Figure 3).

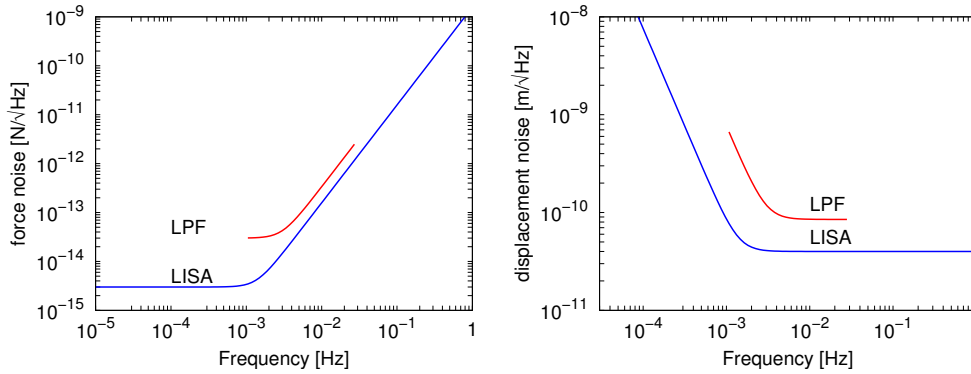


Figure 3: Force and displacement noise budgets for LISA and LISA Pathfinder

## LISA PATHFINDER

LISA requires some novel technology that is under development and cannot be tested on Earth. It is therefore that ESA decided to launch the technology demonstration mission LISA Pathfinder (LPF) to test core technology in a similar space environment as is expected for LISA.

LPF consists of a single satellite that carries two experiments: the ESA payload LTP, and the Disturbance Reduction System (DRS) from NASA. This thesis concentrates on research topics concerning LTP only.

The main aim of LTP is to verify drag-free control of test masses to a level better than  $3 \times 10^{-14} \text{ m s}^{-2}/\sqrt{\text{Hz}}$  in the observation band of 3 – 30 mHz.

A laser interferometer has been included with the capability of measuring the test mass with a displacement accuracy of  $6.3 \text{ pm}/\sqrt{\text{Hz}}$  and an angular resolution of  $20 \text{ nrad}/\sqrt{\text{Hz}}$  [3]. The requirements on the LTP test mass displacement noise are plotted in Figure 4.

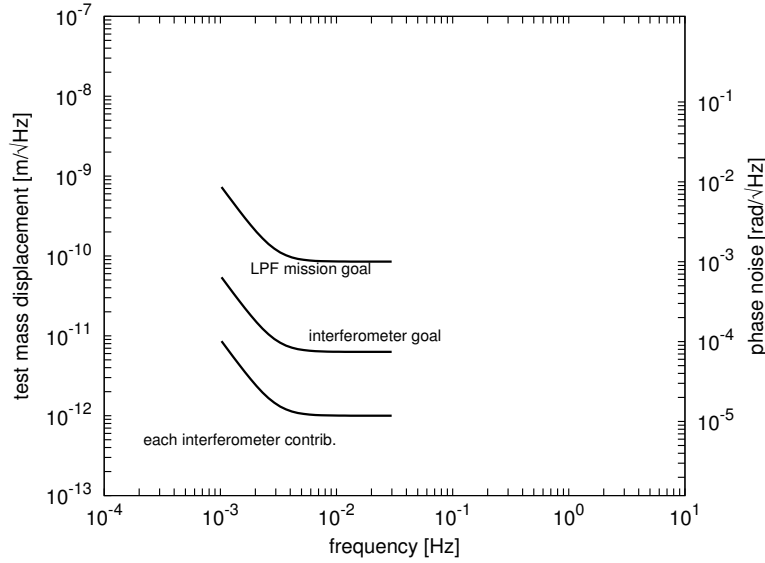


Figure 4: Requirements on the LTP test mass displacement noise

The upper trace at a level of  $85 \text{ pm}/\sqrt{\text{Hz}}$  defines the requirement on the LTP test mass displacement noise under drag-free control. The middle trace at  $6.3 \text{ pm}/\sqrt{\text{Hz}}$  defines the requirement on the accuracy of the optical metrology system for the test mass displacement measurement. The lower trace at  $1 \text{ pm}/\sqrt{\text{Hz}}$  is the noise budget allocated to each noise source of the interferometer.