Conflict-Driven Conditional Termination

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Conditional Termination

- **termination**: does the program terminate for all initial states?

- **conditional termination**: for which initial states does the program terminate?

termination $\equiv$ conditional termination for all initial states

Example

int : x

while $^{1}(x < -2 \vee x > 2)$ {

$\quad ^{2}x := x - 1$

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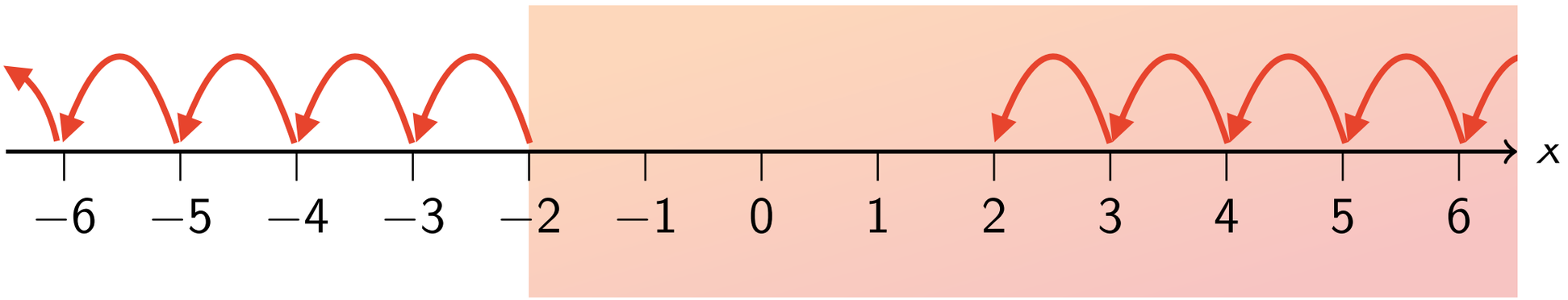

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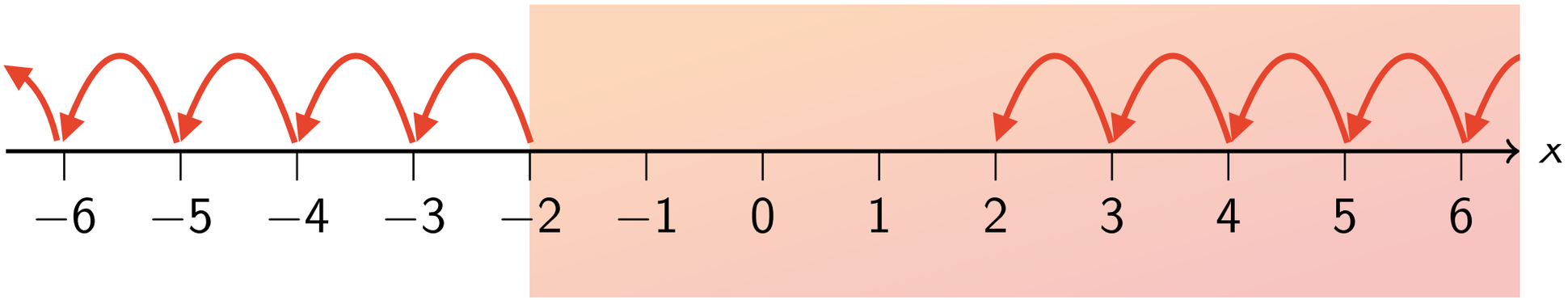
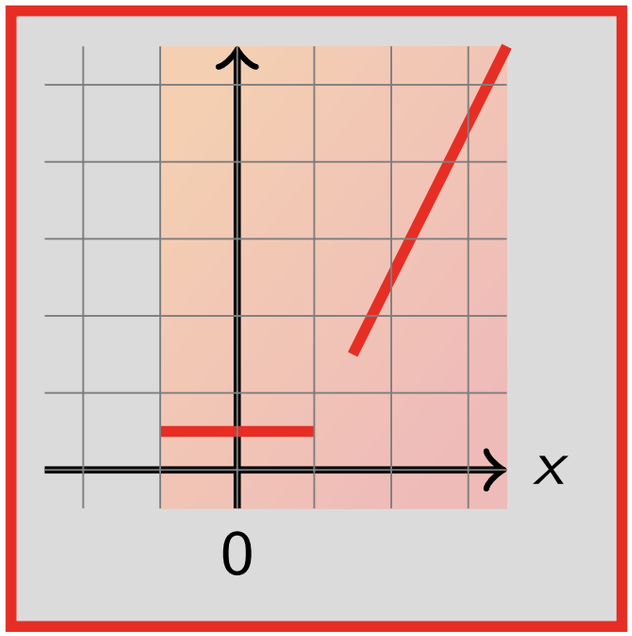


the program **conditionally terminates** for $-2 < x$

Piecewise-Defined Ranking Functions



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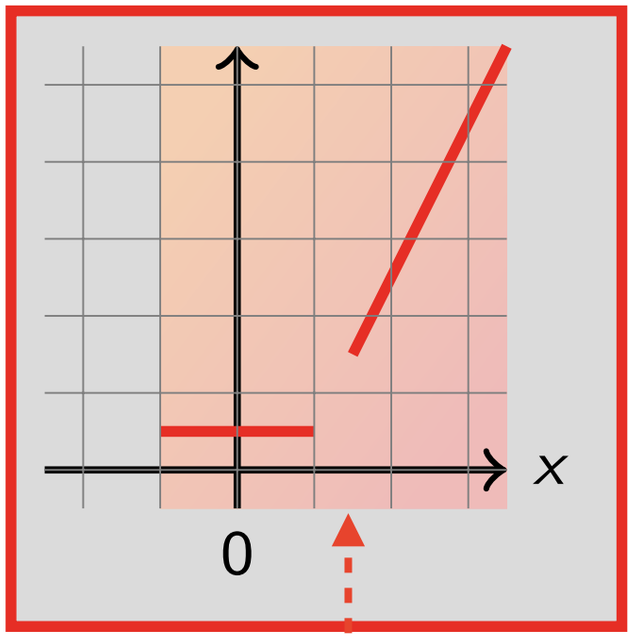

the **domain** of the ranking function represents the terminating states

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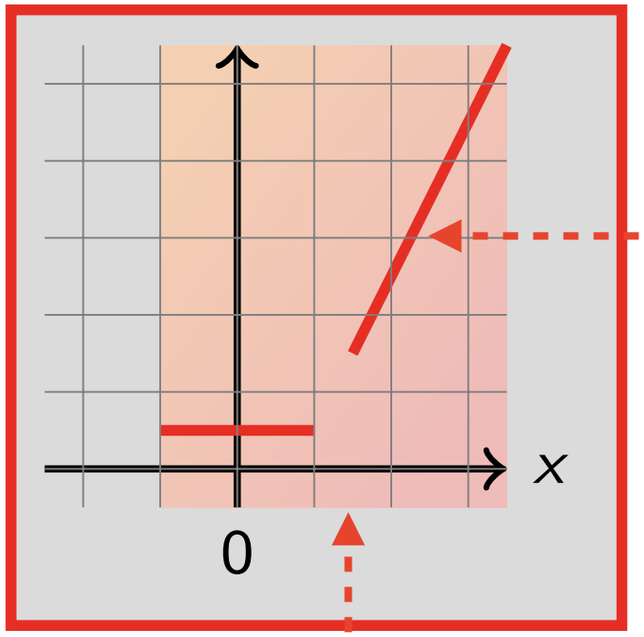

the **value** of the ranking function yields an upper bound on the number of execution steps to termination

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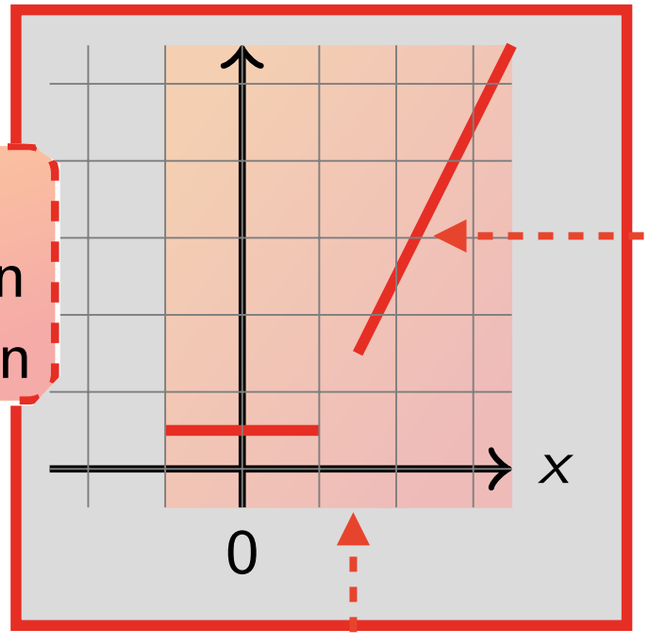
Piecewise-Defined Ranking Functions



outside the domain of the ranking function we have no information

the **value** of the ranking function yields an upper bound on the number of execution steps to termination

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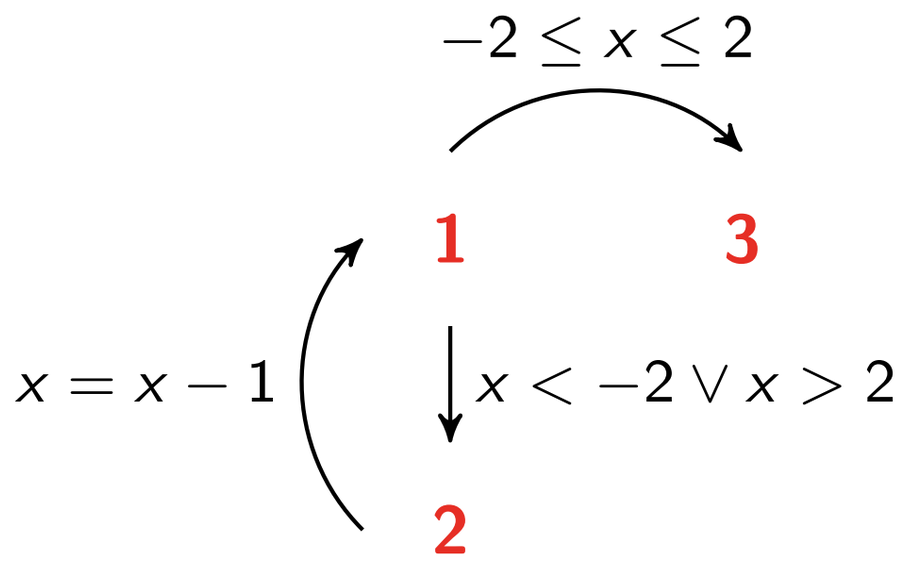

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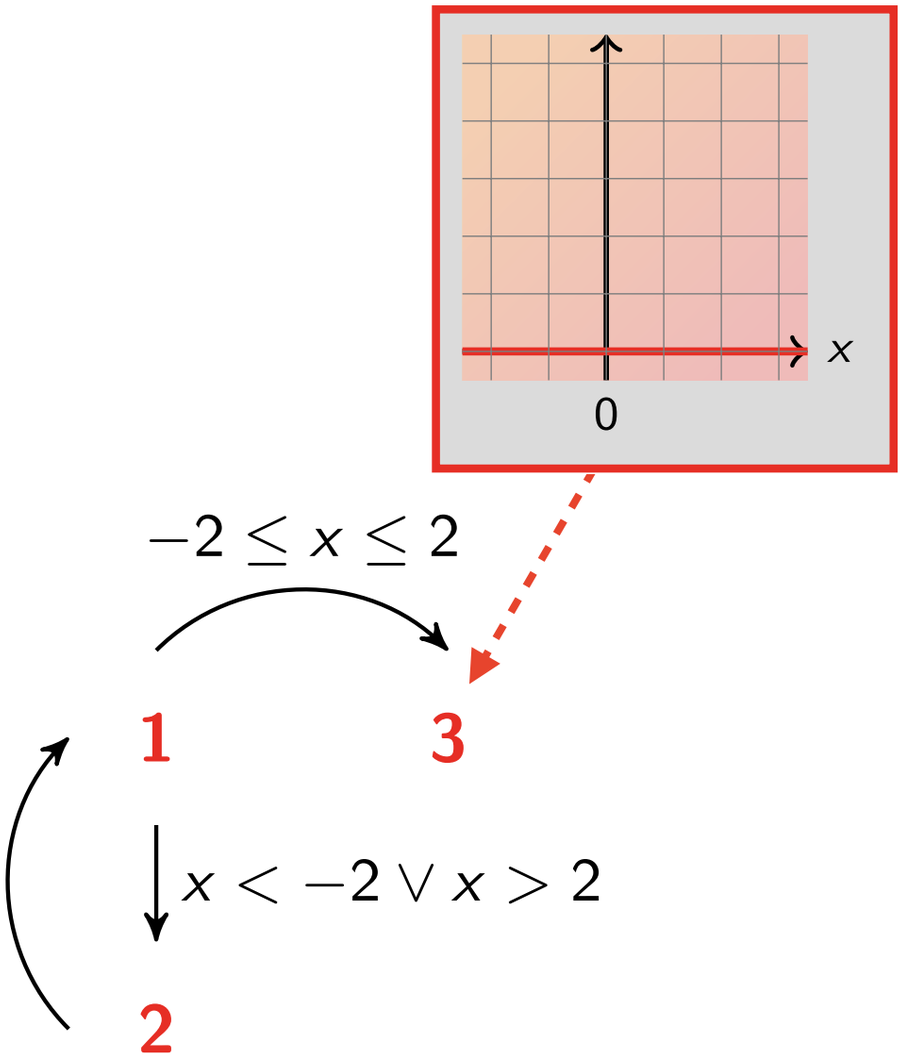

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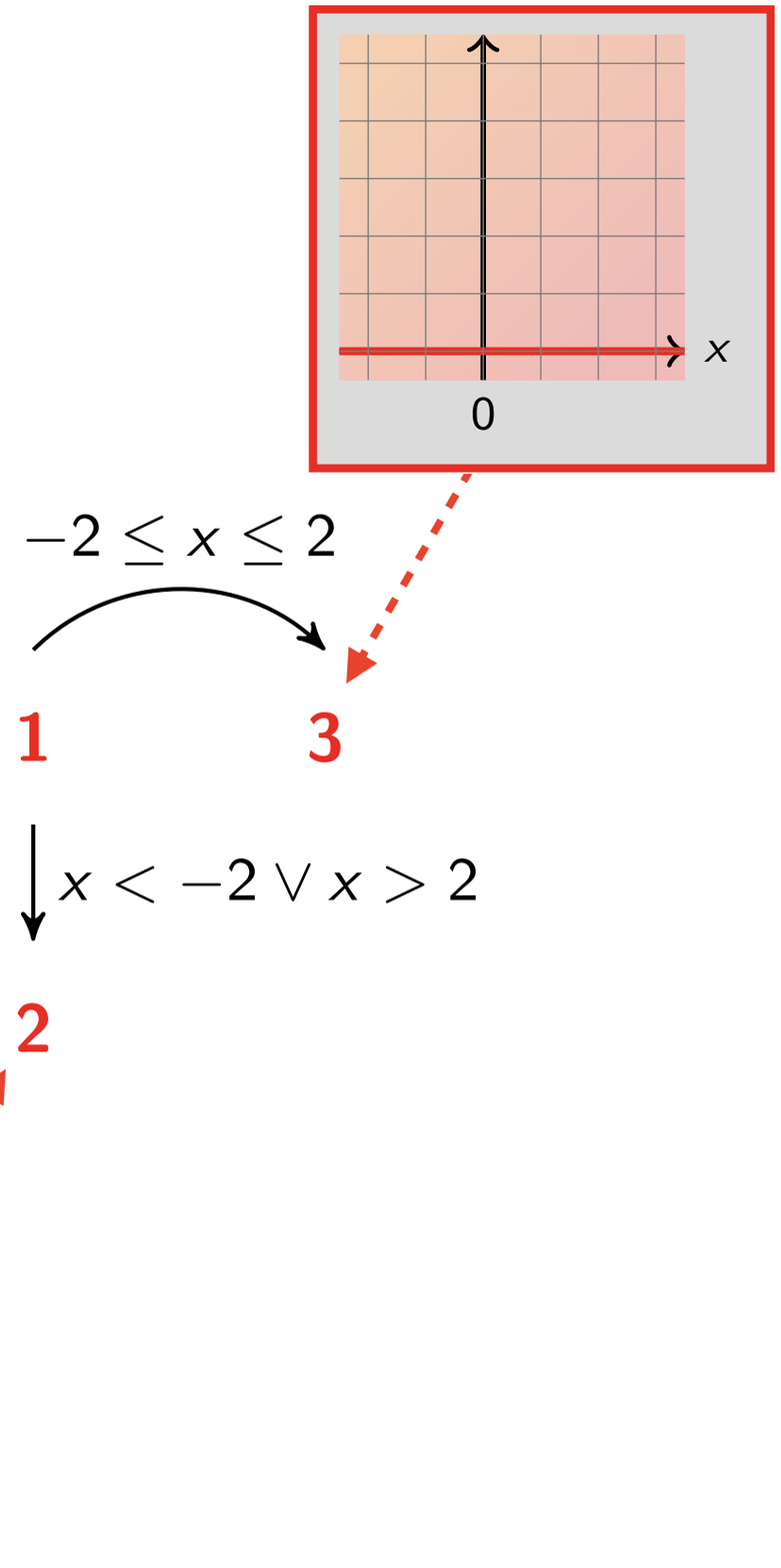

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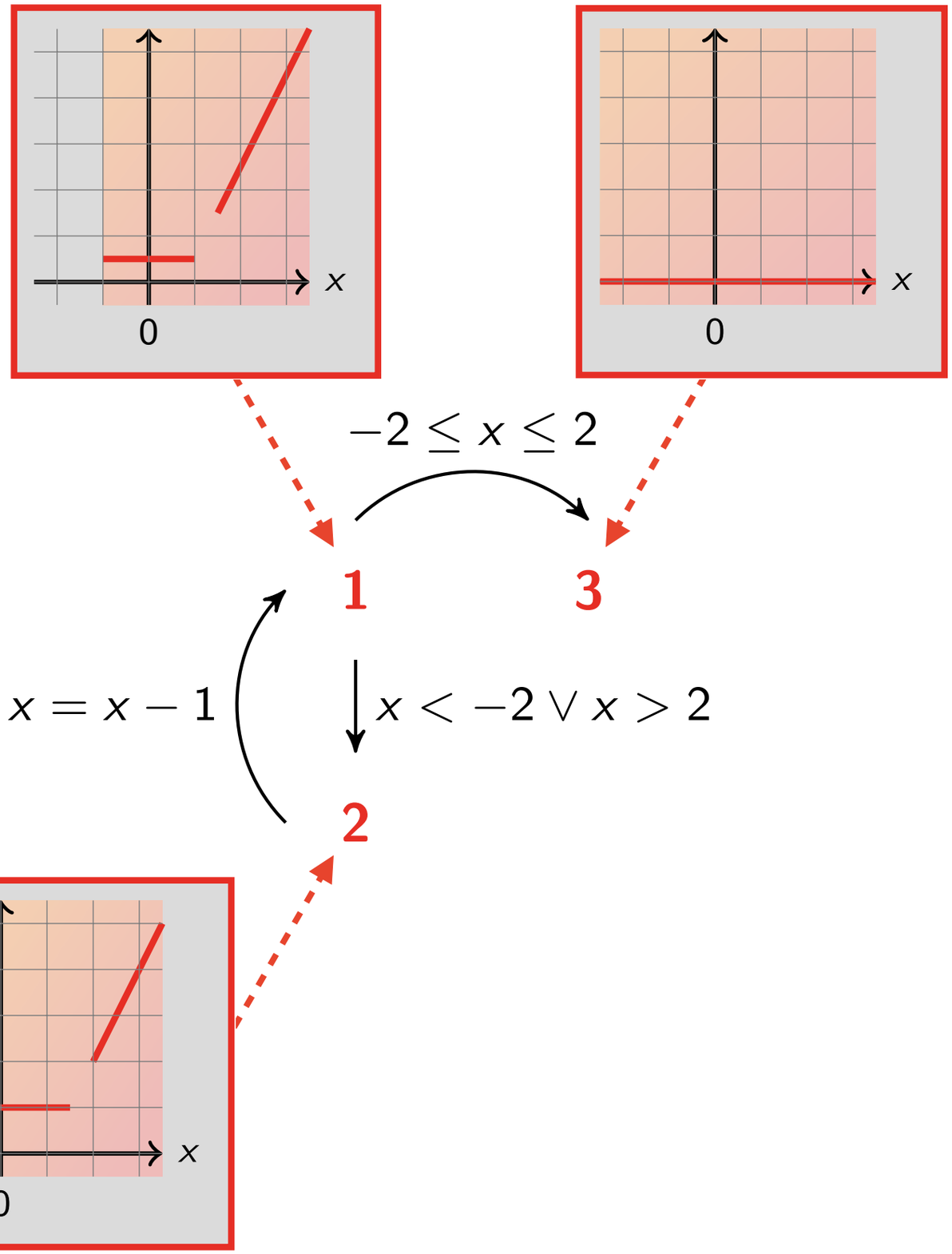

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different *step* values on different **branches**

the analysis **fails**

$$(x \vee z) \wedge (y \vee z) \wedge (\neg x \vee \neg z) \wedge (\neg y \vee z)$$

Bayardo & Schrag - Using CSP Look-Back Techniques to Solve Real World SAT Instances (1997)
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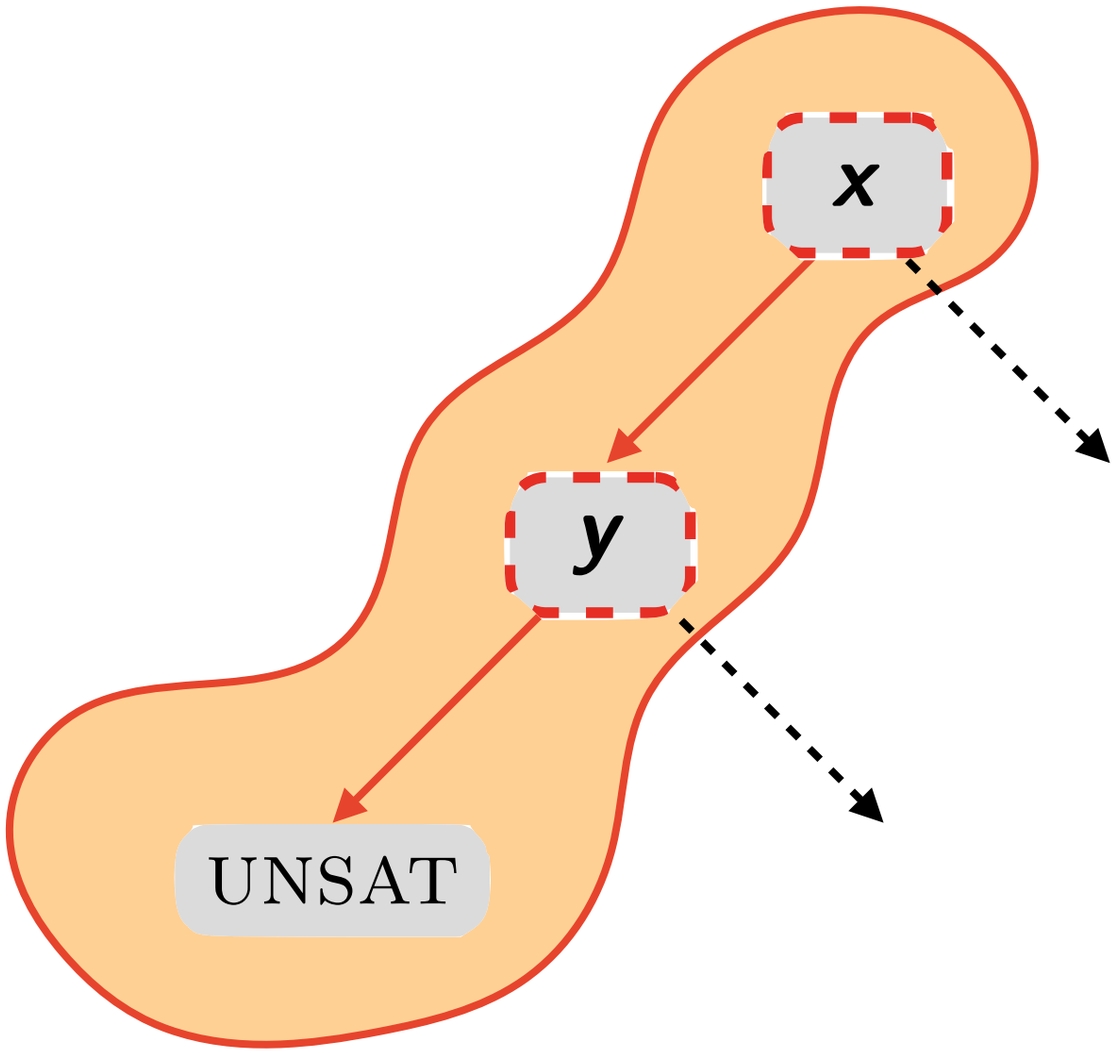

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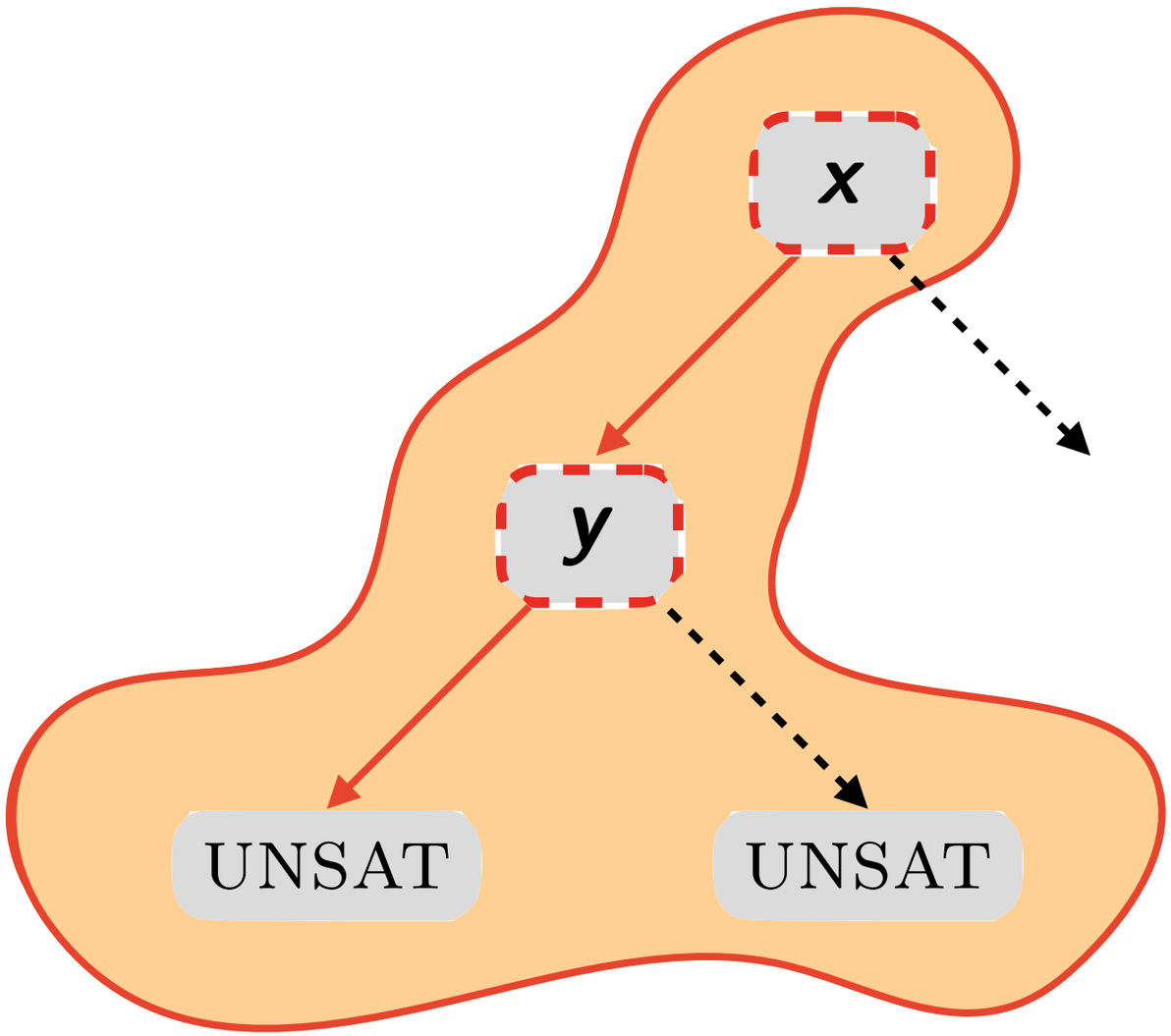

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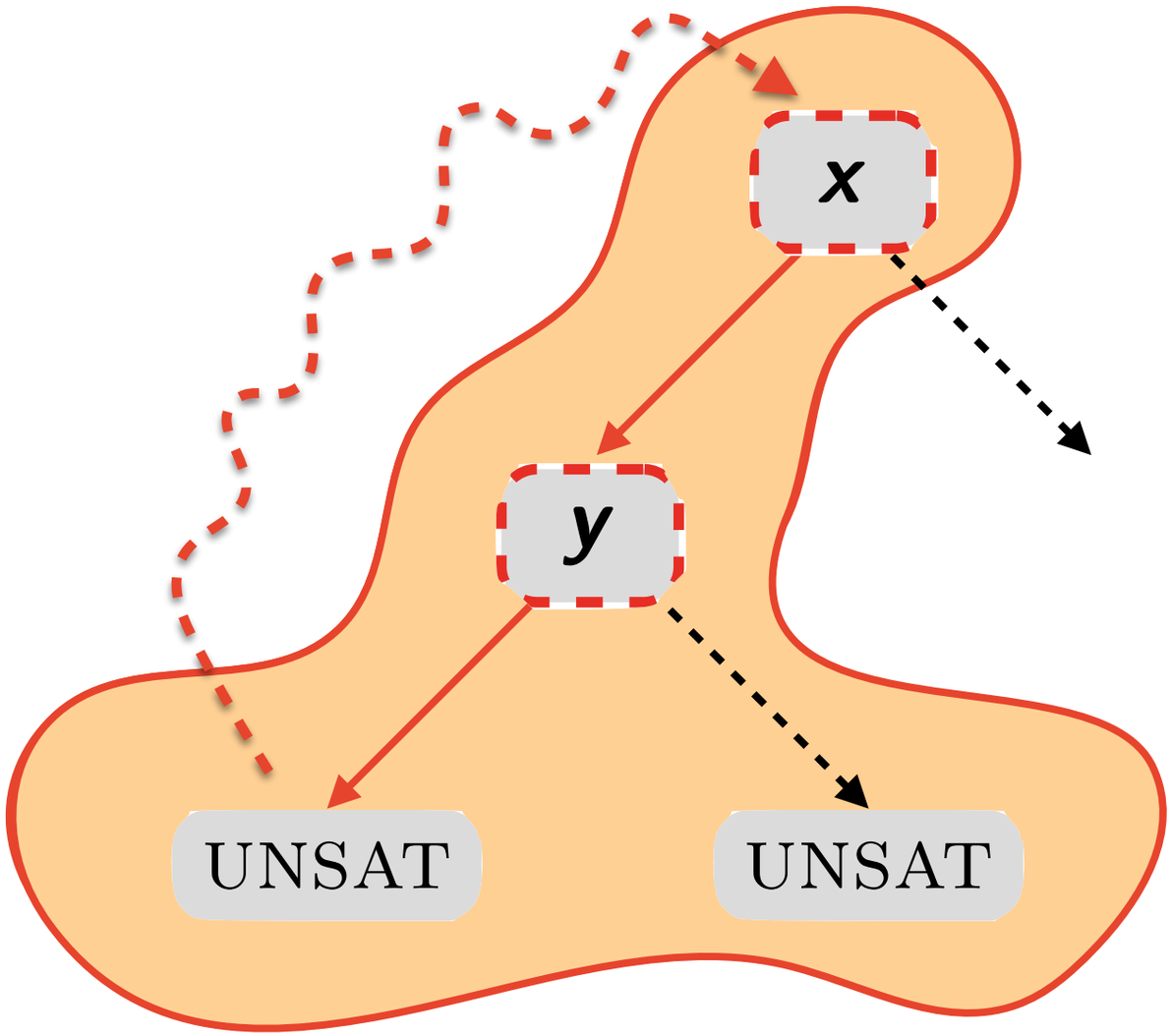

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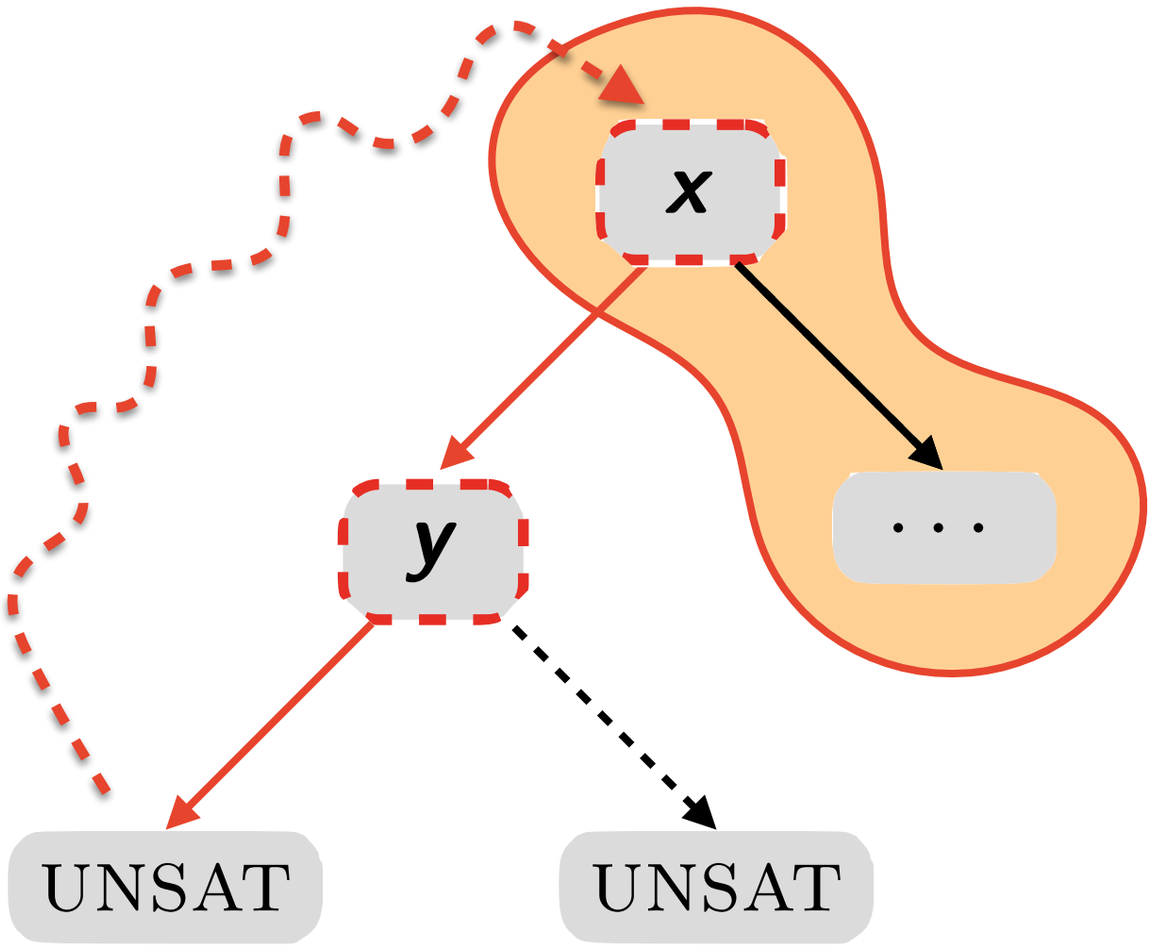

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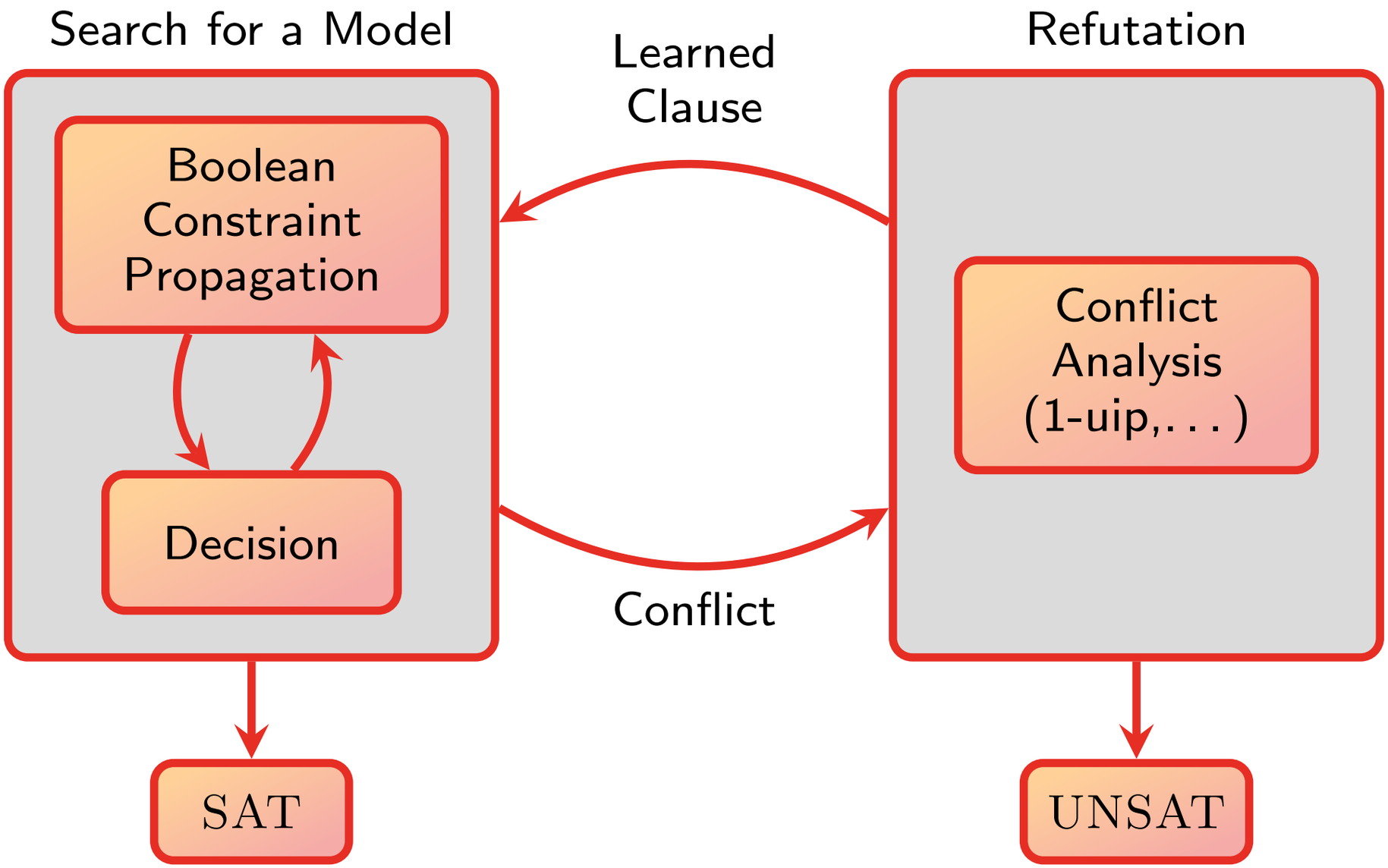

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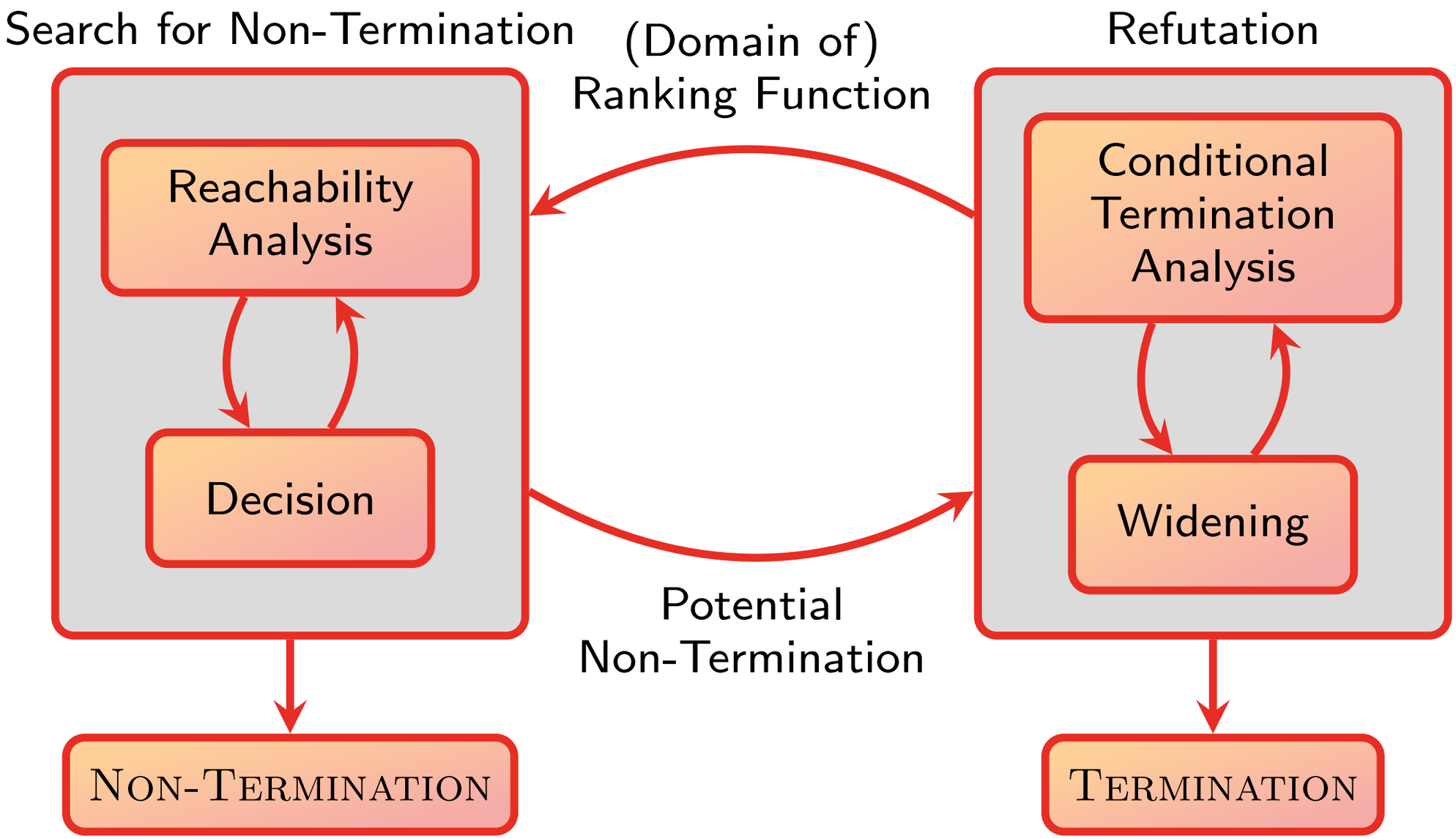
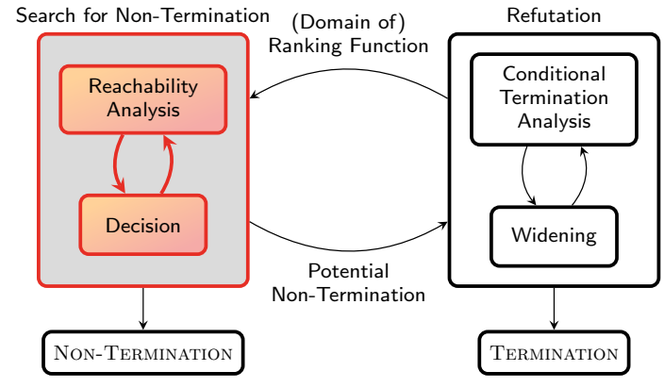

Search for Non-Termination
(Domain of)
Ranking Function
Refutation
Reachability
Analysis
Decision
Conditional
Termination
Analysis
Widening
Potential
Non-Termination
Non-Termination
Termination



Example

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int : x, step
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} else if 3 (x < 0) {
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while 8 (x < −2 ∨ x > 2) {
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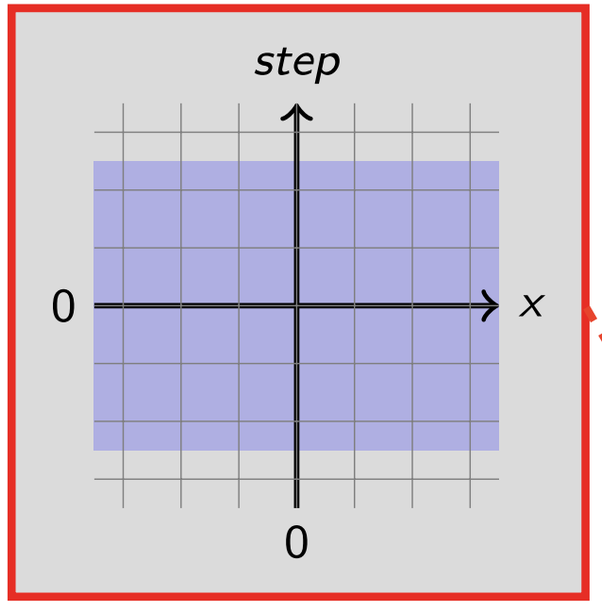
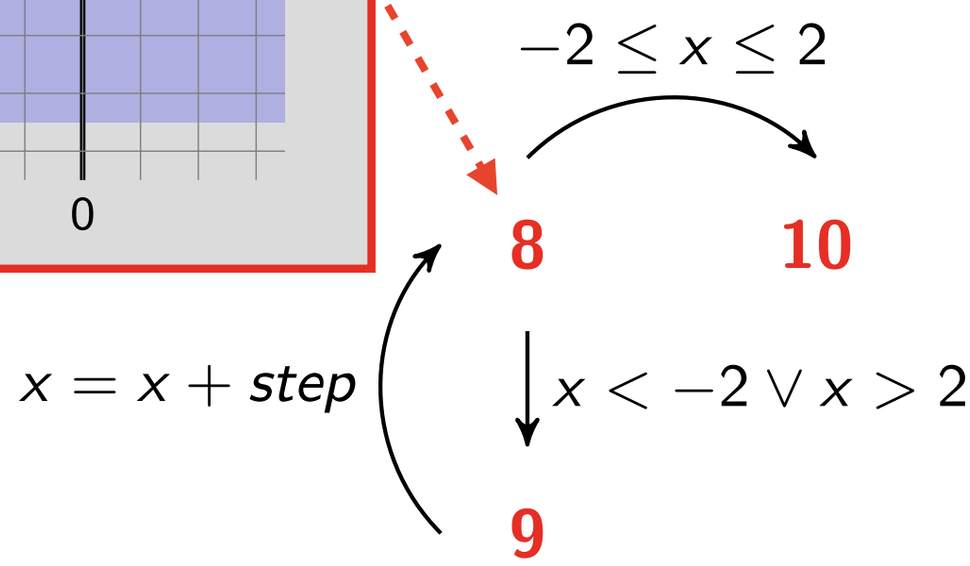


reachable states are potentially non-terminating



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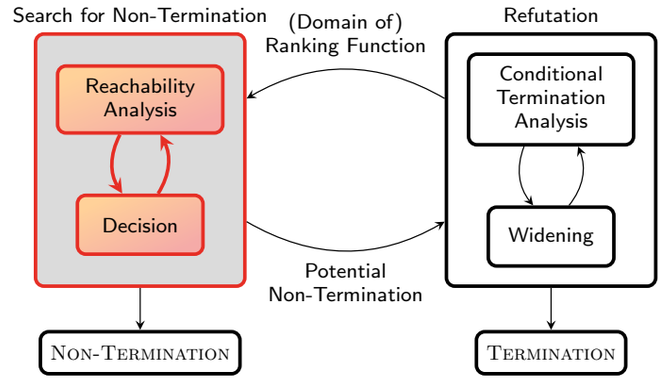
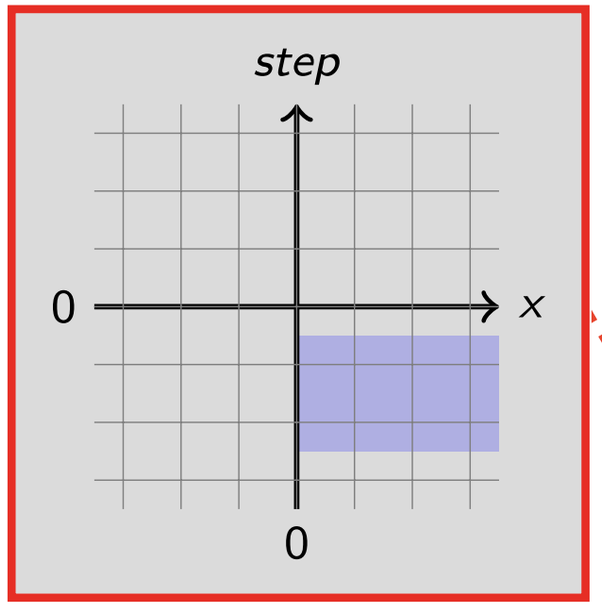
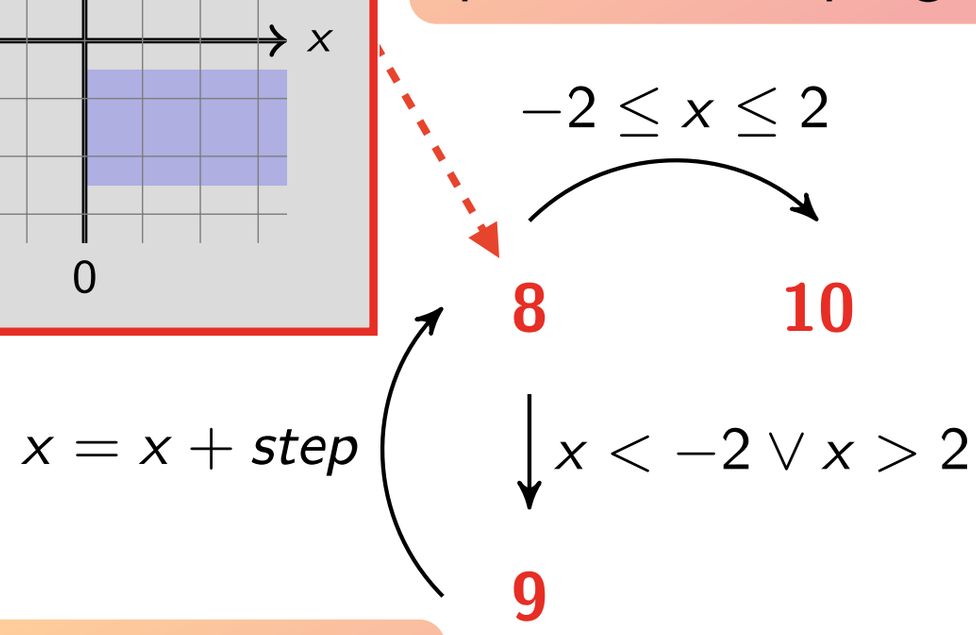
$x \geq 0$





the consequence of a decision is to focus the analyzer on certain paths of the program



a **decision** restricts an abstract element at a program control point

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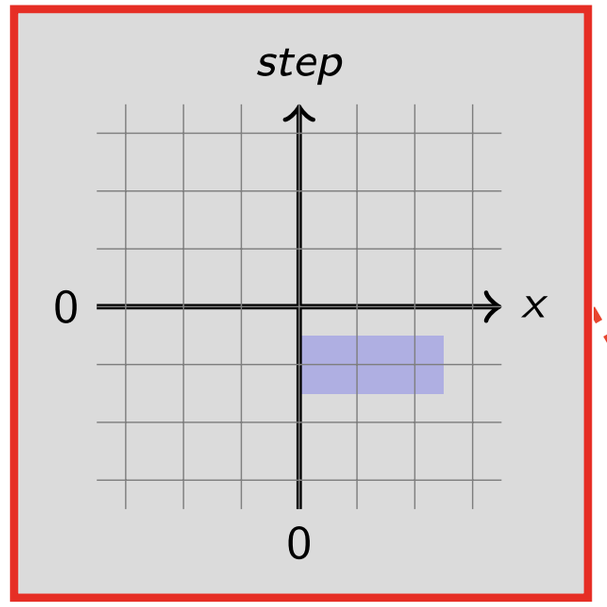

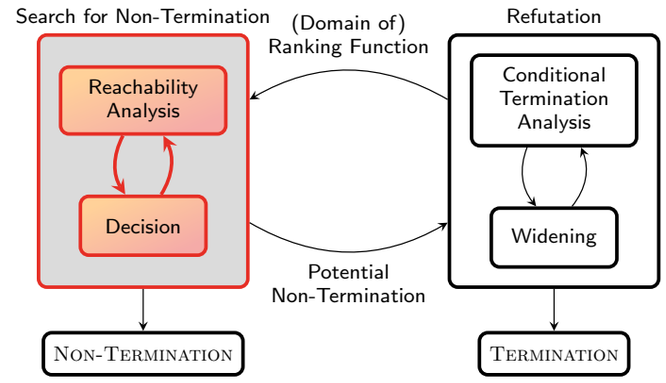

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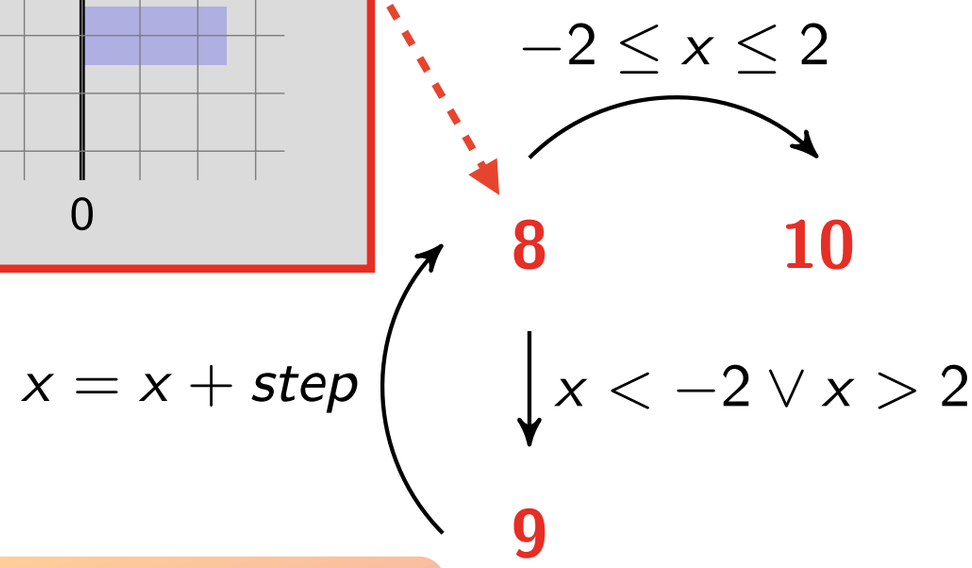

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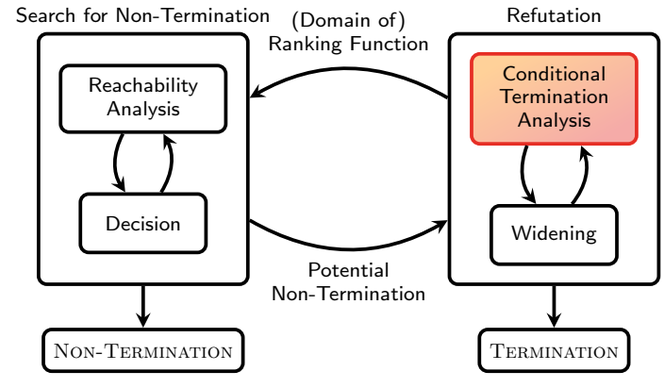

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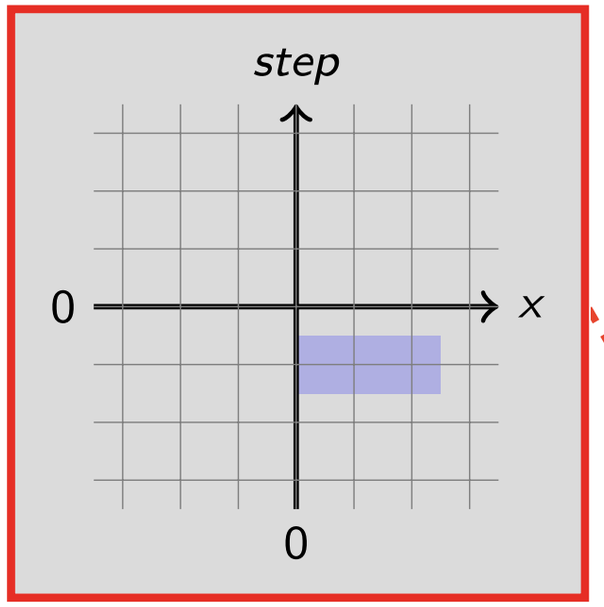
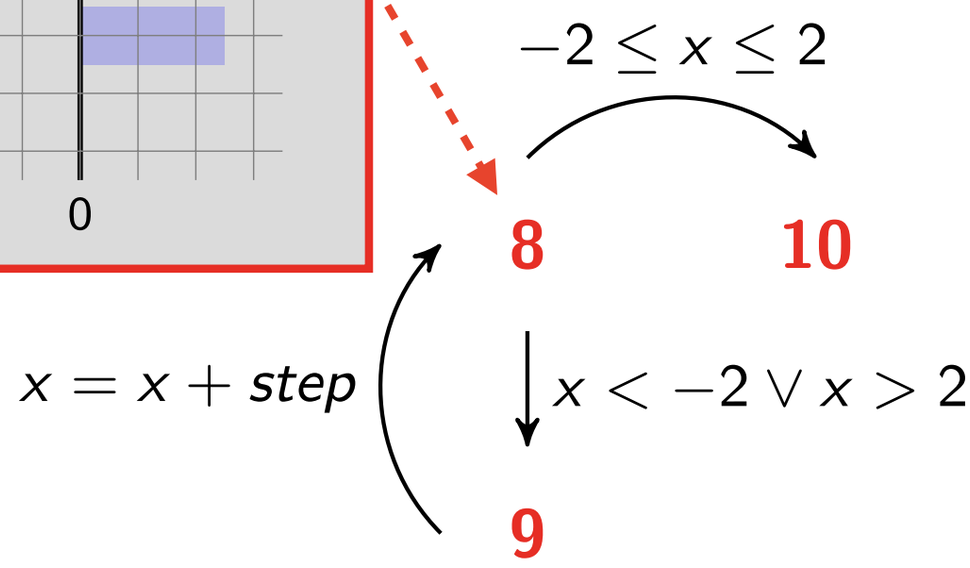
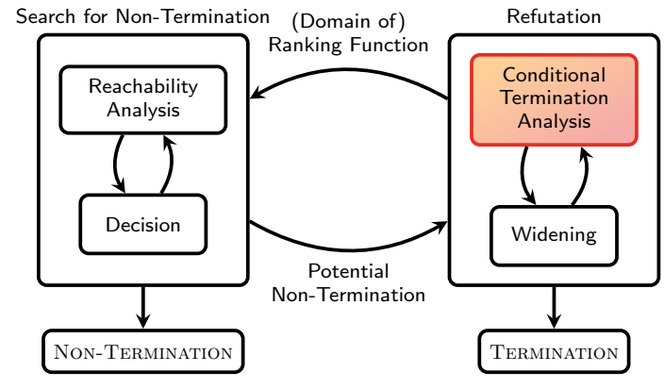




the conditional termination analysis is limited to the states identified by the reachability analysis



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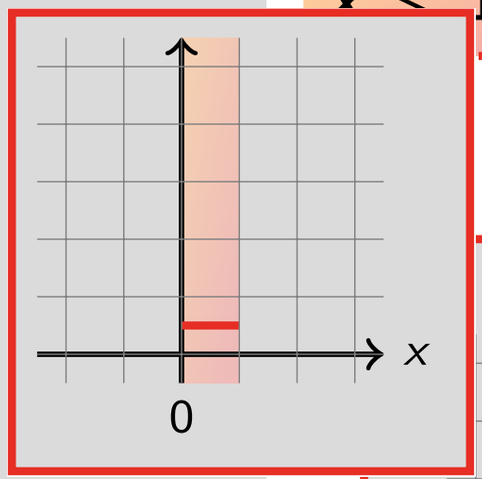

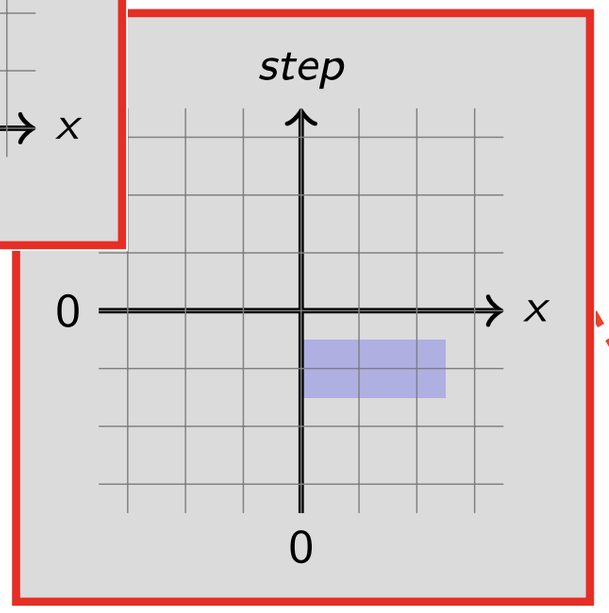

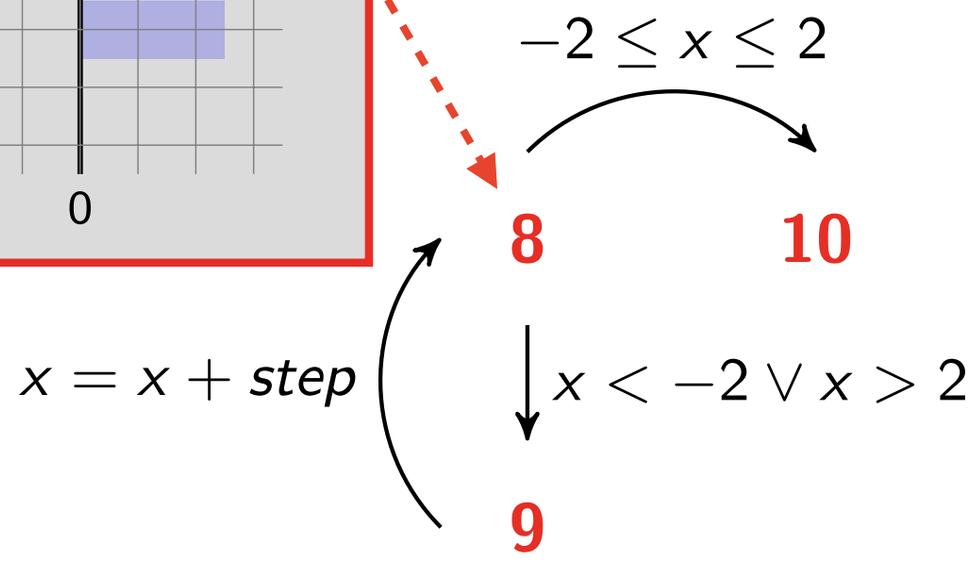

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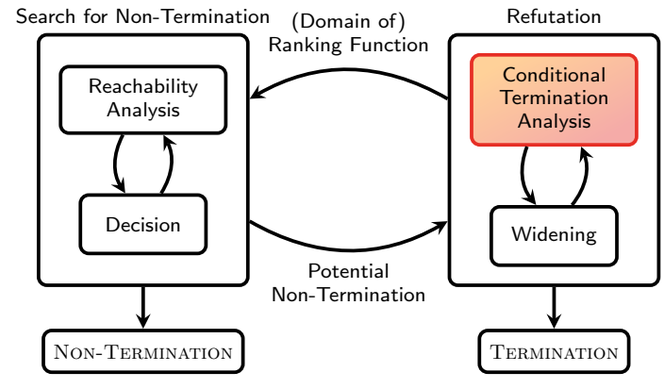

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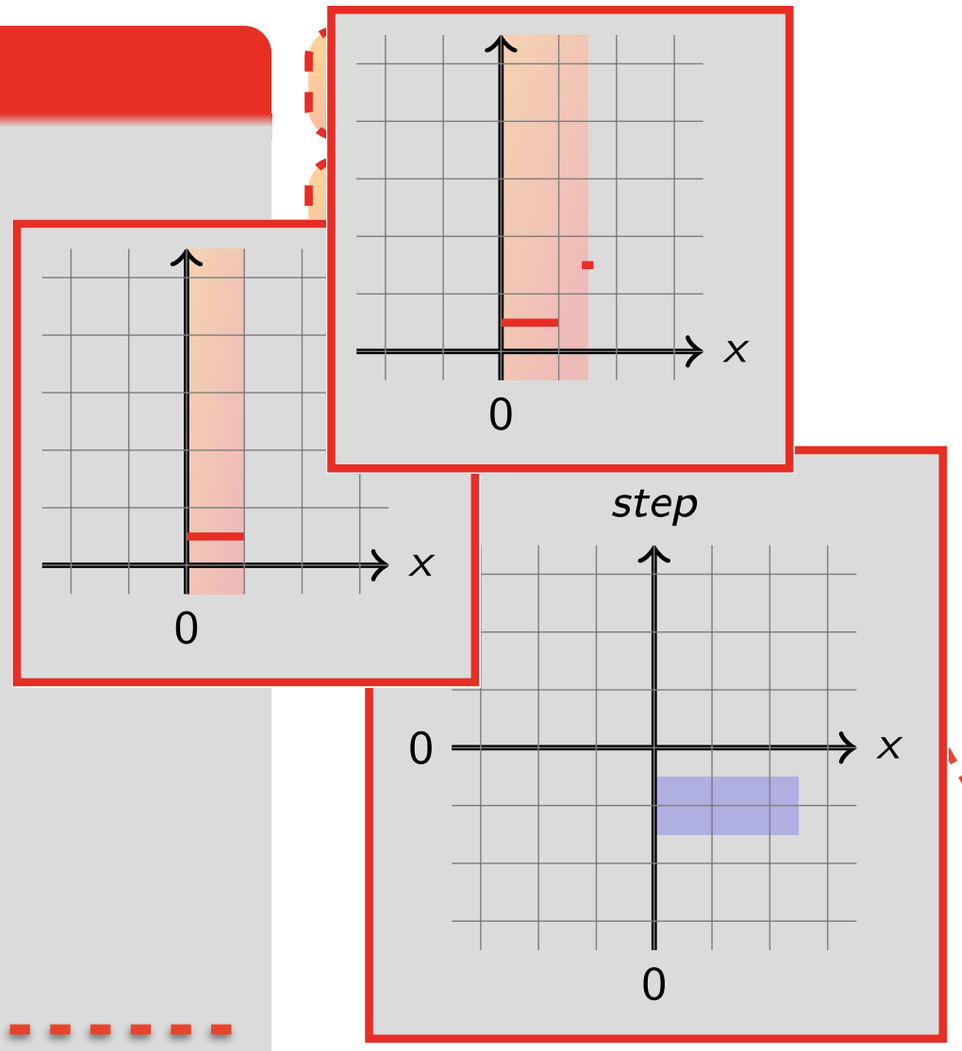

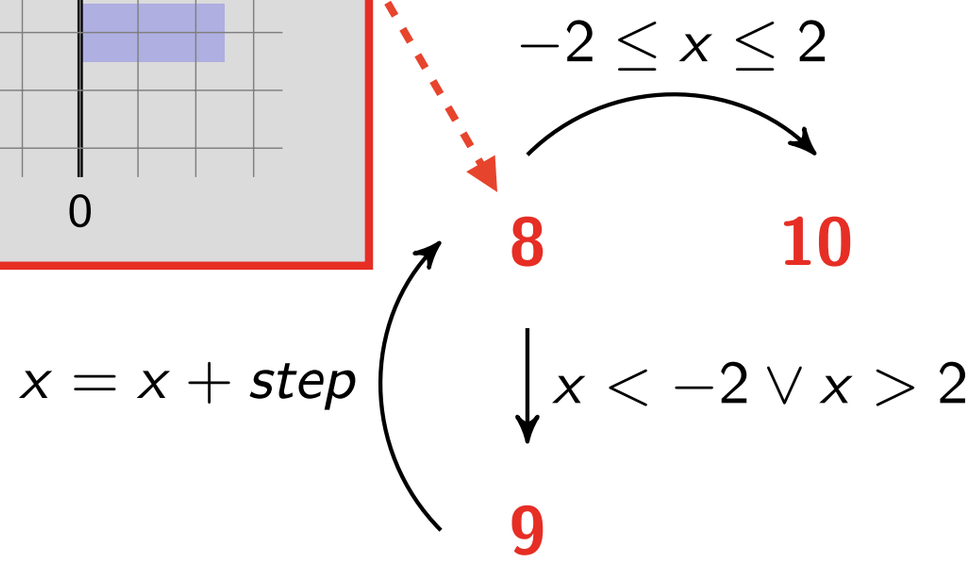

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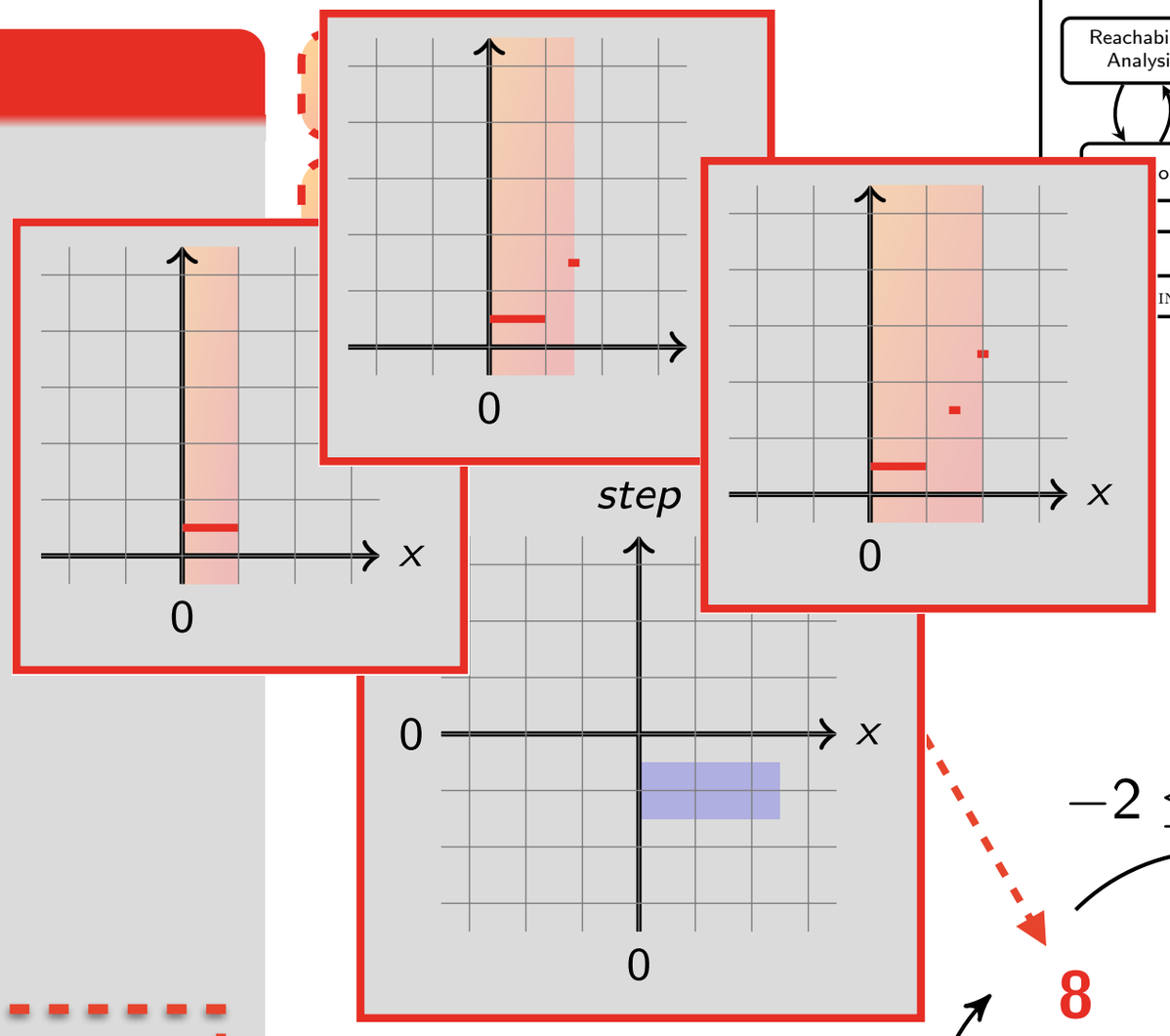
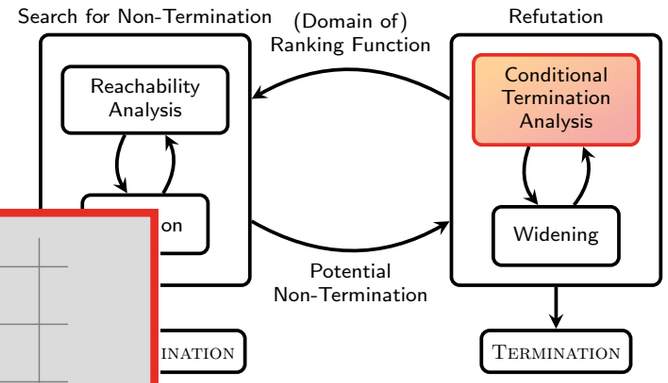
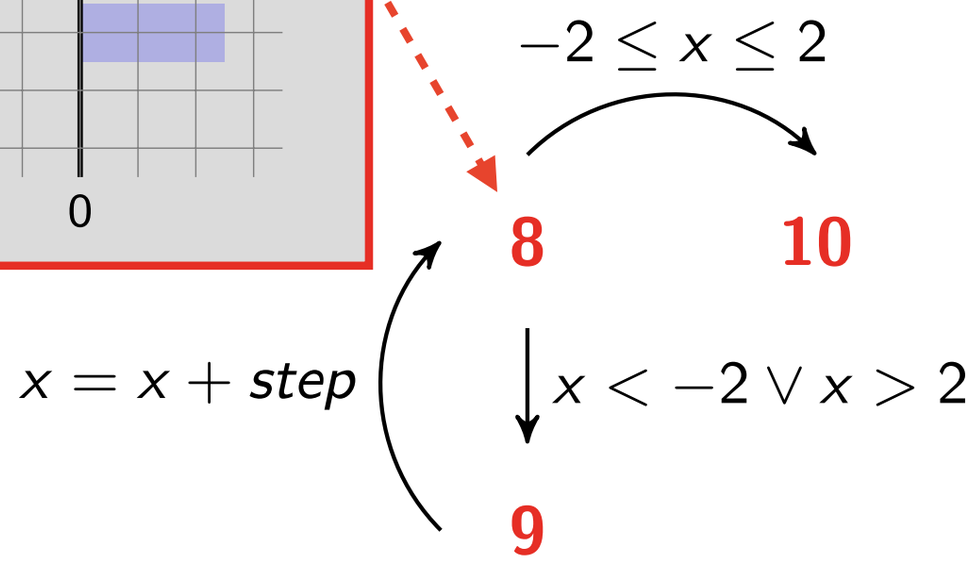
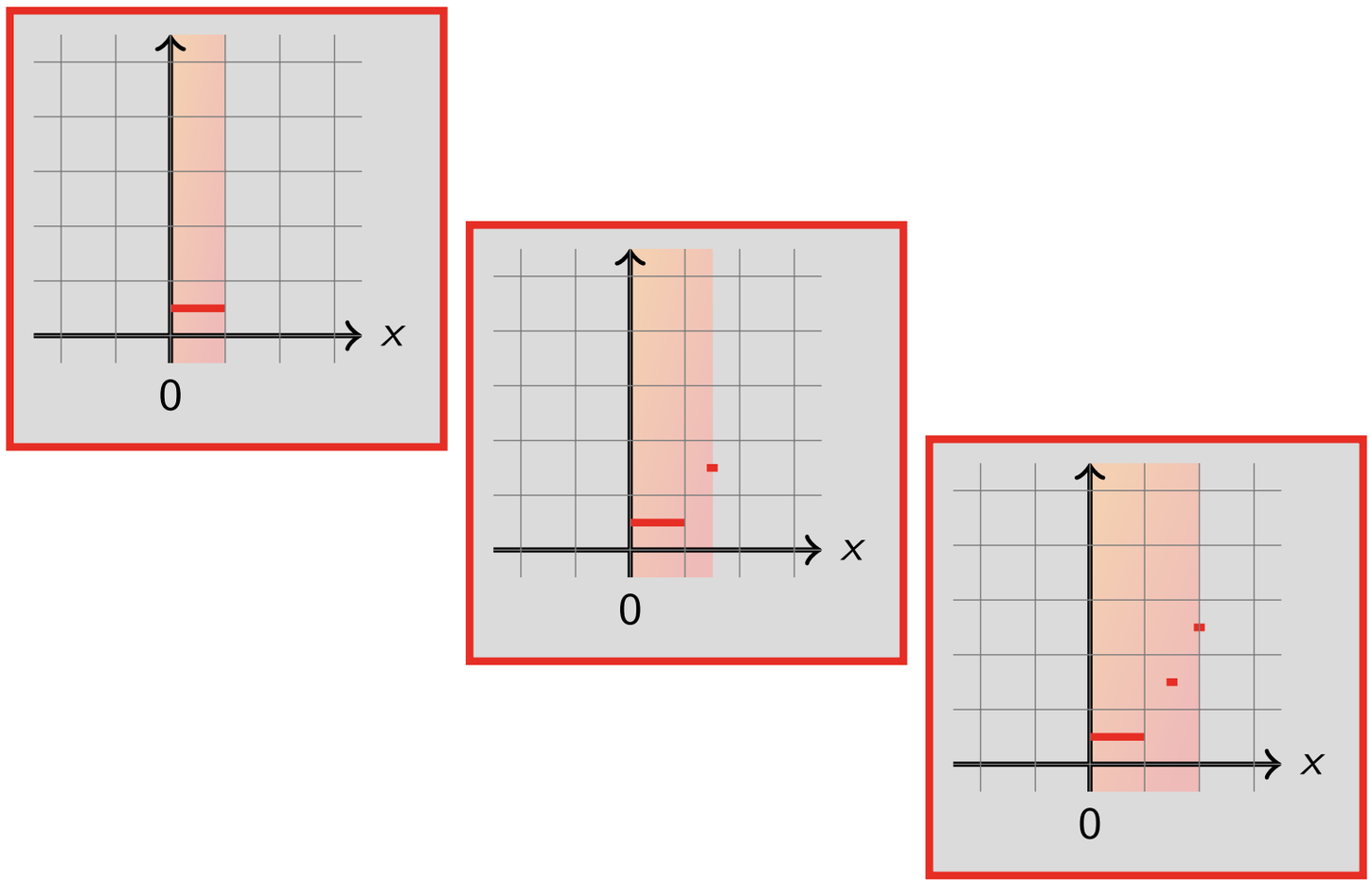






the conditional termination analysis is limited to the states identified by the reachability analysis

Widening



widening is the abstract interpretation approach to generalization

Example

int : x, $step$

if [1]$(x < -10)$ {

  [2]$step := 2$

} else if [3]$(x < 0)$ {

  [4]$step := 1$

} else if [5]$(x > 10)$ {

  [6]$step := -2$

} else {

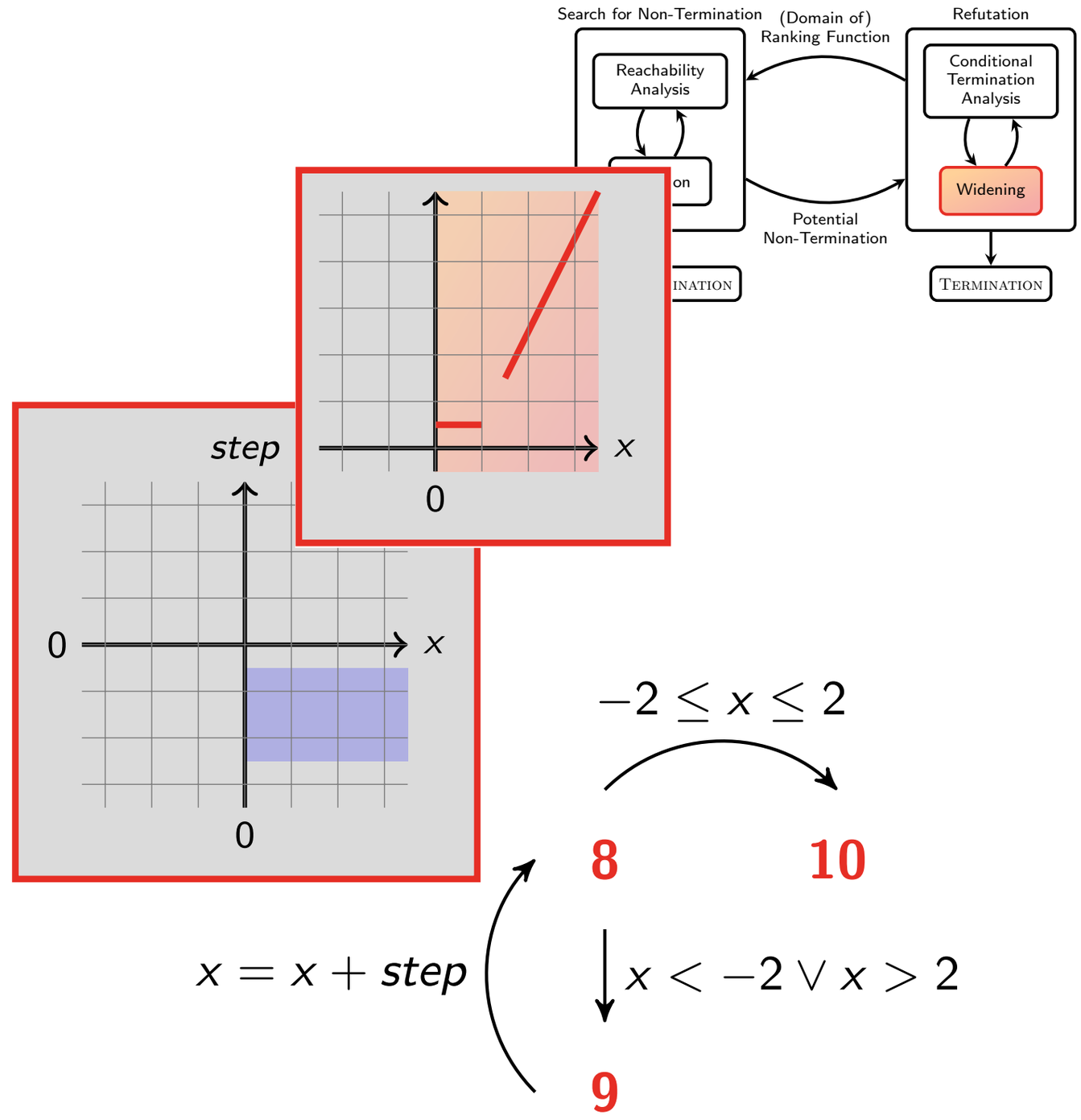
  [7]$step := -1$

}

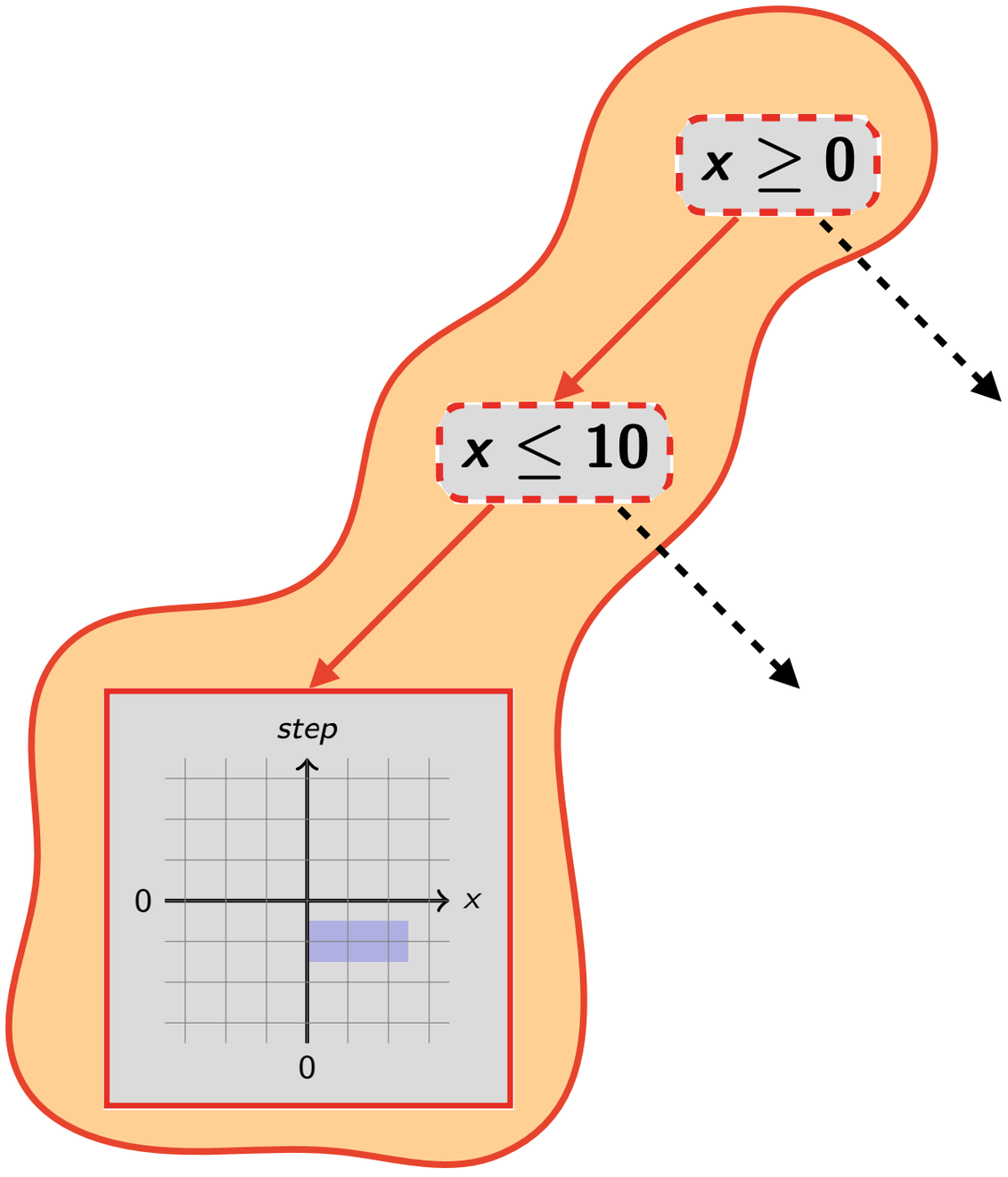
while [8]$(x < -2 \vee x > 2)$ {

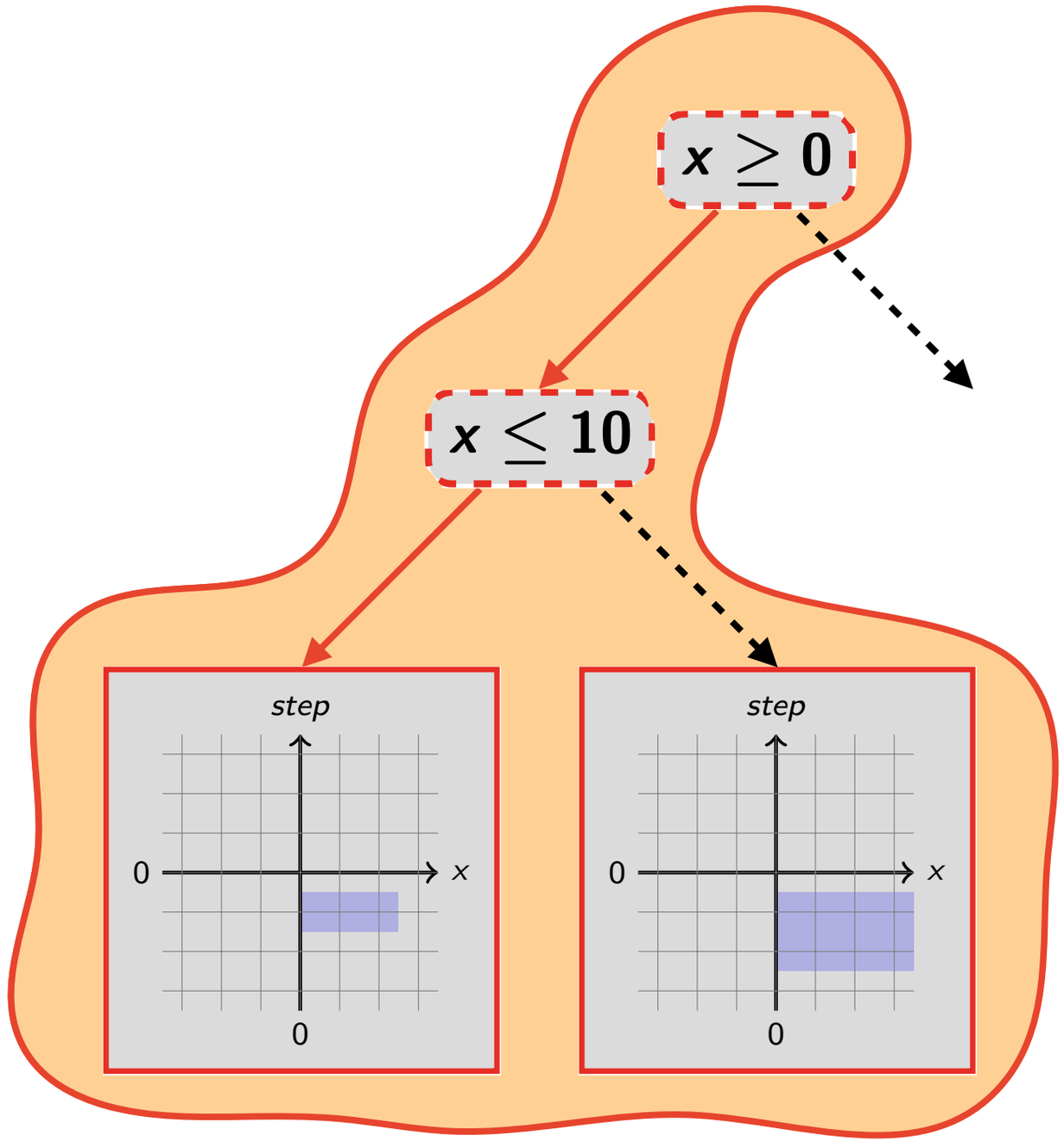
  [9]$x := x + step$

}[10]

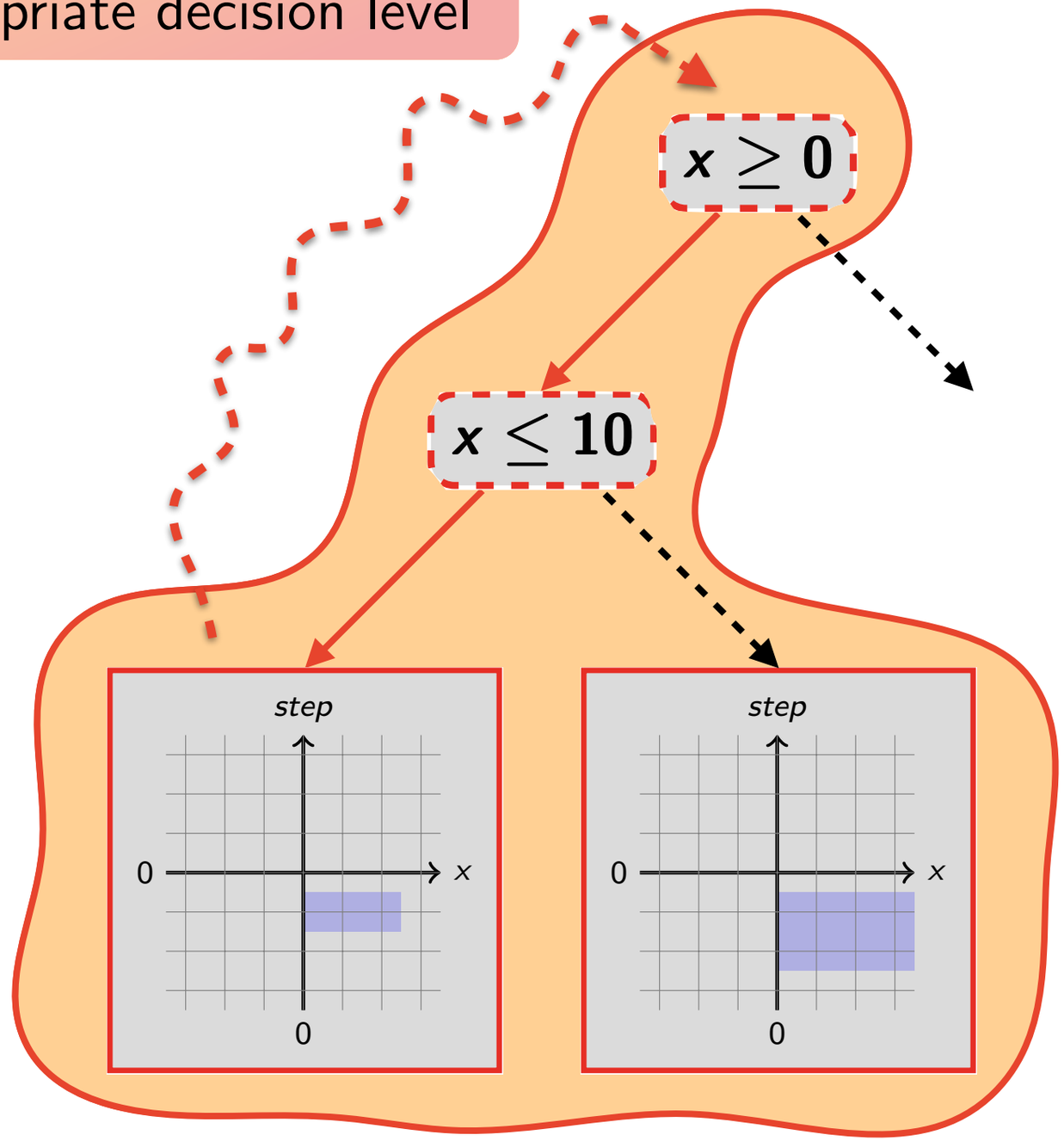


the widening generalizes the **value** and the **domain** of a ranking function

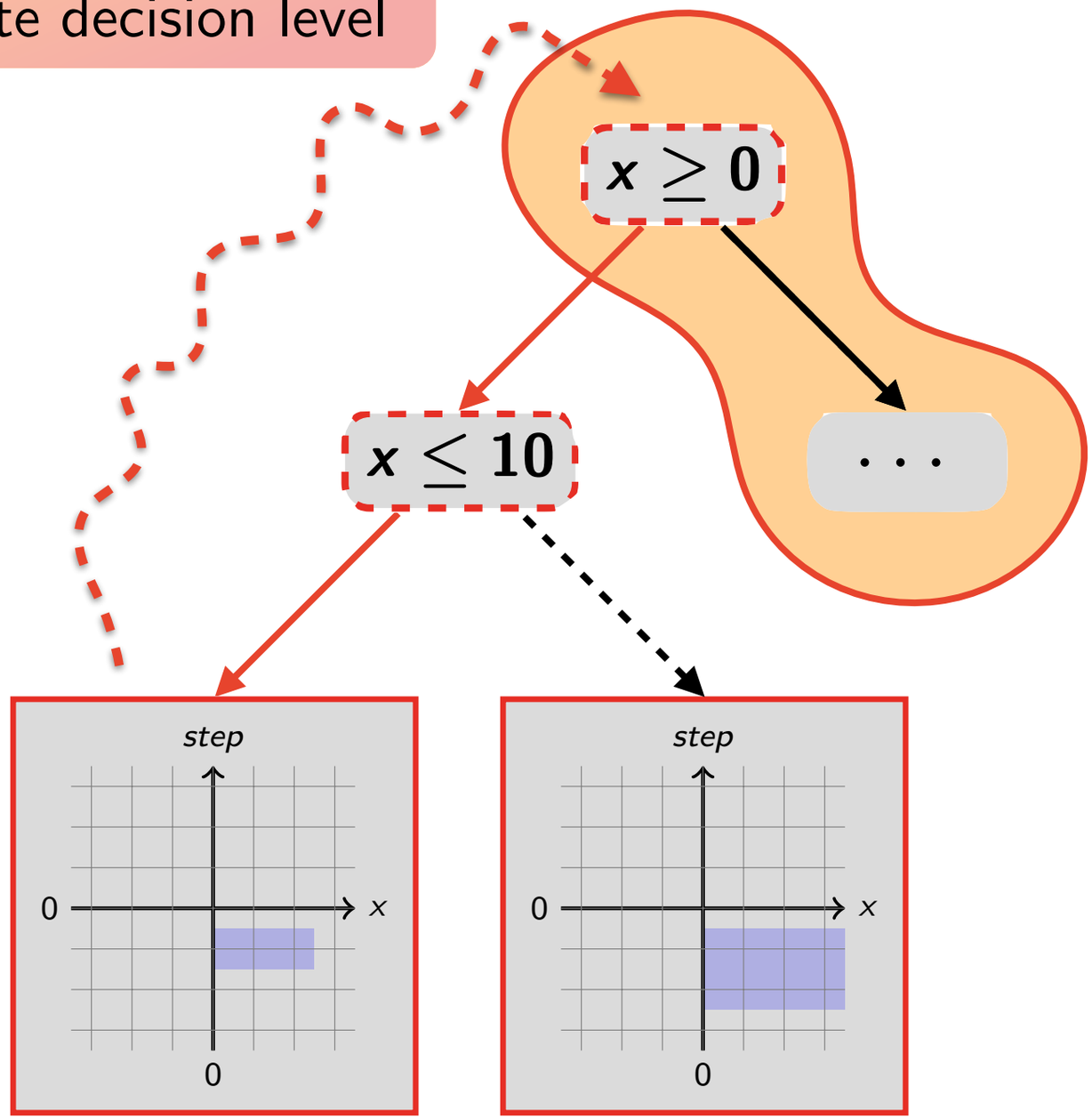

$x \geq 0$
$x \leq 10$
step
0
x
0


$x \geq 0$
$x \leq 10$
step
0
x
0
step
0
x
0

the **domain** of the ranking function guides the **backtracking** at the appropriate decision level

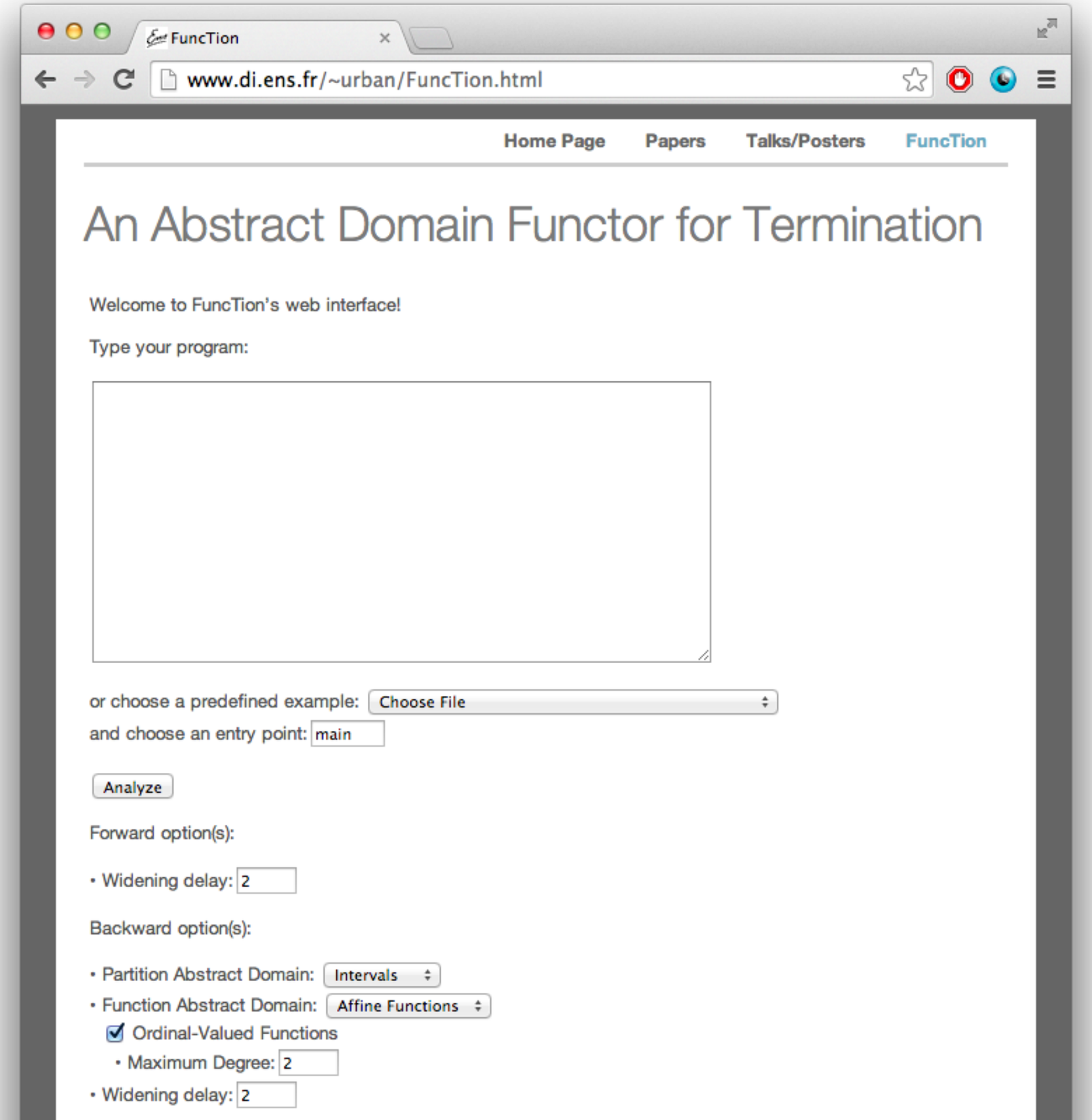


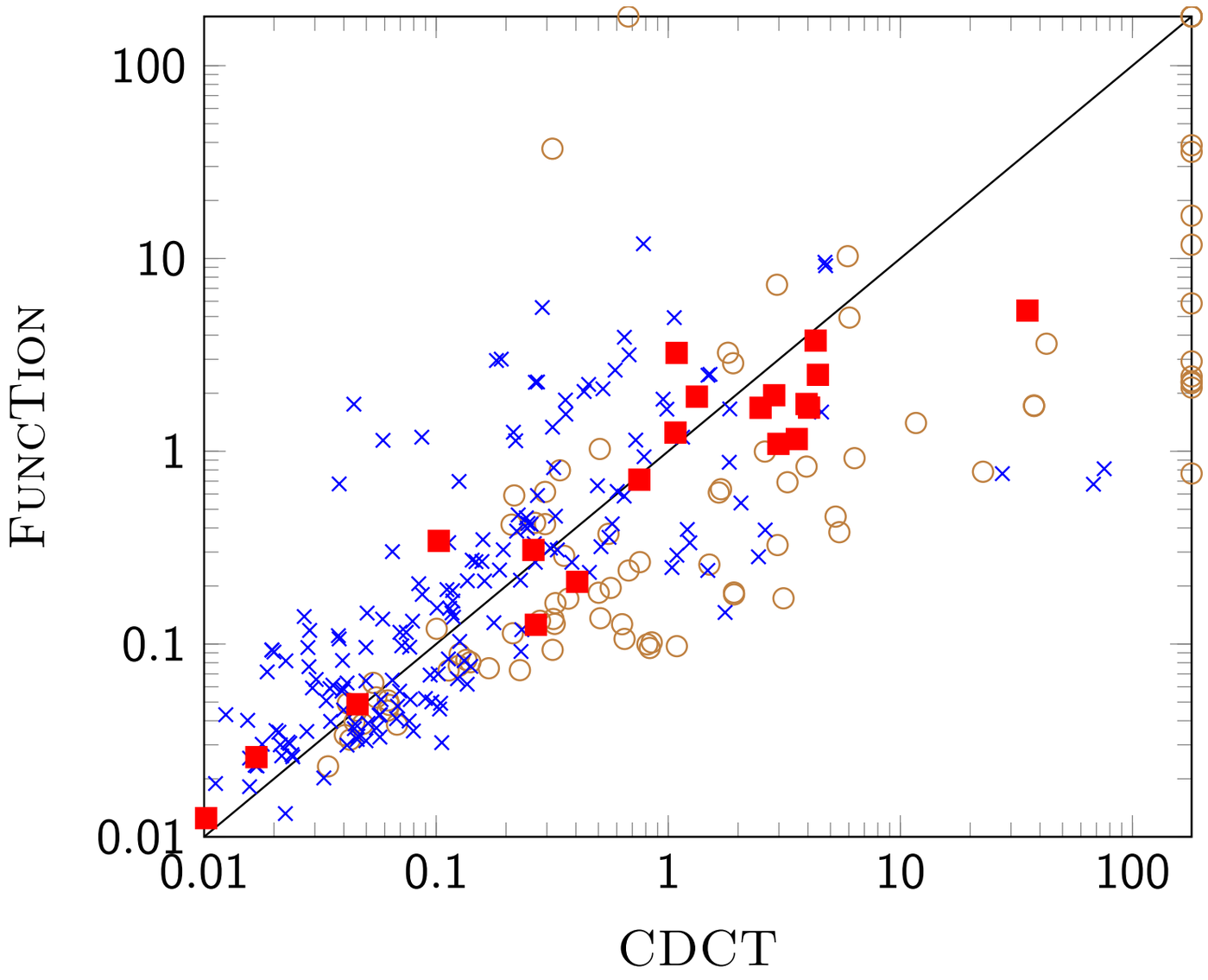
the **domain** of the ranking function guides the **backtracking** at the appropriate decision level



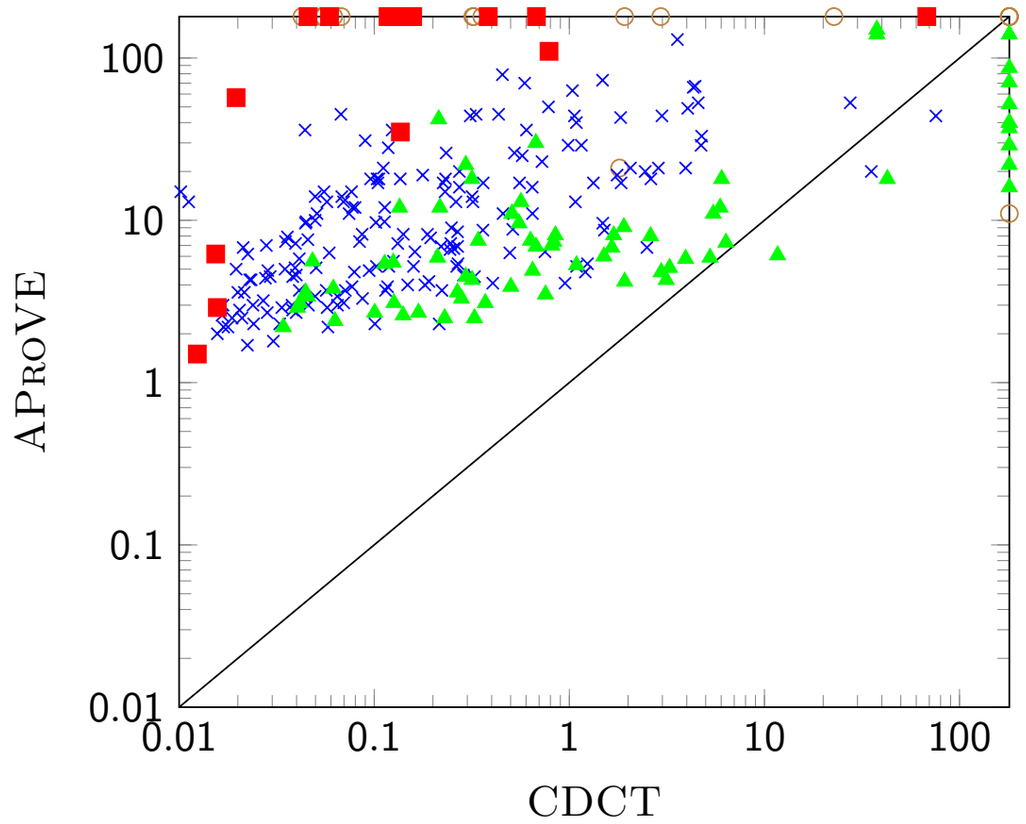
More in the paper...

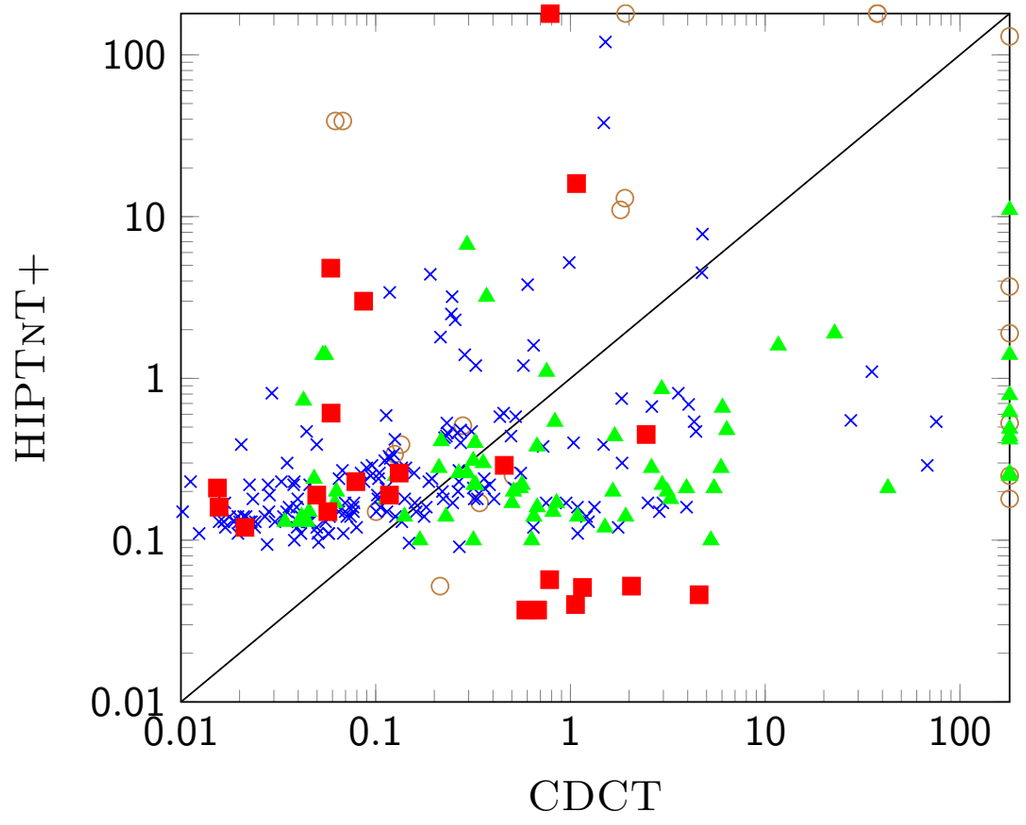
- non-termination as satisfiability via **Büchi's theorem**
- use of a **trail** to represent progress of an abstract interpreter
- **clausal representation** of conditional termination results
- a **generalized unit rule** to integrate clauses in propagation

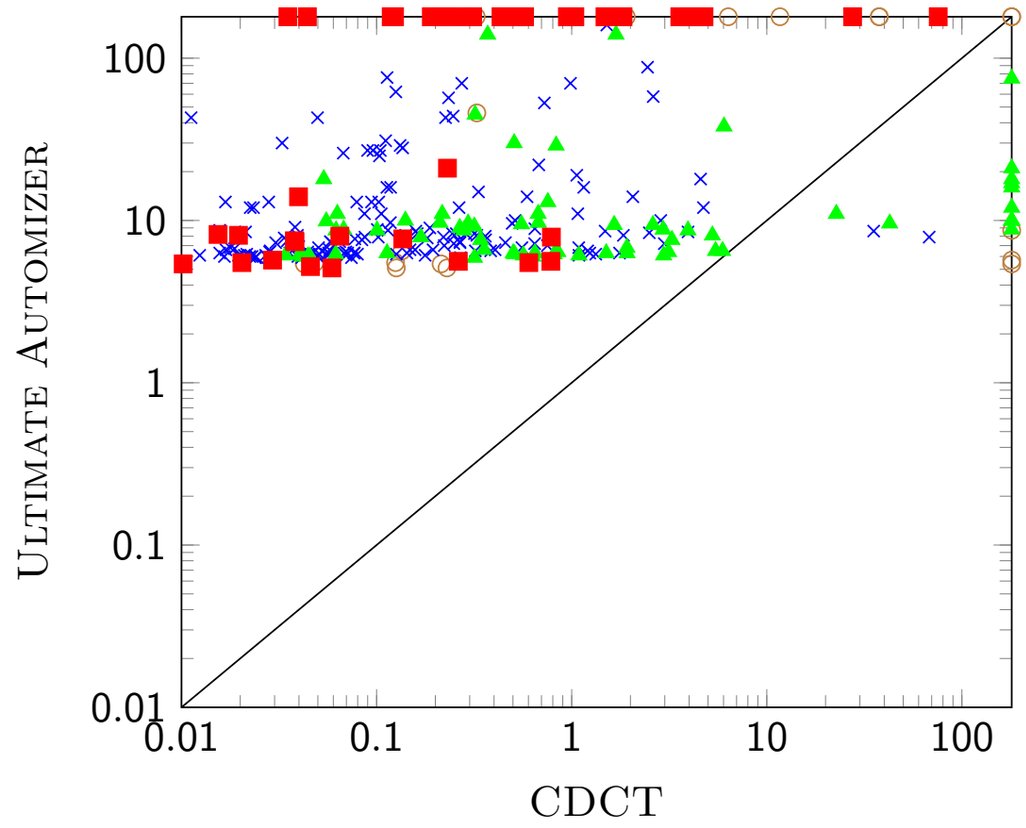

FuncTion
www.di.ens.fr/~urban/FuncTion.html
Home Page
Papers
Talks/Posters
FuncTion
An Abstract Domain Functor for Termination
Welcome to FuncTion's web interface!
Type your program:
or choose a predefined example:
Choose File
and choose an entry point:
main
Analyze
Forward option(s):
• Widening delay:
2
Backward option(s):
• Partition Abstract Domain:
Intervals
• Function Abstract Domain:
Affine Functions
Ordinal-Valued Functions
• Maximum Degree:
2
• Widening delay:
2



- 288 terminating programs
- 8.7% CDCT is more precise (■)
- 65.7% CDCT is faster (×)


AProVE
CDCT
0.01
0.1
1
10
100


HIPTnT+
CDCT
0.01
0.1
1
10
100


Ultimate Automizer
CDCT
0.01
0.1
1
10
100

Conclusion

- **conflict-driven learning** for conditional termination
  - search: potential non-termination
  - refutation: conditional termination analysis
- application to **abstract interpretation-based** analysis
  - potential for application to other termination analyses

Future Work

- better potential **non-termination** analysis
- better **decision heuristics**
- better **widening**
- conflict-driven learning for **liveness properties**