MS4303 Project

Cathaoir Agnew 16171659

February 3, 2020

1 My linear Program

Minimise:

$$z = -1x_1 + 4x_2 - 2x_3 - 5x_4 - 4x_5 - 2x_6 + 5x_7 + 3x_8$$

Constraint Equations:

$$1x_1 - 4x_2 + 2x_3 + 2x_4 - 3x_5 + 1x_6 - 4x_7 + 4x_8 + 1x_9 = -5.71703825$$

$$2x_1 + 0x_2 + 3x_3 + 2x_4 + 1x_5 - 1x_6 + 0x_7 + 1x_8 + 1x_{10} = 117.49007285$$

$$-1x_1 + 0x_2 + 0x_3 + 4x_4 - 1x_5 + 3x_6 - 1x_7 + 1x_8 + 1x_{11} = 78.47199761$$

$$-4x_1 + 1x_2 + 5x_3 - 2x_4 + 5x_5 + 1x_6 - 1x_7 - 1x_8 + 1x_{12} = 27.05784774$$

$$-2x_1 + 1x_2 + 1x_3 - 2x_4 - 3x_5 + 4x_6 - 5x_7 + 2x_8 + 1x_{13} = -61.27608490$$

$$3x_1 - 2x_2 + 4x_3 - 2x_4 - 4x_5 + 3x_6 - 5x_7 + 1x_8 + 1x_{14} = -44.14281050$$

$$3x_1 + 4x_2 - 3x_3 + 4x_4 - 5x_5 + 3x_6 + 1x_7 - 4x_8 + 1x_{15} = 59.07935665$$

$$4x_1 + 3x_2 - 1x_3 + 3x_4 + 3x_5 - 3x_6 - 3x_7 - 3x_8 + 1x_{16} = 12.53792958$$

2 Solution to Optimality

Standard Form Tableau for problem:

$$\mathbf{T_0} = \begin{bmatrix} 0 & -1 & 4 & -2 & -5 & -4 & -2 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.71703825 & 1 & -4 & 2 & 2 & -3 & 1 & -4 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 117.49007285 & 2 & 0 & 3 & 2 & 1 & -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 78.47199761 & -1 & 0 & 0 & 4 & -1 & 3 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 27.05784774 & -4 & 1 & 5 & -2 & 5 & 1 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -61.27608490 & -2 & 1 & 1 & -2 & -3 & 4 & -5 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -44.14281050 & 3 & -2 & 4 & -2 & -4 & 3 & -5 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 59.07935665 & 3 & 4 & -3 & 4 & -5 & 3 & 1 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 12.53792958 & 4 & 3 & -1 & 3 & 3 & -3 & -3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

First step in turning the starting tableau into a canonical from tableau was making the furthest left hand column, (L.H.C), elements all positive. For my first pivot using the dual simplex method, I chose the most negative element in the L.H.C, which was -61.27608490. I then went across the row and calculated the column ratios, considering only negative elements in the tableau for the pivot.The most positive row ratio was in row 6 column 5, so I pivoted there.

Tableau T_1, after first pivot:

$$\mathbf{T_1} = \begin{bmatrix} 153.1902 & 4.0000 & 1.5000 & -4.5000 & 0 & 3.5000 & -12.0000 & 17.5000 & -2.0000 & 0 & 0 & 0 & 0 & -2.5000 & 0 & 0 & 0 \\ -66.9931 & -1.0000 & -3.0000 & 3.0000 & 0 & -6.0000 & 5.0000 & -9.0000 & 6.0000 & 1.0000 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 56.2140 & 0 & 1.0000 & 4.0000 & 0 & -2.0000 & 3.0000 & -5.0000 & 3.0000 & 0 & 1.0000 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ -44.0802 & -5.0000 & 2.0000 & 2.0000 & 0 & -7.0000 & 11.0000 & -11.0000 & 5.0000 & 0 & 0 & 1.0000 & 0 & 2.0000 & 0 & 0 & 0 \\ 88.3339 & -2.0000 & 0 & 4.0000 & 0 & 8.0000 & -3.0000 & 4.0000 & -3.0000 & 0 & 0 & 0 & 1.0000 & -1.0000 & 0 & 0 & 0 \\ 30.6380 & 1.0000 & -0.5000 & -0.5000 & 1.0000 & 1.5000 & -2.0000 & 2.5000 & -1.0000 & 0 & 0 & 0 & 0 & -0.5000 & 0 & 0 & 0 \\ 17.1333 & 5.0000 & -3.0000 & 3.0000 & 0 & -1.0000 & -1.0000 & 0 & -1.0000 & 0 & 0 & 0 & 0 & -1.0000 & 1.0000 & 0 & 0 \\ -63.4728 & -1.0000 & 6.0000 & -1.0000 & 0 & -11.0000 & 11.0000 & -9.0000 & 0 & 0 & 0 & 0 & 0 & 2.0000 & 0 & 1.0000 & 0 \\ -79.3762 & 1.0000 & 4.5000 & 0.5000 & 0 & -1.5000 & 3.0000 & -10.5000 & 0 & 0 & 0 & 0 & 0 & 1.5000 & 0 & 0 & 1.0000 \end{bmatrix}$$

Pivoting again using the dual simplex method, on the most negative element in the L.H.C, row 9 = -79.3762, with the most positive row ratio, considering only negative elements in the row. Pivot on row 9, column 8.

Tableau T_2, after first pivot:

$$\mathbf{T_2} = \begin{bmatrix} 20.8965 & 5.6667 & 9.0000 & -3.6667 & 0 & 1.0000 & -7.0000 & 0 & -2.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.6667 \\ 1.0436 & -1.8571 & -6.8571 & 2.5714 & 0 & -4.7143 & 2.4286 & 0 & 6.0000 & 1.0000 & 0 & 0 & 0 & -0.2857 & 0 & 0 & -0.8571 \\ 94.0122 & -0.4762 & -1.1429 & 3.7619 & 0 & -1.2857 & 1.5714 & 0 & 3.0000 & 0 & 1.0000 & 0 & 0 & 0.2857 & 0 & 0 & -0.4762 \\ 39.0758 & -6.0476 & -2.7143 & 1.4762 & 0 & -5.4286 & 7.8571 & 0 & 5.0000 & 0 & 0 & 1.0000 & 0 & 0.4286 & 0 & 0 & -1.0476 \\ 58.0954 & -1.6190 & 1.7143 & 4.1905 & 0 & 7.4286 & -1.8571 & 0 & -3.0000 & 0 & 0 & 0 & 1.0000 & -0.4286 & 0 & 0 & 0.3810 \\ 11.7389 & 1.2381 & 0.5714 & -0.3810 & 1.0000 & 1.1429 & -1.2857 & 0 & -1.0000 & 0 & 0 & 0 & 0 & -0.1429 & 0 & 0 & 0.2381 \\ 17.1333 & 5.0000 & -3.0000 & 3.0000 & 0 & -1.0000 & -1.0000 & 0 & -1.0000 & 0 & 0 & 0 & 0 & -1.0000 & 1.0000 & 0 & 0 \\ 4.5639 & -1.8571 & 2.1429 & -1.4286 & 0 & -9.7143 & 8.4286 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7143 & 0 & 1.0000 & -0.8571 \\ 7.5596 & -0.0952 & -0.4286 & -0.0476 & 0 & 0.1429 & -0.2857 & 1.0000 & 0 & 0 & 0 & 0 & 0 & -0.1429 & 0 & 0 & -0.0952 \end{bmatrix}$$

The Tableau T_2 is in canonical form. That is, there is no negative values in the L.H.C, the columns of the 7x7 identity matrix appear with cost coefficients 0 on top of these columns. So using colsort.m to rearrange the tableau to get T_c.

$$\mathbf{T_c} = \begin{bmatrix} 20.8965 & 5.6667 & 9.0000 & -3.6667 & 1.0000 & -7.0000 & -2.0000 & 0 & 1.6667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0436 & -1.8571 & -6.8571 & 2.5714 & -4.7143 & 2.4286 & 6.0000 & -0.2857 & -0.8571 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 94.0122 & -0.4762 & -1.1429 & 3.7619 & -1.2857 & 1.5714 & 3.0000 & 0.2857 & -0.4762 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 39.0758 & -6.0476 & -2.7143 & 1.4762 & -5.4286 & 7.8571 & 5.0000 & 0.4286 & -1.0476 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 58.0954 & -1.6190 & 1.7143 & 4.1905 & 7.4286 & -1.8571 & -3.0000 & -0.4286 & 0.3810 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 11.7389 & 1.2381 & 0.5714 & -0.3810 & 1.1429 & -1.2857 & -1.0000 & -0.1429 & 0.2381 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 17.1333 & 5.0000 & -3.0000 & 3.0000 & -1.0000 & -1.0000 & -1.0000 & -1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 4.5639 & -1.8571 & 2.1429 & -1.4286 & -9.7143 & 8.4286 & 0 & 0.7143 & -0.8571 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 7.5596 & -0.0952 & -0.4286 & -0.0476 & 0.1429 & -0.2857 & 0 & -0.1429 & -0.0952 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

To get the canonical form tableau into optimal form, we have to pivot until all the cost coefficients are positive.
The first pivot to optimality occurs in row 2 column 6, because the most negative cost coefficient is in column 6 = -7, and selecting only positive elements in the row, we chose the element with the minimum row ratio.

$$\mathbf{T_3} = \begin{bmatrix} 23.9046 & 0.3137 & -10.7647 & 3.7451 & -12.5882 & 0 & 15.2941 & -0.8235 & -0.8039 & 2.8824 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4297 & -0.7647 & -2.8235 & 1.0588 & -1.9412 & 1.0000 & 2.4706 & -0.1176 & -0.3529 & 0.4118 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 93.3369 & 0.7255 & 3.2941 & 2.0980 & 1.7647 & 0 & -0.8824 & 0.4706 & 0.0784 & -0.6471 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 35.6994 & -0.0392 & 19.4706 & -6.8431 & 9.8235 & 0 & -14.4118 & 1.3529 & 1.7255 & -3.2353 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 58.8934 & -3.0392 & -3.5294 & 6.1569 & 3.8235 & 0 & 1.5882 & -0.6471 & -0.2745 & 0.7647 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 12.2915 & 0.2549 & -3.0588 & 0.9804 & -1.3529 & 0 & 2.1765 & -0.2941 & -0.2157 & 0.5294 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 17.5630 & 4.2353 & -5.8235 & 4.0588 & -2.9412 & 0 & 1.4706 & -1.1176 & -0.3529 & 0.4118 & 0 & 0 & 0 & & 1.0000 & 0 & 0 \\ 0.9420 & 4.5882 & 25.9412 & -10.3529 & 6.6471 & 0 & -20.8235 & 1.7059 & 2.1176 & -3.4706 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 7.6824 & -0.3137 & -1.2353 & 0.2549 & -0.4118 & 0 & 0.7059 & -0.1765 & -0.1961 & 0.1176 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

The second pivot to optimality occurs in row 8 column 3, using simplex method as above.

$$\mathbf{T_4} = \begin{bmatrix} 24.2955 & 2.2177 & 0 & -0.5510 & -9.8299 & 0 & 6.6531 & -0.1156 & 0.0748 & 1.4422 & 0 & 0 & 0 & 0 & 0 & 0.4150 & 0 \\ 0.5323 & -0.2653 & 0 & -0.0680 & -1.2177 & 1.0000 & 0.2041 & 0.0680 & -0.1224 & 0.0340 & 0 & 0 & 0 & 0 & 0 & 0.1088 & 0 \\ 93.2173 & 0.1429 & 0 & 3.4127 & 0.9206 & 0 & 1.7619 & 0.2540 & -0.1905 & -0.2063 & 1.0000 & 0 & 0 & 0 & 0 & -0.1270 & 0 \\ 34.9924 & -3.4830 & 0 & 0.9274 & 4.8345 & 0 & 1.2177 & 0.0726 & 0.1361 & -0.6304 & 0 & 1.0000 & 0 & 0 & 0 & -0.7506 & 0 \\ 59.0216 & -2.4150 & 0 & 4.7483 & 4.7279 & 0 & -1.2449 & -0.4150 & 0.0136 & 0.2925 & 0 & 0 & 1.0000 & 0 & 0 & 0.1361 & 0 \\ 12.4025 & 0.7959 & 0 & -0.2404 & -0.5692 & 0 & -0.2789 & -0.0930 & 0.0340 & 0.1202 & 0 & 0 & 0 & 1.0000 & 0 & 0.1179 & 0 \\ 17.7745 & 5.2653 & 0 & 1.7347 & -1.4490 & 0 & -3.2041 & -0.7347 & 0.1224 & -0.3673 & 0 & 0 & 0 & 0 & 1.0000 & 0.2245 & 0 \\ 0.0363 & 0.1769 & 1.0000 & -0.3991 & 0.2562 & 0 & -0.8027 & 0.0658 & 0.0816 & -0.1338 & 0 & 0 & 0 & 0 & 0 & 0.0385 & 0 \\ 7.7273 & -0.0952 & 0 & -0.2381 & -0.0952 & 0 & -0.2857 & -0.0952 & -0.0952 & -0.0476 & 0 & 0 & 0 & 0 & 0 & 0.0476 & 1.0000 \end{bmatrix}$$

The third pivot to optimality occurs in row 8 column 5, using simplex method as above.

$$\mathbf{T_5} = \begin{bmatrix}
25.6885 & 9.0029 & 38.3628 & -15.8614 & 0 & 0 & -24.1416 & 2.4071 & 3.2065 & -3.6903 & 0 & 0 & 0 & 0 & 0 & 1.8938 & 0 \\
0.7048 & 0.5752 & 4.7522 & -1.9646 & 0 & 1.0000 & -3.6106 & 0.3805 & 0.2655 & -0.6018 & 0 & 0 & 0 & 0 & 0 & 0.2920 & 0 \\
93.0868 & -0.4926 & -3.5929 & 4.8466 & 0 & 0 & 4.6460 & 0.0177 & -0.4838 & 0.2743 & 1.0000 & 0 & 0 & 0 & 0 & -0.2655 & 0 \\
34.3073 & -6.8201 & -18.8673 & 8.4572 & 0 & 0 & 16.3628 & -1.1681 & -1.4041 & 1.8938 & 0 & 1.0000 & 0 & 0 & 0 & -1.4779 & 0 \\
58.3516 & -5.6785 & -18.4513 & 12.1121 & 0 & 0 & 13.5664 & -1.6283 & -1.4926 & 2.7611 & 0 & 0 & 1.0000 & 0 & 0 & -0.5752 & 0 \\
12.4832 & 1.1888 & 2.2212 & -1.1268 & 0 & 0 & -2.0619 & 0.0531 & 0.2153 & -0.1770 & 0 & 0 & 0 & 1.0000 & 0 & 0.2035 & 0 \\
17.9798 & 6.2655 & 5.6549 & -0.5221 & 0 & 0 & -7.7434 & -0.3628 & 0.5841 & -1.1239 & 0 & 0 & 0 & 0 & 1.0000 & 0.4425 & 0 \\
0.1417 & 0.6903 & 3.9027 & -1.5575 & 1.0000 & 0 & -3.1327 & 0.2566 & 0.3186 & -0.5221 & 0 & 0 & 0 & 0 & 0 & 0.1504 & 0 \\
7.7408 & -0.0295 & 0.3717 & -0.3864 & 0 & 0 & -0.5841 & -0.0708 & -0.0649 & -0.0973 & 0 & 0 & 0 & 0 & 0 & 0.0619 & 1.0000
\end{bmatrix}$$

The fourth pivot to optimality occurs in row 4 column 7, using simplex method as above.

$$\mathbf{T_6} = \begin{bmatrix}
76.3053 & -1.0593 & 10.5262 & -3.3836 & 0 & 0 & 0 & 0.6836 & 1.1348 & -0.8962 & 0 & 1.4754 & 0 & 0 & 0 & -0.2866 & 0 \\
8.2751 & -0.9297 & 0.5890 & -0.0984 & 0 & 1.0000 & 0 & 0.1228 & -0.0443 & -0.1839 & 0 & 0.2207 & 0 & 0 & 0 & -0.0341 & 0 \\
83.3457 & 1.4438 & 1.7642 & 2.4453 & 0 & 0 & 0 & 0.3494 & -0.0851 & -0.2634 & 1.0000 & -0.2839 & 0 & 0 & 0 & 0.1541 & 0 \\
2.0967 & -0.4168 & -1.1531 & 0.5169 & 0 & 0 & 1.0000 & -0.0714 & -0.0858 & 0.1157 & 0 & 0.0611 & 0 & 0 & 0 & -0.0903 & 0 \\
29.9075 & -0.0240 & -2.8085 & 5.1002 & 0 & 0 & 0 & -0.6598 & -0.3285 & 1.1909 & 0 & -0.8291 & 1.0000 & 0 & 0 & 0.6501 & 0 \\
16.8064 & 0.3294 & -0.1563 & -0.0611 & 0 & 0 & 0 & -0.0941 & 0.0384 & 0.0617 & 0 & 0.1260 & 0 & 1.0000 & 0 & 0.0173 & 0 \\
34.2150 & 3.0380 & -3.2737 & 3.4801 & 0 & 0 & 0 & -0.9156 & -0.0804 & -0.2277 & 0 & 0.4732 & 0 & 0 & 1.0000 & -0.2569 & 0 \\
6.7100 & -0.6155 & 0.2904 & 0.0617 & 1.0000 & 0 & 0 & 0.0330 & 0.0498 & -0.1595 & 0 & 0.1915 & 0 & 0 & 0 & -0.1325 & 0 \\
8.9654 & -0.2729 & -0.3018 & -0.0846 & 0 & 0 & 0 & -0.1125 & -0.1150 & -0.0297 & 0 & 0.0357 & 0 & 0 & 0 & 0.0092 & 1.0000
\end{bmatrix}$$

The fifth pivot to optimality occurs in row 4 column 4, using simplex method as above.

$$\mathbf{T_7} = \begin{bmatrix}
90.0312 & -3.7879 & 2.9777 & 0 & 0 & 0 & 6.5466 & 0.2163 & 0.5731 & -0.1385 & 0 & 1.8755 & 0 & 0 & 0 & -0.8779 & 0 \\
8.6744 & -1.0091 & 0.3694 & 0 & 0 & 1.0000 & 0.1904 & 0.1092 & -0.0607 & -0.1618 & 0 & 0.2323 & 0 & 0 & 0 & -0.0513 & 0 \\
73.4262 & 3.4158 & 7.2194 & 0 & 0 & 0 & -4.7311 & 0.6871 & 0.3209 & -0.8110 & 1.0000 & -0.5731 & 0 & 0 & 0 & 0.5814 & 0 \\
4.0566 & -0.8064 & -2.2309 & 1.0000 & 0 & 0 & 1.9348 & -0.1381 & -0.1660 & 0.2239 & 0 & 0.1182 & 0 & 0 & 0 & -0.1747 & 0 \\
9.2180 & 4.0889 & 8.5696 & 0 & 0 & 0 & -9.8678 & 0.0446 & 0.5183 & 0.0488 & 0 & -1.4322 & 1.0000 & 0 & 0 & 1.5413 & 0 \\
17.0543 & 0.2801 & -0.2926 & 0 & 0 & 0 & 0.1182 & -0.1025 & 0.0283 & 0.0753 & 0 & 0.1332 & 0 & 1.0000 & 0 & 0.0066 & 0 \\
20.0978 & 5.8444 & 4.4901 & 0 & 0 & 0 & -6.7332 & -0.4349 & 0.4974 & -1.0070 & 0 & 0.0617 & 0 & 0 & 1.0000 & 0.3512 & 0 \\
6.4599 & -0.5657 & 0.4280 & 0 & 1.0000 & 0 & -0.1193 & 0.0415 & 0.0600 & -0.1734 & 0 & 0.1842 & 0 & 0 & 0 & -0.1217 & 0 \\
9.3084 & -0.3411 & -0.4904 & 0 & 0 & 0 & 0.1636 & -0.1242 & -0.1291 & -0.0108 & 0 & 0.0457 & 0 & 0 & 0 & -0.0056 & 1.0000
\end{bmatrix}$$

The sixth pivot to optimality occurs in row 5 column 2, using simplex method as above.

$$\mathbf{T_8} = \begin{bmatrix}
98.5707 & 0 & 10.9164 & 0 & 0 & 0 & -2.5948 & 0.2576 & 1.0532 & -0.0932 & 0 & 0.5488 & 0.9264 & 0 & 0 & 0.5499 & 0 \\
10.9492 & 0 & 2.4842 & 0 & 0 & 1.0000 & -2.2447 & 0.1202 & 0.0672 & -0.1498 & 0 & -0.1211 & 0.2468 & 0 & 0 & 0.3291 & 0 \\
65.7258 & 0 & 0.0607 & 0 & 0 & 0 & 3.5122 & 0.6498 & -0.1121 & -0.8517 & 1.0000 & 0.6233 & -0.8354 & 0 & 0 & -0.7061 & 0 \\
5.8745 & 0 & -0.5408 & 1.0000 & 0 & 0 & -0.0113 & -0.1293 & -0.0638 & 0.2336 & 0 & -0.1642 & 0.1972 & 0 & 0 & 0.1292 & 0 \\
2.2544 & 1.0000 & 2.0958 & 0 & 0 & 0 & -2.4133 & 0.0109 & 0.1268 & 0.0119 & 0 & -0.3503 & 0.2446 & 0 & 0 & 0.3770 & 0 \\
16.4229 & 0 & -0.8796 & 0 & 0 & 0 & 0.7942 & -0.1056 & -0.0073 & 0.0720 & 0 & 0.2313 & -0.0685 & 1.0000 & 0 & -0.0990 & 0 \\
6.9222 & 0 & -7.7587 & 0 & 0 & 0 & 7.3712 & -0.4988 & -0.2435 & -1.0768 & 0 & 2.1088 & -1.4293 & 0 & 1.0000 & -1.8518 & 0 \\
7.7353 & 0 & 1.6137 & 0 & 1.0000 & 0 & -1.4846 & 0.0477 & 0.1317 & -0.1666 & 0 & -0.0140 & 0.1384 & 0 & 0 & 0.0915 & 0 \\
10.0774 & 0 & 0.2245 & 0 & 0 & 0 & -0.6596 & -0.1204 & -0.0858 & -0.0067 & 0 & -0.0738 & 0.0834 & 0 & 0 & 0.1230 & 1.0000
\end{bmatrix}$$

The seventh pivot to optimality occurs in row 7 column 7, using simplex method as above.

$$\mathbf{T_9} = \begin{bmatrix}
101.0075 & 0 & 8.1852 & 0 & 0 & 0 & 0 & 0.0820 & 0.9675 & -0.4723 & 0 & 1.2911 & 0.4232 & 0 & 0.3520 & -0.1019 & 0 \\
13.0572 & 0 & 0.1214 & 0 & 0 & 1.0000 & 0 & -0.0317 & -0.0069 & -0.4777 & 0 & 0.5211 & -0.1885 & 0 & 0.3045 & -0.2348 & 0 \\
62.4275 & 0 & 3.7575 & 0 & 0 & 0 & 0 & 0.8875 & 0.0039 & -0.3387 & 1.0000 & -0.3815 & -0.1543 & 0 & -0.4765 & 0.1762 & 0 \\
5.8852 & 0 & -0.5528 & 1.0000 & 0 & 0 & 0 & -0.1301 & -0.0642 & 0.2319 & 0 & -0.1610 & 0.1950 & 0 & 0.0015 & 0.1264 & 0 \\
4.5207 & 1.0000 & -0.4444 & 0 & 0 & 0 & 0 & -0.1524 & 0.0471 & -0.3406 & 0 & 0.3401 & -0.2234 & 0 & 0.3274 & -0.2293 & 0 \\
15.6771 & 0 & -0.0437 & 0 & 0 & 0 & 0 & -0.0519 & 0.0190 & 0.1880 & 0 & 0.0041 & 0.0855 & 1.0000 & -0.1077 & 0.1006 & 0 \\
0.9391 & 0 & -1.0526 & 0 & 0 & 0 & 1.0000 & -0.0677 & -0.0330 & -0.1461 & 0 & 0.2861 & -0.1939 & 0 & 0.1357 & -0.2512 & 0 \\
9.1295 & 0 & 0.0510 & 0 & 1.0000 & 0 & 0 & -0.0528 & 0.0827 & -0.3835 & 0 & 0.4107 & -0.1495 & 0 & 0.2014 & -0.2814 & 0 \\
10.6968 & 0 & -0.4698 & 0 & 0 & 0 & 0 & -0.1651 & -0.1076 & -0.1031 & 0 & 0.1149 & -0.0445 & 0 & 0.0895 & -0.0427 & 1.0000
\end{bmatrix}$$

The eighth pivot to optimality occurs in row 4 column 10, using simplex method as above.

$$\mathbf{T_{10}} = \begin{bmatrix}
112.9931 & 0 & 7.0594 & 2.0366 & 0 & 0 & 0 & -0.1829 & 0.8368 & 0 & 0 & 0.9633 & 0.8204 & 0 & 0.3552 & 0.1554 & 0 \\
25.1803 & 0 & -1.0172 & 2.0599 & 0 & 1.0000 & 0 & -0.2997 & -0.1391 & 0 & 0 & 0.1895 & 0.2132 & 0 & 0.3077 & 0.0255 & 0 \\
71.0229 & 0 & 2.9501 & 1.4605 & 0 & 0 & 0 & 0.6975 & -0.0898 & 0 & 1.0000 & -0.6165 & 0.1305 & 0 & -0.4742 & 0.3608 & 0 \\
25.3781 & 0 & -2.3836 & 4.3122 & 0 & 0 & 0 & -0.5610 & -0.2768 & 1.0000 & 0 & -0.6941 & 0.8410 & 0 & 0.0066 & 0.5450 & 0 \\
13.1642 & 1.0000 & -1.2562 & 1.4687 & 0 & 0 & 0 & -0.3434 & -0.0472 & 0 & 0 & 0.1037 & 0.0630 & 0 & 0.3297 & -0.0437 & 0 \\
10.9058 & 0 & 0.4044 & -0.8107 & 0 & 0 & 0 & 0.0536 & 0.0710 & 0 & 0 & 0.1346 & -0.0726 & 1.0000 & -0.1090 & -0.0019 & 0 \\
4.6463 & 0 & -1.4008 & 0.6299 & 0 & 0 & 1.0000 & -0.1496 & -0.0735 & 0 & 0 & 0.1847 & -0.0711 & 0 & 0.1366 & -0.1716 & 0 \\
18.8611 & 0 & -0.8630 & 1.6536 & 1.0000 & 0 & 0 & -0.2679 & -0.0235 & 0 & 0 & 0.1446 & 0.1730 & 0 & 0.2040 & -0.0725 & 0 \\
13.3133 & 0 & -0.7156 & 0.4446 & 0 & 0 & 0 & -0.2229 & -0.1361 & 0 & 0 & 0.0434 & 0.0422 & 0 & 0.0902 & 0.0135 & 1.0000
\end{bmatrix}$$

The ninth and final pivot to optimality occurs in row 3 column 8, using simplex method as above.

$$\mathbf{T}^* = \begin{bmatrix} 131.6165 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\ 55.6942 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\ 101.8264 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\ 82.4982 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\ 48.1345 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\ 5.4484 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\ 19.8804 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\ 46.1381 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\ 36.0120 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \end{bmatrix}$$

Optimal solution vector x is

$$\mathbf{x}^* = \begin{bmatrix} 48.1345 \\ 0 \\ 0 \\ 46.1381 \\ 55.6942 \\ 19.8804 \\ 101.8264 \\ 0 \\ 82.4982 \\ 0 \\ 0 \\ 0 \\ 5.4484 \\ 0 \\ 0 \\ 36.0120 \end{bmatrix}$$

Optimal z value $= -131.6165$

3 Sensitivity analysis

3.1 A

3.1.1 (a) change in last non basic variable

The last non-basic variable in the optimal tableau T* is x_{15} or column 16. The minimum row ratio for x_{16} is roughly 98.7816, so going to increase x_{16} by half of 49.3908, which is 49.3908.

$$\mathbf{T_{3A_a}} = \begin{bmatrix} 119.2661 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\ 46.7786 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\ 76.2776 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\ 41.2491 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\ 41.5193 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\ 6.9114 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\ 24.5343 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\ 42.8730 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\ 29.6513 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \end{bmatrix}$$

The new optimal z-value is -119.2661.
The new optimal solution x is:

$$\mathbf{x_{3A_a}} = \begin{bmatrix} 41.5193 \\ 0 \\ 0 \\ 42.8730 \\ 46.7786 \\ 24.5343 \\ 76.2776 \\ 0 \\ 41.2491 \\ 0 \\ 0 \\ 0 \\ 6.9114 \\ 0 \\ 49.3908 \\ 29.6513 \end{bmatrix}$$

To do the check for this result, I added an extra constraint to the tableau T*, where $x_{15} = 49.3908$

$$\mathbf{C_{3A_a}} = \begin{bmatrix} 131.6165 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\ 55.6942 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\ 101.8264 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\ 82.4982 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\ 48.1345 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\ 5.4484 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\ 19.8804 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\ 46.1381 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\ 36.0120 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \\ 49.3908 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

Now pivot on row 8 column 16, we get:

$$\mathbf{C_{3A_{a1}}} = \begin{bmatrix} 119.2661 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0 & 0 \\ 46.7786 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0 & 0 \\ 76.2776 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0 & 0 \\ 41.2491 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0 & 0 \\ 41.5193 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0 & 0 \\ 6.9114 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & 0 & 0 \\ 24.5343 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & 0 & 0 \\ 42.8730 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0 & 0 \\ 29.6513 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0 & 1.0000 \\ 49.3908 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

Comparing the tableaux T_{3A_a} and $C_{3A_{a1}}$, both have an optimal z value of -119.2661.

3.1.2 (b) change in last basic variable

The last basic variable in the optimal tableau T^* is x_{16} or column 17. To increase the basic variable we only consider negative elements in row 9, which are, -0.1648 , -0.1537 , -0.0614. To decide on which element has least effect on z, divide the first row by the last, only considering the negative elements in the row ratios, choosing the least negative row ratio, which was x_{14}. Then getting the row ratio for x_{14} with the l.h.c, the minimum row ratio was roughly 500.6, so I increased x_{14} up to 250.3.

$$\mathbf{T_{3A_b}} = \begin{bmatrix} 73.8480 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\ 29.6751 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\ 271.9982 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\ 176.2960 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\ 24.0672 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\ 23.6063 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\ 11.1417 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\ 40.6764 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\ 51.3763 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \end{bmatrix}$$

The new optimal z-value is -73.8480, and the new optimal solution x is, remembering to manually set $x_{14} = 250.6$:

$$\mathbf{x_{3A_b}} = \begin{bmatrix} 24.0672 \\ 0 \\ 0 \\ 40.6764 \\ 29.6751 \\ 11.1417 \\ 271.9982 \\ 0 \\ 176.2960 \\ 0 \\ 0 \\ 0 \\ 23.6063 \\ 250.2885 \\ 0 \\ 51.3763 \end{bmatrix}$$

To check the above result, I added the constraint x_{16} is equal to $36.012 + (250.6 * .0614) = 51.3763$.

$$\mathbf{C_{3A_{b1}}} = \begin{bmatrix}
131.6165 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\
55.6942 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\
101.8264 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\
82.4982 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\
48.1345 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\
5.4484 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\
19.8804 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\
46.1381 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\
36.0120 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \\
51.3763 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000
\end{bmatrix}$$

Pivoting on row 10, column 17, we get:

$$\mathbf{C_{3A_{b2}}} = \begin{bmatrix}
131.6165 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\
55.6942 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\
101.8264 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\
82.4982 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\
48.1345 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\
5.4484 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\
19.8804 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\
46.1381 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\
-15.3644 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 0 \\
51.3763 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000
\end{bmatrix}$$

Using DSM to get rid of -15.3644, pivot on row 9, column 15.

$$\mathbf{C_{3A_{b3}}} = \begin{bmatrix}
73.8480 & 0 & 8.6876 & 5.8462 & 0 & 0 & 0 & 0 & 0.1935 & 0 & 1.4639 & 0.2238 & 1.1702 & 0 & 0 & 0.7343 & 0 \\
29.6751 & 0 & 0.6352 & 4.2308 & 0 & 1.0000 & 0 & 0 & -0.4569 & 0 & 0.9709 & -0.3357 & 0.4114 & 0 & 0 & 0.3986 & 0 \\
271.9982 & 0 & 1.7121 & -8.0000 & 0 & 0 & 0 & 1.0000 & 1.6970 & 0 & -2.1061 & 0.8182 & -0.7424 & 0 & 0 & -0.9091 & 0 \\
176.2960 & 0 & -1.3986 & -0.0769 & 0 & 0 & 0 & 0 & 0.6573 & 1.0000 & -1.1469 & -0.2517 & 0.4336 & 0 & 0 & 0.0490 & 0 \\
24.0672 & 1.0000 & 0.5524 & 3.6154 & 0 & 0 & 0 & 0 & -0.3497 & 0 & 0.9930 & -0.4406 & 0.2587 & 0 & 0 & 0.3357 & 0 \\
23.6063 & 0 & -0.0909 & -2.0000 & 0 & 0 & 0 & 0 & 0.2727 & 0 & -0.4545 & 0.3636 & -0.1818 & 1.0000 & 0 & -0.1818 & 0 \\
11.1417 & 0 & -0.6387 & 1.4615 & 0 & 0 & 1.0000 & 0 & -0.1865 & 0 & 0.3963 & -0.0350 & 0.0047 & 0 & 0 & -0.0210 & 0 \\
40.6764 & 0 & 0.3508 & 2.5385 & 1.0000 & 0 & 0 & 0 & -0.1166 & 0 & 0.4977 & -0.1469 & 0.2529 & 0 & 0 & 0.1119 & 0 \\
250.2885 & 0 & -3.7028 & -14.8462 & 0 & 0 & 0 & 0 & 2.6853 & 0 & -5.2063 & 2.5035 & -1.3671 & 0 & 1.0000 & -2.0979 & 0 \\
51.3763 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000
\end{bmatrix}$$

The tableau is now in canonical form, its z value matches T_{3A_b}, -73.8480, and x_{14} is now equal to 250.6 in the check tableau.

Comparing the tableaux T_{3A_b} and $C_{3A_{b3}}$, both have an optimal z value of -73.8480.

3.1.3 (c) change in first non basic variable

The first non basic variable in the optimal tableau T^* is x_2 or column 3. To increase the first non basic variable 1 unit above its maximum value, which is the min row ratio of x_2, 24.0743.

Phase 1, pivot on the element which has the min row ratio in row 3, which happens to be also column 3. This will set $x_2 = 24.0743$.

$$\mathbf{T_{3A_{c1}}} = \begin{bmatrix}
-56.9581 & 0 & 0 & -1.4583 & 0 & 0 & 0 & -1.8519 & 1.0518 & 0 & -2.3929 & 2.4386 & 0.5081 & 0 & 1.4899 & -0.7079 & 0 \\
49.6691 & 0 & 0 & 2.5635 & 0 & 1.0000 & 0 & -0.0592 & -0.1701 & 0 & 0.3448 & -0.0231 & 0.2582 & 0 & 0.1442 & 0.1499 & 0 \\
24.0743 & 0 & 1.0000 & 0.4951 & 0 & 0 & 0 & 0.2364 & -0.0304 & 0 & 0.3390 & -0.2090 & 0.0442 & 0 & -0.1607 & 0.1223 & 0 \\
82.7618 & 0 & 0 & 5.4922 & 0 & 0 & 0 & 0.0026 & -0.3493 & 1.0000 & 0.8080 & -1.1922 & 0.9464 & 0 & -0.3765 & 0.8365 & 0 \\
43.4064 & 1.0000 & 0 & 2.0906 & 0 & 0 & 0 & -0.0464 & -0.0855 & 0 & 0.4258 & -0.1588 & 0.1186 & 0 & 0.1277 & 0.1099 & 0 \\
1.1699 & 0 & 0 & -1.0109 & 0 & 0 & 0 & -0.0420 & 0.0833 & 0 & -0.1371 & 0.2192 & -0.0905 & 1.0000 & -0.0440 & -0.0514 & 0 \\
38.3689 & 0 & 0 & 1.3234 & 0 & 0 & 1.0000 & 0.1816 & -0.1161 & 0 & 0.4748 & -0.1081 & -0.0091 & 0 & -0.0885 & -0.0003 & 0 \\
39.6377 & 0 & 0 & 2.0808 & 1.0000 & 0 & 0 & -0.0638 & -0.0497 & 0 & 0.2925 & -0.0358 & 0.2111 & 0 & 0.0652 & 0.0331 & 0 \\
30.5398 & 0 & 0 & 0.7988 & 0 & 0 & 0 & -0.0537 & -0.1579 & 0 & 0.2425 & -0.1062 & 0.0739 & 0 & -0.0248 & 0.1010 & 1.0000
\end{bmatrix}$$

Phase 2, repeating method in A (b), of increasing a basic variable, first only consider negative elements in row 3, and get min row ratios with respect to the l.h.c, all min row ratios are greater than 1. So we now get row ratios of the negative elements with respect to the top row, choosing the least negative row ratio, which corresponds to having the least effect on z. Increasing x_{13} yields the least effect on z.

$$\mathbf{T_{3A_c}} = \begin{bmatrix}
-66.2270 & 0 & 0 & -1.4583 & 0 & 0 & 0 & -1.8519 & 1.0518 & 0 & -2.3929 & 2.4386 & 0.5081 & 0 & 1.4899 & -0.7079 & 0 \\
48.7722 & 0 & 0 & 2.5635 & 0 & 1.0000 & 0 & -0.0592 & -0.1701 & 0 & 0.3448 & -0.0231 & 0.2582 & 0 & 0.1442 & 0.1499 & 0 \\
25.0743 & 0 & 1.0000 & 0.4951 & 0 & 0 & 0 & 0.2364 & -0.0304 & 0 & 0.3390 & -0.2090 & 0.0442 & 0 & -0.1607 & 0.1223 & 0 \\
85.1041 & 0 & 0 & 5.4922 & 0 & 0 & 0 & 0.0026 & -0.3493 & 1.0000 & 0.8080 & -1.1922 & 0.9464 & 0 & -0.3765 & 0.8365 & 0 \\
42.6118 & 1.0000 & 0 & 2.0906 & 0 & 0 & 0 & -0.0464 & -0.0855 & 0 & 0.4258 & -0.1588 & 0.1186 & 0 & 0.1277 & 0.1099 & 0 \\
1.4435 & 0 & 0 & -1.0109 & 0 & 0 & 0 & -0.0420 & 0.0833 & 0 & -0.1371 & 0.2192 & -0.0905 & 1.0000 & -0.0440 & -0.0514 & 0 \\
38.9197 & 0 & 0 & 1.3234 & 0 & 0 & 1.0000 & 0.1816 & -0.1161 & 0 & 0.4748 & -0.1081 & -0.0091 & 0 & -0.0885 & -0.0003 & 0 \\
39.2319 & 0 & 0 & 2.0808 & 1.0000 & 0 & 0 & -0.0638 & -0.0497 & 0 & 0.2925 & -0.0358 & 0.2111 & 0 & 0.0652 & 0.0331 & 0 \\
30.6944 & 0 & 0 & 0.7988 & 0 & 0 & 0 & -0.0537 & -0.1579 & 0 & 0.2425 & -0.1062 & 0.0739 & 0 & -0.0248 & 0.1010 & 1.0000
\end{bmatrix}$$

The new optimal z-value is 66.227, it now costs us to use this combination, and the new optimal solution x is, remembering to manually set x_{14} equal to 6.2210:

$$\mathbf{x_{3A_c}} = \begin{bmatrix} 42.6118 \\ 25.0743 \\ 0 \\ 39.2319 \\ 48.7722 \\ 38.9197 \\ 0 \\ 0 \\ 85.1041 \\ 0 \\ 0 \\ 0 \\ 1.4435 \\ 6.2210 \\ 0 \\ 30.6944 \end{bmatrix}$$

To check the above result, I added the constraint x_3 is equal to 25.0743

$$\mathbf{C_{3A_{c1}}} = \begin{bmatrix}
131.6165 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\
55.6942 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\
101.8264 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\
82.4982 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\
48.1345 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\
5.4484 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\
19.8804 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\
46.1381 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\
36.0120 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \\
25.0743 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Pivoting on row 10, column 3 we get:

$$\mathbf{C_{3A_{c2}}} = \begin{bmatrix}
-64.7911 & 0 & 0 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\
49.4189 & 0 & 0 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\
-4.2297 & 0 & 0 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\
82.7727 & 0 & 0 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\
43.2100 & 1.0000 & 0 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\
0.9922 & 0 & 0 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\
39.1369 & 0 & 0 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\
39.3677 & 0 & 0 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\
30.3125 & 0 & 0 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \\
25.0743 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Using DSM, pivot on row 3, column 15

$$\mathbf{C_{3A_c}} = \begin{bmatrix}
-66.2270 & 0 & 0 & 3.1304 & 0 & 0 & 0 & 0.3395 & 0.7695 & 0 & 0.7489 & 0.5015 & 0.9181 & 0 & 0 & 0.4257 & 0 \\
48.7722 & 0 & 0 & 3.0076 & 0 & 1.0000 & 0 & 0.1529 & -0.1974 & 0 & 0.6488 & -0.2106 & 0.2979 & 0 & 0 & 0.2596 & 0 \\
6.2210 & 0 & 0 & -3.0798 & 0 & 0 & 0 & -1.4708 & 0.1894 & 0 & -2.1087 & 1.3001 & -0.2752 & 0 & 1.0000 & -0.7608 & 0 \\
85.1041 & 0 & 0 & 4.3326 & 0 & 0 & 0 & -0.5512 & -0.2780 & 1.0000 & 0.0140 & -0.7027 & 0.8428 & 0 & 0 & 0.5500 & 0 \\
42.6118 & 1.0000 & 0 & 2.4840 & 0 & 0 & 0 & 0.1414 & -0.1096 & 0 & 0.6951 & -0.3248 & 0.1537 & 0 & 0 & 0.2071 & 0 \\
1.4435 & 0 & 0 & -1.1464 & 0 & 0 & 0 & -0.1067 & 0.0917 & 0 & -0.2298 & 0.2763 & -0.1026 & 1.0000 & 0 & -0.0848 & 0 \\
38.9197 & 0 & 0 & 1.0507 & 0 & 0 & 1.0000 & 0.0514 & -0.0993 & 0 & 0.2881 & 0.0071 & -0.0335 & 0 & 0 & -0.0677 & 0 \\
39.2319 & 0 & 0 & 2.2817 & 1.0000 & 0 & 0 & 0.0321 & -0.0621 & 0 & 0.4301 & -0.1206 & 0.2291 & 0 & 0 & 0.0827 & 0 \\
30.6944 & 0 & 0 & 0.7223 & 0 & 0 & 0 & -0.0903 & -0.1532 & 0 & 0.1902 & -0.0739 & 0.0670 & 0 & 0 & 0.0821 & 1.0000 \\
25.0743 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Comparing the tableaux T_{3A_c} and C_{3A_c}, both have an optimal z value of 66.2270.

3.2 B

My last slack variable in t^* is x_{15}, or column 16. To calculate the range of which this slack variable resources can change while maintaining optimality, I got the row ratio of column 16 with respect to the l.h.c. To get lower bound, I choose the smallest positive row ratio, which was 98.7816. To get the upper bound, consider only negative row ratios and choose the more positive row ratio, which was -183.9429. I then negated both of these to put them into an equality equation.

Range: $-98.7816 \leq a \leq 183.9429$
We can see that we can increase a up to 183.9429/2, which is roughly 91.9715. Then folding we get:

$$\mathbf{T_{3B}} = \begin{bmatrix} 154.6143 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\ 72.2960 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\ 149.4012 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\ 159.3089 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\ 60.4526 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\ 2.7242 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\ 11.2143 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\ 52.2182 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\ 47.8563 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \end{bmatrix}$$

The new optimal z-value is -154.6143, and new optimal x is:

$$\mathbf{X_{3B}} = \begin{bmatrix} 60.4526 \\ 0 \\ 0 \\ 52.2182 \\ 72.2960 \\ 11.2143 \\ 149.4012 \\ 0 \\ 159.3089 \\ 0 \\ 0 \\ 0 \\ 2.7242 \\ 0 \\ 0 \\ 47.8563 \end{bmatrix}$$

For the check, x_{15}, corresponds to a change in x_7 in t_c. So I added 91.9715 onto x_7 and then pivoted to optimality.

$$\mathbf{C_{3B_1}} = \begin{bmatrix} 20.8965 & 5.6667 & 9.0000 & -3.6667 & 1.0000 & -7.0000 & -2.0000 & 0 & 1.6667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0436 & -1.8571 & -6.8571 & 2.5714 & -4.7143 & 2.4286 & 6.0000 & -0.2857 & -0.8571 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 94.0122 & -0.4762 & -1.1429 & 3.7619 & -1.2857 & 1.5714 & 3.0000 & 0.2857 & -0.4762 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 39.0758 & -6.0476 & -2.7143 & 1.4762 & -5.4286 & 7.8571 & 5.0000 & 0.4286 & -1.0476 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 58.0954 & -1.6190 & 1.7143 & 4.1905 & 7.4286 & -1.8571 & -3.0000 & -0.4286 & 0.3810 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 11.7389 & 1.2381 & 0.5714 & -0.3810 & 1.1429 & -1.2857 & -1.0000 & -0.1429 & 0.2381 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 17.1333 & 5.0000 & -3.0000 & 3.0000 & -1.0000 & -1.0000 & -1.0000 & -1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 96.5354 & -1.8571 & 2.1429 & -1.4286 & -9.7143 & 8.4286 & 0 & 0.7143 & -0.8571 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 7.5596 & -0.0952 & -0.4286 & -0.0476 & 0.1429 & -0.2857 & 0 & -0.1429 & -0.0952 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Using simplex method, pivot on 2,6

$$\mathbf{C_{3B_2}} = \begin{bmatrix} 23.9046 & 0.3137 & -10.7647 & 3.7451 & -12.5882 & 0 & 15.2941 & -0.8235 & -0.8039 & 2.8824 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4297 & -0.7647 & -2.8235 & 1.0588 & -1.9412 & 1.0000 & 2.4706 & -0.1176 & -0.3529 & 0.4118 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 93.3369 & 0.7255 & 3.2941 & 2.0980 & 1.7647 & 0 & -0.8824 & 0.4706 & 0.0784 & -0.6471 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 35.6994 & -0.0392 & 19.4706 & -6.8431 & 9.8235 & 0 & -14.4118 & 1.3529 & 1.7255 & -3.2353 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 58.8934 & -3.0392 & -3.5294 & 6.1569 & 3.8235 & 0 & 1.5882 & -0.6471 & -0.2745 & 0.7647 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 12.2915 & 0.2549 & -3.0588 & 0.9804 & -1.3529 & 0 & 2.1765 & -0.2941 & -0.2157 & 0.5294 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 17.5630 & 4.2353 & -5.8235 & 4.0588 & -2.9412 & 0 & 1.4706 & -1.1176 & -0.3529 & 0.4118 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 92.9134 & 4.5882 & 25.9412 & -10.3529 & 6.6471 & 0 & -20.8235 & 1.7059 & 2.1176 & -3.4706 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 7.6824 & -0.3137 & -1.2353 & 0.2549 & -0.4118 & 0 & 0.7059 & -0.1765 & -0.1961 & 0.1176 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Simplex again, pivot 4,5

$$\mathbf{C_{3B_3}} = \begin{bmatrix} 69.6512 & 0.2635 & 14.1856 & -5.0240 & 0 & 0 & -3.1737 & 0.9102 & 1.4072 & -1.2635 & 0 & 1.2814 & 0 & 0 & 0 & 0 & 0 \\ 7.4841 & -0.7725 & 1.0240 & -0.2934 & 0 & 1.0000 & -0.3772 & 0.1497 & -0.0120 & -0.2275 & 0 & 0.1976 & 0 & 0 & 0 & 0 & 0 \\ 86.9238 & 0.7325 & -0.2036 & 3.3273 & 0 & 0 & 1.7066 & 0.2275 & -0.2315 & -0.0659 & 1.0000 & -0.1796 & 0 & 0 & 0 & 0 & 0 \\ 3.6341 & -0.0040 & 1.9820 & -0.6966 & 1.0000 & 0 & -1.4671 & 0.1377 & 0.1756 & -0.3293 & 0 & 0.1018 & 0 & 0 & 0 & 0 & 0 \\ 44.9985 & -3.0240 & -11.1078 & 8.8204 & 0 & 0 & 7.1976 & -1.1737 & -0.9461 & 2.0240 & 0 & -0.3892 & 1.0000 & 0 & 0 & 0 & 0 \\ 17.2081 & 0.2495 & -0.3772 & 0.0379 & 0 & 0 & 0.1916 & -0.1078 & 0.0220 & 0.0838 & 0 & 0.1377 & 0 & 1.0000 & 0 & 0 & 0 \\ 28.2515 & 4.2236 & 0.0060 & 2.0100 & 0 & 0 & -2.8443 & -0.7126 & 0.1637 & -0.5569 & 0 & 0.2994 & 0 & 0 & 1.0000 & 0 & 0 \\ 68.7575 & 4.6148 & 12.7665 & -5.7226 & 0 & 0 & -11.0719 & 0.7904 & 0.9501 & -1.2814 & 0 & -0.6766 & 0 & 0 & 0 & 1.0000 & 0 \\ 9.1788 & -0.3154 & -0.4192 & -0.0319 & 0 & 0 & 0.1018 & -0.1198 & -0.1238 & -0.0180 & 0 & 0.0419 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Simplex again, pivot 5,4

$$\mathbf{C_{3B_4}} = \begin{bmatrix} 95.2817 & -1.4589 & 7.8588 & 0 & 0 & 0 & 0.9260 & 0.2417 & 0.8683 & -0.1107 & 0 & 1.0597 & 0.5696 & 0 & 0 & 0 & 0 \\ 8.9810 & -0.8730 & 0.6544 & 0 & 0 & 1.0000 & -0.1378 & 0.1107 & -0.0434 & -0.1602 & 0 & 0.1847 & 0.0333 & 0 & 0 & 0 & 0 \\ 69.9488 & 1.8733 & 3.9866 & 0 & 0 & 0 & -1.0086 & 0.6703 & 0.1254 & -0.8294 & 1.0000 & -0.0328 & -0.3772 & 0 & 0 & 0 & 0 \\ 7.1879 & -0.2428 & 1.1048 & 0 & 1.0000 & 0 & -0.8986 & 0.0450 & 0.1009 & -0.1695 & 0 & 0.0711 & 0.0790 & 0 & 0 & 0 & 0 \\ 5.1017 & -0.3428 & -1.2593 & 1.0000 & 0 & 0 & 0.8160 & -0.1331 & -0.1073 & 0.2295 & 0 & -0.0441 & 0.1134 & 0 & 0 & 0 & 0 \\ 17.0147 & 0.2625 & -0.3295 & 0 & 0 & 0 & 0.1607 & -0.1027 & 0.0260 & 0.0751 & 0 & 0.1394 & -0.0043 & 1.0000 & 0 & 0 & 0 \\ 17.9972 & 4.9126 & 2.5372 & 0 & 0 & 0 & -4.4845 & -0.4451 & 0.3793 & -1.0181 & 0 & 0.3881 & -0.2279 & 0 & 1.0000 & 0 & 0 \\ 97.9520 & 2.6529 & 5.5599 & 0 & 0 & 0 & -6.4021 & 0.0290 & 0.3363 & 0.0317 & 0 & -0.9292 & 0.6488 & 0 & 0 & 1.0000 & 0 \\ 9.3417 & -0.3263 & -0.4594 & 0 & 0 & 0 & 0.1279 & -0.1240 & -0.1272 & -0.0106 & 0 & 0.0405 & 0.0036 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Simplex, pivot 7,2

$$\mathbf{C_{3B_5}} = \begin{bmatrix} 100.6264 & 0 & 8.6123 & 0 & 0 & 0 & -0.4058 & 0.1095 & 0.9809 & -0.4130 & 0 & 1.1750 & 0.5019 & 0 & 0.2970 & 0 & 0 \\ 12.1794 & 0 & 1.1053 & 0 & 0 & 1.0000 & -0.9348 & 0.0316 & 0.0240 & -0.3411 & 0 & 0.2536 & -0.0072 & 0 & 0.1777 & 0 & 0 \\ 63.0862 & 0 & 3.0192 & 0 & 0 & 0 & 0.7014 & 0.8400 & -0.0193 & -0.4412 & 1.0000 & -0.1808 & -0.2903 & 0 & -0.3813 & 0 & 0 \\ 8.0775 & 0 & 1.2302 & 0 & 1.0000 & 0 & -1.1203 & 0.0230 & 0.1197 & -0.2198 & 0 & 0.0902 & 0.0677 & 0 & 0.0494 & 0 & 0 \\ 6.3576 & 0 & -1.0823 & 1.0000 & 0 & 0 & 0.5031 & -0.1641 & -0.0808 & 0.1584 & 0 & -0.0170 & 0.0975 & 0 & 0.0698 & 0 & 0 \\ 16.0530 & 0 & -0.4651 & 0 & 0 & 0 & 0.4003 & -0.0790 & 0.0058 & 0.1295 & 0 & 0.1187 & 0.0079 & 1.0000 & -0.0534 & 0 & 0 \\ 3.6634 & 1.0000 & 0.5165 & 0 & 0 & 0 & -0.9128 & -0.0906 & 0.0772 & -0.2072 & 0 & 0.0790 & -0.0464 & 0 & 0.2036 & 0 & 0 \\ 88.2334 & 0 & 4.1897 & 0 & 0 & 0 & -3.9805 & 0.2693 & 0.1315 & 0.5815 & 0 & -1.1387 & 0.7718 & 0 & -0.5400 & 1.0000 & 010.5372 & 0 \end{bmatrix}$$

Simplex, 5,10

$$\mathbf{C_{3B_6}} = \begin{bmatrix} 117.2017 & 0 & 5.7906 & 2.6072 & 0 & 0 & 0.9058 & -0.3184 & 0.7703 & 0 & 0 & 1.1306 & 0.7560 & 0 & 0.4789 & 0 & 0 \\ 25.8707 & 0 & -1.2254 & 2.1535 & 0 & 1.0000 & 0.1486 & -0.3219 & -0.1500 & 0 & 0 & 0.2169 & 0.2027 & 0 & 0.3280 & 0 & 0 \\ 80.7911 & 0 & 0.0052 & 2.7848 & 0 & 0 & 2.1024 & 0.3830 & -0.2443 & 0 & 1.0000 & -0.2283 & -0.0189 & 0 & -0.1870 & 0 & 0 \\ 16.8994 & 0 & -0.2716 & 1.3876 & 1.0000 & 0 & -0.4222 & -0.2047 & 0.0076 & 0 & 0 & 0.0666 & 0.2030 & 0 & 0.1463 & 0 & 0 \\ 40.1331 & 0 & -6.8319 & 6.3126 & 0 & 0 & 3.1756 & -1.0361 & -0.5100 & 1.0000 & 0 & -0.1076 & 0.6153 & 0 & 0.4405 & 0 & 0 \\ 10.8545 & 0 & 0.4199 & -0.8177 & 0 & 0 & -0.0110 & 0.0552 & 0.0718 & 0 & 0 & 0.1326 & -0.0718 & 1.0000 & -0.1105 & 0 & 0 \\ 11.9807 & 1.0000 & -0.8994 & 1.3082 & 0 & 0 & -0.2547 & -0.3053 & -0.0285 & 0 & 0 & 0.0567 & 0.0811 & 0 & 0.2949 & 0 & 0 \\ 64.8975 & 0 & 8.1623 & -3.6705 & 0 & 0 & -5.8270 & 0.8718 & 0.4280 & 0 & 0 & -1.0762 & 0.4141 & 0 & -0.7962 & 1.0000 & 0 \\ 13.6781 & 0 & -0.8255 & 0.4940 & 0 & 0 & 0.0785 & -0.2347 & -0.1419 & 0 & 0 & 0.0579 & 0.0366 & 0 & 0.1009 & 0 & 1.0000 \end{bmatrix}$$

Simplex, pivot 8,8

$$\mathbf{C_{3B_7}} = \begin{bmatrix} 140.9051 & 0 & 8.7718 & 1.2665 & 0 & 0 & -1.2225 & 0 & 0.9266 & 0 & 0 & 0.7375 & 0.9073 & 0 & 0.1881 & 0.3652 & 0 \\ 49.8339 & 0 & 1.7885 & 0.7982 & 0 & 1.0000 & -2.0030 & 0 & 0.0080 & 0 & 0 & -0.1805 & 0.3556 & 0 & 0.0340 & 0.3692 & 0 \\ 52.2821 & 0 & -3.5804 & 4.3973 & 0 & 0 & 4.6621 & 0 & -0.4323 & 0 & 1.0000 & 0.2445 & -0.2008 & 0 & 0.1628 & -0.4393 & 0 \\ 32.1388 & 0 & 1.6451 & 0.5257 & 1.0000 & 0 & -1.7905 & 0 & 0.1081 & 0 & 0 & -0.1861 & 0.3002 & 0 & -0.0407 & 0.2348 & 0 \\ 117.2611 & 0 & 2.8686 & 1.9503 & 0 & 0 & -3.7495 & 0 & -0.0013 & 1.0000 & 0 & -1.3866 & 1.1074 & 0 & -0.5057 & 1.1885 & 0 \\ 6.7416 & 0 & -0.0974 & -0.5851 & 0 & 0 & 0.3582 & 0 & 0.0447 & 0 & 0 & 0.2008 & -0.0981 & 1.0000 & -0.0600 & -0.0634 & 0 \\ 34.7099 & 1.0000 & 1.9593 & 0.0227 & 0 & 0 & -2.2955 & 0 & 0.1214 & 0 & 0 & -0.3202 & 0.2262 & 0 & 0.0160 & 0.3502 & 0 \\ 74.4438 & 0 & 9.3629 & -4.2105 & 0 & 0 & -6.6841 & 1.0000 & 0.4910 & 0 & 0 & -1.2345 & 0.4750 & 0 & -0.9133 & 1.1471 & 0 \\ 31.1472 & 0 & 1.3716 & -0.4940 & 0 & 0 & -1.4900 & 0 & -0.0267 & 0 & 0 & -0.2318 & 0.1481 & 0 & -0.1134 & 0.2692 & 1.0000 \end{bmatrix}$$

Final pivot, 3,7

$$\mathbf{C_{3B}} = \begin{bmatrix} 154.6143 & 0 & 7.8330 & 2.4195 & 0 & 0 & 0 & 0 & 0.8133 & 0 & 0.2622 & 0.8016 & 0.8546 & 0 & 0.2308 & 0.2501 & 0 \\ 72.2960 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\ 11.2143 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\ 52.2182 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\ 159.3089 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\ 2.7242 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\ 60.4526 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\ 149.4012 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\ 47.8563 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \end{bmatrix}$$

Comparing the tableaux T_{3B} and C_{3B}, both have an optimal z value of -154.6143.

3.3 C

My first basic variable in t^* is x_2, or column 3. To calculate the range of which this variables price can change while maintaining optimality, I got the row ratio of row 5 with respect to the top row. To get lower bound, I choose the smallest positive row ratio, which was 0.5325. To get the upper bound, consider only negative row ratios and choose the more positive row ratio, which was -4.0115. I then negated both of these to put them into an equality equation.

Range: $-0.5325 \leq q \leq 4.0115$

We can see that we can decrease the price up to $-.05325/2$, which is roughly $-.02663$. I then set the the cost co-efficient of the first basic variable, equal to $-.02663$ and then pivoted on row 5, column 2.

$$\mathbf{T_{3C}} = \begin{bmatrix} 118.7995 & 0 & 7.7807 & 1.8370 & 0 & 0 & 0 & 0 & 0.8376 & 0 & 0.1311 & 0.8548 & 0.8207 & 0 & 0.2052 & 0.2144 & 0 \\ 55.6942 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\ 101.8264 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\ 82.4982 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\ 48.1345 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\ 5.4484 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\ 19.8804 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\ 46.1381 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\ 36.0120 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \end{bmatrix}$$

The new optimal z-value is -118.7995, and optimal x is unchanged:

To check result, I increased the price in T_c and then pivoted to optimality.

$$\mathbf{C_{3C_1}} = \begin{bmatrix} 20.8965 & 5.9329 & 9.0000 & -3.6667 & 1.0000 & -7.0000 & -2.0000 & 0 & 1.6667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0436 & -1.8571 & -6.8571 & 2.5714 & -4.7143 & 2.4286 & 6.0000 & -0.2857 & -0.8571 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 94.0122 & -0.4762 & -1.1429 & 3.7619 & -1.2857 & 1.5714 & 3.0000 & 0.2857 & -0.4762 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 39.0758 & -6.0476 & -2.7143 & 1.4762 & -5.4286 & 7.8571 & 5.0000 & 0.4286 & -1.0476 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 58.0954 & -1.6190 & 1.7143 & 4.1905 & 7.4286 & -1.8571 & -3.0000 & -0.4286 & 0.3810 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 11.7389 & 1.2381 & 0.5714 & -0.3810 & 1.1429 & -1.2857 & -1.0000 & -0.1429 & 0.2381 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 17.1333 & 5.0000 & -3.0000 & 3.0000 & -1.0000 & -1.0000 & -1.0000 & -1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 4.5639 & -1.8571 & 2.1429 & -1.4286 & -9.7143 & 8.4286 & 0 & 0.7143 & -0.8571 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 7.5596 & -0.0952 & -0.4286 & -0.0476 & 0.1429 & -0.2857 & 0 & -0.1429 & -0.0952 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Simplex first, pivot on 2,6

$$\mathbf{C_{3C_2}} = \begin{bmatrix} 23.9046 & 0.5800 & -10.7647 & 3.7451 & -12.5882 & 0 & 15.2941 & -0.8235 & -0.8039 & 2.8824 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4297 & -0.7647 & -2.8235 & 1.0588 & -1.9412 & 1.0000 & 2.4706 & -0.1176 & -0.3529 & 0.4118 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 93.3369 & 0.7255 & 3.2941 & 2.0980 & 1.7647 & 0 & -0.8824 & 0.4706 & 0.0784 & -0.6471 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 35.6994 & -0.0392 & 19.4706 & -6.8431 & 9.8235 & 0 & -14.4118 & 1.3529 & 1.7255 & -3.2353 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 58.8934 & -3.0392 & -3.5294 & 6.1569 & 3.8235 & 0 & 1.5882 & -0.6471 & -0.2745 & 0.7647 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 12.2915 & 0.2549 & -3.0588 & 0.9804 & -1.3529 & 0 & 2.1765 & -0.2941 & -0.2157 & 0.5294 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 17.5630 & 4.2353 & -5.8235 & 4.0588 & -2.9412 & 0 & 1.4706 & -1.1176 & -0.3529 & 0.4118 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0.9420 & 4.5882 & 25.9412 & -10.3529 & 6.6471 & 0 & -20.8235 & 1.7059 & 2.1176 & -3.4706 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 7.6824 & -0.3137 & -1.2353 & 0.2549 & -0.4118 & 0 & 0.7059 & -0.1765 & -0.1961 & 0.1176 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Simplex, pivot on 8,5

$$\mathbf{C_{3C_3}} = \begin{bmatrix} 25.6885 & 9.2692 & 38.3628 & -15.8614 & 0 & 0 & -24.1416 & 2.4071 & 3.2065 & -3.6903 & 0 & 0 & 0 & 0 & 0 & 1.8938 & 0 \\ 0.7048 & 0.5752 & 4.7522 & -1.9646 & 0 & 1.0000 & -3.6106 & 0.3805 & 0.2655 & -0.6018 & 0 & 0 & 0 & 0 & 0 & 0.2920 & 0 \\ 93.0868 & -0.4926 & -3.5929 & 4.8466 & 0 & 0 & 4.6460 & 0.0177 & -0.4838 & 0.2743 & 1.0000 & 0 & 0 & 0 & 0 & -0.2655 & 0 \\ 34.3073 & -6.8201 & -18.8673 & 8.4572 & 0 & 0 & 16.3628 & -1.1681 & -1.4041 & 1.8938 & 0 & 1.0000 & 0 & 0 & 0 & -1.4779 & 0 \\ 58.3516 & -5.6785 & -18.4513 & 12.1121 & 0 & 0 & 13.5664 & -1.6283 & -1.4926 & 2.7611 & 0 & 0 & 1.0000 & 0 & 0 & -0.5752 & 0 \\ 12.4832 & 1.1888 & 2.2212 & -1.1268 & 0 & 0 & -2.0619 & 0.0531 & 0.2153 & -0.1770 & 0 & 0 & 0 & 1.0000 & 0 & 0.2035 & 0 \\ 17.9798 & 6.2655 & 5.6549 & -0.5221 & 0 & 0 & -7.7434 & -0.3628 & 0.5841 & -1.1239 & 0 & 0 & 0 & 0 & 1.0000 & 0.4425 & 0 \\ 0.1417 & 0.6903 & 3.9027 & -1.5575 & 1.0000 & 0 & -3.1327 & 0.2566 & 0.3186 & -0.5221 & 0 & 0 & 0 & 0 & 0 & 0.1504 & 0 \\ 7.7408 & -0.0295 & 0.3717 & -0.3864 & 0 & 0 & -0.5841 & -0.0708 & -0.0649 & -0.0973 & 0 & 0 & 0 & 0 & 0 & 0.0619 & 1.0000 \end{bmatrix}$$

Simplex, pivot on 4,7

$$\mathbf{C_{3C_4}} = \begin{bmatrix} 76.3053 & -0.7930 & 10.5262 & -3.3836 & 0 & 0 & 0 & 0.6836 & 1.1348 & -0.8962 & 0 & 1.4754 & 0 & 0 & 0 & -0.2866 & 0 \\ 8.2751 & -0.9297 & 0.5890 & -0.0984 & 0 & 1.0000 & 0 & 0.1228 & -0.0443 & -0.1839 & 0 & 0.2207 & 0 & 0 & 0 & -0.0341 & 0 \\ 83.3457 & 1.4438 & 1.7642 & 2.4453 & 0 & 0 & 0 & 0.3494 & -0.0851 & -0.2634 & 1.0000 & -0.2839 & 0 & 0 & 0 & 0.1541 & 0 \\ 2.0967 & -0.4168 & -1.1531 & 0.5169 & 0 & 0 & 1.0000 & -0.0714 & -0.0858 & 0.1157 & 0 & 0.0611 & 0 & 0 & 0 & -0.0903 & 0 \\ 29.9075 & -0.0240 & -2.8085 & 5.1002 & 0 & 0 & 0 & -0.6598 & -0.3285 & 1.1909 & 0 & -0.8291 & 1.0000 & 0 & 0 & 0.6501 & 0 \\ 16.8064 & 0.3294 & -0.1563 & -0.0611 & 0 & 0 & 0 & -0.0941 & 0.0384 & 0.0617 & 0 & 0.1260 & 0 & 1.0000 & 0 & 0.0173 & 0 \\ 34.2150 & 3.0380 & -3.2737 & 3.4801 & 0 & 0 & 0 & -0.9156 & -0.0804 & -0.2277 & 0 & 0.4732 & 0 & 0 & 1.0000 & -0.2569 & 0 \\ 6.7100 & -0.6155 & 0.2904 & 0.0617 & 1.0000 & 0 & 0 & 0.0330 & 0.0498 & -0.1595 & 0 & 0.1915 & 0 & 0 & 0 & -0.1325 & 0 \\ 8.9654 & -0.2729 & -0.3018 & -0.0846 & 0 & 0 & 0 & -0.1125 & -0.1150 & -0.0297 & 0 & 0.0357 & 0 & 0 & 0 & 0.0092 & 1.0000 \end{bmatrix}$$

Simplex, pivot 4,4

$$\mathbf{C_{3C_5}} = \begin{bmatrix} 90.0312 & -3.5217 & 2.9777 & 0 & 0 & 0 & 6.5466 & 0.2163 & 0.5731 & -0.1385 & 0 & 1.8755 & 0 & 0 & 0 & -0.8779 & 0 \\ 8.6744 & -1.0091 & 0.3694 & 0 & 0 & 1.0000 & 0.1904 & 0.1092 & -0.0607 & -0.1618 & 0 & 0.2323 & 0 & 0 & 0 & -0.0513 & 0 \\ 73.4262 & 3.4158 & 7.2194 & 0 & 0 & 0 & -4.7311 & 0.6871 & 0.3209 & -0.8110 & 1.0000 & -0.5731 & 0 & 0 & 0 & 0.5814 & 0 \\ 4.0566 & -0.8064 & -2.2309 & 1.0000 & 0 & 0 & 1.9348 & -0.1381 & -0.1660 & 0.2239 & 0 & 0.1182 & 0 & 0 & 0 & -0.1747 & 0 \\ 9.2180 & 4.0889 & 8.5696 & 0 & 0 & 0 & -9.8678 & 0.0446 & 0.5183 & 0.0488 & 0 & -1.4322 & 1.0000 & 0 & 0 & 1.5413 & 0 \\ 17.0543 & 0.2801 & -0.2926 & 0 & 0 & 0 & 0.1182 & -0.1025 & 0.0283 & 0.0753 & 0 & 0.1332 & 0 & 1.0000 & 0 & 0.0066 & 0 \\ 20.0978 & 5.8444 & 4.4901 & 0 & 0 & 0 & -6.7332 & -0.4349 & 0.4974 & -1.0070 & 0 & 0.0617 & 0 & 0 & 1.0000 & 0.3512 & 0 \\ 6.4599 & -0.5657 & 0.4280 & 0 & 1.0000 & 0 & -0.1193 & 0.0415 & 0.0600 & -0.1734 & 0 & 0.1842 & 0 & 0 & 0 & -0.1217 & 0 \\ 9.3084 & -0.3411 & -0.4904 & 0 & 0 & 0 & 0.1636 & -0.1242 & -0.1291 & -0.0108 & 0 & 0.0457 & 0 & 0 & 0 & -0.0056 & 1.0000 \end{bmatrix}$$

Simplex, pivot on 5,2

$$\mathbf{C_{3C_6}} = \begin{bmatrix} 97.9704 & 0 & 10.3583 & 0 & 0 & 0 & -1.9522 & 0.2547 & 1.0195 & -0.0964 & 0 & 0.6420 & 0.8613 & 0 & 0 & 0.4496 & 0 \\ 10.9492 & 0 & 2.4842 & 0 & 0 & 1.0000 & -2.2447 & 0.1202 & 0.0672 & -0.1498 & 0 & -0.1211 & 0.2468 & 0 & 0 & 0.3291 & 0 \\ 65.7258 & 0 & 0.0607 & 0 & 0 & 0 & 3.5122 & 0.6498 & -0.1121 & -0.8517 & 1.0000 & 0.6233 & -0.8354 & 0 & 0 & -0.7061 & 0 \\ 5.8745 & 0 & -0.5408 & 1.0000 & 0 & 0 & -0.0113 & -0.1293 & -0.0638 & 0.2336 & 0 & -0.1642 & 0.1972 & 0 & 0 & 0.1292 & 0 \\ 2.2544 & 1.0000 & 2.0958 & 0 & 0 & 0 & -2.4133 & 0.0109 & 0.1268 & 0.0119 & 0 & -0.3503 & 0.2446 & 0 & 0 & 0.3770 & 0 \\ 16.4229 & 0 & -0.8796 & 0 & 0 & 0 & 0.7942 & -0.1056 & -0.0073 & 0.0720 & 0 & 0.2313 & -0.0685 & 1.0000 & 0 & -0.0990 & 0 \\ 6.9222 & 0 & -7.7587 & 0 & 0 & 0 & 7.3712 & -0.4988 & -0.2435 & -1.0768 & 0 & 2.1088 & -1.4293 & 0 & 1.0000 & -1.8518 & 0 \\ 7.7353 & 0 & 1.6137 & 0 & 1.0000 & 0 & -1.4846 & 0.0477 & 0.1317 & -0.1666 & 0 & -0.0140 & 0.1384 & 0 & 0 & 0.0915 & 0 \\ 10.0774 & 0 & 0.2245 & 0 & 0 & 0 & -0.6596 & -0.1204 & -0.0858 & -0.0067 & 0 & -0.0738 & 0.0834 & 0 & 0 & 0.1230 & 1.0000 \end{bmatrix}$$

Simplex, pivot on 7,7

$$\mathbf{C_{3C_7}} = \begin{bmatrix} 99.8037 & 0 & 8.3035 & 0 & 0 & 0 & 0 & 0.1226 & 0.9550 & -0.3816 & 0 & 1.2005 & 0.4827 & 0 & 0.2648 & -0.0409 & 0 \\ 13.0572 & 0 & 0.1214 & 0 & 0 & 1.0000 & 0 & -0.0317 & -0.0069 & -0.4777 & 0 & 0.5211 & -0.1885 & 0 & 0.3045 & -0.2348 & 0 \\ 62.4275 & 0 & 3.7575 & 0 & 0 & 0 & 0 & 0.8875 & 0.0039 & -0.3387 & 1.0000 & -0.3815 & -0.1543 & 0 & -0.4765 & 0.1762 & 0 \\ 5.8852 & 0 & -0.5528 & 1.0000 & 0 & 0 & 0 & -0.1301 & -0.0642 & 0.2319 & 0 & -0.1610 & 0.1950 & 0 & 0.0015 & 0.1264 & 0 \\ 4.5207 & 1.0000 & -0.4444 & 0 & 0 & 0 & 0 & -0.1524 & 0.0471 & -0.3406 & 0 & 0.3401 & -0.2234 & 0 & 0.3274 & -0.2293 & 0 \\ 15.6771 & 0 & -0.0437 & 0 & 0 & 0 & 0 & -0.0519 & 0.0190 & 0.1880 & 0 & 0.0041 & 0.0855 & 1.0000 & -0.1077 & 0.1006 & 0 \\ 0.9391 & 0 & -1.0526 & 0 & 0 & 0 & 1.0000 & -0.0677 & -0.0330 & -0.1461 & 0 & 0.2861 & -0.1939 & 0 & 0.1357 & -0.2512 & 0 \\ 9.1295 & 0 & 0.0510 & 0 & 1.0000 & 0 & 0 & -0.0528 & 0.0827 & -0.3835 & 0 & 0.4107 & -0.1495 & 0 & 0.2014 & -0.2814 & 0 \\ 10.6968 & 0 & -0.4698 & 0 & 0 & 0 & 0 & -0.1651 & -0.1076 & -0.1031 & 0 & 0.1149 & -0.0445 & 0 & 0.0895 & -0.0427 & 1.0000 \end{bmatrix}$$

Simplex, pivot on 4,10

$$\mathbf{C_{3C_8}} = \begin{bmatrix} 109.4878 & 0 & 7.3939 & 1.6455 & 0 & 0 & 0 & -0.0914 & 0.8494 & 0 & 0 & 0.9356 & 0.8036 & 0 & 0.2674 & 0.1671 & 0 \\ 25.1803 & 0 & -1.0172 & 2.0599 & 0 & 1.0000 & 0 & -0.2997 & -0.1391 & 0 & 0 & 0.1895 & 0.2132 & 0 & 0.3077 & 0.0255 & 0 \\ 71.0229 & 0 & 2.9501 & 1.4605 & 0 & 0 & 0 & 0.6975 & -0.0898 & 0 & 1.0000 & -0.6165 & 0.1305 & 0 & -0.4742 & 0.3608 & 0 \\ 25.3781 & 0 & -2.3836 & 4.3122 & 0 & 0 & 0 & -0.5610 & -0.2768 & 1.0000 & 0 & -0.6941 & 0.8410 & 0 & 0.0066 & 0.5450 & 0 \\ 13.1642 & 1.0000 & -1.2562 & 1.4687 & 0 & 0 & 0 & -0.3434 & -0.0472 & 0 & 0 & 0.1037 & 0.0630 & 0 & 0.3297 & -0.0437 & 0 \\ 10.9058 & 0 & 0.4044 & -0.8107 & 0 & 0 & 0 & 0.0536 & 0.0710 & 0 & 0 & 0.1346 & -0.0726 & 1.0000 & -0.1090 & -0.0019 & 0 \\ 4.6463 & 0 & -1.4008 & 0.6299 & 0 & 0 & 1.0000 & -0.1496 & -0.0735 & 0 & 0 & 0.1847 & -0.0711 & 0 & 0.1366 & -0.1716 & 0 \\ 18.8611 & 0 & -0.8630 & 1.6536 & 1.0000 & 0 & 0 & -0.2679 & -0.0235 & 0 & 0 & 0.1446 & 0.1730 & 0 & 0.2040 & -0.0725 & 0 \\ 13.3133 & 0 & -0.7156 & 0.4446 & 0 & 0 & 0 & -0.2229 & -0.1361 & 0 & 0 & 0.0434 & 0.0422 & 0 & 0.0902 & 0.0135 & 1.0000 \end{bmatrix}$$

Final simplex pivot 3,8

$$\mathbf{C_{3C}} = \begin{bmatrix} 118.7995 & 0 & 7.7807 & 1.8370 & 0 & 0 & 0 & 0 & 0.8376 & 0 & 0.1311 & 0.8548 & 0.8207 & 0 & 0.2052 & 0.2144 & 0 \\ 55.6942 & 0 & 0.2503 & 2.6874 & 0 & 1.0000 & 0 & 0 & -0.1777 & 0 & 0.4296 & -0.0754 & 0.2693 & 0 & 0.1040 & 0.1805 & 0 \\ 101.8264 & 0 & 4.2297 & 2.0939 & 0 & 0 & 0 & 1.0000 & -0.1288 & 0 & 1.4337 & -0.8840 & 0.1871 & 0 & -0.6799 & 0.5173 & 0 \\ 82.4982 & 0 & -0.0109 & 5.4868 & 0 & 0 & 0 & 0 & -0.3490 & 1.0000 & 0.8042 & -1.1900 & 0.9459 & 0 & -0.3748 & 0.8352 & 0 \\ 48.1345 & 1.0000 & 0.1964 & 2.1878 & 0 & 0 & 0 & 0 & -0.0914 & 0 & 0.4924 & -0.1998 & 0.1273 & 0 & 0.0962 & 0.1339 & 0 \\ 5.4484 & 0 & 0.1777 & -0.9229 & 0 & 0 & 0 & 0 & 0.0779 & 0 & -0.0768 & 0.1820 & -0.0826 & 1.0000 & -0.0725 & -0.0296 & 0 \\ 19.8804 & 0 & -0.7680 & 0.9432 & 0 & 0 & 1.0000 & 0 & -0.0927 & 0 & 0.2145 & 0.0524 & -0.0431 & 0 & 0.0349 & -0.0942 & 0 \\ 46.1381 & 0 & 0.2700 & 2.2145 & 1.0000 & 0 & 0 & 0 & -0.0580 & 0 & 0.3841 & -0.0922 & 0.2231 & 0 & 0.0218 & 0.0661 & 0 \\ 36.0120 & 0 & 0.2273 & 0.9114 & 0 & 0 & 0 & 0 & -0.1648 & 0 & 0.3196 & -0.1537 & 0.0839 & 0 & -0.0614 & 0.1288 & 1.0000 \end{bmatrix}$$

Comparing the tableaux T_{3C} and C_{3C}, both have an optimal z value of -118.7995.