<center>## Problem 3</center>

$$z - z_0 = a\left((x-x_0)^2 + (y-y_0)^2\right)$$
$$z = a\left(x^2 - 2xx_0 + x_0^2 + y^2 - 2yy_0 + y_0^2\right) + z_0$$
$$z = a(x^2 + y^2) - 2ax_0x - 2ay_0y + \left(z_0 + ax_0^2 + ay_0^2\right)$$

so
$$p_0 = z_0 + ax_0^2 + ay_0^2$$
$$p_1 = -2ax_0$$
$$p_2 = -2ay_0$$
$$p_3 = a$$

This way we can use linear least-squares fit.

Best fit parameters are the following:
$$a = 1.67 \times 10^{-4}$$
$$x_0 = -1.36$$
$$y_0 = 58.22$$
$$z_0 = 1512.88$$

Noise was found to be ~ 3.7683 using noise $= np.std(\underbrace{z\_data}_{\text{actual}} - \underbrace{z\_pred}_{\text{model}})$

We then use the parameter covariance method to find uncertainty on a.

We can use this to find uncertainty on a, which is 4.54×10^{-9}.

Focal length is $1499.66 \ +/- \ 0.04087$ (i'm assuming this is in mm)