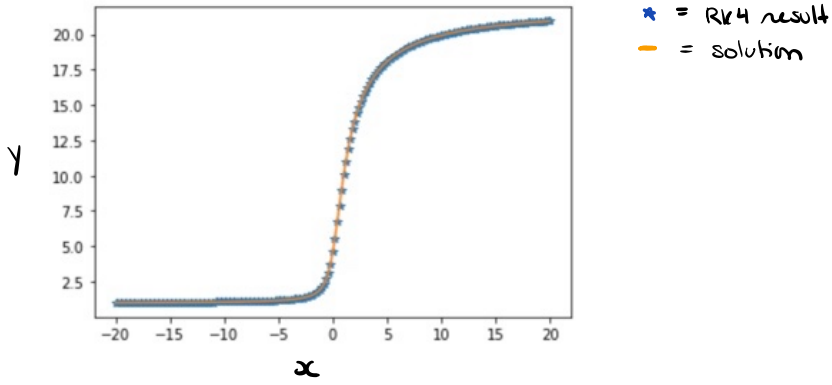
Problem 1

Part A

Integrating $\frac{dy}{dx} = \frac{y}{1+x^2}$ from $x = -20$ to $x = 20$, $y(-20) = 1$

Solution is: $\exp(\arctan(x))$

Graph 1: RK4 Integrator, 200 steps

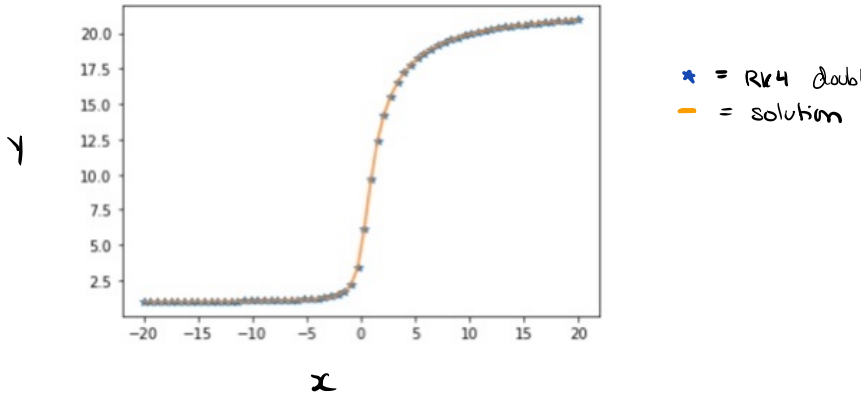


* = RK4 result
— = solution

Part b

Integrating same function, but with double stepper.

Graph 2: Double stepper RK4 integrator, 66 steps



* = RK4 double stepper result
— = solution

* Note: 66 steps because we need same number of function evaluations ($200/3 \approx 66$) → 3 function evaluations per step.

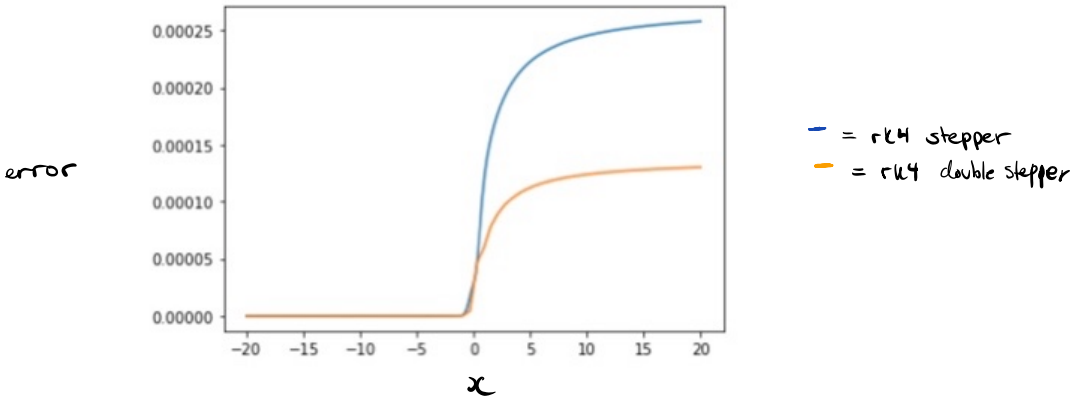
std of rk4_stepper: 1.18×10^{-4}

std of rk4_stepperd: 5.95×10^{-5}

∴ rk4_stepperd is more accurate.

Residuals graph:

Graph 3: Residuals graph comparing methods



From the residuals graph, we see that the rk4 double stepper has significantly less error than the rk4 stepper when we go in the $x > 0$ regime.