Problem 6

a) a proof showing that the expected power spectrum of a random walk goes like k^{-2}:

Correlation function of 2 points separated by δ is proportional to $c-|\delta|$
Doing the fourier transform of $c-|\delta|$: let $c=N$ and $\delta = x$

$$\int_{-N}^{0} (x+N)\, e^{\frac{-2\pi i k x}{N}} dx + \int_{0}^{N} (-x+N) e^{-\frac{2\pi i k x}{N}} dx$$

⋮

Supposed to have a $\frac{1}{k^2}$ behaviour.

OR Say f is a random walk

power spectrum $P[f](k) = |\mathcal{F}[f](k)|^2$

$$\mathcal{F}\left[\frac{df}{dx}\right](k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{df}{dx} e^{-ik} dx$$

Integrate by parts:

$$\frac{1}{\sqrt{2\pi}} f e^{-ikx}\Big|_{-\infty}^{\infty} - \frac{ki}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f e^{-ikx} dx = ki\mathcal{F}[f](k)$$

$$= \frac{k}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f e^{-ikx} dx = -ki\mathcal{F}[f](k)$$

$$\Rightarrow P\left[\frac{df}{dt}\right](x) = \omega^2 P[f](k)$$

not entirely sure...

b)

Graph 1: Power Spectrum of Random Walk.