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Operation Bernhard: A Failed Operation

1 Introduction

Throughout warfare history, there have been multiple incidents of countries counterfeiting the opposition country's currency in an attempt to destabilize the opponent's economy. One such incident was Operation Bernhard, executed by Nazi Germany to destabilize the British Economy during World War II.[1] The idea is that counterfeit money making its way through the economy will cause a devaluation of the currency, leading to other devastating economic results such as hyperinflation.

This paper will model and analyze this strategy by looking specifically at Operation Bernhard, though the results can be applied to other counterfeiting incidents during war as well. The analysis will be structured as follows. First, the report begins by analyzing data from the monetary authorities of Britain and Australia about counterfeit money and its effects. Subsequently, the report proposes a variant of the money-search model that illustrates the impact of this scenario on a model economy and conducts a simulation of the model. Finally, the report concludes that this strategy is ineffective given that the local economy can manipulate its money supply and issue a new currency to evade such tactics.

2 Descriptive Data

The paper will begin by discussing data relevant to counterfeiting schemes. Fig. 1 shows the number of counterfeit British money discovered (in £5, £10, £20, and £50 notes) and withdrawn before circulation between 2005 and 2018. The data are obtained from the Bank of England, which is the British monetary authority, ensuring their accuracy. The inclusion of this dataset is to provide some context into the counterfeiting situation in Britain. Additionally, because Operation Bernhard dealt with counterfeiting British pounds, this dataset is apropos.

As seen in Fig. 1, there are many counterfeit notes detected, especially £20 notes, perhaps because the £20 note is the most commonly used. The total money value of counterfeits often

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[1] Wikimedia Foundation. (2021, June 29). *Operation Bernhard*. Wikipedia. Retrieved November 21, 2021, from https://en.wikipedia.org/wiki/Operation_Bernhard

exceeds £10 million, indicating that counterfeiting is a relatively common strategy. While the data are relatively recent, they still elucidate that the monetary authority attempts to withdraw counterfeit money before it starts circulating (see the line graph) because they will undermine the local currency. However, the data also imply that the monetary authority is often not very successful, as indicated by the low number of notes seized compared to the total number of counterfeit notes. Regardless, one of the goals of the monetary authority is to prevent economic collapse or devastation, and Fig. 1 indicates that one way of doing that is to prevent counterfeits from circulating.

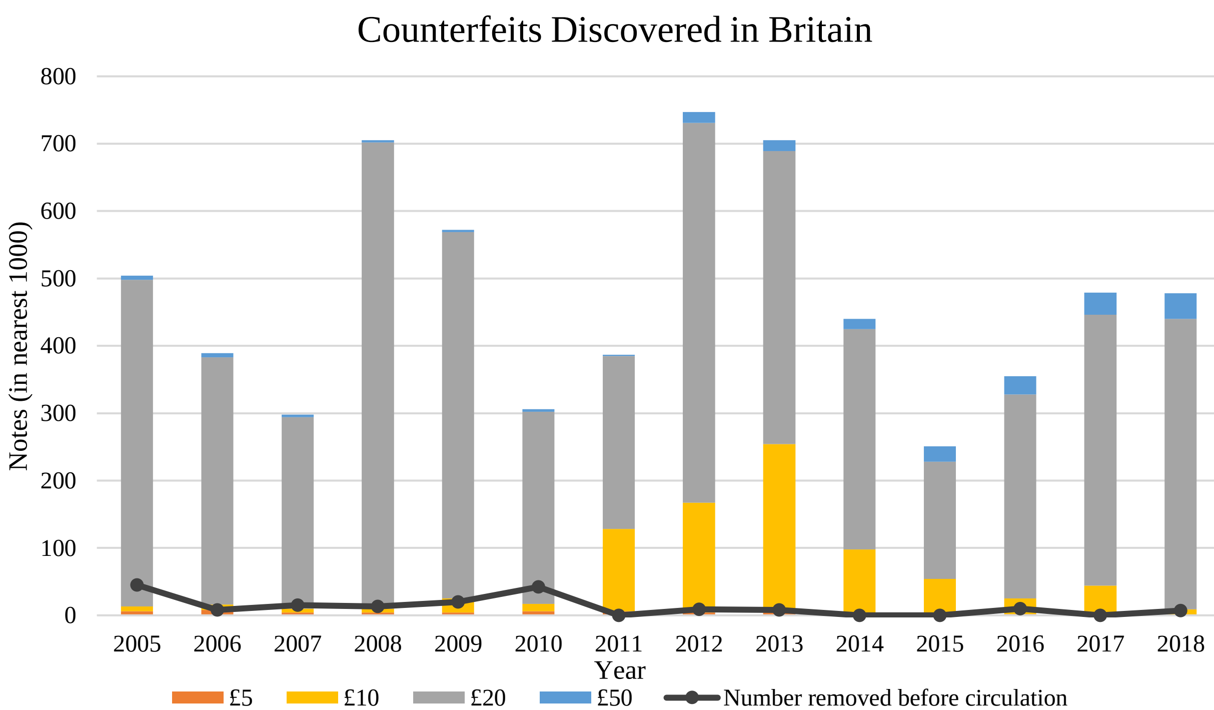

Fig. 1: Number of counterfeit notes discovered in Britain (in the nearest 1000)

Source: Bank of England[2]

Fig. 2 presents the simulation results of a baseline model depicting what happens after a counterfeiting shock occurs. These results are taken from a paper titled 'The Social Costs of Currency Counterfeiting,' published by the Reserve Bank of Australia and by authors Viles, Rush, and Rohling.[3] Since the Reserve Bank of Australia is Australia's monetary authority, this

[2] *Banknote statistics*. Bank of England. (2017, October 5). Retrieved November 21, 2021, from https://www.bankofengland.co.uk/statistics/banknote

[3] Viles, N., Rush, A., & Rohling, T. (2015). The Social Costs of Currency Counterfeiting. *Reserve Bank of Australia*. Retrieved November 30, 2021, from https://www.rba.gov.au/publications/rdp/2015/2015-05/social-costs-currency-counterfeiting.html

paper assumes the results accurately depict what would happen after a counterfeit shock. While Operation Bernhard dealt with Britain and Nazi Germany and not Australia, Fig. 2 provides more context to the incentive behind counterfeits during warfare in general and are therefore included in this paper.

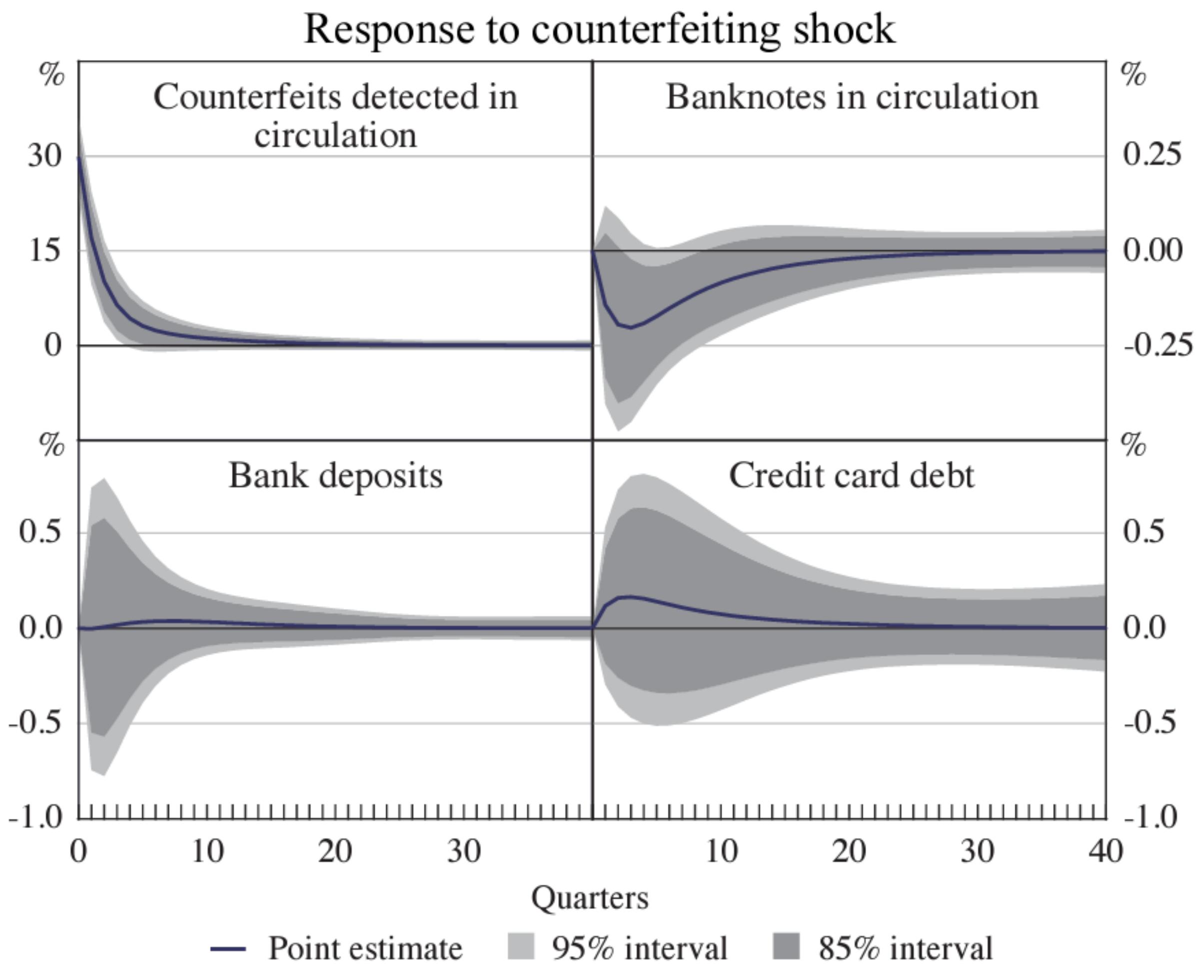

Fig. 2: Impulse Response Functions after a Counterfeiting Shock

Source: "The Social Costs of Currency Counterfeiting," Reserve Bank of Australia[4]

Consider the 'Counterfeits detected in circulation' and the 'Banknotes in circulation' portions of Fig. 2. These two sections do not represent a correlation but, rather, causation because of how the authors conducted the simulation. The source paper specifically simulated 'Banknotes in circulation' as a response to counterfeit shocks.[5] Therefore, these two sections suggest a decrease in banknotes in circulation in response to a detection of counterfeits. As a

[4] Ibid, p. 15; graphs first spotted in a Canvas feedback draft and repurposed for this paper
[5] Ibid, pp. 14-15

result, Fig. 2 corroborates the idea that the local currency is weakened due to the presence of counterfeit money in the economy.

Regarding Operation Bernhard, Nazi Germany pursued a counterfeiting money strategy similar what is depicted in Fig. 1, but on a much larger scale. Fig. 2 implies that Nazi Germany hoped to weaken the British currency by injecting millions of pounds worth of valueless money into the British economy. These two data figures combined contextualize Operation Bernhard and counterfeit money during warfare schemes in general. Such strategies are designed to weaken the local economy and currency and hope to cause economic crises. Moving forward, the rest of this paper will build an economic model and analyze Operation Bernhard and its effectiveness in achieving its goal of undermining the British economy.

3 Model

This paper aims to analyze the use of counterfeiting money to devalue a country's currency in times of war generally, but about Operation Bernhard between Nazi Germany and Britain specifically. In this operation, Nazi Germany is the 'foreign government' in the model; on the other hand, Britain is the 'local government.' The paper utilizes a money-search model on the people of Britain, with two possible agents, namely goods holders and money holders, and an agent can only be one of the two. Furthermore, there are two types of money holders: real money holders and counterfeit money holders.

3.1 Model Construction and Description

Before constructing the model, it is worth relating it to the monetary scenario at hand: counterfeit money in the economy. The standard money-search model depicts agents of different types either having the option to trade goods or buy goods using money. However, in this model, the agents also have a probability of running into a counterfeit money holder and attempting to enter a transaction using counterfeit money. In the event when counterfeit money is detected, the transaction will not occur. The paper designs the model in the manner that counterfeit money can circulate around undetected, and the agents can be left holding money with no value. However, the local government still has the authority to maximize the welfare of the economy. The model in this paper assumes both governments have perfect information on how much counterfeit is circulating and the people's preferences, but these crucial pieces of information are not made available to the people. Therefore, the people themselves will have to determine the legitimacy of the banknote they are about to use. Relating to Operation Bernhard, Nazi Germany wants to destabilize the British economy by devaluing their currency in the manner of reducing the British public's trust in the money. On the other hand, however, the British monetary authority can still choose how much money to put into circulation and will attempt to counter Nazi Germany's efforts to destabilize its economy.

This model utilizes several variables. V_g and V_m are the value functions of the goods holders and the money holders, respectively. The variables π_g and π_m represent the money choice probabilities and can only take two values, either 0 or 1. $\pi_g = 0$ indicates that the goods holder does not want to accept money for their good, and $\pi_g = 1$ is the opposite. Similarly, $\pi_m = 0$ means that the money holder refuses to spend money, and $\pi_m = 1$ means that the money holder wants to spend their money to buy a good. M is the proportion of money holders in the economy, which also translates to the probability of an agent meeting a money holder. The variable x is the probability that the agent likes another agent's good, and the variable y is the opposite. Further in the paper, as the model solves for equilibrium, $x = y$ because they represent a mutual like or dislike between the two agents in the transaction. The model also utilizes two constants: u is the utility gained by an agent from obtaining goods, and r is the interest rate of the economy.

The variables elaborated so far are standard in the money-search model. However, the counterfeit money scenario is not the standard model, and there are additional variables to be accounted for. Let f be the proportion of counterfeit money holders within the money holder population; therefore, the number of counterfeit money holders encompass $M * f$ of the entire population. Additionally, let q be the quality of counterfeit money, which is also the same as the probability of someone detecting the counterfeit in a transaction. If $q = 1$, the counterfeit is very bad such that it will always be detected. This variable cannot take the value of 0 because there is no such thing as a perfect counterfeit.

Goods holder's value function:

$$\begin{aligned} rV_g = {} & (1-M)xy\left(u + \left(V_g - V_g\right)\right) + (1-M)(1-xy)\left(0 + \left(V_g - V_g\right)\right) \\ & + M(1-f)\pi_g\pi_m y\left(0 + \left(V_m - V_g\right)\right) + M(1-f)\left(1 - \pi_g\pi_m y\right)\left(0 + \left(V_g - V_g\right)\right) \\ & + Mfq\left(0 + \left(V_g - V_g\right)\right) + Mf(1-q)\pi_g\pi_m y\left(0 + \left(V_m - V_g\right)\right) \\ & + Mf(1-q)\left(1 - \pi_g\pi_m y\right)\left(0 + \left(V_g - V_g\right)\right) \end{aligned}$$

This is the value function for the goods holders in Britain. The goods holders can experience 7 possible trades, listed as follows:

- Meet a goods holder, double coincidence of wants, trade, no status change;
- Meet a goods holder, no double coincidence of wants, no trade, no status change;
- Meet a real money holder, double coincidence of wants, trade, status changes from goods to money holder;
- Meet a real money holder, no double coincidence of wants, no trade, no status change;
- Meet a counterfeit money holder, counterfeit detected with probability q, no trade, no status change;
- Meet a counterfeit money holder, counterfeit not detected with probability $(1-q)$ double coincidence of wants, trade, status changes from goods to money holder; and
- Meet a counterfeit money holder, counterfeit not detected with probability $(1-q)$, no double coincidence of wants, no trade, no status change.

Money holder's value function

$$\begin{aligned} rV_m = (1-f)\Big[&(1-M)\pi_g\pi_m x\left(u+\left(V_g-V_m\right)\right)+(1-M)\left(1-\pi_g\pi_m x\right)\left(0+(V_m-V_m)\right) \\ &+M(1-f)\left(0+(V_m-V_m)\right)+Mf\left(0+(V_m-V_m)\right)\Big] \\ &+(f)\Big[(1-M)q\left(0+(V_m-V_m)\right)+(1-M)(1-q)\pi_g\pi_m x\left(u+\left(V_g-V_m\right)\right) \\ &+(1-M)\left(1-\pi_g\pi_m x\right)\left(0+(V_m-V_m)\right)+M(1-f)\left(0+(V_m-V_m)\right) \\ &+Mf\left(0+(V_m-V_m)\right)\Big] \end{aligned}$$

On the other hand, this is the value function for the money holders in Britain. Recall that there are two types of money holders: real money holders and counterfeit money holders. Accordingly, the value function for money holders is divided between these two types. The money holders can experience 9 possible trades, listed as follows:

- Money holder has a probability $(1-f)$ of holding real money:
  - Meet a goods holder, double coincidence of wants, trade, status changes from money to goods holder;
  - Meet a goods holder, no double coincidence of wants, no trade, no status change;
  - Meet a real money holder, no trade, no status change; and
  - Meet a counterfeit money holder, no trade, no status change.

- Money holder has a probability f of holding counterfeit money:
  - Meet a goods holder, counterfeit detected, no trade, no status change;
  - Meet a goods holder, counterfeit not detected, double coincidence of wants, trade, status changes from money to goods holder;
  - Meet a goods holder, counterfeit not detected, no double coincidence of wants, no trade, no status change;
  - Meet a real money holder, no trade, no status change; and
  - Meet a counterfeit money holder, no trade, no status change.

3.2 Model Solution

After defining the value functions for the two possible agents in the economy, the next step is to solve the model by first simplifying the value functions into their tractable math versions. A lot of the terms in the equations can be reduced to zero, and therefore the equations are now as follows:

$$rV_g = (1-M)xyu + M\left(V_m - V_g\right)\pi_g\pi_m y(1-fq)$$

$$rV_m = (1-M)\pi_g\pi_m x\left(u + \left(V_g - V_m\right)\right)(1-fq)$$

The next step to solving the model is to find the net gain to becoming a money holder, $V_m - V_g$. Intuitively, this represents the utility gained by a goods holder (or lost, if $V_m - V_g$ is negative) when they enter a transaction and switch their status into a money holder. Utilizing the tractable math versions of the value functions of the agents, the net gain to becoming a money holder is:

$$V_m - V_g = \frac{(1-M)xu\left(\pi_g\pi_m(1-fq) - y\right)}{r + \pi_g\pi_m x(1-fq)}$$

3.2.1 Conditions a Money Holder will Give Up Money

Recall that π_m is the probability of a money holder giving up money. $\pi_m = 1$ means the money holder will spend their money, and $\pi_m = 0$ means that they will not. Therefore, the question becomes: when will a specific money holder choose $\pi_m^i = 1$? Since π_m can only take the values of 0 and 1, a money holder will give up money if the utility for choosing $\pi_m = 1$ is greater than or equal to the utility for choosing $\pi_m = 0$:

$$rV_m|\pi_m^i = 1 \geq rV_m|\pi_m^i = 0$$

$$(1-M)\pi_g\pi_{m=1}^i x\left(u + \left(V_g - V_m\right)\right)(1-fq) \geq (1-M)\pi_g\pi_{m=0}^i x\left(u + \left(V_g - V_m\right)\right)(1-fq)$$

$$(1-M)\pi_g x\left(u + \left(V_g - V_m\right)\right)(1-fq) \geq 0$$

The terms $(1 - M)$, π_g, and x are all probabilities and are greater than or equal to zero. In particular, f and q are also probabilities and are less than or equal to 1, making $(1 - fq)$ greater than or equal to zero. Therefore, for the inequality to hold,

$$u + \left(V_g - V_m\right) \geq 0$$

$$u\left[1 + \frac{(1 - M)x\left(y - \pi_g \pi_m (1 - fq)\right)}{r + \pi_g \pi_m x(1 - fq)}\right] \geq 0$$

Again, because all are probabilities, it can be seen that $-1 \leq y - \pi_g \pi_m (1 - fq) \leq 1$. By extension, $\frac{(1-M)x\left(y-\pi_g\pi_m(1-fq)\right)}{r+\pi_g\pi_m x(1-fq)} \geq -1$. Therefore, the inequality is always satisfied.

The solution for the money holder is thus $\pi_m^i = 1 \; \forall i$, meaning that the individual money holder is always willing to give up money regardless of what other money holders are doing. Intuitively, this makes sense; the money holder, irrespective of whether they hold real or counterfeit money, the money holder is better off spending their money. The money that they hold does not inherently give them any utility and prevents them from producing a good themselves. On the other hand, spending their money on goods gives them utility, furthering the argument that it does not make sense for them to hold on to their money. If the money holder holds counterfeit money, it also makes sense for them to attempt to spend the counterfeit money because they do not bear a direct utility cost. The only 'cost' is that the transaction simply does not happen if they are found out. Taken together, a money holder always wants to spend money.

In context of Operation Bernhard, the British money holders will always want to spend their money to obtain goods. As the foreign government, Nazi Germany has perfect information and knows this, which is the reason why they pursue the counterfeit money strategy. The behavior of the money holders ensures that the counterfeit money planted by Nazi Germany will circulate in the British economy (except when they are detected), causing the intended result of destabilizing the economy.

3.2.2 Conditions a Goods Holder will Accept Money

However, for money to actually flow in the economy, the goods holders have to accept money as a form of payment. Recall that π_g is the probability of a goods holder accepting money, with $\pi_g = 1$ meaning yes and $\pi_g = 0$ meaning no. Like previously, the question becomes: when will a specific goods holder choose $\pi_g^i = 1$? Again, like previously, this means that a goods holder will choose to accept money if the utility for choosing $\pi_g = 1$ is greater than or equal to the utility for choosing $\pi_g = 0$:

$$rV_g|\pi_g^i = 1 \geq rV_g|\pi_g^i = 0$$

$$(1-M)xyu + M\left(V_m - V_g\right)\pi_{g=1}^i \pi_m y(1-fq)$$
$$\geq (1-M)xyu + M\left(V_m - V_g\right)\pi_{g=0}^i \pi_m y(1-fq)$$

$$M\left(V_m - V_g\right)\pi_m y(1-fq) \geq 0$$

Recall from the previous subsection that $\pi_m = 1$. Additionally, M, y, and $(1-fq)$ are all greater than or equal to 0 because they are probabilities. Therefore, for a goods holder to accept money, the inequality that needs to be fulfilled is:

$$\left(V_m - V_g\right) \geq 0$$

Using the equation from earlier, the inequality becomes:

$$\frac{(1-M)xu\left(\pi_g \pi_m (1-fq) - y\right)}{r + \pi_g \pi_m x(1-fq)} \geq 0$$

Like calculations for the money holder, a lot of the terms in this inequality are probabilities and therefore are bounded between 0 and 1, inclusive. Additionally, recall that $\pi_m = 1$. Thus, the required inequality further reduces to:

$$\pi_g(1-fq) - y \geq 0$$

Therefore, if other goods holders reject money, $\pi_g = 0$, and the inequality does not hold. On the other hand, if other goods holders accept money, $\pi_g = 1$, the inequality only holds if and only if:

$$(1-fq) - y \geq 0$$

This economy thus has two Nash Equilibria: one where goods holders collectively do not accept money, and one where they do, constrained by $(1-fq) - y \geq 0$. Observe that the actions of an individual goods holder are heavily influenced by the actions taken by other good holders in the economy. Intuitively, the goods holders will accept money if and only if other goods holders accept money, accounting for the counterfeit money in the economy. The goods holders will only choose to accept money if it benefits them; if nobody accepts money, it makes no sense for them to become a money holder.

In the context of Operation Bernhard, the goods holders in the British economy can either choose to barter among themselves and refuse to accept money, or they can accept money from money holders given the counterfeit constraint. Given that Nazi Germany attempted to destabilize the economy by adding counterfeit money, which will only flow through the economy if money was used, it can be concluded that the economy uses the second Nash Equilibrium. Therefore, in this model, money is accepted by the goods holders, with the condition of holding the constraint.

These solutions of accepting and spending money between the goods and money holders capture the essence of Operation Bernhard. The model emulates what would happen if counterfeit money was injected into the British economy, which is making the goods holders more unwilling to accept money (because there is now a constraint). In essence, during Operation Bernhard, this is akin to prices rising to account for the counterfeit and causing economic devastation, which was the end goal of Nazi Germany. While this paper does not model inflation, it can be seen how Nazi Germany's goal of destabilizing the economy can be achieved by adding counterfeits.

3.2.3 Local Government's Solution

Despite the threat of a foreign government devaluing the currency using counterfeits, the local government will still attempt to maximize their own country's welfare in context where money is used for exchange (can differ across countries). In context of Operation Bernhard, the British monetary authority has the option to choose how much money they will put into circulation, while having perfect information and taking the amount and quality of the counterfeit produced by Nazi Germany as given.

Because this model has two types of agents, welfare can be modeled as follows:

$$rW = (1-M)rV_g + MrV_m$$

Recall that $\pi_m = 1$ and $\pi_g = 1$, with $(1-fq) \geq y$. By solving the algebra, the welfare of the economy in terms of how much money is in circulation is:

$$rW = xyu + Mxu\big((1-fq) - 2y\big) - M^2xu(1-fq) + M^2xyu$$

In order to maximize welfare by choosing M, the partial derivative has to equal zero, $\frac{\partial W}{\partial M} = 0$. Solving for the algebra gives the optimal results that

$$M^* = \frac{(1-fq) - 2y}{2\big((1-fq) - y\big)}$$

$$rW^* = \frac{uy(1-fq)^2}{4\big((1-fq) - y\big)}$$

where $(1-fq) \geq y$ and $0 \leq y \leq 1$.

3.3 Foreign Government's Model

On the other side of the table, the foreign government (which would be Nazi Germany) knows that the British monetary authority will choose to set $M^* = \frac{(1-fq)-2y}{2\left((1-fq)-y\right)}$ depending on what f and q are. Meanwhile, Nazi Germany chooses how much counterfeit to put into the economy, f, and the quality of the counterfeit, q. Since their goal is to demoralize the British as much as possible, a bad economic result in the British economy would provide Nazi Germany with positive utility. This model defines the 'economic result' as the welfare of the economy. Therefore, the lower British welfare is, the higher the utility gained by Nazi Germany. However, Nazi Germany is also the one producing the counterfeits and must incur a cost. The better the counterfeits are, the less detectable they are, and the higher the cost is for Nazi Germany.

In this sense, Nazi Germany acts as a firm. The 'revenue' is the utility gained by reducing the welfare of Britain, and the 'cost' is the utility lost by increasing the quality of the counterfeit (because of how q is coded, this means decreasing the variable q). Additionally, since Nazi Germany cannot control the people of Britain, y is a constant, and so are u and r. Henceforth in this subsection, Nazi Germany will be referred to as the 'firm' for simplicity.

3.3.1 Firm's Production

The firm's revenue increases as W decreases. Recall that $rW^* = \frac{uy(1-fq)^2}{4\left((1-fq)-y\right)}$. The model defines a revenue function for the firm that moves in the opposite direction of W. One such function is its reciprocal. For example, if a variable x increases, its reciprocal $\frac{1}{x}$ decreases. Therefore, this paper defines the firm's revenue as a variation of the reciprocal of W. For simplicity, the function takes the constants u and r out of the equation.

$$\text{revenue} = \frac{u}{4rW} = \frac{(1-fq)-y}{y(1-fq)^2}$$

Next is defining the cost function for the firm. As the counterfeit quality increases (which translates to a decrease in q), the cost to the firm also increases. Therefore, there should also be a negative correlation between the variable q and the cost function. Additionally, q is limited within the bounds $0 < q \leq 1$, because q is a probability that cannot equal to 0 (there is no such thing as a perfect counterfeit). Since $q \neq 0$, the model utilizes a variant of the $\log(x)$ function because $\log(0)$ is undefined. Tweaking the $\log(x)$ function to increase as the variable decreases gives a possible cost function as follows:

$$\text{cost} = q(\log(q) - 1)$$

Note that the derivative of the cost function with respect to q is simply $\log(q)$. Also observe that for the values of $0 < q \leq 1$, $\log(q) < 0$. This means that the cost function has a negative slope in the bounds of $0 < q \leq 1$; i.e., as q decreases, the value of the cost function increases, fulfilling the requirement for the firm's cost function.

The firm's profits come from revenue from lowering welfare minus the cost that it incurs from producing higher quality counterfeit. Combining the firm's revenue and cost functions yields the following profit function:

$$\pi(f, q) = max_{f,q} \frac{(1 - fq) - y}{y(1 - fq)^2} - q(\log(q) - 1)$$

$$\text{s.t.}\,(1 - fq) > y$$

The constraint comes from the constraint in the local government's problem in order to achieve a Nash equilibrium where money flows in the economy. The constraint is strict because $(1 - fq) - y$ appears in the denominator of the profit and welfare functions, and therefore cannot be equal to 0.

3.3.2 Firm's Solution

The firm chooses f and q to maximize their profit function $\pi(f, q)$, taking into account the strict inequality constraint that $(1 - fq) > y$. Solving the model means taking the first-order conditions of f and q with respect to the profit function. However, the solution does not utilize a Lagrangian because the constraint has a strict inequality; using a Lagrangian will cause $(1 - fq) = y$, which is not allowed in the model. Therefore, the solution is as follows:

$$\pi(f, q) = max_{f,q} \frac{(1 - fq) - y}{y(1 - fq)^2} - q(\log(q) - 1)$$

$$\frac{\partial \pi}{\partial f} = \frac{q\big((1 - fq) - 2y\big)}{y(1 - fq)^3} = 0$$

$$\frac{\partial \pi}{\partial q} = \frac{f\big((1 - fq) - 2y\big)}{y(1 - fq)^3} - \log(q) = 0$$

From $\frac{\partial \pi}{\partial f}$, it is implied that $(1 - fq) - 2y = 0$, because $0 < q \leq 1$.

Consider $\frac{\partial \pi}{\partial q}$. Because $(1 - fq) - 2y = 0$, this means that $\log(q) = 0$, and therefore:

$$q^* = 1$$

$$f^* = 1 - 2y$$

However, also consider that $0 \leq f \leq 1$, which implies:

$$0 \leq y \leq \frac{1}{2}$$

Additionally, observe that the constraint $(1 - fq) > y$ is always satisfied for $0 < y \leq \frac{1}{2}$. Therefore, the solution to the firm's problem is to produce a very bad counterfeit ($q = 1$) such that it is always detected by the local economy, and to choose how much counterfeit to put in ($f = 1 - 2y$) dependent on the probability of the people liking goods.

3.4 Model Solution Interpretation

Recall that the foreign government's optimal solutions set the values for f and q, which are taken as given by the local government in determining how much money to put into circulation (M). Observe that the solutions $q^* = 1$ and $f^* = 1 - 2y$ imply that

$$M^* = \frac{(1 - fq) - 2y}{2\big((1 - fq) - y\big)} = 0$$

This means that the optimal amount of money to be put in circulation by the local government (in this case, Britain) is zero; the optimal economy should thus be a pure barter economy with only goods holders.

This may seem like an odd conclusion to draw; however, intuitively, it makes sense. Once Britain realizes that Nazi Germany is attempting to undermine its economy by injecting counterfeit money, the best way to minimize the impact of the counterfeits is to simply not allow them to circulate. Therefore, the easiest way to do this is to take out all money from the economy so that the counterfeits cannot flow, preventing people from holding valueless money and engaging in fraudulent transactions. Having counterfeit money in the economy will only hurt the money holders, regardless of whether they have real or counterfeit money. Thus, the best solution is to simply not have any money holders. In this model, the local economy has the capability of returning to a pure barter economy since every agent can produce goods and trade those instead of money. Money was thus never necessary.

On the other hand, Nazi Germany has perfect information and knows that Britain's best response is to remove all money from circulation. Because of this, there are no incentives for Nazi Germany to produce good quality counterfeit. As long as the threat of counterfeits remain, no matter how bad quality they are, Britain does not allow money to flow. However, Nazi Germany still needs to be in the Nash equilibrium and still must produce a certain number of counterfeits; otherwise, Britain will simply call out on Nazi Germany's bluff. Knowing Britain's next move allows Nazi Germany to gain maximum utility by producing terrible quality counterfeit and incurring minimum cost.

In a real-world context, the model's result of removing money from the economy is similar to what happened in the aftermath of Operation Bernhard. After World War II, the British monetary authority (the Bank of England) withdrew all money with a value of more than five pounds and replaced everything with new paper money.[6] As it relates to the model in this paper, withdrawing all money or stopping money from circulating is equivalent to setting $M = 0$, proving the model's results. However, the model in this paper assumes that every agent in the economy is capable of producing goods and returning to a pure barter economy. In reality, this is not the case because a pure barter economy is not feasible. Therefore, after money was withdrawn, the actual British monetary authority injected money back into the economy with the assumption that the new currency will not be counterfeited. In the model of this paper, this means reverting to the standard money-search model without counterfeits: goods and money holders alike trade with little to no restrictions, and the monetary authority controls how much money flows in the economy.

One key difference between the model's results and what happened in Operation Bernhard was the quality of the counterfeits. While the model concludes that the optimal solution of Nazi Germany is to produce terrible counterfeit, this was not the move that the real Nazi Germany took. In fact, it was the opposite; the actual Operation Bernhard featured astonishingly good quality counterfeit, to the point where "one bank official described them as 'the most dangerous ever seen.'"[7] This discrepancy is perhaps because the model in this paper assumes perfect information in both governments. Because this model's foreign government knows everything that is going to happen, it chooses to minimize costs. On the other hand, the real Nazi Germany does not have that information about Britain. Without knowledge of what the Nash Equilibrium is, the real Nazi Germany must have disregarded costs. The next section of this paper presents simulation results of the model and provides a further explanation to this solution.

[6] Blakemore, E. (2016, March 1). *The Nazis planned to bomb Britain with forged bank notes*. Smithsonian Magazine. Retrieved November 21, 2021, from https://www.smithsonianmag.com/smart-news/nazis-planned-bomb-britain-forged-bank-notes-180958258/.
[7] Wikimedia Foundation. (2021, June 29). *Operation Bernhard*. Wikipedia. Retrieved November 21, 2021, from https://en.wikipedia.org/wiki/Operation_Bernhard

4 Simulation

This paper will now simulate how foreign government (Nazi Germany) utility and local economic (British) welfare as the variable q changes between 0 and 1. The simulation will be run against varying levels of the probability of liking a good, y, with the range $0 \leq y \leq \frac{1}{2}$. The optimal solution of $f^* = 1 - 2y$ is implied and utilized in all simulations. This section begins with utility graphs, followed by a graph plotting welfare.

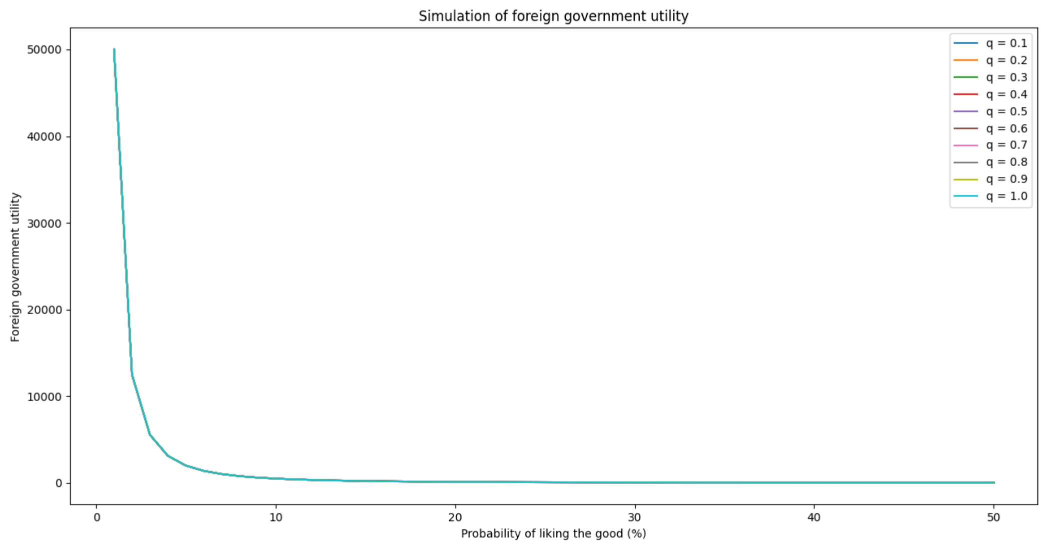

Fig. 3: The utility as $0 < y \leq 0.50$, plotted against different values of q

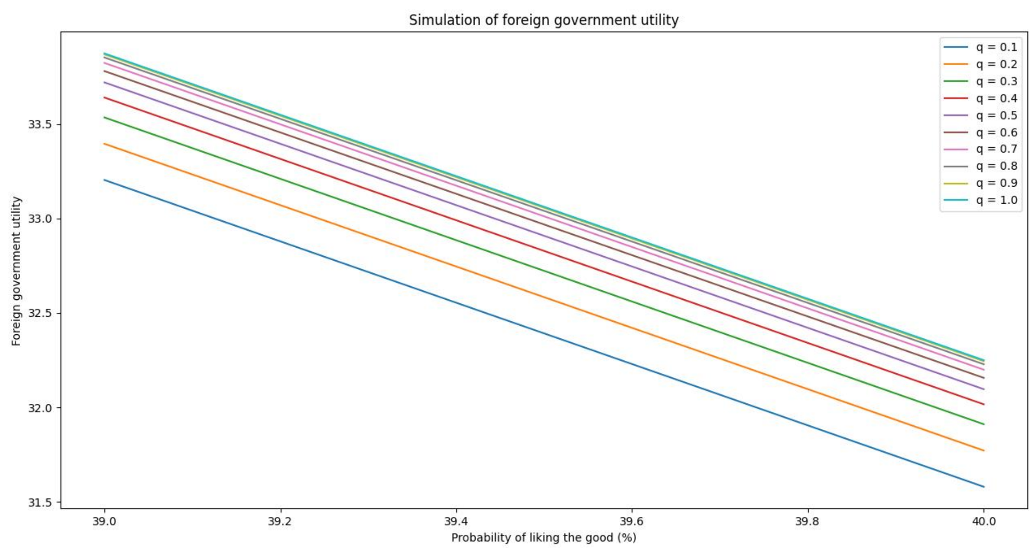

Fig. 4: The utility as $0.39 \leq y \leq 0.40$, plotted against different values of q

Fig. 3 shows that the foreign government (Nazi Germany) utility decreases and approaches 0 as the value of y increases. Note that the graph looks like one line and the level curves are barely visible, indicating that the utility with different values of q are very similar. To that end, the simulation will also be run against the value of y within the bounds of $0.39 \leq y \leq 0.40$ (in the sense that it is zooming into the graph).

Fig. 4 shows a zoomed-in version of Fig. 3 within the values of $0.39 \leq y \leq 0.40$. As seen in the graph, $q = 1$ yields the highest utility, which corroborates the results from the model in the previous section—therefore, highest utility is achieved when the quality of the counterfeit is very bad. Additionally, Fig. 4 provides more detail: the value of utility is seen sloping down and not equal to zero, as opposed to what Fig. 3 may seem to imply.

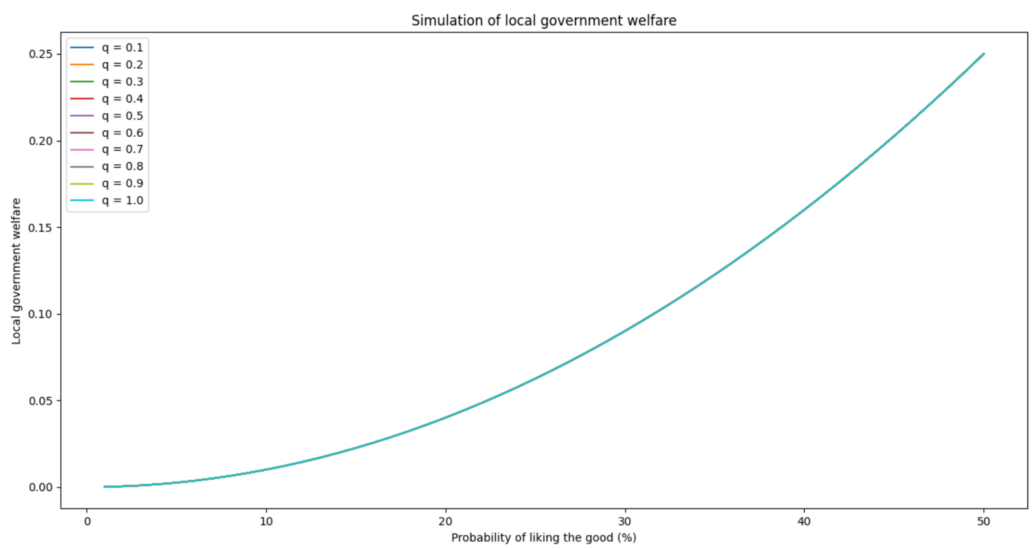

Fig. 5: The welfare as $0 < y \leq 0.50$, plotted against different values of q

Fig. 5 depicts the relationship between welfare and y with varying values of q. It shows that for all possible values of q, the welfare of the economy increases as y increases. Observe that the level curves are, again, not visible. Unlike Fig. 3 where there was a slight difference, however, there is no difference in welfare as q changes between $0 < q \leq 1$ and the graph is simply one curve. This indicates that the economy reaches equilibrium and maximizes its welfare while adapting to the counterfeit situation, independent from the quality of the counterfeits.

Intuitively, as y increases, more people like one another's good, and bartering goods becomes easier. When y is low, it is more difficult to achieve a double coincidence of wants, and the local monetary authority has to inject more money into the economy to compensate. Using money for trades only requires the money holder to like the good because the goods holder will almost always accept money as payment, making such a transaction easier to take place. Therefore, there is a negative relationship between y and M. Going along the same logic, welfare increases as y increases because this means that more transactions will happen, causing more utility to be gained by the agents in the economy.

In terms of Operation Bernhard, this simulation proves that Nazi Germany is better off choosing to produce the lowest quality counterfeit possible. Since welfare moves independently from the quality of the counterfeit (see Fig. 5), theoretically, there is no point for the foreign government (Nazi Germany) to produce higher quality ones. The foreign government would essentially be incurring more costs for no additional utility.

However, continuing the discussion from the model section of this paper, the real Nazi Germany did not have this information when conducting the real Operation Bernhard. Without knowing that the quality of counterfeits does not affect the British welfare, Nazi Germany decided to go all-out in producing high-quality counterfeits, hoping that it will move the needle on British welfare.[8] However, as this paper shows, that is not the optimal solution because all that does is increase costs. Indeed, the model simulation shows that in equilibrium, the counterfeit efforts by Nazi Germany will be perfectly counteracted by the actions of the British monetary authority. In both real and theory, the British government counteracted Nazi Germany's efforts by removing all money in the economy and cutting counterfeits from circulation, ensuring that economic welfare is always maximized. Essentially, unless the end goal was to cause a change in currencies, there is no point for Nazi Germany to counterfeit money; if it only increases costs, why bother? History acknowledges that Operation Bernhard was "a scheme that didn't work,"[9] corroborating what the simulation concludes.

5 Conclusion

Nazi Germany executed Operation Bernhard to undermine the British economy, but the model presented in this paper shows that it would not have succeeded. Ultimately, Nazi Germany's optimal solution would have been to produce terrible quality counterfeit to minimize costs as the British monetary authority has full control of the country's monetary system and

[8] Ibid
[9] Blakemore, E. (2016, March 1). *The Nazis planned to bomb Britain with forged bank notes*. Smithsonian Magazine. Retrieved November 21, 2021, from https://www.smithsonianmag.com/smart-news/nazis-planned-bomb-britain-forged-bank-notes-180958258/.

money supply. This control enables the British monetary authority to adjust the supply of money in the economy, counteracting Nazi Germany's counterfeiting efforts. Eventually, the optimal solution would be for the British monetary authority to simply withdraw all the money from the economy to stop the circulation of the counterfeits and re-issue a new currency, which was not the intention of Nazi Germany.

Nazi Germany's optimal solution of producing a certain amount of very bad quality counterfeits might seem counterintuitive, but it is necessary if it anticipates the British government's actions. However, its counter-intuitiveness also proves that this counterfeiting strategy does not work; if the optimal solution is only to goad the local government with the threat of counterfeits but then produce terrible ones, why bother counterfeiting in the first place? As history will have it, Operation Bernhard was indeed a failed operation, and counterfeiting money in such a manner might not be the best wartime strategy.

6 Works Cited

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