13.4 A Supervised Random Walk

DATASCI W261

Machine Learning at Scale

Link Prediction Problem

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- Here, given a snapshot of a network, need to infer:
  - Which interactions among members is likely
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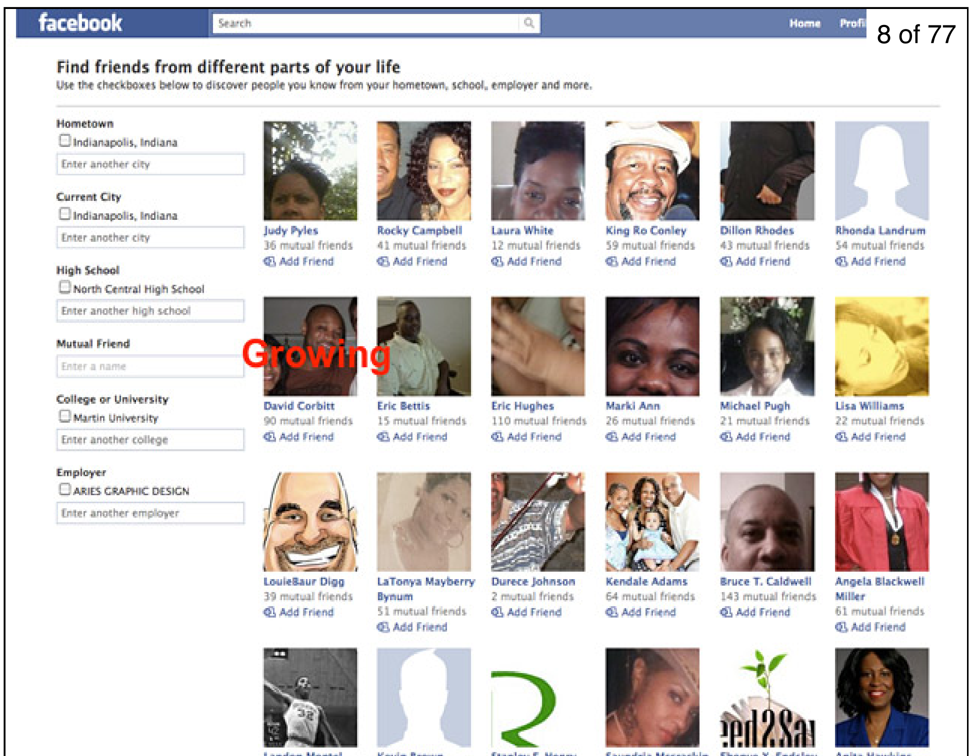
- Challenge: Combining information from network with rich node and edge attribute data



How do we combine network structure with node and edge features?

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- Idea: To "guide" random walk using supervised learning

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- Problem formulated as part supervised machine learning and part random walking with restarts

Friend Graphs: Random Walk With Restarts

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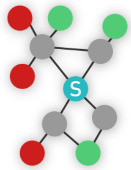
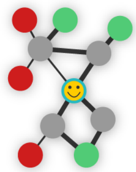
Personalized PageRank



Network

$$Q_{uv} = (1-\alpha)Q'_{uv} + \alpha\mathbf{1}(v=s).$$

Weighted graph



Set edge strengths

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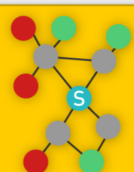

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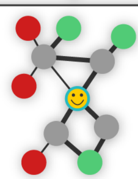

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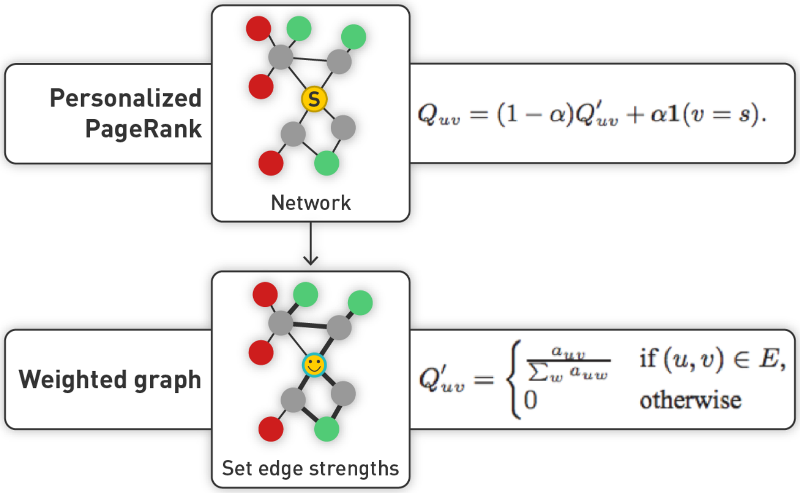

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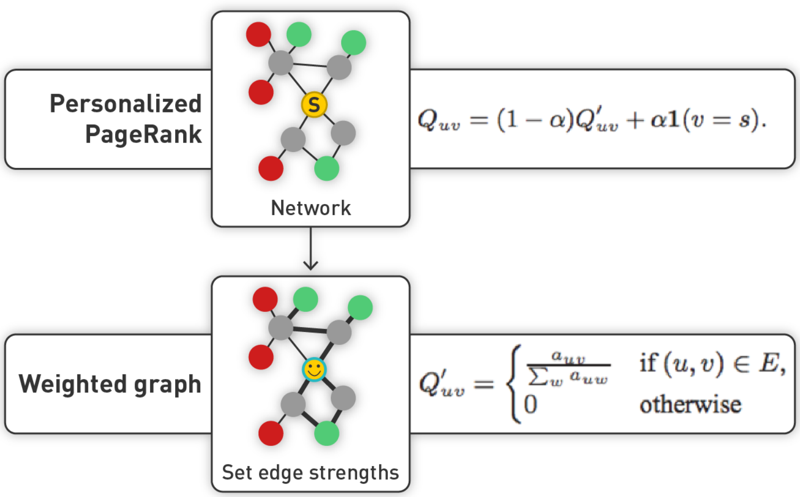

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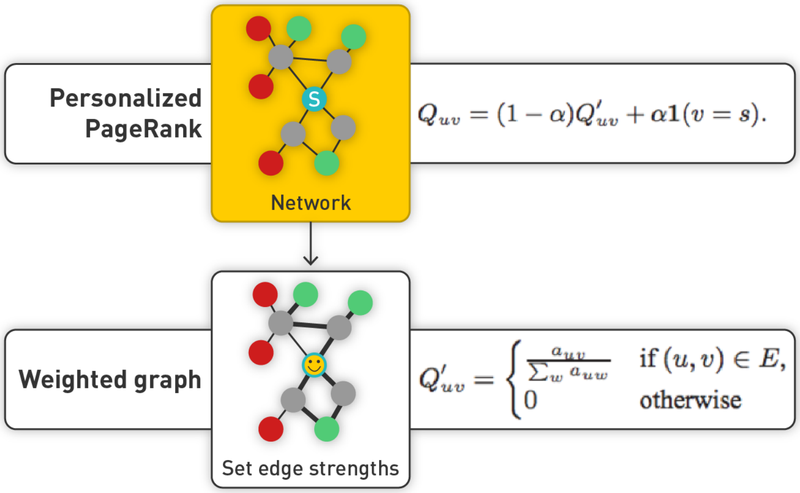

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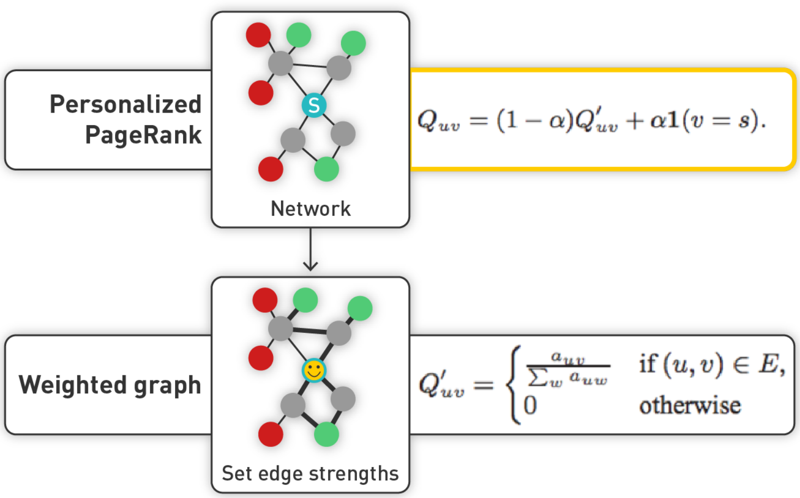

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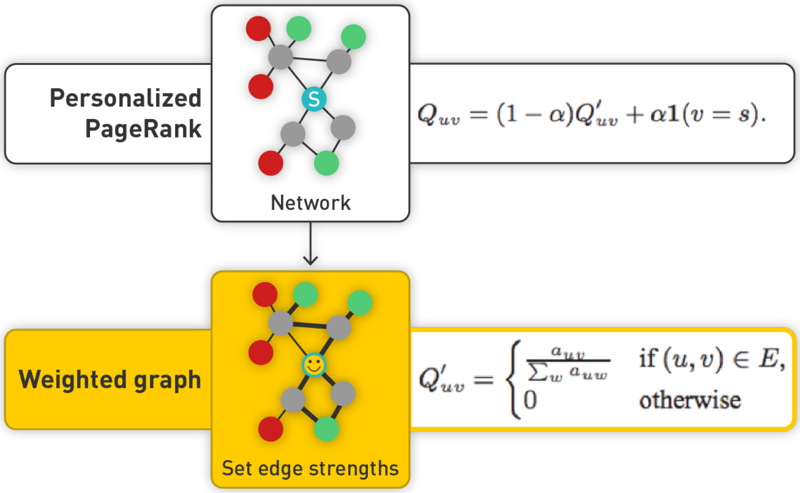

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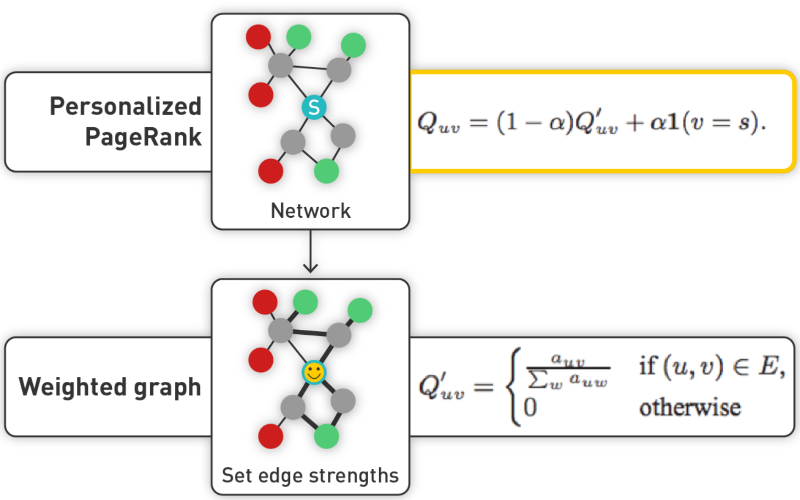

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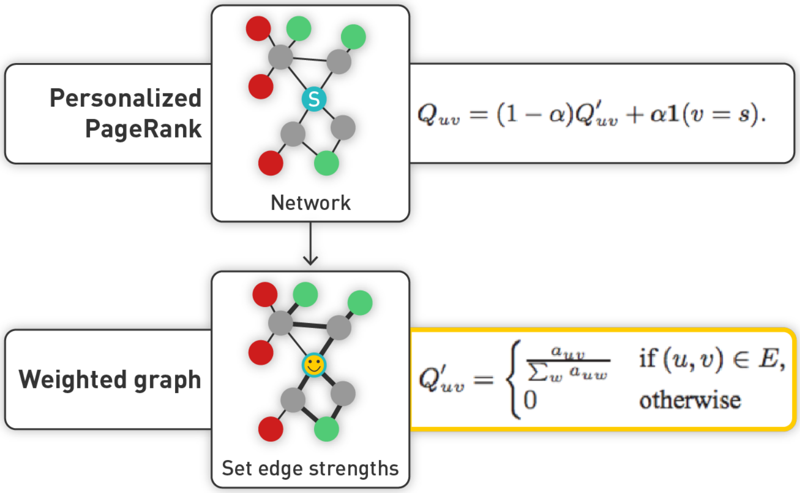

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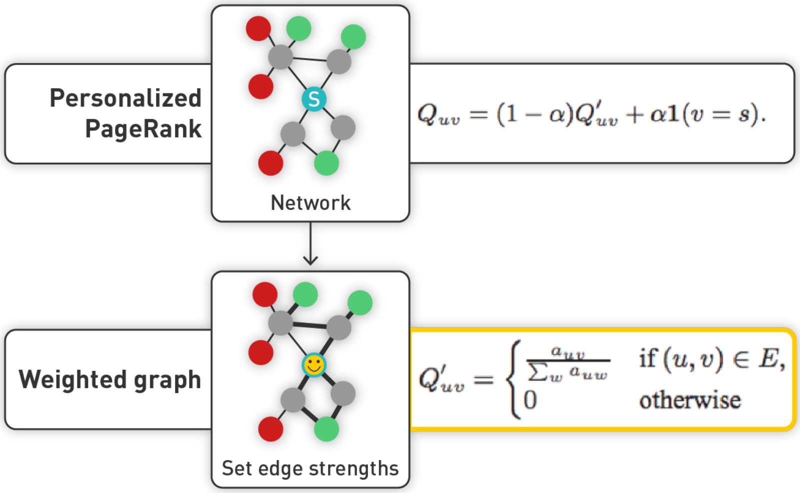

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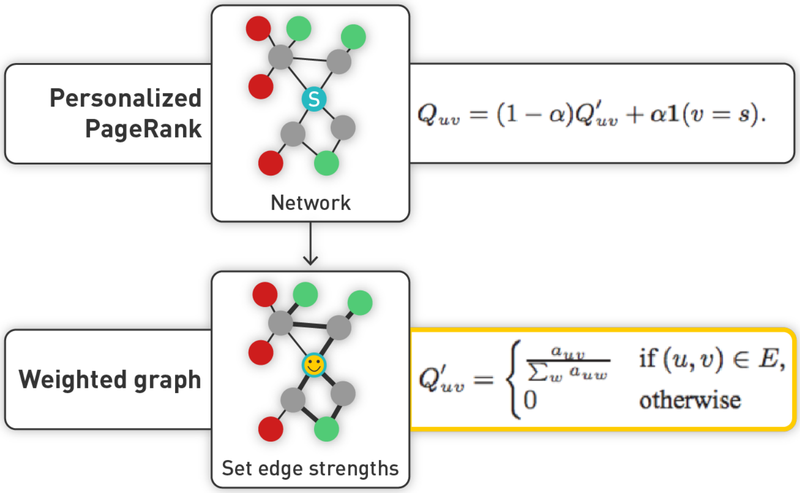

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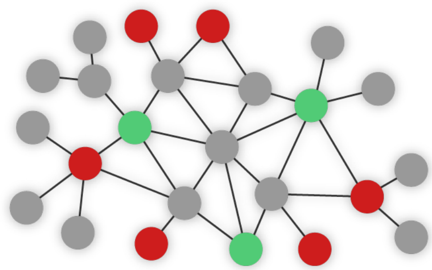

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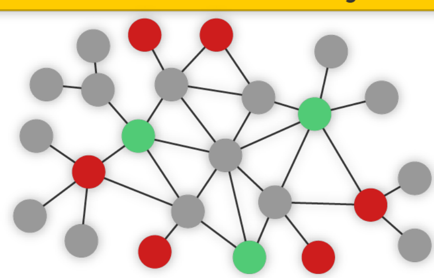

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- Pooled all positive and negative examples for users, put in training and test set

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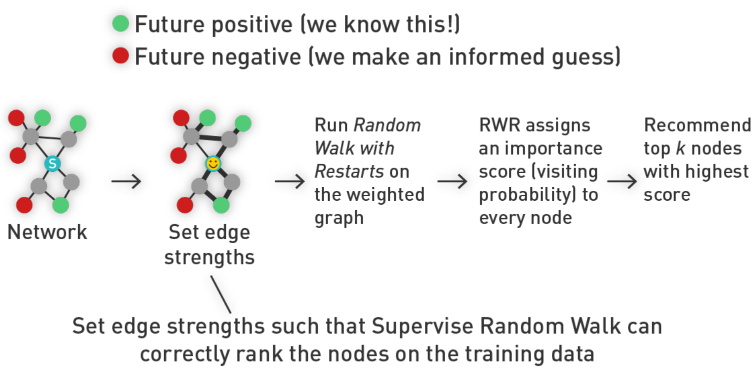
- All features rescaled to have mean 0 and standard deviation 1; also constant feature of value 1

Hybrid Model



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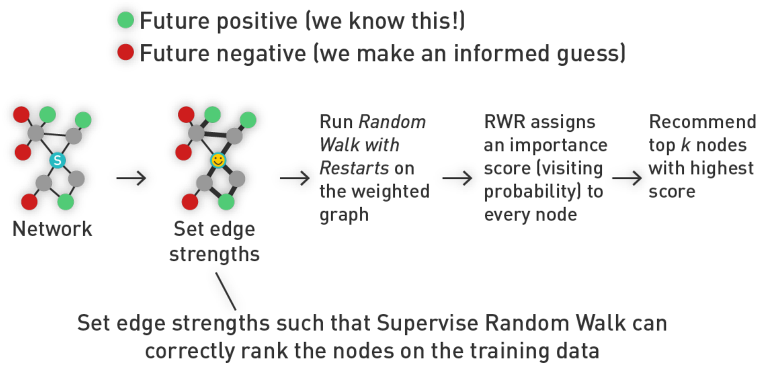

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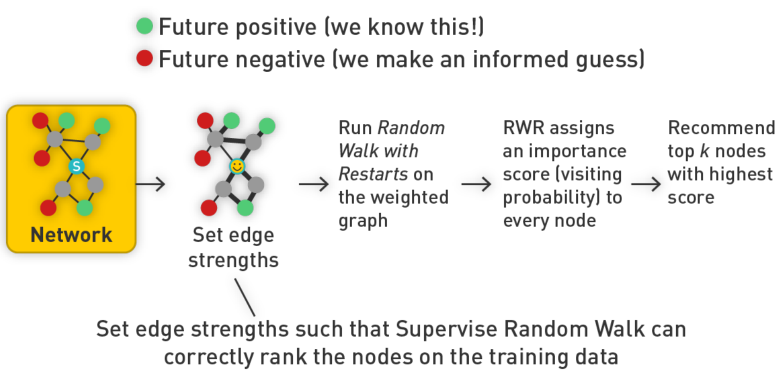

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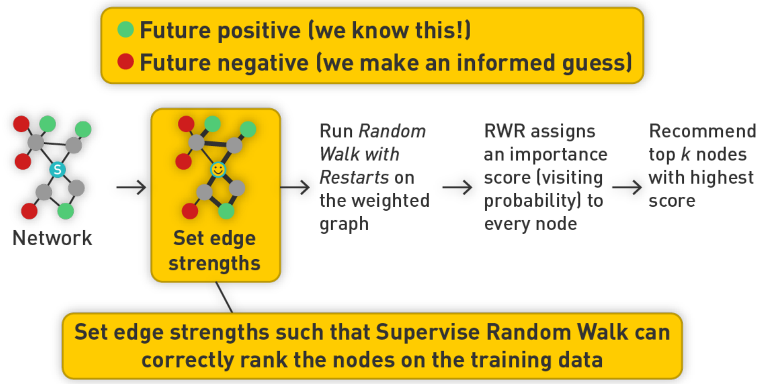

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- Individual with positive and negative examples to reweight links and social graph

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- Teleport to s (our node of interest) at every opportunity.

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Positive nodes

Negative nodes

Penalty for violating constraint $r_p > r_n$

r_x … score of node x on a weighted graph with edge weights $f_\theta(x, y)$

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- Repeat process many times.

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