ORDERS OF GROWTH

COMPUTER SCIENCE MENTORS 61A

March 7 to March 11, 2016

1. In big-O notation, what is the runtime for `foo`?
   (a)
   ```
   def foo(n):
       for i in range(n):
           print('hello')
   ```

   (b) What's the runtime of `foo` if we change `range(n)`:
      i. To `range(n / 2)`?
      ii. To `range(10)`?
      iii. To `range(10000000)`?

2. What is the order of growth in time for the following functions? Use big-O notation.
   (a)
   ```
   def strange_add(n):
       if n == 0:
           return 1
       else:
           return strange_add(n - 1) + strange_add(n - 1)
   ```

   (b)
   ```
   def stranger_add(n):
       if n < 3:
           return n
       elif n % 3 ==  0:
           return stranger_add(n - 1) + stranger_add(n - 2) +
              stranger_add(n - 3)
       else:
           return n
   ```

(c)
```
def waffle(n):
    i = 0
    sum = 0
    while i < n:
        for j in range(50 * n):
            sum += 1
        i += 1
    return sum
```

(d)
```
def belgian_waffle(n):
    i = 0
    sum = 0
    while i < n:
        for j in range(n ** 2):
            sum += 1
        i += 1
    return sum
```

(e)
```
def pancake(n):
    if n == 0 or n == 1:
        return n
    # Flip will always perform three operations and return
      -n.
    return flip(n) + pancake(n - 1) + pancake(n - 2)
```

(f)
```
def toast(n):
    i = 0
    j = 0
    stack = 0
    while i < n:
        stack += pancake(n)
        i += 1
    while j < n:
        stack += 1
        j += 1
    return stack
```

3. Consider the following functions:

```
def hailstone(n):
    print(n)
    if n < 2:
     return
    if n % 2 == 0:
         hailstone(n // 2)
    else:
         hailstone((n * 3) + 1)

def fib(n):
    if n < 2:
       return n
    return fib(n - 1) + fib(n - 2)

def slow(n):
    i, j, k = 0, 0, 0
    while i < n:
        while j < n:
            while k < n:
                fib(k)
                k += 1
            fib(j)
            j += 1
        fib(i)
        i += 1

def foo(n, f):
    return n + f(500)
```

In big-O notation, describe the runtime for the following:

(a) `foo(10, hailstone)`

(b) `foo(3000, fib)`

(c) `foo(99999999999, slow)`

4. **Fast Exponentiation:** in this problem, we will examine a real-world algorithm used to improve the speed of calculating exponents.

   (a) First, express the runtime of the naive exponentiation algorithm in big-O notation.

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n - 1)
```

   (b) Now, express the runtime of the fast exponentiation algorithm in big-O notation.

```
def fast_exp(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n // 2))
    else:
        return b * fast_exp(b, n - 1)
```

   (c) What about this slightly modified version of `fast_exp`?

```
def fast_exp(b, n):
    for _ in range(50 * n):
        print("Killing time")
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n // 2))
    else:
        return b * fast_exp(b, n - 1)
```

5. **Mysterious loops:** What is the order of growth in time for the following functions? Use big-O notation.

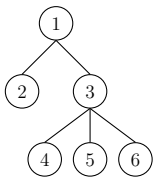
   (a)
```
def mystery(n):
    for i in range(n):
        while i % 2 != 0:
            print(i)
            i = i - 1
        print("Done")
```

   (b)
```
def fun(n):
    for i in range(n):
        for j in range(n * n):
            if j == 4:
                return -1
            print("Fun!")
```

6. **Orders of Growth and Trees:** Assume we are using the non-mutable Tree implementation introduced in discussion. Consider the following function:

```
def word_finder(t, n, word):
    if label(t) == word:
        n -= 1
        if n == 0:
            return True
    for child in children(t):
        if word_finder(child, n, word) == True:
            return True
    return False
```

   (a) What does this function do?
   Hint: A path is a sequence of connected nodes. For example, here are four paths in the tree below: $1 \to 2$, $1 \to 3 \to 4$, $1 \to 3 \to 5$, $1 \to 3 \to 6$



   (b) If a tree has n total nodes, what is the total runtime for all searches in big-O notation?

7. **Orders of Growth and Linked Lists:** Consider the following linked list function:

```
def insert_at_end(lst, x):
    if lst.rest is Link.empty:
        lst.rest = Link(x)
    else:
        insert_at_end(lst.rest, x)
```

(a) What does this function do?

(b) Say we want to repeatedly insert some numbers into the end of a linked list:

```
def insert_many(lst, n):
    for i in range(n):
        insert_at_end(lst, i)
```

i. Assume `lst` is initially length 1. How long will it take to do the first insertion? The second? The nth?

ii. In big-O notation, What is the total runtime to do all the inserts? (total runtime of `insert_many`)