ORDERS OF GROWTH

COMPUTER SCIENCE MENTORS 61A

March 7 to March 11, 2016

1. In big-O notation, what is the runtime for `foo`?

   (a)
   ```
   def foo(n):
       for i in range(n):
           print('hello')
   ```

   (b) What's the runtime of `foo` if we change `range(n)`:

      i. To `range(n / 2)`?

      ii. To `range(10)`?

      iii. To `range(10000000)`?

2. What is the order of growth in time for the following functions? Use big-O notation.

   (a)
   ```
   def strange_add(n):
       if n == 0:
           return 1
       else:
           return strange_add(n - 1) + strange_add(n - 1)
   ```

   (b)
   ```
   def stranger_add(n):
       if n < 3:
           return n
       elif n % 3 ==  0:
           return stranger_add(n - 1) + stranger_add(n - 2) +
              stranger_add(n - 3)
       else:
           return n
   ```

(c)
```
def waffle(n):
    i = 0
    sum = 0
    while i < n:
        for j in range(50 * n):
            sum += 1
        i += 1
    return sum
```

(d)
```
def belgian_waffle(n):
    i = 0
    sum = 0
    while i < n:
        for j in range(n ** 2):
            sum += 1
        i += 1
    return sum
```

(e)
```
def pancake(n):
    if n == 0 or n == 1:
        return n
    # Flip will always perform three operations and return
      -n.
    return flip(n) + pancake(n - 1) + pancake(n - 2)
```

(f)
```
def toast(n):
    i = 0
    j = 0
    stack = 0
    while i < n:
        stack += pancake(n)
        i += 1
    while j < n:
        stack += 1
        j += 1
    return stack
```

3. Consider the following functions:

```
def hailstone(n):
   print(n)
   if n < 2:
    return
   if n % 2 == 0:
        hailstone(n // 2)
   else:
        hailstone((n * 3) + 1)

def fib(n):
   if n < 2:
      return n
   return fib(n - 1) + fib(n - 2)

def slow(n):
    i, j, k = 0, 0, 0
    while i < n:
        while j < n:
            while k < n:
                fib(k)
                k += 1
            fib(j)
            j += 1
        fib(i)
        i += 1

def foo(n, f):
    return n + f(500)
```

In big-O notation, describe the runtime for the following:

(a) `foo(10, hailstone)`

(b) `foo(3000, fib)`

(c) `foo(99999999999, slow)`

4. **Fast Exponentiation:** in this problem, we will examine a real-world algorithm used to improve the speed of calculating exponents.

   (a) First, express the runtime of the naive exponentiation algorithm in big-O notation.

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n - 1)
```

   (b) Now, express the runtime of the fast exponentiation algorithm in big-O notation.

```
def fast_exp(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n // 2))
    else:
        return b * fast_exp(b, n - 1)
```

   (c) What about this slightly modified version of `fast_exp`?

```
def fast_exp(b, n):
    for _ in range(50 * n):
        print("Killing time")
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n // 2))
    else:
        return b * fast_exp(b, n - 1)
```

5. **Mysterious loops:** What is the order of growth in time for the following functions? Use big-O notation.

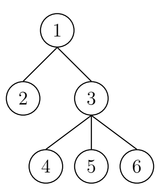
   (a)
```
def mystery(n):
    for i in range(n):
        while i % 2 != 0:
            print(i)
            i = i - 1
        print("Done")
```

   (b)
```
def fun(n):
    for i in range(n):
        for j in range(n * n):
            if j == 4:
                return -1
            print("Fun!")
```

6. **Orders of Growth and Trees:** Assume we are using the non-mutable Tree implementation introduced in discussion. Consider the following function:

```
def word_finder(t, n, word):
    if label(t) == word:
        n -= 1
        if n == 0:
            return True
    for child in children(t):
        if word_finder(child, n, word) == True:
            return True
    return False
```

   (a) What does this function do?
   Hint: A path is a sequence of connected nodes. For example, here are four paths in the tree below: $1 \rightarrow 2$, $1 \rightarrow 3 \rightarrow 4$, $1 \rightarrow 3 \rightarrow 5$, $1 \rightarrow 3 \rightarrow 6$



   (b) If a tree has n total nodes, what is the total runtime for all searches in big-O notation?

7. **Orders of Growth and Linked Lists:** Consider the following linked list function:

```
def insert_at_end(lst, x):
    if lst.rest is Link.empty:
        lst.rest = Link(x)
    else:
        insert_at_end(lst.rest, x)
```

(a) What does this function do?

(b) Say we want to repeatedly insert some numbers into the end of a linked list:

```
def insert_many(lst, n):
    for i in range(n):
        insert_at_end(lst, i)
```

i. Assume `lst` is initially length 1. How long will it take to do the first insertion? The second? The nth?

ii. In big-O notation, What is the total runtime to do all the inserts? (total runtime of `insert_many`)

Computer Science Mentors CS61A Spring 2016: Jerry Chen and Tiffany Perumpail, with
Albert Chen, Amanda Bui, Anthony Dela Paz, Anders Lewis, Ching Fang, Colin Schoen, Cynthia Shen, Kevin Lin,
Kimberly Bourque, Kunal Munshani, Meha Bakshi, Regina Xu, and Ryan Chang