TREES, MUTABLE STRUCTURES, AND GROWTH

COMPUTER SCIENCE MENTORS 61A

September 26 to September 30, 2016

1 Trees

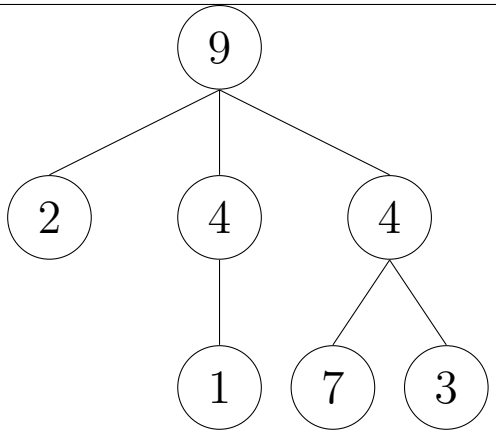
Things to remember

```
def tree(root, branches=[]):
    return [root] + list(branches)

def root(t):
    return t[0]

def branches(t): # Always returns a list of trees
    return t[1:]
```

1. Draw the tree that is created by the following statement:

```
tree(4,
     [tree(5, []),
      tree(2,
           [tree(2, []),
            tree(1, [])]),
      tree(1, []),
      tree(8,
           [tree(4, [])])])
```

2. Construct the following tree and save it to the variable `t`.



3. What would this output?

```
>>> root(t)

>>> branches(t)[2]

>>> branches(branches(t)[2])[0]
```

4. Write the Python expression to get the integer 2 from t.

5. Write the function `sum_of_nodes` which takes in a tree and outputs the sum of all the elements in the tree.

```
def sum_of_nodes(t):
    """
    >>> t = Tree(...) # Tree from question 2.
    >>> sum_of_nodes(t) # 9 + 2 + 4 + 4 + 1 + 7 + 3 = 30
    30
    """
```

2 Mutation

1. **What Would Python Display?**

```
>>> a = [1, 2]
>>> a.append([3, 4])
>>> a

>>> b = list(a)
>>> a[0] = 5
>>> a[2][0] = 6
>>> b

>>> a.extend([7])
>>> a += [8]
>>> a += 9

>>> a
```

Challenge problem:

```
>>> b[2][1] = a[2:]
>>> a[2][1][0][0]
```

2. Given a list of lists `lst_of_lsts` and some element `elem`, append elem to every list in `lst_of_lsts`.

```
def append_to_all(lst_of_lsts, elem):
    """
    >>> l = [[1, 0, 5], [2, 6, 4], [8, 3]]
    >>> append_to_all(l, 7)
    >>> l
    [[1, 0, 5, 7], [2, 6, 4, 7], [8, 3, 7]]
    """
```

3. Given some list `lst`, possibly a deep list, mutate lst to have the accumulated sum of all elements so far in the list. If there is a nested list, mutate it to similarly reflect the accumulated sum of all elements so far in the nested list. Return the total sum of lst.

   Note: You may find it useful to use the `isinstance` function, which returns True for `isinstance(l, list)` if `l` is a list and False otherwise.

```
def accumulate(lst):
    """
    >>> l = [1, 5, 13, 4]
    >>> accumulate(l)
    23
    >>> l
    [1, 6, 19, 23]
    >>> deep_l = [3, 7, [2, 5, 6], 9]
    32
    >>> deep_l
    [3, 10, [2, 7, 13], 32]
    """
```

3 Growth

1. In big-O notation, what is the runtime for `foo`?
   (a)
   ```
   def foo(n):
       for i in range(n):
           print('hello')
   ```

   (b) What's the runtime of `foo` if we change `range(n)`:

      i. To `range(n / 2)`?

      ii. To `range(10)`?

      iii. To `range(10000000)`?

2. **Orders of Growth and Trees:** Assume we are using the non-mutable tree implementation introduced earlier. Consider the following function:
   ```
   def word_finder(t, n, word):
       if root(t) == word:
           n -= 1
           if n == 0:
               return True
       for branch in branches(t):
           if word_finder(branch, n, word):
               return True
       return False
   ```

   (a) What does this function do?

   (b) If a tree has n total nodes, what is the total runtime for all searches in big-O notation?

3. What is the order of growth in time for the following functions? Use big-O notation.
   (a)
   ```
   def strange_add(n):
       if n == 0:
           return 1
       else:
           return strange_add(n - 1) + strange_add(n - 1)
   ```

(b)
```
def stranger_add(n):
    if n < 3:
        return n
    elif n % 3 ==  0:
        return stranger_add(n - 1) + stranger_add(n - 2) +
           stranger_add(n - 3)
    else:
        return n
```

(c)
```
def waffle(n):
    i = 0
    sum = 0
    while i < n:
        for j in range(50 * n):
            sum += 1
        i += 1
    return sum
```

(d)
```
def belgian_waffle(n):
    i = 0
    sum = 0
    while i < n:
        for j in range(n ** 2):
            sum += 1
        i += 1
    return sum
```

(e)
```
def pancake(n):
    if n == 0 or n == 1:
        return n
    # Flip will always perform three operations and return
       -n.
    return flip(n) + pancake(n - 1) + pancake(n - 2)
```

(f)
```
def toast(n):
    i = 0
    j = 0
    stack = 0
    while i < n:
        stack += pancake(n)
        i += 1
    while j < n:
        stack += 1
        j += 1
    return stack
```