

# RECURSION AND HIGHER ORDER FUNCTIONS

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COMPUTER SCIENCE MENTORS 61A

February 15 to February 19, 2016

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**1 Lists**

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1. Draw box-and-pointer diagrams for the following.

```
>>> a = [1, 2, 3]
>>> a
```

**Solution:**

```
[1, 2, 3]
```

```
>>> a[2]
```

**Solution: 3**

```
>>> b = a
>>> a = a + [4, 5]
>>> a
```

**Solution:**

```
[1, 2, 3, 4, 5]
```

```
>>> b
```

**Solution:**

```
[1, 2, 3]
```

```
>>> c = a
>>> a = [4, 5]
>>> a
```

**Solution:**

```
[4, 5]
```

```
>>> c
```

**Solution:**

```
[1, 2, 3, 4, 5]
```

**Solution:** Box and pointer diagram on Python Tutor.

2. Write a function that takes in a list `nums` and returns a new list with only the primes from `nums`. Assume that `is_prime(n)` is defined. You may use a `while` loop, a `for` loop, or a list comprehension.

**def** all\_primes(num):

**Solution:**

```
result = []
for i in nums:
    if is_prime(i):
        result = result + [i]
return result
```

List comprehension:

```
return [x for x in nums if is_prime(x)]
```

## 2 Data Abstraction

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1. The following is an **Abstract Data Type (ADT)** for elephants. Each elephant keeps track of its name, age, and whether or not it can fly. Given our provided constructor, fill out the selectors:

```
def elephant(name, age, can_fly):  
    """  
    Takes in a string name, an int age, and a boolean can_fly.  
    Constructs an elephant with these attributes.  
    >>> dumbo = elephant("Dumbo", 10, True)  
    >>> elephant_name(dumbo)  
        "Dumbo"  
    >>> elephant_age(dumbo)  
        10  
    >>> elephant_can_fly(dumbo)  
        True  
    """  
    return [name, age, can_fly]  
def elephant_name(e):
```

**Solution:**

```
    return e[0]
```

```
def elephant_age(e):
```

**Solution:**

```
    return e[1]
```

```
def elephant_can_fly(e):
```

**Solution:**

```
    return e[2]
```

2. This function returns the correct result, but there's something wrong about its implementation. How do we fix it?

```
def elephant_roster(elephants):  
    """  
    Takes in a list of elephants and returns a list of their  
    names.  
    """  
    result = []  
    for elephant in elephants:  
        result = result + [elephant[0]]  
    return result
```

**Solution:**

elephant[0] is a Data Abstraction Violation (DAV).  
We should use a selector instead.

3. Fill out the following constructor for the given selectors.

```
def elephant(name, age, can_fly):
```

**Solution:**

```
    return [[name, age], can_fly]
```

```
def elephant_name(e):  
    return e[0][0]  
def elephant_age(e):  
    return e[0][1]  
def elephant_can_fly(e):  
    return e[1]
```

4. How can we write the fixed `elephant_roster` function for the constructors and selectors in the previous question?

**Solution:** No change is necessary to fix `elephant_roster` since using the `elephant` selectors “protects” the roster from constructor definition changes.

5. (Optional) Fill out the following constructor for the given selectors.

```
def elephant(name, age, can_fly):
    """
    >>> chris = elephant("Chris Martin", 38, False)
    >>> elephant_name(chris)
        "Chris Martin"
    >>> elephant_age(chris)
        37
    >>> elephant_can_fly(chris)
        False
    """
    def select(command)
```

**Solution:**

```
        if command == "name":
            return name
        elif command == "age":
            return age
        elif command == "can_fly":
            return can_fly
        return "Breaking abstraction barrier!"

    return select
def elephant_name(e):
    return e("name")
def elephant_age(e):
    return e("age")
def elephant_can_fly(e):
    return e("can_fly")
```

### 3 Trees

#### Things to remember

```
def tree(root, branches=[]): # ALWAYS OUTPUTS A TREE
    for branch in branches:
        assert is_tree(branch), 'branches must be trees'
    return [root] + list(branches)

def root(t): # ALWAYS OUTPUTS A NUMBER
    return t[0]
```

```
def branches(t): # ALWAYS OUTPUTS A LIST
    return t[1:]
```

1. Draw the tree that is created by the following statement:

```
tree(4,
     [tree(5, []),
      tree(2,
           [tree(2, []),
            tree(1, [])])],
     tree(1, []),
     tree(8,
          [tree(4, [])]))
```

**Solution:** TREE DRAWING HERE

2. Given the following, construct it using Python. Save it to the variable `t`.

TREE DRAWING HERE

**Solution:**

```
t = tree(9, [tree(2, []),
             tree(4, [tree(1, [])]),
             tree(4, [tree(7, []),
                     tree(3, [])])])
```

3. What would this output?

```
>>> root(t)
```

**Solution:** 9

```
>>> branches(t)[2]
```

**Solution:**

```
tree(4, [tree(7, []), tree(3, [])])
```

```
>>> branches(branches(t)[2])[0]
```

**Solution:**

```
tree(7, [])
```

4. Write the Python expression to get the integer 2 from t.

**Solution:**

```
root(branches(t)[0])
```

5. Write the function `sum_of_nodes` which takes in a tree and outputs the sum of all the elements in the tree.

```
def sum_of_nodes(t):  
    """  
    >>> t = Tree(...) # Tree from question 2.  
    >>> sum_of_nodes(t) # 9 + 2 + 4 + 4 + 1 + 7 + 3 = 30  
    30  
    """
```

**Solution:**

```
sum = 0  
for branch in branches(t):  
    sum += sum_of_nodes(branch)  
sum += root(t)  
return sum
```

Alternative solution:

```
return root(t) +\  
    sum([sum_of_nodes(b) for b in branches(t)])
```