Verification of Moment Conditions Identifying Structural Vector Autoregressions

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Bayesian Nonparametric Network Workshop

contributions

Objectives.

To verify the sources of identification of SVARs using the spike′n′slab prior in the BETEL framework

Contributions.

- Present various sources of model identification as moment conditions **MCs**
- Adapt spike′n′slab prior for overidentifying **MCs**
- Develop MCMC sampler facilitating estimation and inference
- Verify sources of identification of the tax shock

MC validity verification

MC validity verification

Two types of MCs.

assumed:	$\mathbb{E}\left[g(y_{it}, \boldsymbol{\theta})\right] = 0$
to be verified:	$\mathbb{E}\left[g(y_{it}, \boldsymbol{\theta})\right] = \gamma_j$

MC validity.

valid:	$\gamma_j = 0$
not valid:	$\gamma_j \neq 0$

Chib, Shin, Simoni (2018, JASA)

MC validity verification

MC to be verified.

$$\mathbb{E}\left[g(y_{it}, \boldsymbol{\theta})\right] = \gamma_j$$

spike'n'slab prior.

$$\gamma_j \mid \kappa_j \sim (1 - \kappa_j)\delta_0 + \kappa_j \mathcal{N}\left(0, \underline{\sigma}_\gamma^2\right)$$
$$\kappa_j \mid \underline{p} \sim \mathcal{B}ernoulli\left(\underline{p}\right)$$

$$\kappa_j = \mathcal{I}(\gamma_j = 0) \in 0, 1$$
δ_0 – Dirac mass at 0

MC validity verification

spike'n'slab prior.

$$\gamma_j \mid \kappa_j \sim (1-\kappa_j)\delta_0 + \kappa_j \mathcal{N}\left(0, \underline{\sigma}_\gamma^2\right)$$
$$\kappa_j \mid \underline{p} \sim \mathcal{B}ernoulli\left(\underline{p}\right)$$

MC validity.

valid: $\kappa_j = 0 \quad \Rightarrow \quad p(\gamma_j \mid \kappa_j = 0) = \delta_0$

not valid: $\kappa_j = 1 \quad \Rightarrow \quad \gamma_j \mid \kappa_j = 1 \sim \mathcal{N}\left(0, \underline{\sigma}_\gamma^2\right)$

MC validity verification

BETEL Framework.

$$p\left(\boldsymbol{\theta}, \boldsymbol{\gamma}, \boldsymbol{\kappa} \mid \mathbf{Y}\right) \propto \widehat{L}\left(\mathbf{Y}; \boldsymbol{\theta}, \boldsymbol{\gamma}\right) p\left(\boldsymbol{\theta}\right) p\left(\boldsymbol{\gamma} \mid \boldsymbol{\kappa}\right) p\left(\boldsymbol{\kappa}\right)$$

Empirical Likelihood.

likelihood: $\widehat{L}\left(\mathbf{Y}; \boldsymbol{\theta}, \boldsymbol{\gamma}\right) = \prod_{t=1}^{T} p_t^*\left(\boldsymbol{\theta}, \boldsymbol{\gamma}\right)$

as optimum of: $\min_{p_1, \ldots, p_T} \prod_{t=1}^{T} p_t \log p_t$

subject to: $\sum_{t=1}^{T} p_t = 1$ and $\sum_{t=1}^{T} p_t g(y_t, \boldsymbol{\theta}, \boldsymbol{\gamma}) = 0$

Schennach (2005, BIOMET)

MC validity verification

$$p\left(\boldsymbol{\theta}, \boldsymbol{\gamma}, \boldsymbol{\kappa} \mid \mathbf{Y}\right) \propto \widehat{L}\left(\mathbf{Y}; \boldsymbol{\theta}, \boldsymbol{\gamma}\right) p\left(\boldsymbol{\theta}\right) p\left(\boldsymbol{\gamma} \mid \boldsymbol{\kappa}\right) p\left(\boldsymbol{\kappa}\right)$$

MC validity verification.

sample S draws from the posterior using MCMC

$$\left\{\boldsymbol{\theta}^{(s)}, \boldsymbol{\gamma}^{(s)}, \boldsymbol{\kappa}^{(s)}\right\}_{s=1}^{S}$$

estimate MC validity posterior probability

$$\widehat{\Pr}\left[\kappa_j = 0 \mid \mathbf{Y}\right] = \frac{1}{S} \sum_{s} \kappa_j^{(s)}$$

reject MC validity $\gamma_j = 0 \quad \text{if} \quad \widehat{\Pr}\left[\kappa_j = 0 \mid \mathbf{Y}\right] < c$

identifying SVARs via MCs

identifying SVARs via MCs

Structural VAR.

reduced form: $\mathbf{y}_t = \mathbf{A}_+\mathbf{x}_t + \boldsymbol{\varepsilon}_t$

structural form: $\mathbf{A}\boldsymbol{\varepsilon}_t = \mathbf{B}\mathbf{u}_t$

structural shocks: $\mathbf{u}_t \sim \left(\mathbf{0}_N, \text{diag}(\boldsymbol{\sigma}^2)\right)$

identifying SVARs via MCs

reduced form: $\mathbf{y}_t = \mathbf{A}_+ \mathbf{x}_t + \boldsymbol{\varepsilon}_t$

$$\begin{aligned} \mathbb{E}\left[\varepsilon_{nt}\right] &= 0 && \forall n \\ \mathbb{E}\left[x_{it}\varepsilon_{nt}\right] &= 0 && \forall i, n \end{aligned}$$

structural shocks: $\mathbf{u}_t \sim \left(\mathbf{0}_N, \mathrm{diag}\left(\boldsymbol{\sigma}^2\right)\right)$

$$\mathbb{E}\left[u_{nt}^2\right] - \sigma_n^2 = 0 \qquad \forall n$$

structural form: $\mathbf{A}\boldsymbol{\varepsilon}_t = \mathbf{B}\mathbf{u}_t$

$$\mathbb{E}\left[u_{nt}u_{mt}\right] = 0 \qquad \forall n, m : n < m$$

tax shock identification

tax shock identification

Tax Policy Model by Mertens & Ravn (2016, JME)

$$\mathbf{A}\boldsymbol{\varepsilon}_t = \mathbf{B}\mathbf{u}_t$$

$$\begin{bmatrix} 1 & 0 & -\theta_{gdp} \\ 0 & 1 & -\gamma_{gdp} \\ -\rho_{ttr} & -\rho_{gs} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^{ttr} \\ \epsilon_t^{gs} \\ \epsilon_t^{gdp} \end{bmatrix} = \begin{bmatrix} 1 & \theta_{gs} & 0 \\ \gamma_{ttr} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_t^{ttr} \\ u_t^{gs} \\ u_t^{gdp} \end{bmatrix}$$

Identification.

exclusion restrictions Blanchard & Perotti (2002, QJE)

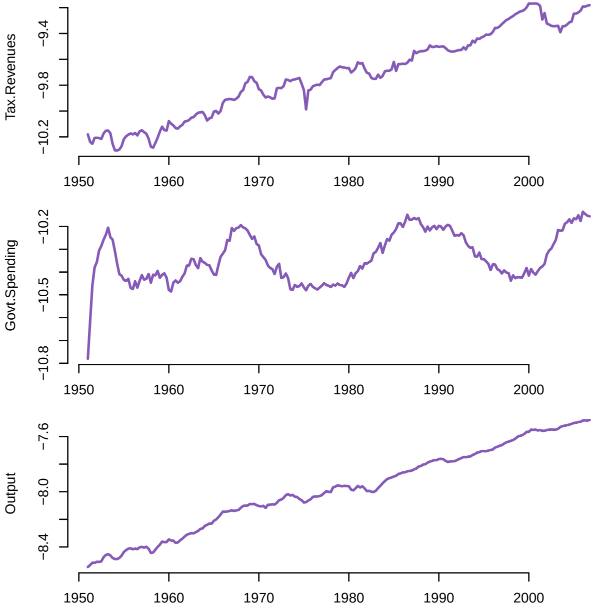
sign restrictions Mountford & Uhlig (2009, JAE)

narrative measure as an instrument Mertens & Ravn (2016, JME)

time-varying volatility Lewis (2021, RESTUD)

verified volatility: Lütkepohl, Shang, Uzeda, Woźniak (2023)

tax shock identification



tax shock identification verification

tax shock identification verification

$$\begin{bmatrix} 1 & 0 & -\theta_{gdp} \\ 0 & 1 & -\gamma_{gdp} \\ -\rho_{ttr} & -\rho_{gs} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^{ttr} \\ \epsilon_t^{gs} \\ \epsilon_t^{gdp} \end{bmatrix} = \begin{bmatrix} 1 & \theta_{gs} & 0 \\ \gamma_{ttr} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_t^{ttr} \\ u_t^{gs} \\ u_t^{gdp} \end{bmatrix}$$

parameter restrictions Blanchard & Perotti (2002, QJE)

$$\theta_{gdp} - 3.13 = \gamma_1 \qquad \text{(calibration)}$$

$$\gamma_{gdp} = \gamma_2 \qquad \text{(exclusion)}$$

$$\gamma_{ttr} = \gamma_3 \qquad \text{(exclusion)}$$

tax shock identification verification

$$\begin{bmatrix} 1 & 0 & -\theta_{gdp} \\ 0 & 1 & -\gamma_{gdp} \\ -\rho_{ttr} & -\rho_{gs} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^{ttr} \\ \epsilon_t^{gs} \\ \epsilon_t^{gdp} \end{bmatrix} = \begin{bmatrix} 1 & \theta_{gs} & 0 \\ \gamma_{ttr} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_t^{ttr} \\ u_t^{gs} \\ u_t^{gdp} \end{bmatrix}$$

instrumental variable z_t Mertens & Ravn (2016, JME)

$\mathbb{E}[z_t u_t^{ttr}] = \gamma_4$ (relevance)

$\mathbb{E}[z_t u_t^{gs}] = \gamma_5$ (exogeneity)

$\mathbb{E}[z_t u_t^{gdp}] = \gamma_6$ (exogeneity)

tax shock identification verification

$$\begin{bmatrix} 1 & 0 & -\theta_{gdp} \\ 0 & 1 & -\gamma_{gdp} \\ -\rho_{ttr} & -\rho_{gs} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^{ttr} \\ \epsilon_t^{gs} \\ \epsilon_t^{gdp} \end{bmatrix} = \begin{bmatrix} 1 & \theta_{gs} & 0 \\ \gamma_{ttr} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_t^{ttr} \\ u_t^{gs} \\ u_t^{gdp} \end{bmatrix}$$

asymmetric kurtosis Lanne & Luoto (2019, JBES)

$$\mathbb{E}[u_t^{ttr.3} u_t^{gs}] = \gamma_7$$

$$\mathbb{E}[u_t^{ttr.3} u_t^{gdp}] = \gamma_8$$

$$\mathbb{E}[u_t^{gs.3} u_t^{ttr}] = \gamma_9$$

$$\mathbb{E}[u_t^{gs.3} u_t^{gdp}] = \gamma_{10}$$

$$\mathbb{E}[u_t^{gdp.3} u_t^{ttr}] = \gamma_{11}$$

$$\mathbb{E}[u_t^{gdp.3} u_t^{gs}] = \gamma_{12}$$

tax shock identification verification

$$\begin{bmatrix} 1 & 0 & -\theta_{gdp} \\ 0 & 1 & -\gamma_{gdp} \\ -\rho_{ttr} & -\rho_{gs} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^{ttr} \\ \epsilon_t^{gs} \\ \epsilon_t^{gdp} \end{bmatrix} = \begin{bmatrix} 1 & \theta_{gs} & 0 \\ \gamma_{ttr} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_t^{ttr} \\ u_t^{gs} \\ u_t^{gdp} \end{bmatrix}$$

symmetric kurtosis Lanne & Luoto (2019, JBES)

$$\mathbb{E}[u_t^{ttr.2} u_t^{gs.2}] - 1 = \gamma_{13}$$

$$\mathbb{E}[u_t^{ttr.2} u_t^{gdp.2}] - 1 = \gamma_{14}$$

$$\mathbb{E}[u_t^{gs.2} u_t^{gdp.2}] - 1 = \gamma_{15}$$

MC validity verification

Simulation Study Summary.

looking at false discovery rate and false negative rate by Castillo & Roquain (2020, AOS)

DGPs bivariate Gaussian SVAR(0) with $T \in \{250, 750\}$ and

- symmetric kurtosis and exclusion restrictions
- asymmetric kurtosis, IV, and exclusion restrictions

performance summary

- FDR excellent for exclusion and higher-order conditions (especially for $T = 750$)
- FDR v reasonable for IV relevance and exogeneity
- v strong performance on FNR

tax shock identification verification

The estimation results indicate that

$$\widehat{\Pr}\left[\kappa_j = 0 \mid \mathbf{Y}\right] = 1 \qquad \forall j$$

Sources of tax shock identification.

parameter restrictions by Blanchard & Perotti **YES!**

instrumental variable by Mertens & Ravn **NO!**

time-varying volatility by Lewis **NO!**

non-normality by Lanne & Luoto **YES!**

to-do list

- Improve the efficiency of the sampler
  Chaudhuri, Mondal, Yin (2017, JRSSB)
- Accommodate conditional MCs
  Chib, Shin, Simoni (2018, JASA)
- Include more MCs
  Lewis (2021, RESTUD), Klein & Vella (2010, JOE)
- Include more IVs
  Ramey (2016, HOM)
- **R** package

- Verify the sources of SVARs identification using the MC framework
- Use spike′n′slab prior to infer MCs′ validity
- Develop MCMC sampler facilitating estimation and inference
- Verify sources of identification of the tax shock