Time-Varying Parameter Distribution Regression: Inflation Target at Risk

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Contents

Inflation Forecasting

Let π_t denote the (annualized) quarterly inflation rate at time t.

- **Philips Curve Model:** Blanchard et al. (2015)

$$\pi_{t+h} = \mu + \lambda\pi^*_{t-1} + (1-\lambda)\pi^{LTE}_t + \theta\left(u_t - u^*_t\right) + \gamma\left(\pi^R_t - \pi_t\right) + \epsilon_{t+h},$$

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- **Unobserved Component Stochastic Volatility Model**: Stock and Waston (2007):

$$\begin{aligned}
\pi_t &= \tau_t + \eta_t, \quad \eta_t \sim N\left(0, \sigma^2_{\eta,t}\right) \\
\tau_t &= \tau_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \left(0, \sigma^2_{\varepsilon,t}\right) \\
\ln\sigma^2_{\eta,t} &= \ln\sigma^2_{\eta,t-1} + \nu_{\eta,t}, \quad \nu_{\eta,t} \sim N(0,\gamma) \\
\ln\sigma^2_{\epsilon,t} &= \ln\sigma^2_{\epsilon,t-1} + \nu_{\epsilon,t}, \quad \nu_{\epsilon,t} \sim N(0,\gamma)
\end{aligned}$$

Estimation: Bayesian MCMC.

Motivation and Background

- **Inflation Risk**: Understanding inflation risks is a fundamental task of central banks to maintain the price stability.

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- **Tail Risks using Quantile Regression:** See L'opez-Salido and Loria (2019), Banerjee et al.(2020), Korobilis et al.(2021)

  Eg. L'opez-Salido and Loria (2019):

$$Q_{\pi_{t+h}}(\tau) = \mu + \lambda \pi^*_{t-1} + (1-\lambda)\pi_t^{LTE} + \theta\left(u_t - u_t^*\right) + \gamma\left(\pi_t^R - \pi_t\right) + \epsilon_{t+h}$$

Inflation and Deflation Risks

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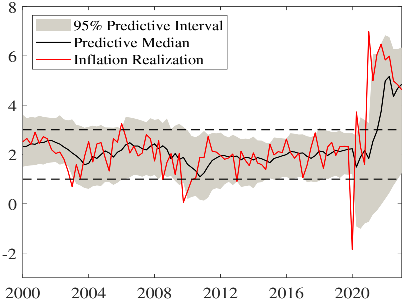

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Basic Distribution Regression (Univariate)

Distribution Regression (Williams and Grizzle,1972; Foresi and Peracchi, 1995; Chernozhukov et al.,2013)

For any $y \in \mathscr{Y}$,

$$F_{Y|X}(y|x) = \Lambda\big(\phi(x)^\top \theta(y)\big), \tag{1}$$

where

- $\Lambda(\cdot)$ is a known link function (eg. probit, logit, etc.)
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Estimation: Binary choice model for $\mathbb{1}\{Y \le y\}$ using MLE, given that $F_{Y|X}(y|x) = \mathbb{E}[\mathbb{1}\{Y \le y\}|x]$.

Model Specification: Time-Varying Parameter Distribution Regression

Let $Y_t \in \mathscr{Y}$ be the outcome variable, $X_t \in \mathscr{X}$ be a $d_0 \times 1$ vector of covariates.

A time-varying parameter distribution regression (TVP-DR) is defined as

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$$\beta_{y,t} = \beta_{y,t-1} + e_{yt}, e_{yt} \sim N\left(\mathbf{0}_d, \Sigma_{y,t}\right) \tag{3}$$

where $\Sigma_y = \operatorname{diag}\left(\sigma^2_{y,1}, \sigma^2_{y,2}, \ldots, \sigma^2_{y,d}\right)$, and $\beta_{y,1} \sim N\left(\mathbf{0}_d, \Sigma_y\right)$.

Contents

Model Estimation

Estimation: MCMC algorithm based on classical Bayesian theory for binary regression models. In this paper, we focus on the probit link function setting, that is,

$$F_{Y_t}(y|X_t) = \Phi\left(X_t^\top \beta_{y,t}\right) \tag{4}$$

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Reparameterization:

- a latent Guassian state-space model
- a high-dimensional static regression

Reparameterization: Latent State-Space Model

For $1\!\!1\{Y_t \leq y\}$, we can consider a latent variable $Y^*_{y,t}$.

- $Y^*_{y,t}$ satisfies $1\!\!1\{Y_t \leq y\} = 1\!\!1\{Y^*_{y,t} \geq 0\}$, and defined by the following latent Gaussian state-space model,

$$\begin{aligned} Y^*_{y,t} &= X_t^\top \beta_{y,t} + \epsilon_{y,t}, \quad && \epsilon_{y,t} \sim N(0,1), \\ \beta_{y,t} &= \beta_{y,t-1} + e_{y,t}, \quad && e_{y,t} \sim N(\mathbf{0}_d, \Sigma_{y,t}). \end{aligned} \tag{5}$$

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- Equivalently, $Y^*_{y,t}$ can be described as,

$$Y^*_{y,t} \sim \begin{cases} N_{[0,\infty)}(X_t^\top \beta_{y,t}, 1) & \mathbb{1}\{Y_t \leq y\} \\ N_{(-\infty,0)}(X_t^\top \beta_{y,t}, 1) & \mathbb{1}\{Y_t > y\}, \end{cases} \tag{6}$$

where N_A denotes a truncated normal distribution on set A.

Reparameterization: High-dimensional Static Regression

We stack $\beta_{y,t}$, $e_{y,t}$ and $\Sigma_{y,t}$ for all $t = 1, \ldots, T$ as follows,

$$\boldsymbol{\beta}_y = (\beta_{y,1}^\top, \beta_{y,2}^\top, \ldots, \beta_{y,T}^\top)^\top \in \mathbb{R}^{Td},$$
$$\boldsymbol{e}_y = (e_{y,1}^\top, e_{y,2}^\top, \ldots, e_{y,T}^\top)^\top \in \mathbb{R}^{Td},$$
$$\boldsymbol{\Omega}_y = diag(\Sigma_{y,1}, \ldots, \Sigma_{y,T}) \in \mathbb{R}^{Td \times Td}.$$

- According to the random walk assumption,

$$\boldsymbol{H}\boldsymbol{\beta}_y = \boldsymbol{e}_y \sim N(\mathbf{0}_{Td}, \boldsymbol{\Omega}_y), \tag{7}$$

where $\boldsymbol{H} := (\mathbb{I}_T - \mathbb{I}_{T,-1}) \otimes \mathbb{I}_d$.

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- Gaussian latent state-space model can be written as

$$\boldsymbol{Y}_y^* = \boldsymbol{X}\boldsymbol{\beta}_y + \boldsymbol{\epsilon}_y, \quad \boldsymbol{\epsilon}_y \sim N(\boldsymbol{0}, \mathbb{I}_T), \tag{8}$$
$$\boldsymbol{\beta}_y | \boldsymbol{\Omega}_y \sim N\big(\boldsymbol{0}_d, (\boldsymbol{H}^\top \boldsymbol{\Omega}_y^{-1} \boldsymbol{H})^{-1}\big), \tag{9}$$

  where $\boldsymbol{X} = diag(X_1^\top, \ldots, X_T^\top) \in \mathbb{R}^{T \times Td}$.

MCMC Algorithm

Bayesian MCMC Algorithm:

1. Sample $Y^*_{y,t}$ from the following truncated normal distributions

$$Y^*_{yt} \sim \begin{cases} N_{[0,\infty)}(x_t'\beta_{yt}, 1), & \mathbb{1}\{Y_t \leq y\} \\ N_{(-\infty,0)}(x_t'\beta_{yt}, 1), & \mathbb{1}\{Y_t > y\} \end{cases}$$

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2. Sample the TVP from its conditional posterior distribution

$$\boldsymbol{\beta}_y | \boldsymbol{X}, \boldsymbol{Y}^*_y, \boldsymbol{\Omega}_y \sim N(\boldsymbol{\mu}_y, \boldsymbol{V}_y),$$

where

$$\boldsymbol{V}_y = \left(\boldsymbol{X}^\top \boldsymbol{X} + \boldsymbol{H}^\top \boldsymbol{\Omega}_y^{-1} \boldsymbol{H}\right)^{-1},$$

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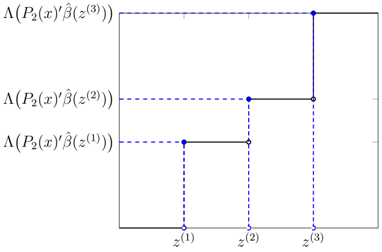
3. Sample Covariance parameters Σ_y via inverse-gamma posterior distributions

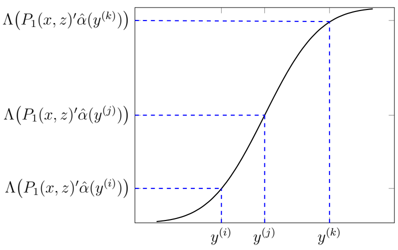
$$\sigma^2_{y,i} | \boldsymbol{\beta}_y \sim \mathscr{I}G\left(\nu_{y,i} + \frac{T}{2}, S_{y,i} + \frac{1}{2}\sum_{t=2}^{T}(\beta_{y,t,i} - \beta_{y,t-1,i})^2 + \beta^2_{y,1,i}\right).$$

Monotonicity of $F_{Y_t}(y \mid X_t)$

Construct the entire distribution:



(1.1) DR for Discrete



(1.2) DR for Continuous

Figure: Implementation of DR

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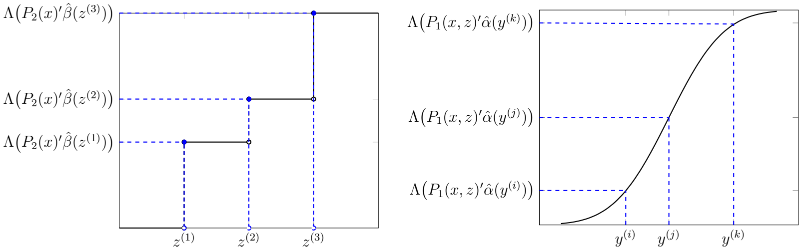

(1.1) DR for Discrete

(1.2) DR for Continuous

Figure: Implementation of DR

Note: Monotoncity guaranteed by the rearrangement method (Chernozhukov. et. al, 2009).

Monotonicity of $F_{Y_t}(y \mid X_t)$

The monotonicity of the conditional distribution at $y_j \in \mathscr{Y}, j = 1, \ldots, K$, can be ensured by imposing the following constrain:

$$\boldsymbol{X}\boldsymbol{\beta}_{y_1} \preceq \ldots \preceq \boldsymbol{X}\boldsymbol{\beta}_{y_K}. \tag{10}$$

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That implies if we were to sample

$$\boldsymbol{\beta} = [\boldsymbol{\beta}_1, \ldots., \boldsymbol{\beta}_K]$$

simultaneously, this is subject to $T(K-1)$ constraints. That is

$$(M \otimes \mathbf{X})\, vec(\boldsymbol{\beta}) \succeq \mathbf{0}_{T(K-1)} \tag{11}$$

where

$$M = [\mathbf{0}_{K-1}, \mathbb{I}_{K-1}] - [\mathbb{I}_{K-1}, \mathbf{0}_{K-1}].$$

MCMC for sampling $\boldsymbol{\beta}$ simultaneously

Sample

$$\boldsymbol{\beta}_{y_j} | \left\{ \boldsymbol{\beta}_{y_{j-1}}, \boldsymbol{\beta}_{y_{j+1}} \right\}$$

from multivariate Gaussian with linear constraints on both side.

- **Constraints:** The monotonicity of CDF ensured by the following constrain:

$$X_t^\top \beta_{y_{j-1},t} \leq X_t^\top \beta_{y_j,t} \leq X_t^\top \beta_{y_{j+1},t}. \tag{12}$$

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Let $X_t = (1, x_2, x_3, \ldots, x_d) \in \mathbb{R}^d$ and $\beta_{y,t} = (\beta_{y,t,1}, \ldots, \beta_{y,t,d})^\top$,

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$$X_t^\top \beta_{y_j,t} = \beta_{y_j,t,1} + x_2 \beta_{y_j,t,2} + \ldots + x_d \beta_{y_j,t,d},$$

Constraint on the intercept $\beta_{y_j,t,1}$,

$$\begin{cases} \beta_{y_j,t,1} \ge X_t^\top \beta_{y_{j-1}t} - x_2 \beta_{y_j,t,2} - \ldots - x_d \beta_{y_j,t,d}, \\ \beta_{y_j,t,1} \le X_t^\top \beta_{y_{j+1}t} - x_2 \beta_{y_j,t,2} - \ldots - x_d \beta_{y_j,t,d}. \end{cases}$$

MCMC for sampling $\boldsymbol{\beta}_y$ sequentially

Two steps sampling of $\boldsymbol{\beta}_y$ based on

$$\boldsymbol{\beta}_y \mid \bullet \sim N(\boldsymbol{\mu}_y, \boldsymbol{V}_y)$$

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$$\boldsymbol{\beta}_y^{(1)} \mid \boldsymbol{\beta}_y^{(2)}, \bullet \sim N(\boldsymbol{\mu}_y^*, \boldsymbol{V}_y^*), \quad s.t. \quad \mathbf{l} \leq \boldsymbol{\beta}_y^{(1)} \leq \mathbf{u}$$

where

$$\boldsymbol{V}_y^* = (M_1 \boldsymbol{V}_y^{-1} M_1')^{-1}$$
$$\boldsymbol{\mu}_y^* = \boldsymbol{V}_y^* \left(M_1 \left(\boldsymbol{V}_y^{-1} (\boldsymbol{\mu}_y - M_2' \boldsymbol{\beta}_y^{(2)}) \right) \right)$$

Contents

U.S. Inflation

Let P_t be the quarterly consumer price index (CPI) at time t, we define the h-periods (annualized) CPI inflation as

$$\pi_t^h := (400/h)\ln(P_t/P_{t-h})$$

- **TVPDR Model**

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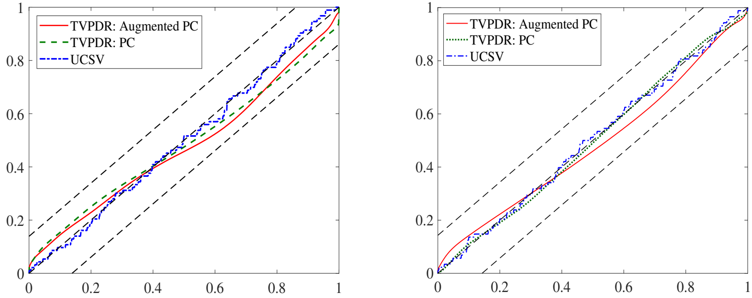
- **Data:** quarterly CPI (for all urban consumers: all items less food and energy (Index 1982-84=100)) from 1982:Q1 to 2023:Q2.
- Out-of-sample: 2000:Q1 to 2023:Q2. (expand rollowing window)

Out-of-sample Performance: PITs Plot

PIT test (Rossi and Sekhposyan, 2019):

Figure: PITs



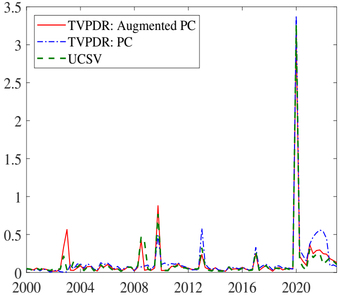
(3.1) $h = 1$ (3.2) $h = 4$

Out-of-sample Performance: Quantile Score

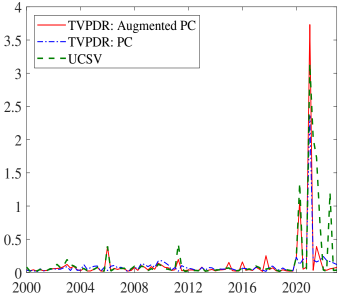
Quantile Score:

$$QS_t^h(\tau) = [\pi_{t+h}^h - \widehat{Q}_\tau(\pi_{t+h}^h|x_t)]\,\mathbb{1}\{\pi_{t+h}^h \le \widehat{Q}_\tau(\pi_{t+h}^h|x_t)\},$$

Figure: $h = 1$



(4.1) 5%



(4.2) 95%

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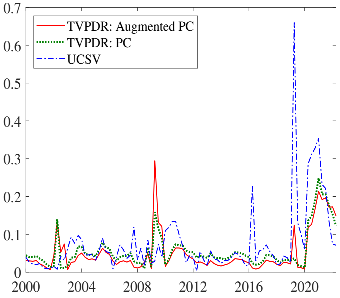
Figure: $h = 4$



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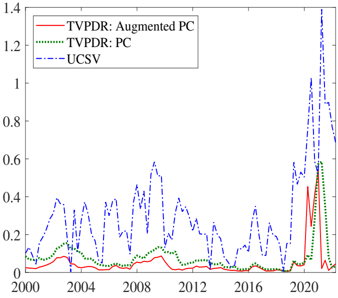

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Inflation Risk Analysis

Risk Measures of Killian and Manganelli (2007): Let $[\underline{\pi}, \overline{\pi}]$ be the preferred range of inflation,

$$\text{Deflation: } DR_t(\underline{\pi}, \alpha) := -\int_{-\infty}^{\underline{\pi}} (\underline{\pi} - y)^{\alpha} dF_{\pi}(y|x_t), \quad \alpha > 0$$

$$\text{Excessive inflation: } EIR_t(\bar{\pi}, \gamma) := \int_{\bar{\pi}}^{\infty} (y - \bar{\pi})^{\gamma} dF_{\pi}(y|x_t), \quad \gamma > 0,$$

where

- $\alpha \geq 0$ and $\gamma \geq 0$ measure the degree of risk aversion of the economic agent.

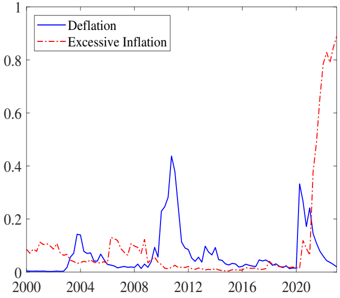
Inflation Risk Analysis

- $\alpha = \gamma = 0$:

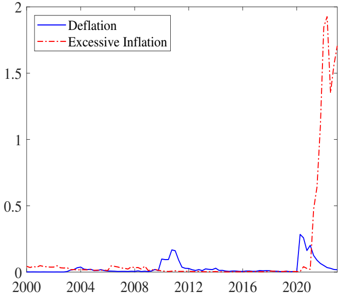
$$P(\pi_{t+1} < 1\% | x_t),\ \ P(\pi_{t+1} > 3\% | x_t)$$

- $\alpha = \gamma = 1$:

$$\mathbb{E}[\pi_{t+1} | \pi_{t+1} < 1\%, x_t],\ \ \mathbb{E}[\pi_{t+1} | \pi_{t+1} > 3\%, x_t]$$



(4.3) $\alpha = \gamma = 0$



(4.4) $\alpha = \gamma = 1$

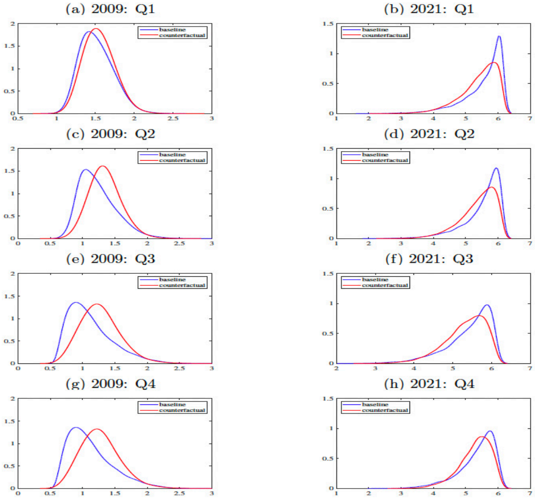
Inflation and Unemployment Trade-off

Table: True and Modified Unemployment Gap (%)

GFC	t	2009:Q1	2009:Q2	2009:Q3	2009:Q4
	True	3.39	4.43	4.77	5.08
	Modified	-1.61	-0.57	-0.23	0.08
Covid-19	t	2021:Q1	2021:Q2	2021:Q3	2021:Q4
	True	1.73	1.47	0.68	-0.25
	Modified	6.73	6.47	5.68	4.75

Inflation and Unemployment Trade-off

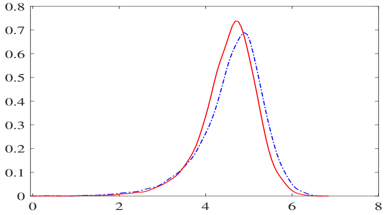
Figure 5: The Effect of Unemployment Gap on Inflation Distribution



Up-to-date Forecasting and Analysis

Effect of Increasing Unemployment Gap in 2023:Q3 by 5%: from -0.86% to 4.14%, unemployment rate becomes 8.2%.

Figure: Inflation Distribution in 2023:Q4



Measures	Mean	$P(\pi_{t+1} > 3\%)$	$P(\pi_{t+1} > 4\%)$	$P(\pi_{t+1} > 5\%)$	$P(\pi_{t+1} > 6\%)$
Baseline	4.6270	0.9696	0.8361	0.3171	0.0084
Counterfactual	4.5424	0.9767	0.8346	0.2273	0.0034

Thank you !

Q&A