# Normal modes splitting estimations from recent large earthquakes

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**abstract**

Recent large earthquakes recorded by the global seismic network are studied to compute and determinate the splitting characteristics of normal mode multiplets. Our ultimate goal is to use these data together with multiple-frequency travel times to invert for structure of the Earth's mantle. In a first effort, we analyzed nearly 90 earthquakes in the period 2000-2010 using a mode stripping technique. This allowed us to do a rigorous quality control. We strip multiplets of different spectra computed from more 30000 recordings of 80 events. Since we desire to get informations about the lower mantle, the maximum angular order studied is 9 for a maximum frequency of 4 mHz.

In the second step, we use the autoregressive estimation technique of Masters et al. (GJI, 2000), which allows for solving the splitting matrix system without knowledge of the earthquake source.

We present results from the first step that show how we use the technique to strip multiplets of different spectra computed from three components of multiple events in the period of study to select suitable data for subsequent autoregressive estimation. New estimations of splitting are also computed for a variety of multiplets. We also present splitting estimates from the autoregressive technique and make a comparison with earlier data, as well as with recent efforts by other groups to re-determine mode splitting.
We check the robustess of this estimates using a 'jackknifing' test in which we repeatedly remove a random fraction (10\%) of the data and recompute the coefficients.

## Method

To present simply the theory, we consider an isolated multiplet and we suppose we have many recordings $u_j(t)$ of an event. We can write:

$$u_j(t) = \sum_{k=1}^{2l+1} R_{jk} a_k(t) e^{i\omega t} \quad (1)$$

or

$$u(t) = R.a(t)e^{i\omega t} \quad (2)$$

The $j$'th row of $R_{jk}$ is a $2\ell+1$ vector of the spherical harmonics which describes the motion of the spherical-earth singlets at the $j$'th receiver and is readily calculated. $\omega$ is the multiplet degenerate frequency and $a(t)$ is a slowly varying function of time given by

$$a(t) = exp(iHt).a(0) \quad (3)$$

where $a(0)$ is a $2\ell+1$ vector of spherical-earth singlet excitation coefficients that can be computed if the source mechanism of the event is known. H is the 'splitting matrix' of the multiplet and incorporates all the information about 3-D structure

$$H_{mm'} = (a + mb + m^2c)\delta_{mm'} + \sum \gamma_s^{mm'} c_s^{m-m'} + \sum \gamma_s^{mm'} d_s^{m-m'} \quad (4)$$

where $a$,$b$ and $c$ describe the effects of rotation and ellipticity (Dahlen,1968), $\gamma_s^{mm'}$ are integrals over three spherical harmonics easy to compute (e.g. Dahlen and Tromp,1998) and $c_s^t$ and $d_s^t$ the structure coefficients.

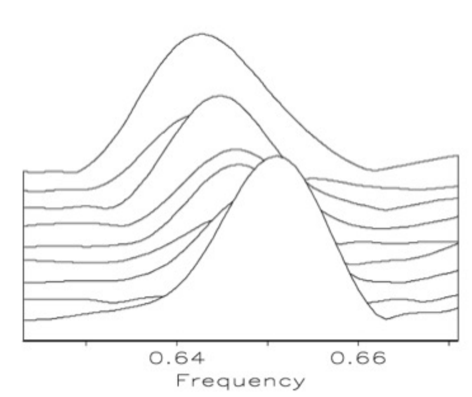
Now we form the 'receiver strips' for each event:

$$b(t) = R^{-1}.u(t) = exp[i(H + I\omega)t].a(0) \quad (5)$$

We actually work in the frequency domain using spectra of Hanning-tapered records in a small frequency band about a multiplet of interest

Example for the multiplet 00S04

Sumatra 2004




As shown by Masters and al.(2000), the receiver strips, b(t), satisfy a recurrence in time given by

$$b(t+\delta t) = R^{-1}.u(t+\delta t) = exp[i(H + I\omega)(t+\delta t)].a(0) = P(\delta t).b(t) \quad (6)$$

where

$$P(\delta t) = exp[i(H + I\omega)\delta t] \quad (7)$$

We recover P from the data and to get H, we use an eigenvalue decomposition of P. We calculate the spectra of the data at different time lags $\delta t$ and we form the receiver strips. We get a matrix system where each row is a $2\ell+1$ vector of receiver strips at a particular frequency:

$$B_{n+1} = B_n.P^T \quad (8)$$

Finally we get H from P using an eigenvalue decomposition of P.

$$P(\delta t) = Uexp[i\lambda\delta t]U^{-1} \quad (9)$$

where $\lambda = \Omega + I(\omega - \omega_g)$; then $H = U\Omega U^{-1}$. We use unique representation $H = E + iA$ where $E = 1/2(H + H^H)$ the real elastic part and $iA = 1/2(H - H^H)$ the imaginary part (superscript H is Harmitian transpose). We can write

$$E_{mm'} = \sum_s \gamma_s^{mm'} c_s^{m-m'} \quad (10)$$

$$A_{mm'} = \sum_s \gamma_s^{mm'} ds^{m-m'} \quad (11)$$

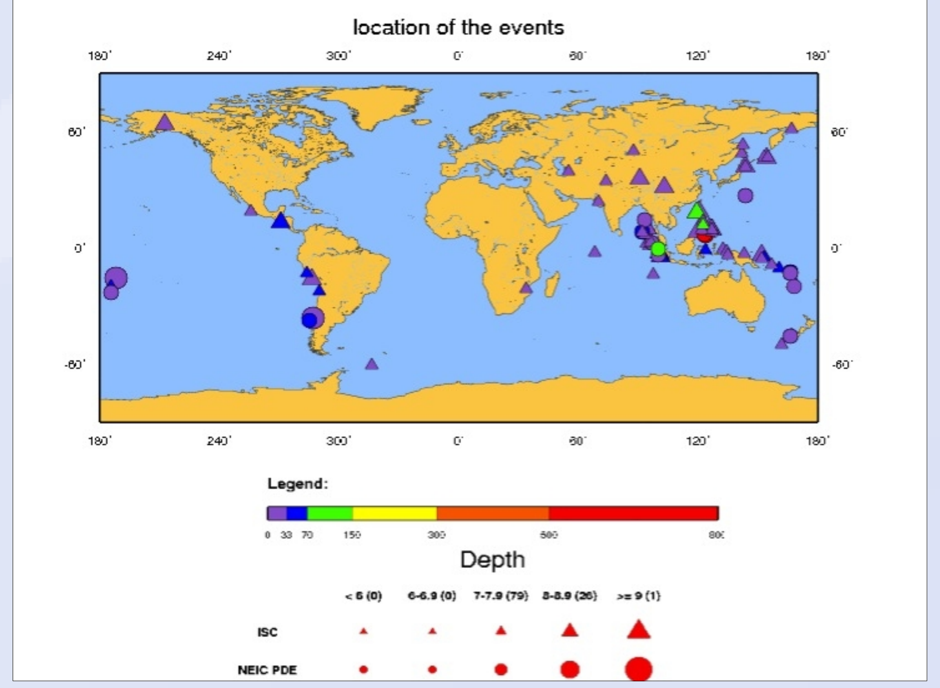
with $c_s^t$ the elastic structure coefficients and $d_s^t$ the anelastic structure coefficients.

## Definitions

Large earthquake are capable of exciting terrestrial free oscillations. A single mode of oscillations (singlet) depends on three integer numbers, n a radial number, l harmonic degree and m azimuthal order. Associated with every pair of n and l are 2l+1 azimuthal orders. 2l+1 singlets, also called multiplet, oscillate at the same frequency in a spherically symmetric Earth model. Potential deviations from spherical symmetry (rotation , ellipticity and aspherical structure) can perturbate frequency in the multiplet (splitting).

To determinate splitting characteristics of multiplets, data from recents large events recorded by global seismic network are currently analyzed though the splitting matrix estimation.

The coefficients of this multiplets are computed by a linear inversion from the splitting matrix and compared with measurements made by previous studies, Li and al. (1991) and He and Tromp (1996).




## Results

We present on the right side, three examples of splitting matrix and their coefficients that are compared with the estimates given by He and Tromp (1996) and Li and al. (1991).

The first example is the multiplet 00S09. The splitting matrix is decomposed into its elastic (E) and anelastic (A) parts, both of which are Hermitian. We use 49 earthquakes to determine this matrix. The diagonal of the matrix is caused by zonal (axisymmetric) structure (t=0), as constrained by the selection rules for a mode. The contribution from t= +/-2 structure are also indicated on imaginary part of (E).
The second example is the matrix of the multiplet 04S04. This matrix is computed with 35 earthquakes.
The last example is the matrix of the multiplet 05S05 that is computed with 21 earthquakes. The earthquakes used for the computation are shown on the map, depending on the matrix, we don't use the same earthquakes but some are always used like the Sumatra earthquake (2004) and the Chili earthquake (2007).

The coefficients are computed for s=2 and s=4 with t varying between –s and s.
The plots for 04S04 also include the estimates given by Laske and al. (2000).

## Conclusion

In this study and for the first time, a collection of 25 multiplets coefficients are computed with the autoregressive method.

We used the jacknifing method to obtain (still preliminary) estimates of the standard errors in the coefficients, shown as vertical bars in the figures.

The comparison with the results by Li et al. (1991) and He and Tromp (1996) show important disagreements for the coefficients obtained with different methods. Since all three studies neglect cross-coupling - assumed to be negligible for the mantle modes observed here - this cannot be the major cause of the differences. In contrast to the earlier studies, we allow for 3D Q structure, and this could explain part of the difference. But we suspect the largest difference is due to the fact that (1) the early studies used far fewer spectra and (2) require accurate knowledge of the moment tensor.
Unfortunately, these differences may mask any differences due to the methods used, so we are at this point not (yet) able to say whether the autoregressive method is more stable than older methods.

We plan to use the observed splitting coefficients in combination with multiple-frequency travel times of S and P waves in future global inversions.

References :

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Li X., Giardini D. and Woodhouse J., Large Scale three Dimensionnal even-degree Structure of the Earth from Splitting of long-period Normal Modes, JGR (1991) vol.96, 551- 577
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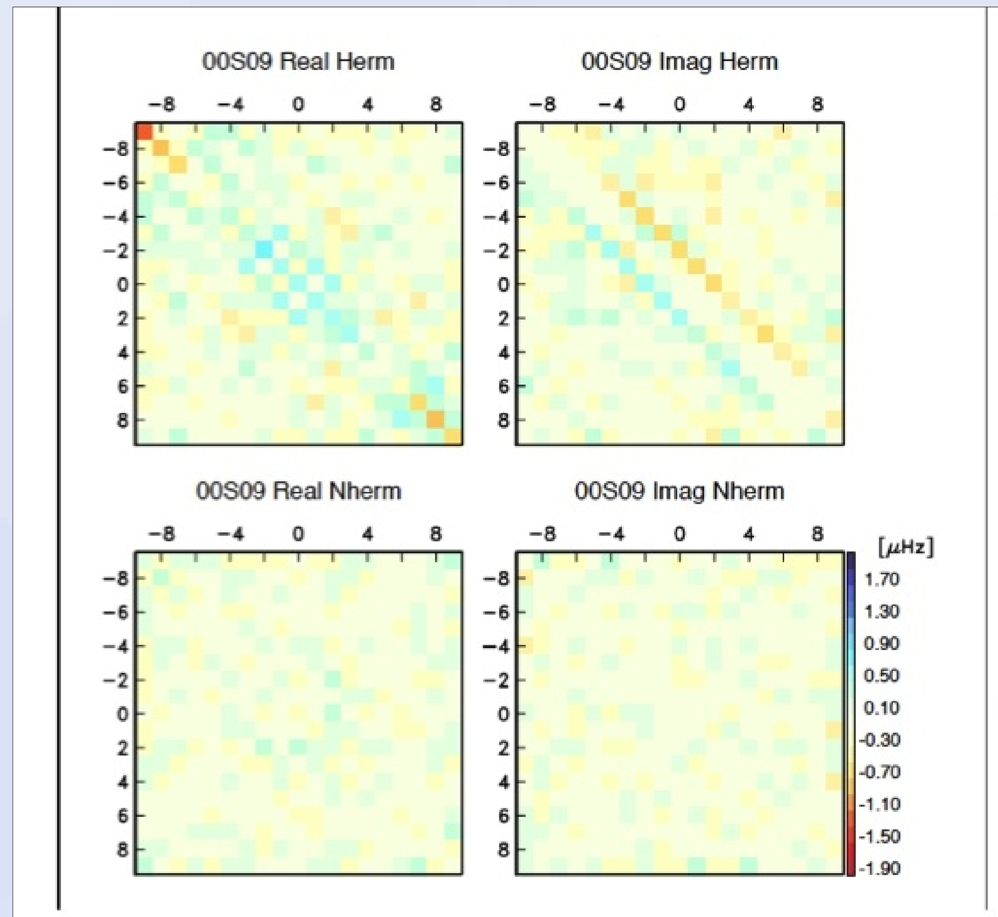
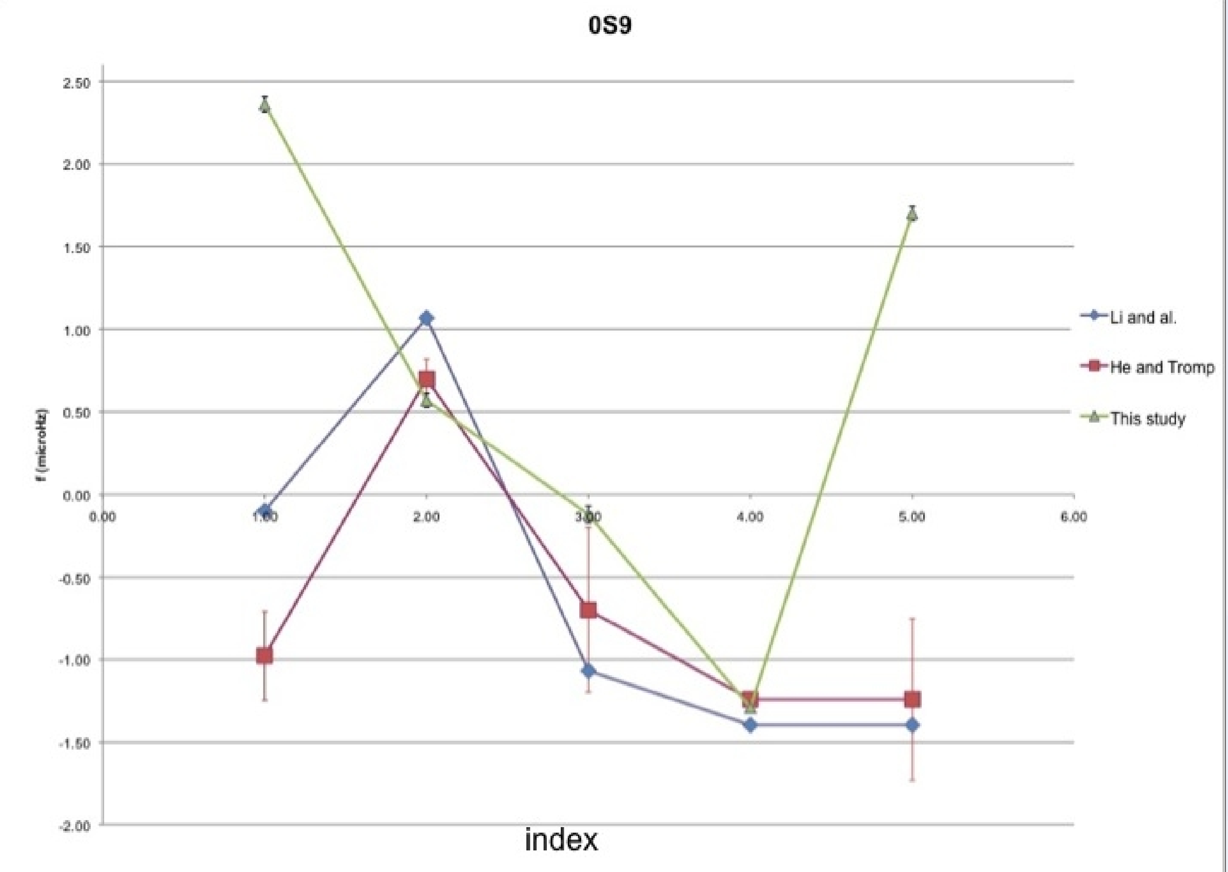
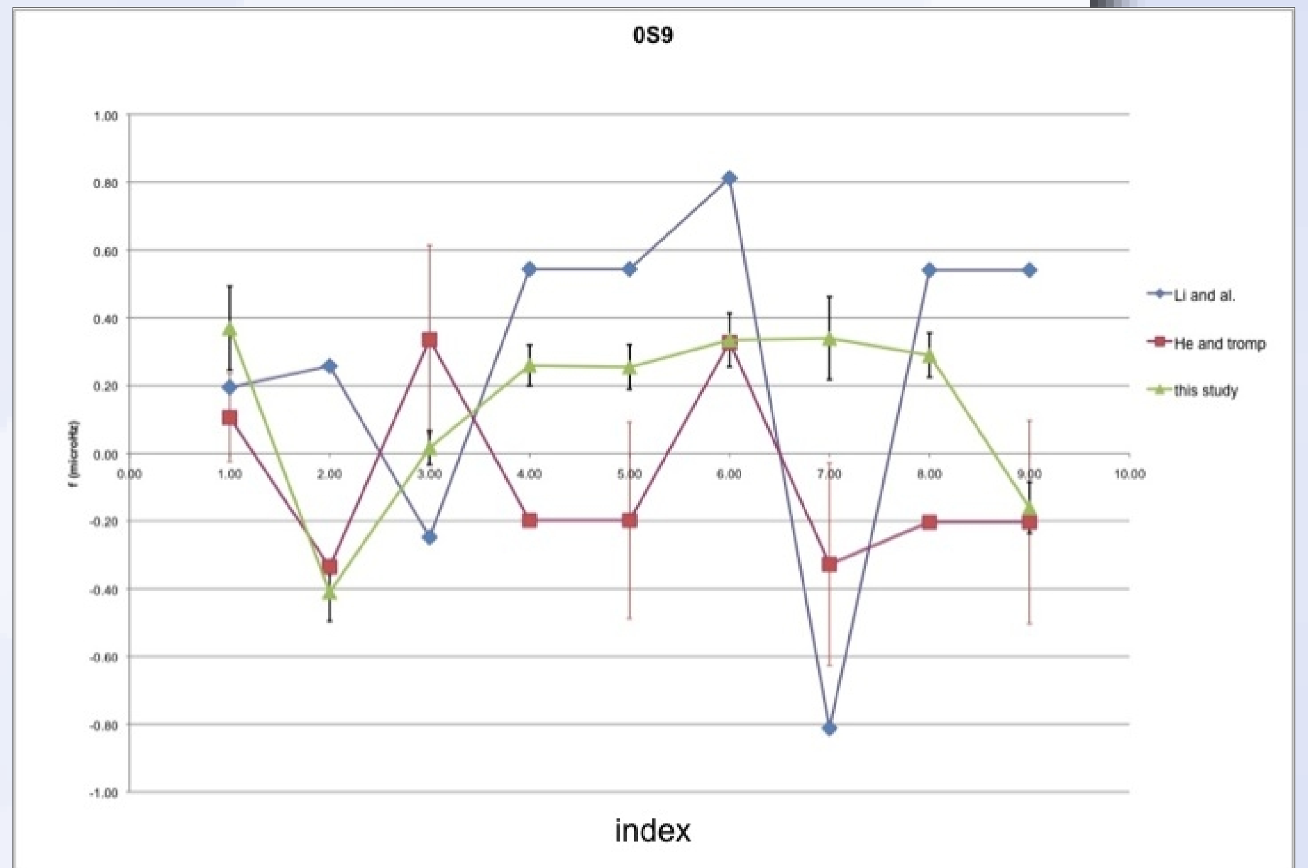
We have beneffited from discussions with some members of the Globalseis group at Geoazur.



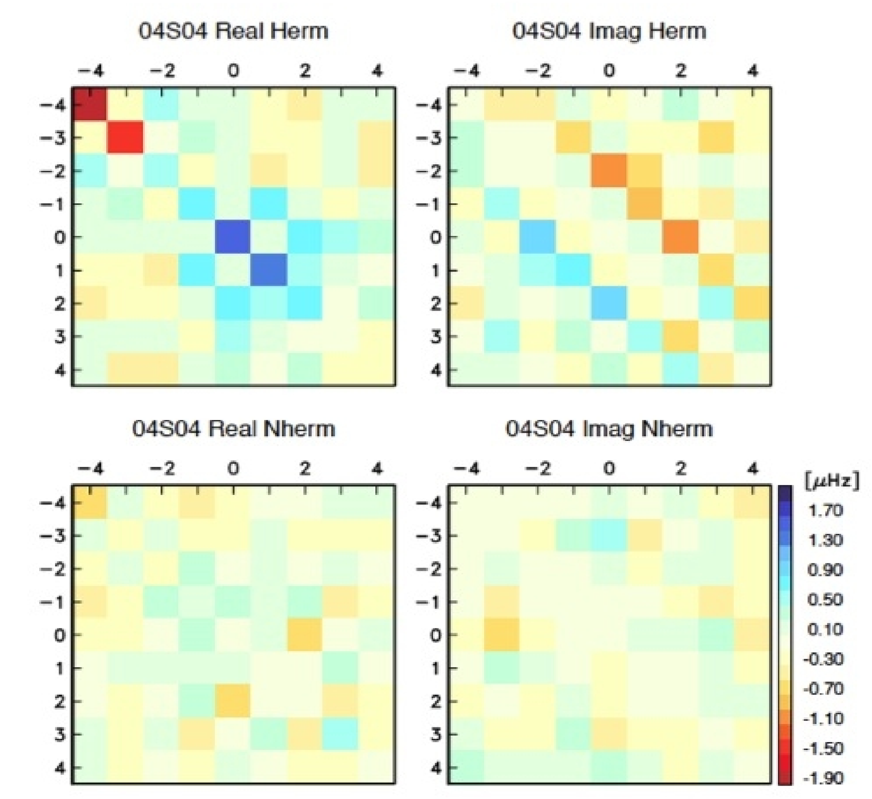

Splitting matrix for 00S09



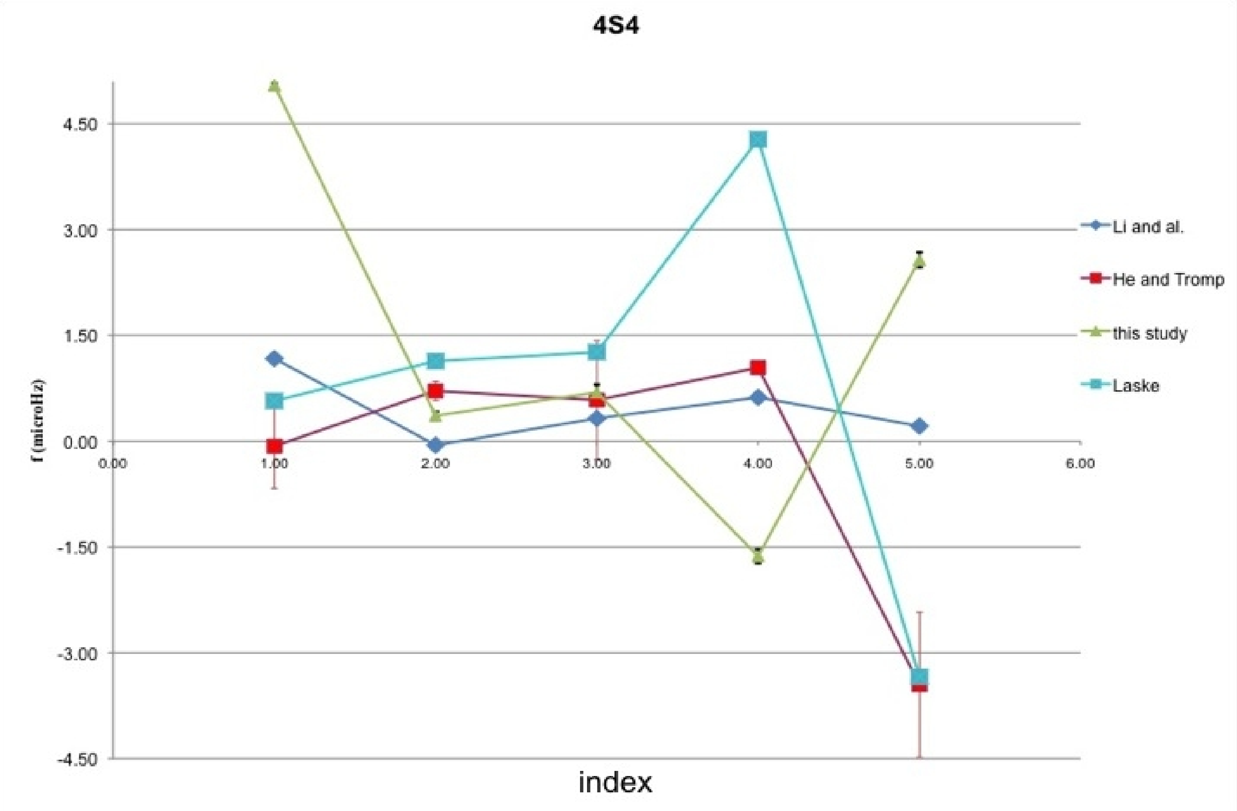

index=1 corresponds to $C_{2,0}$, index=2 is $C_{2,1}$, index=3 is $C_{2,-1}$, etc.



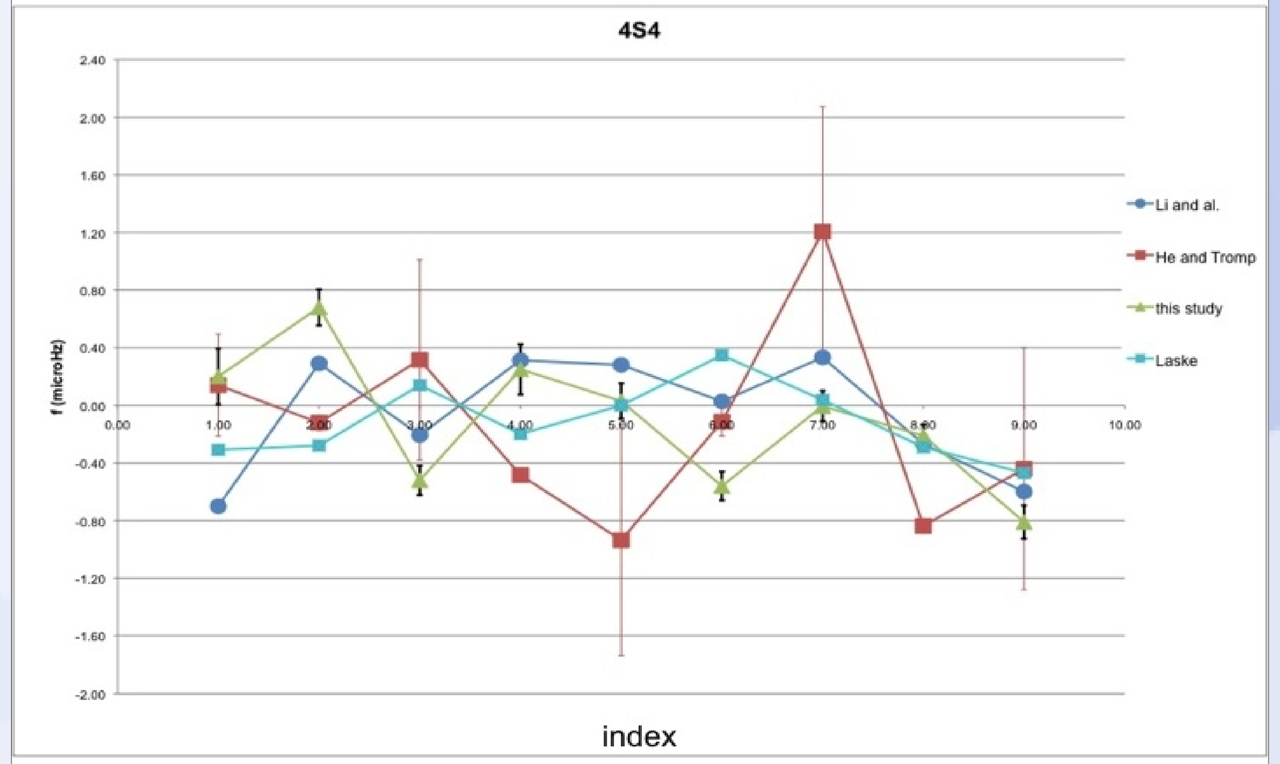

index=1 corresponds to $C_{4,0}$, index=2 is $C_{4,1}$, index=3 is $C_{4,-1}$, etc.



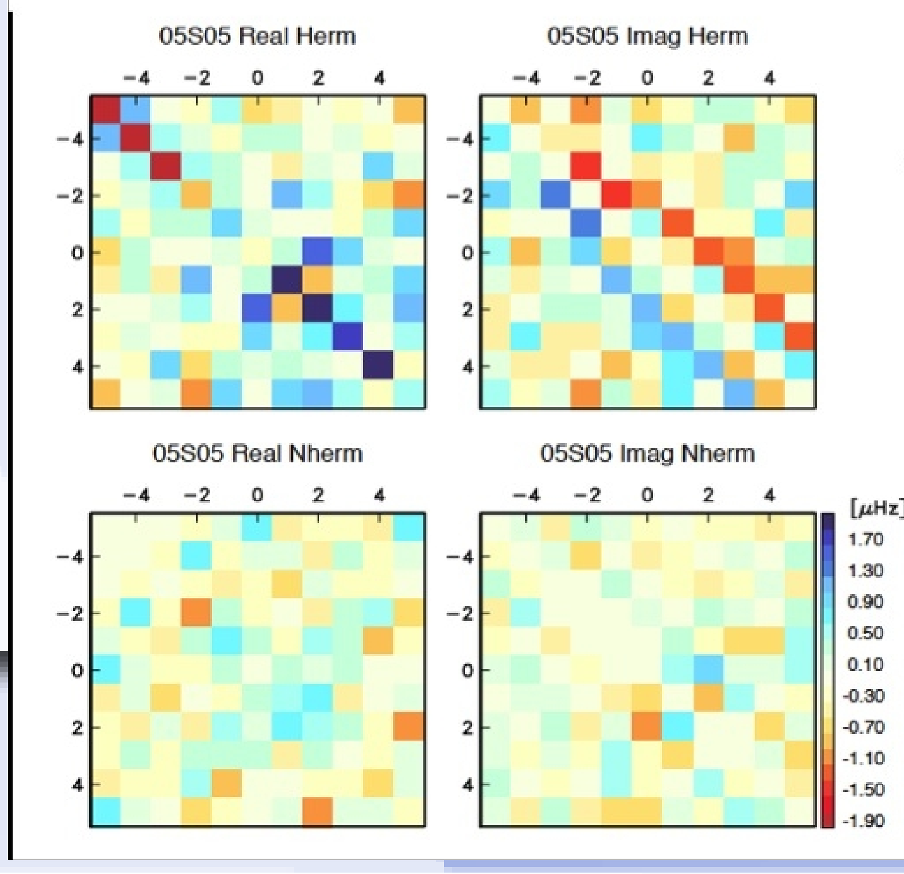

Splitting matrix for 04S04



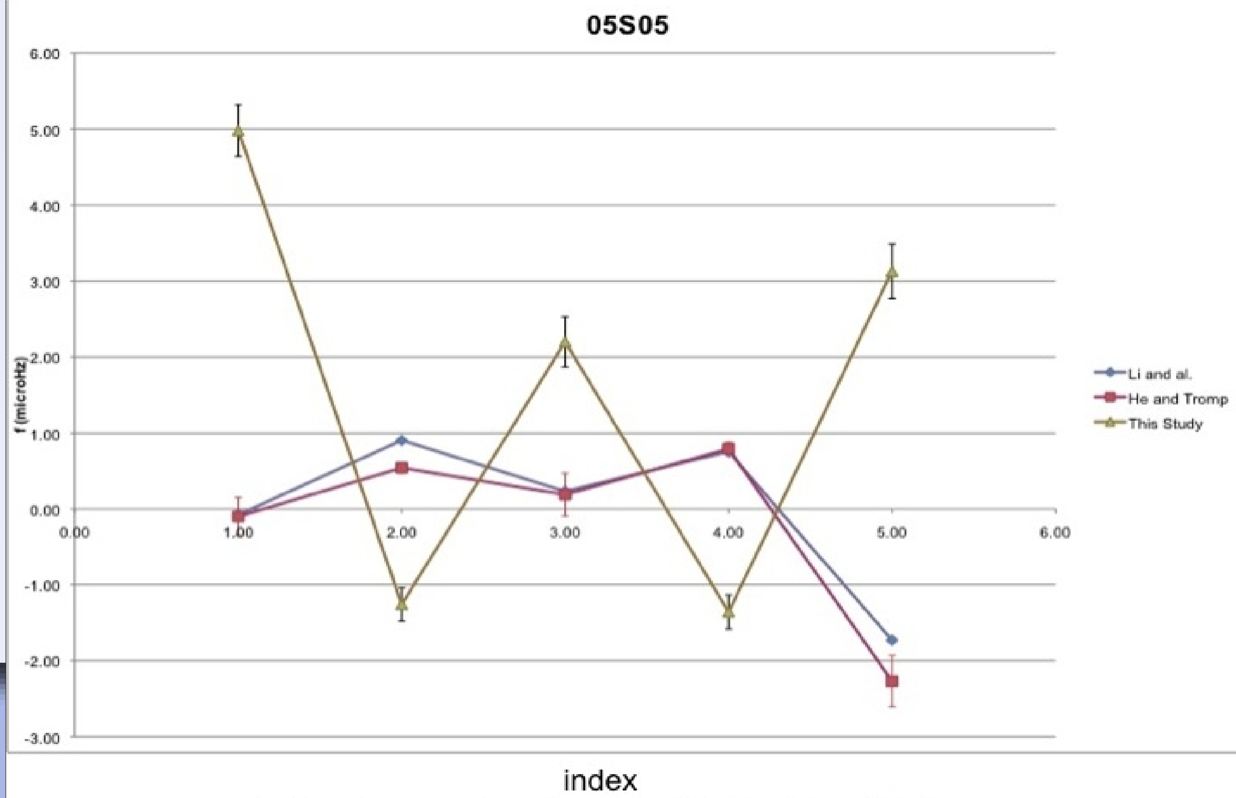

index=1 corresponds to $C_{2,0}$, index=2 is $C_{2,1}$, index=3 is $C_{2,-1}$, etc.



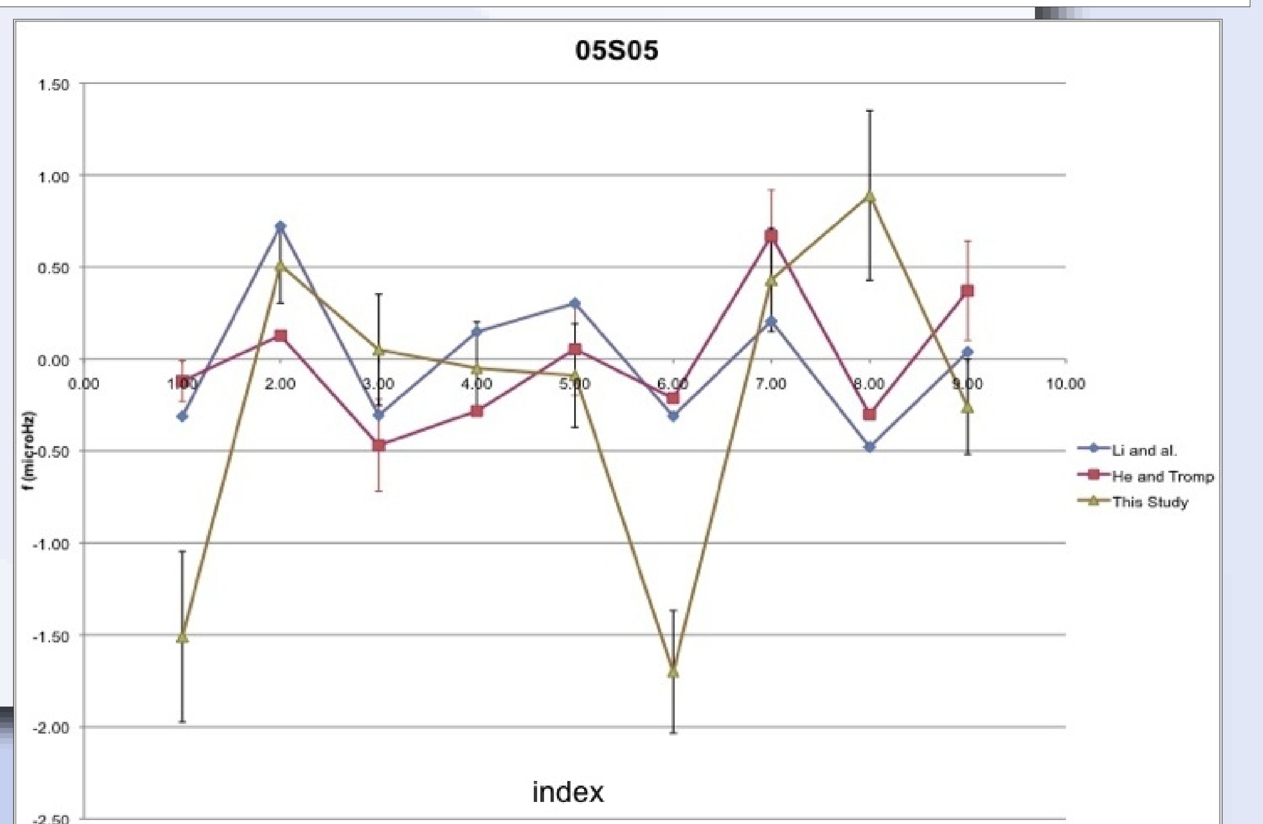

index=1 corresponds to $C_{4,0}$, index=2 is $C_{4,1}$, index=3 is $C_{4,-1}$, etc.




Splitting matrix for 05S05




index=1 corresponds to $C_{2,0}$, index=2 is $C_{2,1}$, index=3 is $C_{2,-1}$, etc.




index=1 corresponds to $C_{4,0}$, index=2 is $C_{4,1}$, index=3 is $C_{4,-1}$, etc.