This portrait depicts Tycho Brahe in 1586, at the age of 46. On the arch around him are the crests of important noble fam

CHAPTER 3

The New Astronomy

On January 11, 1600, Kepler set off from Graz to meet Tycho Brahe. After a journey of about ten days, the party arrived in Prague, the seat of the Holy Roman Emperor. High on the hill overlooking the city sat the Hradschin, the emperor's sprawling compound incorporating castle, cathedral, palace, and imperial offices. Clustered around it, as though drawn to the seat of power, were the palaces of aristocrats and ambassadors in the district called Hradcany. The Lesser Town, home of courtiers and craftsmen, spilled down the hillside to the Molda. Across the long stone bridge, the city spread out into the patrician Old Town, and further into New Town. Compared to Graz, it was a busy and chaotic place, with numerous open markets among the city's narrow, stinking streets. Prague was an important city in its own right as capital of prosperous Bohemia. But the presence of the emperor had also attracted a diverse international community of ambassadors, aristocrats, power-seekers, and hangers-on, as well as scholars, alchemists, artists, and skilled craftsmen.

Tycho did not live in the city but in Benatky castle in the countryside northeast of the city, which had been put at his disposal by the emperor. Consequently, it took some time before Kepler could send out a note announcing his presence in Prague. The next day, Tycho sent his son, Tycho Jr., and a trusted associate, Franz Tengnagel, into town with instructions to bring Kepler back out with them in the carriage.

The meeting of Johannes Kepler and Tycho Brahe on February 4, 1600, is extraordinarily significant in the history of science. The two men could not have been more different. Tycho was a nobleman, self-assured, domineering, and combative. Kepler was a commoner, sincere, reflective, peace-loving, and unassuming. Yet they fit together like a lock and key. Tycho was the observer, with a lifetime's accomplishment behind him in the form of 20 volumes of astronomical observations stretching back more than 35 years. Kepler was the young theorist, with one slim, highly speculative volume to his credit. Both were brilliant, and each one's skills complemented the other's. But neither was there by choice. Tycho had abandoned his native Denmark after a haughty dispute with his patron the king, and now was an expatriot, if not an exile. Kepler had fled oppressive atmosphere of religious intolerance in Styria. The bringing together of these two men at this place and time would change astronomy.

Tycho Brahe was an unusual man, to say the least. The first thing that would have struck Kepler about him was that he had a prosthetic nose made of gold and silver blended to a flesh color, a souvenir of a duel dating back to Tycho's student days. He had close-cropped, receding reddish hair, with a trim beard overhung by a large handlebar mustache. His personality was regal and overbearing.

Tycho had been born into the highest level of Danish society, a small class of aristocratic families that owned and ran the country. With the lavish support of the Danish crown, he had established an unprecedented observatory, Uraniborg, and engaged scores of scholars and craftsmen to aid him in his investigations of the heavens. Tycho had spent most of the last twenty years holed up on his private island devoting his attention—and a great deal of the king of Denmark's gold—to a total reformation of astronomical theory founded on an unprecedentedly complete and accurate collection of observations of the planets. He had trained assistants, cultivated instrument makers, and sent agents to collect astronomical books and manuscripts. Then, just when his 20 years of astronomical activity seemed to be coming to fruition, his royal support had suddenly eroded and he had had to leave Denmark in search of a new patron. After a couple years of uncertainty, he had secured the support of a most important and devoted patron, the Holy Roman Emperor Rudolf II.

When Kepler arrived, Benatky castle was abuzz with activity. Tycho was never comfortable without his great astronomical instruments set up, and masons and carpenters were in the process of making modifications to the castle to accommodate them. Interconnected instrument bays were being erected along the bluff overlooking the river Iser and the flat plain to the south. Here, he would regather his forces and found a "New Uraniborg."

A large and varied staff was being assembled to assist Tycho in his efforts. In addition to Tycho Jr. and Franz Tengnagel, there was Christian Severin Longomontanus, a talented Danish astronomer who had spent his entire career working for Tycho. Johannes Müller, mathematician to the elector of Brandenburg, and his family arrived the following month, pushing Kepler further down in the hierarchy. With Tycho's common-law wife, Kirsten Jørgensdatter, their other children, various other assistants, and servants, the small castle was crowded.

Kepler was rather bewildered by the scene and felt lost amid Tycho's large household. Nor was their collaboration what he had hoped for. He came to use Tycho's superior observational data to test and develop his polyhedral cosmo-

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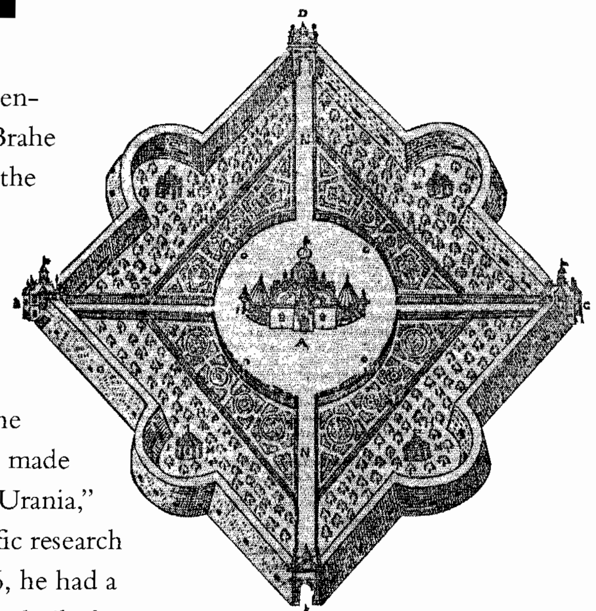
URANIBORG OBSERVATORY



In 1575, the king of Denmark granted Tycho Brahe the Island of Hven in the Øresund (the strait between present-day Denmark and Sweden) along with enough money to build and run an observatory. During the next twenty years, Tycho made Uraniborg, "the castle of Urania," into Europe's first scientific research institute. Starting in 1576, he had a Renaissance castle custom-built there to correspond to his needs. Its main feature was two second-story observing decks where instruments were permanently installed under removable conical roofs. There was also a library, where Tycho had a huge brass globe five feet in diameter, on which the positions of stars were patiently engraved when they were known with sufficient accuracy. In the basement, there were 16 furnaces of different kinds for alchemical experiments. Eight small rooms under the roof gables on the third floor housed assistants and students.

Tycho had instrument shops, where he was constantly producing more refined and accurate instruments, and a printing press, so he could publish his findings. In the watchtower of his castle wall, he even had a jail. Elsewhere on the island, he built his own paper mill and fishponds. Later on, Tycho decided that it would be better to have a separate observatory, where larger instruments could be installed down out of the wind. This subterranean observatory, called Stjerneborg, "the castle of the stars," housed Tycho's largest and most sophisticated instruments.

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logical hypothesis from the *Mysterium cosmographicum*. But he found Tycho to be secretive with his data. For his part, Tycho was not about to give his data away, and he did not particularly trust Kepler, especially given his suspicious connection with Ursus.

Tycho was at the stage in his career when he needed to spend time analyzing his many years of observations to distill accurate planetary theories from the raw data. For this, he needed many assistants to do the calculations. Tycho assigned Kepler to work under Longomontanus's supervision on the theory of Mars. The situation was galling to Kepler. He found himself lost in the commotion at Benatky, picking up dribs and drabs of greatly-desired information as Tycho casually let fall a reference to the location of a planet's apogee (its furthest distance from the earth) or node (where its orbit intersected the sun's orbit) as he held court during dinner in the crowded second floor dining room.

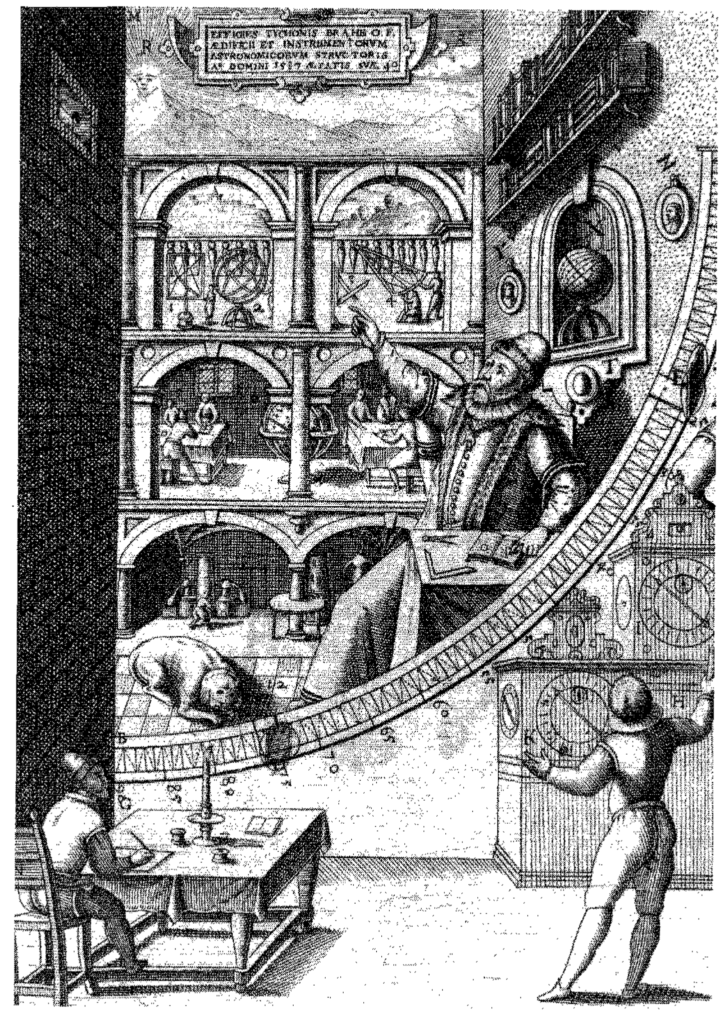
Even though Kepler could not get on with the development of the polyhedral hypothesis, for which he needed data on all of the planets, there was still work he could do on his planet-moving force hypothesis, for which he could use just the Mars observations. And within a few months he had come up with some remarkable confirmation. If the planets were really moved by a force coming from the sun, then the geometry of the planets' theories should reflect that. First, he found that, no matter what he tried, Mars's orbit had to take into account the sun's actual position, which made sense if it was the source of motion. Second, and more importantly, by ingeniously manipulating the observations of Mars, Kepler was able to investigate the earth's orbit, and he found that the earth shared the physically non-uniform motion of the other planets; it too speeded up when it approached the sun and slowed down as it receded. Astronomers had never previously understood that the theory of the earth was so similar to the theories of other planets. In fact, in the *Mysterium cosmographicum*, Kepler had had to acknowledge that the planet-moving force hypothesis did not work with the earth's orbit. Now, suddenly, the earth's motion confirmed the planet-moving force hypothesis. Though the result was immensely gratifying to Kepler, Tycho Brahe, like Maestlin, objected vehemently to his use of physical analysis in the derivation of planetary theory.

The first summer of Kepler's collaboration with Tycho Brahe was also marred by some friction between the two astronomers over Kepler's status and professional prospects. Facing the extreme uncertainty of how events would unfold in Styria, Kepler pressured Tycho for a formal position and contract. To Tycho, Kepler's demands were an affront. He had problems of his own collecting his salary from the emperor and pushing on with the renovation of Benatky. However, he was working behind the scenes to secure an imperial salary for Kepler, by having the emperor officially summon Kepler to assist Tycho Brahe for two years in his astronomical work. During this time, Kepler would continue to receive his 200 florin salary as district mathematician in Styria, and the emperor would supplement it with another 100 florins. Since the request to release Kepler for this assignment would come from the emperor, it was believed the representatives of the Estates of Styria would not be able to turn it down.

With his future prospects looking much better, Kepler prepared to return home in May. As a final gesture of goodwill, Tycho arranged for Kepler to travel with his third cousin, Frederick Rosenkrantz, as far as Vienna. They departed on June 1. Rosenkrantz would have had tales to tell as they traveled southeast through Bohemia into Austria. Like his cousin, Rosenkrantz was a Danish nobleman whose relations with his native land were strained. He had fled Denmark after getting a young lady-in-waiting pregnant, but had been captured and sentenced to the loss of two fingers and his nobility. But then, the sentence had been commuted to service in the Christian campaign against the

Islamic Turks, who had advanced through the Balkans and were threatening Austria's southern border. After stopping to visit his cousin at Benatky, he was traveling to Vienna to join the Austrian troops. Unbeknownst to him, Rosenkrantz was already being immortalized in a way. In 1592, when on a diplomatic mission to England with another cousin of Tycho's, Knud Gyldenstierne, he had made an impression on the young playwright William Shakespeare and had earned himself a bit part in *Hamlet*.

The hopeful joy that attended the preliminary results of



In this mural, printed in Tycho's Mechanica, *Tycho points to the heavens. A cutaway image of Uraniborg shows the observing decks, the library with the great celestial globe, and the alchemical furnaces in the basement.*

his research on Mars and the prospect of returning to Prague to continue his work with Tycho Brahe quickly dissipated upon Kepler's return to Graz. The Styrian councilors were not well disposed toward releasing Kepler to return to Prague. Kepler's astronomical speculations were out of place in the uneasy atmosphere that gripped Styria. It would be better, they concluded, if Kepler were to turn his attention to something useful, like going to Italy to study medicine and then returning to practice as a physician.

That summer, Kepler tried to interest Archduke Ferdinand to hire him as his personal mathematician, as his cousin the emperor had done with Tycho Brahe, but Ferdinand had other plans. On July 27, 1600, a notice appeared: an ecclesiastical commission was coming to Graz. At 6 A.M. on July 31, all citizens would present themselves for an examination of their faith. Anyone who was not Catholic or did not pledge to convert to Catholicism would be expelled from the country. Archduke Ferdinand himself accompanied the commissioners. They set up a large table in the middle of the church. During the course of three days, one by one, more than a thousand people approached the table and declared themselves. When Kepler's turn came, he declared himself a Lutheran and unwilling to convert. His name was inscribed on the list of banished men, 15th of 61. He was given six weeks and three days to be out of the country.

Kepler began to make preparations to leave. He only had to figure out where to go. The arrangement with Tycho Brahe was ruined, for it presupposed his receiving the greater part of his pay from Styria. Desperately, he wrote to Maestlin, again asking whether some "little professorship" might be found for him at Tübingen. Not hearing from Maestlin and having no other options, he would head back to Prague. He had been advised that Tycho would find a way to take care of him, and, indeed, Tycho responded to his distress by writing that the collapse of their arrangement did not matter; Kepler should not hesitate but should return with confidence.

On September 30, 1600, two weeks beyond the expulsion deadline, Kepler left Graz with his wife and daughter and two wagons containing all their possessions. His stay in Graz was over.

Kepler had grave misgivings about returning to Tycho's service. He was too proud and insecure to depend entirely on Tycho's mercy, but there was nowhere else to go. Underway, he was struck with a terrible fever. When he arrived in Prague on October 19, Baron Hoffmann took him in, a sick, exhausted, and depressed man. When Maestlin finally reported that there was no prospect of a job for him in Tübingen, Kepler was shattered. He replied with pathetic resignation, "I cannot describe what paroxysm of melancholy your letter caused me . . . For here in Prague I have found everything uncertain, even my life. The only certainty is staying here until I get well or die." A serious cough joined the fever, and Kepler feared he had tuberculosis. His wife became ill as well.

When he was finally well enough to go to work, he found Tycho's circumstances substantially changed as well. Tycho had abandoned his unfinished "New Uraniborg" at Benatky for cramped quarters in the city. When the plague that had gripped Prague the previous year had receded, Emperor Rudolf II and his court had returned, and the emperor desired the presence of his astrologer, Tycho Brahe. It was precisely the kind of work Tycho disliked—it was difficult to convince the emperor of the limit of astrological prognostications—but it was essential to satisfy his patron. He had packed up the instruments and was doing his best to accommodate them at his new home in the city. Kepler and his family were also squeezed in somewhere when they left Baron Hoffmann's. Tycho's personnel had also changed. Longomontanus had left him after many years of service and returned to Denmark to make a career for himself independent of Tycho. None of the various Germans Tycho had tried to attract, including Johannes Müller, had worked out either.

Kepler's fever raged on intermittently for months through the spring of 1601, and he was unable to work much on his Mars research. His fever subsided only that summer, during a visit back to Styria. Old Jobst Müller had died, and Kepler went back to look after his wife's inheritance, hoping to convert her assets into cash. His effort was fruitless, but after he returned from his four month visit around the end of August, he felt really well and rested. When he returned to Prague, Tycho had a scheme to secure for him a formal imperial appointment. The fact of the matter was, Kepler was about the only assistant Tycho had left. Longomontanus was gone. Tengnagel had married Tycho's daughter Elizabeth that summer and gone off to Deventer, Holland, taking another assistant, Johannes Erikson, with him.

Putting a great deal of faith in Kepler, Tycho took him to court and introduced him to the emperor, a strange, shy man with round childlike eyes set in a face anchored by a prominent chin, the characteristic feature of the Hapsburg family. Tycho presented a plan to compile a great set of astronomical tables and asked permission to name them *The Rudolfine Tables*, after the emperor. It was a grand gesture. The great astronomical tables, like the *Alphonsine* (Ptolemaic) or the *Prutenic* (Copernican), had been named after their sponsors, ensuring them a kind of immortality. If Tycho's lived up to their promise, they would be a magnificent monument indeed. Rudolf liked the idea very much. The only thing Tycho would require would be a salary for his assistant, Johannes Kepler.

Tycho worked as astrologer to Holy Roman Emperor Rudolf II (below) and proposed a great set of astronomical tables that were named the Rudolfine Tables *in his honor.*

The paperwork for Kepler's salary does not seem even to have been put in motion when it became obsolete. Ever since moving into

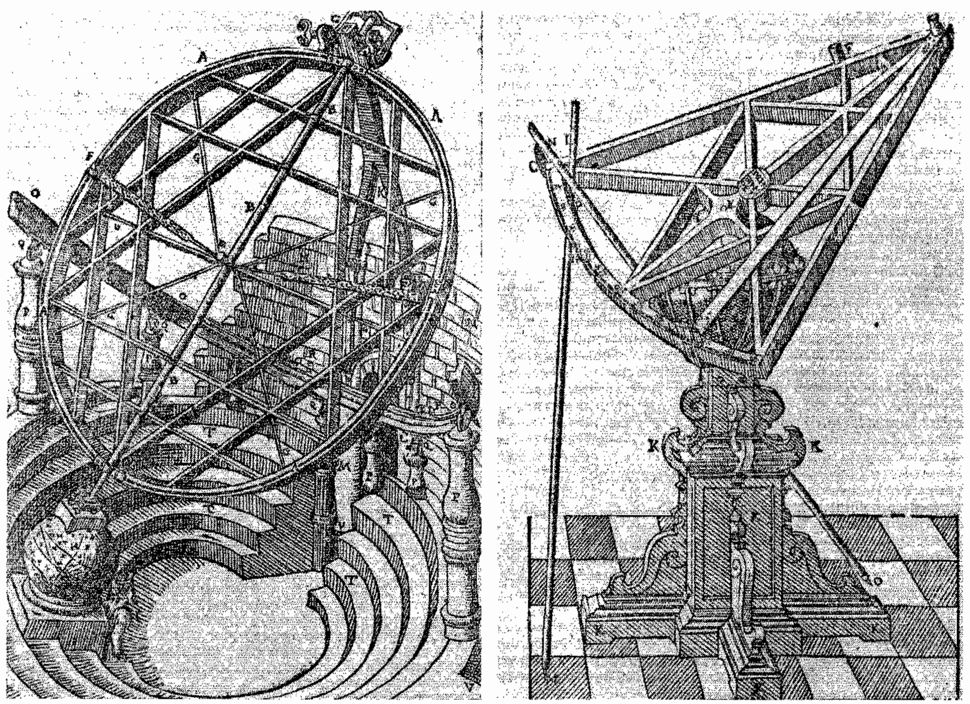
the city, Tycho's social circle had expanded, and without much observing being done at night, he got back into the noble pastime of attending parties where a great deal of hard drinking was done. On October 13, 1601, he attended a party at the house of Peter Vok Rozmberk. In order to avoid some breach of etiquette, Tycho remained seated at the table for far longer than his bladder allowed. It was a fatal miscalculation. By the time he got home, he could no longer urinate, and it quickly became clear that he was in serious trouble. It is impossible to know precisely what afflicted Tycho. Passing even a little urine was excruciatingly painful, and as the wastes built up in his body, he suffered from what Kepler called "intestinal fever," probably what we now call uremia. He passed sleepless nights in agony. Knowing that he would die, he spoke to Kepler, and begged him to present his research in the Tychonic system of the world rather than the Copernican. Then he became delirious, repeating over and over, "Let me not be seen to have lived in vain." Finally, as Kepler inscribed on the final page of Tycho's observation log,

> On October 24, 1601, when his delirium had subsided for a few hours, amid the prayers, tears, and efforts of his family to console him, his strength failed and he passed away very peacefully.
>
> At this time, then, his series of celestial observations was interrupted, and the observations of thirty-eight years came to an end.

On November 4, draped in black cloth and decorated in gold with the Brahe coat of arms, Tycho's casket was carried by twelve imperial officers, all of them noblemen, in a procession to the Protestant Tyne Cathedral. Accompanying it were black banners carrying lists of his titles and accomplishments in gold letters. His riderless horse and men carrying his arms and armor followed behind. Then followed a parade of people: noblemen, barons, ambassadors, his assistants, including Kepler, Tycho's family, and distinguished citizens. A solid wall of humanity lined the route as the procession snaked through the city. And in the church, there was scarcely any room to be found. Tycho was interred in the nave, his grave marked by a magnificent red marble frieze depicting him in full armor. He rests there still.

Kepler had scarcely any time to ponder his future. Within two days, he was informed the he would become the emperor's new mathematician, with responsibility to care for Tycho's instruments and to complete Tycho's unfinished publications, the most important of which would be the *Rudolfine Tables*. At the time of his appointment, Kepler was the obvious choice. There was no other qualified candidate around, and only weeks earlier Tycho had presented him as his primary collaborator in the *Rudolfine Tables*. Still, the emperor recognized that Tycho's instruments and observing logs rightly belonged to his heirs, so he simply bought them for the extraordinary sum of 20,000 florins—an amount sufficient to pay Kepler's previous salary in Styria for a century

Tycho's renowned instruments include the Great Equitorial Armillary (left) and the Trigonal Sextant (right). After Tycho's death, Emperor Rudolf II bought these instruments and made Kepler responsible for looking after them.



or to buy a half dozen country estates in Bohemia. However, money at the court of Rudolf II had an unearthly quality. The emperor promised whatever he wanted; collecting it from the imperial treasury was another matter. Kepler consistently had problems collecting even his 500 florin annual salary.

The terms of Kepler's succession to Tycho's position sowed the seeds for a conflict with Tycho Brahe's heirs that would exercise a significant influence on the form of Kepler's future scientific work. When Tengnagel returned from England the following summer, he discovered that the heirs had hardly received any money from the treasury. Tengnagel was a nobleman, and Tycho's son-in-law by virtue of his marriage to Elizabeth, so he represented the family's interests. First, he thought he could exert some pressure by suing to get the observing logs back until they were paid for. But then it occurred to him that there was money in the *Rudolfine Tables* project. In October 1602, he managed to get responsibility for the *Rudolfine Tables* transferred to himself, at double Kepler's salary. To add insult to injury, he accused Kepler of sloth and saw to it that someone was assigned to check up on what Kepler was doing.

At this point, the emperor had two mathematicians, and he may well have wondered what he was paying Kepler to do. Kepler was compelled to name the books he would compose to justify his continuing employment. It was a fateful moment, for in grasping among his half-finished projects Kepler named what would be two of the most significant books of seventeenth-century science. As he described the situation in a letter to a friend,

> . . . because I have had my diligence called into doubt, I have assumed the obligation for two works. The one to be ready for Easter 1603 will be Commentaries on the Theory of Mars (or whatever else the name might be), or The Key to a Universal Astronomy . . . The other, to be completed within 8 weeks, will be the Astronomiae pars optica [The Optical Part of Astronomy].

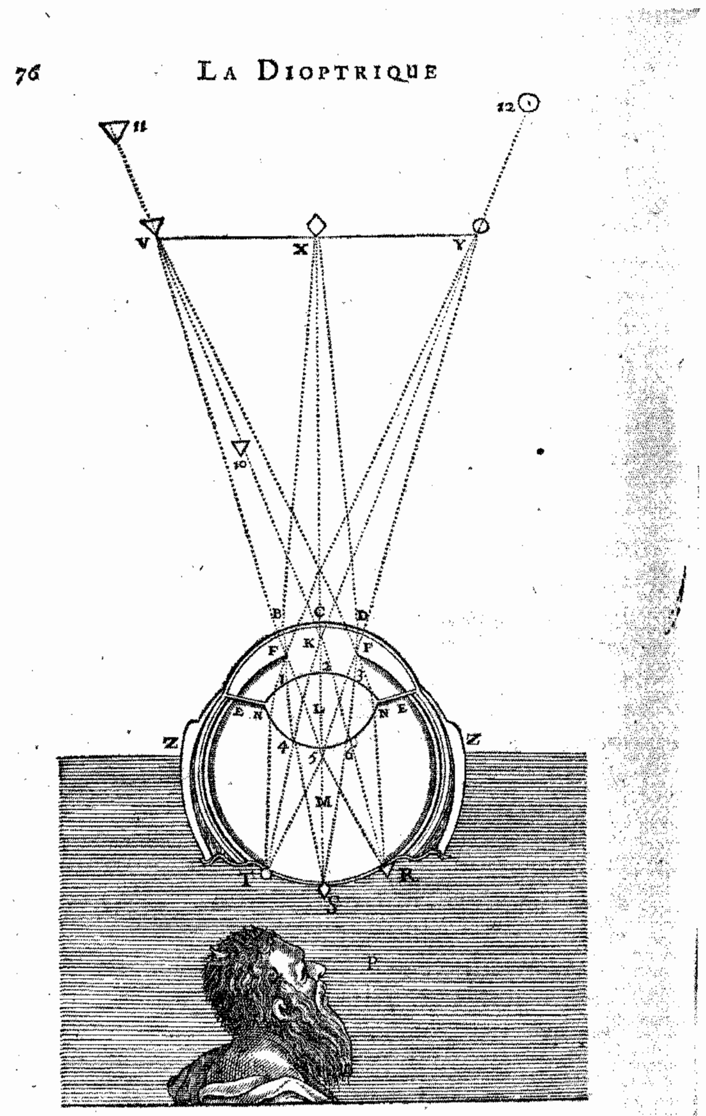
The *Astronomiae pars optica*, whose full title ended up being *Ad Vitellionem paralipomena, quibus astronomiae pars optica traditur* (Supplements to Witelo, in which the Astronomical Part of Astronomy is Treated), had its origin in an essay Kepler had composed in the summer of 1600 on the formation of pinhole images. Earlier that year, Tycho had told him about his observations of a partial solar eclipse, in which the moon passed in front of the sun without completely covering it. Tycho had observed the event without looking directly at the sun by allowing sunlight to fall through a pinhole onto a white screen, where it formed an image of the eclipsed sun. Using his measurements, Tycho concluded that total solar eclipses, in which the moon would completely cover the sun, were impossible. Kepler was skeptical regarding this claim, for there were ample accounts of total solar eclipses in the historic record. After observing a partial solar eclipse for himself from Graz on July 10, 1600, Kepler had carefully analyzed the formation of pinhole images and come to the correct conclusion that their accuracy depended on the size of the aperture. This finding explained Tycho's incorrect conclusion about the possibility of solar eclipses; his pinhole image of the sun was distorted by the size of the aperture and slightly too large, which had led Tycho to believe the moon could never totally cover it. Kepler's essay was a nice little supplement, as he put it, to Witelo's *Optics*, the standard 13th-century treatise on optical theory.

Kepler's essay had clear implications for astronomical observations, and two years later, he grasped for it as something that he could easily get ready for publication within a few weeks. But here, Kepler's legendary inability to focus on one problem at a time got the better of him. First, he wanted to add the other elements of optics that were relevant to astronomy, such as atmospheric refraction. And then he got the work tangled up with a comprehensive treatise on eclipses and the sizes and distances of the sun and moon that he had also been working on. Ultimately, he decided that he could not really write on astronomical observation without taking into account the function of the human eye.

He managed to disentangle the treatise on the sizes and distances of the sun and moon and put it aside, but the material on the function of the eye was a great success. In addition to pinhole images, he was able to publish the first correct account of vision and the eye. For centuries, how we see, a complicated question involving the nature of light, geometrical optics, and the anatomy of the eye, had been a topic of investigation by natural philosophers and optical theorists. By rigorously building on his other optical analyses, Kepler realized that, rather than rays of light being "captured" somehow in the fluid of the eyeball, the lens in the eye *projects* the image of the outside world onto the surface of the retina. Kepler's optical princles dictated that such an image is formed upside down and backwards on the retina; how that image is taken into the mind and rectified to be right side up and frontwards, he could not say. With the help of his new account of vision, he was able to add to this a precise description of the function of different eyeglasses in the correction of nearsightedness and farsightedness. Finally, in the introductory chapter on the nature of light, he was able to deduce the correct relationship for the intensity of light as a function of its distance from the source. Reasoning that light spreads out from a point in a sphere, he concluded that its intensity should be proportional to the area of that sphere, or the intensity should be inversely proportional to the square of the distance.

Some of the problems Kepler attacked did not yield, such as a precise account of the theory of refraction, the bending of light rays as they pass from one medium to another. Nevertheless, starting from an analysis of a limited problem in Witelo's *Optics*, he ended up with such a thorough reworking of optical theory that his *The Optical Part of Astronomy* became the foundation work of seventeenth-century optical theory. It was not a bad performance for some-

Kepler's theory of retinal vision as depicted in René Descartes's Dioptrique. The triangle (V) is projected onto the retina at R and the circle (Y) at T. Thus the image of the outside world is projected upside down and backwards onto the retina.



one who had to "justify his employment."

Kepler had originally promised the *Astronomiae pars optica* in time for Christmas 1602. As it swelled to become a tome of 450 pages, publication was delayed, and he did not finally present the finished manuscript to the emperor until January 1604. It was printed in Frankfurt, and appeared in time for the great Frankfurt book fair in the fall of 1604.

When the book was out of his hands, Kepler turned his attention to the other project he had promised the emperor, a work he referred to as *The Commentaries on Mars*, or *The Key to a Universal Astronomy*. It was an odd genre for an astronomical book; a single planet's motion had never before been the subject of a book-length treatment. This tight focus was in some sense contrived, because Tengnagel had snatched away responsibility for the larger project of coming up with tables for all of the planets. But Kepler realized that it would be a work of great significance, for it was by using observations of Mars that he had discovered that a major change was needed in the theory of the earth's orbit. He recognized the significance of this finding even before committing to the *Commentaries*, writing in a letter, "In short, I have beheld the sun in the theory of Mars as though in a mirror, in that I see how and to what extent it affects all planets. I take from Mars the example for treating the others. And thus I hope presently for all the best for every part of astronomy."

So far, Kepler's work on Mars had vindicated his novel, physical approach to deriving the orbits of the planets. He had had the hunch, based upon his notion of a planet-moving force coming forth from the sun, that the earth's orbit had to be like the other planets, so that it would move faster when it was closer to the sun and slower when it was further away. And he had been right. If he could demonstrate that his "celestial physics" was valid, he would be able to argue that only the Copernican system of the universe made physical sense and that it was true. Since, as he believed, the heliocentric system was a material symbol of God in His creation, establishing its truth continued to have an important religious dimension as well.

Since Tycho's death, Kepler's Mars research had taken a turn toward becoming ever more physical. He had started with the planet-moving force hypothesis put forward in the *Mysterium cosmographicum*, but he had realized that his formulation was flawed. He had then begun using the simple principle that a planet's speed around the sun was inversely proportional to its distance: the closer it is, the faster it goes around. But how could one describe the ensuing motion mathematically? It was a difficult question, because on an eccentric orbit the planet would be slightly changing its distance, and therefore its speed, all the way around. A modern mathematician would use calculus to calculate the effect, but it had not been invented yet.

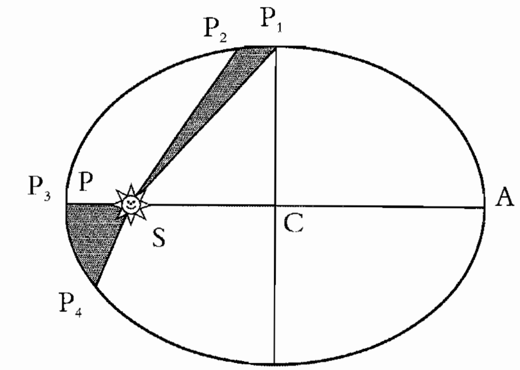
Kepler first took a brute-force approach. He calculated the distance from the sun to Mars at each and every degree around an eccentric circular orbit and used the sum of those distances as a measure of the time it took to get from one place to another. It was a tedious and unsatisfactory experience, but as he was thinking about it, he remembered that Archimedes, the Greek mathematician who lived at the turn of the second century B.C., had used a similar trick with sums of distances to calculate the area of a circle. Surely, the area swept out by Mars as it went around its orbit would be a good measurement of the sum of the distances. In this way, Kepler came to the approximating principle that the area swept out by a planet as it goes around its orbit will be equal in equal time intervals. This later came to be known, through an historical accident, as Kepler's "second law" of planetary motion, even though he came upon it first.

He tried applying his new area law on a circular orbit that was slightly eccentric, so that the sun was located off center along an axis that passed through the center of the circle and defined Mars's closest and furthest positions from the sun. When he did so, he realized that, in his calculated position, Mars was spending too much time along the sides of its orbit away from the axis. It would have to be speeded up there, which meant that the orbit would have to be squeezed in a little along the sides to redistribute that area, or time, into other parts of the orbit. As he put it, it was as though you held a fat-bellied sausage and squeezed it in the

KEPLER'S FIRST TWO LAWS

Kepler's first two laws are:

1. The planets move in elliptical orbits, with the sun at one focus.
2. The line connecting the planet and the sun sweeps out equal areas in equal times.



The two laws are illustrated in the figure above, which shows the elliptical orbit of a planet, with the sun at the focus marked S. The size of an ellipse is usually given in terms of the *semimajor axis*, which is half the long axis PA, or the distance PC. And the eccentricity is defined as the ratio of the distance of the sun from the center to the size of the semimajor axis, or $e = \frac{SC}{PC}$. The ellipse depicted above has an extraordinarily large eccentricity compared to the orbits of the planets, which would look like perfect circles if shown at this scale.

Kepler's second law states the area swept out by the planet as it moves along its orbit from position P_1 to P_2 (the shaded area P_1SP_2) must be the same as the area swept out in an equal time interval as it moves from P_3 to P_4 (area P_3SP_4). As is apparent in the diagram, this means that when a planet's distance to the sun is smaller, it must move correspondingly further around its orbit in the same time interval. Consequently, the planet moves most quickly around perihelion (P) and most slowly around aphelion (A), or as Kepler would have expressed it, it moves most quickly when it is near the source of the planet-moving force and most slowly when it is furthest away.

middle, forcing the meat out into the ends. Thus, by exercising his physical intuition, Kepler came to the conclusion that the orbit must be some kind of oval, rather than a perfect circle. The rest of his effort would be devoted to determining which oval, in conjunction with his area law, produced positions that agreed with Tycho's exquisitely accurate observations.

The task of determining exactly which oval was appropriate and how to generate it was a torturously complex process that took all of 1604. Kepler wrote to Longomontanus that he had tried it twenty different ways. Eventually, he resorted to using an ellipse as an approximation of a likely oval orbit. Ellipses are a subset of ovals that have mathematical properties that make them much easier to use, especially when calculating areas. With this approximating elliptical orbit and the area law, the error was almost precisely opposite what it had been with a circular orbit. The sausage had been squeezed too much. He then concluded that the correct orbit must lie somewhere in between.

This new in-between ellipse had the interesting feature that the sun precisely occupied one of its foci. Indeed, Kepler's interest in this new orbit had originally been spurred by considering exactly how far in from a circle Mars had come at the point one quarter of the way around its orbit. He knew that it should come in about half as far as his previous attempt, and then suddenly realized that, measured from the sun, there was a concise trigonometrical way to describe that distance, and it entailed an elliptical orbit. Moreover, he understood precisely how Mars's distance from the sun changed throughout its orbit, and this particular ellipse resolved a nagging problem about the accuracy of his area law approximation. This flood of considerations came to him at once. It was, he wrote, "as if I were roused from a dream and saw a new light." Thus, he came to his "first law" of planetary motion, that the orbits of the planets are ellipses with the sun at one focus.

The book was not supposed to be merely about a new theory for Mars. It was the debut of a whole new physical approach to astronomical theory, which happened to be based on Kepler's Mars research. Therefore he entitled it *Astronomia nova AITIOLOGHTOΣ, seu physica coelestis, tradita commentariis de motibus stellae Martis*, that is, A New Astronomy Based upon Causes, or Celestial Physics, Treated by Means of Commentaries on the Motions of Mars. He knew it could not *prove* that his physical astronomy or the Copernican system was true. Mathematical astronomers would be all too willing to disregard what he thought would be the most compelling feature of the book, the physical basis of his new astronomical theories. His journey through astronomical theory carries the weight of his argument. "No other approach," he wrote, "would succeed than that founded upon the very physical causes of the motions." In the end, the argument is rhetorical: the fact that Kepler discovered the ellipse and the area law following certain hunches does not logically establish that his reasoning is correct.

Around Easter 1605, Kepler realized that Mars's orbit was an ellipse, but he still had much more to write. Before Kepler could publish he also had to settle with Tengnagel, who had the right to approve any of Kepler's work based on Tycho's observations. The prospect of having Tengnagel meddling in his work was almost more than Kepler could bear, especially since Tengnagel had essentially abandoned his work on the *Rudolfine Tables*. Kepler agreed to let Tengnagel write a preface to his book, in which Tengnagel admonished the reader "not to be swayed by anything of Kepler's, especially his liberty in disagreeing with Brahe in physical arguments."

Everything went slowly, and it was not until 1609 that the *Astronomia nova* finally appeared. The emperor had reserved the right to distribute every copy of his personal mathematician's work, but in the end Kepler had to turn the entire edition over to the printer to cover unpaid costs. It was not a very auspicious launching for what would turn out to be one of the most important astronomical works in history.

The *Astronomia nova* was a tall, handsome—if slightly austere—volume of some 340 pages. It is considered to be Kepler's masterpiece. It is a work of great mathematical genius and breathtaking inventiveness. His contentious point that knowledge of the planet's motions can only be determined by consideration of the physical cause of those motions eventually came to be recognized as true. It is interesting, however, that, though he showed that astronomy should be physical, his particular physics was ultimately discarded. In the generations after his death, it came to be recognized that there is no force coming from the sun that pushes the planets around. The celestial mechanics that Isaac Newton developed is entirely different. The planets' tendency is to continue moving in straight lines, and the gravitational attraction of the sun pulling them in constrains them to move around it. But even with this different physics, Kepler's first two laws necessarily follow: the orbits of the planets are elliptical with the sun at one focus and the area swept out by a planet is equal in equal times.

The great work of the *Astronomia nova* now completed, Kepler took a break from his studies, and his mind turned to Galileo. How would the Italian, who also sought physical proofs of Copernicus's system, react to Kepler's painstaking presentation of his physical astronomy? Little did he know that Galileo was not studying the *Astronomia nova* but making the astronomical discoveries that would make him the talk of Europe and secure his reputation for all time.

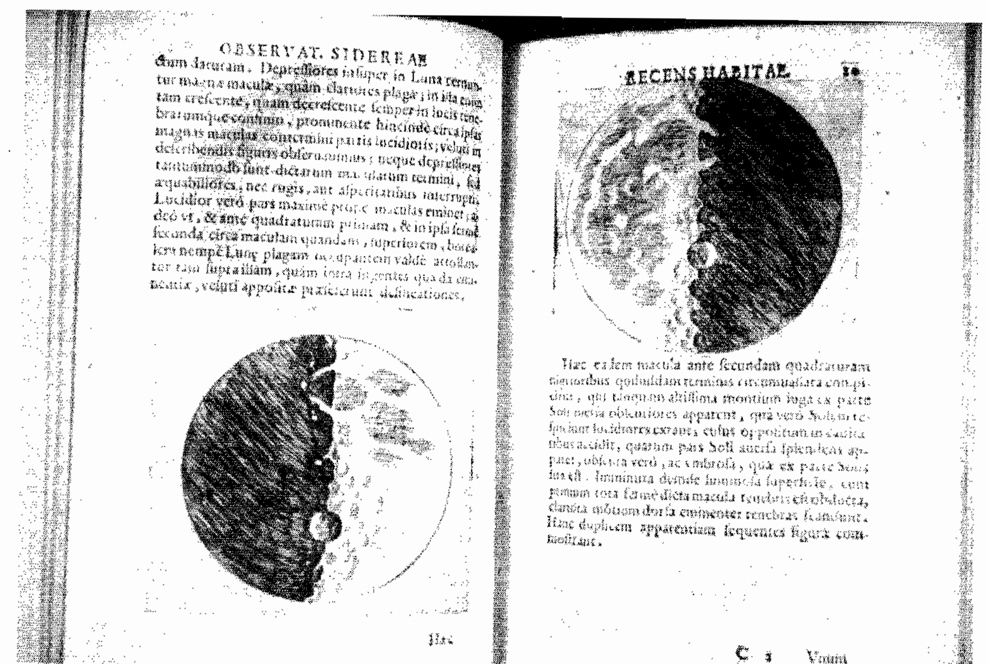
On March 15, 1610, the startling news came to Prague that Galileo had discovered four new planets. Kepler's friend the imperial councilor Johann Matthäus Wackher von Wackenfels was so excited by the report that he stopped his carriage at Kepler's house and called him down to the street to tell him. The two were so overcome that they could scarcely talk. They babbled and laughed in excitement at the

news. Kepler was excited but ashamed and confused as well. What did the discovery mean for his polyhedral hypothesis? He had already determined the necessary number of planets, and there was room for no more.

Galileo's book had not even left the press when the news first flew to Prague. The first copy of his *Sidereus nuncius* (The Starry Messenger) (1610) to reach the city belonged to the curious emperor, who lent it to his mathematician for his opinion. Kepler was immediately relieved. The new planets were previously-unknown satellites of Jupiter, discovered by Galileo using the newly invented telescope. In addition to Jupiter's moons, Galileo demonstrated definitively that the surface of the moon was rough and earthlike. He also turned his telescope to the stars, revealing thousands that had previously remained unseen. The Milky Way he resolved into a myriad of stars whose faint light combined into the nebulous streak across the sky.

The advent of telescopic observation would open a new era for astronomy. In the meantime, Galileo's announce-

These images from Galileo's Sidereus nuncius *depict the surface of the moon as viewed through the telescope. Galileo's observations showed that the moon was mountainous and earthlike.*



ment was so radical that many could scarcely believe it. Because Kepler was imperial mathematician, his opinion would carry some weight and lend Galileo important credibility. Galileo sent a copy of his book along with a letter asking for Kepler's judgment to the Tuscan ambassador in Prague, who had the book delivered to Kepler. On April 13, Kepler visited the ambassador's residence, where Galileo's request was read to him. An official courier was returning to Tuscany within a week, and Kepler promised his reply would be ready for the return trip. He finished his letter to Galileo on April 19.

So many other people were anxious to know what Kepler had said that he had the letter printed as a small 35-page book with the title *Dissertatio cum nunceo sidereo* (A Conversation with the Starry Messenger) (1610). It was an unusual work. Kepler did not have a telescope, so he could not confirm the observations. (Try as he might he could not get Galileo to send him one and eventually had to borrow one to see the new phenomena for himself.) In the meantime, the most Kepler could do to lend Galileo support was establish the plausibility of what Galileo had reported, beginning with the telescope itself. In some ways the principle of magnifying images using a combination of lenses had been alluded to in previous optical theory. But it was something Kepler had missed in his *Astronomia pars optica*. Five months later, he had cracked the problem and the next year Kepler published the first detailed optical theory of two lens systems in his *Dioptrice* (1611), including a superior telescope design using two convex lenses, now called the "astronomical" or "Keplerian" telescope.

Otherwise, Kepler could only respond enthusiastically to Galileo's discoveries and speculate about their meaning. With regard to Galileo's account of lunar geography, Kepler admitted that he was totally convinced by Galileo's observations and analysis of mountains and craters on the moon, and he speculated that the cratered appearance was due perhaps to the moon being light and porous (which according to Kepler's physical astronomy would explain its rapid revolution around the earth). Or perhaps the craters were great circular ramparts built by lunar inhabitants, in whose shade they could shelter during the inhospitable 14 days of continuous sunlight on the moon's surface.

Jupiter's moons were by far the most spectacular of Galileo's discoveries. For Kepler, they were significant because they had implications in favor of heliocentrism. First, the fact that Jupiter also had moons seemed to remove the objection that the earth could not travel around the sun without losing its moon. Also, the fact that the moons revolved in the plane of Jupiter's rotation implied that the moons were being swept around by a planet-moving force coming from Jupiter, just as Kepler had suggested in the *Astronomia nova* that the moon is moved by the earth's rotation. Finally, Jupiter's moons suggested to Kepler that Jupiter must be inhabited by intelligent beings. Why else would God have endowed Jupiter with this feature we cannot see?

With the publication of the *Dissertatio,* Kepler became the first astronomer to come out publicly in favor of Galileo and his discoveries. The support of the imperial mathematician helped to subdue the sniping Galileo faced from his critics. Yet, in return, Kepler received from Galileo scarcely a word of thanks and no acknowledgment of Kepler's more substantial achievements in astronomy. Though Kepler tried a few more times to engage the Italian in correspondence, apart from one inconsequential note some 17 years later, he never heard from Galileo again. Though two of the greatest astronomers in history lived at the same time, and even communicated, there was barely a connection between them. In his unassuming way, Kepler never complained of Galileo's offensive disregard. And Galileo apparently took little notice of Kepler's reform of astronomical theory.

It was now 1611; Kepler was 39 years old. In the 11 years since coming to Prague, Johannes Kepler had grown from an insecure refugee into a leading figure in the imperial capital's learned circles and a man of international scientific reputation. His status as scientific heir to Tycho Brahe and the flow of important works that issued from his pen lent him an air of astronomical omniscience that the English poet John Donne described in his satire *Ignatius his Conclave* (1611), where he wrote that "ever since Tycho Brahe's death [Kepler] hath received it into his care, that no new thing should be done in heaven without his knowledge."

The same period that saw Kepler's rise to fame saw the decline of his eccentric patron into madness. Observing the emperor's governance from afar before moving to Prague, Kepler had marveled at the emperor's "Archimedian manner," a kind of dynamic immobility, as he saw it, in which the emperor nonetheless managed to maintain a long, stalemated war against the Ottoman Turks and, at the same time, kept the empire's fractious states from disintegrating. But since Kepler's arrival in Prague in 1600, the emperor's pathological shyness and extreme stubbornness had given way to isolation, paralyzing indecision, and paranoia. Turning away from the world, he became a recluse, shut in among his precious collections in Hradschin, a virtual prisoner in his own castle. It was widely reported that his erratic mental state had deteriorated into insanity.

As Rudolf's intransigence and inactivity began to imperil the house of Hapsburg and the empire, a conspiracy was hatched against him. At a secret meeting of the Austrian Hapsburgs in Vienna in April 1606, the family agreed to recognize Rudolf's estranged, ambitious younger brother Matthias as head of the family. Two years later, Matthias moved against his brother under force of arms, leading an army of 20,000 men from Vienna, through Moravia, into Bohemia, and to within a day's march of Prague. Facing certain defeat, the emperor capitulated. He ceded Matthias the kingdom of Hungary and archduchies of Austria and Moravia effective immediately, retaining only Bohemia,

Silesia, and Lusatia to himself, though Rudolf had to ensure Matthias's succession as king of Bohemia after his death.

The weakened emperor now faced pressure from the powerful Protestant representatives of the Bohemian Estates, the representative assembly in Bohemia, who exacted from him a Letter of Majesty (1609) guaranteeing freedom of religion. Chaffing under the concessions they had extorted and descending into insanity, Rudolf made a desperate bid to regain control of his country and its capital. The following winter, Rudolf inexplicably invited his cousin, Archduke Leopold V, bishop of Passua, to invade Bohemia. Leopold's army pillaged its way through Bohemia to Prague, invading and looting the Hradcany and the Lesser Town.

A spirited defense by Protestant troops (who for their part also looted Catholic churches and monasteries in the Old Town) and a large bribe put an end to Leopold's attack, but Rudolf was finished. In the midst of the crisis, the Protestant representatives sided with Matthias. Rudolf was deposed, and on May 23, 1611, Matthias was crowned king of Bohemia. Deranged and powerless, the emperor lived out his days in the Hradschin, where he died within a year on January 20, 1612.

Kepler stayed loyal to his patron to the end. Though he was consulted for advice, he did his best to keep astrology out of the gullible emperor's troubled mind. And when the emperor's enemies approached Kepler, he spun the astrological analysis in the emperor's favor, predicting long life for him and trouble for Matthias. Still, Kepler could see that the situation was deteriorating. He took care to have backup plans ready and found a promising situation in the Upper Austrian capital of Linz. After the emperor's death, nothing held Kepler in Prague, and in the middle of April he left the city for Linz.