spheres inscribed in the figures to the spheres circumscribing them, and what is calculated is the harmonic proportions which come close to them. For only in the tetrahedron is the diameter of the inscribed sphere expressible, that is as one third of the circumscribed sphere; but in the cubic marriage the proportion, which is unique to that case, is similar to lines which are expressible only in square. For the diameter of the inscribed sphere is to the diameter of the circumscribing sphere in the semitriple proportion.[29] And if you compare the actual proportions with each other, the proportion of the tetrahedric spheres is the square of the proportion of the cubic spheres. In the dodecahedric marriage, the proportion of the spheres is again unique, but inexpressible, a little greater than 4:5. Therefore, the harmonic proportions which are close to the proportion of the cubic and octahedric spheres are the following: 1:2 as the next greatest, and 3:5 as the next smallest; while the harmonies which are close to the proportion of the dodecahedric spheres are 4:5 and 5:6, the next smallest, and 3:4 and 5:8, the next greatest.

But if for particular reasons 1:2 and 1:3 are to be appropriated to the cube, as the proportion of the cube's spheres to the proportion of the tetrahedron's spheres, so the harmonies 1:2 and 1:3, which are allotted to the cube, will be to 1:4 and 1:9 which have to be allotted to the tetrahedron, if indeed it is right to use this analogy; for these proportions are the squares of the harmonies mentioned. And because 1:9 is not harmonic, its place will be taken by the nearest harmonic, 1:8, for the tetrahedron. But to the dodecahedric marriage, using this analogy, will belong approximately 4:5 and 3:4. For just as the proportion of the cubic spheres is approximately the cube of that of the dodecahedric, so also the cubic harmonies 1:2 and 1:3 are approximately the cubes of the harmonies 4:5 and 3:4. For 4:5 cubed is 64:125; and 1:2 is 64:128. Similarly 3:4 cubed is 27:64, and 1:3 is 27:81.

(Saturn and Jupiter) and the octahedric planets (Venus and Mercury) are less exactly like their figures, because the extreme intervals recede from the ratio of the figures, though the mean intervals are in agreement. Finally, the ratios of the radii of the circumscribed and inscribed spheres of the dodecahedron and the icosahedron are inexpressible. Correspondingly, the intervals of the dodecahedric planets (Mars and Earth) and the icosahedric planets (Earth and Venus) abandon the ratios of their figures, though they do not approach closer to those of any other figure.

[29] That is, the ratio $1:\sqrt{3}$.

CHAPTER III.

Summary of Astronomical Theory, Necessary for the Study of the Heavenly Harmonies.

To start with, readers should know that the ancient astronomical hypotheses of Ptolemy, in the way in which they have been expounded in the *Theoricae* of Peurbach[30] and in the other writers of Epitomes,[31] have been totally excluded from this discussion, and put out of mind; for they do not convey truthfully either the arrangement of the bodies in the world or the commonwealth of the motions.

In their place I cannot do other than substitute solely the opinion of Copernicus on the world, and, if it were possible, persuade everyone to believe it. Yet it is still a new idea to the common herd of scholars, and a doctrine which to many is quite absurd to hear, that the Earth is one of the planets, and is carried among the stars round an unmoving Sun. Therefore, those who are shocked at the novelty of this opinion should know that these speculations about harmonies also find a place in the hypotheses of Tycho Brahe, because that author has everything else which relates to the arrangements of the bodies and the combination of their motions in common with Copernicus.[32] It is only the Copernican annual motion of the Earth which he transfers to the whole system of the planetary spheres, and to the Sun, which occupies the middle by the agreement of both authors. For by this transference of the motion it nevertheless comes about that the Earth,

The hypotheses of Copernicus.

Hypotheses of Tycho Brahe.

Comparison of the two.

[30] G. Peurbach, *Theoricae novae planetarum*. See the English translation by E.J. Aiton (1987).

[31] The best known is *Epytoma Joannis de Monte Regio in Almagestum Ptolomei*; reprinted in F. Schmeidler (1972), 55–274.

[32] Both here and on the title page of Book V Kepler emphasizes that the theory he describes is equally applicable to the systems of Copernicus and Tycho Brahe, which he regarded as kinematically equivalent. In 1616, Copernicus' *De revolutionibus* had been suspended pending corrections by the Holy Congregation of the Index and on 10 May 1619, the first part of Kepler's *Epitome astronomiae Copernicanae*, published in 1618, was also placed on the Index. Johannes Remus Quietanus, the Imperial physician-in-ordinary, who communicated this news to him, remarking that Galileo would like to have a copy (KGW 17, p. 362), and also the Venetian Vincenzo Bianchi (KGW 17, p. 319), assured him that his books could still be read in Italy by scholars and astronomers, and that in fact the prohibition would increase their influence. Kepler himself addressed an open letter to the foreign booksellers, especially those in Italy, asking that the censors should examine the new evidence he had produced in favor of Copernicus, unfortunately too late to prevent the prohibition. In the meantime he requested the booksellers to restrict the sales of his *Harmonice mundi* to the highest clergy and most eminent mathematicians and philosophers (KGW 6, pp. 543–544).

if not in that vast and immense space within the sphere of the fixed stars, yet at least in the system of the planetary world, takes the same position at any given time according to Brahe as Copernicus gives it. In fact, just as someone who draws a circle on paper moves the writing leg of his compasses round, whereas someone who fastens his paper or tablet to a revolving wheel describes the same circle, without moving the leg of his compasses or his pen, on the tablet as it moves round; in the same way in this case for Copernicus indeed the Earth traces out a circle by a real motion of its own body, passing in between the circles of Mars on the outside and Venus on the inside; but for Tycho Brahe the whole planetary system (in which among the others are also the circles of Mars and Venus) turns, like the tablet on the wheel, applying to the motionless Earth, as if to the pen of the man who turns the wheel, the blank space between the circles of Mars and Venus.[33] The effect of this motion of the system is that the Earth marks on it the same circle round the Sun, intermediate between those of Mars and Venus, while itself it remains motionless, as according to Copernicus it marks out by a true motion of its own body, with the system at rest. Therefore, as harmonic study considers the motions of the planets as eccentric, as if viewed from the Sun,[34] it may readily be understood that if an observer were on the Sun, even though it were in motion, to him the Earth, although it were at rest (to make a concession already to Brahe), would nevertheless appear to be going around an annual circuit, placed in between the planets, and also in an intermediate period of time. Hence if there is a man whose confidence is too weak for him to be able to accept the motion of the Earth among the stars, nevertheless he will be able to rejoice in the marvelous study of this absolutely divine mechanism, if he applies whatever he is told about the daily motions of the Earth on its eccentric to their appearance from the Sun, the same appearance as Tycho Brahe shows, with the Earth at rest.

Harmonic study valid in the hypotheses of Brahe.

However, true enthusiasts for the Samian philosophy[35] have no just cause to grudge such people this share in a most delightful speculation,

Much more valid in the Copernican ones.

[33] The analogy is not exact, for in the case of the rotating wheel both Earth and Sun would be at rest, whereas the Sun should be allowed to circulate about the Earth.

[34] Kepler seeks the celestial harmonies in the motions of the planets as seen from the sun, because he believes these to be the true motions; that is, the actual motions free from the distortion of combination with the earth's motion. The same harmonies would be found in the Tychonic system, because the motions as seen from the sun were in fact the true motions, even if followers of Tycho would not have regarded them as such. Ptolemy, however, had only the apparent motions as seen from the earth, and for this reason it was impossible for him to have discovered the celestial harmonies. In order to avoid confusion, it should be noted that Kepler also uses the expression "true motion" to refer to the angular motion as seen from the center of the eccentric. The harmonies are based on the appearance of this true motion as seen from the sun, that is, the apparent angular velocity with respect to the sun.

[35] That is, the heliocentricism of Aristarchus of Samos, to whom the first statement of the Copernican hypothesis has been attributed. See T. Heath (1981).

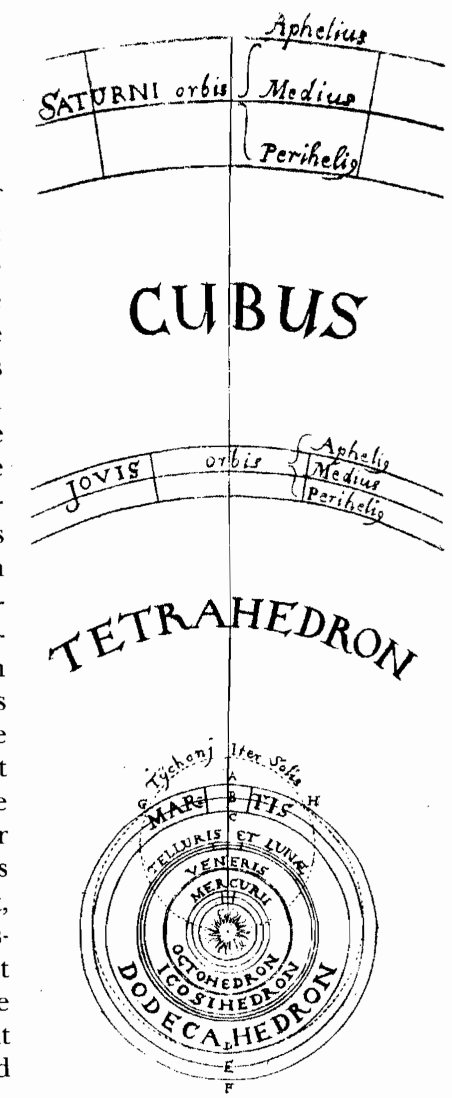
inasmuch as their joy will be many times more perfect, that is from the complete perfection of the speculation, if they do also accept the immobility of the Sun and furthermore the movement of the Earth.

First, therefore, readers should take it as absolutely settled today among all astronomers that all the planets go round the Sun, with the exception of the moon, which alone has the Earth as its center; and its orbit or course is not large enough to be capable of being drawn in its correct proportion to the rest in the following plan. Therefore the Earth is added to the other five as a sixth, which either by its own motion, with the Sun at rest, or without moving itself while the whole system of the planets revolves, itself also marks out a sixth circle round the Sun.

I. The bodies of which the circuits establish the harmonies.

Second, it is also settled that all the planets are eccentric, that is, they change their distances from the Sun, in such a way that on one side their orbits are furthest from the Sun, in the other they come closest to the Sun. In the attached diagram three circles have been constructed for each of the planets. None of them indicates the actual eccentric path of the planet, but the middle one in fact, for instance *BE* in the case of Mars, is equivalent to the eccentric orbit, with respect to its longer diameter; but the actual orbit, for instance *AD*, touches the upper of the three, *AF*, on one side *A*, and the lower *CD* on the opposite side *D*. The circle *GH* which is sketched out by dots, and drawn through the center of the Sun, indicates the path of the Sun according to Tycho Brahe. If it moves on this path absolutely all the points in the whole planetary system here depicted proceed on an equivalent path, each on its own. And if one point on it, that is to say the center of the Sun, stops in one part of its circle, as here at the lowest point, then absolutely all points of the system will stop, each at the lowest parts of their own circles. Also the three circles of Venus on account of the restricted space have merged into one, contrary to my intention.

II. The variation in the separation of these bodies.



Orbit of SATURN at { *aphelian* / *mean* / *perihelian* }

CUBE

Orbit of JUPITER at { *aphelian* / *mean* / *perihelian* }

TETRAHEDRON

Tychonic path of Sun
A
G *B* *H*
of MARS *C*
of Earth and Moon
of VENUS
of MERCURY

OCTOHEDRON
ICOSIHEDRON
DODECAHEDRON
D
E
F

III. The reason for the number of the bodies which make the harmonies.

Third, the reader should remember what I published in *The Secret of the Universe*, 22 years ago, that the number of the planets, or of courses round the Sun, was taken by the most wise Creator from the five regular solid figures, about which Euclid so many centuries ago wrote a book which is called the *Elements*, on account of its being made up of a series of Propositions. However, the fact that there cannot be more regular solids, that is, that regular plane figures cannot be congruent in a solid in more than five ways, has been made clear in Book II of this work.

IV. The reason for the size of the spheres which pass between the solids.

Fourth, as far as the proportion of the planetary orbits is concerned, between pairs of neighboring orbits indeed it is always such as to make it readily apparent that in each case the proportion is close to the unique proportion of the spheres of one of the solid figures, that is to say the proportion of the circumscribed sphere of the figures to the inscribed sphere. However, it is not definitely equal, as I once dared to promise for eventually perfected astronomy. For after the final verification of the intervals, from the observations of Brahe, I discovered the following facts: if the vertices of the cube are indeed applied to the inside circle of Saturn, the centers of the faces almost touch the middle circle of Jupiter; and if the vertices of the tetrahedron rest on the inside circle of Jupiter, the centers of the faces of the tetrahedron almost touch the outside circle of Mars.[36] In the same way the vertices of the octahedron, which rise from any of the circles of Venus (as they are all three compressed into a very narrow gap) are penetrated by the centers of the faces of the octahedron, which go down more deeply below the outside circle of Mercury, yet do not reach as far as the middle circle of Mercury. Finally, the closest of all to the proportions of the dodecahedric and icosahedric spheres, which are equal to each other, are the proportions or intervals between the circles of Mars and the Earth, and between those of the earth and Venus, which are similarly equal to each other, if we reckon from the inside circle of Mars to the middle circle of the Earth, but from the middle circle of the Earth to the middle circle of Venus. For the mean distance of the Earth is the mean proportional between the smallest circle of Mars and the middle one of Venus. However, these two proportions between the circles of the planets are still greater than are those of the pairs of circles in the figures belonging to the spheres, to the extent that the centers of the faces of the dodecahedron do not touch the outer circle of the Earth, nor the centers of the faces of the icosahedron the outer circle of Venus. Yet this gap is not filled up by

[36] For a perfect fit, the circumscribed sphere should correspond to the perigee of the outer planet and the inscribed sphere to the apogee of the inner planet. Thus the tetrahedron fits perfectly in the space between Jupiter and Mars. The spouses cube and octahedron both penetrate their planetary spheres. On the other hand, the spouses dodecahedron and icosahedron leave a gap. Kepler's attempt to rationalize the discrepancies is described in note 28.

adding the semidiameter of the moon's orbit above the greatest interval of the Earth and subtracting it from below the smallest interval. However, I discover another proportion in the figures: namely that if the augmented dodecahedron to which I have given the name of Echinus (Hedgehog), that is to say the one formed from twelve quinquagonal stars, and therefore very close to the five regular solids, if, I say, it places its twelve points on the inner circle of Mars, then the sides of the pentagons which are individually the bases of the rays or points touch the middle circle of Venus.[37] In brief, the cube and octahedron which are spouses do penetrate their planetary spheres somewhat; the dodecahedron and icosahedron which are spouses do not altogether follow theirs, whereas the tetrahedron exactly touches both. In the first case there is a deficiency, in the second an excess, and in the last an equality in the intervals of the planets. Page 396.

From that fact it is evident that the actual proportions of the planetary distances from the Sun have not been taken from the regular figures alone; for the Creator, the actual fount of geometry, who, as Plato wrote, practices eternal geometry, does not stray from his own archetype.[38] And that could certainly be inferred from the very fact that all the planets change their intervals over definite periods of time, in such a way that each one of them has two distinctive distances from the Sun, its greatest and its least; and comparison of distances from the Sun between pairs of planets is possible in four ways, either of the greatest distances, or of the least, or of the distances on opposite sides when they are furthest from each other, or when they are closest. Thus the comparisons between pair and pair of neighboring planets are twenty in number, whereas on the other hand there are only five solid figures. However, it is fitting that the Creator, if He paid attention to the proportion of the orbits in general, also paid attention to the proportion between the varying distances of the individual orbits in particular, and that that attention should be the same in each case, and that one should be linked with another. On careful consideration, we shall plainly reach the following conclusion, that for establishing both the diameters and the eccentricities of the orbits in conjunction, more basic principles are needed in addition to the five regular solids.

V. Astronomical axioms of the true motions.

Fifth, to come to the motions, between which harmonies are established, I again impress on the reader that it was shown by me in my *Commentaries on Mars*, from the thoroughly reliable observations

[37] The echinus fills the gap between Mars and Venus. This derivative solid thus provides an explanation of the actual magnitude of the discrepancy between the planetary distances and the interpolation of the dodecahedron and icosahedron. Although the requirements of the celestial harmony necessitated an adjustment of the distances as indicated by the interpolation of these polyhedra, the geometrical archetype nevertheless included an element from which the required deviations could be derived *a priori.*

[38] The expression is not actually to be found in Plato's writings. See Plutarch, *Convivia*, viii, 2.

of Brahe, that equal daily arcs on one and the same eccentric are not traversed at the same speed;

1. but that these differing *times expended on equal parts of the eccentric observe the proportion of their own distances from the Sun*, the fount of motion;[39] and in turn, that supposing equal times, say one natural day in each case,
2. *the true daily arcs of a single eccentric orbit* corresponding with them *have a proportion to each other which is the inverse of the proportion of the two distances from the Sun.*
3. At the same time, however, it was shown by me that *the orbit of a planet is elliptical,*
4. *and the Sun*, the fount of motion, *is at one of the focuses of that ellipse*;
5. and thus it comes about that the planet, when it has completed out of its whole circuit *a quadrant from its aphelion, is at precisely its mean distance from the Sun*, between its greatest at aphelion and its least at perihelion.
6. From these two axioms the conclusion is drawn that the daily *mean motion* of the planet on its eccentric *is the same as the true* daily arc of its eccentric, at those moments at which the planet is *at the end of the quadrant of its eccentric as reckoned from the aphelion*, even though that true quadrant still appears smaller than a proper quadrant.
7. Furthermore, it follows that *any two really true daily arcs of the eccentric, which are truly at equal distances, one from the aphelion and the other from the perihelion, added together are equal to two mean daily arcs*;[40]
8. and in consequence, since the proportion of the circles is the same as that of their diameters, that *the proportion of one mean daily arc to the sum of all the mean daily arcs* making up the whole circuit, which are equal to each other, *is the same as that of a mean daily arc to the sum of all the true* eccentric arcs, which are the same in number but unequal to each other. We need to have this knowledge of the true eccentric daily arcs, and of the true motions, beforehand so that we can now grasp through them the apparent motions, supposing the eye to be at the Sun.

[39] Kepler here alludes to his distance law, based on his physical explanation of planetary motion, according to which the velocity of a planet in its orbit is inversely proportional to its distance from the sun. On the relation of Kepler's distance and area laws, see E.J. Aiton (1969).

[40] This is only approximately true but for small eccentricities the error is insignificant. According to the distance law, the sum of the two arcs s_1 and s_2 (with corresponding distances r_1 and r_2) is $(2r^2s)/(r_1r_2)$, where s and r are the mean arc and distance respectively. The sum of the two arcs would be $2s$ if r were taken to be the geometric mean of r_1 and r_2 instead of the arithmetic mean.

VI. Astronomical axioms of the apparent motions or angles as seen from the Sun, which are included daily between lines originating from the center of the Sun.

Sixth, as far indeed as concerns the apparent arcs as seen from the Sun, it has been known even from the time of ancient astronomy that of the true motions, even those which are equal to each other, one which has moved further away from the center of the world (such as one which is at *aphelion*) *seems to be smaller*, to an observer at that center; and one which is nearer, such as *one which is at perihelion, also seems to be greater.* Since therefore in addition the true daily arcs are also greater when close, on account of their motion's being faster, but lesser when at a distance at aphelion on account of the slowness of the motion,

2. hence I have shown in the *Commentaries on Mars* that *the proportion of the apparent daily arcs on a given eccentric is fairly precisely the square of the inverse proportion of their distances from the Sun.*[41]
   Thus if a planet on a particular one of its days, when it is at aphelion, were at a distance of 10 parts from the Sun, in any units, and on the opposite day, when it is at perihelion, at a distance of 9 parts, in similar units, it is certain that at aphelion its apparent forward motion as seen from the Sun will be to its apparent motion at perihelion as 81 to 100.
3. However, that is true with the following reservations; first, *that the eccentric arcs are not large*, so that they do not participate in different distances which vary considerably, that is so that they do not produce an appreciable difference in the distance of their ends from the apsides;
4. second, *that the eccentricity is not very large*, for the larger the eccentricity, that is to say the larger the arc is, the more the angle which it appears to subtend is increased, beyond the bound set by its closeness to the Sun, *by Theorem 8 of Euclid's Optics.*[42]
5. However, it *is of no importance in small arcs and at a great distance*, as I have commented in my *Optics*, Chapter XI. But there is another reason for me to comment on this point.
6. For *arcs of the eccentric near the mean anomalies are viewed obliquely from the center of the Sun*, and this obliquity diminishes the apparent size,
7. whereas on the contrary *arcs near the apsides present themselves*

[41] Let the daily motions (that is, the apparent arcs) at aphelion and in the mean distances be M_a and M. Then, if S_a and S are the corresponding true arcs, and R_a and R the corresponding distances, $S_a/S = (M_a/M)(R_a/R)$. Owing to the weakening of the solar force, $S_a/S = R/R_a$. Hence $M_a/M = R^2/R_a^2$. Similarly $M/M_p = R_p^2/R^2$, where M_p and R_p are the daily motion and distance respectively at perihelion. Combining the two results, it follows that $M_a/M_p = R_p^2/R_a^2$. See *Astronomia nova*, Chapter 32.

[42] *Euclides optica et catoptrica . . . eadem Latine reddita per I. Penam* (Paris, 1557). Cf. the French translation by Paul ver Eecke, *Euclide, L'Optique et la catoptrique* (Paris/Bruges, 1938), pp. 6–7.

normally to an observer, so to speak, placed on the Sun.[43] Therefore, when the eccentricity is very large, the sensible damage is done to the proportion of the motions, if we apply the mean diurnal motion undiminished to the average distance, as if it appeared from the average distance to be the size which it actually is, as will be apparent below in the case of Mercury. All this is related at greater length in the *Epitome of Copernican Astronomy*, Book V. Nevertheless it had to be recalled here as well, because it concerns the actual terms of the heavenly harmonies, considered separately on their own.

VII. Rejection of the motions which are apparent to observers on Earth.

Seventh, if anyone should bring to mind the daily motions not as they appear to observers from the Sun, so to speak, but from the Earth, with which Book VI of the *Epitome of Copernican Astronomy* deals, he should know that no account of them whatever is taken in this proceeding, and definitely none should be. For the Earth is not the fount of their motions, and cannot be, for those motions degenerate not only into mere rest or apparent standstill, but into definite retrogression, as far as the deceptive appearance is concerned. On that basis all the infinity of proportions is attributable to all the planets simultaneously and equally. Therefore, to make certain what proportions are established as their own by the daily motions of the true eccentric orbits individually (even though they are themselves still apparent, to an observer, so to speak, on the Sun, the fount of motion), this fantasy, common to all five, of an adventitious annual motion must be removed from those proper motions, whether it arises from the motion of the Earth itself, according to Copernicus, or from the annual motion of the whole system, according to Tycho Brahe; and the motions proper to each planet, stripped of inessentials, must be brought into view.[44]

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[43] This difference played an important role in Kepler's definitive clarification of the area law, when he demonstrated that his original inverse-distance law had to be understood to mean that the trans-radial velocity of the planet was inversely proportional to the distance and not the velocity in the orbit itself. In this formulation, the inverse-distance law is equivalent to the area law. See Aiton (1969), 87–88.

[44] Kepler here reiterates his view that the celestial harmonies are to be sought in the true motions of the planets, which are those that would be seen from the sun. It would be possible to recognize the harmonies in the Tychonic system, because this differs from the Copernican only by the addition of an illusory annual motion of the whole system, which is easily abstracted. From the fact that the celestial harmonies would only be perceived from the sun, it would seem to follow that Kepler regarded them as objects of the mind and not the senses. While the earth-soul could perceive and be influenced by the astrological aspects (the manifestation of harmony in nature), it would seem that only intelligent minds could understand and recognize the celestial harmonies, once they had been brought to light. And as Kepler remarked in his introduction to Book V, God had waited six thousand years for this to happen. However, in Chapter IV, he played with the idea of an instinctive recognition of the celestial harmonies, conveyed in some way which he does not specify, along with rays of light from the Sun. Cf. note 59.

VIII. What is the proportion of the periodic times to the distances from the Sun of any pair of planets?

Eighth, up till now we have dealt with the various elapsed times or arcs of one and the same planet. Now we must also deal with the motions of pairs of planets compared with each other. Here note the definitions of the terms which we are going to need. We shall call the *nearest apsides* of two planets the perihelion of the upper one and the aphelion of the lower one, notwithstanding the fact that they are tending not to the same side of the world, but to different, and perhaps opposite sides.

2. By *extreme motions* understand the slowest and the fastest of the whole planetary circuit.
3. By *converging or approaching motions*, those which are at the nearest apsides of the two, that is at the perihelion of the upper planet and the aphelion of the lower;
4. by *diverging or receding motions*, those which are at opposite apsides, that is at the aphelion of the upper planet and the perihelion of the lower. Again, therefore, a part of my *Secret of the Universe*, put in suspense 22 years ago because it was not yet clear, is to be completed here, and brought in at this point. For when the true distances between the spheres were found, through the observations of Brahe, by continuous toil for a very long time, at last, at last, the genuine proportion of the periodic times to the proportion of the spheres—

   only at long last did she look back at him as he lay motionless,
   But she looked back and after a long time she came;[45]

   and if you want the exact moment in time, it was conceived mentally on the 8th March in this year one thousand six hundred and eighteen, but submitted to calculation in an unlucky way, and therefore rejected as false, and finally returning on the 15th of May and adopting a new line of attack, stormed the darkness of my mind. So strong was the support from the combination of my labor of seventeen years on the observations of Brahe and the present study, which conspired together, that at first I believed I was dreaming, and assuming my conclusion among my basic premises. But it is absolutely certain and exact that *the proportion between the periodic times of any two planets is precisely the sesquialterate proportion of their mean distances, that is, of the actual spheres*,[46] though with this in mind, that *the arithmetic mean*

For in the *Commentaries on Mars*, Ch. XLVIII, p. 232, I have proved that *this arithmetic mean is either the actual diameter of the circle which is equal in length to the elliptical orbit or slightly smaller.*

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[45] Vergil, *Eclogue* I, 27 and 29.

[46] In the *Mysterium cosmographicum*, physical considerations led Kepler to a relation between the periodic times and the mean distances expressed by the formula $(T_2 - T_1)/T_1 = 2(r_2 - r_1)/r_1$, where T_1, T_2 represent the periodic times and r_1, r_2 the mean distances and r_2 is greater than r_1. See Duncan (1981), pp. 201 and 249. In the *Astronomia nova* (Chapter 39), this was modified to $T_1 : T_2 = r_1^2 : r_2^2$. As he here relates, he discovered the correct law, $T_1^2 : T_2^2 = r_1^3 : r_2^3$, on 15 May 1618. An *a priori* (physical) explanation of the law was first given in the third part of the *Epitome astronomiae*

between the two diameters of the elliptical orbit is a little less than the longer diameter. Thus if one takes *one third of the proportion from the period*, for example, of the Earth, which is one year, and the same from *the period* of Saturn, thirty years, that is, *the cube roots*, and one doubles that proportion, *by squaring* the roots, *he has in the resulting numbers the exactly correct proportion of the mean distances* of the Earth and Saturn *from the Sun.* For the cube root of 1 is 1, and the square of that is 1. Also the cube root of 30 is greater than 3, and therefore the square of that is greater than 9. And Saturn at its average distance from the Sun is a little higher than nine times the average distance of the Earth from the Sun. The use of this theorem will be necessary in Chapter IX for the derivation of the eccentricities.

IX. How large a space any planet traverses relatively to another in any given time.

Ninth, if you want to measure the actual completely true daily paths of each planet through the aethereal air, with, so to speak, a ten foot rule, two proportions will have to be combined, one that of the true (not the apparent) daily arcs of the eccentric, the other that of the average distances of each planet from the Sun, because it is the same as that of the width of the orbits. *That is, the true daily arc of each planet must be multiplied by the semidiameter of its orbit.* When that has been done, the resulting figures will be convenient for investigating whether those paths make harmonic proportions.

X. How from the true paths, and the true distances of the planets from the Sun, is found the apparent motion as from the Sun, the subject of celestial harmony.

Tenth, to find definitely the apparent size of any such daily path, to an eye placed, so to speak, on the Sun, although the same thing can immediately be sought through astronomy, yet it will also be revealed if you multiply the proportion of the paths by the inverse proportion not of the mean, but of the true distances as they are at any point of the eccentrics: *by multiplying the path of the upper by the distance of the lower from the Sun, and correspondingly the path of the lower by the distance of the upper from the Sun.*

XI. How from the apparent diurnal motions (seen, so to speak, from the Sun), are elicited the distances of the planets from the Sun.

Eleventh, in the same way also from given apparent motions, at aphelion for one planet and at perihelion for the second, or the other way round, the proportions of the distances are elicited, of one at aphelion to that of the second at perihelion. In this case, however, the mean motions must be known in advance, that is the inverse pro-

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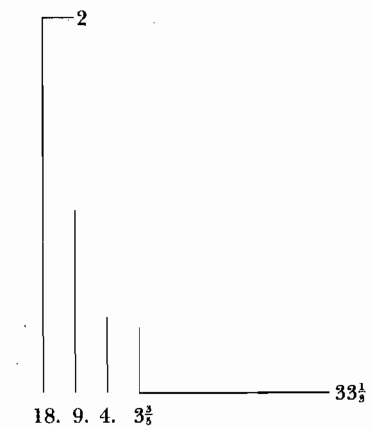
Copernicanae (KGW 7, p. 307), published in 1621. There he explains that the periodic times depend on four factors. Of these, the length of the orbital path (proportional to the mean distance r) and the strength of the solar force (inversely proportional to r) would by themselves combine to give the relation described in the *Astronomia nova*. The other two factors are the mass of the planet, which on the basis of archetypal reasons he took to be proportional to $\sqrt{r}$, and the volume (measuring the ability of the planet to assimilate the solar force), which on the basis of observational evidence he felt he could take proportional to r. In accordance with his Aristotelian dynamics, where speed was proportional to force and inversely proportional to resistance (represented by the mass), the four factors combined to give the third or harmonic law.

portion of the periodic times, from which the proportion of the orbits is elicited, by Number VIII stated above: then *by taking the mean proportional between one or the other apparent motion and their mean, it turns out that this mean proportional is to the semidiameter*, which has already been revealed, of the *orbit, as is the mean motion to the separation or distance, which is required.* Let the periodic times of two planets be 27 and 8. Then the proportion of the mean daily motion of the former to the latter is as 8 to 27. Hence the semidiameters of the orbits will be as 9 to 4. For the cube root of 27 is 3; that of 8 is 2; and the squares of these roots are 9 and 4. Now let the apparent motion at aphelion of one be 2, and at perihelion of the other 33 and a third. The mean proportionals between the mean motions 8 and 27, and these apparent motions, will be 4 and 30. Therefore, if the mean 4 gives an average distance for the planet of 9, then a mean motion of 8 yields a distance at aphelion of 18, corresponding with an apparent motion of 2. And if the other mean, 30, gives an average distance for the other planet of 4, then its mean motion of 27 gives its distance at perihelion as $3\frac{3}{5}$. Therefore, I say that its distance at aphelion is to its distance at perihelion as 18 to $3\frac{3}{5}$. From that it is evident that the harmonies dictated between the extreme motions of the two, and the periodic times prescribed in each case, entail the extreme and average distances, and so also the eccentricities.[47]



XII. The proportion of the mean motion to the extreme motions.

Twelfth, from the receding extreme motions of one and the same planet it is possible to find *the mean motion. For in this case it is not precisely the arithmetic mean between the extreme motions, nor precisely the geometric mean; but it is less than the geometric mean by the same amount as the geometric mean is less than the mean between the two.*[48] Let there be two

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[47] The calculation is easier to follow when formulated in modern algebraic notation. Let R_p, r_p (where R_p is greater than r_p) be the distances of the two planets at perihelion, R_a, r_a the corresponding distances at aphelion, and R, r the respective mean distances. Also let M_p, m_p be the motions at perihelion, M_a, m_a the motions at aphelion and M, m the mean motions. Given M_a/m_p and M/m, Kepler calculates r_p/R_a. As the mean daily motions are inversely proportional to the periodic times, the third (harmonic) law gives (1) $m/M = (R/r)^{3/2}$. From Number 6 of this chapter (see note 41), it follows that (2) $M_a/M = R^2/R_a^2$ and $m_p/m = r^2/r_p^2$. For a given ratio of periodic times, namely 27:8, Kepler takes the mean distances as $R = 9$, $r = 4$. The corresponding values of the motions are $M = 8$, $m = 27$. Then taking $M_a = 2$, $m_p = 33\frac{1}{3}$, he calculates the auxiliary quantities $M_1 = \sqrt{M_a M} = 4$ and $m_1 = \sqrt{m_p m} = 30$. Using (2), $R_a = MR/M_1 = 18$ and $r_p = mr/m_1 = 3\frac{3}{5}$. Hence $r_p{:}R_a = 1{:}5$.

[48] Using the notation of the previous note for the inner planet, and in addition taking $G = \sqrt{m_a m_p}$ and $A = \frac{1}{2}(m_a + m_p)$, Kepler's formula becomes $m = G - \frac{1}{2}(A -$

extreme motions, 8 and 10. The mean motion will be less than 9 and also less than the square root of 80 by half the difference between the two, that is between 9 and the square root of 80. Thus if the motion at aphelion is 20 and at perihelion 24, the mean motion will be less than 22, and also less than the square root of 480 by half the difference between that root and 22. The application of this theorem is in what follows.

XIII. The relationship of the proportion of the distances between the two planets and the Sun to the proportion of the apparent motions of each of them.

Thirteenth, from what has already been stated is proved the proposition, which will be very necessary to us, that as the proportion of the mean motions in the two planets, so is the inverse of the square root of the cube of the proportion of the orbits. Thus *the proportion* of two *apparent converging extreme motions is always less than the sesquialterate of the proportion*[49] *of the distances corresponding with those* extreme motions; *and by the same amount as, multiplied together, the* two *proportions of two corresponding distances to the* two *mean distances or to the semidiameters of the* two *orbits come to less than the square root of the proportion of the orbits, the proportion of* the two extreme *converging motions is greater than the proportion of the corresponding distances; whereas if that product exceeded the square root of the proportion of the orbits, then the proportion of the converging motions would be less than the proportion of their distances.*[50]

Let the proportion of the orbits be *DH:AE*, and the proportion of the mean motions *HI:EM*, the sesquialterate of the inverse of the former. Let the distance of the orbit, that is *CG*, be at its smallest in

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G). Although Kepler offers no justification, the result may be established as follows. Multiplying the relations $\frac{m}{m_a} = \frac{r_a^2}{r^2}$ and $\frac{m}{m_p} = \frac{r_p^2}{r^2}$ gives $\frac{r_a r_p}{r^2} = \frac{m}{G}$.
Then adding the same relations gives

$$\frac{m(m_a + m_p)}{m_a m_p} = \frac{r_a^2 + r_p^2}{r^2} \quad \text{or} \quad \frac{2mA}{G^2} = \frac{(r_a + r_p)^2 - 2r_a r_p}{r^2}; \quad \text{that is } \frac{mA}{G^2} = 2 - \frac{m}{G}.$$

Hence

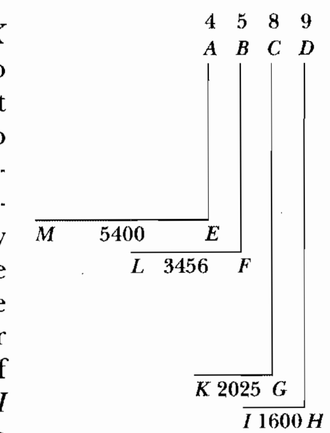
$$m = \frac{2G^2}{A + G} = G\left(1 + \frac{A - G}{2G}\right)^{-1} = G - \tfrac{1}{2}(A - G),$$

neglecting the square and higher powers of $(A - G)$.

[49] This means the inverse proportion.

[50] First, it needs to be observed that Kepler takes a proportion to be greater according to the difference of the quotient of the numbers forming the proportion from 1. Thus the first part of Kepler's proposition—the proportion of two apparent converging extreme motions is always less than the 3/2th power of the inverse proportion of the distances—would be formulated in modern notation as $M_p/m_a > (r_a/R_p)^{3/2}$, since the quotients are less than 1. Similarly, the second part of the proposition may be formulated $M_p/m_a < r_a/R_p$ when $(rR_p)/(Rr_a) > (r/R)^{1/2}$ (that is, when $r_a/R_p < (r/R)^{1/2}$), and $M_p/m_a > r_a/R_p$ when $(rR_p)/(Rr_a) < (r/R)^{1/2}$ (that is, when $r_a/R_p > (r/R)^{1/2}$. From $M_p/M = R^2/R_p^2$ and $m_a/m = r^2/r_a^2$, it follows that $(M_p m)/(M m_a) = (R^2 r_a^2)/(r^2 Rp^2)$. Using the harmonic law $M/m = (r/R)^{3/2}$, this becomes $M_p/m_a = (R/r)^{1/2} \cdot (r_a^2/R_p^2)$. Since $R/r > R_p/r_a$ and therefore $(R/r)^{1/2} > (R_p/r_a)^{1/2}$, it follows that $M_p/m_a > (r_a/R_p)^{3/2}$. If now $r_a/R_p < (r/R)^{1/2}$, it follows from $M_p/m_a = (R/r)^{1/2} \cdot (r_a^2/R_p^2)$ that $M_p/m_a < r_a/R_p$, and if $r_a/R_p > (r/R)^{1/2}$, that $M_p/m_a > r_a/R_p$.

the former case, and of the orbit in the latter case, that is *BF*, at its greatest; and let the product of the proportions *DH:CG* and *BF:AE* be in the first instance less than the square root of DG:*AE*. Also let the apparent motion of the upper planet at perihelion be *GK*, and of the lower at aphelion *FL*, so that they are extreme converging motions. I say that the proportion *GK:FL* is greater than the inverse of the proportion *CG:BF*, but less than its sesquialterate. For the proportion of *HI* to *GK* is the square of the proportion of *CG* to *DH*; and the proportion of *FL* to *EM* is the square of the proportion of *AE* to *BF*. Therefore, the two proportions multiplied together, that of *HI* to *GK* and of *FL* to *EM*, come to the square of the proportions of *CG* to *DH* and of *AE* to *BF* multiplied together. But the proportions of *CG* to *DH* and of *AE* to *BF* multiplied together are less than the square root of the proportion of *AE* to *DH* by a definite amount, as in the assumptions.



Therefore, the proportions of *HI* to *GK* and of *FL* to *EM* multiplied together are also less than the square of the square root, that is, less than the whole proportion of *AE* to *DH*, by a factor which is the square of the previous deficiency. But *HI* to *EM* is the sesquialterate of the proportion of *AE* to *DH*, by VIII previously stated. Then less than the square of the deficiency divided into the sesquialterate of the proportion, or in other words the proportions of *HI* to *GK* and of *FL* to *EM* divided into the proportion of *HI* to *EM* leave as quotient more than the square root of the proportion of *AE* to *DH*, by the square of the amount in excess. But they yield as quotient the proportion of *GK* to *FL*. Therefore, the proportion of *GK* to *FL* is more than the square root of the proportion of *AE* to *DH* by the square of the factor in excess. But the proportion of *AE* to *DH* is made up of three proportions, those of *AE* to *BF*, of *BF* to *CG*, and of *CG* to *DH*. Also the proportion of *CG* to *DH* together with that of *AE* to *BF* is less than the square root of that of *AE* to *DH*, by a deficiency of the simple factor. Therefore, the proportion of *BF* to *CG* is more than the square root of that of *AE* to *DH*, by the simple factor. But the proportion of *GK* to *FL* was also more than the square root of that of *AE* to *DH*, in fact by the square of the excess factor. However, the square of the excess is greater than the simple factor. Therefore, the proportion of the motions *GK* to *FL* is greater than the proportion of the corresponding distances, *BF* to *CG*.

Clearly it is shown in the same way that in the opposite case, if the planets come close to each other at *G* and *F*, beyond the mean separations at *H* and *E*, in such a way that the proportion of the mean separations *DH*, *AE* loses more than its square root, then the proportion of the motions *GK* to *FL* becomes less than the proportion of their distances, *BF* to *CG*. For nothing more needs to be done than to change the words *greater* to *lesser*, *more* to *less*, *excess factor* to *deficiency*, and the other way round.

In the numbers quoted, the square root of 4:9 is 2:3, and 5:8 is still greater than 2:3 by a factor of 15:16 in excess. Also the proportion 8:9 squared is the proportion 1600:2025, that is 64:81; and the proportion 4:5 squared is the proportion 3456:5400, that is 16:25; and lastly the sesquialterate of the proportion 4:9 is the proportion 1600:5400, that is 8:27. Therefore, also the proportion 2025 to 3456, that is 75:128, is still greater than 5:8, that is 75:120, by an excess factor of the same amount (120:128, that is), 15:16. Hence *the proportion of the converging motions, 2025:3456, exceeds the inverse proportion of the corresponding distances, 5:8, by the same factor as the latter exceeds the square root of the proportion of the orbits, 4:9.* Or, which comes to the same thing,[51] *the proportion of the two converging distances is the mean between the square root of the proportion of the orbits and the inverse proportion of the corresponding motions.*

From that, however, we may infer that *the proportion of the diverging motions is much greater than the sesquialterate of the proportion of the orbits*, since the sesquialterate is multiplied by the square of the proportions of the distance at aphelion to the mean distance, and of the mean distance to that at perihelion.

[51] For his numerical example, Kepler takes $DH = R = 9$, $AE = r = 4$, $CG = R_p = 8$, $BF = r_a = 5$, $HJ = M = 1600$, $EM = m = 5400$, $GK = M_p = 2025$ and $FL = m_a = 3456$. Then $r/R = 4/9$, so that $(r/R)^{1/2} = 2/3$, while $r_a/R_p = 5/8$. Hence the proportion $r_a{:}R_p$ is greater (in Kepler's terms) that the proportion $r{:}R$, the excess being the proportion 15:16, obtained by dividing 2/3 into 5/8. Again $(R_p/R)^2 = 64/81 = 1600/2025 = M/M_p$ and $(r/r_a)^2 = 16/25 = 3456/5400 = m_a/m$. Also $(r/R)^{3/2} = 8/27 = 1600/5400 = M/m$. Hence the proportion $M_p{:}m_a = 2025{:}3456 = 75{:}128$ is greater (in Kepler's terms) than the proportion $r_a{:}R_p = 5{:}8$, the excess being 15:16, obtained by dividing 5/8 into 75/128. Thus the excess of $M_p{:}m_a$ over $r_a{:}R_p$ equals the excess of $r_a{:}R_p$ over $r^{1/2}{:}R^{1/2}$.

CHAPTER IV.

In What Features Relating to the Motions of the Planets Have the Harmonic Proportions been Expressed by the Creator, and How?

When therefore the fantasy of retrogressions and stations has disappeared, and the planets' proper motions, in their own true eccentric orbits, have been stripped to essentials, there still remain in the planets the following distinct features: 1. their distances from the Sun; 2. their periodic times; 3. their daily eccentric arcs; 4. the daily times expended on their arcs; 5. their angles at the Sun, or apparent daily arcs to observers, so to speak, on the Sun. And again, all of these (except for the periodic times) are variable right around their orbit, most indeed at the mean longitudes, and least in fact at the extremities, when they have just turned away from one of them and are returning towards the opposite one. Hence when the planet is lowest and closest to the Sun, and therefore expends as little time as possible on one degree of its eccentric, and on the other hand completes its greatest daily arc of the eccentric in a single day, and appears fastest from the Sun, then its motion persists for a while in this vigorous state, without sensible variation, until when the perihelion has been passed the planet has begun to increase its linear distance from the Sun. Then at the same time it also expends a longer time on the degrees of its eccentric, or if you consider the motion of a single day, it makes less progress on each following day, and also appears much slower from the Sun, until it approaches its upper apsis, making its distance from the Sun the greatest. For then it also expends the longest time of all on one degree of its eccentric, or on the other hand completes its smallest arc in one day, and also makes its appearance much smaller and the smallest in its whole circuit.

Lastly, all these features belong either to any one planet at different times, or to different planets; so that if we suppose an infinity of time, all the states of the orbit of one planet can coincide at the same moment of time with all the states of the orbit of another planet, and can be compared; and then the complete eccentrics indeed, compared with each other, have the same proportion as their semidiameters, or their average distances, whereas the arcs of the two eccentrics, designated as equal or by the same number, nevertheless have unequal true lengths in the proportion of the whole eccentrics. For example, one degree on the sphere of Saturn is almost twice as large as one degree on the sphere of Jupiter. And on the other hand, the daily arcs of the eccentrics, expressed in astronomical numbers, do not show the same proportion as the true paths, which the globes complete through the aethereal air in one day because single degrees each represent on the wider circle of the superior planet a section of its path which is larger, but on the narrower circle of the inferior planet a section which is smaller. Hence a sixth aspect for consideration is now added, concerning the daily paths of the two planets.

First, therefore, let us take the second of the features listed, that is to say the periodic times of the planets, which comprise the assembled totals of all the times expended on all the degrees of the whole circuit, long, average, and small.[52] And it has been observed from antiquity up to our own time that the planets complete their journeys around the Sun as follows in the table.

	Days	*Sixtieths of a day*	*Therefore mean daily motions: Minutes*	*Seconds*	*Third minutes*
Saturn	10759	12	2	0	27
Jupiter	4332	37	4	59	8
Mars	686	59	31	26	31
Earth with Moon	365	15	59	8	11
Venus	224	42	96	7	39
Mercury	87	58	245	32	25

In these periodic times there are therefore no harmonic proportions, which is readily apparent if the larger periods are continually divided by two, and the smaller ones are continually doubled, so that with the intervals of a diapason suppressed we can look for those which are within a single diapason.[53]

	Saturn	*Jupiter*	*Mars*	*Earth*	*Venus*	*Mercury*	
	10759. 12						
Halves	5379. 36	4332. 37				87. 58	Doubles
	2689. 48	2166. 19			224. 42	175. 56	
	1344. 54	1083. 10	686. 59	365. 15	449. 24	351. 52	
	672. 27	541. 35					

All the last numbers, as you see, are repugnant to harmonic proportions, and seem similar to inexpressibles. For let the number of

[52] Kepler here sets out to explain why harmonies are not to be found in several relations in which they might have been expected to occur, such as the periodic times, the bulks or volumes of the planets, the aphelion and perihelion distances, and finally, the true daily paths at aphelion and perihelion. This leaves the apparent daily motions at aphelion and perihelion as seen from the sun (that is, the angular velocities with respect to the sun) in which at last he locates the celestial harmonies.

[53] Each division of the period by 2 raises the musical interval by an octave. Similarly, multiplication by 2 lowers the interval by an octave. For example, taking one sixteenth of the period of Saturn, namely 672.27 days, raises the interval by four octaves. Then comparing this number of days with the period of Mars gives a ratio of 117:120 approximately. This represents a musical interval within a single octave but it is quite clearly not a consonance.

days for Mars, 687, be measured in units in which it represents 120, which stand for a division of a string. In these units Saturn will be represented by a little more than 117, taking a sixteenth part; Jupiter by less than 95, taking an eighth; the Earth by less than 64; Venus by more than 78, taking double; Mercury by over 61, taking quadruple. Yet these numbers do not make any harmonic proportion with 120; but the neighboring numbers 60, 75, 80, and 96 do. Similarly in units in which Saturn comes to 120, Jupiter comes to about 97, the Earth over 65, Venus more than 80, Mercury less than 63. And in units in which Jupiter comes to 120, the Earth comes to less than 81, Venus less than 100, Mercury less than 78. Also in units in which Venus comes to 120, the Earth comes to less than 98, Mercury to more than 94. Lastly in units in which the Earth comes to 120, Mercury comes to less than 116. But if this free selection of proportions had been valid, they would have been absolutely perfect harmonies, without excesses or deficiencies. God the Creator is therefore not discovered to have intended to introduce harmonic proportions among these sums of times expended added together into periodic times.

Proportion of the planetary globes to each other.

And since it is a very probable conjecture (inasmuch as it depends on geometrical proofs, and on the theory of the causes of the planetary motions set out in the *Commentaries on Mars*) that the bulk of the bodies of the planets are in the proportion of their periodic times,[54] so that the globe of Saturn is about thirty times greater than the globe of the Earth, Jupiter twelve times, Mars less than twice, the Earth greater than one and a half times the globe of Venus, and four times greater than the globe of Mercury, then these proportions of the bodies will not be harmonic either.

Since, however, God has established nothing without geometrical beauty unless it is bound up with some other, prior law of necessity, we readily infer that the periodic times get their durations, and therefore the moving bodies also their bulks, from something which has prior existence in the Archetype. It is to express it that these, as they appear, disproportionate bulks and periods are fitted to this measure. But I have said that the periods are the sum of the times expended, very long, medium, and very slow. The geometrical harmonizations must therefore be found either in these times, or in something prior to them in the mind of the Maker, perhaps. Now the proportions of the expended times are bound up with the proportions of the daily

[54] At this time, Kepler supposed, for archetypal reasons which he does not specify, that the surface area of a planet was proportional to the distance, so that the volume was proportional to the periodic time. In the part of the *Epitome* (KGW 7, pp. 281–282) published in 1620, he abandoned this hypothesis in favor of that of Remus Quietanus, which seemed to be in better accord with the observations; namely, that the volume was in proportion to the distance. It was on the basis of this new hypothesis that he propounded a causal explanation of the harmonic law in the part of the *Epitome* published in 1621 (KGW 7, p. 307).

arcs, because the arcs are in the inverse proportion of the times. Again, we have stated that the proportions of the times expended and the distances of any one planet are the same. As far as individual planets are concerned, therefore, discussion of these three, the arcs, the times expended on equal arcs, and the remoteness of the arcs from the Sun, or the distances, will be one and the same. And because all these are as it happens variable in the case of the planets, there can be no doubt that if they have been assigned any geometrical beauty, by the sure design of the Maker, they acquire it at their extremes, as at their distances in aphelion and perihelion, and not so much at the mean distances in between. For given the proportions of the extreme distances, the design does not need to fit the intermediate proportions to a definite number; for they follow automatically, by the necessity of the planetary motion from one extreme, through all the intermediate points, to the other extreme.

Therefore, the extreme distances are as follows, worked out from the very accurate observations of Tycho Brahe, by the method explained in the *Commentaries on Mars*, by the most persistent exertions of seventeen years.

Distances Compared with Harmonic Intervals

Proportions of pairs			*Distance at:*		*Proportions for individual ones*
Divergent	*Convergent*				
		Of Saturn:	Aphelion	10052.a	More than a minor tone $\frac{10000}{9000}$
			Perihelion	8968.b	Less than a major tone $\frac{10000}{8935}$
a d $\frac{2}{1}$	b c $\frac{5}{3}$				
		Of Jupiter:	Aphelion	5451.c	No melodious proportion, but about 11/10, not melodious, or the square root of 6/5, which is harmonic.
			Perihelion	4949.d	
c f $\frac{4}{1}$	d e $\frac{3}{1}$				
		Of Mars:	Aphelion	1665.e	Here $\frac{1020}{1388}$ would be harmonic, $\frac{6}{5}$ and $\frac{1665}{1332}$ would be $\frac{5}{4}$.
			Perihelion	1382.f	
e h $\frac{5}{3}$	f g $\frac{27}{20}$				
		Of Earth:	Aphelion	1018.g	Here $\frac{1020}{980}$ would be a diesis $\frac{25}{24}$; therefore it does not cover a diesis.
			Perihelion	982.h	
the square root of g k $\frac{2}{1}$ i.e., $\frac{10000}{7071}$	h $\frac{27}{20}$				
		Of Venus:	Aphelion	729.i	Less than a comma and a half; more than a third of a diesis.
			Perihelion	719.k	
i m $\frac{12}{5}$	k l $\frac{243}{160}$				
		Of Mercury:	Aphelion	470.l	More than an oversize fifth $\frac{243}{160}$; less than the harmonic $\frac{8}{5}$.
			Perihelion	307.m	

Then there is no single planet of which the extreme distances hint at harmonies, except for Mars and Mercury.

But if you compare the extreme distances of different planets with each other, some light of harmony now begins to shine forth. For the divergent extremes of Saturn and Jupiter make a little more than a diapason; their convergent extremes the mean between major and minor sixths. Similarly the divergent extremes of Jupiter and Mars embrace about a double diapason, and their convergent extremes about a diapason and a diapente. However, the divergent extremes of the Earth and Mars have embraced rather more than a major sixth, and their convergent extremes an oversize diatessaron. In the following couple of the Earth and Venus again there is the same oversize diatessaron between their convergent extremes, but between their divergent extremes we are deserted by harmonic proportion; for it is less than half a diapason (if we may use the phrase), that is, less than the semiduplicate proportion.[55] Lastly, between the divergent extremes of Venus and Mercury the proportion is a little less than the combination of a diapason and a minor third; between their convergent extremes is an oversize diapente, and a little over.

The stages by which the true celestial harmonies were reached. The analogy between a convex orbit and a circular line is the same as between solid figures and harmonies, also the same as between a body and the motion of a body.

Therefore, although one interval departs a little too far from the harmonic proportions, yet this good result was an invitation to proceed further. Now the following was my reasoning. First, these distances, insofar as they are lengths without motion, are not appropriate to be examined for harmonies, because the harmonies are more intimately connected with motion, on account of its swiftness and slowness. Second, in the case of the same distances, insofar as they are diameters of spheres, it is easy to believe that the ratio of the five regular solids should be taken in preference, by analogy. For the ratio of the solid geometrical bodies to the celestial spheres, either enclosed on all sides by celestial matter, as antiquity would have it, or to be enclosed by the accumulation of a great many successive rotations, is also the same as that of the plane figures which are inscribed in a circle (and which are the figures which generate the harmonies) to the celestial circles of the motions, and to the other spaces in which the motions occur. Therefore, if we are seeking harmonies, let us seek for them not in these latter distances, as they are the semidiameters of spheres, but in the former distances, as they are the measures of the motions, that is, rather in the actual motions. Certainly no other distances can be taken as the semidiameters of the spheres, but the average distances; whereas we are dealing with the extreme distances. Therefore, we are not dealing with the distances in respect of their spheres, but in respect of the motions.

For these reasons, then, since I had gone over to comparison of the extreme motions, at first the proportions of the motions remained the same in magnitude as those of the distances were previously, except that they were inverted. Hence some proportions were also found

[55] That is, the proportion $\sqrt{2}$:1.

between the motions, as previously, to be unmelodic, and foreign to the harmonies. However, again I thought that I deserved that result, inasmuch as I was comparing arcs of the eccentric with each other, which are not expressed or counted by a measure of the same size, but are counted in degrees and minutes which are different in size for different planets. Also they do not anywhere show the apparent size which the numerical value of each indicates, except only at the center of each eccentric, which is not supported by any body; and similarly also it is incredible that there should be any sensation or natural instinct in that position in the world which could grasp this apparent size, or rather it is even impossible, if I was comparing the eccentric arcs of different planets, with respect to their apparent sizes at their own centers, which are different in different cases. However, if the different apparent magnitudes were compared, they ought to be apparent at a single position in the world, in such a way that that which has the opportunity of comparing them would be situated at that position of their common appearance. Therefore, I judged that the apparent sizes of these eccentric arcs should either be put out of my mind or represented in a different way. But if I were to put the apparent sizes out of my mind, and turn my attention to the actual daily paths of the planets, I saw that I should have to apply the precept which I stated in Number IX of the previous chapter.[56] Therefore, on multiplying the daily arcs of the eccentrics by the mean distances of the orbits, the following paths resulted.[57]

Thus Saturn completes hardly a seventh of the path of Mercury, and what Aristotle in Book II of his *De Caelo* judged agreeable to reason, that always that which is nearer to the Sun completes a greater distance than that which is further—which cannot be brought about in the ancient astronomy.

		Daily motions. *Min. Sec.*	*Average distances.*	*Daily paths.*
Of Saturn	at Aphelion	1. 53.	9510.	1075
	at Perihelion	2. 7.		1208
Of Jupiter	at Aphelion	4. 44.	5200.	1477
	at Perihelion	5. 15.		1638
Of Mars	at Aphelion	28. 44.	1524.	2627
	at Perihelion	34. 34.		3161
Of Earth	at Aphelion	58. 6.	1000.	3486
	at Perihelion	60. 13.		3613
Of Venus	at Aphelion	95. 29.	724.	4148
	at Perihelion	96. 50.		4207
Of Mercury	at Aphelion	201. 0.	388.	4680
	at Perihelion	307. 3.		7148

[56] The point of this precept was to have a common measure of the true daily paths, applicable to all planets, and this is obtained by forming the product of the true daily arcs (measured in minutes and seconds of angle subtended at the center of the eccentric) by the mean distance of the planet from the sun.

[57] The true daily path in the third column is obtained by multiplying the true daily motion in seconds of arc by the average distance in the second column and then dividing by 1000 for convenience. The resulting numbers represent the true daily paths for all the planets in a common measure.

Thus Saturn completes scarcely a seventh part of the path of Mercury; and the result is what Aristotle in Book II of his *De Caelo* (*On the Heaven*)[58] judged to be in agreement with reason, that the planet which is nearer to the Sun always completes a greater distance than the one which is further away, which is impossible to attain in the ancient astronomy.

Therefore, as far as the daily paths of individuals are concerned, the proportions which they comprise ought to be the same in magnitude as those which were previously in the distances, but inverted in kind, because the eccentric arcs, as has been stated, have the inverse proportion of their own distances from the Sun.

However, if we consider the extreme paths of the pairs, either divergent or convergent, there is much less appearance of anything harmonic than previously when we had considered the actual arcs.

And indeed if we should ponder the matter more carefully, it will be apparent that it is not very likely that the most wise Creator should have taken thought most of all for harmonies between the actual planetary paths. For if the proportions of the paths are harmonic, all the other features of the planets will be constrained, and linked to the paths, so that there will be no room for taking thought for harmonies elsewhere. But who will benefit from harmonies between the paths, or who will perceive these harmonies? There are two things which reveal to us harmonies in natural occurrences, either light or sound. The former is received through the eyes, or hidden senses analogous to eyes, the latter through the ears; and the mind seizing on these emanations distinguishes either by instinct (on which plenty has been said in Book IV) or by astronomical or harmonic reasoning between melodic and unmelodic. In fact, no sounds exist in the heaven, and the motion is not so turbulent that a whistling is produced by friction with the heavenly air. There remains light. If it is to teach us anything about the paths of the planets, it will teach us that either the eyes, or some sensory organ analogous to them, are located in a certain position; and for the light to inform us immediately of its own accord, it seems that the sensory organ must be there in its presence. Therefore, there will be an organ of sensation all over the world, that is to say in such a way that one and the same is present to the motions of all the planets. For that way which was traversed by dint of observations, by way of long drawn out wanderings in geometry and arithmetic, of the proportions of the spheres and the rest which had to be learnt beforehand, to reach these actual paths, is too long for some natural instinct, to influence which it seems to be fitting that the harmonies were introduced.

Therefore, assembling all these points into a single review I have rightly concluded that we should dismiss the true paths of the planets

[58] Aristotle, *De caelo*, 291 a 29–291 b 10.

through the aethereal air, and turn our eyes to the apparent daily arcs, all indeed apparent to one definite and prominent position in the world, that is to say to the actual solar body, the source of the motion of all the planets. Also we should look not how high any particular planet is from the Sun, nor what space it traverses in a single day—for that is rational and astronomical, not instinctive—but how large an angle the daily motion of each planet subtends at the actual body of the Sun, or how large an arc on one common circle drawn about the Sun, such as the ecliptic, it seems to complete on any particular day. Thus this appearance, brought by the agency of light to the body of the Sun, can along with the light itself flow straight to living creatures, who share in this instinct, just as in the fourth book we have stated that the pattern of the heaven flows to a foetus by the agency of the rays.[59]

Therefore, the Tychonic astronomy teaches us (abstracting from the proper motion of the planets the parallaxes of the annual orbit, which impart to them the semblance of stations and retrogressions) that the daily motions of the planets in their own orbits (as they appear, so to speak, to those watching on the Sun) are as follows:

Harmonies of pairs.						*Apparent daily paths.*			
Div.		*Conv.*				*Min. Sec.*		*Individuals' own harmonies.* *Min. Sec.*	
				Saturn	at Aphelion	1.46.a	Between	1.48.	is $\frac{4}{5}$ a major third.
					at Perihelion	2.15.b	and	2.15.	
a d	$\frac{1}{3}$	b c	$\frac{1}{2}$						
				Jupiter	at Aphelion	4.30.c	Between	4.35.	is $\frac{5}{6}$ a minor third.
					at Perihelion	5.30.d	and	5.30	
c f	$\frac{1}{8}$	d e	$\frac{5}{24}$						
				Mercury	at Aphelion	26.14.e	Between	25.21	is $\frac{2}{3}$ a diapente.
					at Perihelion	38. 1.f	and	38.1	
e h	$\frac{5}{12}$	f g	$\frac{2}{3}$						
				Earth	at Aphelion	57. 3.g	Between	57.28	is $\frac{15}{16}$ a semitone.
					at Perihelion	61.18.h	and	61.18	
g k	$\frac{3}{5}$	h i	$\frac{5}{8}$						
				Venus	at Aphelion	94.50.i	Between	94.50	is $\frac{24}{25}$ a diesis.
					at Perihelion	97.37.k	and	98.47	
i m	$\frac{1}{4}$	k l	$\frac{3}{5}$						
				Mars	at Aphelion	147. 0.l	Between	164. 0	is $\frac{5}{12}$ a diapason and minor third
					at Perihelion	384. 0.m	and	394. 0	

[59] Here Kepler seems to suggest that the reception of the celestial harmonies by living creatures is instinctive, like that of the aspects, the harmonies being conveyed in some way along with the light from the sun. Of the various possible locations of the celestial harmonies, Kepler indicates that the one which places them in the apparent motions (as seen from the sun) is the one which would need the least amount of calculation and discursive reasoning for their recognition. In other words, this is the most suitable location for an instinctive recognition.

Notice that the great eccentricity of Mercury makes the proportion of the motions differ considerably from the square of the proportion of the separations.[60] For if you make the proportion of the motion at aphelion to the mean motion, 245 minutes 32 seconds, that is the square of the proportion of the mean separation, taken as 100, to the separation at aphelion, 121, then the resulting motion at aphelion is 167; and if you make the proportion of the motion at perihelion to the same mean motion, that is the square of the proportion of 100 to the distance at perihelion, 79, the motion at perihelion will be made 393. In both cases it is greater than I have supposed here, naturally because the mean motion at the mean anomaly being viewed very obliquely does not appear as great, that is to say not 245 minutes 32 seconds but smaller by about 5 minutes. Therefore, the motions at aphelion and at perihelion will also be found to be smaller. However, it will be less so for the motion at aphelion, and more so for the motion at perihelion, on account of the theorem in Euclid's *Optics*, in accordance with my warning in the preceding chapter, under Number VI.

Therefore, I could assume mentally that between these apparent extreme motions of individual planets there would be harmonies, and their distances would be melodic, and that indeed from the proportions of their daily eccentric arcs, set out above, since I there saw that square roots of harmonic proportions reigned everywhere, whereas I knew that the proportion of the apparent motions was the square of that of the eccentric motions. But we may verify what is stated by actual observation, indeed without reasoning, as you see in the next table. For the proportions of the apparent motions of individual planets come very close to harmonies. Thus Saturn and Jupiter embrace a very little more than thirds, major and minor: there is an excess in the former case of 53:54, in the latter of 54:55 or less, that is to say about one and a half commas; the Earth embraces a very little more, that is to say 137:138 more, scarcely half a comma, than a semitone; Mars somewhat (that is to say 29:30, which is close to 35:35 or 35:36) less than a diapente; Mercury occupies, over the diapason, nearer a minor third than a tone, that is to say it has less by about 38:39, which is about two commas, in other words about 34:35 or 35:36. Venus alone occupies something smaller than any of the melodic intervals, and is itself just a diesis; for its proportion is between two and three commas, and exceeds two thirds of a diesis, being about 34:35, almost 35:36, a Diesis diminished by a comma.

The Moon also enters into consideration here.[61] For it is found that its hourly motion at apogee in quadrature, that is to say when

[60] Kepler has shown that, for small eccentricities, the apparent angular velocity, as seen from the sun, is inversely proportional to the square of the distance from the sun. Cf. Chapter III, number six and note 41.

[61] The proportion of the moon's apparent motion is taken to be that as seen from the earth.

it is slowest of all, is 26 minutes 26 seconds. At perigee at the syzygies, that is to say when it is fastest of all, it is 35 minutes 12 seconds. By this ratio a diatessaron is formed with great exactness. For a third part of 26′26″ is 8′49″—four times which is 35′15″. And notice that the harmony of diatessaron is found nowhere else among the apparent motions. Notice also the analogy of the Fourth in the harmonies with quadrature in the phases.[62] These, then, are found in the motions of individual planets.

What the harmonies are between the approaching and receding motions of the pairs.

But among the extreme motions of the pairs of planets compared with each other, the clearest light is thrown at once as soon as we look at the heavenly harmonies, whether you compare the receding extreme motions with each other, or the approaching. For between the approaching motions of Saturn and Jupiter the proportion is exactly double, or a diapason; between their receding motions, it is a very little more than triple, or a diapason with a diapente. For of 5 minutes 30 seconds, a third part is 1 minute 50 seconds, whereas Saturn has instead of that 1 minute 46 seconds. Therefore, the planetary proportion has one diesis over, or something a little less, that is 26:27 or 27:28; and when Saturn is approaching to within less than a single second from aphelion, the excess will be 34:35, the size of the proportion between the extreme motions of Venus. Between the diverging and converging motions of Jupiter and Mars reign the triple diapason, and the third a double diapason above, though not perfectly. For an eighth part of 38 minutes 1 second is 4 minutes 45 seconds, whereas Jupiter has 4 minutes 30 seconds. Between those numbers there is still a difference of 18:19, which is the mean between 15:16 and 24:25, a semitone and a diesis, that is to say very nearly a perfect limma, 128:135. Similarly, a fifth part of 26 minutes 14 seconds is 5 minutes 15 seconds, whereas Jupiter has 5 minutes 30 seconds. Therefore, the deficiency from the fivefold proportion here is about 21:22, the amount of the excess in the other proportion previously, that is about a diesis, 24:25. The harmony 5:24 which takes in a minor instead of a major third over the second octave comes rather near. For of 5′30″ a fifth part is 1′6″, and taking twenty four times that produces 26′24″, with which 26′14″ makes no more than half a comma.[63] Mars has been allotted a very small proportion with the Earth, very exactly the sesquialterate, or a diapente; for a third part of 57 minutes 3 seconds is 19 minutes 1 second, and double that is 38 minutes 2 seconds, the very number which Mars has, that is 38 minutes 1 second. As their greater proportion they have been allotted a diapason with

See Book III, Chapter IV, page 182

[62] The absence of a primary consonance like the diatessaron from the celestial harmonies would, of course, have been a serious problem for Kepler, so he was pleased to find this harmony in the apparent motion of the moon and a reason for its location here in the analogy with the quadratures in the phases.

[63] For purposes of calculation, Kepler later represents half a comma by the proportion 157:158.

a minor third, 5:12, a little less nearly perfect. For a twelfth part of 61 minutes 18 seconds is 5 minutes $6\frac{1}{2}$ seconds, and taking five times that gives 25 minutes 33 seconds, whereas instead of that Mars has 26 minutes 14 seconds. The deficiency is therefore about a narrow diesis, that is 35:36. However, the Earth and Venus have been allotted harmonies in common, the greatest 3:5 and the least 5:8, which are sixths, major and minor, again not quite perfect. For a fifth part of 97 minutes 37 seconds is 19 minutes 31 seconds, and three times that comes to 58 minutes 34 seconds which is more than the motion of the Earth at aphelion by 34:35, which is almost 35:36, the amount by which the planetary proportion exceeds the harmonic. Similarly, an eighth part of 94 minutes 50 seconds is 11 minutes 51 seconds +, and five times that is 59 minutes 16 seconds which is as nearly as possible equal to the mean motion of the Earth. Hence in this case the planetary proportion is less than the harmonic by 29:30, or 30:31, which again is nearly 35:36, a narrow diesis; and to that extent this smallest of their proportions approaches the harmony of diapente. For a third part of 94 minutes 50 seconds is 31 minutes 37 seconds, and twice that is 63 minutes 14 seconds, from which the motion of the Earth at perihelion, 61 minutes 18 seconds, is deficient by the tiny amount of 31:32, so that the planetary proportion occupies exactly the mean between the neighboring harmonic proportions. Lastly, the proportions allotted to Venus and Mercury are as the greatest a double diapason, and as the least a hard sixth, though these are not absolutely perfect. For a fourth part of 384 is 96 minutes 0 seconds, whereas Venus has 94 minutes 50 seconds. Therefore, it approaches the fourfold within about one comma. Similarly, a fifth part of 164 minutes is 32 minutes 48 seconds, and taking three times that makes 98 minutes 24 seconds, whereas Venus has 97 minutes 37 seconds. Therefore, the planetary proportion is in excess by about two thirds of a comma, that is 126:127.

These, then, are the harmonies with each other allocated to the planets; and there is none of the direct comparisons (that is to say between convergent and divergent extreme motions) which does not come very close to some harmony, so that if strings were tuned in that way, the ears would not easily be able to detect the imperfection, except for the excess of the single one between Jupiter and Mars.[64]

[64] Kepler here takes the maximum imperfection "which the ears would not easily be able to detect" as the diesis 24:25. Only in the case of the divergent motions of Jupiter and Mars is the imperfection greater than a diesis. In this case it is a limma 128:135, which is the mean between a diesis and a semitone. Although the imperfections are small, they are still not as small as Kepler could have wished. For an imperfection as large as a diesis would not be tolerated in musical performance, where the largest acceptable imperfection is the comma 80:81, which is less than a third of the diesis.

What the harmonies are between the motions of the pairs on the same side.

Now it follows that if we compare the motions on the same side[65] we shall not be likely to stray far from the harmonies in that case either. For on multiplying the 4:5 times 53:54 of Saturn by the intermediate proportion 1:2 the combined product is 2:5 times 53:54, which is the proportion between the motions at aphelion of Saturn and Jupiter.[66] Multiply by the 5:6 times 54:55 of Jupiter: the product is 5:12 times 54:55 which is the proportion between the motions at perihelion of Saturn and Jupiter. Similarly multiply the 5:6 times 54:55 of Jupiter by the following intermediate proportion, 5:24 divided by 157:158; the result is 1:6 divided by 35:36, the proportion between the motions at aphelion.[67] Multiply the same, 5:24 divided by 157:158, by the 2:3 divided by 29:30 of Mars; the result is 5:36 divided by 24:25, about, that is 125:864 or nearly 1:7, the proportion between the motions at perihelion: in fact this alone so far is unmelodic.[68] Multiply the third of the intermediate proportions, 2:3,[69] by the 2:3 divided by 29:30 of Mars: it comes out as 4:9 divided by 29:30, that is 40:87, another unmelodic interval between the motions at aphelion. If instead of the proportion for Mars you multiply by the Earth's 15:16 times 137:138 you will obtain 5:8 times 137:138, the proportion between them at perihelion.[70] And if you multiply the fourth of the intermediate proportions, 5:8 divided by 30:31, or 2:3 times 31:32, by the Earth's 15:16 times 137:138, you will find the product is very nearly 3:5, the proportion between the motions at aphelion of the Earth and Venus.

[65] That is, comparing the motions of the two planets at aphelion or at perihelion.

[66] Kepler has already shown that the proportion of the aphelion and perihelion motions of Saturn exceeds the major third 4:5 by 53:54, or about one and a half commas. Combining these intervals with the almost exact octave 1:2 between the perihelion motion of Saturn and the aphelion motion of Jupiter gives an interval 53:54 in excess of 2:5 for the proportion of the motions of Saturn and Jupiter at aphelion. The interval between these aphelion motions thus exceeds the consonance of an octave and a major third by about one and a half commas. In this case, therefore, the imperfection is well within the limit of a diesis that Kepler is willing to accept. The further calculations described by Kepler proceed in the same way. By adopting this method of calculation, instead of simply calculating the proportion of the motions from the table given earlier, Kepler is able to estimate the imperfections in the new intervals by using those already found in the intervals of the motions of each planet and the intervals of the divergent and convergent motions. Intervals are combined, it should be noted, by multiplying the proportions.

[67] That is, the proportion of the aphelion motions of Jupiter and Mars. This is equivalent to a musical interval of about two commas less than two octaves and a fifth.

[68] The proportion of the perihelion motion of Jupiter and the aphelion motion of Mars is 5:24 divided by the imperfection 157:158. Combining this with the proportion of the aphelion and perihelion motions of Mars, 2:3 divided by the imperfection 29:30, gives the proportion of the perihelion motions of Jupiter and Mars which, as Kepler notes, corresponds to the dissonant interval 1:7.

[69] That is, the proportion of the perihelion motion of Mars and the aphelion motion of the Earth.

[70] Thus the proportion of the perihelion motions of Mars and the Earth corresponds to a consonant interval, with an imperfection of about half a comma.

For a fifth part of 94 minutes 50 seconds is 18 minutes 58 seconds, and three times that is 56 minutes 54 seconds, whereas the Earth has 57 minutes 3 seconds.[71] If you multiply the same proportion by the 34:35 of Venus,[72] you obtain a product of 5:8, the proportion between the motions at perihelion. For an eighth part of 97 minutes 37 seconds is 12 minutes 12 seconds +, and taking five times that gives a return of 61 minutes 1 second, whereas the Earth has 61 minutes 18 seconds.

Lastly, if you multiply the last of the intermediate proportions, 3:5 times 126:127, by Venus' 34:35, the combined product will be 24:25 times 3:5, and the result is a dissonant interval, made of the two combined, between the motions at aphelion. Nevertheless, if you multiply by Mercury's proportion, 5:12 divided by 38:39, now it will fall short of 1:4, or the double diapason, by as nearly as possible a complete diesis, for the proportion between the motions at perihelion.

Therefore, perfect harmonies are found between the convergent extreme motions of Saturn and Jupiter, a diapason; between the converging extremes of Jupiter and Mars, a double diapason together with nearly a soft third; between the converging extremes of Mars and the Earth, a diapente, and between their motions at perihelion a soft sixth; between the motions of the Earth and Venus at aphelion, a hard sixth, and at perihelion a soft sixth; between the converging extremes of Venus and Mercury a hard sixth, and between their divergent extremes or even between their motions at perihelion, a double diapason.[73] Hence without detriment to the astronomy developed most subtly of all from the observations of Brahe, it seems that the residual very tiny discrepancy can be absorbed, especially in the motions of Venus and Mercury.

However, you will notice that where there is not a perfect major harmony, as between Jupiter and Mars, there alone I have detected a very nearly perfect intermediate placing of the solid figure, since the separation of Jupiter at perihelion is very nearly three times that of Mars at aphelion, so that this pair aspires in its distances to the perfect harmony which it has not got in its motions.[74] You will no-

[71] The imperfection is about a quarter of a comma.

[72] Kepler has already shown that the proportion of the aphelion and perihelion motions of Venus corresponds to a diesis diminished by a comma; that is, the interval 34:35.

[73] With the exception of the proportion of the perihelion motions of Venus and Mercury, where the imperfection is a diesis, the proportions mentioned here by Kepler represent harmonies that are either perfect or well within a comma of absolute perfection; in other words, they represent intervals acceptable as perfect in musical performance.

[74] Kepler explains discrepancies in the interpolation of the regular polyhedra between the planetary spheres as a consequence of the requirements of the celestial harmony. Conversely, where the harmonies fail, there must be a reason, and this he located in the interpolation of the solids. Nothing is the result of chance or

tice further that the greater planetary proportion of Saturn and Jupiter exceeds the harmonic proportion, that is to say the threefold, by almost the same amount as is Venus' own proportion; and the deficiency in the common greater proportion of Mars and the Earth is also almost the same as that in the two common proportions of the extremes of the Earth and Venus, convergent and divergent. You will notice thirdly that among the superior planets there are almost fixed harmonies between the convergent motions, but among the inferior planets between motions in the same direction.[75] And notice fourthly that between the motions at aphelion of Saturn and the Earth there are very nearly five diapasons; for a thirty second part of 57 minutes 3 seconds is 1 minute 47 seconds, whereas the motion of Saturn at aphelion amounts to 1 minute 46 seconds.

Further, there is a great distinction between the harmonies which have been set out between individual planets, and between planets combined. For the former cannot indeed exist at the same moment of time, whereas the latter can absolutely; because the same planet when it is situated at its aphelion cannot at the same time also be at its perihelion which is opposite, but of two planets one can be at its aphelion and the other at its perihelion at the same moment of time.[76] Then the proportion of simple melody or monody, which we call choral music and which was the only kind known to the ancients, to the melody of several voices, called figured and the invention of recent centuries, is the same as the proportion of the harmonies which are indicated by individual planets to the harmonies which they indicate in combination. Further, then, in Chapters V and VI the individual planets will now be compared with the choral music of the ancients, and its properties will be demonstrated in the motions of the planets; but in the chapters which follow it will be demonstrated that the planets in combination match modern figured music.

accident. So, where harmony is lacking in certain motions, such as those of Jupiter and Mars to which Kepler refers, the planets must have been arranged in accordance with some other principle; in this case the harmony of distances arising from the interpolation of the polyhedra. The geometrical archetype is a composite one, consisting of the regular polyhedra and the harmonic proportions.

[75] That is, the motions of the two planets at aphelion or at perihelion.

[76] The notes of the harmonic intervals represented by the single planets can only be sounded in succession, as in a melody consisting of a single line. The notes of the harmonies represented by pairs of planets, however, can be sounded simultaneously, as in the polyphonic music that Kepler believed to be a recent invention.

CHAPTER V.

That the Positions in the System, or the Notes[77] of the Musical Scale, and the Kinds of Melody, Hard and Soft, Have Been Expressed in the Apparent (to observers on the sun, so to speak) Planetary Motions.

Therefore, that between these twelve terms or motions of the six planets which revolve round the Sun there exist upwards, downwards, and in every direction proportions which are harmonic, or very close to such within an imperceptible fraction of the smallest melodic interval, has been proved so far by numbers which have been sought in the former case from astronomy and in the latter from harmony. However, just as in the third Book we first extracted the individual harmonic proportions separately in the first Chapter, and only after that in the second Chapter we assembled all that there were of them into one common system or musical scale, or rather we divided one diapason of them, which embraces the remaining ones in its dominion, through those remaining ones into steps or positions, so that by this procedure we produced a scale; so also now, when we have found the harmonies which God Himself embodied in the world, the next thing is for us to see whether the individual harmonies stand separately, so that they have no affinity with the rest, or whether in fact they all agree with each other? However, it is easy without further inquiry to conclude that these harmonies are fitted together with the utmost skill so that they support each other mutually as if within a single structure, and no single one clashes with another, inasmuch as we see that in such a many-sided comparison of their terms harmonies never fail to occur. For if all were not fitted to all to form a single scale, it could easily have come about (as has happened here and there, when necessity is so pressing) that several dissonances occurred. Thus if anyone established a major sixth between the first and second term, and between the second and third a third, also major, without regard to the previous interval, in that case he would be admitting between the first and the third a dissonance and an unmelodic interval, 12:25.[78]

[77] The word "clavis" (literally "key") here means "note."

[78] Kepler's purpose here is simply to show that if harmonies are combined without regard to the way in which they fit together to form a system, dissonances would be generated accidentally.