Staple here!
CMSC 27100 Autumn 2014
Acknowledgment:

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Section: 1

Homework # 7

Problem 1, Section 7.3 - 18 (pg 477)

Suppose that a Bayesian spam filter is trained on a set of 500 spam messages and 200 messages that are not spam. The word exciting appears in 40 spam messages and in 25 messages that are not spam. Would an incoming message be rejected as spam if it contains the word exciting and the threshold for rejecting spam is 0.9?

Under the assumption that spam and non-spam emails have an equally likely chance of being received, the probability that an email will be spam given that it contains the word *exciting* is

$$p(A|B) = \frac{p(B|A)}{p(B|A) + p(B|\overline{A})}$$

Substituting in values,

$$p(A|B) = \frac{\frac{40}{500}}{\frac{40}{500} + \frac{25}{200}} = 0.39$$

This is not above the 0.90 threshold thus an incoming message containing the word *exciting* will not get rejected.

Problem 2, Section 7.3 - 16 (pg 476)

Ramesh can get to work in three different ways: by bicycle, by car, or by bus. Because of commuter traffic, there is a 50% chance that he will be late when he drives his car. When he takes the bus, which uses a special lane reserved for buses, there is a 20% chance that he will be late. The probability that he is late when he rides his bicycle is only 5%. Ramesh arrives late one day. His boss wants to estimate the probability that he drove his car to work that day. (I use A, B, and C to denote the events of getting to work via car, bike, and bus.)

a. Suppose the boss assumes that there is a 1/3 chance that Ramesh takes each of the three ways he can get to work. What estimate for the probability that Ramesh drove his car does the boss obtain from Bayes theorem under this assumption?

$$p(A|S) = \frac{p(S|A)}{p(S|A) + p(S|B) + p(S|C)} = \frac{0.50}{0.50 + 0.05 + 0.20} = \frac{2}{3}$$

b. Suppose the boss knows that Ramesh drives 30% of the time, takes the bus only 10% of the time, and takes his bicycle 60% of the time. What estimate for the probability that Ramesh drove his car does the boss obtain from Bayes theorem using this information?

$$p(A|S) = \frac{0.50 \times 0.30}{0.50 \times 0.30 + 0.05 \times 0.60 + 0.20 \times 0.10} = 0.75$$

Problem 3, Section 7.4 - 38 (pg 493)

Suppose that the number of cans of soda pop filled in a day at a bottling plant is a random variable with an expected value of 10,000 and a variance of 1000.

a. Use Markovs inequality (Exercise 37) to obtain an upper bound on the probability that the plant will fill more than 11,000 cans on a particular day.

$$p(X(s) \geq 11,000) \leq \frac{10,000}{11,000} \approx 0.91$$

b. Use Chebyshevs inequality to obtain a lower bound on the probability that the plant will fill between 9000 and 11,000 cans on a particular day.

$$p(|X(s) - 10,000| \geq 1,000) \leq \frac{1,000}{1,000^2} = 0.001$$

Problem 4, Chapter 7 Supplementary Exercises - 10 (pg 496)

Suppose that a pair of fair dodecahedral dice is rolled.

a. What is the expected value of the sum of the numbers that come up?
By symmetry,
$$E(X) = 13$$

b. What is the variance of the sum of the numbers that come up?
By symmetry,
$$V(X) = \frac{2}{144}\sum_{n=2}^{13}(n-1)(13-n)^2 = 23.83$$

Problem 5, Chapter 7 Supplementary Exercises - 24 (pg 497)

Suppose that A and B are events with probabilities $p(A) = 2/3$ and $p(B) = 1/2$. (All examples are for one coin toss)

a. What is the largest $p(A \cap B)$ can be? What is the smallest it can be? Give examples to show that both extremes for $p(A \cap B)$ are possible.

$$p(A \cap B) = p(A)p(B|A)$$

If A and B are disjoint, we have a minimum because $p(B|A) = 0$. EXAMPLE: A = Getting heads and B = Getting tails

$$p(A \cap B) = p(A)p(B|A) = p(A) \times 0 = 0$$

If $A = B$, we have the maximum case beacause $p(B|A) = 1$. EXAMPLE: A = Getting heads and B = Getting heads.
$$p(A \cap B) = p(A)p(B|A) = p(A) = 0.50$$

b. What is the largest $p(A \cup B)$ can be? What is the smallest it can be? Give examples to show that both extremes for $p(A \cup B)$ are possible.

$$p(A \cup B) = p(A) + p(B) - P(A \cap B)$$

If A and B are disjoint, then we have the maximum because we are not subtracting any overlap. EXAMPLE: A = Getting heads and B = Getting tails.

$$p(A \cup B) = p(A) + p(B) - P(A \cap B) = p(A) + p(B) + p(\emptyset) = p(A) + p(B) = 1$$

If $A = B$, then we have the minimum because we are subtracting away the probability of one of the events because it they are the same and it overlaps. EXAMPLE: A = Getting heads and B = Getting tails.

$$p(A \cup B) = p(A) + p(B) - P(A \cap B) = p(A) + p(A) - p(A) = p(A) = 0.50$$