Uncertainty and Persistence*

A Bayesian Update Semantics for Probabilistic Expressions

Deniz Rudin

drudin@ucsc.edu

DRAFT OF JUNE 12, 2015

COMMENTS WELCOME // CORRESPOND BEFORE CITING

1 Introduction

A vein of recent papers in both linguistics and philosophy have proposed probabilistic truth conditions for epistemic modals (Lassiter 2011 *et seq.*, Yalcin 2010 *et seq.*, Swanson 2006 *et seq.*, Moss 2015). Probabilistic truth conditions for other natural language phenomena have been proposed as well; for example Kaufmann (2004 *et seq.*) proposes a probabilist semantics for conditionals, and McCready & Ogata (2007) propose a probabilist semantics for evidentials. It is not clear how to incorporate these probabilistic truth conditions into traditional models of dynamic semantics (like Heim 1982, Kamp 1981, or Veltman 1996), which takes sentence meanings to be sets of worlds, and perform updates via set-theoretic operations like intersection. This paper will propose an all-purpose Bayesian Update Semantics for probabilistic expressions, with the following goals in mind:

- to treat all sentence meanings as the same kind of formal object, on which the same operations are performed
- to ensure that traditional propositions behave exactly as they behave in traditional update systems
- to derive the difference in behavior between traditional propositions and expressions of intermediate probability from their content

The meaning of there goals will become clearer throughout the paper.

Before describing my proposal for Bayesian Update Semantics, I will first introduce the data that I am interested in capturing, and articulate the project of this paper a little more finely,

*Thanks above all to Pranav Anand for countless hours of productive and enjoyable conversation, without which this paper would not exist. For reading and commenting on various incarnations of this paper, thanks to Adrian Brasoveanu, Donka Farkas, Dan Lassiter, Margaret Kroll, Karen Duek and Kelsey Kraus. Thanks also to Floris Roelofsen, for helpful discussion and a pleasant walk. Especially grateful thanks to Chris Barker & Dylan Bumford, who were this paper's saviors at one point in its history. All errors are mine.

in §2. In §3 I will explore the data in more detail, and extract from it three desiderata for probabilistic update systems for natural language semantics. In §4 I describe a typical dynamic update system, drawn primarily from Veltman (1996); many core concepts for any update semantics will be introduced here and carried forward through the rest of the paper. I will move on to a discussion of recent attempts to provide an update semantics for probabilistic expressions, and their failings, in §5. My proposal for Bayesian Update Semantics lives in §6; its similarities to previous dynamic update systems are discussed briefly in §7, and in §8 I wrap up.

2 Laying the Foundation

This paper concerns sentences of the following form:

(1) a. It might be raining.
b. My great grandfather probably had two kids.
c. There's a 25-30% chance that Nixon faked Watergate.
d. It's extremely unlikely that we're all just brains in vats.

Those sentences bear an intuitive relationship to these ones:

(2) a. It's raining.
b. My great grandfather had two kids.
c. Nixon faked Watergate.
d. We're all just brains in vats.

The sentences in (2) are all statements about what the world is like; the sentences in (1) are expressions of some intermediate degree of credence about whether or not the sentences in (2) are accurate descriptions of what the world is like. The sentences in (1) have been derived from the sentences in (2) via the addition of the operators *might*, *probably*, *25-30% chance* and *extremely unlikely*. Because the addition of these operators causes sentences to morph from confident descriptions of reality to expressions of intermediate credence toward those descriptions, I will call these operators CREDAL OPERATORS.

I will assume a standard propositional logic, and assign to it the responsibility of generating sentences like those in (2):

(3) The Language $\mathcal{L}$ (Syntax):
a. Any atomic proposition $p \subseteq \mathrm{W}$ is a formula of $\mathcal{L}$
Where W is some finite set of possible worlds
b. If φ and ψ are formulas of $\mathcal{L}$, then so are $(\varphi \wedge \psi)$ and $\neg(\varphi)$
c. Nothing else is a formula of $\mathcal{L}$

I will refer to formulas of $\mathcal{L}$, i.e. sentences like (2), as PROPOSITIONS. $\mathcal{L}$ can be extended via the addition of credal operators to generate sentences like those in (1):

(4) The Language $\mathcal{CL}$ (Syntax):
  a. Every formula of $\mathcal{L}$ is a formula of $\mathcal{CL}$
  b. For any formula φ of $\mathcal{L}$ and any C in $\mathfrak{C}$, Cφ is a formula of $\mathcal{CL}$
     Where $\mathfrak{C}$ is the set of all credal operators
  c. If Φ and Ψ are formulas of $\mathcal{CL}$, then so is $(\Phi; \Psi)$
  d. Nothing else is a formula of $\mathcal{CL}$

(5) Semantics for $\mathcal{L}$ and $\mathcal{CL}$:
  a. $[\![p]\!] = \{\mathrm{w} \in \mathrm{W} : p \text{ is true in w}\}$
  b. $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$
  c. $[\![\neg\varphi]\!] = \mathrm{W} - [\![\varphi]\!]$
  d. $[\![\mathrm{C}\varphi]\!] = [\![\mathrm{C}]\!]([\![\varphi]\!])$

Throughout the paper, I will omit denotation brackets wherever doing so will cause no confusion.

I will refer to formulas of $\mathcal{CL}$ that are not also formulas of $\mathcal{L}$, perhaps clunkily, as EXPRESSIONS OF SUBJECTIVE UNCERTAINTY, borrowing terminology from Swanson (2011). To avoid confusion, I will use the variables φ and ψ to represent propositions (i.e. formulas of $\mathcal{L}$), and I will use the variables Φ and Ψ to represent formulas of $\mathcal{CL}$ (both propositions and expressions of subjective uncertainty).

The definitions above allow for one credal operator to be added to a formula of $\mathcal{L}$; stacked credal operators are not a part of the language $\mathcal{CL}$. Stacked credal operators do exist in natural language:

(6)
  a. Bob might be a likely hire. Moss (2015)
  b. Bob is probably a possible hire. Moss (2015)
  c. John definitely must be home. Anand & Brasoveanu (2009)
  d. That absolutely might make sense for a bus driver. Anand & Brasoveanu (2009)

It would be trivial to define (4-b) recursively to allow for operator stacking; however, in this paper I will restrict my attention to formulas of $\mathcal{CL}$, in which at most one credal operator is allowed to scope over any proposition.[1] See Moss (2015), Anand & Brasoveanu (2009), Grosz (2009) for various views on the nature of doubled credal operators.

There are many theories of expressions of subjective uncertainty on the market (just a taste: Kratzer 1977 *et seq.*, Lassiter 2011 *et seq.*, Moss 2015, Rothschild 2012, Swanson 2006, Yalcin 2007 *et seq.*). By far the most-studied credal operator is *might* and its compatriots *possible* and *can* (Hacking 1967, Teller 1972, DeRose 1991, Kratzer 1977 *et seq.*, von Fintel & Gillies

[1] I also ignore in this paper interactions between credal operators and negation, and embedded credal operators.

2011, Stephenson 2007, MacFarlane 2011, Karttunen 1972, Yanovich 2014, Veltman 1996, Willer 2013); there is some controversy over whether *must* is an expression of subjective uncertainty or not: Karttunen (1972), Kratzer (1981, 1991) and Lassiter (2014b) argue that $must\varphi$ expresses less than total conviction in φ; Stone (1994) and von Fintel & Gillies (2010) argue that $must\varphi$ simply asserts φ, while also marking its evidentiary source as indirect. Navigating this controversy is far beyond the scope of this paper, so I will simply ignore *must* here.

In this paper I will argue for a set of desiderata about how expressions of subjective uncertainty should behave in dynamic, update-based semantic systems (Heim 1982, 1983, Veltman 1996, Gillies 2004, Beaver 2001); once these desiderata have been established, I will go on to evaluate extant implementations of expressions of subjective uncertainty in such models (principally Veltman 1996, Willer 2010, 2013, and Yalcin 2012), and, once these implementations have been found wanting relative to the desiderata I propose, I will describe a Bayesian Update Semantics as a general-purpose way to implement probabilistic expressions in dynamic semantics. My desiderata center around the contention that expressions of subjective uncertainty express beliefs which are *not* persistent (i.e. beliefs that won't necessarily survive the addition of new information that is compatible with all previous information), and propositions express beliefs that *are* persistent; I will argue that this behavior is not captured by any extant dynamic implementation of these expressions, but that if we make the move of treating updates in a dynamic semantics as Bayesian updates, i.e. as conditionalization, then expressions of subjective uncertainty will behave the way we want them to without altering the way propositions behave.

I will make the standard assumption throughout this paper that an assertion comprises a proposal that all interlocutors agree to behave as though the asserted content is true (Stalnaker 1978, Farkas & Bruce 2010). For the sake of simplicity, I will consider only solipsistic information states, not common grounds, and I will only consider scenarios in which interlocutors sincerely believe what is asserted to them; these constraints are imposed only to keep examples from becoming unmanageably complex, and they can be removed to no devastating effect on my arguments or solutions.

Nota bene: the focus on dynamic implementation in this paper leaves no room for detailed consideration of purely truth-conditional work; the most gaping omission here is Kratzer's (1977 *et seq.*) seminal theory. Her theory lacks a dynamic component, and so it's difficult to say what predictions it makes relative to the desiderata I've established here. The theory has been voluminously criticized and voluminously defended in the literature, and engaging with it at the level of thoroughness required is outside the scope of this paper.

3 Uncertainty and Malignance

I'll begin this section with an example:

(7) Bill is a chemist. Five minutes ago, he combined two substances, x and y,

in an opaque soundproof container. He believes based on previous experimentation that there is a 30% chance that combining x and y will cause a small explosion within a minute of their combination—the other 70% of the time they simply coagulate harmlessly. So, he believes that on this particular occasion, his concoction *might* have exploded. He opens the container, and sees only a coagulated mass. Now, Bill believes that on this particular occasion, his concoction *didn't* explode.

This scenario describes, in some sense, an example of BELIEF REVISION. Call the proposition that Bill's concoction exploded φ. At the beginning of the scenario, Bill believes that φ *might* be true; by the end of the scenario, Bill believes that φ is false. So his beliefs about φ have changed.

However, this belief revision did not involve Bill's discovery of any errors in his reasoning, or his discovery that any information that he had previously taken to be factual was in fact spurious. The belief revision was *error-free*. I'll call such error-free revisions of belief BENIGN:

(8) For any agent **A**, any proposition φ, and any times t, t', if **A**'s information state at t licenses a different degree of credence toward φ than **A**'s information state at t', and at no time between t and t' did **A** discover an error in her reasoning or an error in the evidence available to her, we call the shift in **A**'s beliefs about φ a BENIGN belief revision.

In other words, in cases of benign belief revision, one's credal stance toward a proposition has changed, but one has not thrown out as false any previously believed facts, or reclassified as invalid any lines of reasoning previously thought to be valid. This is opposed, of course, to belief revision that is predicated on the discovery of errors. Consider this variation on the above example:

(9) Bill is an amateur chemist. Five minutes ago, he combined two substances, x and y, in an opaque soundproof container; he believes based on an article that he read in his Junior Chemist magazine that combining x and y always causes an explosion. So, he believes that on this particular occasion, his concoction has exploded. He opens the box, and sees that no explosion occurred. Now, Bill believes that on this particular occasion, his concoction didn't explode. He had read the article too hastily; it contained the correct 30/70 figures.

Just like (7), the scenario in (9) describes an occurrence of belief revision; however, it is not error-free. Again, call the proposition that Bill's concoction exploded φ. At the beginning of the scenario, Bill believes that φ is true; by the end of the scenario, Bill believes that φ is false. However, unlike in (7), in this case Bill's belief revision requires the revelation that something that he previously took to be a fact was actually not true. I'll call cases of belief revision that are predicated on the discovery of errors of reasoning or the discovery of the spuriousness of what was previously taken to be factual TRUE belief revision—true because in these cases one's beliefs about what is *true* have been revised.

I've selected these two examples for the following reason: the possibility of benign belief revision is characteristic of beliefs that express uncertainty about the truth or falsity of a proposition. Surely this is not a controversial claim—part of what it means to be uncertain whether a proposition is true is to be open both to the possibility that it is true and to the possibility that it is false. However, benign belief revision is impossible in principle with respect to a belief that a proposition is true, or that it is false. I'll refer to this phenomenon as THE BELIEF REVISION ASYMMETRY.

It's not enough, however, to say that uncertainty about whether a proposition is true must be able to give way both to certainty that it is true and certainty that it is false; the kinds of benign belief revision available relative to uncertainty about the truth of some proposition are much richer than that. Uncertainty must also be able to morph into other degrees of uncertainty. Let me explain:

(10) Say that it's election day, and Bill is a statistician tasked with calculating the results of the election. He is fed information about election results slowly, as different precincts report in; Bill recalculates the candidates' chances of victory given each new report. After the first report, Bill calculates that there is a 65-70% chance that the democratic candidate won the election. After the second report, Bill calculates that there is a 42-45% chance that the democratic candidate won the election. After the third report, Bill calculates that there is a 29-32% chance that the democratic candidate won the election. After the fourth report, Bill calculates that there is less than a 1% chance that the democratic candidate won the election.

In this example, call the proposition that the democratic candidate won the election φ. As more and more precincts report in, Bill's information improves, and his understanding of likelihood that φ is true shifts. After the first report, Bill believes that φ is probably true. After the fourth report, Bill believes that φ is extremely unlikely to be true. The point that this example makes is a simple extension of the point made in the previous section: benign belief revision doesn't simply take the form of movement from uncertainty to certainty; beliefs can also be revised benignly from one range of uncertainty about a proposition to another, potentially non-overlapping range of uncertainty about the same proposition. I'll call this phenomenon SLIPPERY INTERMEDIATE CREDENCES.

My contention in this paper is that these phenomena about belief revision are reflected in linguistic behavior involving expressions of subjective uncertainty: that cases like these, if modified to be suitably linguistic, can provide us with desiderata for the success of models of the contribution of expressions of the form *might*φ, *probably*φ, *extremely-unlikely*φ, and so on. Consider this variation on (7):

(11) Bill is a chemist. Five minutes ago, he combined two substances, x and y, in an opaque soundproof container. He believes based on previous experimentation that there is a 30% chance that combining x and y will cause a small explosion within a minute of their combination—the other 70% of the time they simply coagulate harmlessly. So, he believes that on this particular occasion, his concoction *might* have exploded. He expresses this belief to Frank, who is in the other room, by shouting 'My concoction might have exploded! Let me check!' He opens the container, and sees only a coagulated mass. Now, Bill believes that on this particular occasion, his concoction *didn't* explode. He expresses this belief to Frank, who is in the other room, by shouting 'It didn't explode!'

In this case, Frank has no access to any of the evidence that Bill has; he only has access to what Bill tells him, namely that the concoction might have exploded, and, later, that it didn't. We'll assume that Frank believes Bill to be an honest and informed interlocutor, and accepts both of his assertions at face value. In this case, Frank first incorporates the information conveyed by 'My concoction might have exploded,' and then goes on to incorporate the information conveyed by 'It didn't.' There is nothing strange about a situation like this; it would not be appropriate for Frank to respond to the second statement by saying something like 'Hey wait a minute, I thought you said it *might* have exploded!'

We see something very different when we look at a linguistically enriched version of (9):

(12) Bill is an amateur chemist. Five minutes ago, he combined two substances, x and y, in an opaque soundproof container; he believes based on an article that he read in his Junior Chemist magazine that combining x and y always causes an explosion. So, he believes that on this particular occasion, his concoction has exploded. He expresses this belief to Frank, who is in the other room, by shouting 'My concoction has exploded by now!' He opens the box, and sees that no explosion occurred. Now, Bill believes that on this particular occasion, his concoction didn't explode. He expresses this belief to Frank, who is in the other room, by shouting 'It didn't explode!'

In this case, it would be entirely appropriate for Frank to feel as though Bill has contradicted himself; Frank can perfectly well respond with 'Hey wait a minute, I thought you said it *did* explode!'

Expressing uncertainty about whether or not φ is true and then, after gathering information, following up by asserting that φ is false doesn't seem anomalous or contradictory. However, expressing certainty that φ is true and then, after gathering information, following up by asserting that φ is false does. This provides us with our first desideratum for linguistic models of the affect of expressions of subjective uncertainty:

(13) **Desideratum 1: The Belief Revision Asymmetry**
Expressions of subjective uncertainty about φ must be followable without contra-

diction by the proposition $\neg\varphi$; the proposition φ must *not* be followable without contradiction by the proposition $\neg\varphi$.

We can linguisticize the example in (10) too:

(14) Say that it's election day, and Bill is a statistician tasked with calculating the results of the election. He is fed information about election results slowly, as different precincts report in; Bill recalculates the candidates' chances of victory given each new report. After the first report, Bill calculates that there is a 65-70% chance that the democratic candidate won the election. He tells Frank, who is in the other room: 'The democratic candidate probably won! But let's wait for more results before celebrating.' After the second report, Bill calculates that there is a 42-45% chance that the democratic candidate won the election. He tells Frank: 'More results are in! The chances that the democratic candidate won are lower now.' After the third report, Bill calculates that there is a 29-32% chance that the democratic candidate won the election. He tells Frank: 'More results are in! The democratic candidate probability didn't win.' After the fourth report, Bill calculates that there is less than a 1% chance that the democratic candidate won the election. He tells Frank: 'It's extremely unlikely that the democratic candidate won.'

We see again that the evolution of Bill's beliefs relative to new statistical evidence is mirrored by the evolution of Frank's beliefs relative to new linguistic evidence: Frank can incorporate each new statement about the probability that φ is true with no sense of contradiction with previous linguistic expressions. But if Bill were to tell Frank that the democratic candidate won, and then were to tell Frank that more results are in and now the democratic candidate lost, Frank would not be able to accept the information being relayed to him as coherent. This provides us with a second desideratum for linguistic models of the effect of expressions of subjective uncertainty:

(15) **Desideratum 2: Slippery Intermediate Credences**
Expressions of some range of uncertainty about φ must be followable without contradiction by expressions of some other range of uncertainty about φ—potentially a non-overlapping range.

There's a final desideratum to propose for these models. Consider the simple observation that assertions of expressions of uncertainty can provide new information:[2]

(16) **A**: I don't follow baseball at all. What can you tell me about tomorrow's game?
a. **B**: Team X will probably win.
b. **B**: It is extremely unlikely that Team Y will win.

[2] Example due to Dan Lassiter, p.c.

In these cases, it is quite clear that **B** has given **A** information that she didn't already have. If she takes **B** to be honest and informed, and accepts his statements at face value, then her expectations about the outcome of the game have been changed: she will be less surprised by a victory by Team X, and more surprised by a victory by Team Y. A linguistic model of the effect of an assertion of an expression of subjective uncertainty should capture the fact that such assertions can, at least in principle, be INFORMATIVE—that is to say, they can have a substantive effect on the information state of an addressee. This supplies us with our third and final desideratum:

(17) **Desideratum 3: Informativity**
Expressions of subjective uncertainty must be (potentially) informative updates.

Dynamic, update-based semantic systems have been designed and studied relative to their behavior wrt a number of different aspects of natural language: quantification (Groenendijk et al. 1996), anaphora (Kamp 1981), presupposition (Heim 1982, 1983), conditionals (Gillies 2004), expressions of belief and of knowledge (Heim 1992, Beaver 2001); this paper is the first study of which I am aware that investigates the behavior of such systems relative to expressions of subjective uncertainty with the desiderata above in mind.

I will evaluate three previous systems before proposing my own. In §4, I will discus Veltman (1996)'s Update Semantics for *might*, and in §5, I will discuss Willer (2010, 2013)'s implementation of Veltman's *might* in a structurally-enriched version of Update Semantics, and Yalcin (2012)'s extension of Willer's system to probability spaces. My proposed Bayesian Update Semantics is put forward in §6.

4 Tests in Update Semantics

Veltman (1996) (henceforth just Veltman) provides a denotation for *might* in his Update Semantics—a semantics that models the meaning of a proposition as the effect of accepting its assertion on an agent's information state, and relates the structure of an agent's information state to which propositions they can cooperatively assert. Many of the notions put forward in this section will be carried forward through the rest of the paper, and this section will also provide us with a more precise vocabulary in which to couch the generalizations made in the previous section. The basic update system works like this:

(18) a. An information state I is a set of worlds
b. $[\cdot]$ is an update function; we read $\varphi[\psi]$ as 'φ updated with ψ'
c. For all formulas of $\mathcal{L}$ φ, $I[\varphi] = I \cap \varphi$
d. For all formulas Φ and Ψ of $\mathcal{CL}$: $I[\Phi; \Psi] = I[\Phi][\Psi]$

Veltman assumes all of this machinery except for conjunction; he assumes an intersective conjunction operator. The treatment of dynamic conjunction as sequential update is developed and defended by Stalnaker (1974) and Heim (1982), and it's a necessary assumption for us to make, because we'll be interested in conjoining things that do not represent sets of worlds.

Information states represent the set of worlds compatible with some agent's beliefs. Propositions also denote sets of worlds, and updates with sets of worlds are intersective. The maximally uninformed information state (the information state of somebody who knows nothing) is the set of all possible worlds W, and information growth is modeled as the shrinkage of the information state—the more information we have, the fewer possibilities are compatible with our beliefs. The empty set is called THE CONTRADICTORY INFORMATION STATE (sometimes referred to as the 'absurd' information state in the literature); if an update results in the empty set, something has gone wrong—no worlds are compatible with all of our information. In the case that a proposed update would result in the absurd information state, we expect agents to reject that update if they are confident in their previous information, or reject some of their previous information if they are confident in the truth of the proposed update.

I will call updates that don't result in the contradictory information state CANONICAL updates; and I will call information states that are derivable from I via canonical updates canonical EVOLUTIONS of I:

(19) CANONICAL UPDATES AND EVOLUTIONS:
  a. φ is a canonical update to I iff $I[\varphi] \neq \varnothing$
  b. I' is a canonical evolution of I iff $I' \neq \varnothing$ and there is some update sequence $[\varphi_1] \ldots [\varphi_{\mathrm{n}}]$ such that $I[\varphi_1] \ldots [\varphi_{\mathrm{n}}] = I'$

If there is no way for an agent to canonically update with a proposition that they have incontrovertible evidence for, then true belief revision will be necessary to accommodate that new information. So the notion of canonical update can help us clarify the nature of the belief revision asymmetry. We want the effect of accepting an assertion of an expression of uncertainty about φ not to preclude the possibility of canonical update with $\neg\varphi$ or with φ.

We can take the norm of assertion to be LICENSING:

(20) LICENSING:[3]
  I LICENSES φ iff $I[\varphi] = I$

An agent can cooperatively assert some utterance φ iff an update with φ would have no effect on that agent's information state; i.e. iff they believe φ to be true. The reason for this is straightforward: an assertion of φ is taken to be a proposal to update with φ; an agent can only cooperatively propose that her interlocutors update with φ if she already believes φ herself.

[3]Veltman defines this as 'acceptance'. I've diverged in terminology here because I want to capture the fact that an information state with this relationship to φ *licenses* an assertion of φ.

Veltman doesn't analyze finer-grained expressions of uncertainty, but he does analyze *might*. His analysis of *might* takes it to be a 'test' on information states. A test is an update that returns an information state unchanged if certain conditions are met, and returns the empty set otherwise:

(21) $I[might\varphi] = \begin{cases} I \text{ if } I[\varphi] \neq \varnothing \\ \varnothing \text{ ow} \end{cases}$

In Veltman's terms, $might\varphi$ is a 'consistency test': it tests whether the information state is consistent with φ. As Veltman observes, this produces a good result wrt the belief revision asymmetry characteristic of expressions of subjective uncertainty—because update with sets of worlds is intersective, those updates will be persistent:

(22) PERSISTENCE:
Φ is PERSISTENT iff whenever I licenses Φ, all possible canonical evolutions of I also license Φ.

There is no canonical update in this system that adds worlds to an information state; just intersective updates and tests. Therefore, once an intersective update with a proposition φ has been performed, that information state and all of its canonical evolutions will be subsets of φ, and therefore license φ. In systems without canonical ways to add worlds to information states, intersective updates will always be persistent updates.

However, update with $might\varphi$ will not necessarily be persistent. An information state licenses $might\varphi$ iff it has at least one φ-world in it. If such an information state also has at least one non-φ world in it, then it can be canonically updated with $\neg\varphi$; after this update the information state will no longer license $might\varphi$. Therefore, not all formulas of $\mathcal{CL}$ comprise persistent expressions.

The notion of persistence gives us another way of formally rephrasing the belief revision asymmetry:

(23) THE (IM)PERSISTENCE ASYMMETRY:
Propositions must comprise persistent updates; expressions of subjective uncertainty must not comprise persistent updates.

Or to put it another way:

(24) THE CANONICAL EVOLUTION ASYMMETRY:
Updating an information state I with an expression of subjective uncertainty about φ should not preclude in principle the possibility of canonical evolutions of I licensing $\neg\varphi$.

Given Veltman's system, we can have an update with *might*φ followed by an update with $\neg\varphi$. To see this is simple. Assume that we have an information state containing both φ and non-φ worlds:

(25) a. $I = \{w_1, w_2, w_3\}$
b. $\varphi = \{w_1\}$

First, we update I with *might*φ; this update returns I unchanged, because I has a φ-world in it.

(26) $I[might\varphi] = I$

We can now update with $\neg\varphi$:

(27) $I[might\varphi][\neg\varphi] = \{w_2, w_3\}$

The φ-worlds are removed, and the result is not the contradictory information state. $I[might\varphi][\neg\varphi]$ is a licit update sequence.

So far so good. Observe also that an update with $\neg\varphi$ will not be followable by an update with *might*φ: the update with $\neg\varphi$ removes all φ-worlds from the information state, so the update with *might*φ will produce the absurd information state. Again, so far so good. Veltman's system derives the (im)persistence asymmetry for the operator *might*. How can this system be extended to incorporate the credal operators *probably* and *extremely-unlikely*?

I'll follow Yalcin (2010, 2012) in assuming that *probably*φ means that there is a higher than 50% chance that φ is true; I'll assume, somewhat arbitrarily, that *extremely-unlikely*φ means that there is less than a 5% chance that φ is true. Veltman's system as it stands has no way of expressing things like this. However, if we enrich this model with Yalcin (2010)'s semantics for probability operators, we can extend the 'test' mechanic to a wide variety of credal operators. Yalcin (2012) describes just such a system, which he names 'sharp context probabilism'; I'll present this model now.

Understanding Yalcin's model requires some background in Bayesian probability. Specifically, we must understand the concept of a probability measure, the concept of conditional probability, and the concept of conditionalizing a probability measure on a proposition.

4.1 Begin Bayesian Excursus

4.1.1 Probability Measures

(28) Given some finite set of worlds W, $\mu : \wp(W) \rightarrow [0,1]$ is a probability measure iff
a. REALISM: $\mu(W) = 1$
b. FINITE ADDITIVITY: $[\forall\varphi,\psi \subseteq W : \varphi \cap \psi = \varnothing](\mu(\varphi) + \mu(\psi) = \mu(\varphi \cup \psi))$

A probability measure is a function assigning to each subset of W some real number in the interval [0,1]. We will consider here only those measures that assign the value 1 to W (i.e., that express certainty that the actual world is somewhere in W), and that behave additively (i.e., that take the probability of the union of any two disjoint propositions to be the probability of one plus the probability of the other).

We'll call the set of all measures meeting the conditions in (89) $\mathfrak{M}$.

4.1.2 Conditional Probability

It will be necessary to talk about the probability of φ *given* ψ for any two propositions φ and $\psi \subseteq$ W. This is written: $\mu(\varphi|\psi)$.

(29) $$\mu(\varphi|\psi) = \frac{\mu(\varphi \cap \psi)}{\mu(\psi)}$$

To provide a concrete example, consider a measure μ which gives the following values to φ and ψ:

(30)

	φ	$\neg\varphi$	
ψ	.4	.1	.5
$\neg\psi$	.1	.4	.5
	.5	.5	

The overall value assigned to φ by μ is .5; the overall value assigned to ψ is also .5. We call the overall probability assigned to a proposition its MARGINAL PROBABILITY. The probability assigned to the intersection of φ and ψ is .4. Given this, if we want to know what the probability of φ would be if ψ were true (all else being equal), we can apply the formula in (29):

(31) $$\mu(\varphi|\psi) = \frac{\mu(\varphi \cap \psi)}{\mu(\psi)} = \frac{.4}{.5} = .8$$

Crucially, if the probability assigned to some proposition φ by μ is 0, there is no conditional probability that can assign to φ any value other than 0. Consider a measure μ that gives the following values to φ and ψ:

(32)

	φ	$\neg\varphi$	
ψ	0	.3	.3
$\neg\psi$	0	.7	.7
	0	1	

μ assigns 0 to φ (and therefore, by additivity and realism, assigns $\neg\varphi$ 1); there's a 30% chance that ψ is true. What is the probability of φ given ψ?

(33) $$\mu(\varphi|\psi) = \frac{\mu(\varphi \cap \psi)}{\mu(\psi)} = \frac{0}{.3} = 0$$

If a measure μ assigns 0 to φ, then $\mu(\varphi|\psi) = 0$ *for all* ψ. The numerator will always be 0.

4.1.3 Conditionalizing a Measure on a Proposition

To conditionalize a measure μ on a proposition φ is to derive from it a new measure μ' that assigns to all propositions ψ the value that μ assigned to ψ given φ. μ conditionalized on φ is written $\mu\restriction_{\varphi}$.

(34) $\mu\restriction_{\varphi}$ = that measure μ' s.t. $\forall\psi \subseteq$ W, μ'$(\psi) = \mu(\psi|\varphi)$

I'll make one comment here before moving on: it's impossible to conditionalize a measure on a proposition to which that measure assigns the value 0. Let's consider again a measure μ that gives the following values to φ and ψ:

(35)

	φ	$\neg\varphi$	
ψ	0	.3	.3
$\neg\psi$	0	.7	.7
	0	1	

To conditionalize this measure on φ would result in a measure that assigns to all ψ the value $\mu(\psi|\varphi)$.

(36) $$\mu(\psi|\varphi) = \frac{\mu(\psi \cap \varphi)}{\mu(\varphi)} = \frac{0}{0} = ??$$

All such conditional probabilities have the denominator 0. So no such measure exists.

To summarize to important points:

- If a measure assigns to some proposition φ the value 0, there is no way to conditionalize that measure such that the result does not give φ 0
- Likewise, by additivity and realism, if a measure assigns to some proposition φ the value 1, there is no way to conditionalize that measure such that the result does not give φ 1
- It is impossible to conditionalize a measure on a proposition to which that measure assigns the value 0

End Bayesian Excursus

Continuing with Yalcin (2010)'s semantics for epistemics: Instead of assuming that an information state is a set of worlds, we can define it like so:

(37) $I = \langle s,\mu \rangle$ is an information state iff:

a. $s \subseteq \mathrm{W}$

b. $\mu \in \mathfrak{M} \wedge \mu(s) = 1$

An information state is now treated as a pair of a set of worlds and some probability measure that assigns 1 to that set of worlds (i.e. that expresses certainty that the actual world is somewhere in that set). Updating with propositions now includes a conditionalization step:

(38) *Updates involve Conditionalization*:
$\forall \varphi \subseteq \mathrm{W}$, $I[\varphi] = \langle \hat{s},\mu\restriction_{\hat{s}} \rangle$
Where $\hat{s} = s[\varphi]$

So: when an information state is updated with a new proposition, that proposition is intersected with the information state's set of worlds, and the information state's probability measure is conditionalized on the result. Intersecting with φ kicks out all non-φ worlds from s; the probability measure then redistributes probability such that everything outside of s is assigned the value 0.

With this technology in hand, we can now understand Yalcin (2012)'s test-based semantics for credal operators; he proposes a probabilistic test-based denotation for *probably*, and it can be treated as a template for other operators like *extremely-unlikely*; Yalcin (2012) takes credal operators to be tests targeting an information state's probability measure:

(39) a. $I[probably \varphi] = \begin{cases} I \text{ iff } \mu(\varphi) > .5 \\ \varnothing \text{ ow} \end{cases}$

b. $I[extremely\text{-}unlikely \varphi] = \begin{cases} I \text{ iff } \mu(\varphi) < .05 \\ \varnothing \text{ ow} \end{cases}$

An information state is unchanged by accepting an assertion of *probably*φ or *extremely-unlikely*φ if its probability measures assigns an appropriate value to φ; anomaly results in all other cases. Again, we derive the (im)persistence asymmetry: updates like either of these can be canonically followed by updates with either $\neg\varphi$ or φ.

However, this system achieves this result precisely by way of treating expressions of subjective uncertainty as uninformative in principle. This is due to the nature of the 'test' form of update: an information state is returned unaltered if it meets the specifications of the test.

(40) Informativeness:
An update with φ is INFORMATIVE iff $I[\varphi] \neq I$; it is REDUNDANT otherwise.

If an update with an expression of subjective uncertainty is successful, then it is redundant; by definition it is not possible for a test to meaningfully alter an information state. If we model all successful updates with expressions of subjective uncertainty as having no discernible effect on information states, then we are completely incapable of satisfying Desideratum 3: Informativity. The mechanism of the 'test' appears to have failed us.

The mechanism of the test is not a good way of capturing the (im)persistence asymmetry if we consider such tests within a Veltmanian model, in which an information state is a single set of worlds (or, in Yalcin (2012)'s extension, a single pair of a set of worlds and a probability measure). In the next section, I'll consider systems that treat expressions of subjective uncertainty as Veltmanian tests in a model where information states are taken to be *sets* of sets of worlds, or *sets* of pairs of a set of worlds and a probability measure. In these models, tests can derive the (im)persistence asymmetry while also allowing updates with expressions of subjective uncertainty to be informative.

5 Superstates and Substates

Willer (2010, 2013) develops a model in which information states are taken to be sets of sets of worlds. To formalize:

(41) *Willerian Information States:*[4]
I is a Willerian Information State iff $\forall i \in I$, $i \subseteq \mathrm{W}$

I'll use the term SUPERSTATE to refer to a Willerian information state, and the term SUBSTATE to refer to an element of a Willerian information state. Updates to superstates are defined like so:

(42) *Updates to Information States:*
$I[\Phi] = \{i \subseteq \mathrm{W} : \exists i' \in I(i'[\Phi] = i)\}$

In other words, to update an information state is to perform pointwise updates on each of its substates. Updates with sets of worlds are defined at the level of the substate in the Veltmanian fashion:

(43) *Updates with sets of worlds:*
$\forall \varphi \subseteq \mathrm{W}$, $i[\varphi] = i \cap \varphi$

[4] Willer's model excludes the empty set from information states. This is of no crucial importance, and I've omitted that restriction here.

Willer takes *might* to be Veltmanian as well:

(44) *Updates with mightφ:*

$$i[might\varphi] = \begin{cases} i \text{ iff } s_\mathrm{i} \cap \varphi \neq \varnothing \\ \varnothing \text{ ow} \end{cases}$$

This is a very interesting way to model *might*, for two reasons. The first is that Willer's implementation takes Veltman's *might*, which is uninformative in principle, and coerces informativity out of it. Let's see how. Based on (42) and (44), an update to a superstate I with *might*φ will always deliver some subset of I, namely an information state containing all those substates of I that contain at least one φ-world. Consider the information state in (45), containing four worlds w, u_1, u_2, and u_3, where w is in φ and u1-u3 are not in φ. I is the powerset of {w, u1, u2, u3}:

(45)
$$I = \left\{ \begin{array}{c} \{\mathrm{w}, \mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3\} \\ \{\mathrm{w}, \mathrm{u}_1, \mathrm{u}_2\} \quad \{\mathrm{w}, \mathrm{u}_1, \mathrm{u}_3\} \quad \{\mathrm{w}, \mathrm{u}_2, \mathrm{u}_3\} \quad \{\mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3\} \\ \{\mathrm{w}, \mathrm{u}_1\} \quad \{\mathrm{w}, \mathrm{u}_2\} \quad \{\mathrm{w}, \mathrm{u}_3\} \quad \{\mathrm{u}_1, \mathrm{u}_2\} \quad \{\mathrm{u}_1, \mathrm{u}_3\} \quad \{\mathrm{u}_2, \mathrm{u}_3\} \\ \{\mathrm{w}\} \quad \{\mathrm{u}_1\} \quad \{\mathrm{u}_2\} \quad \{\mathrm{u}_3\} \\ \varnothing \end{array} \right\}$$

An update with *might*φ will remove from the information state all substates that contain only non-φ worlds. The total world-content of the information state is unchanged; if you take the grand union of all the substates, it is the same after the update with *might*φ as it was before. However, the structure of the information state is perceptibly changed; prior to the update, it was downward closed (or closed under the subset relation); after the update it is not. This is because I contained elements that contained no φ-worlds; these elements are removed by the update with *might*φ. I have represented elements of I that are removed by the update with *might-*φ as struck-out sets:

(46)
$$I[might\text{-}\varphi] = \left\{ \begin{array}{c} \{\mathrm{w}, \mathrm{u}_1, \mathrm{u}_2\} \quad \{\mathrm{w}, \mathrm{u}_1, \mathrm{u}_3\} \quad \{\mathrm{w}, \mathrm{u}_2, \mathrm{u}_3\} \quad \cancel{\{\mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3\}} \\ \{\mathrm{w}, \mathrm{u}_1\} \quad \{\mathrm{w}, \mathrm{u}_2\} \quad \{\mathrm{w}, \mathrm{u}_3\} \quad \cancel{\{\mathrm{u}_1, \mathrm{u}_2\}} \quad \cancel{\{\mathrm{u}_1, \mathrm{u}_3\}} \quad \cancel{\{\mathrm{u}_2, \mathrm{u}_3\}} \\ \{\mathrm{w}\} \quad \cancel{\{\mathrm{u}_1\}} \quad \cancel{\{\mathrm{u}_2\}} \quad \cancel{\{\mathrm{u}_3\}} \\ \varnothing \end{array} \right\}$$

The update with *might*φ has removed stuff from I, so it meets our definition of informativeness. Willer chooses to associate structures like the above, in which every substate in an information state contains at least one φ-world, as representing 'live' possibility associated with φ.[5]

[5] This may be too strong: there are cases in which *might*φ is followed by an explicit dismissal of φ as irrelevant:

(i) Running the Large Hadron Collider might destroyer the universe, but the chance is so astronomically low that we don't need to worry about it.

Willer's model is just as effective at deriving the belief revision asymmetry as Veltman's. Consider what happens if we take the information state above and update it with $\neg\varphi$:

(47) $$I[\textit{might-}\varphi][\neg\varphi] = \left\{ \begin{array}{ccc} \{u_1, u_2\} & \{u_1, u_3\} & \{u_2, u_3\} \\ \{u_1\} & \{u_2\} & \{u_3\} \\ & \varnothing & \end{array} \right\}$$

When we remove all of the φ-worlds from each individual substate, the result is a downward-closed set containing only non-φ worlds. Because update is pointwise within the superstate, the distinction between test updates and intersective updates allows updates to be non-monotonic at the level of the superstate, while requiring them to be monotonic at the level of the substate. In other words, though all substates can only stay the same or shrink as the result of an update, elements that have been taken out of the superstate can later be reintroduced: I contained the set $\{u_1, u_2\}$; that set was knocked out of $I[\textit{might}\varphi]$ due to the absence of φ-worlds in it; and in $I[\textit{might}\varphi][\neg\varphi]$ it was reintroduced, by removing the φ-world from $\{w, u_1, u_2\}$. In fact, every element of $I[\textit{might-}\varphi][\neg\varphi]$ (other than the empty set) is an element that was *removed* from I by the update with *might*φ.

We can also show that if we reverse the order of the updates, the update sequence is incoherent:

(48) $$I[\neg\varphi] = \left\{ \begin{array}{ccc} & \{u_1, u_2, u_3\} & \\ \{u_1, u_2\} & \{u_1, u_3\} & \{u_2, u_3\} \\ \{u_1\} & \{u_2\} & \{u_3\} \\ & \varnothing & \end{array} \right\}$$

The update with $\neg\varphi$ removes all φ-worlds from all substates.

(49) $$I[\neg\varphi][\textit{might}\varphi] = \{ \varnothing \}$$

The update with *might*φ then kicks out all states with no φ worlds in them, which, because of the update with $\neg\varphi$, is all of them but the empty set.

Willer is only concerned with *might*, but it's easy to see how a model like this could be extended to deal with finer-grained expressions of subjective uncertainty. Yalcin (2012) does exactly this. We can take a Willer model and extend it to a Willer/Yalcin model by treating elements of I not as sets of worlds but as pairs of a set of worlds and a probability measure. Let's define a substate like so:

(50) $i = \langle s,\mu \rangle$ is a potential substate iff:

a. $s \subseteq W$

This certainly doesn't seem contradictory. Whether or not this is a problem for the meaning that Willer ascribes to this structural configuration depends on whether you model the effect of 'astronomically low chance' or 'we don't need to worry about it' as altering this characteristic structure. I think it might be better to identify this structural configuration with something like 'acknowledged' possibility.

b. $\mu \in \mathfrak{M} \wedge \mu(s) = 1$

So now instead of a set of sets of worlds, we have a set of pairs of a set of worlds and a probability measure. The maximally uninformed information state would be the set of all potential substates i. We can describe information states that are not maximally uninformed in terms of CREDENCE STRUCTURES: for example, if every probability measure in an information state assigns .7 to φ, we say that that information state's structural credence toward φ is 70%. And likewise, if no probability measure in the information state assigns less than .1 to φ, we say that that information state's structural credence toward φ is at least 10%.

As before, updates to a superstate are simply passed down to its substates; updates to substates behave as discussed in the previous section:

(51) *Updates with sets of worlds*:
$\forall \varphi \subseteq \mathrm{W},\ i[\varphi] = \langle \hat{s}, \mu\restriction_{\hat{s}} \rangle$
Where $\hat{s} = s[\varphi]$

Tests work as expected:

(52) *Updates with might-φ*:
$$i[might\varphi] = \begin{cases} i \text{ iff } s_{\mathrm{i}} \cap \varphi \neq \varnothing \\ \varnothing \text{ ow} \end{cases}$$

Just as before, an update with *might*φ leaves a substate unchanged if it is compatible with φ, and reduces it to the empty set otherwise. And just as before, we can treat other expressions of subjective uncertainty as comprising tests on the level of the substate:

(53) a. $$i[probably\varphi] = \begin{cases} i \text{ iff } \mu(\varphi) > .5 \\ \varnothing \text{ ow} \end{cases}$$

b. $$i[extremely\text{-}unlikely\varphi] = \begin{cases} i \text{ iff } \mu(\varphi) < .05 \\ \varnothing \text{ ow} \end{cases}$$

The Willer/Yalcin model gives us information states that are very rich in their representation of uncertainty: when we start at the maximally uninformed information state, we start with not only all subsets of W in the information state, but also with all possible credence distributions over those sets. However, as we update with expressions of subjective uncertainty, this winnows down the possible credence distributions over the sets of worlds in the information state.

Updates with expressions of subjective uncertainty are all (potentially) informative in this model, because, as tests, they kick out states which don't match the appropriate degree of credence, lending a signature credence structure to information states that they have updated. The (im)persistence asymmetry is derived exactly as before: an update with intermediate credence toward φ will kick out states without the appropriate degree of credence;

a subsequent update with $\neg\varphi$ will intersect and conditionalize, reintroducing states which give no credence to φ. And if we change the order of the updates, an update with $\neg\varphi$ will create an information state all of whose substates assign 0 probability to φ; a subsequent update with an intermediate degree of credence towards φ will simply remove all of these states, and the empty set will be the result.

Yalcin (2012) is skeptical of exactly that attribute of this system that I take to be its chief virtue: the fact that it derives this asymmetry. I'll explore his reasons for discomfort now. Yalcin (2007) presents data like the following, which he refers to as EPISTEMIC CONTRADICTION:

(54) a. #It is raining and it might not be raining.
b. #It is not raining and it might be raining.

The badness of the cases in (54) is clear from the perspective of Willer's system: $\forall I$, $I[\neg\varphi][might\varphi] = \varnothing$; in other words, $\neg\varphi$ followed by $might\varphi$ will never be a licit update sequence. I'll call update sequences that always lead to the empty set INCOHERENT:

INCOHERENCE:
$\forall\Phi,\Psi$, if $\forall I : I[\Phi][\Psi] = \varnothing$, then the update sequence $[\Phi][\Psi]$ is INCOHERENT.

Note that incoherence is not commutative; one feature of dynamic conjunction is that an incoherent update sequence may be perfectly licit if the order of updates is reversed. Willer's system generates the pleasant result that if you reverse the order of the updates with $\neg\varphi$ and $might\varphi$, you'll have a (potentially) licit update sequence. This prediction cashes out the intuition that I have taken to be a crucial desideratum for any theory of the semantics of *might*: the possibility of canonical evolution to both φ and $\neg\varphi$ after an update with $might\varphi$. Yalcin (2012), however, argues that allowing $I[might\varphi][\neg\varphi]$ to be a licit update sequence is not necessarily something that we want. In support of this, consider the fact that reversing conjunct order doesn't seem to render these sentences non-anomalous:

(55) a. #It might not be raining and it's raining.
b. #It might be raining and it's not raining.

I believe this argument to be based on a too-narrow view of the space of possible explanations for contradictoriness in a dynamic semantics. It is certainly the case that any incoherent update sequence should give rise to contradiction; but it is not the case that contradiction results *only* if a proposed update sequence is incoherent. As Willer (2013) notes, there is no possible information state that licenses both conjuncts of sentences like those in (55). I'll call this INCOMPATIBILITY:

(56) INCOMPATIBILITY:
$\forall\Phi,\Psi$, Φ and Ψ are INCOMPATIBLE iff $\neg\exists I : I[\Phi] = I \wedge I[\Psi] = I$.

My sense is that there's a very simple, intuitive pragmatic reason why we should expect assertions of incompatible conjuncts to seem self-contradictory almost all of the time. By definition, if two propositions are incompatible, then it's impossible to believe both at the same time. Because it is impossible to simultaneously believe two incompatible propositions, it is impossible to simultaneously assert them if we assume sincerity as a criterion for cooperative assertion. It's no surprise that a conjunction of $might\varphi$ with $\neg\varphi$ seems self-contradictory: if an addressee is reasoning about the information package that the speaker has just proposed that they update with, it will seem glaringly self-contradictory that the two updates proposed are incompatible with each other—that updating with the second conjunct will eradicate the belief expressed by the first. How could the speaker, at the time of utterance, have sincerely believed both conjuncts? I'm making an assumption here, which is that the speaker asserts the conjunction smoothly, giving the impression that the entire utterance was planned in advance. My pragmatic approach to explaining why update sequences which are incompatible but not incoherent will usually sound contradictory therefore makes a prediction, namely that if the speaker receives, in some manner accessible to the addressee, some new information relevant to φ in between the expression of the first and second conjunct, the anomaly associated with these conjunctions will disappear. This is indeed the case:

(57) [*Context: the speaker is watching a tape of yesterday's horseraces. The horse she is rooting for is in the lead during the first conjunct; prior to the second conjunct, another horse overtakes it and crosses the finish line. 'My horse' and 'it' are coreferent.*]
My horse might have won... and it didn't.

In case this context seems jerry-rigged to eliminate contradictoriness in conjunctions, note that contexts like this don't alleviate the badness associated with truly incoherent update sequences:

(58) [*Context: the speaker is watching a detective show. The suspects are in a room, and the detective is about to reveal the killer. The first conjunct is uttered immediately before the reveal; the second conjunct is uttered immediately after. 'That guy' and 'he' are coreferent.*]
#That guy was the killer... and he wasn't.

Because of the purported epistemic contradiction facts, Yalcin (2012) advocates for a static version of Willer's semantics that enforces total monotonicity at the level of the superstate—and therefore enforces the persistence of all statements of credence. Here's Yalcin's definition of update:

(59) $I[\Phi] = \{i \in I : i[\Phi] = i\}$

This update function, instead of percolating updates through a superstate's substates via pointwise update, instead just kicks out of an information state all of its substates that do not already license the expression being updated with. This update function forces all updating to be strictly monotonic on the level of the superstate.

Yalcin (2012) only considers tests which set lower bounds: *might* and *probably*. If we make the conservative extension of admitting into this system operators like *exactly*70%, as in 'There's an exactly seventy percent chance that my concoction has exploded', the definition of update in (59) predicts that such an update will not allow the credence expressed toward a proposition to slide around once it has been accepted—once a 70% chance, always a 70% chance. By updating with *exactly*$70\%\varphi$, we close off not only the possibility of canonical evolution to $\neg\varphi$, but also the possibility of canonical evolution to φ! One of the great strengths of an analysis like Yalcin's, that models linguistic expressions of probability by assigning them a probability-theoretic meaning, is that it allows us to formalize precise meanings for all kinds of probability talk, like 'The chance that my concoction exploded is no less that seventy percent, and no greater than seventy-one,' or 'The chance that my concoction exploded is very high, but is less than ninety percent.' Unfortunately, Yalcin's update operation in (59) predicts that the effect of updating with either of these sentences will preclude the possibility of canonical evolution to φ.

Yalcin's system is designed as a model of common ground, but if we take it to be a potential model for the information states of individuals, then things get even more dire: the staticized update function as applied to an individual information state predicts that not only will it be a contradiction for an individual speaker to say *might*φ; $\neg\varphi$, but that if one person tells you that they might have won an award, and another person, who is on the award-deciding panel, whispers into your ear that the first person didn't win it, but hasn't yet been informed of this, you would have no canonical way to incorporate that new information. By eliminating the possibility of *might*φ and $\neg\varphi$ update sequences, Yalcin's staticized update function punishes not only conjunctions like *might*φ; $\neg\varphi$, but also non-conjunctive update sequences of that form.

As Yalcin acknowledges, his system generates the result that accepting an update with *might*φ; *might*$\neg\varphi$, instead of leaving one open to either possibility, will close off the possibility of canonical evolution to either φ or $\neg\varphi$. But I've argued above that the problem with Yalcin's proposal for a staticized, strictly monotonic system runs much deeper than its pathological predictions for sentences like *might*φ; *might*$\neg\varphi$; it actually treats all expressions of uncertainty as forever committing us to the degree of uncertainty expressed. Yalcin's response to his system's overgeneration of contradictions is to say that maybe we fall back to the truly dynamic pointwise update function if doing so is necessary to avoid an update resulting in the empty set. Unfortunately, if we jump back to the dynamic update system whenever using Yalcin's static update function would result in the empty set, this hobbles the system so thoroughly that it no longer does what Yalcin designed it to do: it no longer predicts epistemic contradictions. Yalcin's explanation of epistemic contradictions is that they are cases in which the using the static update function produces the empty set; so in this hybrid model we would simply shunt to the dynamic system, where they don't. The primary source of predictive power in an update system is the predictions it makes about which update sequences will result in the empty set; so Yalcin's retreat is virtually equivalent to rejecting the static system entirely.

I've argued that Yalcin's criticism of Willer's system for deriving the belief revision asymmetry is misguided, and that his alternative proposal is unworkable. The Willer/Yalcin system is very close to ideal as a system for dealing with expressions of subjective uncertainty: by allowing updates to be non-monotonic on the level of the superstate, it allows updates with expressions of subjective uncertainty to be impersistent; by forcing updates to be monotonic on the level of the substate, it ensures that updates with propositions will be persistent. However, the system does not quite satisfy all of the desiderata outlined at the beginning of this paper: intermediate credences in this system are not slippery enough!

To make this concrete, let's see what happens when an information state is updated with *probably*φ, and then subsequently updated with *probably*$\neg\varphi$. I'll start with a toy information state I with 9 classes of substates in it. All substates contain both φ-worlds and non-φ-worlds. The substates differ in the values that their probability measures assign to φ. Because we're only interested in the value assigned to φ in this example, I'll simply represent each class of substate by reference to the value that it's probability measure assigns to φ. So the class of all substates whose probability measure assigns the value .1 to φ I'll represent by simply writing $\mu(\varphi) = .1$:

(60) $$I = \left\{ \begin{array}{l} \mu(\varphi) = .1 \\ \mu(\varphi) = .2 \\ \mu(\varphi) = .3 \\ \mu(\varphi) = .4 \\ \mu(\varphi) = .5 \\ \mu(\varphi) = .6 \\ \mu(\varphi) = .7 \\ \mu(\varphi) = .8 \\ \mu(\varphi) = .9 \end{array} \right\}$$

For simplicity's sake I've included in I only substates whose probability measure assigns exactly $.n$ to φ, for some n from 1 to 9. It is trivial to see that the argument developed here extends to information states containing an arbitrarily dense array of values assigned to φ by various measures.

If we update this information state with the test *probably*φ (recall the definition in (53)), that will remove all substates that assign .5 or less to φ:

(61) $$I[probably\varphi] = \left\{ \begin{array}{l} \cancel{\mu(\varphi) = .1} \\ \cancel{\mu(\varphi) = .2} \\ \cancel{\mu(\varphi) = .3} \\ \cancel{\mu(\varphi) = .4} \\ \cancel{\mu(\varphi) = .5} \\ \mu(\varphi) = .6 \\ \mu(\varphi) = .7 \\ \mu(\varphi) = .8 \\ \mu(\varphi) = .9 \end{array} \right\} = \left\{ \begin{array}{l} \mu(\varphi) = .6 \\ \mu(\varphi) = .7 \\ \mu(\varphi) = .8 \\ \mu(\varphi) = .9 \end{array} \right\}$$

The result of the update will be an information state containing only substates whose measures assign values greater than .5 to φ. If we try to update this resulting information state with *probably*-$\neg\varphi$, that will remove all substates that assign values greater than .5 to φ, and anomaly will result:

$$\text{(62)} \qquad I[\mathit{probably}\varphi][\mathit{probably}\text{-}\neg\varphi] = \left\{ \begin{array}{l} \cancel{\mu(\varphi) = .6} \\ \cancel{\mu(\varphi) = .7} \\ \cancel{\mu(\varphi) = .8} \\ \cancel{\mu(\varphi) = .9} \end{array} \right\} = \{\varnothing\}$$

The Willer/Yalcin model is successful in deriving the (im)persistence asymmetry because it stipulates that propositions and expressions of subjective uncertainty are different kinds of updates. But treating expressions of subjective uncertainty as tests doesn't allow the system to behave with the degree of flexibility necessary to account for slippery intermediate credences. It would be nice to have a system that can derive that distinction from the content of expressions of subjective uncertainty versus the content of propositions, while treating both as the same kind of update (Rothschild 2012). In the next section I will propose such a system, Bayesian Update Semantics, and show that it satisfies all three of the desiderata with which this paper began.

6 Bayesian Update Semantics

The Willer/Yalcin system gets close to satisfying the desiderata for expressions of subjective uncertainty, but it runs into problems with sequential updates with non-overlapping probability ranges; credences aren't quite as slippery and impersistent as they should be. The positive results of the Willer/Yalcin system are derived via the formal mechanism of treating expressions of subjective uncertainty as tests while treating propositions as sets of worlds. That is to say, the difference in persistence properties between expressions of subjective uncertainty and propositions is stipulated, not derived: it's a result of the decision to treat the two kinds of expressions as triggering different kinds of updates.

I propose that we treat conditionalization, not intersection, as the basic update operation. In this section I propose a novel dynamic semantic system, Bayesian Update Semantics, which preserves the positive results of the Willer/Yalcin system while avoiding its failings, and derives the difference in persistence properties between expressions of subjective uncertainty and propositions from their content, while treating them as the same kind of object, to which the same operations are applied.

I'll begin by observing that if we assume a finite set of worlds, treating substates as pairs of a set of worlds and a probability measure becomes redundant: the set of worlds can in fact be derived from the probability measure by taking the intersection of all propositions to which the measure assigns the value 1. I'll call the set i derived from a measure μ in this fashion μ's WORLD-IMAGE. The Willer/Yalcin model uses two different operations, one targeting probability measures, and the other targeting their world-images: tests and intersections, respectively. I propose that we should think of updates as conditionalization, pure and simple; this will be isomorphic to the Willer/Yalcin model's treatment of propositional updates, but for updates with expressions of subjective uncertainty, conditionalization gives us the slippery credences instead of the rigidity of test-based updates.

The first step in converting the Willer/Yalcin model into Bayesian Update Semantics is to take an information state to simply be a set of probability measures, defined over W:

(63) *Bayesian Update Semantics: Information States*
  a. Any $I \subseteq \mathfrak{M}$ is an information state
  b. The maximally uninformed information state is $\mathfrak{M}$

Treating an information state as a set of probability measures gives us an intuitive way to cash out what it means for an agent to be certain or uncertain about various propositions: if all probability measures in an agent's information state assign the same value to φ, we say that that agent is CERTAIN about the likelihood of φ; if different measures in an agent's information state assign different values to φ, then we say that that agent is UNCERTAIN about the likelihood of φ.[6] We can read the range of likelihoods that an agent currently entertains about φ by finding the measure in their information state that gives the lowest value to φ, and the measure that gives the highest. Credence structures in this system are no different than in the Willer/Yalcin system.

In a model like this, we can take assertions to be proposals to associate a proposition with a probability range. The semantic content of any expression is of the following form:

(64) *Bayesian Update Semantics: Credal Operators:*
  $\forall C \in \mathfrak{C}$, C is a function of the form $\lambda\varphi.\langle\varphi, \textsc{int}\rangle$
  Where INT is a real interval and $\varphi \subseteq W$

Here are some example denotations:

(65) a. $[\![\textit{might}]\!] = \lambda\varphi.\langle\varphi, (0,1]\rangle$
  b. $[\![\textit{probably}]\!] = \lambda\varphi.\langle\varphi, (.5,1]\rangle$
  c. $[\![\textit{extremely-unlikely}]\!] = \lambda\varphi.\langle\varphi, [0,.1)\rangle$

I'll assume that asserting a proposition comprises a proposal to associate that proposition with probability 1.[7]

[6] Note that the concept of (un)certainty in terms of credence structures is fundamentally separate from the notion of an expression of subjective uncertainty.

[7] This is compatible with the traditional notion that a proposition denotes the set of worlds in which it is true. We could say that the semantic meaning of a proposition is exactly that set, and that the association between a proposition and probability 1 is a fact about the pragmatics of assertion, not about the semantics of propositions. The difference is more or less notational: the Bayesian Update Semantics that I outline in

If an agent accepts an assertion of an expression, that is to say, agrees to associate φ with the probability range specified by the expression, they update their information state with that probability range assignment. Here is an update function that can accomplish this:

(66) *Bayesian Update Semantics: Updates to Superstates:*
$I[[\![\Phi]\!]] = \{\mu : \exists\mu' \in I \text{ s.t. } \mu \in \mu'[[\![\Phi]\!]]\}$

Just as before, updating the superstate takes the form of pointwise update of substates.

(67) *Updates to Substates (to be revised):*
$\mu[\langle\varphi, \textsc{int}\rangle] = \{\mu' : \exists\psi \in \uparrow(\mu, \varphi, \textsc{int}) \text{ s.t. } \mu\restriction_{\psi} = \mu'\}$
Where $\uparrow(\mu, \varphi, \textsc{int})$ $\{\psi \subseteq \mathrm{W} : \mu\restriction_{\psi}(\varphi) \in \textsc{int}\}$

To break it down a little for easier comprehension:

(68) *The update function, piece by piece:*
Updating a measure μ with an ascription of a probability range INT to a proposition φ generates a set of measures μ' such that:

a. μ' assigns to φ a value in INT:
$\mu'(\varphi) \in \textsc{int}$

b. μ' is derived from μ via conditionalization:
$\exists\psi \subseteq \mathrm{W} \text{ s.t. } \mu\restriction_{\psi} = \mu'$

I'll add one more clause to this before moving on to a discussion of how this semantics satisfies the desiderata that I outlined at the beginning of this paper. Consider an information state I that has been updated with the information that there's an exactly 70% chance that φ is true. Let's say that φ is the proposition that the democratic candidate won today's election. An information state that has been updated with this information is an information

this paper is isomorphic to a split system that treats propositional assertions in the manner of Yalcin (2012) and employs my new machinery only for expressions of subjective uncertainty.

What is more problematic is trying to understand the difference between these two sentences:

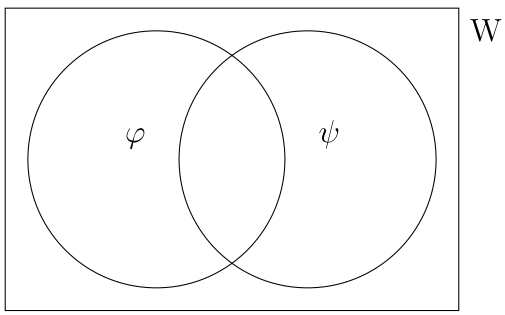
(i) a. Paul came to the party.
b. Paul definitely came to the party.

If we make the intuitive move of saying that *definitely* is a credal operator that associates the proposition it scopes over with the probability range 1, and we accept my proposal that asserting a proposition pragmatically comprises a request for interlocutors to associate that proposition with probability 1, then we would expect these sentences to be identical. Are they? If so, then we have an inroads to cashing out the default pragmatics of assertion: we could say that the default pragmatics of asserting a proposition associates that proposition with a covert *definitely* operator; i.e. $[\![\varphi]\!] = [\![\textit{definitely}\varphi]\!] = \langle\varphi, [1,1]\rangle$. On this view, the sentences in (i) are semantically identical; they differ only wrt the covertness of the definitely operator. If there are substantial empirically attested differences in the behavior or effects of the sentences in (i), we could say that precisely because asserting a proposition already associates it with probability 1, adding the *definitely* operator is strictly redundant, and therefore serves some other pragmatic purpose, a la Barker & Taranto (2003)'s analysis of *clearly*. Further empirical work is clearly required here.

state comprising only measures that assign probability of .7 to φ, though they may differ wildly in the way they spread that probability across its subsets and the subsets of its complement. Now let's say that this information state gets updated with the information that the probability of ψ is 1. Let's say that ψ is the proposition that I just cut my fingernails.

φ is clearly not disjoint with ψ; it's possible for them to both be true at the same time. So we know that they have a nonempty intersection, and neither is contained within the other:

(69)



ψ is also clearly independent of φ; that is to say, the outcome of ψ does not affect the probability of φ, and vice-versa. Let's say for the sake of simplicity that prior to the update with ψ, this information state was characterized by 50% credence in ψ. In that case, because φ and ψ are independent, every measure in I will look like this wrt φ and ψ:

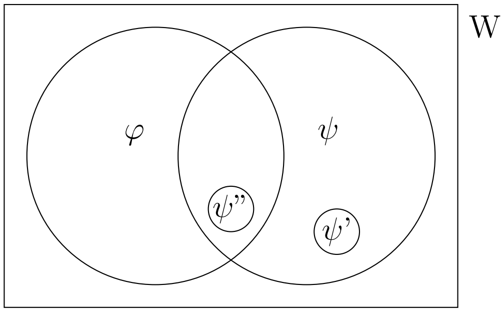
(70)

	φ	$\neg\varphi$	
ψ	.35	.15	.5
$\neg\psi$	.35	.15	.5
	.7	.3	

The conditional probability of φ given ψ is $\frac{.35}{.5}$, or .7; the conditional probability of φ given $\neg\psi$ is $\frac{.35}{.5}$, or .7. Likewise, the conditional probability of ψ given φ is $\frac{.35}{.7}$, or .5, and the conditional probability of ψ given $\neg\varphi$ is $\frac{.15}{.3}$, or .5. The probability of ψ is not affected by the outcome of φ, and vice versa.

This is the situation as it stands when the information state is updated with the attribution of the value 1 to ψ. The update function in (67) will return the set of all probability measures that (a) assign 1 to ψ and (b) can be derived via conditionalization from measures already in the information state. There are any number of propositions on which we could conditionalize measures in I to produce a measure that assigns a value of 1 to ψ. Consider the following two:

(71)



ψ’ is a measure that is contained within ψ, and is disjoint with φ. So, assuming an arbitrary value for the probability of ψ’, it has the following relationships with ψ and φ:

(72) μ’s values for φ, ψ and ψ’:

	ψ’	$\neg\psi$’	
ψ	.2	.3	.5
$\neg\psi$	.0	.5	.5
	.2	.8	

	ψ’	$\neg\psi$’	
φ	.0	.7	.7
$\neg\varphi$	.2	.1	.3
	.2	.8	

The marginal values of ψ’ and $\neg\psi$’ are irrelevant; what matters is that the probability of ψ given ψ’ is $\frac{.2}{.2} = 1$, and that the probability of φ given ψ’ is $\frac{0}{.2} = 0$. Because μ can be conditionalized on ψ’ to produce a measure that gives ψ the value 1, the update function in (67) will therefore include $\mu\restriction_{\psi'}$ in the information state that results from an update with an ascription of probability 1 to ψ. So despite the fact that all measures in the pre-update information state assigned .7 to φ, the information state that results from an update with a completely independent proposition will include measures that assign 0 to φ.

ψ” is a measure that lies in the intersection of φ and ψ; it is contained within both propositions. Assuming an arbitrary value for the probability of ψ”, it has the following relationships with ψ and φ:

(73) μ’s values for φ, ψ and ψ”:

	ψ”	$\neg\psi$”	
ψ	.5	.0	.5
$\neg\psi$	.0	.5	.5
	.5	.5	

	ψ”	$\neg\psi$”	
φ	.5	.2	.7
$\neg\varphi$	.0	.3	.3
	.5	.5	

Again, the marginal values of ψ” and its complement are irrelevant. $\mu(\psi|\psi") = \frac{.5}{.5} = 1$; $\mu(\varphi|\psi") = \frac{.5}{.5} = 1$. Because μ can be conditionalized on ψ” to produce a measure that gives ψ the value 1, the update function in (67) will therefore include $\mu\restriction_{\psi"}$ in the information state the results from an update with an ascription of probability 1 to φ. So despite the fact that all measures in the pre-update information state assigned .7 to φ, the information state that results from an update with a completely independent proposition will include measures that assign 1 to φ.

In other words, this model currently predicts that if I tell you that I just cut my fingernails, and you believe me, then suddenly you're no longer sure that the democratic candidate probably won the election. We need our update function to be constrained in some way, to keep credence structures from being completely destroyed by each new update. As the image in (71) suggests, we want it to be constrained by making sure that we don't derive new measures by conditionalizing the measures currently in the information state on whatever will get the target result, but instead want to conditionalize prudently on the largest possible propositions that will derive the target results. We want to conditionalize as minimally as possible, because we want credence structures in the information state to survive updates not related to those credence structures. A small alteration will take us a long way; perhaps all the way:

(74) *Bayesian Update Semantics: Updates to Substates (final):*
$\mu[\langle\varphi, \textsc{int}\rangle] = \{\mu' : \exists\psi \in \textsc{max}(\uparrow(\mu, \varphi, \textsc{int}))$ s.t. $\mu\restriction_\psi = \mu'\}$
Where $\uparrow(\mu, \varphi, \textsc{int})$ $\{\psi \subseteq W : \mu\restriction_\psi(\varphi) \in \textsc{int}\}$, and
For some set of propositions S, $\textsc{max}(S) = \{\psi \in S : \neg\exists\psi' \in S$ s.t. $\psi \subset \psi'\}$

This new update function will rule out measures conditionalized on propositions like ψ' and ψ" from the new information state: they are properly included within a proposition which we can use to conditionalize measures to achieve the target value for ψ: ψ itself.

Now that we have a workable update function under our belts, let's look at some schematic test cases of how this works:

Let's begin by assuming the maximally uninformed information state, $\mathfrak{M}$, which includes the full range of measures wrt φ: measures that give φ 1 (φ-μs), measures that give φ 0 ($\neg\varphi$-μs), and measures that give φ some intermediate value (?φ-μs). What happens to each of these classes of measures in the information state if we update with $\langle\varphi$, [1,1]$\rangle$?

6.1 Updating with certainty that φ is true

6.1.1 The φ-μs

If an information state I is updated with $\langle\varphi$, [1,1]$\rangle$, the set of φ-μs in I is carried over to the updated information state unchanged: they can all be vacuously conditionalized to produce a measure that gives 1 to φ.

(75) μ's values for φ and W, for all φ-μs:

	φ	$\neg\varphi$	
W	1	0	1
∅	0	0	0
	1	0	

All μs in $\mathfrak{M}$ give 1 to W and 0 to $\varnothing$. All φ-μs give 1 to φ and 0 to $\neg\varphi$, by definition. So for any φ-μ, $\mu(\varphi|\mathrm{W}) = \frac{1}{1} = 1$. There are no propositions that are not subsets of W, again by definition, so W will be the only proposition selected by the MAX function to conditionalize the φ-μs on. And for any μ, $\mu{\restriction}_{\mathrm{W}} = \mu$. Therefore, the update function will, for each φ-μ in I, simply add that φ-μ unaltered to $I[\langle\varphi, [1,1]\rangle]$.

6.1.2 The $\neg\varphi$-μs

If an information state I is updated with $\langle\varphi, [1,1]\rangle$, the set of $\neg\varphi$-μs in I will be eradicated completely in the updated information state. As discussed in the Bayesian Excursus (§4.1), there is no way to conditionalize these measures such that they assign anything other than 0 to φ. I've presented values here for φ and some arbitrary proposition ψ:

(76) μ's values for φ and ψ, for all $\neg\varphi$-μs:

	φ	$\neg\varphi$	
ψ	0	.5	.5
$\neg\psi$	0	.5	.5
	0	1	

$\mu(\varphi|\psi) = \frac{0}{.5} = 0$. The marginal values for ψ don't matter; what matters is that because this is a $\neg\varphi$-μ, for any ψ at all the numerator of the $\mu(\varphi|\psi)$ will always be 0, and so $\mu(\varphi|\psi) = 0$ for all ψ, regardless of what probability the measure assigns to ψ. There is no ψ on which these measures can be conditionalized to result in a measure that gives φ 1, so for any I, $I[\langle\varphi, [1,1]\rangle]$ will bear no trace of the $\neg\varphi$-μs in I.

6.1.3 The ?φ-μs

If an information state I is updated with $\langle\varphi, [1,1]\rangle$, the ?$\varphi$-$\mu$s in I will be collapsed into the set of φ-μs in the updated information state. In most cases, ?φ-μs will all be conditionalized on φ and therefore collapsed into the set of φ-μs in the resulting information state. Let's say we're dealing with an arbitrarily chosen ?φ-μ that assigns .5 to φ:

(77) μ's values for φ:

	φ	$\neg\varphi$	
φ	.5	0	.5
$\neg\varphi$	0	.5	.5
	.5	.5	

For a μ that assigns .5 to φ, $\mu(\varphi|\varphi) = \frac{.5}{.5} = 1$.

All μs in $\mathfrak{M}$ assign 0 to $\varphi \cap \neg\varphi$ $(= \varnothing)$; and $\varphi \cap \varphi = \varphi$, so whatever a measure assigns to φ, it also assigns to $\varphi \cap \varphi$. So for any μ that assigns a non-zero value to φ, $\mu(\varphi|\varphi)$ $= 1$. Conditionalizing a ?φ-μ on φ will usually be the minimally structure-altering way to produce a measure that assigns 1 to φ, and so each measure in the set of ?φ-μs in I will be conditionalized on φ and added to $I[\langle\varphi,\ [1,1]\rangle]$.[8]

6.1.4 Summing up

To put it simply: updating an information state I with an ascription of 1 to φ will kick out all of I's $\neg\varphi$-μs, leave all of I's φ-μs alone, and conditionalize all of I's ?φ-μs on φ.

To put it extremely simply:

(78) $I[\langle\varphi,\ [1,1]\rangle] = \{\mu \in \mathfrak{M} : \exists\mu\text{'} \in I \text{ s.t. } \mu\text{'}\restriction_{\varphi} = \mu\}$

It's not necessary to discuss the operation of updating an information state with an ascription of 0 to φ independently of updates with ascriptions of 1 to φ; the results will simply be inverted, because updating with an ascription of 0 to φ is identical to updating with an ascription of 1 to $\overline{\varphi}$: the φ-μs will be eradicated, the $\neg\varphi$-μs will be carried over, and the ?φ-μs will be collapsed into the $\neg\varphi$-μs via conditionalization on $\overline{\varphi}$. In other words:

(79) $I[\langle\varphi,\ [0,0]\rangle] = \{\mu \in \mathfrak{M} : \exists\mu\text{'} \in I \text{ s.t. } \mu\text{'}\restriction_{\overline{\varphi}} = \mu\}$

6.2 Updating with intermediate ranges for φ

Things get more complicated when we update with ascriptions of intermediate ranges to φ.

6.2.1 The φ μs and $\neg\varphi$-μs

If an information state I is updated with any ascription of a probability range to φ that includes neither 0 nor 1, both the φ-μs and the $\neg\varphi$-μs in I will be eliminated from the updated information state, because there is no way to conditionalize them such that φ does not receive 1 or 0, respectively (see the Excursus in §4.1 for details).

6.2.2 The ?φ-μs

The ?φ-μs in an information state I that is updated with an ascription of some intermediate probability range to φ will behave differently depending on whether or not they already assign to φ a value in that interval. Those that do will be carried over into the new information state unaltered; they can be vacuously conditionalized on W. For those ?φ-μs that don't already assign to φ a value in that interval, things will get a little trickier.

[8] There are cases in which φ will not be the unique proposition selected by MAX to conditionalize a ?φ-μ on; however, in all such cases the result of conditionalizing on the proposition(s) chosen by MAX will be equivalent to conditionalizing on φ, so I'll ignore such cases here for the sake of simplicity.

The update function in (74) generates for each μ in I a set of measures that assign a value in the target range to φ and are derivable from μ via conditionalization. The function MAX has insured that this is a singleton set until this point; if the target value for φ is 1 or 0 there is always a unique minimally structure-altering way (or a cluster of equivalent ways) to conditionalize a measure such that the result assigns that value to φ. That is not the case when conditionalizing a measure to achieve the assignment of some intermediate target value to φ.

Because there is not necessarily a unique minimally structure-altering way to conditionalize a measure such that the result assigns some intermediate value to φ, updates with ascriptions of intermediate values can introduce some uncertainty into an information state's credence structure that was not previously there. However, the MAX function will keep that introduction of uncertainty as manageable as possible.

The plurality of potential ways of achieving a desired intermediate target value represents the fact that if someone simply asserts that there's a 70% chance that φ is the case, they have not provided some crucial information about what facts about the world should prompt this shift in credence toward φ; they have not provided their grounds. Updates with ascriptions of 1 or 0 to φ don't introduce uncertainty because they tell you exactly what observation has been made about the world: in the case of an ascription of 1 to φ, the observation has been made that φ obtains; in the case of an ascription of 0 to φ, the observation has been made that $\overline{\varphi}$ obtains. Updates with ascriptions of intermediate values to φ, however, do introduce uncertainty because they don't include information about what observation has been made about the world that prompts the modification of credence toward φ.

This model doesn't include any representation of world-knowledge—it makes no claims about what the more or less plausible reasons are to adjust one's degree of credence toward some proposition φ. Its update function simply returns the set of all possible minimally structure-altering ways to update one's credence to hit the target value. In practice, we expect actual interpreters of linguistic expressions to alter their information states in much more conservative ways—we expect that they will generate only the set of measures that result from conditionalizing on propositions which plausibly could have been the observation that a speaker made which caused them to assert an intermediate degree of credence toward φ. In other words: we expect people to respond to assertions of some intermediate degree of credence toward φ not by saying, in effect, 'well, I don't know what changed about the world to prompt my interlocutor's shift of credence toward φ, so now all bets are off!' Instead, we expect people to respond to assertions of some intermediate degree of credence toward φ by guessing at what facts their interlocutors might plausibly have obtained to alter their credence toward φ, and conditionalizing the measures in their information states on those plausible facts.[9]

The practical ramifications of this for the model presented here is that, because ascriptions of intermediate ranges of values to φ potentially introduce uncertainty to an information state, we expect there to be pressure on speakers to indicate their evidence sources (e.g. 'A

[9]This could potentially be cashed out by way of formal tools encoding relations of plausible causality, such as probabilistic graphical models.

bunch more precincts are reporting in, so now there's a 45% chance that the democratic candidate won'), and we expect listeners to be more likely to ask speakers for their grounds for asserting an ascription of an intermediate range of values to φ (assuming that the grounds were not provided) than they would be to ask speakers for their grounds for asserting that φ is either true or false.

6.2.3 Summing up

Updating an information state I with an ascription of an intermediate range of values to φ will remove all φ-μs and $\neg\varphi$-μs from I, leave all μs that already assign to φ a value in the target range alone, and conditionalize all other μs in as structure-preserving a fashion as possible to produce measures that assign to φ values in the target range.

6.3 Success relative to desiderata

I began this paper by proposing three desiderata for dynamic implementations of expressions of subjective uncertainty. I've repeated them here:

(80) **Desideratum 1: The Belief Revision Asymmetry**
Expressions of subjective uncertainty about φ must be followable without contradiction by the proposition $\neg\varphi$; the proposition φ must *not* be followable without contradiction by the proposition $\neg\varphi$.

(81) **Desideratum 2: Slippery Intermediate Credences**
Expressions of some range of uncertainty about φ must be followable without contradiction by expressions of some other range of uncertainty about φ—potentially a non-overlapping range.

(82) **Desideratum 3: Informativity**
Expressions of subjective uncertainty must be (potentially) informative updates.

This system satisfies desideratum 1 with flying colors. Any information state that has been updated with an ascription of 1 to φ will contain only measures that assign 1 to φ; attempting to update with an ascription of anything other than 1 to φ will result in anomaly, because there is no way to conditionalize those measures such that they no longer assign 1 to φ. So updating with an assertion of the proposition φ will be persistent; there is no canonical evolution that no longer licenses an assertion of φ.

Consider an information state I containing measures that assign a full range of values to φ: measures that assign 1 to φ, measures that assign 0 to φ, and everything in between. If I is updated with an ascription of some intermediate range of values to φ, the result will contain many measures that assign to φ a value that is neither 0 nor 1. If that resulting information state is updated with an ascription of 1 to φ, those measures will be conditionalized on φ and passed on to the new information state; if it is updated with an ascription of 0 to φ, those measures will be conditionalized on $\overline{\varphi}$ and passed on to the new information state. No anomaly results in either case; an assertion of an expression of subjective uncertainty about φ can be followable by either φ or $\neg\varphi$.

Likewise, this system satisfies desideratum 2: an update to an information state I with an ascription of an intermediate range of values to φ will produce a new information state containing only measures that assign to φ a value in that range; if updated with an ascription of a new, potentially non-overlapping range of values to φ, those measures will be conditionalized so as to assign to φ a value in the new range. Anomaly does not necessarily result.

Finally, this system satisfies desideratum 3: it is possible for an update with an ascription of an intermediate range of values to φ to alter an information state.

7 How radically different is Bayesian Update Semantics from a traditional Update Semantics?

The source of desiderata 2 and 3 is the intuition that an assertion of a proposition behaves differently from an assertion of an expression of subjective uncertainty; this is the same intuition that underlies the use of the formal mechanism of the test to model such expressions in the work of Veltman (1996), Willer (2010, 2013) and Yalcin (2012). The fact that the system proposed in this paper satisfies those desiderata shows that the distinction between propositions and expressions of subjective uncertainty is at work here as well. However, in the system presented here, unlike in test-based models of expressions of subjective uncertainty, this distinction was not stipulated in any way; it simply falls out of the difference in content between propositions and expressions of subjective uncertainty. With conditionalization as the basic update operation, we get slippery intermediate credences and persistent propositions for free.

But at what cost did this unification come? This system might seem remarkably different from a traditional intersection-based dynamic semantics, but it really isn't. In fact, a homomorphism can be defined between the system presented here and Willer (2013)'s system, if we restrict ourselves to the language $\mathcal{L}$ plus the credal operator *might*, defined as follows:

(83) $[\![might\varphi]\!] = \langle \varphi,\ (0,1] \rangle$

Informally: $might\varphi$ ascribes to φ any nonzero value.

For any information state I, we can define a Willerian representation of I as the set of all ψ such that ψ is the world-image of some measure in I. If we update I with $\langle \varphi,\ [1,1] \rangle$, that will remove all measures that assign 0 to φ, leave untouched all measures that assign 1 to φ, and conditionalize all other measures on φ—in terms of the effects on the Willerian representation of I, this is equivalent to pointwise intersection of substates with φ.

For any information state I, updating I with $might\varphi$ as defined above will remove from I all measures that assign 0 to φ, and vacuously conditionalize all other measures on W. In terms of the effects on the Willerian representation of I, this is equivalent to pointwise testing of substates for consistency with φ.

Likewise, a homomorphism can be defined between Bayesian Update Semantics and Veltman (1996)'s Update Semantics, if we restrict our attention to the language $\mathcal{L}$: for any Bayesian Update Semantics information state I, a corresponding Veltmanian information state can be generated by taking the grand union of the set of all world-images of a measure μ in I. Updating I with $\langle\varphi, [1,1]\rangle$, for any $\varphi \subseteq$ W, will, via pointwise conditionalization of all measures in I, remove all non-φ worlds from the world-images of those measures, and therefore remove all non-φ worlds from their grand union.

To put it simply: Bayesian Update Semantics behaves identically to Veltman (1996)'s Update Semantics in the way it treats updates with propositions. The differences lie only in the way that Bayesian Update Semantics treats sentences that do not express certainty that a proposition is true.

8 Conclusion

Bayesian Update Semantics was designed to account for some tricky facts about the nature of expressions of subjective uncertainty—namely, that the beliefs they express are robustly impersistent. However, the system proposed here has much broader applications. Probabilistic truth conditions have recently been proposed for a variety of natural language phenomena: by Lassiter (2011) for scalar epistemic adjectives; by Kaufmann (2004, 2005, 2009) for conditionals; by McCready & Ogata (2007) for evidential particles. Other than Yalcin (2012), I know of no attempt to provide an update semantics for probabilistic expressions that would allow them to be incorporated into dynamic semantic systems like Veltman (1996), Heim (1982, 1983), or Kamp (1981). Bayesian Update Semantics fills this gap: it's an all-purpose dynamic semantics for probabilistic expressions, that can be used as an out-of-the-box dynamic system for dynamic semanticists interested in modeling probabilistic expressions.

This paper has been an attempt to explain the meaning of sentences like the following, in terms of the effects that believing they are true has on an information state:

(84) a. It might be raining.
b. My great grandfather probably had two kids.
c. There's a 25-30% chance that Nixon faked Watergate.
d. It's extremely unlikely that we're all just brains in vats.

In one recent strand of argumentation in the philosophical literature, sentences like the above are referred to as 'nonfactual' (Yalcin 2011, Rothschild 2012; see also Swanson 2006, 2011, 2015 and Moss 2015), and contrasted with their 'factual' counterparts:

(85) a. It's raining.
b. My great grandfather had two kids.
c. Nixon faked Watergate.
d. We're all just brains in vats.

Traditional models of dynamic semantics, which treat update as intersective, and therefore treat all information growth as monotonic, are difficult to reconcile with the impersistent nature of expressions of subjective uncertainty—hence the 'factual'/'nonfactual' split. Rothschild (2012) proposes a reconciliation of sorts between factual and nonfactual expressions: for both groups of expressions, asserting an expression is a recommendation to all interlocutors to believe the semantic content of that expression.

This proposal is taken up by Swanson (2015) and Moss (2015) in terms of a constraint-based treatment of the meaning of expressions of subjective uncertainty: accepting such an assertion is accepting the constraint imposed by the semantic value of the expression on one's possible distributions of credence. In Swanson's terms, when a speaker makes an assertion, she 'advises her addressees to conform their credences to the semantic value' of that assertion.

This paper can be seen as a cashing out of Swanson's implicit IOU about how exactly conforming one's credences to meet the constraint imposed by a set of possible values might work; it can also be seen as a formal realization of of Rothschild's wish for reconciliation between the two types of expressions. In the system presented here, 'factual' and 'nonfactual' expressions, or propositions and expressions of subjective uncertainty, in the terminology I've employed, are treated as the same kind of formal object incorporated into an information state via the same update function. Bayesian Update Semantics does not stipulate the difference between the two classes of expressions in any way; the difference falls out naturally from differences in the content of each type of expression.

Appendix A: Bayesian Update Semantics in one page

(86) Syntax for $\mathcal{L}$ and $\mathcal{CL}$:

a. Any $p \in \mathcal{A}$ is a formula of $\mathcal{L}$
Where $\mathcal{A}$ is the set of all atomic propositions
b. If φ and ψ are formulas of $\mathcal{L}$, then so are $(\varphi \wedge \psi)$ and $\neg(\varphi)$
c. Nothing else is a formula of $\mathcal{L}$
d. Every formula of $\mathcal{L}$ is a formula of $\mathcal{CL}$
e. For any formula φ of $\mathcal{L}$ and any C in $\mathfrak{C}$, Cφ is a formula of $\mathcal{CL}$
Where $\mathfrak{C}$ is the set of all credal operators
f. If Φ and Ψ are formulas of $\mathcal{CL}$, then so is $(\Phi; \Psi)$
g. Nothing else is a formula of $\mathcal{CL}$

(87) Semantics for $\mathcal{L}$ and $\mathcal{CL}$:

a. $[\![p]\!] = \{\mathrm{w} \in \mathrm{W} : p \text{ is true in w}\}$
b. $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$
c. $[\![\neg\varphi]\!] = \mathrm{W} - [\![\varphi]\!]$
d. $[\![\mathrm{C}\varphi]\!] = [\![\mathrm{C}]\!]([\![\varphi]\!])$

(88) *Credal Operators*
$\forall \mathrm{C} \in \mathfrak{C}$, C is a function of the form $\lambda\varphi.\langle\varphi, \textsc{int}\rangle$
Where $\textsc{int}$ is a real interval and $\varphi \subseteq \mathrm{W}$
Example denotations:

a. $[\![\textit{might}]\!] = \lambda\varphi.\langle\varphi, (0,1]\rangle$
b. $[\![\textit{probably}]\!] = \lambda\varphi.\langle\varphi, (.5,1]\rangle$

(89) *Probability Measures*
Given some finite set of worlds W, $\mu : \wp(\mathrm{W}) \to [0,1]$ is a probability measure iff

a. REALISM: $\mu(\mathrm{W}) = 1$
b. FINITE ADDITIVITY: $[\forall\varphi,\psi \subseteq \mathrm{W} : \varphi \cap \psi = \varnothing](\mu(\varphi) + \mu(\psi) = \mu(\varphi \cup \psi))$

(90) *Information States*

a. Any $I \subseteq \mathfrak{M}$ is an information state
Where $\mathfrak{M}$ is the set of all probability measures
b. The maximally uninformed information state is $\mathfrak{M}$

(91) *The Update Function*

a. $I[\Phi] = \{\mu : \exists\mu' \in I \text{ s.t. } \mu \in \mu'[\Phi]\}$
b. $\mu[\Phi; \Psi] = \mu[\Phi][\Psi]$
c. $\mu[\varphi] = \mu[\langle\varphi, [1,1]\rangle]$
d. $\mu[\langle\varphi, \textsc{int}\rangle] = \{\mu' : \exists\psi \in \textsc{max}(\uparrow(\mu, \varphi, \textsc{int})) \text{ s.t. } \mu\restriction_{\psi} = \mu'\}$
Where $\uparrow(\mu, \varphi, \textsc{int})$ $\{\psi \subseteq \mathrm{W} : \mu\restriction_{\psi}(\varphi) \in \textsc{int}\}$, and
For some set of propositions S, $\textsc{max}(\mathrm{S}) = \{\psi \in \mathrm{S} : \neg\exists\psi' \in \mathrm{S} \text{ s.t. } \psi \subset \psi'\}$

Appendix B: More on Epistemic Contradiction

One of Yalcin (2007)'s arguments that epistemic contradictions really are contradictions is that they, unlike the Moore paradoxes that they resemble, are still bad under suppose; this turns out to be true regardless of conjunct order:

(92) a. #Suppose it is raining and it might not be raining.
b. #Suppose it is not raining and it might be raining.
c. #Suppose it might not be raining, and it's raining.
d. #Suppose it might be raining, and it's not raining.

Willer (2013) has this to say:

> Insofar as the point of a supposition is to adopt a state of information that supports a certain hypothesis, we then expect that supposing "It might not be raining and it is raining" is just as odd as supposing "It is raining and it might not be raining."

That's all Willer says about this; I'll make this argument at greater length and in greater detail here. The idea is that there's something about the suppositional project that makes conjunctions under 'suppose' behave differently than conjunctions normally behave; in a dynamic semantics, conjunction generally is translated as sequential update; however, in a suppositional project, the goal seems to be more like world-building; in other words, when a speaker instructs an addressee to suppose a list of conjuncts, the implicit directive is something like: 'construct the largest possible information state that licenses all of these'. If conjuncts under suppose are incompatible, then this project is impossible, and anomaly results.

However, it's possible to force the interpretation of suppositional projects to be narrative/sequential; in this case, conjunction seems to behave like sequential update:

(93) Suppose that it might be raining, and you go outside to check, and it's not raining.

This sounds perfectly fine! But for truly illicit update sequences, narrativizing the suppositional project does not alleviate the badness:

(94) a. #Suppose that your dad was born in Greece and your dad wasn't born in Greece.
b. #Suppose that your dad was born in Greece, and you receive a new familial record, and your dad wasn't born in Greece.

As Yalcin notes, epistemic contradictions are also bad in the antecedents of conditionals, and this is independent of conjunct order as well:

(95) a. #If it is raining and it might not be raining, then I'll bring an umbrella just in case.

b. #If it is not raining and it might be raining, then I'll bring an umbrella just in case.
c. #If it might be raining and it's not raining, then I'll bring an umbrella just in case.
d. #If it might not be raining and it's raining, then I'll bring an umbrella just in case.

The same patterns I noted with *suppose* are characteristic of conditional antecedents:

(96) a. If it might be raining, and you go outside to check, and it's not raining, then you can open all the windows.
b. #If your dad was born in Greece, and you receive a new familial record, and your dad wasn't born in Greece, then you must be very surprised.

If the antecedent is made to seem narrative, the anomaly associated with conjoined incompatible updates disappears; the anomaly associated with incoherent update sequences remains.

Finally, as Dorr and Hawthorne (2013) note, these conjunctions are bad when embedded in disjunctions:

(97) a. #Either it's raining and it might not be raining, or it's a good day for a picnic.
b. #Either it's not raining and it might be raining, or it's a good day for a picnic.
c. #Either it might be raining and it's not raining, or it's a good day for a picnic.
d. #Either it might not be raining and it's raining, or it's a good day for a picnic.

Once again, we see the same pattern:

(98) a. Either it might have been raining and you went outside to check and it wasn't raining, or you haven't been doing your job.
b. #Either your dad was born in Greece and you received a new familial record and your dad wasn't born in Greece, or you haven't been doing your job.

Narrativizing the conjunction again rescues the incompatible updates; the incoherent updates remain bad. It seems that we've discovered a fact about the pragmatics of embedded conjunction in at least some contexts: often the implicit project suggested by embedded conjunction is to build an information state compatible with all conjuncts. It follows from the definition of incompatibility that incompatible updates will be anomalous relative to this project. Developing, clarifying, formalizing and defending the proposal that embedded conjunction often gives rise to 'world-building' pragmatics would be material for a paper in itself; my only goal in bringing it up here is to have identified some salient differences between the behavior in these contexts of updates that I've classified as incoherent and updates that I've classified as merely incompatible, thereby discrediting or at least complicating the claim that the anomalousness of $might\varphi$ plus $\neg\varphi$ sequences in these contexts is due to the fact that those sequences are truly incoherent. In fact, I believe that I have found support in my brief examination of these contexts for the notion that these sequences behave exactly as we would expect incompatible but non-incoherent update sequences to behave.

References

Anand, Pranav & Adrian Brasoveanu. 2009. Modal concord as modal modification. *Proceedings of Sinn und Bedeutung 14*, Vienna.

Barker, Chris & Gina Taranto. 2003. The paradox of asserting clarity. *Proceedings of the Western Conference on Linguistics (WECOL) 2002*, California State University.

Beaver, David. 2001. *Presupposition and assertion in dynamic semantics.* Studies in Logic, Language and Information, CSLI Publications.

DeRose, Keith. 1991. Epistemic possibilities. *The Philosophical Review*, 100(4): 581-605.

Dorr, Cian & John Hawthorne. 2013. Embedding epistemic modals. *Mind* 122: 867-913.

Farkas, Donka & Kim Bruce. 2010. On reacting to assertions and polar questions. *Journal of Semantics* 27: 81-118.

von Fintel, Kai & Anthony Gillies. 2010. Must ... stay ... strong! *Natural Language Semantics* 18(4): 351-383.

von Fintel, Kai & Anthony Gillies. 2011. 'Might' made right. In Andy Egan & Brian Weatherson (eds.), *Epistemic modality.* Oxford University Press: 108-130.

Gillies, Anthony. 2004. Epistemic conditionals and conditional epistemics. *Noûs* 38(4): 585-616.

Hacking, Ian. 1967. Possibility. *The Philosophical Review*, 76(2): 143-168.

Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases.* UMass Amherst dissertation.

Heim, Irene. 1983. On the projection problem for presuppositions. *Proceedings of the second west coast conference on formal linguistics*: 114-125.

Heim, Irene. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics* 9: 183-221.

Groenendijk, Jeroen, Martin Stokhof & Frank Veltman. 1996. Coreference and modality. In S. Lappin (ed.), *The Handbook of Contemporary Semantic Theory*, pp. 179-213. Blackwell.

Grosz, Patrick. 2009. Grading modality: A new approach to modal concord and its relatives. *Proceedings of Sinn und Bedeutung* 14: 185-201.

Kamp, Hans. 1981. A theory of truth and semantic representation. In J. Groenendijk, T. Janssen & M. Stokhof (eds.), *Formal Methods in the Study of Language.*

Karttunen, Lauri. 1972. Possible and must. In J. Kimball (ed.), *Syntax and semantics*, vol. 1, 120.

Kaufmann, Stefan. 2004. Conditioning against the grain: Abduction and indicative conditionals. *Journal of Philosophical Logic* 33: 583-606.

Kaufmann, Stefan. 2005. Conditional predictions: A probabilistic account. *Linguistics and Philosophy* 28: 181-231.

Kaufmann, Stefan. 2009. Conditionals right and left: Probabilities for the whole family. *Journal of Philosophical Logic* 38: 1-53.

Kratzer, Angelika. 1977. What 'must' and 'can' must and can mean. *Linguistics and Philosophy* 1: 337-355.

Kratzer, Angelika. 1981. The notional category of modality. In H. Eikmeyer & H. Rieser (eds.), *Words, worlds, and contexts: New approaches in world semantics.* de Gruyter.
Kratzer, Angelika. 1991. Modality. In A. von Stechow & D. Wunderlich (eds.) *Semantics: An International Handbook of Contemporary Research*: 639-650. de Gruyter.
Kratzer, Angelika. 2012. *Modals and conditionals.* Oxford University Press.
Lassiter, Daniel. 2011. *Measurement and modality: the scalar basis of modal semantics.* NYU dissertation.
Lassiter, Daniel. 2014. Epistemic comparison, models of uncertainty, and the disjunction puzzle. *Journal of Semantics.*
Lassiter, Daniel. 2014. The weakness of *must*: In defense of a Mantra. *Proceedings of SALT 24*: 597-618..
MacFarlane, John. 2011. Epistemic modals are assessment-sensitive. In Andy Egan & Brian Weatherson (eds.), *Epistemic modality.* Oxford University Press.
McCready, Eric & Norry Ogata. 2007. Evidentiality, modality and probability. *Linguistics and Philosophy* 30: 147-206.
Moss, Sarah. 2015. On the Semantics and Pragmatics of Epistemic Vocabulary. *Semantics & Pragmatics* 8.
Rothschild, Daniel. 2012. Expressing credences. *Proceedings of the Aristotelian Society.*
Stalnaker, Robert. 1974. Pragmatic presuppositions. *Context and Content.* Oxford University Press: 47-62.
Stalnaker, Robert. 1978. Assertion. In P. Cole (ed.), *Syntax and Semantics: 9.* Academic Press: 31532.
Stephenson, Tamina. 2007. Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy 30*: 487-525.
Stone, Matthew. 1994. The reference argument of epistemic *must.* *Proceedings of IWCS 1*:181-190.
Swanson, Eric. 2006. *Interactions with context.* MIT dissertation.
Swanson, Eric. 2011. How not to theorize about the language of subjective uncertainty. In Andy Egan & Brian Weatherson (eds.), *Epistemic modality.* Oxford University Press.
Swanson, Eric. 2015. The application of constraint semantics to the language of subjective uncertainty. *Journal of philosophical logic.*
Teller, Paul. 1972. Epistemic possibility. *Philosophia* 2(4): 303-320.
Veltman, Frank. 1996. Defaults in update semantics. *Journal of Philosophical Logic* 25(3): 221-261.
Willer, Malte. 2010. *Modality in flux.* UT Austin dissertation.
Willer, Malte. 2013. Dynamics of epistemic modality. *Philosophical Review*, 122(1): 45-92.
Yalcin, Seth. 2007. Epistemic modals. *Mind* 116: 983-1026.
Yalcin, Seth. 2010. Probability operators. *Philosophy Compass* 5(11): 916-937.
Yalcin, Seth. 2012. Context Probabilism. *Proceedings of the 18th Amsterdam Colloquium*: 12-21.