Weak Constraints (1 of 3)

- A weak constraint has the form

  :~ F. [Weight @ Level]

where F is a conjunction of literals (atoms and their negations)

- **Weight** is an integer and **Level** is a nonnegative integer
  - When F is satisfied, we say the weak constraint is violated, and we apply a penalty of Weight at this Level — the higher level it is, the higher priority we consider

Weak Constraints (2 of 3)

- Let Π be a program $\Pi_1 \cup \Pi_2$, where Π_1 is a usual ASP program and Π_2 is a set of weak constraints.

- We call I a stable model of Π if it is a stable model of Π_1 .

- For every stable model I of Π and any nonnegative integer l, the penalty of I at level L, denoted by $Penalty_{\Pi}(I, L)$, is defined as

$$\sum_{\substack{:\sim\ F[w@l] \in \Pi_2,\\ I \models F}} w.$$

- ex:

```
{p; q}.
:~ p.   [10@0]
:~ q.   [5@1]
```

Weak Constraints (3 of 3)

- For any two stable models I and I' of Π, we say I is **dominated** by I' if
  - there is some level L such that $Penalty_{\Pi}(I', L) < Penalty_{\Pi}(I, L)$ and
  - for all integers $K > L$, $Penalty_{\Pi}(I', K) = Penalty_{\Pi}(I, K)$
- A stable model of Π is called **optimal** if it is not dominated by another stable model of Π

In clingo

```
% test

{p;q}.
:~ p. [10@0]
:~ q. [5@1]
```

```
$ clingo test

Answer: 1
Optimization: 0 0
OPTIMUM FOUND


Models        : 1
  Optimum     : yes
Optimization  : 0 0
```

```
$ clingo test --opt-mode=enum 0
Solving...
Answer: 1


Optimization: 0 0
Answer: 2
q
Optimization: 5 0
Answer: 3
p
Optimization: 0 10
Answer: 4
p q
Optimization: 5 10
OPTIMUM FOUND


Models          : 4
```