# Fourier properties 傅立叶特性

## Linearity 线性特征

$$a x_1[n] + b x_2[n] \Leftrightarrow a X_1[k] + b X_2[k]$$

Prove.

$$\mathrm{DFT}(a x_1[n] + b x_2[n])$$

$$= \sum_{n=0}^{N-1} (a x_1[n] + b x_2[n]) \cdot e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} a x_1[n] \cdot e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} b x_2[n] e^{-j2\pi kn/N}$$

$$= a \sum_{n=0}^{N-1} x_1[n] \cdot e^{-j2\pi kn/N} + b \sum_{n=0}^{N-1} x_2[n] e^{-j2\pi kn/N}$$

$$= a X_1[k] + b X_2[k]$$

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linear combination of two signals $x_1$, $x_2$ corresponding to the linear combination of the spectrum domain of the two signals.

Fourier properties 傅立叶特性

shift: $x[n-n_0] \Leftrightarrow e^{-j2\pi k n_0/N} X[k]$

displacing samples of a signal

Prove.

$$\mathrm{DFT}(x[n-n_0])$$

$$= \sum_{n=0}^{N-1} x[n-n_0] \cdot e^{-j2\pi kn/N}$$

$$= \sum_{m=-n_0}^{N-1-n_0} x[m] e^{-j2\pi k(m+n_0)/N} \quad (m = n-n_0)$$

$$= \sum_{m=0}^{N-1} x[m] e^{-j2\pi km/N} \cdot e^{-j2\pi k n_0/N}$$

$$= e^{-j2\pi k n_0/N} \sum_{m=0}^{N-1} x[m] e^{-j2\pi km/N}$$

$$= e^{-j2\pi k n_0/N} X[k]$$

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Shift will not affect the spectrum magnitude value. it will affect the phase value.

# Fourier properties 傅立叶特性

Symmetry:

$x[n]$ real $\Leftrightarrow \underbrace{\Re\{X[k]\}}_{\text{real part}}$ even and $\underbrace{\Im\{X[k]\}}_{\text{imaginary part}}$ odd

$\Leftrightarrow |X[k]|$ even and $\underbrace{\angle X[k]}_{\text{phase}}$ odd

$x[n]$ real and even $\Leftrightarrow \Re\{X[k]\}$ even and $\Im\{X[k]\} = 0$

$\Leftrightarrow |X[k]|$ even and $\angle X[k] = n\pi$ (including 0)

# Fourier properties 傅立叶特性

Convolution : $x_1[n] * x_2[n] \Leftrightarrow X_1[k] \overset{\text{product}}{\times} X_2[k]$

( how two signal area overlap over the time)

Prove.

$$DFT\ (x_1[n] * x_2[n])$$

$$= \sum_{n=0}^{N-1} (x_1[n] * x_2[n]) \cdot e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} x_1[m]\, x_2[n-m] \cdot e^{-j2\pi kn/N}$$

$$= \sum_{m=0}^{N-1} x_1[m] \cdot \underline{\sum_{n=0}^{N-1} x_2[n-m]\, e^{-j2\pi kn/N}} \quad \text{shift}$$

$$= \left(\sum_{m=0}^{N-1} x_1[m]\, e^{-j2\pi km/N}\right) \cdot X_2[k]$$

$$= X_1[k]\, X_2[k]$$

## Fourier properties 傅立叶特性

### Energy conservation.

$$\sum_{n=-N/2}^{N/2-1} |x[n]|^2 = \frac{1}{N} \sum_{k=-N/2}^{N/2-1} |X[k]|^2$$

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### Amplitude in decibels (dB)

$$20 * \log_{10}(\mathrm{abs}(X))$$

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### Phase unwrapping

represent a phase spectrum in a way that is easier to visualize and understand.

By comparing the previous phase data. adding $\pi$ whenever there is a discontinity.

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### Zero padding

By zero padding more samples. we will get more interpolated values in between. Therefore resulting into a smother spectrum.

# Fourier properties 傅立叶特性

## Fast Fourier Transform

Taking advance of symmetric

breaks down recursively the DFT of a power of 2 size into two pieces of size $N/2$.

DFT complexity $N^2$

FFT complexity $N \log N$

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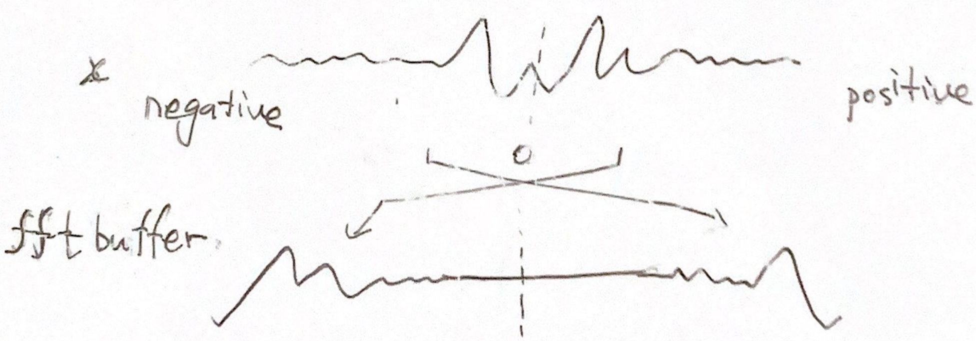
## FFT and zero-phase windowing

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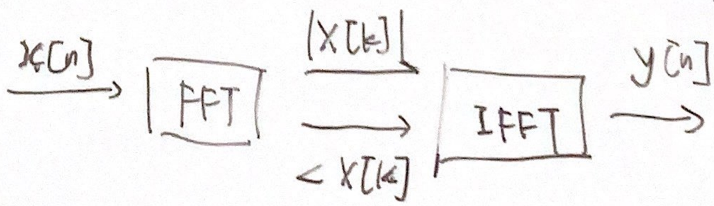
## Analysis / synthesis




· only take positive part is enough. due to the symmectric properties.