基本数学知识

- Sinusoidal functions
- Complex numbers
- Euler's formula
- Complex sinusoids
- Scalar product of sequences
- Even and odd functions
- Convolution

一、Sinusoidal functions - 正弦函数.

$$x[n] = A\cos(\omega nT + \varphi) = A\cos(2\pi f nT + \varphi)$$

A: amplitude 振幅

ω: angular frequency in radians/seconds.

角频率, 弧度制, 单位为 radians/seconds.

$f = \omega / 2\pi$: frequency in Hertz (cycles/seconds)

频率. 单位为Hz. 表示 cycles/seconds.

φ: initial phase in radians. 初始相位. 弧度制)

n. time index 采样次数

$T = 1/f_s$: sampling period in seconds ($t = nT = n/f_s$)

采样周期. 多久采样一次

二 Complex numbers 复数

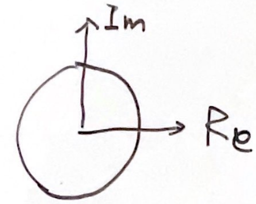
$(a+jb)$ a, b: real numbers 实部

$j = \sqrt{-1}$ imaginary unit 虚部 $x^2+1=0$

Complex plane

Re (real axis)

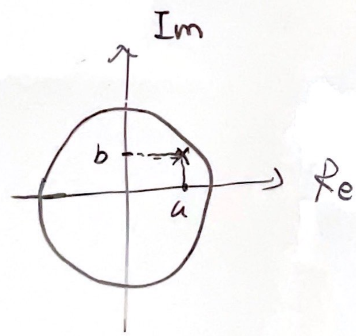
Im (imaginary axis)



单位圆

Rectangular form

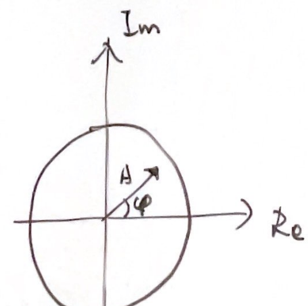
$(a+jb)$



Polar form 极坐标

$A = \sqrt{a^2+b^2}$

$\varphi = \text{atan2}\,\frac{b}{a}$



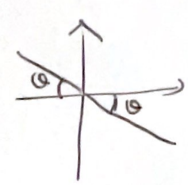
($\text{atan2} = \text{atan}\,\frac{b}{a}$、反正切函数

但是 atan2 可以处理 $a=0$ 而 $b\neq 0$ 的情况)

where

if $(a>0)$ $\text{atan2}(\frac{b}{a}) = \arctan\frac{b}{a}$

else if $(a<0)$ $\text{atan2}(\frac{b}{a}) = \arctan\frac{b}{a} - \pi$

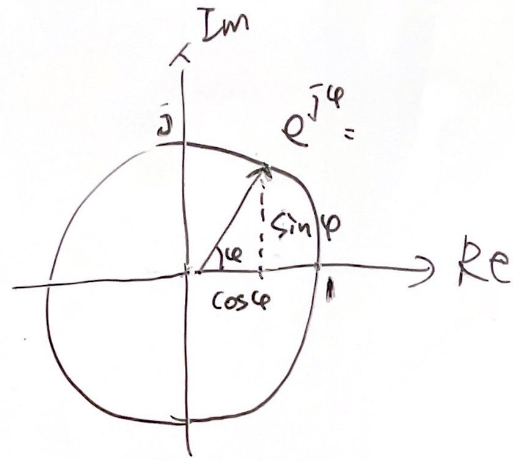


三. Euler's formula 欧拉公式

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$$

$$\sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j}$$



j是虚数单位，φ是实数

推导过程

$$e^{j\varphi} = \cos\varphi + j\sin\varphi \quad ①$$

$$e^{-j\varphi} = \cos\varphi - j\sin\varphi \quad ②$$

由①+②得

$$e^{j\varphi} + e^{-j\varphi} = 2\cos\varphi$$

$$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$$

由①-②得

$$e^{j\varphi} - e^{-j\varphi} = 2j\sin\varphi$$

$$\sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j}$$

欧拉恒等式

$\varphi = \pi$ $\quad e^{j\pi} = -1.$ $\quad e^{j\pi} + 1 = 0$

四. complex sinewave 复合正弦信号 变化形式

$$\tilde{x}[n] = Ae^{j(wnT+\varphi)} = \underline{Ae^{j\varphi} \cdot e}^{j(wnT)} \quad \underline{xe^{j(wnT)}}$$

$$= A\cos(wnT+\varphi) + jA\sin(wnT+\varphi$$

Real sinewave 实部正弦信号

$$x[n] = A\cos(wnT+\varphi) = A\left(\frac{e^{j(wnT+\varphi)} + e^{-j(wnT+\varphi)}}{2}\right)$$

可以由复合正弦信号表示 一个复合正弦信号可以表示一个实部正弦信号

$$= \frac{1}{2}xe^{j(wnT)} + \frac{1}{2}x^*e^{-j(wnT)} = \frac{1}{2}\tilde{x}[n] + \frac{1}{2}\tilde{x}^*[n]$$

$$= \mathcal{R}\{\tilde{x}[n]\}$$

↑

实数

x 和 x^* 表示不同的数

五 Scalar (dot) product of sequences

$$\langle x, y \rangle = \sum_{n=0}^{N-1} x[n]\, \underline{\underline{y^*[n]}}$$

复共轭

example

$x[n] = [0, j, 1]$　　$y[n] = [1, j, j]$

虚部共轭，故取反

$$\langle x, y \rangle = 0 \times 1 + j \times (\underline{-j}) + 1 \times (\underline{-j})$$

$$= 1 - j$$

Orthogonality of sequences

$$x \perp y \iff \langle x, y \rangle = 0$$

example:

$x[n] = [2, 2]$; $y[n] = [2, -2]$

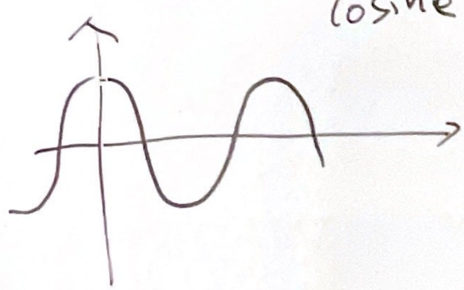
$$\langle x, y \rangle = 2 \times 2 + 2 \times -2 = 4 - 4 = 0$$

六 Even and odd function 奇偶函数

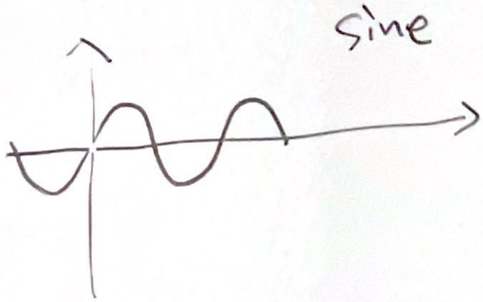
偶函数 $f(-x) = f(x)$

奇函数 $f(-x) = -f(x)$

cosine (偶函数 even)



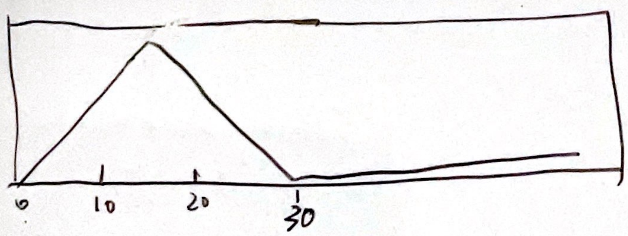
sine (奇函数 odd)



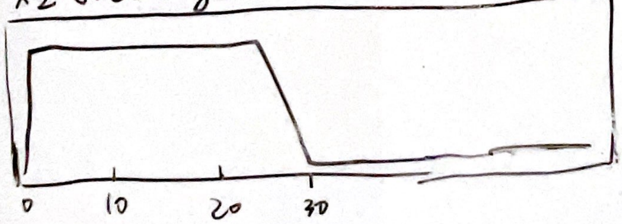
七 Convolution — 卷积

$$y[n] = (x_1[n] * x_2[n])_n$$
$$= \sum_{m=0}^{N-1} x_1[m]\, x_2[n-m]$$

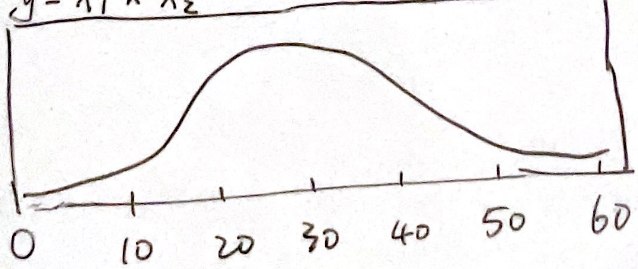
x_1 (triangle)



x_2 (rectangle



$y = x_1 * x_2$



y是 x_1和x_2的结合。卷积类似计算交叉相关性
(cross correlation)，常见的滤波算法