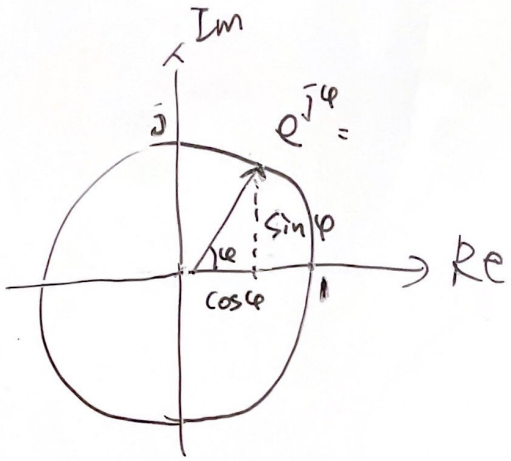
三. Euler's formula 欧拉公式

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$$

$$\sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j}$$



j是虚数单位，φ是实数

推导过程

$$e^{j\varphi} = \cos\varphi + j\sin\varphi \quad ①$$

$$e^{-j\varphi} = \cos\varphi - j\sin\varphi \quad ②$$

由①+②得

$$e^{j\varphi} + e^{-j\varphi} = 2\cos\varphi$$

$$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$$

由①-②得

$$e^{j\varphi} - e^{-j\varphi} = 2j\sin\varphi$$

$$\sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j}$$

欧拉恒等式

$\varphi = \pi$ $\quad e^{j\pi} = -1$ $\quad e^{j\pi} + 1 = 0$