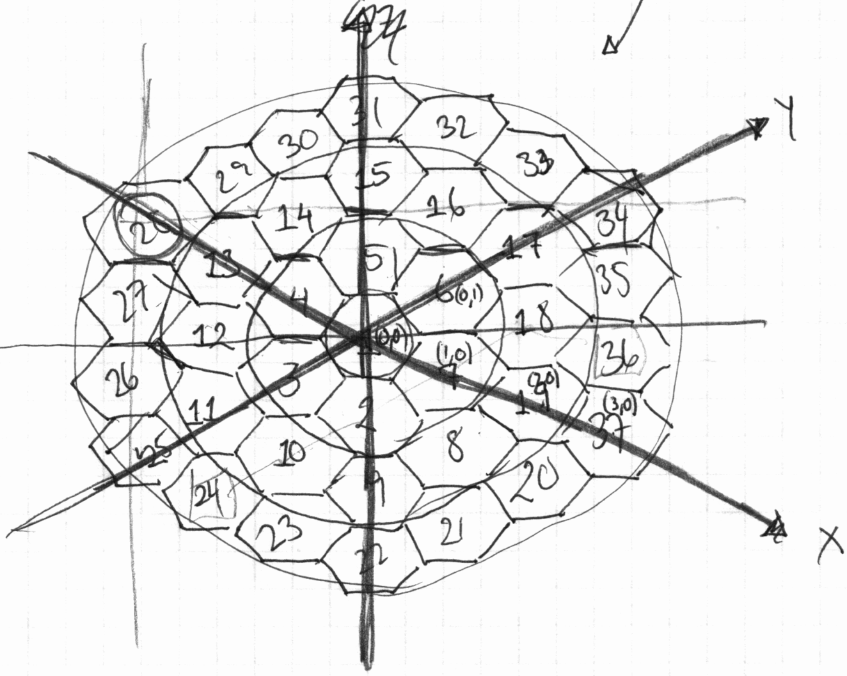
Input: $n_1, n_2 \in \mathbb{N}_{\geq 1}$

Output: Shortest path between $c(n_1)$ and $c(n_2)$ when positioned like this,



~~Metrik~~: Point: $(x, y, \cancel{z})$

Ex: ~~$c(18) = (2,2,0)$; $c(20) = (2,2,-2)$; $c(23) = (-1,-1,$~~

$c(18) = (1,1,\cancel{0})$; $c(20) = (3,-1,\cancel{0})$; $c(23) = (2,-3)$?

Metrik: $d((x,y),(x',y')) = ?$ $= |x'-x| + |y'-y|$

Ex: $d((1,-3),(2,1)) = 5$

$c(24)$, $c(36)$

Just convenient; no $\sqrt{(x'-x)^2 + (y'-y)^2}$:)

"~~Short-cutting isn't possible in hex world~~"

Series
1, 6, 12, 18, ...
Sum
1, 7, 19, ... MAX

$c(n) = (x,y)$; $c = ?$ ~~$d(c(1), c(n))$ $1+$~~

$1 + \sum_{d=0}^{n} 6d = 1 + 6\frac{n(n+1)}{2} = 3n^2 + 3n + 1$ = "max on ring n"

$\sum_{1}^{10} \frac{n(n+1)}{2}$

Renaming... $3r(r+1)+1 = n$

$\Leftrightarrow 3r^2 + 3r + (1-n) = 0 \Leftrightarrow \frac{-3 \pm \sqrt{12n-3}}{2 \cdot 3} \Leftrightarrow r = \frac{\pm\sqrt{12n-3} - 3}{6}$

"$D = b^2 - 4ac$": $3^2 - 4 \cdot 3 \cdot (1-n) = 9 - 12(\ldots$

$9 - 12(1-n) = 9 - 12 + 12n = 12n - 3$

$r(n) = \left\lceil \frac{\sqrt{12n-3} - 3}{6} \right\rceil$

$7 \cdot 12 = 84$

$r(7) = \left\lceil \frac{\sqrt{12 \cdot 7 - 3} - 3}{6} \right\rceil = \left\lceil \frac{9 - 3}{6} \right\rceil = 1$

$r(8) = \left\lceil \frac{\sqrt{12 \cdot 8 - 3} - 3}{6} \right\rceil = 2.$ I trust.

$(r(n), 0) = c(\text{max on ring } r(n))$..

How long around (anti)clockwise will I find n? Bleh

Assume $n, n-1$: $r(n) = r(n-1)$

$c(n) =: (x,y)$; $c(n-1) = ?$

<u>EX</u>: $n = 19$. $c(19) = (2,0)$

$(2,0), (1,1), (0,2), (-1,2), (-2,-2), (-2,-1), (-2,0), (-1,-1), \cancel{(0,)}, \ldots$

1. quadrant: $(r,0), (r-1,1), (r-2,2), \ldots, \cancel{(0,r)}$, (a)
2. quadrant: $(0,r), (-1,r), (-2,r), \ldots, \cancel{(-r,r)}$ (b)

   $(-r,r), (-r,r-1), (-r,r-2), \ldots, \cancel{(-r,0)}$ (c)
3. quadrant: $(-r,0), (-r+1,-1), (-r+2,-2), \ldots, \cancel{(0,-r)}$ (d)
4. quadrant: $(0,-r), (1,-r), (2,-r), \ldots, \cancel{(r,-r)}$ (e)

   $(r,-r), (r,-r+1), (r,-r+2), \ldots, \cancel{(r,0)}$ (f)

<u>ALG</u>: "int2coord"

1. $r := r(n)$
2. $m := 3r^2 + 3r + 1$
3. case $m - n =: d$

   when $\in [0, r]$ # (a)

   $(r-d, d)$

   when $\in [r+1, 2r]$ # (b)

   $(-(d-r), r)$

   ⋮

<u>Ex</u> $c(16) = ?$

$16 \cdot 12$ / 72 / 120 / 192

$13^2 = 169$

$12^2 = 144$

$14^2 = 196$

1. $r := r(16) = \left\lceil \frac{\sqrt{12n-3} - 3}{6} \right\rceil = \left\lceil \frac{\sqrt{189} - 3}{6} \right\rceil = \left\lceil \frac{? - 3}{6} \right\rceil = \cancel{\ldots} = \lceil 1.? \rceil = 2$ ✗ ☺ 0-indexed..

   ~ 14 ; $\sim \frac{11}{6}$
2. $m := 19$.
3. $d := 19 - 16 = 3$; case (b) : $(-(3-2), 2) = (-1, 2)$ ✓ ☺

Constant time to do "int 2 coord".

Basicly done. Programming...

Done ~~DISTANCE~~

<u>Ex</u> $d(c(10), c(15)) = d((1,-2), (-2,2))$

My distance was completely wrong!

$c(15) - c(10) = (-2,2) - (1,-2) = (-3,4) = (x,y)$

"Along z-axis" ... $\overset{\text{sign}}{S}(x) \neq S(y) \Rightarrow d(c(a), c(b)) = \max\{|x|, |y|\}$

$S(x) = S(y) \Rightarrow d(c(a), c(b)) = |x| + |y|$

where $(x,y) = c(b) - c(a)$