

The Pumping Lemma for CFL's

Statement
Applications

Intuition

- ◆ Recall the pumping lemma for regular languages.
- ◆ It told us that if there was a string long enough to cause a cycle in the DFA for the language, then we could “pump” the cycle and discover an infinite sequence of strings that had to be in the language.

Intuition – (2)

- ◆ For CFL's the situation is a little more complicated.
- ◆ We can always find **two** pieces of any sufficiently long string to “pump” in tandem.
 - ◆ **That is:** if we repeat each of the two pieces the same number of times, we get another string in the language.

Statement of the CFL Pumping Lemma

For every context-free language L

There is an integer n , such that

For every string z in L of length $\geq n$

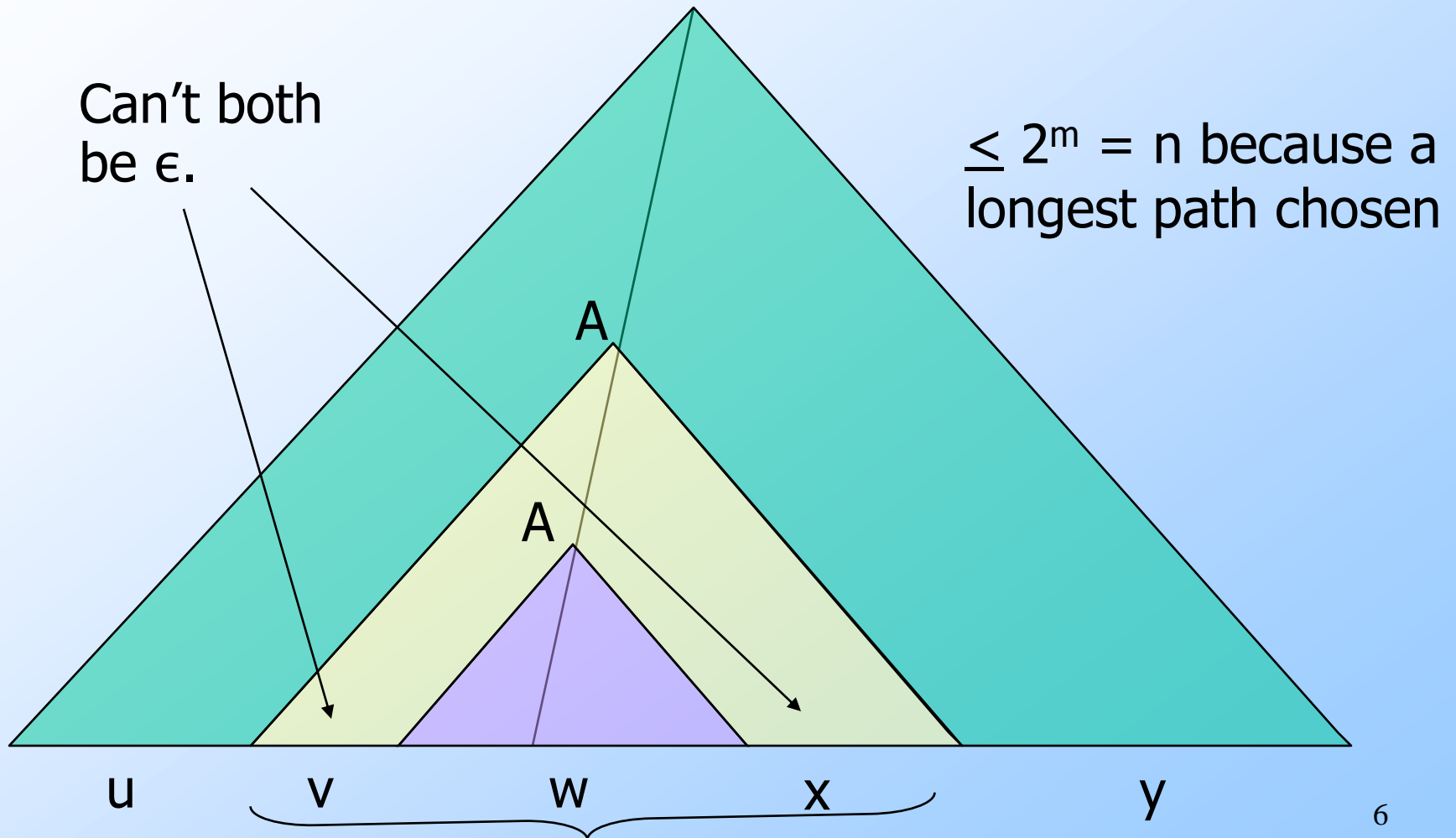
There exists $z = uvwxy$ such that:

1. $|vwx| \leq n$.
2. $|vx| > 0$.
3. For all $i \geq 0$, uv^iwx^iy is in L .

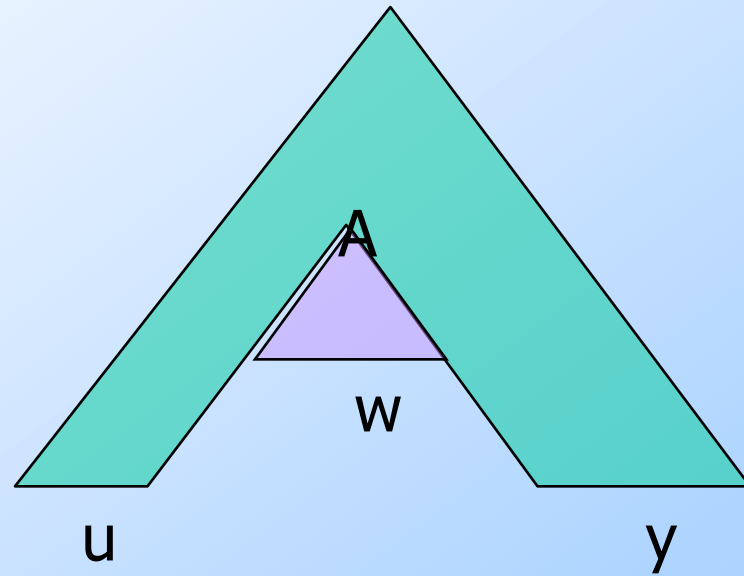
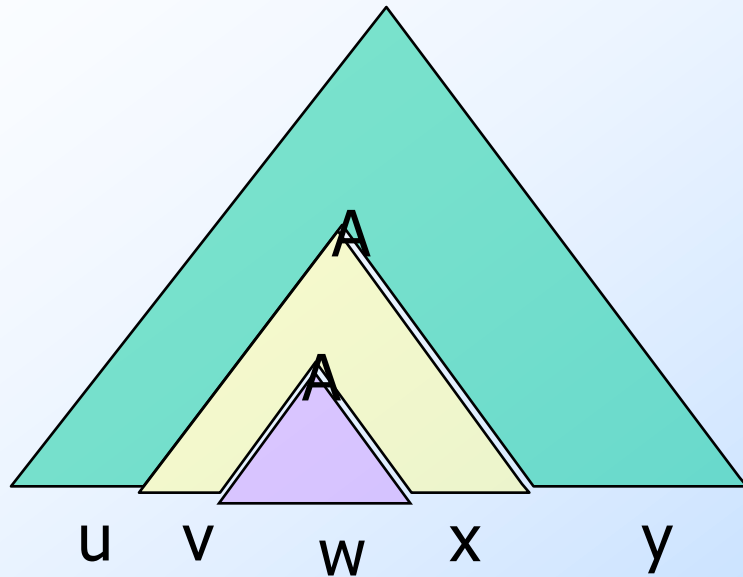
Proof of the Pumping Lemma

- ◆ Let the grammar have m variables.
- ◆ Pick $n = 2^m$.
- ◆ Let $|z| \geq n$.

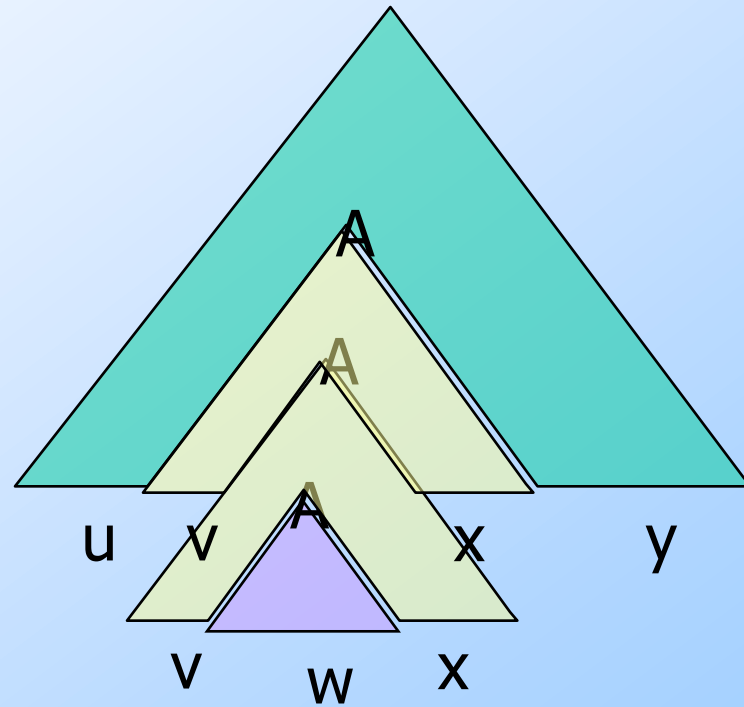
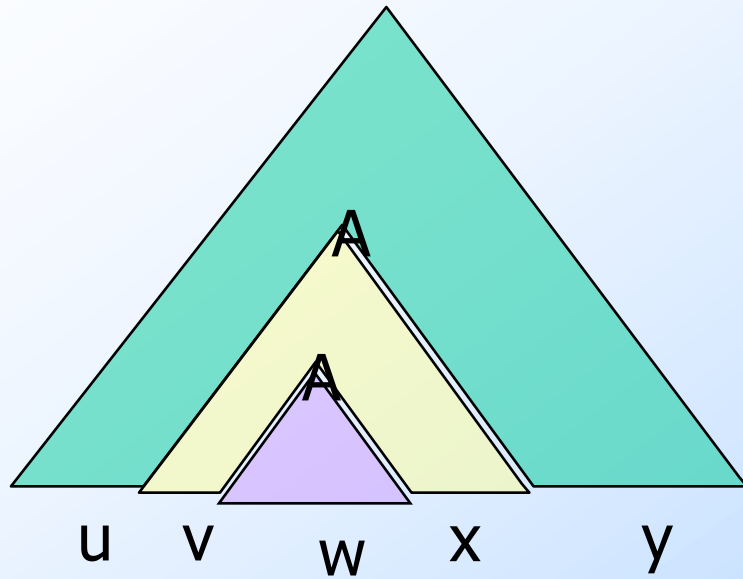
Parse Tree in the Pumping-Lemma **Proof**



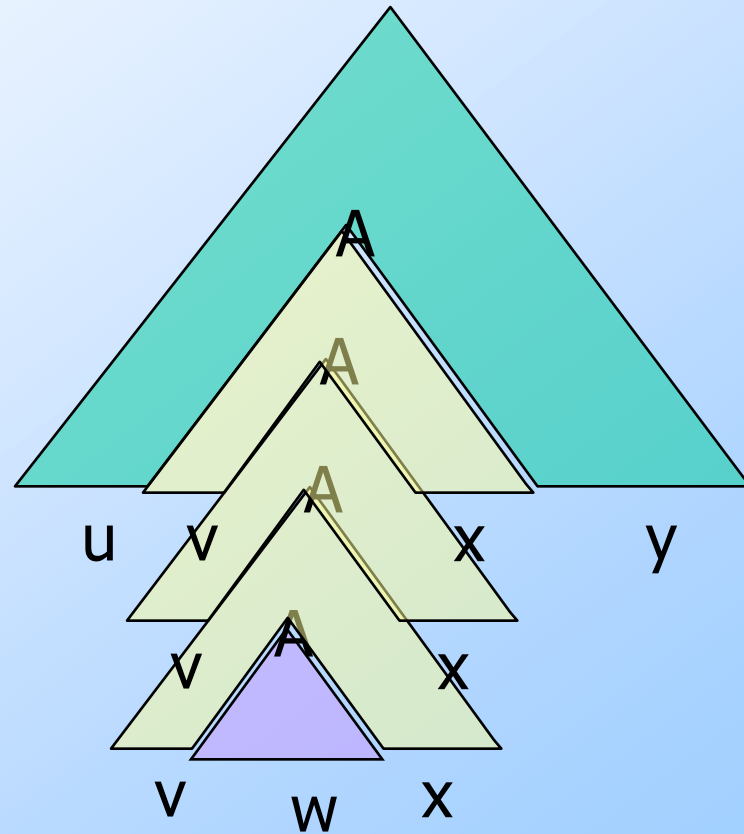
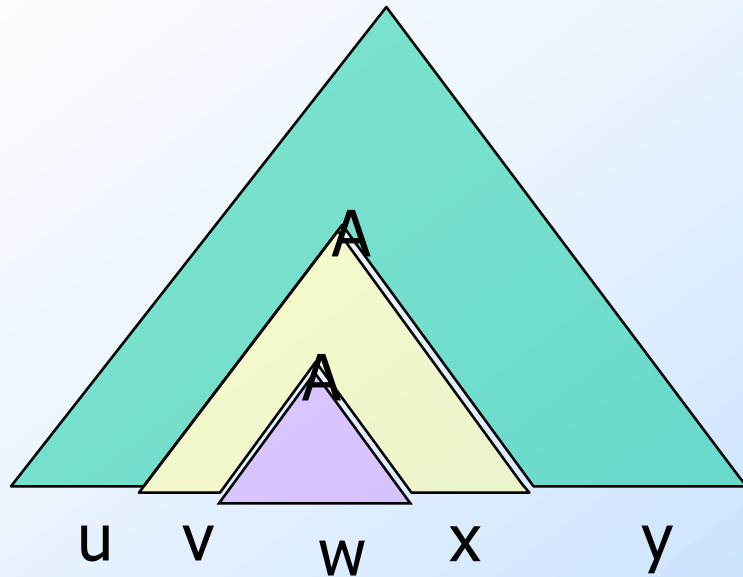
Pump Zero Times



Pump Twice



Pump Thrice Etc., Etc.



Using the Pumping Lemma

- ◆ Non-CFL's typically involve trying to match two pairs of counts or match two strings.
- ◆ **Example:** The text uses the pumping lemma to show that $\{ww \mid w \text{ in } (\mathbf{0+1})^*\}$ is not a CFL.

Using the Pumping Lemma – (2)

- ◆ $\{0^i10^i \mid i \geq 1\}$ is a CFL.
 - ◆ We can match one pair of counts.
- ◆ But $L = \{0^i10^i10^i \mid i \geq 1\}$ is not.
 - ◆ We can't match two pairs, or three counts as a group.
- ◆ **Proof** using the pumping lemma.
- ◆ Suppose L were a CFL.
- ◆ Let n be L 's pumping-lemma constant.

Using the Pumping Lemma – (3)

- ◆ Consider $z = 0^n 1 0^n 1 0^n$.
- ◆ We can write $z = uvwxy$, where $|vwx| \leq n$, and $|vx| \geq 1$.
- ◆ **Case 1:** vx has no 0's.
 - ◆ Then at least one of them is a 1, and uwv has at most one 1, which no string in L does.

Using the Pumping Lemma – (4)

- ◆ Still considering $z = 0^n 1 0^n 1 0^n$.
- ◆ **Case 2:** vx has at least one 0.
 - ◆ vwx is too short ($\text{length} \leq n$) to extend to all three blocks of 0's in $0^n 1 0^n 1 0^n$.
 - ◆ Thus, uwy has at least one block of n 0's, and at least one block with fewer than n 0's.
 - ◆ Thus, uwy is not in L .