Signal Scaling

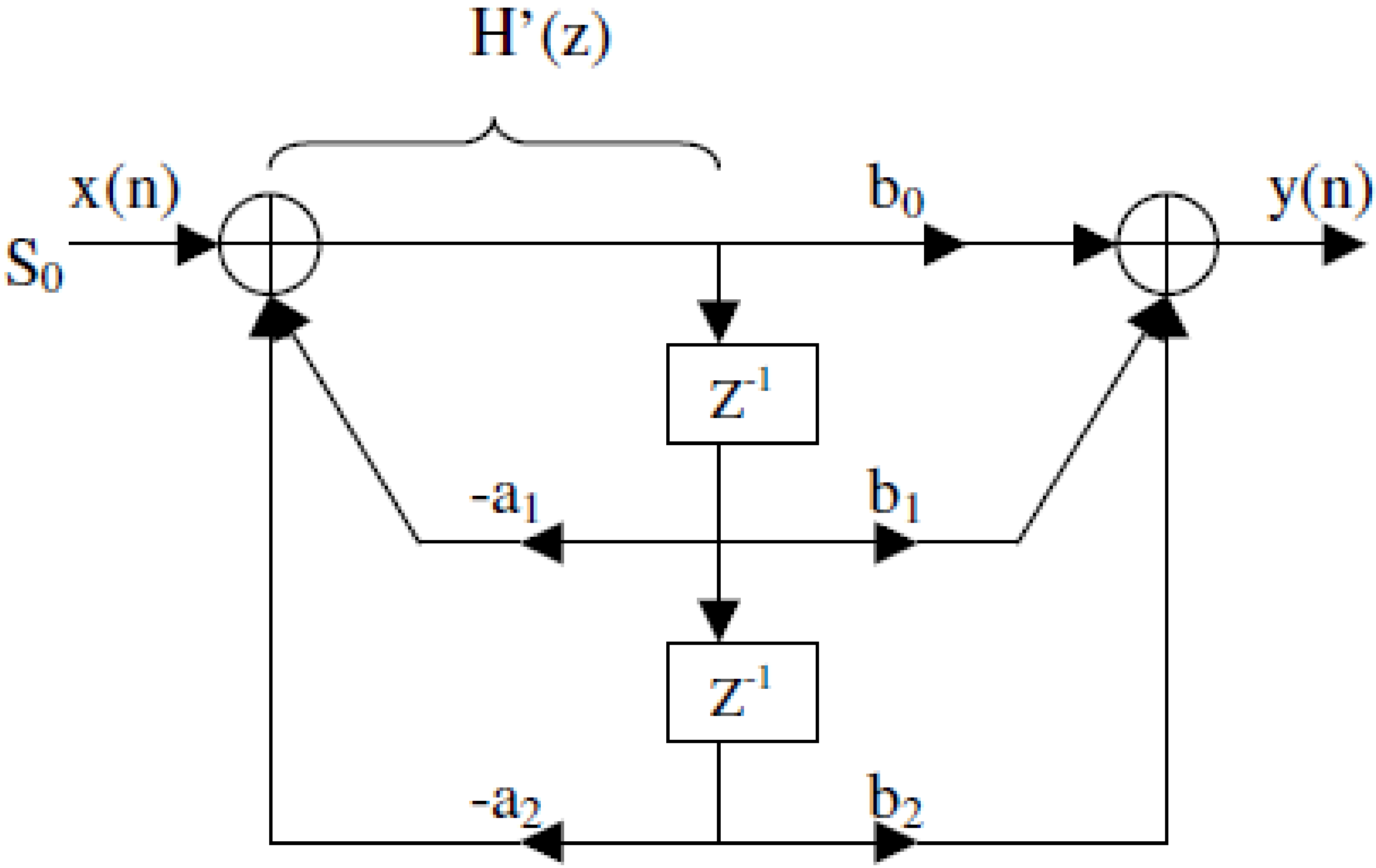
I.Nelson
SSN College of Engineering

Signal Scaling

- Saturating the adder characteristics may cause undesirable signal distortion due to the nonlinearity of the clipper.

- In order to limit the non-linear distortion, it is important to scale the input signal and unit sample response between the input and any internal summing node in the system, so that the occurrence of overflow is reduced.

- Let us consider the second order IIR filter.



- A scale factor S_0 is introduced between the input x(n) and the adder 1, to prevent overflow at the output adder1. The overall transfer function is

$$H(z) = S_0 \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = S_0 \frac{N(z)}{D(z)}$$

and

$$H'(z) = \frac{W(z)}{X(z)} = \frac{S_0}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{S_0}{D(z)}$$

- If the instantaneous energy in the output sequence w(n) is less than the input sequence finite energy, then there will be no overflow.

- We have,

$$W(z) = \frac{S_0 X(z)}{D(z)} = S_0 X(z) S(z)$$

where

S(z) = 1/ D(z)

- Therefore,

$$w(n) = \frac{S_0}{2\pi}\int X(e^{j\omega})S(e^{j\omega})e^{j\omega n}\,d\omega$$

$$w^2(n) = \frac{S_0^2}{4\pi^2}\left|\int X(e^{j\omega})S(e^{j\omega})e^{j\omega n}\,d\omega\right|^2$$

- Using Schwartz inequality,

$$w^2(n) \le S_0^2\left[\frac{1}{2\pi}\int_{2\pi}\left|S(e^{j\omega})\right|^2 d\omega\right]\left[\frac{1}{2\pi}\int_{2\pi}\left|X(e^{j\omega})\right|^2 d\omega\right]$$

- Applying Parseval's theorem we get,

$$w^2(n) \le \sum_{n=0}^{\infty} x^2(n) S_0^2 \left[\frac{1}{2\pi} \int_{2\pi} \left| S(e^{j\omega}) \right|^2 d\omega \right]$$

- Let z = $e^{j\omega}$, therefore,

$$w^2(n) \le \sum_{n=0}^{\infty} x^2(n) S_0^2 \left[\frac{1}{2\pi j} \oint_C \left| S(z) \right|^2 z^{-1} dz \right]$$

$$\le \sum_{n=0}^{\infty} x^2(n) S_0^2 \left[\frac{1}{2\pi j} \oint_C S(z) S(z^{-1}) z^{-1} dz \right]$$

- We can obtain $w^2(n) \le \sum_{n=0}^{\infty} x^2(n)$

when, $$S_0^2\left[\frac{1}{2\pi j}\oint_C S(z)S(z^{-1})z^{-1}dz\right]=1$$

$$S_0^2 = \frac{1}{\frac{1}{2\pi j}\oint_C S(z)S(z^{-1})z^{-1}dz}$$

$$S_0^2 = \frac{1}{\frac{1}{2\pi j}\oint_C \frac{1}{D(z)D(z^{-1})}z^{-1}dz} = \frac{1}{I}$$

where $$I = \frac{1}{2\pi j}\oint_C \frac{1}{D(z)D(z^{-1})}z^{-1}dz$$