FA to RE

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Conversion of DFA to RE

- The conversion of FA to RE is possible with three different methods. They are :
  - Regular Expression equation method – $R_{ij}^{(k)}$
  - Arden's Theorem.
  - State elimination technique.

RE equation method $R_{ij}^{(k)}$

- **Theorem**

For every DFA $A = (Q, \Sigma, \delta, S, F)$, there is a regular expression R, such that $L(R) = L(A)$.

- **Proof**

Let L be the set accepted by the DFA

$A = (\{q1, q2, ...qn\}, \Sigma, \delta, q1, F)$ with q1 being the start state.

Let $R_{ij}^{(k)}$ be the regular expression describing the set of all strings x *such that* $\delta(q_i, x) = q_j$ going through intermediate states $\{q_1, q_2, ...q_K\}$ *only*

$R_{ij}^{(k)}$ will be defined inductively. Note that

$$L\left(\bigcup_{j \in F} R_{1j}^{(n)}\right) = L(A)$$

RE equation method $R_{ij}^{(k)}$

- **Basis**

K = 0, i.e., no intermediate states.

$R_{ij}^{(0)}$ denotes a set of strings which is either ε(or) single symbol.

Case 1 : *i ≠ j*

$R_{ij}^{(0)} = \{a \mid \delta(q_i, a) = q_j\}$ denotes set of symbols a such that $\delta(q_i, a) = q_j$

Case 2 : *i = j*

$R_{ij}^{(0)} = R_{ii}^{(0)} = (\{a \mid \delta(q_i,a) = q_j\} \cup \{\varepsilon\})$ denotes set of all symbols a such that a (or) ε.

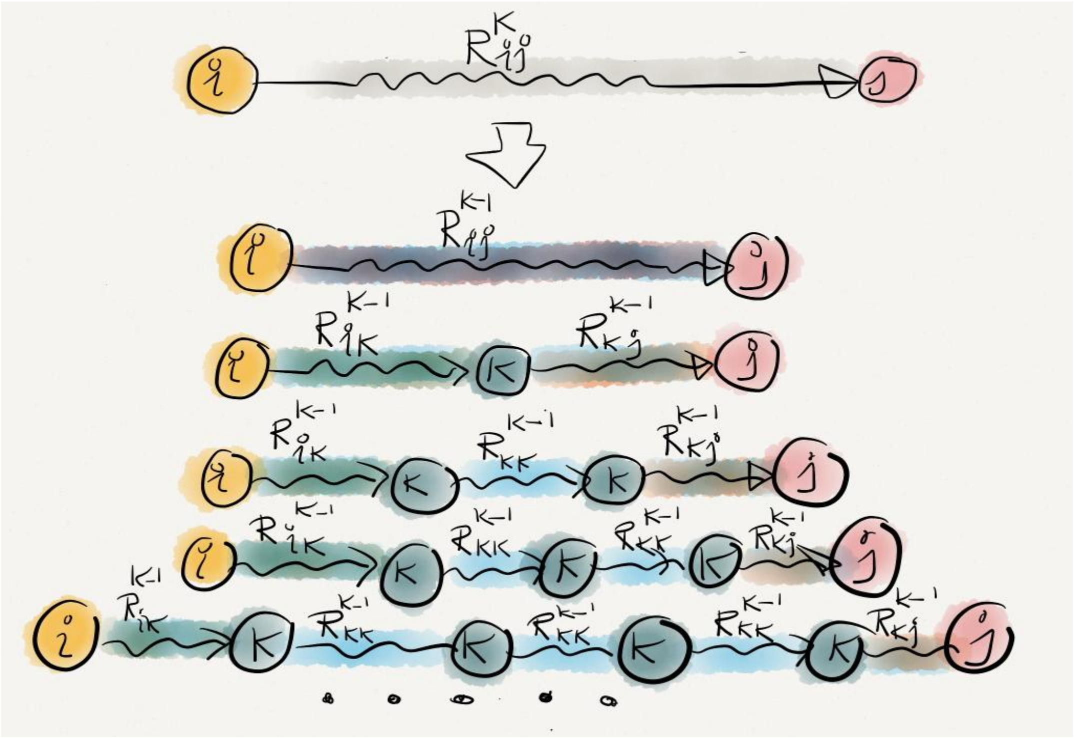
$R_{ii}^{(0)} = a+\varepsilon$

RE equation method $R_{ij}^{(k)}$

- **Induction**

  It involves regular expression operations : union, concatenation and closure.

$$R_{ij}^{(k)} = R_{ik}^{(k-1)} + (R_{kk}^{(k-1)})^{*} R_{kj}^{(k-1)}$$



RE equation method $R_{ij}^{(k)}$

The observation of this proof is that regular expression

$$L(A) = \bigcup_{q_j \text{ in } F} R_{1j}^{(n)}$$

where $R_{1j}^{(n)}$ denotes the labels of all paths from q_1 to q_j

where $F = \{q_{j1}, q_{j2}, \ldots\ldots q_{jp}\}$, so

$$L(A) = R_{1j1}^{(n)} + R_{1j2}^{(n)} + \ldots.. R_{1jp}^{(n)}$$

Arden's Theorem

- Let **P** and **Q** be two regular expressions. If **P** does not contain null string, then **R = Q + RP** has a unique solution that is **R = QP***
- **Proof** −

$R = Q + (Q + RP)P$ [After putting the value $R = Q + RP$]

$= Q + QP + RPP$

When we put the value of **R** recursively again and again, we get the following equation −

$R = Q + QP + QP^2 + QP^3 \ldots\ldots$

$R = Q\ (\varepsilon + P + P^2 + P^3 + \ldots.)$

$R = QP^*$ [As P^* represents $(\varepsilon + P + P2 + P3 + \ldots.)$]

Hence, proved.

Arden's Theorem

$$q_1 = q_1\alpha_{11} + q_2\,\alpha_{21} + \ldots + q_n\,\alpha_{n1} + \varepsilon$$

$$q_2 = q_1\,\alpha_{12} + q_2\,\alpha_{22} + \ldots + q_n\,\alpha_{n2}$$

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$$q_n = q_1\,\alpha_{1n} + q_2\,\alpha_{2n} + \ldots + q_n\,\alpha_{nn}$$

Summary

- Definition of Pumping lemma – Regular Language
- Application of pumping lemma

Reference

- Hopcroft J.E., Motwani R. and Ullman J.D, “Introduction to Automata Theory, Languages and Computations”, Second Edition, Pearson Education, 2008