Time Domain Analysis of LTI – CT Systems

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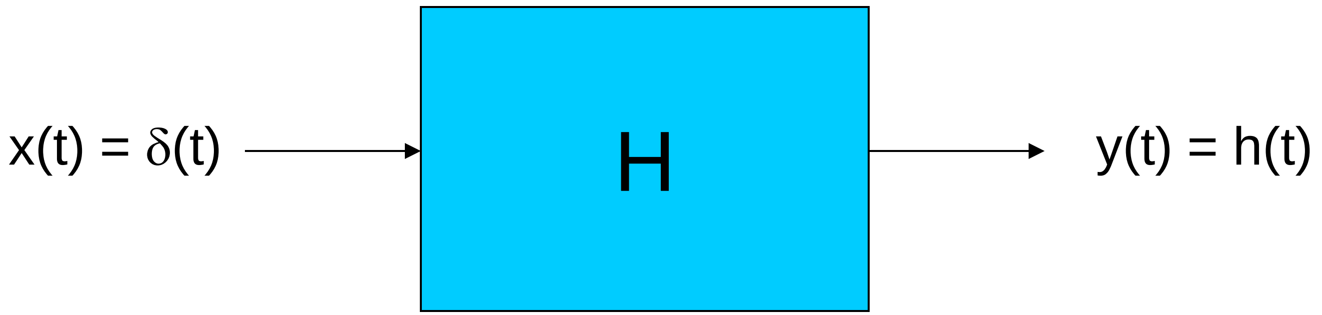
Overview

- LTI – CT systems
- Causality
- Stability
- Impulse response
- System function / Transfer function
- Frequency response

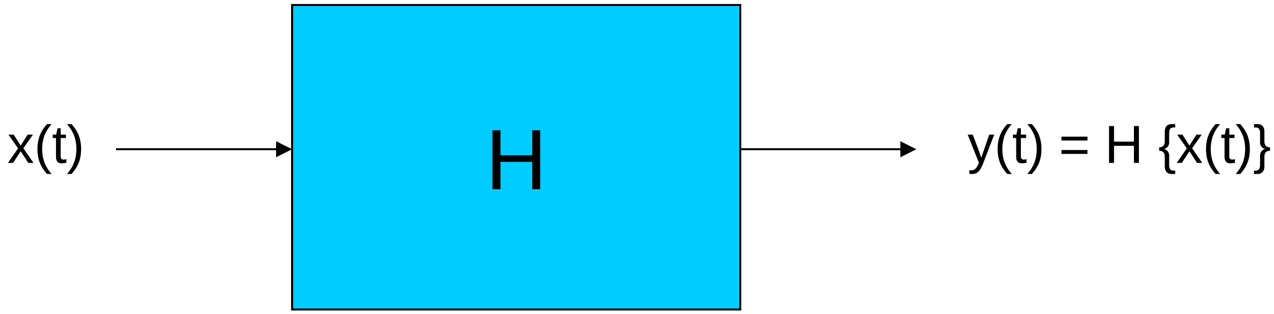
Impulse response

- The response of an LTI system due to an impulse input.
- denoted by h(t)



Convolution integral

- Let x(t) be an arbitrary input applied to the system H.
- response from the system is denoted as y(t)



- The output of the system for the given input x(t) is expressed as

$$
\begin{aligned}
y(t) &= H\left\{x(t)\right\} \\
&= H\left\{\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau\right\} \\
&= \int_{-\infty}^{\infty} x(\tau)H\left\{\delta(t-\tau)\right\}d\tau \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= x(t) * h(t)
\end{aligned}
$$

This is the convolution Integral of any system and * denotes convolution.

Properties of Convolution

- Commutative Property

$$x(t)*h(t)=h(t)*x(t)$$

- Distributive Property

$$x(t)*(h_1(t)*h_2(t))=(x(t)*h_1(t))*(x(t)*h_2(t))$$

- Associative Property

$$x(t)*[h_1(t)+h_2(t)]=x(t)*h_1(t)+x(t)*h_2(t)$$

Causality & Stability

Using Impulse response h(t) of the systems , Causality and Stability are defined as follows

Causality: For system to be causal, then

$$h(t) = 0 \qquad \forall t < 0$$

- Stability:

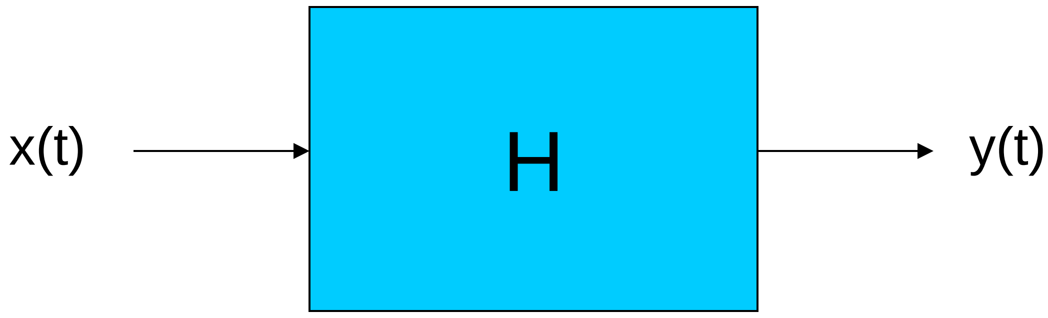
For a system to be stable, the impulse response should be absolutely integrable.

$$\int_{-\infty}^{\infty} |\mathbf{h}(\tau)| \mathbf{d}\tau < \infty$$

Transfer Function

- Let the Laplace transform of x(t) be X(s) and the Laplace of the output y(t) be Y(s). Now the transfer function of the system is defined as the ratio of Y(s) to X(s) when the initial conditions are zero. It is denoted by H(s).

$$H(s) = \frac{Y(s)}{X(s)}$$



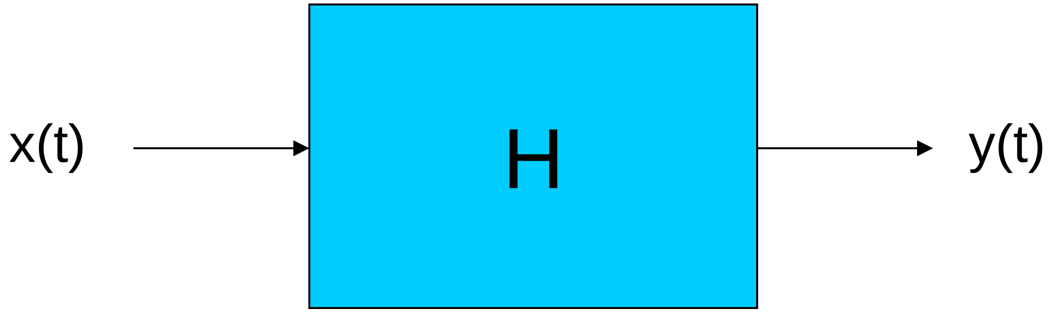
Frequency Response

- Let the Fourier transform of x(t) be X(jω) and the Fourier of the output y(t) be Y(jω). Now the frequency response of the system is given by the ratio of Y(j ω) to X(jω) when the initial conditions are zero. It is denoted by H(j ω).

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

- $\left|H(j\omega)\right|$ is the magnitude response and

$\angle H(j\omega)$ is the phase response.



Convolution Integral

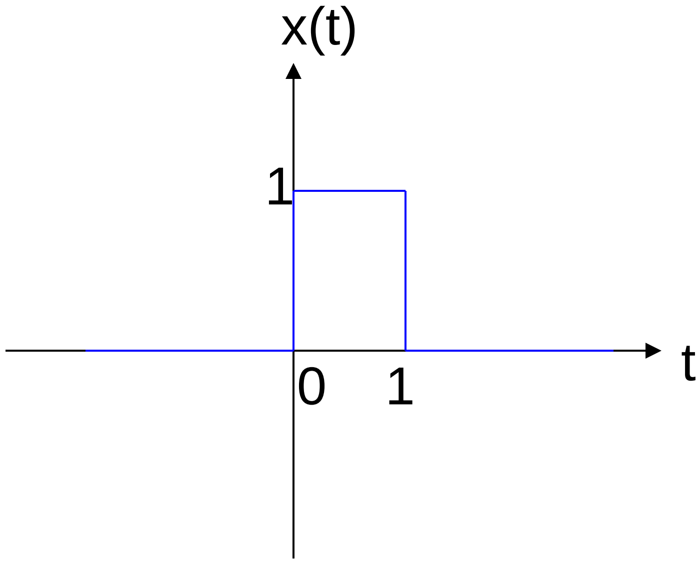
- Let y(t) $= x(t) * h(t)$

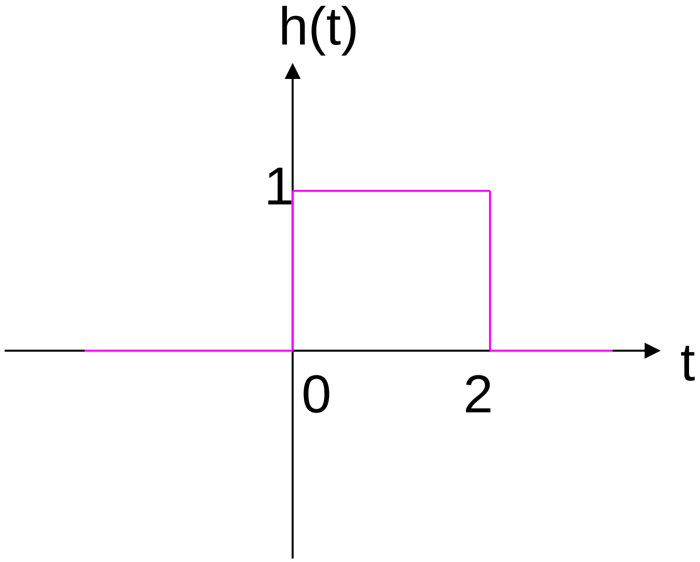
$$= \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

- It consists of the following mathematical operations.
  - Reflection
  - Shifting
  - Multiplication
  - Addition

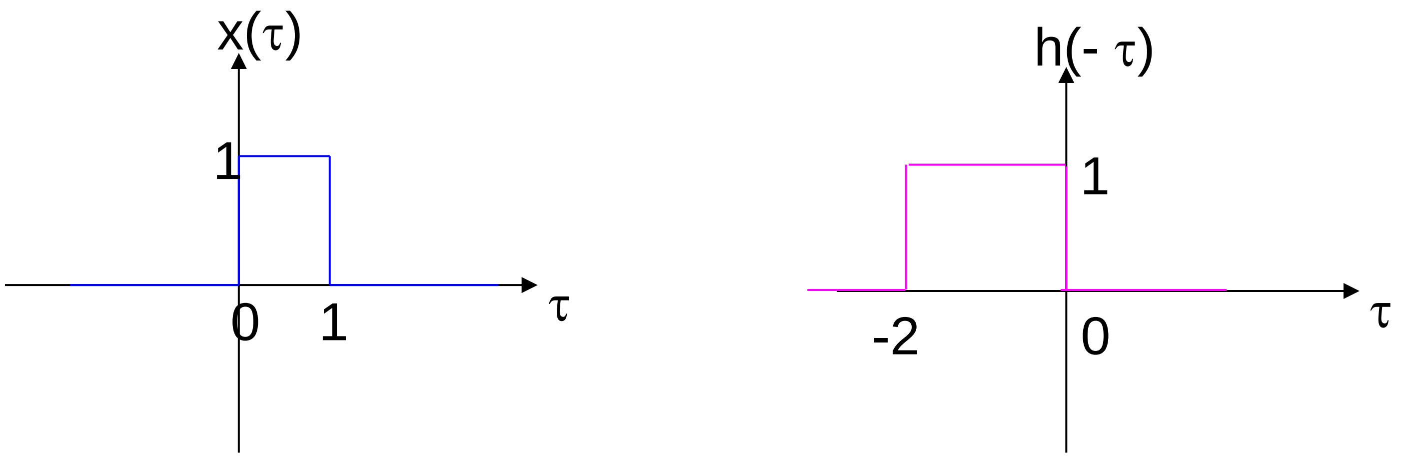
Example

- Let $x(t) = 1 \; ; \; 0<t<1$
  $\qquad 0 \; ; \;$ elsewhere
- Let $h(t) = 1 \; ; \; 0<t<2$
  $\qquad 0 \; ; \;$ elsewhere



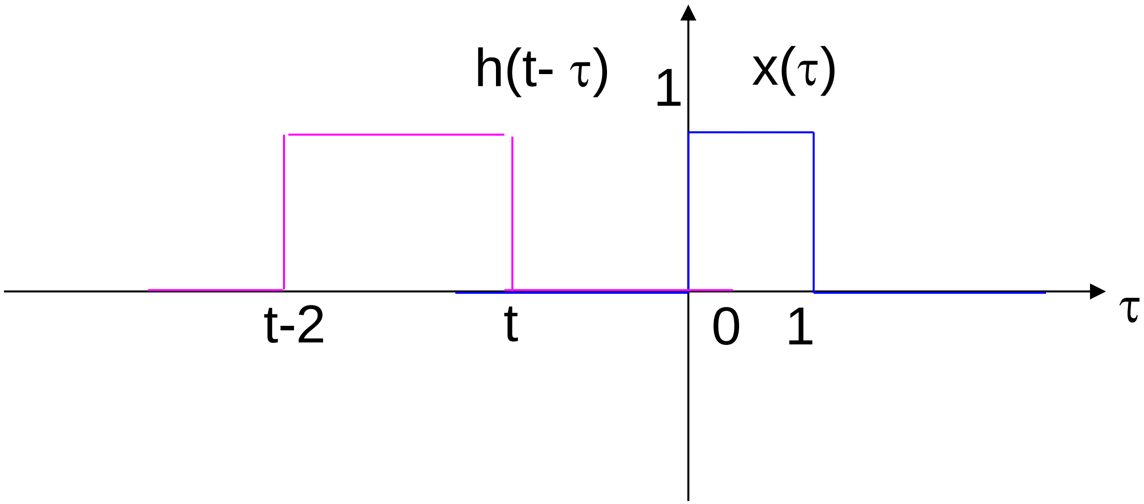


- Sketch x(τ) and h(-τ) as shown



- Shift h(-τ) by t units
- Depending on the given signals, consider various ranges for t, multiply h(t- τ) and x(τ)
- Integrate and get the solution.

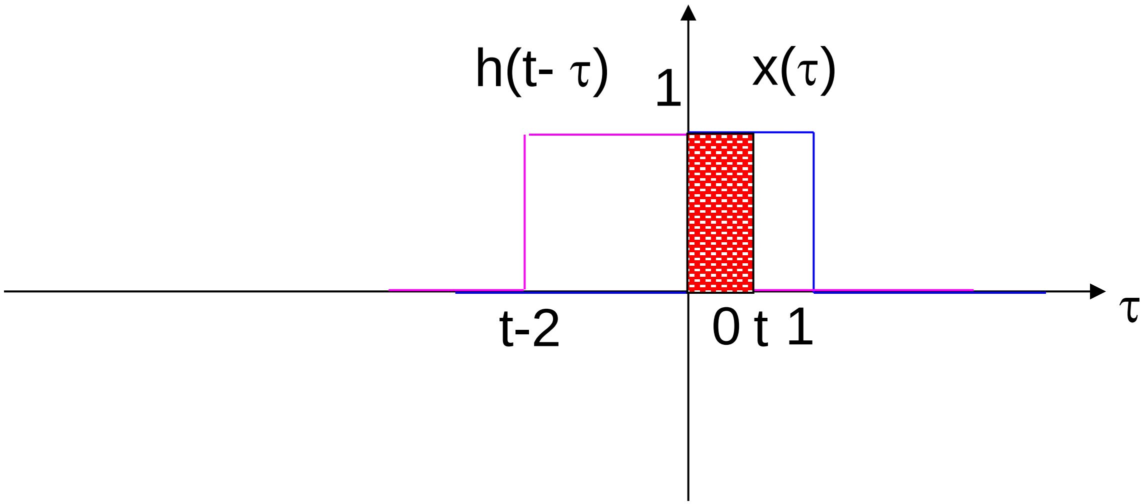
Case (i) : -∞ ≤ t ≤ 0



$$y(t) = 0$$

$$\boxed{y(t) = 0; -\infty \leq t \leq 0}$$

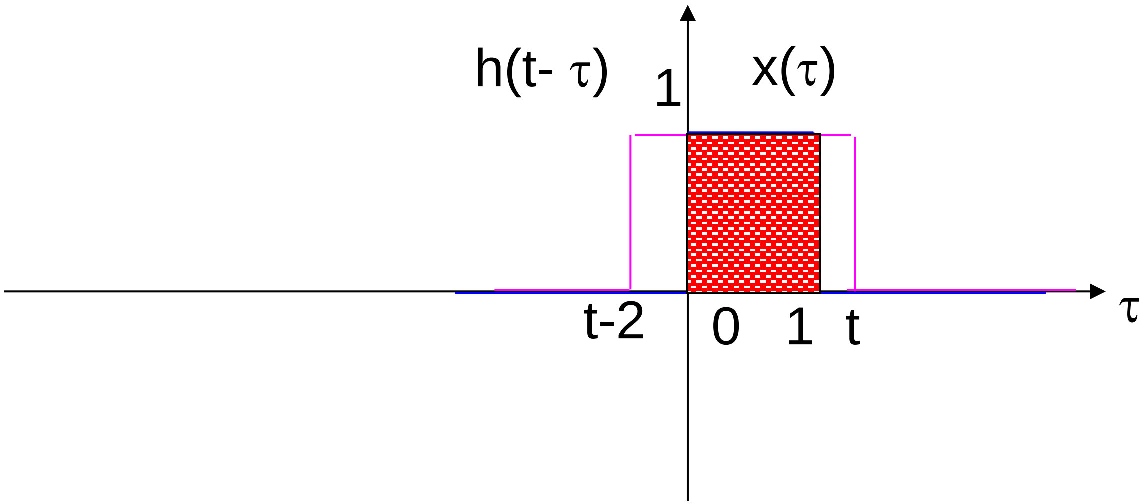
Case (ii) : $0 \leq t \leq 1$



$$y(t) = \int_0^t 1.d\tau = t$$

$$\boxed{y(t) = t; 0 \leq t \leq 1}$$

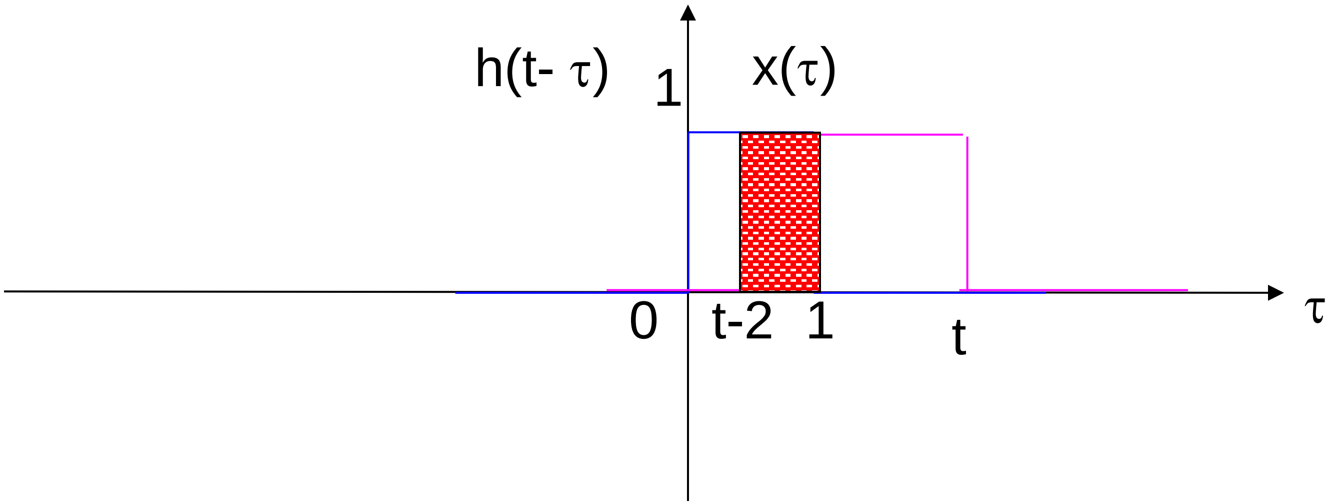
Case (ii) : $1 \leq t \leq 2$



$$y(t) = \int_0^1 1.d\tau = 1$$

$$\boxed{y(t) = 1; 1 \leq t \leq 2}$$

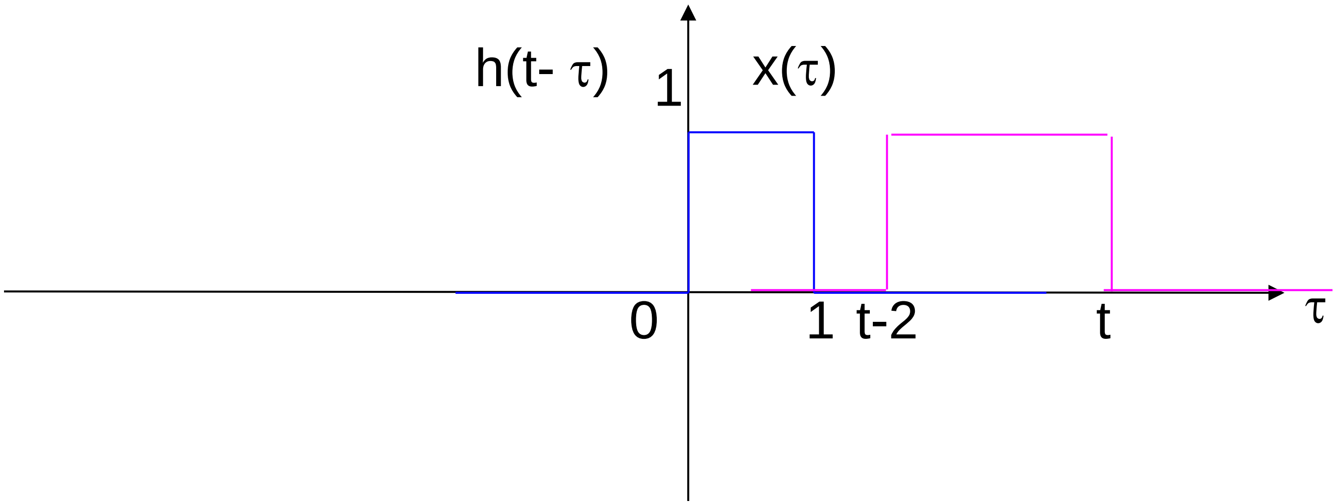
Case (iii) : $2 \leq t \leq 3$



$$y(t) = \int_{t-2}^{1} 1.d\tau = 3 - t$$

$$\boxed{y(t) = 3 - t; 2 \leq t \leq 3}$$

Case (iv) : $3 \leq t \leq \infty$

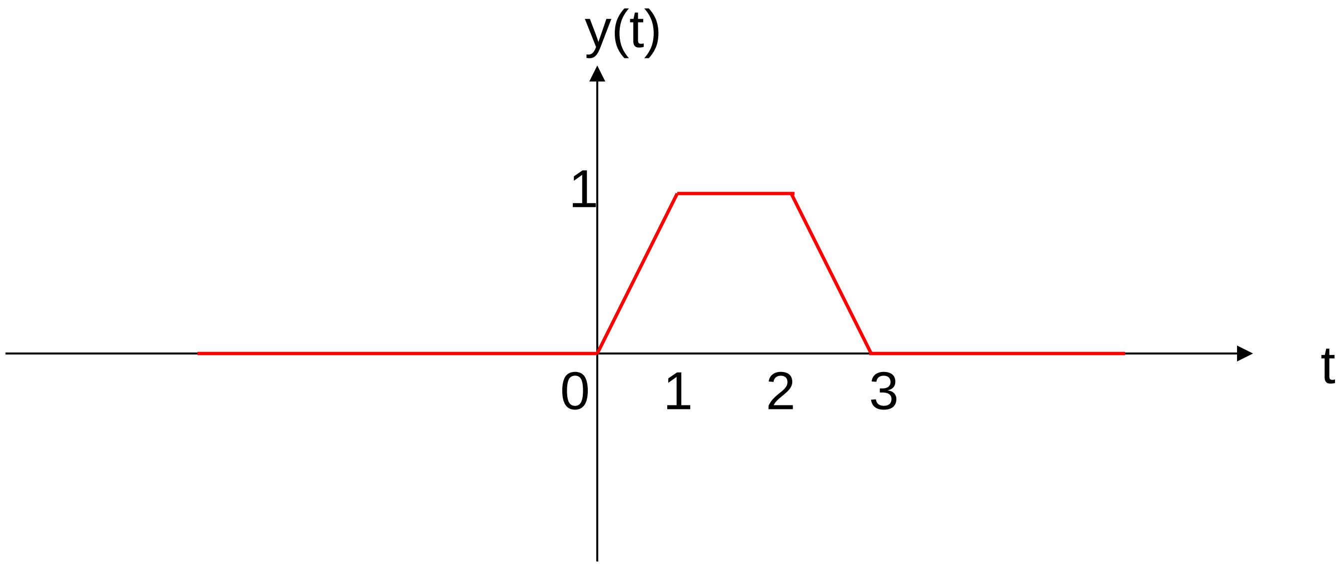


$$y(t) = 0$$

$$\boxed{y(t) = 0; 3 \leq t \leq \infty}$$

- Solution

$$
\begin{aligned}
y(t) &= 0; -\infty \le t \le 0 \\
&= t; 0 \le t \le 1 \\
&= 1; 1 \le t \le 2 \\
&= 3 - t; 2 \le t \le 3 \\
&= 0; 3 \le t \le \infty
\end{aligned}
$$



Problems

Find the convolution of the given signals using graphical method

$$1.\ x(t) = e^{-at}u(t); h(t) = u(t)$$

$$2.\ x(t) = h(t) = 1; 0 \le t \le T$$

$$= 0; elsewhere$$