Zero Input Limit Cycle Oscillations

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Zero – input Limit Cycle Oscillations

- When a stable IIR digital filter is excited by a finite input sequence, that is constant, the output will ideally decay to zero.

- However, the nonlinearities due to the finite – precision arithmetic operations often cause periodic oscillations to occur in the output. Such oscillations in recursive systems are called zero input limit cycle oscillations.

- Consider the first order IIR filter with difference equation,

$$y(n) = x(n) + \alpha\, y(n-1)$$

- Let $\alpha = ½$ and the data register length is 3 bits plus a sign bit.

- If the input is,

$$x(n) = \begin{cases} 0.75 & ; \quad n = 0 \\ 0 & ; \quad otherwise \end{cases}$$

- If rounding is applied after the arithmetic operation then the limit cycle behavior can be shown in the table.

n	x(n)	y(n-1)	α y(n-1)	Q(α y(n-1))	y(n) = x(n) + Q(α y(n-1))
0	0.75	0	0.0000…00	0.000	3/4
1	0	3/4	0.0110…00	0.011	3/8
2	0	3/8	0.00110..00	0.010	1/4
3	0	1/4	0.0010…00	0.001	1/8
4	0	1/8	0.00010..00	0.001	1/8
5	0	1/8	0.00010..00	0.001	1/8

- If $\alpha = -½$

n	x(n)	y(n-1)	α y(n-1)	Q(α y(n-1))	y(n) = x(n) + Q(α y(n-1))
0	0.75	0	1.0000…00	1.000	- 3/4
1	0	- 3/4	0.0110…00	0.011	3/8
2	0	3/8	1.00110..00	1.010	- 1/4
3	0	- 1/4	0.0010…00	0.001	1/8
4	0	1/8	1.00010..00	1.001	-1/8
5	0	- 1/8	0.00010..00	0.001	1/8

Dead Band

- The limit cycles occur as a result of the quantization effects in multiplications. The amplitudes of the output during a limit cycle are confined to a range of values that is called the dead band of the filter.

- Let us consider a single pole IIR system whose difference equation is given by

$$y(n) = x(n) + \alpha\, y(n-1)$$

- After rounding the product term, we have,

$$y_q(n) = x(n) + Q[\alpha\, y(n-1)]$$

- During the limit cycle oscillations,

$$\begin{aligned} Q[\alpha\, y(n-1)] &= y(n-1) \quad \text{for} \quad \alpha > 0 \\ &= -\, y(n-1) \quad \text{for} \quad \alpha < 0 \end{aligned}$$

- By the definition of rounding, we have,

  $|Q[\alpha\, y(n-1)] - \alpha y(n-1)| \leq 2^{-b}/2$

- Substituting for Q[α y(n-1)], we get,

  $|\pm\, y(n-1) - \alpha y(n-1)| \leq 2^{-b}/2$

$$y(n-1) \leq \frac{2^{-b}}{2(1-|\alpha|)}$$

- The above equation is the dead band for the first order filter.