Introduction of Digital Signal Processing

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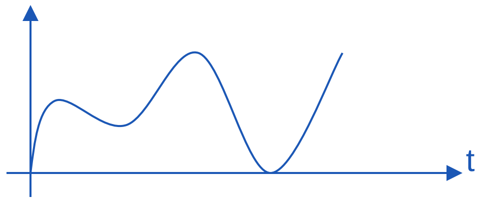
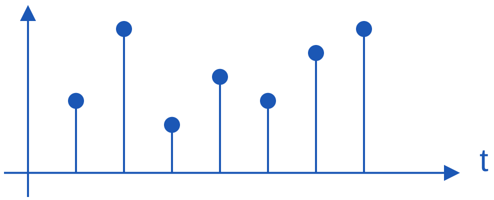
Signal:

A Signal is a function representing a physical quantity or variable, and typically it contains information about the behavior or nature of the phenomenon.

Types of Signals:

1. Continuous – Time and Discrete – Time Signals





2. Analog and Digital Signals

3. Real and Complex Signals

4. Deterministic and Random Signals

5. Even and Odd Signals

   For Even, $x(-t) = x(t)$ and for Odd, $x(-t) = -x(t)$

6. Periodic and Non-periodic Signals

   $x(t+T) = x(t)$

7. Energy and Power Signals

Basic Signals:

1. The Unit Step function
2. The Unit Impulse or Delta function
3. The Unit Ramp function
4. The Complex Exponential Signals
5. The Sinusoidal Signals

Basic Operations performed on the Signals:

1. Time reversal
2. Shifting
3. Scaling (Amplitude and Time)
4. Adding
5. Multiplying

System:

A System is a mathematical model of a physical process that relates the input (excitation) signal to the output (response) signal.

Types of Systems:

1. Continuous – Time and Discrete – Time Systems
2. Systems with Memory and without Memory
3. Causal and Noncausal Systems
4. Linear and Nonlinear Systems (Additivity and Homogeneity)
5. Time – Invariant and Time – Varying Systems
6. Stable Systems
7. Feedback Systems

Continuous Time – Linear Time – Invariant Systems

- The response is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$
- Properties of Convolution integral are
  1. Commutative
  2. Associative
  3. Distributive
- Properties of Continuous – Time LTI Systems
  1. Systems with or without Memory
  2. Causality
  3. Stability

- The CT – LTI Systems are described by differential equations.

DiscreteTime – Linear Time – Invariant Systems

- The response is $y[n] = \Sigma\, x[k]\, h[n-k]$
- Properties of Convolution sum are
  1. Commutative
  2. Associative
  3. Distributive
- Properties of Discrete – LTI Systems
  1. Systems with or without Memory
  2. Causality
  3. Stability

DiscreteTime – Linear Time – Invariant Systems (Contd):

The DT – LTI Systems are described by difference equations

$$\sum_{k=0}^{N} a_k \, y[n-k] = \sum_{k=0}^{M} b_k \, x[n-k]$$

Recursive Equation – IIR

$$y[n] = (1/a_0) \left\{\sum_{k=0}^{M} b_k \, x[n-k] - \sum_{k=1}^{M} a_k \, y[n-k] \right\}$$

Non recursive Equation – FIR

$$y[n] = (1/a_0) \left\{\sum_{k=0}^{M} b_k \, x[n-k] \right\}$$

Laplace Transform:

- It is used to solve the CT – LTI – Differential equations
- H(s) = Y(s) / X(s)

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt$$

Z Transform:

- It is used to solve the DT – LTI – Differential equations
- H(z) = Y(z) / X(z)

$$H[z] = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$

Fourier Analysis – CT – LTI System:

Fourier Series:

- It converts the time – domain signals into frequency – domain signals.
- Fourier series decomposes a periodic function into a sum of simple oscillating functions, namely sines and cosines.
- Complex Exponential Fourier Series, Trigonometric Fourier Series and Harmonic Form Series are the Fourier Series representation

Fourier Series (Contd):

- It is known that a periodic signal x(t) has a Fourier series representation, if it satisfies the following Dirichlet conditions:
    1. x(t) is absolutely integrable over any period, that is, $\int_{T_0}|x(t)|dt < \infty$
    2. x(t) has a finite number of maxima and minima within any finite interval of t
    3. x(t) has a finite number of discontinuities within any finite interval of t and each these discontinuities is finite.

Fourier Series (Contd):

- The Average power of a periodic signal x(t) over any period is, $P = (1 / T_0) \int_{T_o} |x(t)|^2 dt$
- If x(t) is represented by the complex exponential Fourier series, then

$$\int_{T_o} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2$$

- This is called Parseval's theorem or Parseval's identity for the Fourier series

Fourier Transform:

The Fourier Transform pair is

$$x(t) \leftrightarrow X(\omega)$$

$$X(\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = F^{-1}\{X(\omega)\} = \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}dt$$

Fourier Transform (Contd):

The sufficient conditions for the convergence of $X(\omega)$ are

1. x(t) is absolutely integrable over any period, that is, $\int_{-\infty}^{\infty} |x(t)| dt < \infty$
2. x(t) has a finite number of maxima and minima within any finite interval of t
3. x(t) has a finite number of discontinuities within any finite interval of t and each these discontinuities is finite.

   (Note: Even though the above Dirichlet conditions does not satisfy for impulse functions, Fourier Transforms can be used to analyze)

Fourier Transform (Contd):

- The Fourier Transform can be converted into Laplace Transform by substituting $s = j\omega$
- The Parseval's identity or Parseval's theorem of Energy theorem for Fourier Transform is

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = (1/2\pi)\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

- $|X(\omega)|^2$ is referred as energy density spectrum of x(t)
- The frequency response of a continuous – time – LTI System is given by

$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega)\, H(\omega)$$