Fast Fourier Transform
- Decimation in Time

I.Nelson

SSN College of Engineering

Fast Fourier Transform (FFT)

Why FFT?

DFT	FFT
Multiplications – N^2	$(N/2)\log_2 N$
Additions – $N(N-1)$	$N \log_2 N$

- Symmetry Property: $W_N^{k+(N/2)} = -W_N^k$
- Periodicity Property: $W_N^{k+N} = W_N^k$

Two types of FFT

- Decimation in Time
- Decimation in Frequency

Steps of radix – 2 DIT – FFT algorithm

1. The number of input samples $N=2^M$, where, M is an integer.
2. The input sequence is shuffled through bit – reversal.
3. The number of stages in the flow graph is given by $M = \log_2 N$.
4. Each stage consists of N/2 butterflies.
5. Inputs / Outputs for each butterfly are separated by 2^{m-1} samples, where m represents the stage index, i.e., for first stage m=1 and for second stage m=2 and so on.
6. The number of complex multiplications is given by $(N/2)\log_2 N$.
7. The number of complex additions is given by $N\log_2 N$.
8. The twiddle factor exponents are a function of the stage index m and is given by

$K = Nt/2^m$; $t= 0, 1, 2, 3,\ldots\ldots\ldots,2^{m-1}-1$

9. The number of sets or sections of butterflies in each stage is given by the formula 2^{M-m}.
10. The exponent repeat factor (ERF), which is the number of times the exponent sequence associated with m repeated is given by 2^{M-m}.

Problem1: Draw the Flow graph of 16 – point DIT – FFT.

Solution: Using the steps,

1. The Number of input samples, N = 16.
2. The input sequence is shuffled through bit – reversal shown below and applied as input to the flow graph.
3. The number of stages $M = \log_2 16 = 4$.
4. The number of butterflies per stage is N/2 = 8.
5. The inputs / outputs for each butterfly in stage m is separated by 2^{m-1} samples

   Stage 1 – Inputs / Outputs for each butterfly are separated by 1 sample.
   Stage 2 – Inputs / Outputs for each butterfly are separated by 2 samples.
   Stage 3 – Inputs / Outputs for each butterfly are separated by 4 samples.
   Stage 4 – Inputs / Outputs for each butterfly are separated by 8 samples.

6. The number of complex multiplications is given by

$$(N/2)\log_2 N = 8\log_2 16 = 32$$

7. The number of complex additions is given by $16\log_2 16 = 64$.

8. The twiddle factor exponents for each stage are given by
   $K = Nt/2^m$; t= 0, 1, 2, 3,………,$2^{m-1}-1$

   For Stage 1 the exponent is 0.
   For Stage 2 the exponent are 0,4.
   For Stage 3 the exponent are 0,2,4,6.
   For Stage 4 the exponent are 0,1,2,3,4,5,6,7.

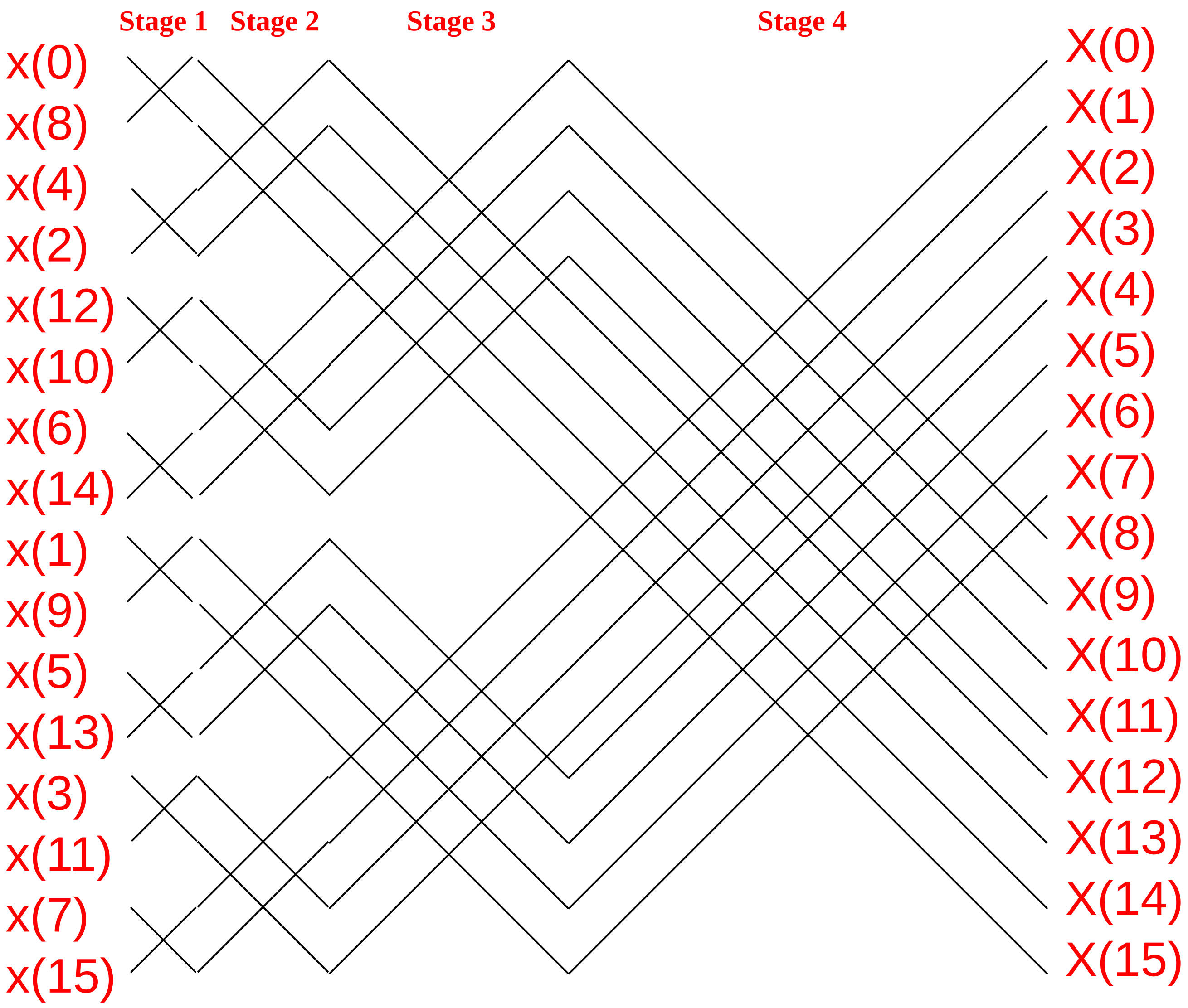
9. The number of sets or sections of butterflies in each stage is given by 2^{M-m}.

   For Stage 1 the number of sets of butterflies are $2^{4-1} = 8$
   For Stage 2 the number of sets of butterflies are $2^{4-2} = 4$
   For Stage 3 the number of sets of butterflies are $2^{4-3} = 2$
   For Stage 4 the number of sets of butterflies are $2^{4-4} = 1$

10. The exponent repeat factor (ERF), which is the number of times the exponent sequence associated with m is repeat is given by 2^{M-m}.

   For Stage 1, ERF = 8
   For Stage 2, ERF = 4
   For Stage 2, ERF = 2
   For Stage 2, ERF = 1

Index	Binary Representation	Bit reversed Order	Bit reversed index
0	0000	0000	0
1	0001	1000	8
2	0010	0100	4
3	0011	1100	12
4	0100	0010	2
5	0101	1010	10
6	0110	0110	6
7	0111	1110	14
8	1000	0001	1
9	1001	1001	9
10	1010	0101	5
11	1011	1101	13
12	1100	0011	3
13	1101	1011	11
14	1110	0111	7
15	1111	1111	15


Stage 1
Stage 2
Stage 3
Stage 4
x(0)
x(8)
x(4)
x(2)
x(12)
x(10)
x(6)
x(14)
x(1)
x(9)
x(5)
x(13)
x(3)
x(11)
x(7)
x(15)
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)
X(8)
X(9)
X(10)
X(11)
X(12)
X(13)
X(14)
X(15)

Problem3: Find the DFT of a sequence x(n) = {1,2,3,4,4,3,2,1} using DIT algorithm.

Solution:

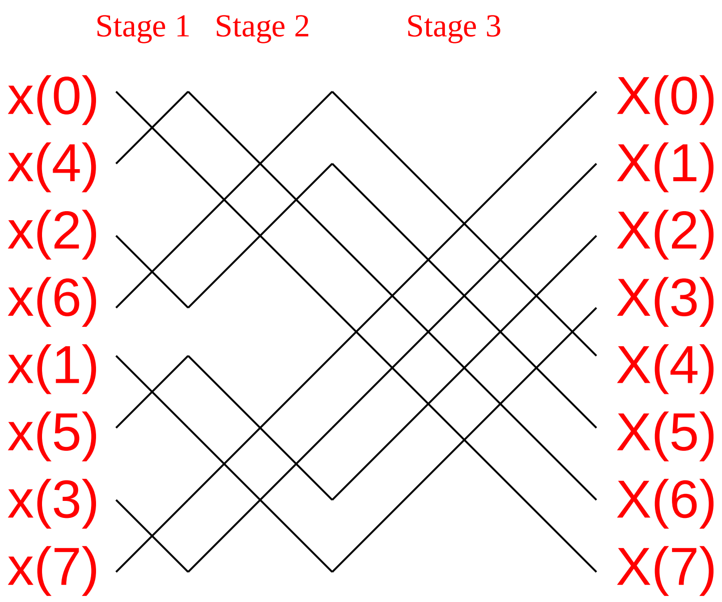
The twiddle factors associated with the flow graph are

$W_8^0 = 1$

$W_8^1 = e^{-j2\pi/8} = e^{-j\pi/4} = 0.707 - j0.707$

$W_8^2 = e^{-j4\pi/8} = e^{-j\pi/2} = -j$

$W_8^3 = e^{-j6\pi/8} = e^{-j3\pi/4} = -0.707 - j0.707$



Input	Output of Stage1	Output of Stage2	Output of Stage3
x(0) = 1	1+4=5	5+5=10	10+10=20
x(4) = 4	1-4=-3	-3+(-j)*1=-3-j	(-3-j)+(0.707-j0.707)(-1-3j) = -5.828-j2.414
x(2) = 3	3+2=5	5-5=0	0
x(6) = 2	3-2=1	-3-(-j)*1=-3+j	(-3+j)+(-0.707-j0.707)(-1+3j) = -0.172-j0.414
x(1) = 2	2+3=5	5+5=10	10-10=0
x(5) = 3	2-3=-1	-1+(-j)*3=-1-3j	(-3-j)-(0.707-j0.707)(-1-3j) = -0.172+j0.414
x(3) = 4	4+1=5	5-5=0	0
x(7) = 1	4-1=3	-1-(-j)*3=-1+3j	(-3+j)-(-0.707-j0.707)(-1+3j) = -5.828+j2.414

X(k) = {20, -5.828-j2.414, 0, -0.172-j0.414, 0, -0.172-j0.414, 0, -5.828+j2.414}