

# TURING MACHINE

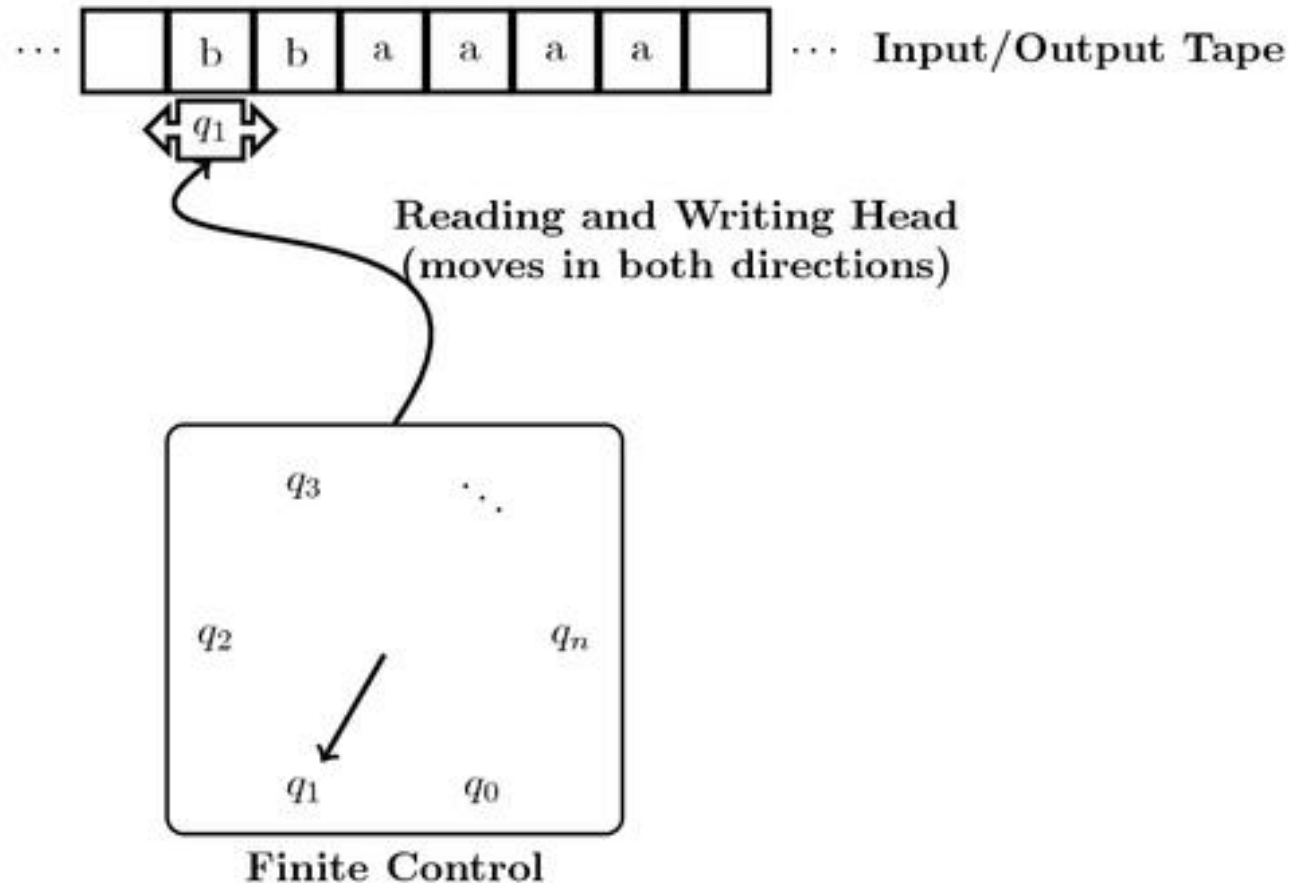
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# Devices of Increasing Computational Power

- ▶ So far:
  - Finite Automata – good for devices with small amounts of memory, relatively simple control
  - Pushdown Automata – stack-based automata
- ▶ But both have limitations for even simple tasks, too restrictive as general purpose computers
- ▶ Enter the **Turing Machine**
  - More powerful than either of the above
  - Essentially a finite automaton but with unlimited memory
  - Although theoretical, can do everything a general purpose computer of today can do
    - If a TM can't solve it, neither can a computer (Undecidable problems)

# Turing Machine

- ▶ A TM consists of a finite control (i.e. a finite state automaton) that is connected to an infinite tape.



# Notion for the Turing Machine

- ▶ A move of Turing machine includes:
  - Change state;
  - Write a tape symbol in the cell scanned;
  - Move the tape head left or right.

# Formal Definition

- ▶ A Turing machine (TM) is a 7-tuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F) \text{ where}$$

- $Q$  – A finite set of states of the finite control
- $\Sigma$  – A finite set of input symbols
- $\Gamma$  – A set of tape symbols, with  $\Sigma$  being a subset
- $q_0$  - The start state, in  $Q$
- $B$  - The blank symbol in  $\Gamma$ , *not* in  $\Sigma$  (should not be an input symbol)
- $F$  - The set of final or accepting states

# Formal Definition

- ▶  $\delta$ : a transition function  $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\}$
- ▶ Example  $\delta(q, X) = (p, Y, D)$ 
  - $q$  - The current state, in  $Q$
  - $X$  - A tape symbol being scanned
  - $p$  - The next state, in  $Q$
  - $Y$  - The tape symbol written on the cell being scanned, used to replace  $X$
  - $D$  - Either  $L$  (left) or  $R$  (right) telling the move direction of the tape head

# Representation of TM

- ▶ Turing Machines are represented in 3 ways
  - Instantaneous Descriptions
  - Transition Table
  - Transition Diagram

# Instantaneous Descriptions

- ▶ The *instantaneous description* (ID) of a TM is represented by
- ▶  $X_1 X_2 \dots X_{i-1} q X_i X_{i+1} \dots X_n$  in which
  - $q$  is the current state
  - The tape head is scanning the  $i$ th symbol from the left
  - $X_1 X_2 \dots X_n$  is the portion of the tape between the leftmost and the rightmost nonblank symbols



# Instantaneous Descriptions

- ▶ Moves of a TM  $M$  denoted by  $\vdash_M$  or  $\vdash$  as follows:

If  $\delta(q, X_i) = (p, Y, L)$

$$\begin{array}{l} X_1 X_2 \dots X_{i-1} q X_i X_{i+1} \dots X_n \vdash \\ X_1 X_2 \dots X_{i-2} p X_{i-1} Y X_{i+1} \dots X_n \end{array}$$

- ▶ Right moves are defined similarly.

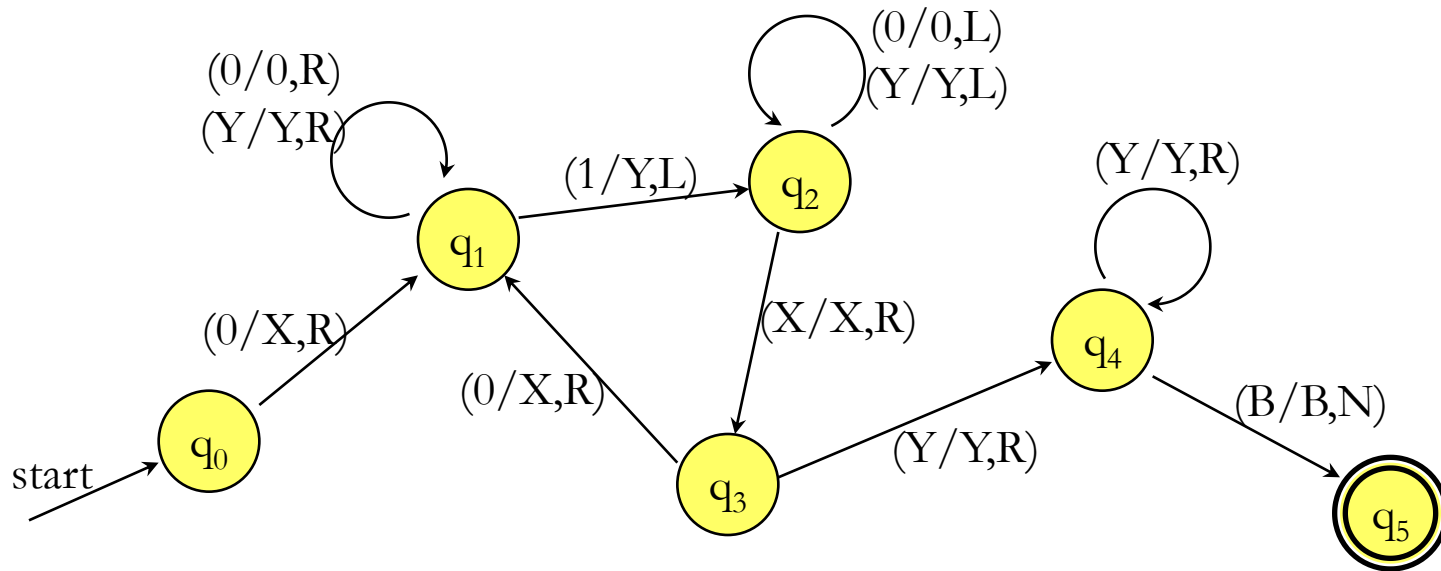
# Transition Table

►  $L = \{0^n 1^n \mid n \geq 1\}$

<div></div>	0	1	X	Y	B
$q_0$	$(q_1, X, R)$	-	-	-	-
$q_1$	$(q_1, 0, R)$	$(q_2, Y, L)$	-	$(q_1, Y, R)$	-
$q_2$	$(q_2, 0, L)$	-	$(q_3, X, R)$	$(q_2, Y, L)$	-
$q_3$	$(q_1, X, R)$	-	-	$(q_4, Y, R)$	-
$q_4$	-	-	-	$(q_4, Y, R)$	$(q_5, B, N)$
$q_5$	-	-	-	-	-

- : undefined and the machine halts.

# Transition Diagram



# Language Acceptance of TM

▶ Let  $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$  be a TM.

▶ The language accepted by  $M$  is

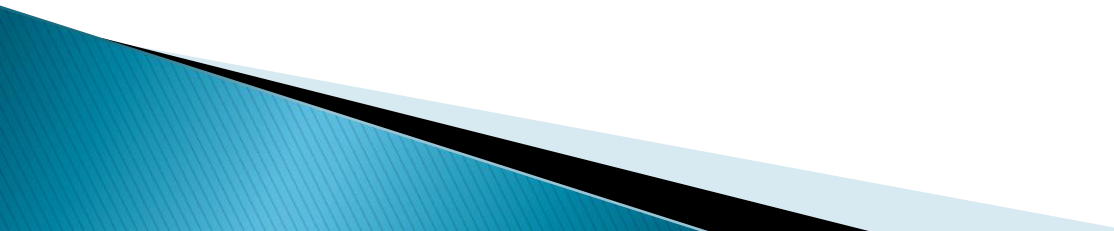
$$L(M) = \{w \mid w \in \Sigma^* \text{ and } q_0 w \vdash \alpha p \beta \text{ with } p \in F, \\ \alpha, \beta \in \Gamma^* \}$$

▶ Turing machine can accept the string by entering accepting state

▶ TM can reject the string by entering non-accepting state.

▶ TM can enter an infinite loop so that it never halts.

# Designing a TM

- ▶ The fundamental objective in scanning a symbol by R/W head is to 'know' what to do in the future.
  - ▶ The machine must remember the past symbols scanned.
  - ▶ Change the states only when there is a change in the written symbol or when there is a change in the movement of R/W head.
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# Summary

- ▶ Definition of Turing Machine
- ▶ Model of Turing Machine
- ▶ ID of Turing Machine
- ▶ Language of a Turing Machine

# Test Your Knowledge

- ▶ Which of the following statements is/are FALSE?
  1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.
  2. Turing recognizable languages are closed under union and complementation.
  3. Turing decidable languages are closed under intersection and complementation.
  4. Turing recognizable languages are closed under union and intersection.
- A. 1 and 4 only
- B. 1 and 3 only
- C. 2 only
- D. 3 only

# Test Your Knowledge

▶ Which of the following is true for the language

$$L = \{a^p / p \text{ is prime}\}$$

A. It is not accepted by a Turing Machine

B. It is regular but not context-free

C. It is context-free but not regular

D. It is neither regular nor context-free, but accepted by a Turing machine



# Reference

- ▶ Hopcroft J.E., Motwani R. and Ullman J.D, “Introduction to Automata Theory, Languages and Computations”, Second Edition, Pearson Education, 2008