DFA to RE

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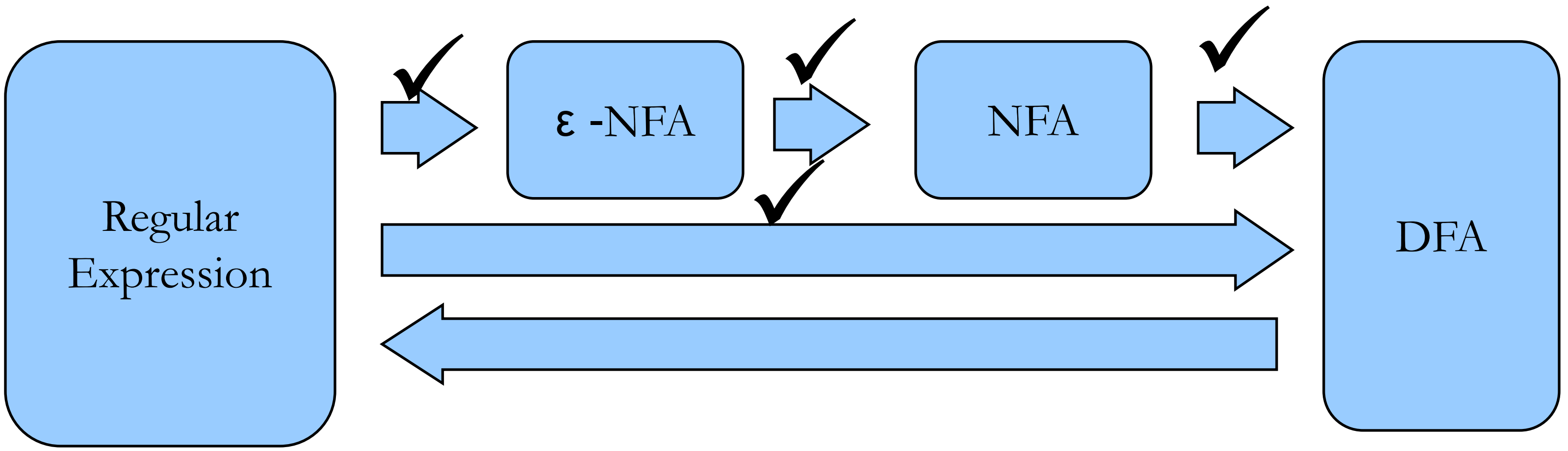
AP/CSE

Road map



Conversion of DFA to RE

1. Regular Expression equation method - $R_{ij}^{(k)}$
2. Arden's Theorem.
3. State elimination technique.

RE Equation Method - $R_{ij}^{(k)}$

- **Theorem**

  If L=L(M) for some DFA M= ($\mathcal{Q}$, $\sum$, δ, S, F), then there is a regular expression r such that L= L(r).

- **Proof**

  Let L be the set accepted by the DFA

  Given a DFA M = ($\mathcal{Q}$, $\sum$, δ, S, F), where $\mathcal{Q}=\{q_1, q_2, \ldots, q_n\}$, i.e., $|\mathcal{Q}| = n$.

RE Equation Method - $R_{ij}^{(k)}$

$R_{ij}^{(K)}$ → RE describing the set of all strings x such that $\delta(q_i, x) = q_j$ going through intermediate states $\{q_1, q_2, \ldots\ldots q_K\}$ only.

- **Basis**

  K = 0 → no intermediate states.

  $R_{ij}^{(0)}$ → a set of strings which is either ε(or) single symbol.

RE Equation Method - $R_{ij}^{(k)}$

- Case i

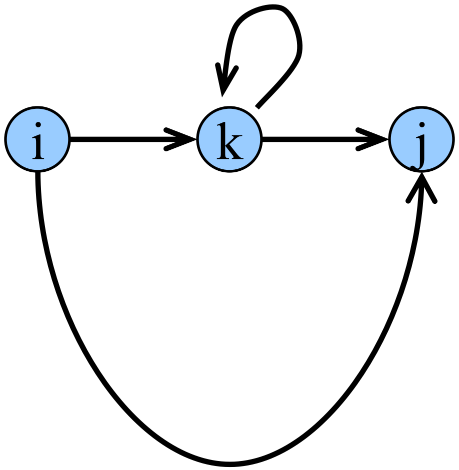
$$R_{ij}^{0} = \{a \in \Sigma \mid \delta(q_i, a) = q_j\} \qquad \text{if } i \neq j$$

- Case ii

$$R_{ij}^{0} = \{a \in \Sigma \mid \delta(q_i, a) = q_j\} \cup \{\varepsilon\} \qquad \text{if } i = j$$

RE Equation Method - $R_{ij}^{(k)}$

- Induction
  - It involves regular expression operations : union, concatenation and closure.
  - $R_{ij}^{k} = R_{ik}^{k-1} (R_{kk}^{k-1})^* R_{kj}^{k-1} + R_{ij}^{k-1}$



RE Equation Method - $R_{ij}^{(k)}$

- The observation of this proof is that regular expression

$$L(M) = \{w \in \Sigma^* \mid \delta(q_1, w) = q_j \in F\}$$

$$= \cup_{q_j \in F} R_{1j}^{\,n}$$

where $R_{1j}^{(n)}$ denotes the labels of all paths from q_1 to q_j

where $F = \{q_{j1}, q_{j2}, \ldots\ldots q_{jp}\}$,

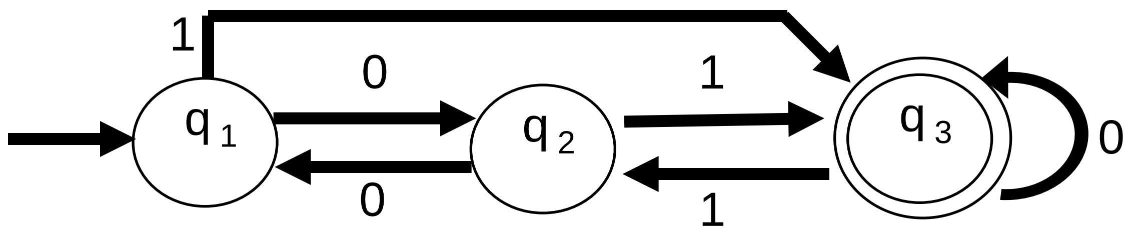
so $L(M) = R_{1j1}^{(n)} + R_{1j2}^{(n)} + \ldots.. R_{1jp}(n)$

Identities for Regular Expressions

I1	$\varphi + R = R$
I2	$\varphi R = R\varphi = \varphi$
I3	$\lambda R = R\lambda = R$
I4	$\lambda^* = \lambda$
I5	$R + R = R$
I6	$R^*R^* = R^*$
I7	$RR^* = R^*R$
I8	$(R^*)^* = R^*$
I9	$\lambda + RR^* = R^* = \lambda + R^*R$
I10	$(PQ)^*P = P(QP)^*$
I11	$(P + Q)^* = (P^*Q^*)^* = (P^* + Q^*)^*$
I12	$(P + Q)R = PR + QR$ and $R(P + Q) = RP + RQ$

Example

- Find a regular expression representing the set L over an alphabet $\Sigma = \{0, 1\}$ accepted by the following DFA M.



Example

	k=0	k=1	k=2
r(1,1,k)	e	e	0(00)*0+ e
r(1,2,k)	0	0	0(00)*
r(1,3,k)	1	1	0(00)*(1+01)+1
r(2,1,k)	0	0	(00)*0
r(2,2,k)	e	00+e	(00)*
r(2,3,k)	1	1+01	(00)*(1+01)
r(3,1,k)	∅	∅	1(00)*0
r(3,2,k)	1	1	1(00)*
r(3,3,k)	0+ e	0+ e	1(00)*(1+01)+0+ e

Example

$r_{1,3}^{3} = r_{1,3}^{2} + r_{1,3}^{2} (r_{3,3}^{2})^* r_{3,3}^{2}$

$= (0(00)^*(1+01)+1)+(0(00)^*(1+01)+1)(1(00)^*(1+01)+0+\varepsilon)^*(1(00)^*(1+01)+0+\varepsilon)$

$= (0(00)^*(1+01)+1)\ (1(00)^*\ (1+01)+0)^*$

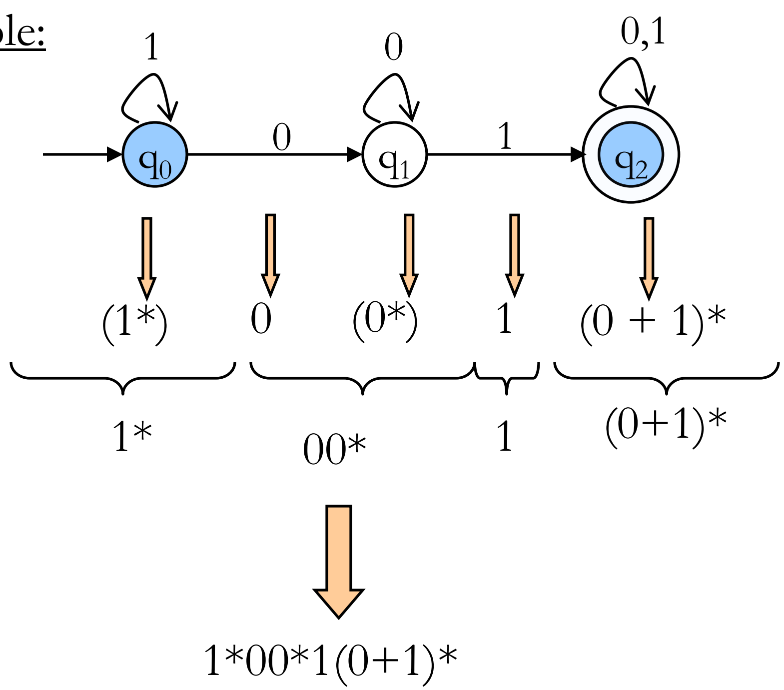
$= (0^*1)\ (1(00)^*\ (1+01)+0)^*$

$= (0^*1)\ (10^*1+0)^*$

DFA to RE construction

- Informally, trace all distinct paths (traversing cycles only once)
  from the start state to *each of the* final states
  and enumerate all the expressions along the way

<u>Example:</u>



Arden's Theorem

- Lemma:

  A solution of $x = S+Rx$ is $x = SR^*$. Furthermore, if $\varepsilon \notin L(R)$ then this is the only solution of the equation $x = Rx + S$.

- Proof

$$x = S+Rx$$

$$= S + RSR^* \qquad x = SR^*$$

$$= S(\varepsilon + RR^*) \qquad R^* = R^*R + \varepsilon$$

$$= SR^*$$

So $x = SR^*$ is a solution of $x = S+Rx$

Principle of Arden's Theorem

- No ε moves
- Only one start state say q_1
- Its states are $q_1, q_2 \ldots\ldots q_n$
- α_{ij} denotes the set of labels of edges from q_i to q_j.

If there is no edge $\alpha_{ij} = \varphi$.

$q_1 = q_1\alpha_{11} + q_2\alpha_{21} + \ldots.. + q_n \alpha_{n1} + \varepsilon$

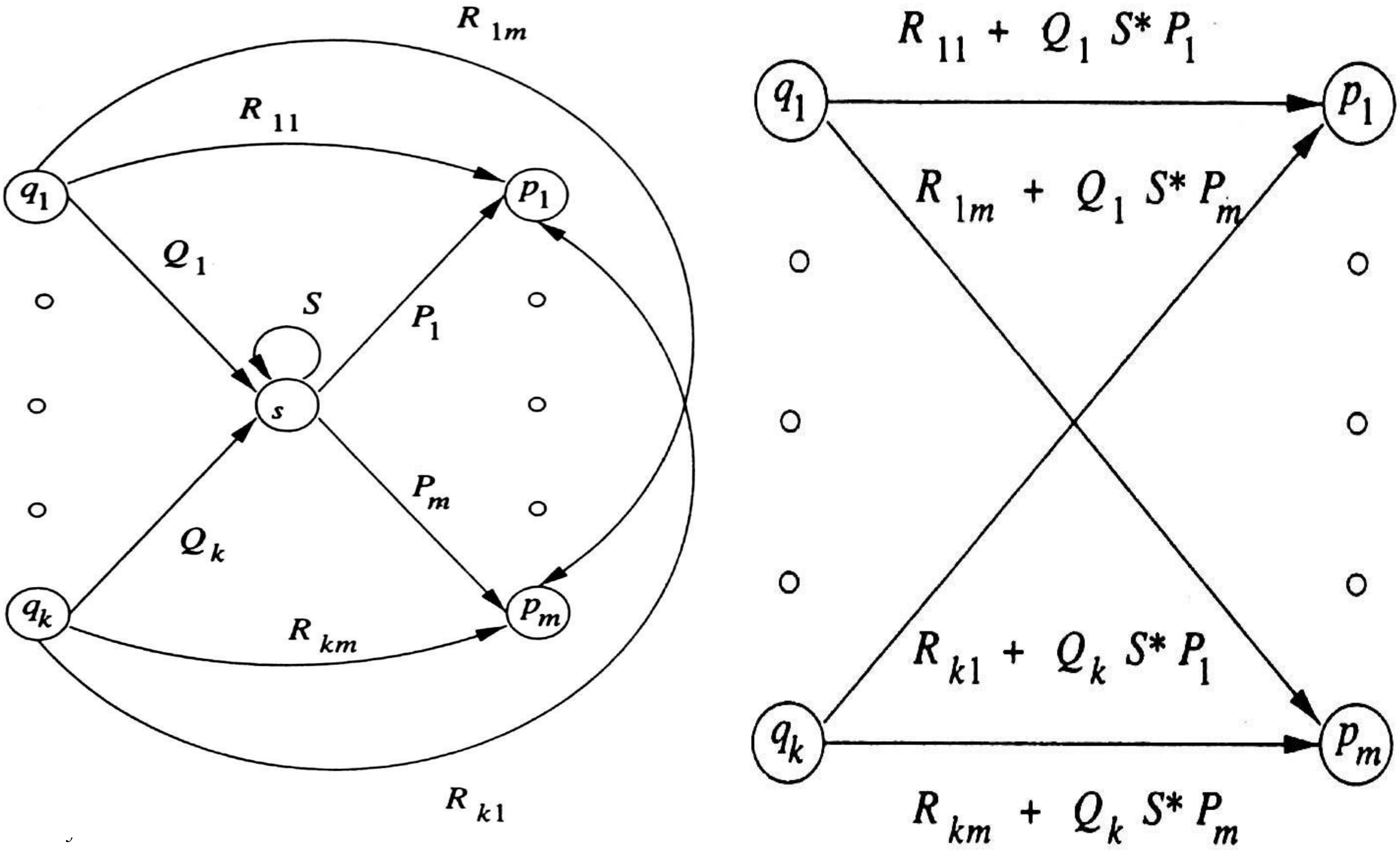
$q_2 = q_1 \alpha_{12} + q_2 \alpha_{22} + \ldots.. + q_n\alpha_{n2}$

$q_n = q_1 \alpha_{1n} + q_2\alpha_{2n} + \ldots.. + q_n\alpha_{nn}$

State Elimination Method

- S → intermediate state
- Predecessor of S → q1, q2 … qk
- Successor of S → p1, p2, … pm

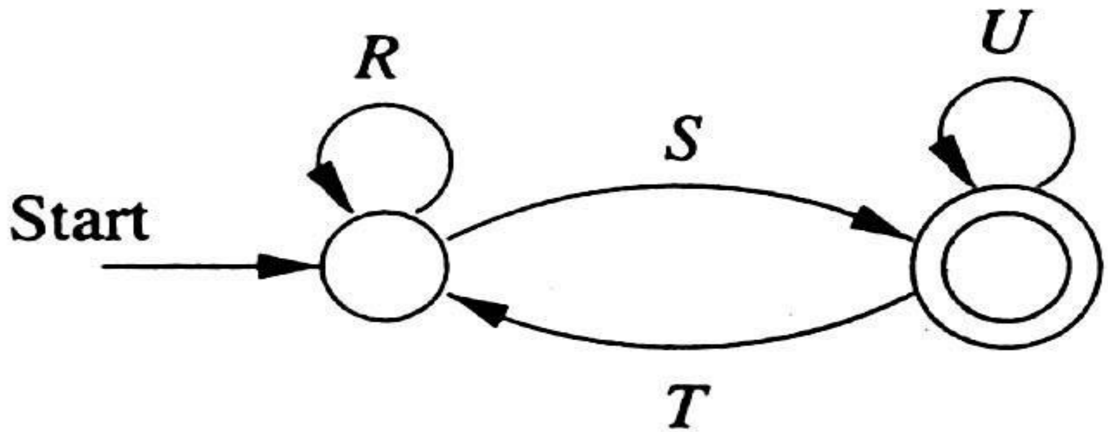
S before & after elimination



Steps

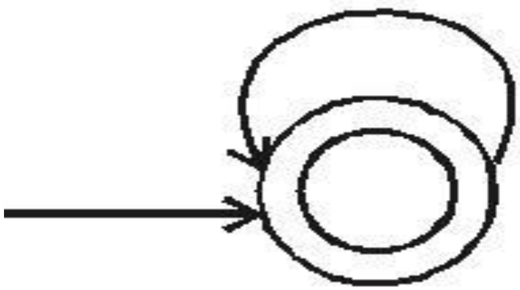
1. Eliminate all states except q and the start state q_0
2. $q \neq q0$

- $(R + SU^*T)^*SU^*$



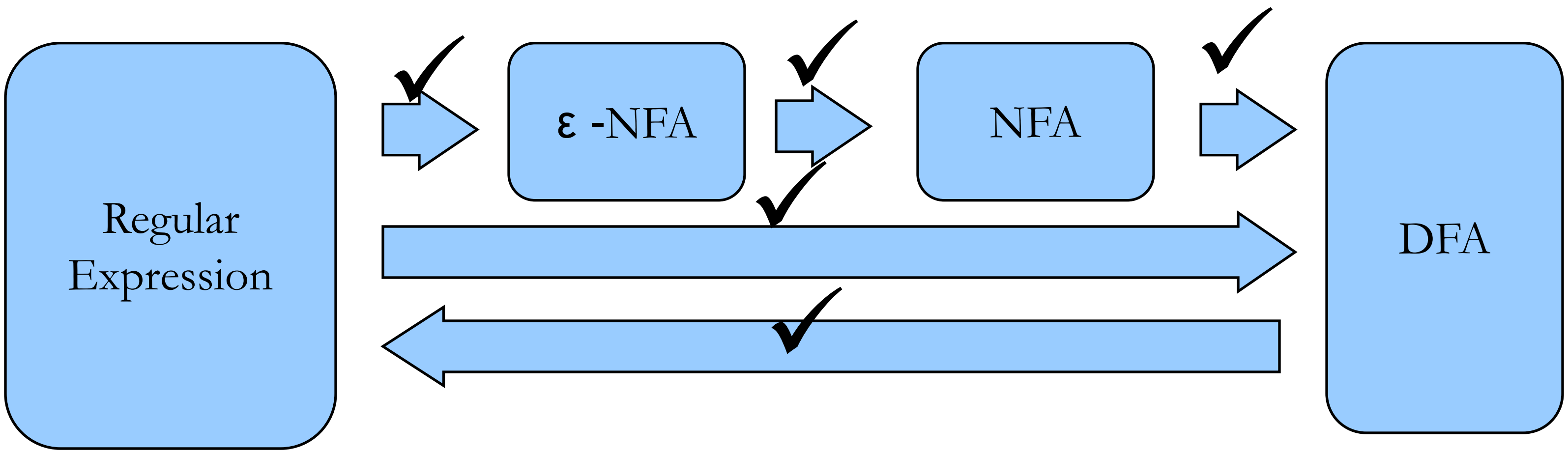
Steps

3. Start state = final state



4. Union of all expressions derived from 2 and 3

Road map



A language is regular iff it is accepted by a DFA, NFA, eNFA, or regular expression