Problems

I.Nelson
SSN College of Engineering

1. Derive the relation between Ωc, Ωp and Ωs.

2. Design a low pass Butterworth filter to meet the he following specifications.

$$F_p = 6KHz \ \& \ F_s = 10\,KHz, \ \delta_p = \ \delta_s = 0.1$$

Answer:

$$H(s) = \frac{1}{s^6 + 3.8637\,s^5 + 7.4641\,s^4 + 9.141\,s^3 + 7.4641\,s^2 + 3.8637\,s + 1}$$

3. Determine the Butterworth poles for the specifications

$A_p = 1\text{dB}$ for $\Omega \le 4$ rad /sec

$A_s \ge 20$ dB for $\Omega \ge 8$ rad /sec

Answer: The Butterworth poles are

$$(s+1)(s+.309+j.951)(s+.309-j.951)(s+.809+j.5878)(s+.809-j.5878)$$

4. Find the transfer function of a third order Chebyshev low pass filter prototype with a passband ripple of 1.5 dB.

Answer:

$$H(s)=\frac{0.3892}{\mathbf{s}^3+0.8402\,\mathbf{s}^2+1.103\mathbf{s}+0.3892}$$

5. Find the Chebyshev poles for the specifications

$$F_p = 6KHz \ \& \ F_s = 10\,KHz,\ \ \delta_p = \delta_s = 0.1$$

Answer:

$$(s+0.2895)(s+.234+j.612)(s+.234-j.612)(s+.0895+j.9901)(s+.0895-j.9901)$$

6. Find the Chebyshev transfer function for the specifications

A_p = 1dB for $\Omega \leq 4$ rad /sec

$A_s \geq 20$ dB for $\Omega \geq 8$ rad /sec

Answer:

$$H(s) = \frac{0.512}{s^3 + 1.0214\,s^2 + 1.2716\,s + 0.5162}$$

7. Use impulse invariant method to design a digital filter from an analog prototype that has a system function.

$$H(s) = \frac{s+a}{(s+a)^2 + b^2}$$

Answer:

$$H(z) = \frac{1 - e^{-aT}\cos(bT)z^{-1}}{1 - 2e^{-aT}\cos(bT)z^{-1} + e^{-2aT}z^{-2}}$$

8. Use impulse invariant method to design a digital lowpass Butterworth filter to meet the following specifications.

$$0.9 \le \left|H(e^{j\omega})\right| \le 1 \qquad \left|\omega\right| \le 0.2\pi$$

$$\left|H(e^{j\omega})\right| \le 0.2 \qquad 0.3\pi \le \omega \le \pi$$

Answer:

The digital lowpass Butterworth transfer function is

$$H(s) = \frac{0.0007\,z^{-1} + 0.0105\,z^{-2} + 0.0168\,z^{-3} + 0.0042\,z^{-4} + 0.0001\,z^{-5}}{1 - 30343\,z^{-1} + 5.015\,z^{-2} - 4.2153\,z^{-3} + 2.0703\,z^{-4} - 0.5593\,z^{-5} + 0.0646\,z^{-6}}$$

9. Using Bilinear transform method Design a digital lowpass Butterworth filter with the following specifications:

(a) 3 dB attenuation at the passband frequency of 1.5 kHz

(b) 10 dB stopband attenuation at the frequency of 3 kHz

(c) Sampling frequency of 8,000 Hz.

Answer:

$$H(z) = \frac{0.4006 + 0.4006\, z^{-1}}{1 - 0.1989\, z^{-1}}$$

10. Use Bilinear transformation to design a digital lowpass Butterworth filter to meet the following specifications.

$$0.707 \le \left|H(e^{j\omega})\right| \le 1 \qquad |\omega| \le \frac{\pi}{2}$$

$$\left|H(e^{j\omega})\right| \le 0.2 \qquad \frac{3\pi}{4} \le \omega \le \pi$$

Answer:

The digital lowpass Butterworth transfer function is

$$H(z) = \frac{0.2929\left(1 + z^{-1}\right)^2}{1 + 0.1716\, z^{-2}}$$

11. Design a Chebyshev lowpass filter with the specifications, A_p= 1dB ripple in the passband $0 \le \omega \le 0.2\pi$ and A_s = 15 dB attenuation in the stopband $0.3\pi \le \omega \le \pi$, using bilinear transformation.

Answer:

$$H(z) = \frac{0.001836\left(1+z^{-1}\right)^4}{\left(1-1.499\,z^{-1}+0.8482\,z^{-2}\right)\left(1-1.5548\,z^{-1}+0.6493\,z^{-2}\right)}$$

12. Design a 1dB ripple digital highpass Chebyshev filter with the following specifications:

As=6.3 dB in the stopband for $0 \le \omega \le 10$ rad/sec

and the passband edge frequency is ωp=15 rad/sec.

Answer:

$$H(z) = \frac{0.6902\left(1-z^{-1}\right)^2}{1-1.4678\,z^{-1}+0.6298\,z^{-2}}$$

13. Using bilinear transformation, design a digital bandpass Butterworth filter with the following specifications.

Ap = 2 dB in the passband 800 Hz ≤ f ≤ 1000 Hz

As = 20 dB in the stopband 0 ≤ f ≤ 400Hz & 2000Hz ≤ f.

The sampling frequency is 8000 Hz.

Answer: The digital bandpass Butterworth transfer function is

$$H(z)=\frac{0.004837\left(1-z^{-1}\right)^{4}}{1-30936\,z^{-1}+8.2587\,z^{-2}-11.214\,z^{-3}+10.778\,z^{-4}-7.3866\,z^{-5}+3.573\,z^{-6}-1.1135\,z^{-7}+0.187\,z^{-8}}$$

14. Design a digital bandstop Chebyshev filter with the following specifications:

As= 15 dB in the stopband 550 to 1000 Hz and the Ap= 1 dB in the passband dc to 275 Hz and 2KHz to ∞

Answer:

$$H(z)=\frac{0.0.3732\left(1-3.2176\,z^{-1}+4.588\,z^{-2}-3.2176\,z^{-3}+z^{-4}\right)}{1-1.8869\,z^{-1}+1.429\,z^{-2}-0.8007\,z^{-3}+0.3292\,z^{-4}}$$

15. Design a digital bandstop Butterworth filter with the following specifications:

1. Center frequency of 2.5 kHz
2. Passband width of 200 Hz and ripple of 3 dB
3. Stopband width of 50 Hz and attenuation of 10 dB

Sampling frequency of 8,000 Hz.

Answer: The digital bandpass Butterworth transfer function is

$$H(z) = \frac{0.9259 + 0.7078\,z^{-1} + 0.9259\,z^{-2}}{1 - 0.7078\,z^{-1} + 8.518\,z^{-2}}$$

16. Design a Butterworth & Chebyshev low pass filter specifications.

Passband ripple is 2 dB for $\Omega \le 10$ rad/sec

Stopband attenuation is 20 for $\Omega \ge 30$ rad/sec

Answer:

The Butterworth Lowpass filter transfer function is

$$H(s) = \frac{2713.602}{s^3 + 27.896\,s^2 + 398.1s + 2713.602}$$

The Chebyshev Lowpass filter transfer function is

$$H(s) = \frac{65.378}{s^2 + 8.0382\,s + 82.306}$$

17. Design a Chebyshev highpass filter with the following specifications

Ap = 1 dB for $\Omega \geq 40$ rad/sec

As = 20 dB for $\Omega \leq 15$ rad/sec

Answer:

The Butterworth Highpass filter transfer function

$$H(s) = \frac{s^4}{s^4 + 69.616 s^3 + 2423.159 s^2 + 49408.01 s + 503712.39}$$

The Chebyshev Highpass filter transfer function

$$H(s) = \frac{s^3}{s^3 + 100.826 s^2 + 3218.653 s + 130264.868}$$

18. Design a Butter worth & Chebyshev band pass filter specifications. Passband ripple is 2.4 dB & Stopband attenuation is 20 dB and the passband edge frequencies are 1000 rad/sec, and 2000 rad/sec and stopband edge frequencies are 450 rad/sec and 4000 rad/sec.

Answer:

The Butterworth Lowpass filter transfer function is

$$H(s) = \frac{1.2312 \times 10^{6} s^{2}}{s^{4} + 1569\, s^{3} + 5.2312 \times 10^{6} s^{2} + 3.1384 \times 10^{6} s + 4 \times 10^{6}}$$

The Chebyshev Lowpass filter transfer function is

$$H(s) = \frac{5.821 \times 10^{5} s^{2}}{s^{4} + 731.249\, s^{3} + 4.768 \times 10^{6} s^{2} + 1.463 \times 10^{9} s + 4 \times 10^{12}}$$