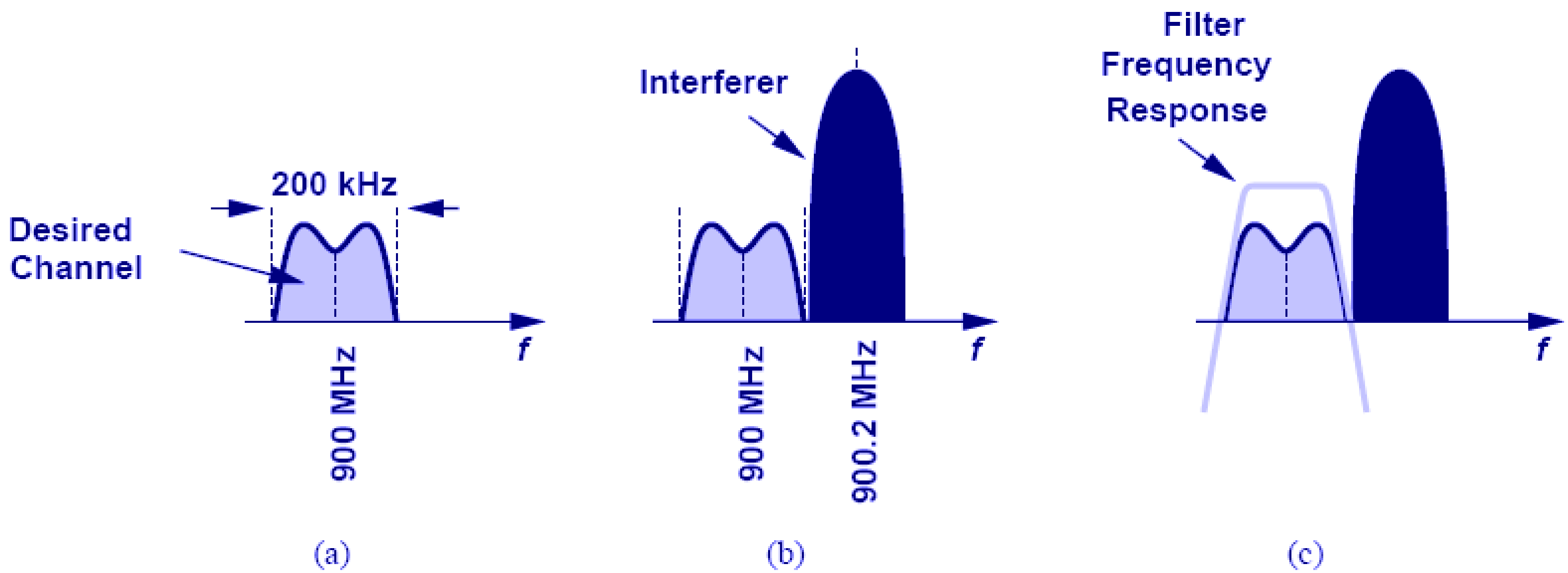
Filter Fundamentals

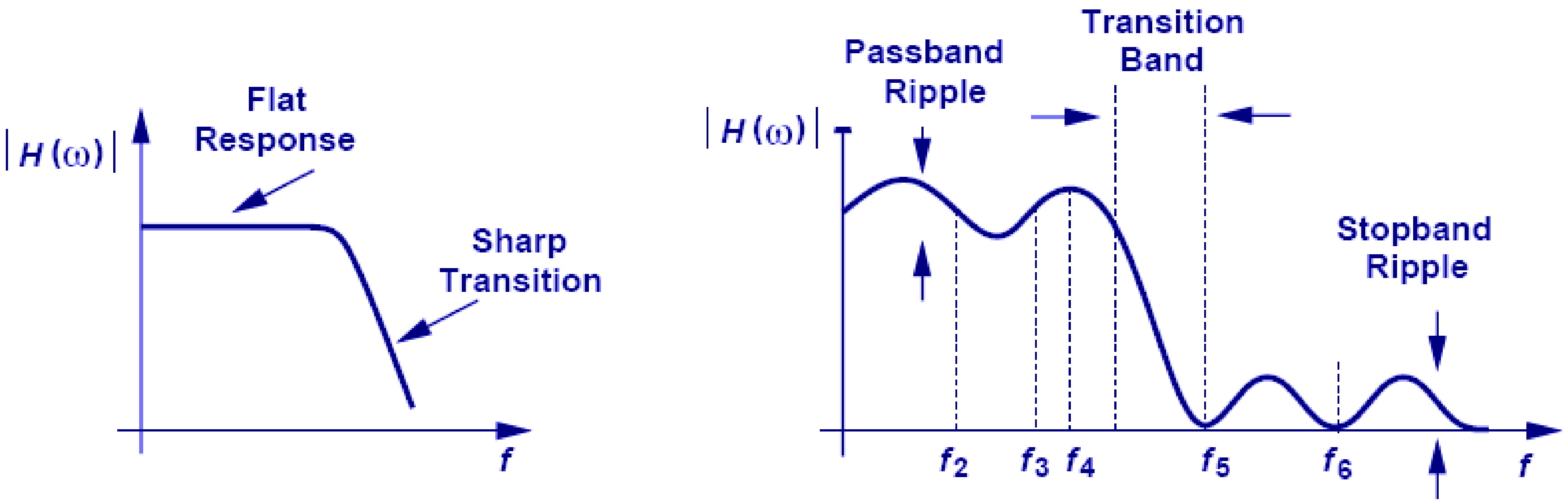
I.Nelson
SSN College of Engineering

Why We Need Filters



- In order to eliminate the unwanted interference that accompanies a signal, a filter is needed.

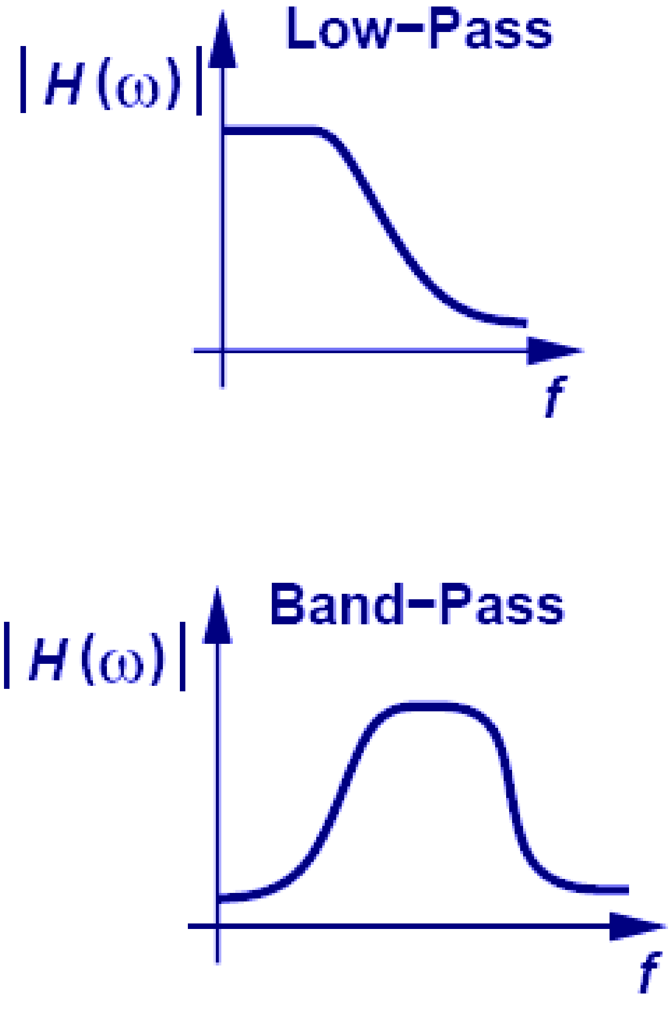
Filter Characteristics



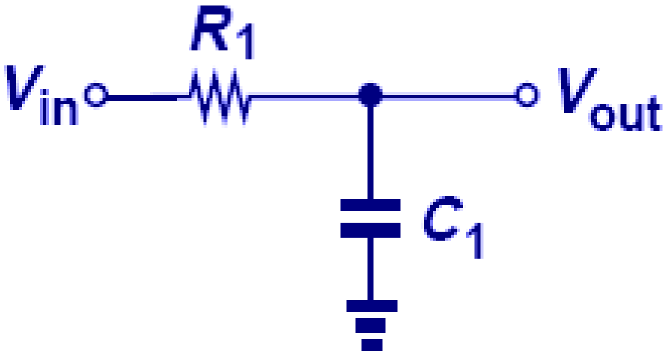
(a) (b)

- Ideally, a filter needs to have a flat pass band and a sharp roll-off in its transition band.
- Realistically, it has a rippling pass/stop band and a transition band.

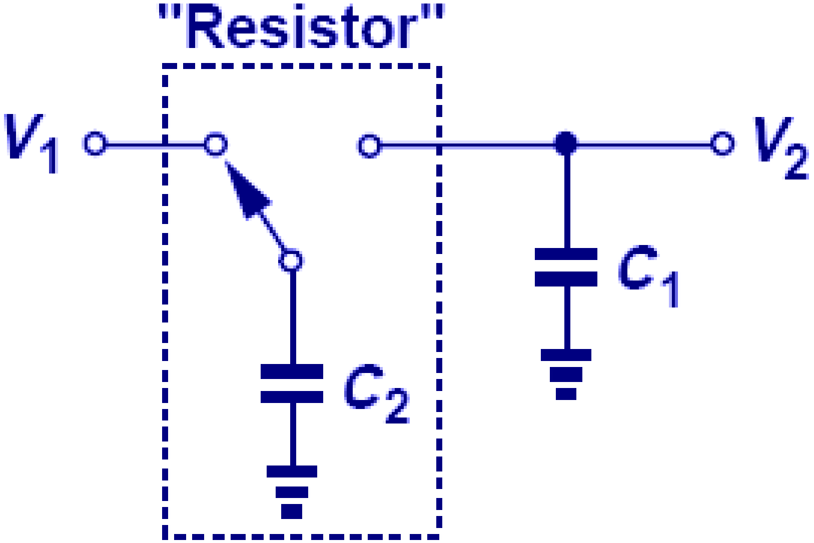
Classification of Filters I



Classification of Filters II



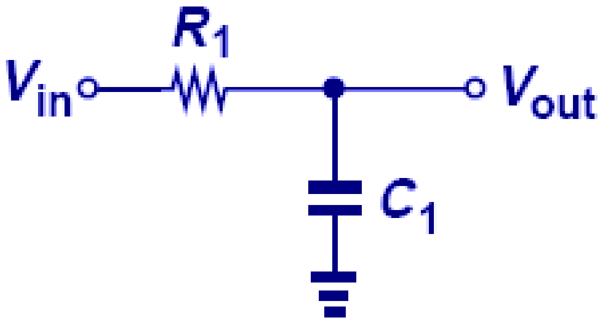
$$R_{eff} = \frac{1}{C_2 f}$$



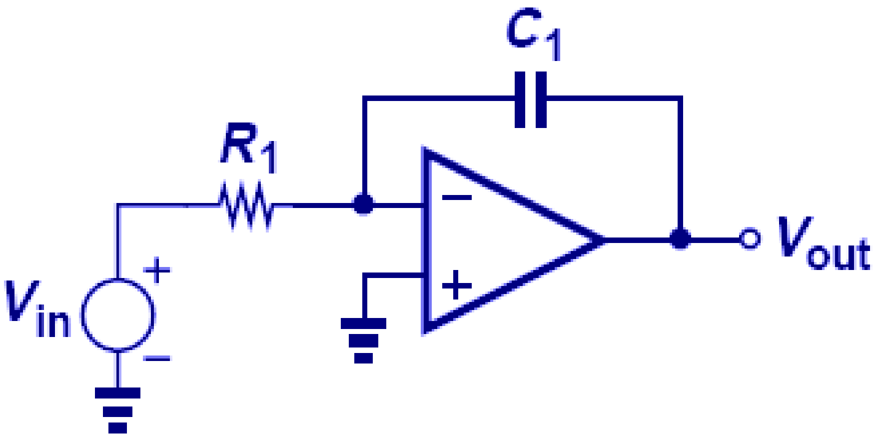
Continuous-time

Discrete-time

Classification of Filters III

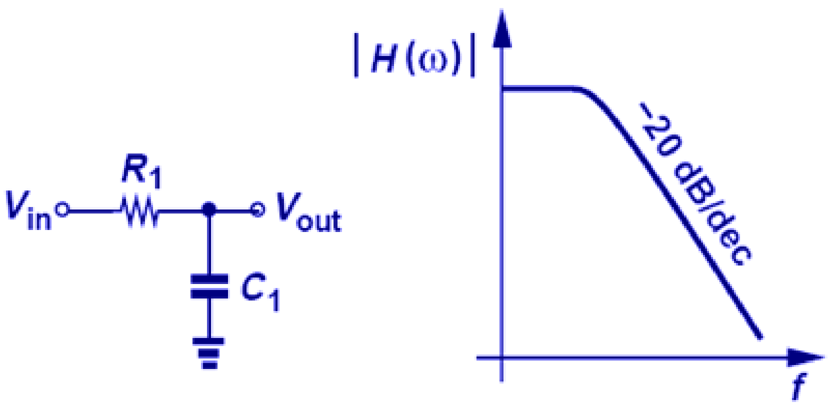


Passive

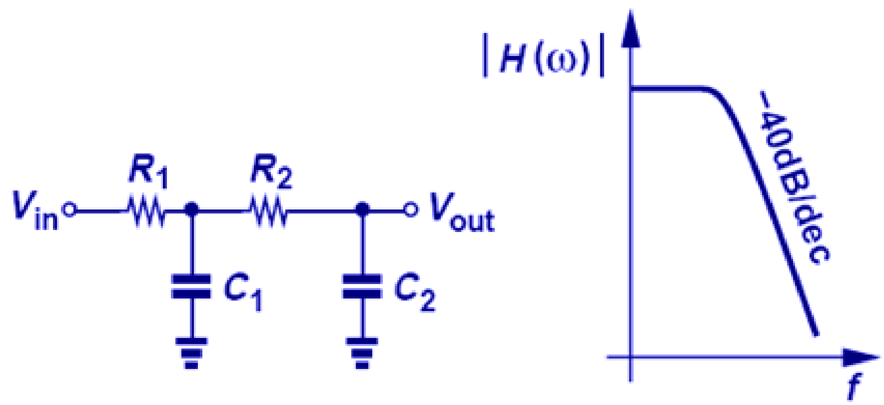


Active

Filter Transfer Function



A



B

- Filter (a) has a transfer function with -20dB/dec roll-off
- Filter (b) has a transfer function with -40dB/dec roll-off, better selectivity.

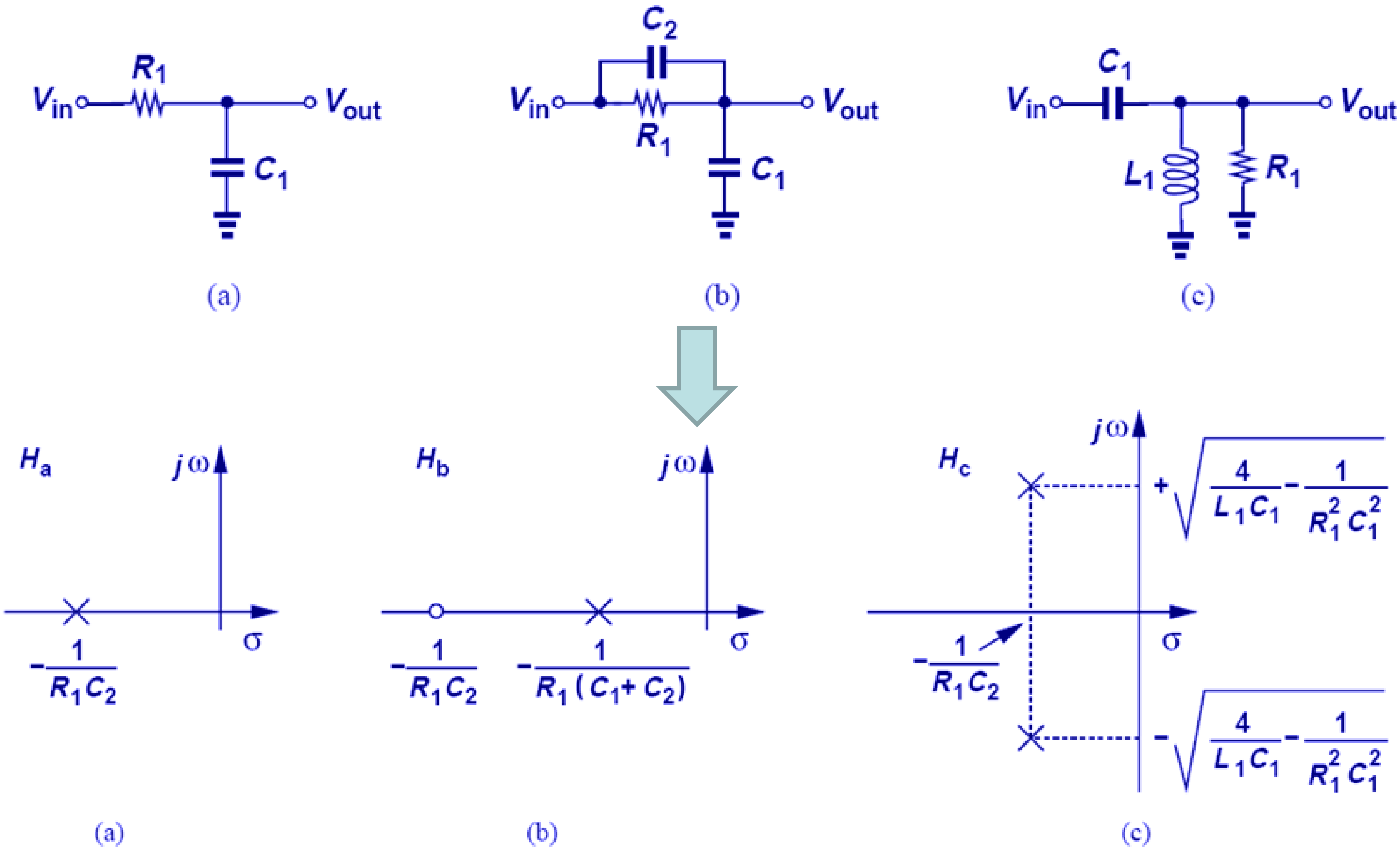
General Transfer Function

$$H(s) = \alpha \frac{(s - Z_1)(s - Z_2)\cdots(s - Z_m)}{(s - P_1)(s - P_2)\cdots(s - P_m)}$$

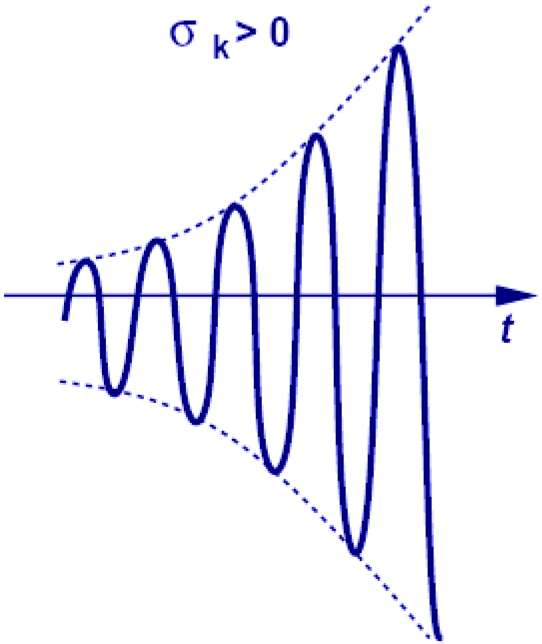
Z_m=m'th zero

P_n =n'th pole

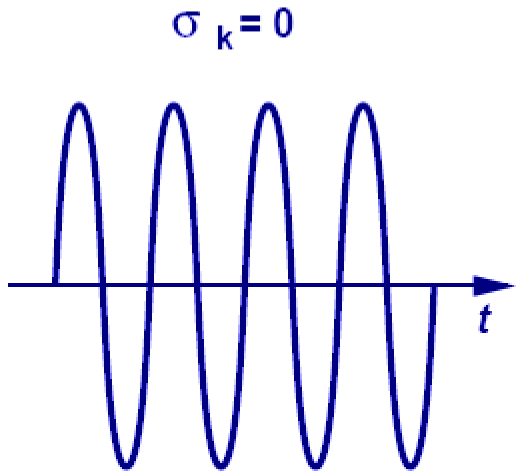
Pole-Zero Diagram



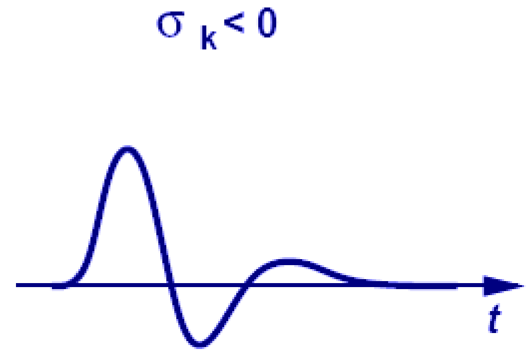
Position of the Poles



Poles on the RHP
Unstable
(no good)

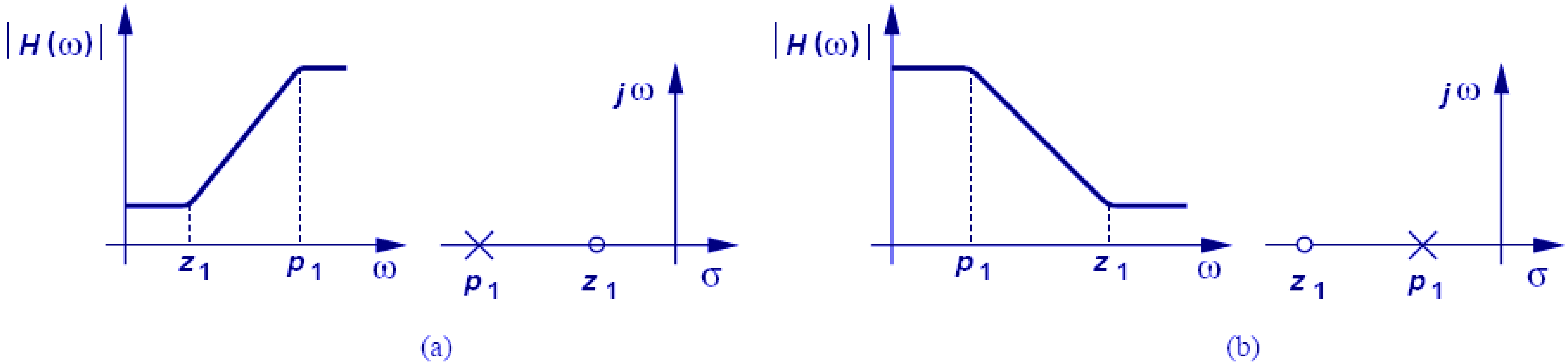


Poles on the jω axis
Oscillatory
(no good)



Poles on the LHP
Decaying
(good)

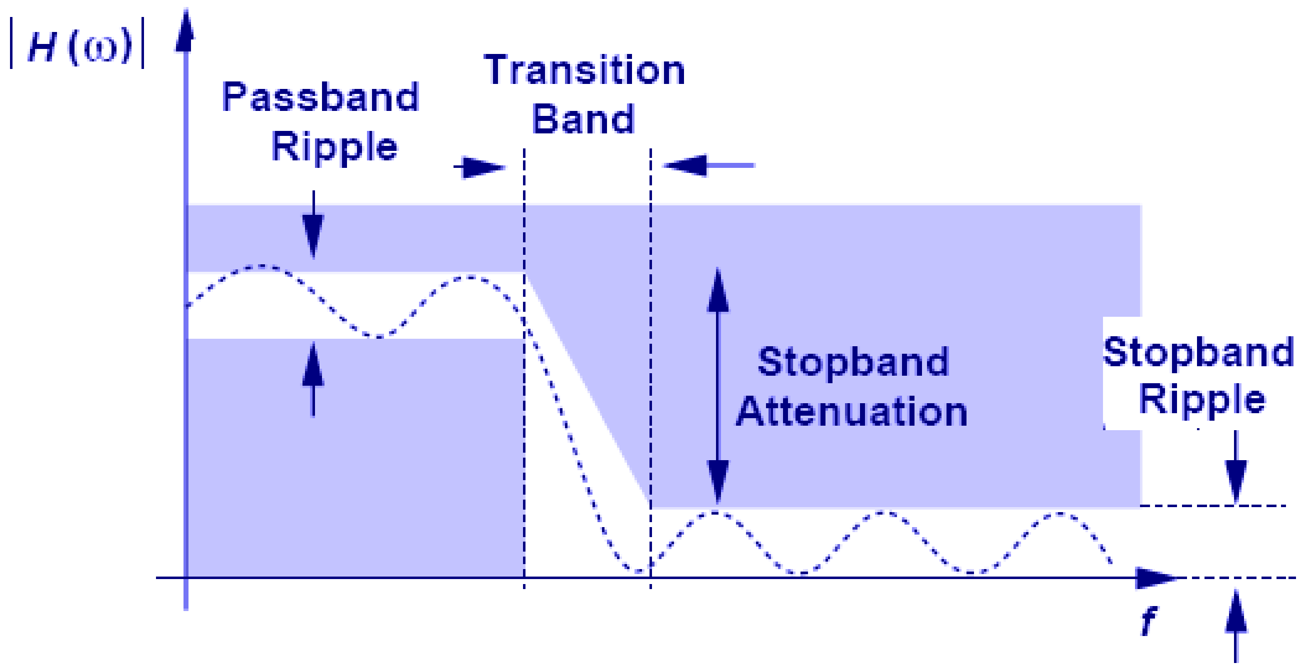
First-Order Filters



$$H(s) = \alpha \frac{s + z_1}{s + p_1}$$

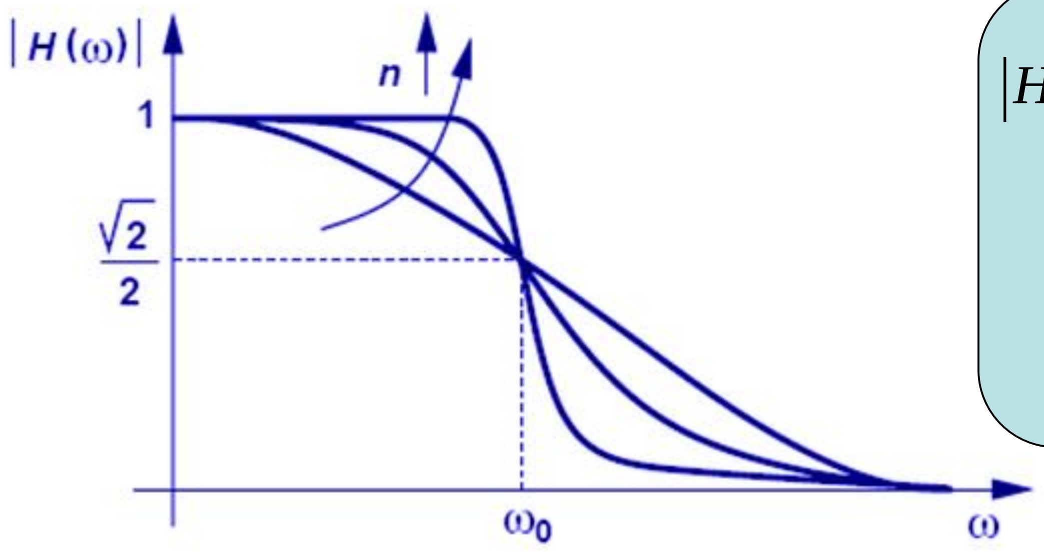
- First-order filters are represented by the transfer function shown above.
- Low/high pass filters can be realized by changing the relative positions of poles and zeros.

Frequency Response Template



- With all the specifications on pass/stop band ripples and transition band slope, one can create a filter template that will lend itself to transfer function approximation.

Butterworth Response

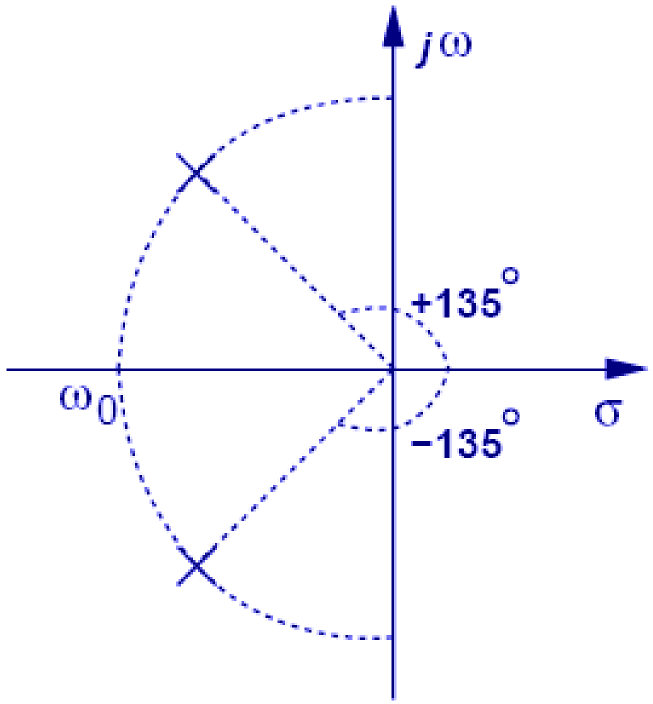


$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_0}\right)^{2n}}}$$

$\omega_{-3dB}=\omega_0$, for all n

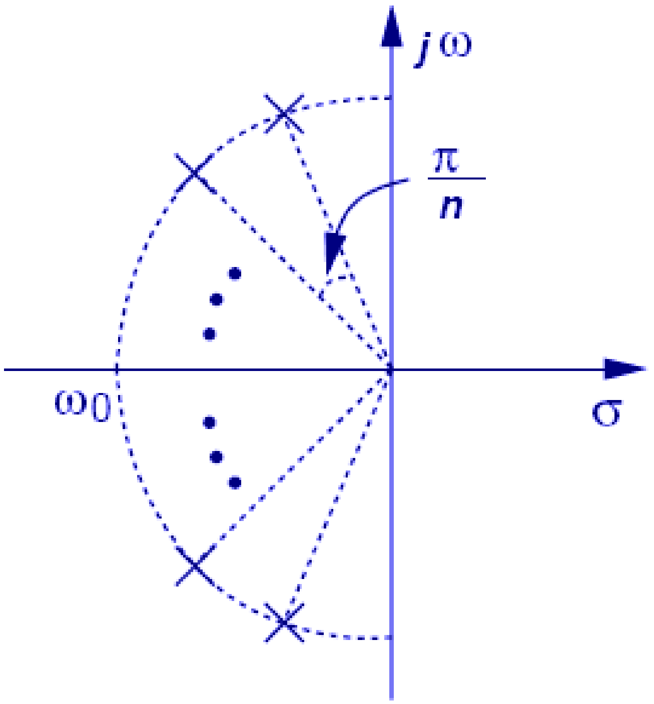
- The Butterworth response completely avoids ripples in the pass/stop bands at the expense of the transition band slope.

Poles of the Butterworth Response



(a)

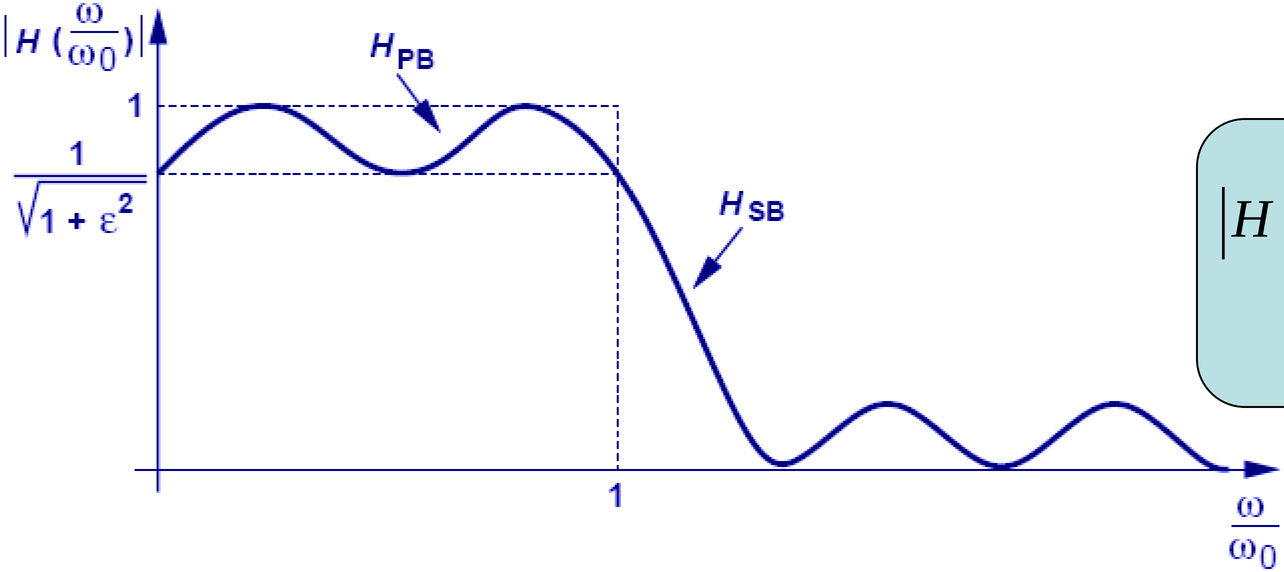
2nd-Order



(b)

nth-Order

Chebyshev Response

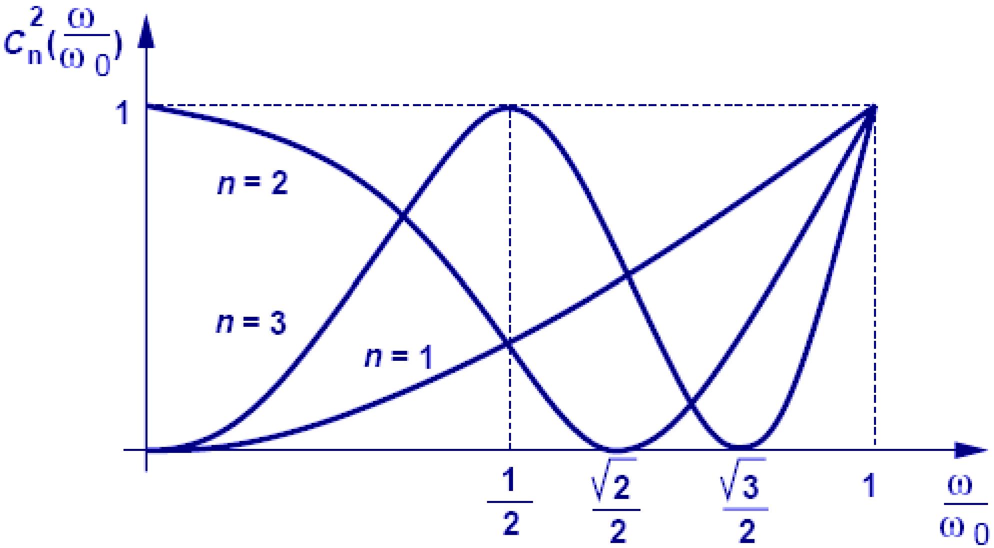


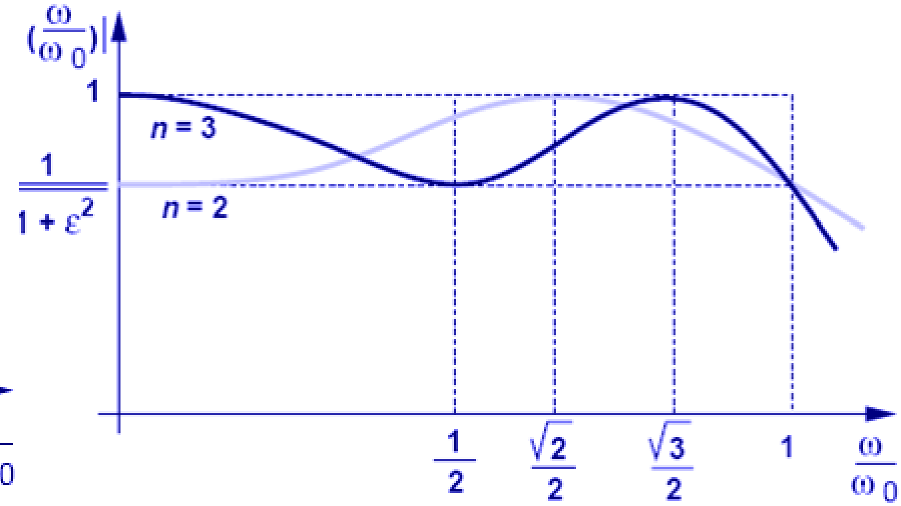
$$|H(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 C_n^2\left(\frac{\omega}{\omega_0}\right)}}$$

Chebyshev Polynomial

- The Chebyshev response provides an "equiripple" pass/stop band response.

Chebyshev Polynomial





Chebyshev Polynomial for n=1,2,3

Resulting Transfer function for n=2,3

$$C_n\left(\frac{\omega}{\omega_0}\right) = \cos\left(n\cos^{-1}\frac{\omega}{\omega_0}\right), \omega < \omega_0$$

$$= \cosh\left(n\cosh^{-1}\frac{\omega}{\omega_0}\right), \omega > \omega_0$$