Infinite Impulse Response Filter – Impulse Invariant Method

I.Nelson

SSN College of Engineering

Design of IIR digital filters from analog filters:

If the conversion technique is to be effective, it should possess the following desirable properties.

- The $j\Omega$ axis in the s-plane should map into the unit circle in the z-plane. Thus there will be a direct relationship between the two frequency variables in the two domain.
- The LHP of the s-plane should map into the inside of the unit circle in the z-plane. Thus a stable analog filter will be converted to a stable digital filter.

Impulse Invariant Method:

In impulse invariant method, the IIR filter is designed such that the unit impulse response h(n) of digital filter is the sampled version of the impulse response of analog filter.

The Z – transform of an infinite impulse response is given by

$$H(z) = \sum_{n=0}^{\infty} h(n)\, z^{-n}$$

(Or)

$$H(z)\Big/_{z = e^{sT}} = \sum_{n=0}^{\infty} h(n)\, e^{-nsT}$$

Let us consider the mapping of points from s – plane to z – plane implied by the relation $z=e^{sT}$.

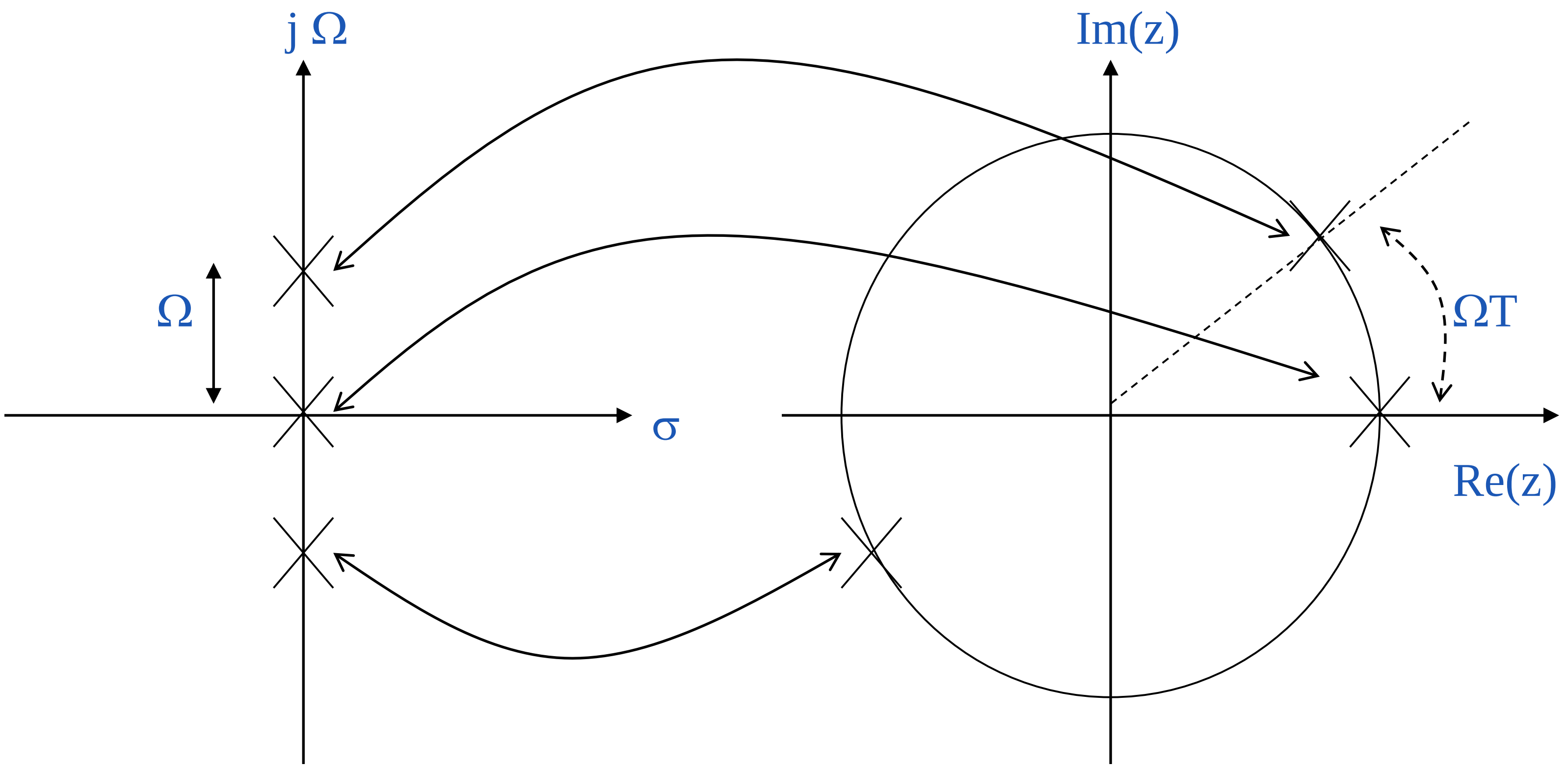
Substituting $s = \sigma + j\Omega$ and expressing the complex variable z in polar form as $z = r\, e^{j\omega}$, we get,

$r\, e^{j\omega} = e^{(\sigma + j\Omega)T} = e^{\sigma T}\; e^{\,j\Omega T}$

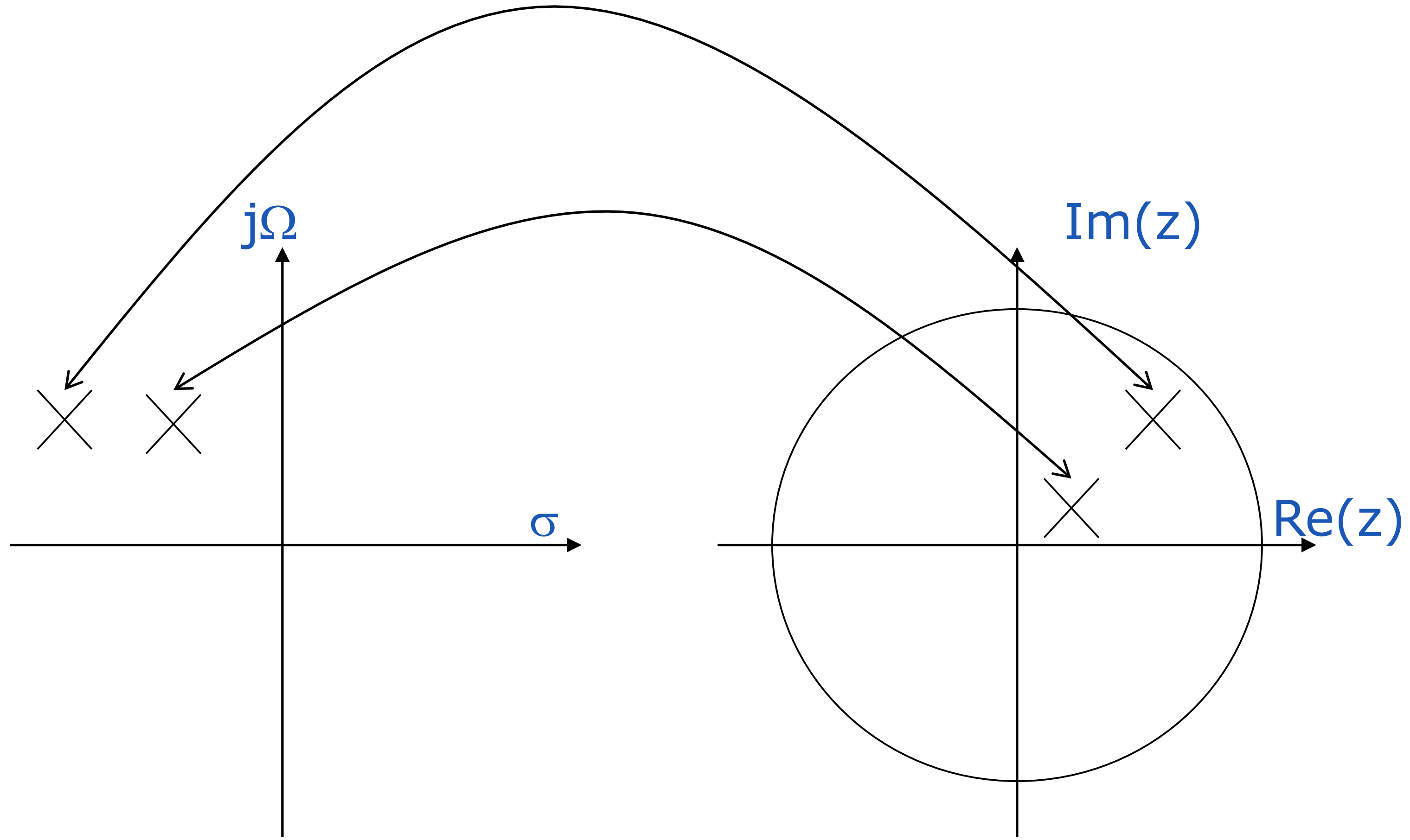
$r = e^{\sigma T}$ - magnitude is $e^{\sigma T}$ and angle is 0

$e^{j\omega} = e^{\,j\Omega T}$ - magnitude is 1 and angle is ΩT

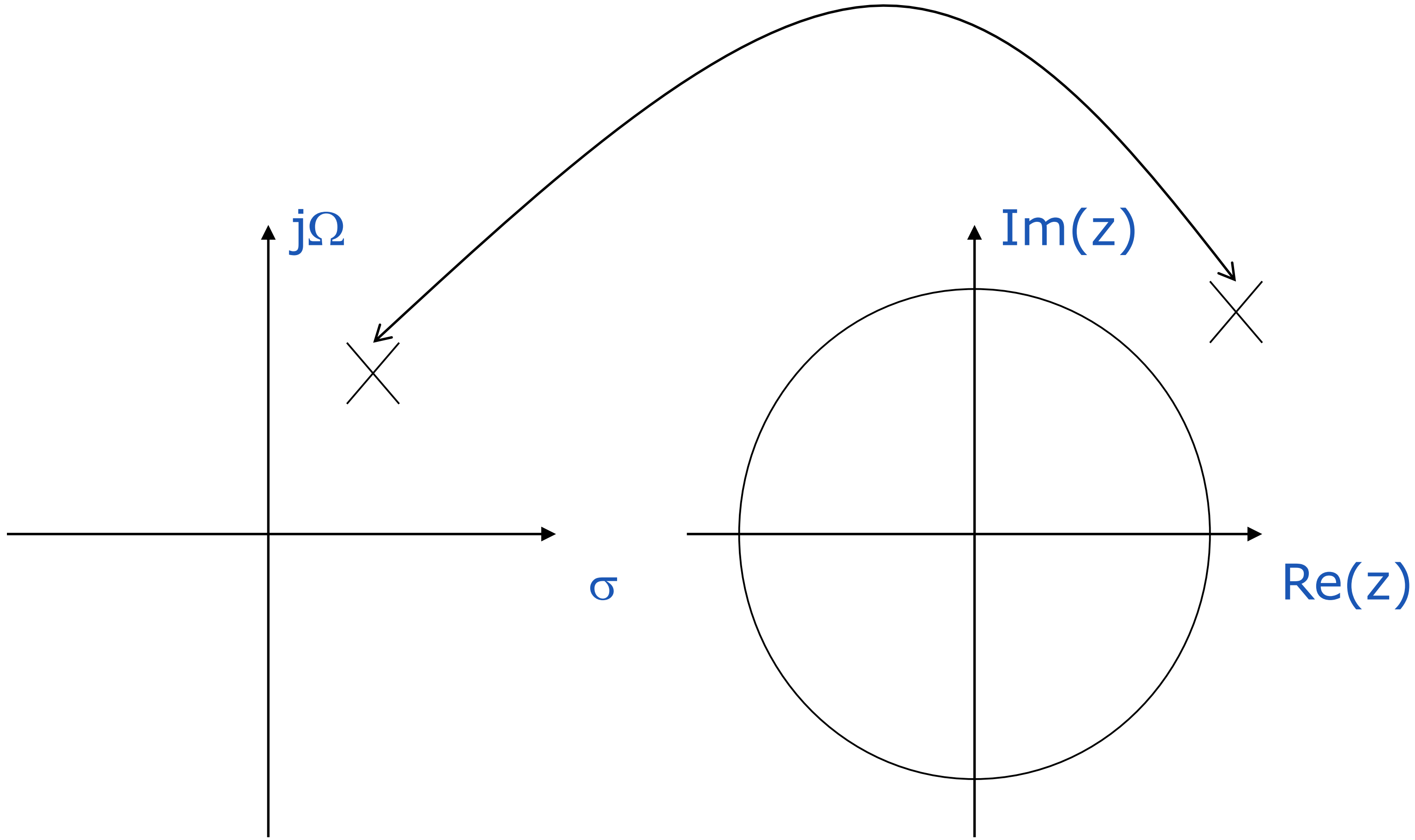
Poles having σ = 0



Poles having $\sigma < 0$

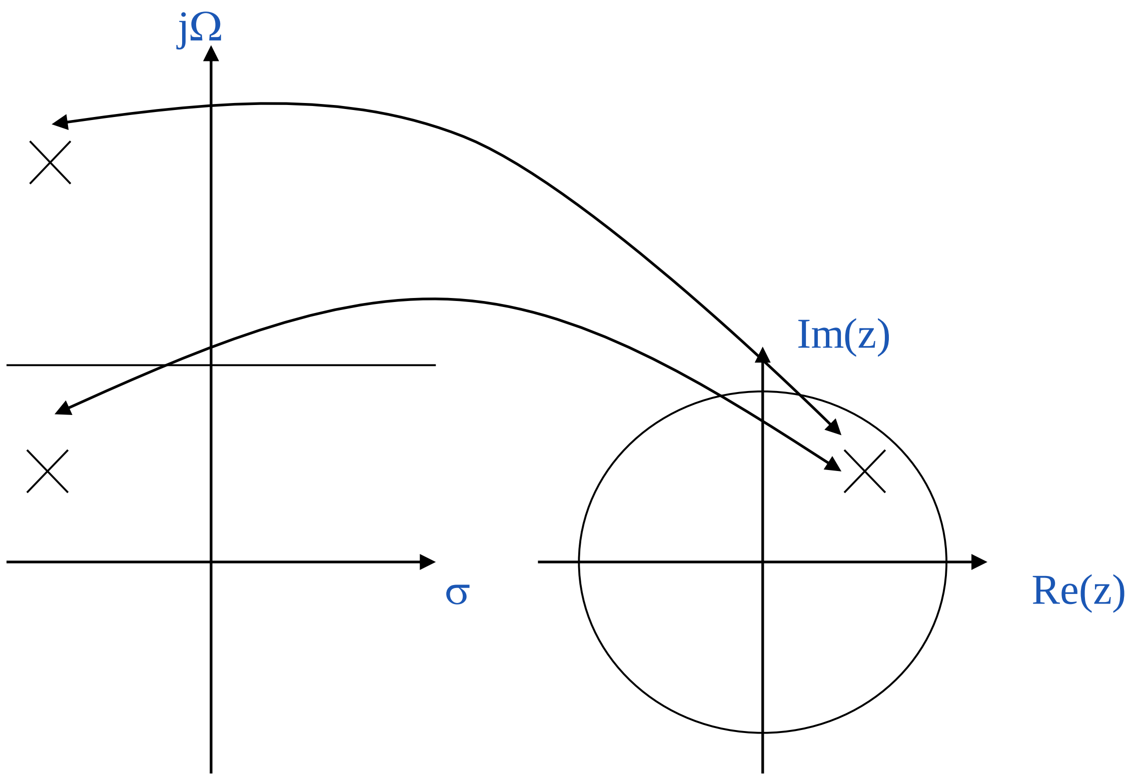


Poles having $\sigma > 0$



Note:

- Two poles with identical real parts but imaginary components differs by $2\pi/T$ – Many to one mapping.



- S – plane poles having imaginary parts greater than π/T or less than $-\pi/T$ cause aliasing, when sampling analog signals.

Let $H_a(s)$ be the system function of an analog filter.

$$H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k}$$

Taking inverse Laplace transform, we get,

$$h_a(t) = \sum_{k=1}^{N} c_k e^{p_k t}$$

If we sample $h_a(t)$ periodically at t = nT, we have

$$h(n) = h_a(nt) = \sum_{k=1}^{N} c_k e^{p_k nT}$$

applying Z – transform,

$$H(z) = \sum_{n=0}^{\infty} \sum_{k=1}^{N} c_k e^{p_k nT} z^{-n}$$

$$H(z)=\sum_{k=1}^{N} c_k \sum_{n=0}^{\infty} e^{p_k nT} z^{-n} = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_k T} z^{-1}}$$

For high sampling rates, i.e., for small T, the digital filter gain is high.

Therefore,

$$H(z)=\sum_{k=1}^{N} \frac{T c_k}{1-e^{p_k T} z^{-1}}$$

Note: Due to aliasing, the impulse invariant method is appropriate for the design of low pass and band pass filters only. It is unsuccessful for implementing High pass digital filters.

Steps to design a digital filter using Impulse Invariant method

For the given specifications find $H_a(s)$, the transfer function of an analog filter.

Select the sampling rate of the digital filter T seconds per sample.

Express the analog filter transfer function as the sum of single pole filters.

$$H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k}$$

Compute the Z – transform of the digital filter by using the formula,

$$H(z) = \sum_{k=1}^{N} c_k \sum_{n=0}^{\infty} e^{p_k nT} z^{-n} = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}$$

For high sampling rates,

$$H(z) = \sum_{k=1}^{N} \frac{T\, c_k}{1 - e^{p_k T} z^{-1}}$$