MA6566 - Discrete Mathematics

Prepared by

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L: 45, T: 15, TOTAL: 60 PERIODS

TEXT BOOKS:

1. Kenneth H.Rosen, "Discrete Mathematics and its Applications", Special Indian edition, Tata McGraw-Hill Pub. Co. Ltd., New Delhi, (2007). (For the units 1 to 3, Sections 1.1 to 1.7 , 4.1 & 4.2, 5.1 to 5.3, 6.1, 6.2, 6.4 to 6.6, 8.1 to 8.5).

2. Trembly J.P and Manohar R, "Discrete Mathematical Structures with Applications to Computer Science", Tata McGraw–Hill Pub. Co. Ltd, New Delhi, 30th Re-print (2007).(For units 4 & 5 , Sections 2-3.8 & 2-3.9,3-1,3-2 & 3-5, 4-1 & 4-2).

REFERENCES:

1. Ralph. P. Grimaldi, "Discrete and Combinatorial Mathematics: An Applied Introduction", Fourth Edition, Pearson Education Asia, Delhi, (2002).

2. Thomas Koshy, ”Discrete Mathematics with Applications”, Elsevier Publications, (2006).

3. Seymour Lipschutz and Mark Lipson, ”Discrete Mathematics”, Schaum’s Outlines, Tata McGraw-Hill Pub. Co. Ltd., New Delhi, Second edition, (2007).

UNIT I - LOGIC AND PROOFS

Propositional Calculus

1. Define a If-statement.
2. Define conjunction.
3. Define disjunction.
4. Define negation.
5. Define conditional statements and bi-conditional statement.
6. Construct the truth table for $\neg(\neg P \wedge \neg Q)$
7. Define tautology.
8. Define contradiction.
9. Define NAND and NOR.
10. Define functionally complete.
11. Define duals.
12. Define normal forms, PCNF and PDNF
13. Write down the truth table for the compound statement and state whether it’s a tautology $(P \wedge (P \leftrightarrow Q)) \rightarrow Q$

Logical Identities

14. I_1. Idempotent Laws: $P \vee P \equiv P,\ P \wedge P \equiv P$
15. I_2. Commutative Laws: $P \vee Q \equiv Q \vee P,\ P \wedge Q \equiv Q \wedge P$
16. I_3. Associative Laws: $P \vee (Q \vee R) \equiv (P \vee Q) \vee R, P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
17. I_4.Distributive Laws: $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R), P \wedge (Q \vee R) \equiv (P \vee Q) \vee (P \wedge R)$
18. I_5. Absorption Laws: $P \vee (P \wedge Q) \equiv P, P \wedge (P \vee Q) \equiv P$
19. I_6. Demorgan’s Laws: $\neg(P \vee Q) \equiv \neg P \wedge \neg Q, \neg(P \wedge Q) \equiv \neg P \vee \neg Q$

20. I_7. Double Negation: $P \equiv \neg(\neg P)$

21. I_8. : $P \wedge \neg P = F, P \vee \neg P = T$

22. I_9. : $\begin{array}{l} P \vee T = T, P \wedge T = P \\ P \vee F = P, P \wedge F = F \end{array}$

23. I_{10}. : $(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q) \equiv \neg P$

24. I_{11}. Contra-positive: $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

25. I_{12}. : $P \Rightarrow Q \equiv (\neg P \vee Q)$

26. Show that $P \to (Q \to P) \Leftrightarrow \neg P \to (P \to Q)$

27. Show that $\begin{array}{l} (i) \quad \neg(P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q) \\ (ii) \quad \neg(P \leftrightarrow Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q) \end{array}$

28. Show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

29. Establish $\neg(P \wedge Q) \to (\neg P \vee (\neg P \vee Q)) \Rightarrow (\neg P \vee Q)$

30. Show that $(P \to Q) \wedge (R \to Q) and (P \vee R) \to Q$ are equivalent formulae

31. Show that $\{\wedge, \neg\}$ and $\{\vee, \neg\}$ are functionally complete.

32. Show that $\{\uparrow\}$ and $\{\downarrow\}$ are functionally complete.

33. Obtain the PDNF for $\neg P \vee Q$

34. By not using the truth table directly find PDNF for $\begin{array}{l} (i) P \leftrightarrow Q \\ (ii) \neg P \vee Q \end{array}$

35. Obtain the PCNF of the formula $(\neg P \to R) \wedge (Q \leftrightarrow P)$

36. Find the PDNF and PCNF of the formula $P \vee (\neg P \to (Q \vee (\neg Q \to R)))$

37. Obtain the PDNF of $P \to ((P \to Q) \wedge \neg(\neg Q \vee \neg P))$

<u>Theory of inference</u>

38. We say that from a set of premises {H_1, H_2, … H_n} a conclusion C follows logically if $\{H_1 \wedge H_2 \wedge ... \wedge H_n\} \Rightarrow C$

39. When a conclusion is derived from a set of premises by using the accepted rule of reasoning then such a process of derivation is called a deduction, or a formal proof, and the argument is called a valid argument

40. Rule P: We may introduce a premise any point in a derivation.

41. Rule T: We may introduce a formula S in a derivation if S is tautologically implied by anyone or more of the preceding formulae in derivation.
42. Rule CP: If we can derive S from R and a set of premises, then we may derive $R \to S$ from the set of premises alone.
43. Inconsistent: A set of formulae $\{H_1, H_2, \dots H_m\}$ is said to be inconsistent if their conjunction implies a contradiction, that is $\{H_1 \wedge H_2 \wedge \dots \wedge H_m\} \Rightarrow R \wedge \neg R$ for some formulae R.
44. Consistent: A set of formulae $\{H_1, H_2, \dots H_m\}$ is said to be consistent if it is not inconsistent.
45. Indirect Method of Proof: The technique of indirect method of proof runs as follows:Introduce a negation of the desired conclusion as a new premise.
46. Form the new premise, together with the given premise, derive a contradiction
47. Assert the desired conclusion as a logical inference from the premises

<u>Implications</u>

48. I_1. $P \wedge Q \Rightarrow P$ (Simplification)
49. I_2. $P \wedge Q \Rightarrow Q$ (Simplification)
50. I_3. $P \Rightarrow P \vee Q$ (Addition)
51. I_4. $Q \Rightarrow P \vee Q$ (Addition)
52. I_5. $\neg P \Rightarrow P \to Q$
53. I_6. $Q \Rightarrow P \to Q$
54. I_7. $\neg(P \to Q) \Rightarrow P$
55. I_8. $\neg(P \to Q) \Rightarrow \neg Q$
56. I_9. $P, Q \Rightarrow P \wedge Q$
57. I_{10}. $\neg P, P \vee Q \Rightarrow Q$ (Disjunction syllogism)
58. I_{11}. $P, P \to Q \Rightarrow Q$ (Modus Ponens)
59. I_{12}. $\neg Q, P \to Q \Rightarrow \neg P$ (Modus Tollens)
60. I_{13}. $P \to Q, Q \to R \Rightarrow P \to R$ (Hypothetical syllogism)
61. I_{14}. $P \vee Q, P \to R, Q \to R \Rightarrow R$ (Dilemma)
62. I_{15}. $P \to R, Q \to R \Rightarrow (P \vee Q) \to R$

63. Demonstrate that S is a valid inference from the premises $P \to \neg Q, Q \vee R, \neg S \to P$ and $\neg R$.

64. Show that $R \vee S$ is a valid conclusion from the premises $C \vee D, C \vee D \to \neg H, \neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to (R \vee S)$

65. Show that $P \to S$ can be derived from the premises $\neg P \vee Q, \neg Q \vee R, R \to S$

66. Show that if $p \to q, q \to r, \neg(p \wedge r)$ and $p \vee r$, then r.

67. Using indirect method of proof, derive $P \to \neg S$ fro $P \to Q \vee R, Q \to \neg P, S \to \neg R, P$.

68. Using indirect method of proof, show that $p \to q, q \to r, \neg(p \wedge r), p \vee r \Rightarrow r$

69. Determine the validity of the following argument:

My father praises me only if I can be proud of myself. Either I do well in sports or I cannot be proud of myself. If I study hard, then I cannot do well in sports. Therefore, if father praises me, then I do not study well.

70. Show that the following set of premises is inconsistent:

If the contract is valid, then John is liable for penalty. If John is liable for penalty, he will go bankrupt. If the bank will loan him money, he will not go bankrupt. As a matter of fact, the contract is valid and the bank will loan him money.

71. Show that each of the following sets of premises is inconsistent:

    1. If Jack misses many classes through illness, then he fails in high school
    2. If Jack fails in high school, then he is uneducated
    3. If Jack reads a lot of books, then he is not uneducated
    4. Jack misses many classes through illness and reads a lot books.

<u>Predicate Calculus</u>

72. An open statement is a declarative sentence which contains one or more symbols, is not a if-statement produces a if-statement when each of its symbols is replaced by a specific object from a designed set. The symbols appearing in open statements are called pronouns. The set of objects which the symbols in an open statement can represent is the universe of the open statement.

73. Quantifiers

‘all true’ means the same as ‘none false’
‘all false’ means same as ‘none true’
‘not all true’ means same as ‘atleast one false’
‘not all false’ means same as ‘atleast one true’.

The following quantified statements are equivalencies:

$(\forall x)(P(x)) \leftrightarrow \neg(\exists x)(\neg P(x))$

$(\forall x)(\neg P(x)) \leftrightarrow \neg(\exists x)(P(x))$

$\neg(\forall x)(P(x)) \leftrightarrow (\exists x)(\neg P(x))$

$\neg(\forall x)(\neg P(x)) \leftrightarrow (\exists x)(P(x))$

74. Write the following sentences in the closed form:
    1. Some people who trust others are rewarded
    2. If anyone is good, then John is good
75. Write the symbolic form for the following statements:
    1. All cats have tails
    2. No cat has a tail
    3. Some cats have tails
    4. Some cats have no tails
76. Show that $(\forall x)(P(x) \wedge Q(x)) \leftrightarrow ((\forall x)(P(x)) \wedge (\forall x)Q(x))$ is a logically valid statement.
77. Show that $(\forall x)(P(x)) \vee (\forall x)(Q(x)) \leftrightarrow (\forall x)(P(x) \vee Q(x))$ is logically valid.
78. Show by counter example $(\forall x)(P(x) \vee Q(x)) \leftrightarrow (\forall x)P(x) \vee (\forall x)Q(x)$ is not valid.
79. <u>Theory of Inference for Predicate Calculus</u>
    1. Rule US(Universal Specification)
    2. Rule UG(Universal Generalization)
    3. Rule ES(Existential Specification)
    4. Rule EG(Existential Generalization)
80. Verify the validity of the following argument: Every living thing is a plant or an animal. Johns goldfish is alive and its not a plant. All animals have hearts. Therefore, John goldfish has a heart.
81. Show that from $\begin{array}{l}(i)(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall y)(M(y) \rightarrow W(y)) \\ (ii)(\exists y)(M(y) \wedge \neg W(y))\end{array}$, the conclusion $(\forall x)(F(x) \rightarrow \neg S(x))$ follows.
82. Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$
83. Using CP or otherwise obtain the following implication $(\forall x)(P(x) \rightarrow Q(x)), (\forall x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (\forall x)(R(x) \rightarrow \neg P(x))$

84. In the universe of everything, symbolize:

1. All men are giants. No men are giants.
2. Some men are giants.
3. Some men are not giants.
4. Some men are giants but not intelligent.
5. Not all intelligent man are giants.
6. Every intelligent man is not a giant.

85. Verify the validity of the following argument:

1. If two sides of a triangle are equal, then two opposite angles are equal. Two sides of a triangle are not equal. Therefore, the opposite angles are not equal.
2. If there is a meeting, then catching the bus is difficult. If they arrive in time then catching the bus is not difficult. They arrived on time. Therefore there was no meeting.
3. If an integer is divisible by 10 then it is divisible by 2. If an integer is divisible by 2 then it is divisible by 3. Therefore, an integer divisible by 10 is divisible by 3.
4. If one person is more successful than another, then he has worked harder to deserve success. John has not worked harder than peter. Therefore, John is not more successful than peter.

UNIT II - COMBINATORICS

86. Prove that $1/1.3 + 1/3.5 + 1/5.7 + ... + 1/(2n - 1)(2n + 1) = n/(2n + 1)$ for all $n \geq 1$.
87. Prove that $1/1.3 + 1/3.5 + 1/5.7 + ... + 1/(2n - 1)(2n + 1) = n/(2n + 1)$ for all $n \geq 1$.
88. For every positive integer n prove that $1 + 1/\sqrt{2} + 1/\sqrt{3} + ... + 1/\sqrt{n} > 2(\sqrt{(n+1)} - 1)$.
89. Let x be any real number greater than -1. Prove that $(1 + x)^n \geq 1 + nx$ for all $n \geq 0$.
90. If $u_1 = 1$, $u_2 = 5$ and $u_{n+1} = 5u_n - 6u_{n-1}$ for all $n \geq 2$, then prove that $u_n = 3^n - 2^n$ for all $n \geq 1$.
91. Using mathematical induction prove that $a - b$ is a factor of $a^n - b^n$ for all positive integers n.
92. A simple polygon with n sides, $n \geq 3$, can be triangulated into $(n - 2)$ triangles.
93. Using mathematical induction prove that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.
94. Show that $n < 2^n$, $n \geq 1$.
95. Using mathematical induction prove that if $n \geq 1$, then $1.1! + 2.2! + 3.3! + \cdots + n.n! = (n+1)! - 1$.
96. Use mathematical induction to prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$ for $n \geq 2$.
97. Use mathematical induction, prove that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9, for $n \geq 1$.
98. In how many ways can 8 papers in an examination be arranged so that the two mathematics papers are not consecutive?
99. Find the number of permutations of the letters of the word MATHEMATICS. Also find the number of arrangements beginning and ending with the same letter.

100. The password for a computer system consists of eight distinct alphabetic characters. Find the number of passwords possible that

1. end in the string MATH
2. begin with the string CREAM contain the word COMPUTER as a substring.

101. In how many of the permutations of 10 things taken 4 at a time will

1. one thing always occur.
2. never occur.

102. Suppose there are 6 boys and 5 girls.

1. In how many ways can they sit in a row.
2. In how many ways can they sit in row if the boys and girls sit together.
3. In how many ways they can sit in a row if the girls are to sit together and the boys do not sit together.
4. How many seating arrangements are there with no two girls sitting together.

103. How many bit strings of length 10 contain

1. exactly 4 1's.
2. atmost 4 1's.
3. atleast 4 1's.
4. an equal number of 0's and 1's

104. A Survey of 100 students with respect to their choice of the ice cream flavours vanilla, chocolate and strawberry shows that 50 students like vanilla, 43 like chocolate, 28 like strawberry, 13 like vanilla and chocolate, 11 like chocolate and strawberry, 12 like strawberry and vanilla, and 5 like all of them. Find the number of students who like

1. vanilla only
2. chocolate only
3. strawberry only
4. chocolate but not strawberry
5. chocolate and strawberry but not vanilla
6. vanilla or chocolate but not strawberry Also find the number of students who do not like any of these flavours.

105. In a survey of 120 passengers, an airline found that 52 enjoyed wine with their meals, 75 enjoyed mixed drinks and 62 enjoyed iced tea. 35 enjoyed any given pair of these

beverages and 20 passengers enjoyed all of them. Find the number of passengers who enjoyed

1. only tea
2. only one of the three
3. exactly two of the 3 beverages none of the drinks

106. A sample of 80 people have revealed that 30 like to see films in theatres and 57 like to see TV programmes at home. Find the number of people who like both.

107. Cardinality

   1. Inclusion-exclusion principle
   2. $|A \cup B| = |A| + |B| - |A \cap B|$
   3. Addition Principle
   4. $|A \cup B| = |A| + |B|$ where $A \cap B = \phi$
   5. A set with n elements has 2^n subsets

108. The number of derangements of n objects is

$$D_n = n![1 - 1/1! + 1/2! - 1/3! + 1/4! - ... + (-1)^n(1/n!)]$$

109. Explain the Pigeonhole principle.

110. In a group of 6 people, show that there are 3 mutual friends or 3 mutual foes in the group.

111. Find the minimum number of students needed to make sure that *5* of them take the same engineering course *ECE*, *CSE*, *EEE* and *MECH*.

112. Show that among (n+1) positive integers not exceeding 2n, there must be an integer that divides one of the other integers.

113. Solve the recurrence relation $a_n = a_{n-1} + (n-1)$, $n \geq 1$ and $a_1 = 0$.

114. Solve the recurrence relation for the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, ...

115. Solve the recurrence relation $a_n = 2\,(a_{n-1} - a_{n-2})$, $n \geq 2$ and $a_0 = 1$, $a_1 = 2$.

116. Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} + 4(n+1)3^n$, where $a_0 = 2$, $a_1 = 3$.

117. Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3.2^n + 7.3^n$ where $n \geq 0$ and $a_0 = 1$, $a_1 = 4$.

118. Solve $a_n - 3a_{n-1} = 2$ for all $n \geq 1$, $a_o = 2$, using generating function method.

119. Solve $a_{n+2} - 2a_{n+1} + a_n = 2^n$, $a_0 = 2$, $a_1 = 1$ by generating function method.

120. Solve the recurrence relation $a_n - 2a_{n-1} = 2^n$, $a_0 = 2$.

121. Define recursively the factorial function .

122. Let a_n denote the number of subsets of the set $S=\{1,2,...,n\}$ that do not contain consecutive integers, where $n \geq 0$. When $n=0$, $S=\phi$, find an explicit formula for a_n.

123. Using generating functions, solve the Fibonacci recurrence relation $F_n=F_{n-1}+F_{n-2,}$ where $F_1=1=F_{2.}$

124. The number of r-permutations of n distinct elements satisfies the recurrence relation $P(n,r)=P(n-1,r)+rP(n-1,r-1)$, where $0<r<n$.

125. Use method of generating function to solve the recurrence relation $a_n = 3a_{n-1} + 1$, $n \geq 1$ given that $a_0 = 1$.

126. Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} + 3n\ 2^n$, $n \geq 1$ given that $a_0 = 4$.

127. Find the sequence having the expression $\frac{3-5x}{1-2x-3x^2}$ as generating function.

128. Find the sequence whose generating function is $\frac{6-29x}{1-11x+30x^2}$ using partial fraction.

UNIT III - GRAPH THEORY

129. Explain the Konigsberg problem.
130. Let e denote the number of edges of a graph G with n vertices $v_1, v_2, .., v_n$.
     Then prove that $\sum_{i=1}^{n} \deg(v_i) = 2e$.
131. The number of odd degree vertices in a graph is an even integer.
132. Let G be a graph with n vertices and e edges. Let M and m denote the maximum and minimum of the degrees of vertices in G, respectively. Prove that $m \leq 2e/n \leq M$.
133. The length of a simple path between any two distinct vertices of a connected graph with n vertices is at most n-1.
134. Let A be the adjacency matrix of a connected graph with n vertices $v_1, v_2, ..., v_n$ and k a positive integer less than or equal to n -1. The ij^{th} entry of the matrix A^k gives the number of paths of length k from v_i to v_j.
135. A connected graph G is Eulerian if and only if every vertex of G has even degree.
136. A connected graph contains an Eulerian path, but not an Eulerian circuit, if and only if it has exactly two vertices of odd degree.
137. Give an example of a graph which is
     1. Eulerian but not Hamiltonian.
     2. Hamiltonian but not Eulerian.
     3. both Eulerian and Hamiltonian.
     4. non Eulerian and non Hamiltonian.
138. For any simple bipartite graph G with e edges and v vertices, prove that $e \leq v^2/4$.

139. Prove that any self complementary graph has $4n$ or $4n+1$vertices.

140. Show that the graphs with the following adjacency matrices are isomorphic.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

141. If G is a simple graph with n vertices and k-components, then the number of edges is at most $(n-k)\,(n-k+1)\,/\,2$.

142. If G is a graph with n vertices such that the minimum degree of a vertex is $(n-1)/2$, prove that G is connected.

143. Show that a simple graph G is bipartite iff it has no circuit (or cycle) of odd length.

144. A connected graph G is Eulerian if and only if every vertex of G is of even degree.

145. If a graph G has m edges and n vertices and if $m \geq (1/2)(n^2-3n+6)$. Prove that G has a Hamilton circuit.

146. Does there exist a graph with 13 vertices each of degree 3?

147. A regular graph G has 10 edges and degree of any v is 5, find the number of vertices.

148. Whether the following graphs are isomorphic or not.

149. Test the isomorphism of the graphs by considering their adjacency matrices.

UNIT IV - ALGEBRAIC STRUCTURES

150. Define Algebraic Structure.
151. Define Semi group and monoid. Give examples.
152. Show that the inverse of an element if it exists in a Monoid (M, *,e) is unique.
153. Define a Cyclic Monoid.
154. Let R be the set of real numbers and * be the operation defined by a * b = a + b + 3ab. Show that (R, *) is a semi group
    1. Is it a monoid ?
    2. Specify the identity elements
    3. which elements have inverse and what are they?
155. Define semi group homomorphism, semi group isomorphism. Give examples.
156. Define monoid homomorphism, monoid isomorphism. Give examples.
157. State and prove the properties of homomorphism.
158. Define sub-semi group and sub-monoid.
159. For a commutative monoid (M, *), show that the set of all idempotent elements form a sub-monoid.
160. Define a Group. Examples.
161. Explain the additive group Z_5 and draw a Cayley table.
162. Define abelian group. Give examples.
163. State and prove reversal laws.
164. State and prove Cancellation laws for groups.
165. Show that in a group G
    1. the equation a * x = b has the unique solution $x = a^{-1} * b$.
    2. the equation y * a = b has the unique solution $y = b * a^{-1}$
166. Show that $(Z_n, +_n)$ is an abelian group.
167. It every element in a group is its own inverse, prove that G is abelian. Is the converse is true?
168. If *G* is abelian group, then for all a, b $\in$ G, show that $(a * b)^n = a^n * b^n$ for every integer *n*.
169. If $(a * b)^2 = a^2 * b^2$ for all $a, b \in G$, show that *G* is abelian.
170. Define order of a group, order of an element and give examples.
171. Show that a and a^{-1} have the same order.
172. Define sub-group. Give examples.

173. Show that a subset H of a group G is a subgroup iff $ab^{-1} \in H$, for all $a,b \in H$.
174. Show that in a finite group G, a non-empty subset H is a sub-group if and only if $ab \in H$, for all $a, b \in H$.
175. Show that intersection of two subgroup is a subgroup.
176. Let G be a group and $a \in G$. Show that the subset $\{a\, x\, a^{-1} / x \in G\}$ is a sub group of G.
177. Let G be a non-abelian group. If $H = \{x \in G / x\, y = y\, x\ \forall\ x, y \in G\}$. Then, show that H is a subgroup of G (H is called centre of G).
178. Show that if H and K are two subgroups of G, then HK is a subgroup of G if and only if $HK=KH$.
179. Define Cyclic Group. Give examples.
180. Show that every Cyclic Group is abelian.
181. Show that every subgroup of a Cyclic group is Cyclic.
182. Show that every group of order 4 is abelian.
183. Define left coset and right coset. Give example.
184. Show that the set of all left cosets of H in G form a partition of G.
185. Show that there is a bijection between any two left cosets of H in G.
186. State and Prove Lagrange's Theorem.
187. If group G is of prime order what are its only possible subgroups? Explain.
188. If G is a finite group with $o(G)=n$. Show that $o(a) \mid o(G)$.
189. Show that every group of prime order is Cyclic (and hence abelian).
190. Let H be a subgroup of G. Define '~' G as $a, b \in G$, $a \sim b$ if and only if $ab^{-1} \in H$, show that '~' is an equivalence relation.
191. Define Normal subgroups. Give examples.
192. For a subgroup H, show that the following statements are equivalent
    1. $aH = Ha, \forall\ a \in G$
    2. $a^{-1} Ha = H, \forall\ a \in G$
    3. $a^{-1} Ha \subseteq H, \forall\ a \in G$
193. Show that the intersection of two Normal subgroups is a Normal subgroup.
194. If H is a subgroup of G and N is a Normal subgroup of G. Show that HN is a subgroup of G. If both H and N are Normal subgroups, then HN is also Normal.
195. Define Quotient group.
196. Define group homomorphism, group isomorphism. Give examples.
197. Show that a group homomorphism preserves subgroups.
198. Define Kernel of a homomorphism $f : G_1 \rightarrow G_2$. Show that Kernel is Normal subgroup of G_1.
199. Let $f : G_1 \rightarrow G_2$ be a group homomorphism. Show that f is one-one if and only if Ker $f = \{e\}$.
200. State and prove fundamental theorem of homomorphism.
201. Define Symmetric group. Draw the Cayley table for S_3.
202. Define Cycle, transposition.

203. Show that every permutation $\sigma \in S_n$ can be written as a product of a finite number of transpositions.
204. Define alternating group of on n symbols.
205. Show that the set of all even permutation is a group S_n.
206. State and Prove Cayley representation theorem.
207. Define direct product of groups, show that the direct product of two groups is a group.
208. Define Ring, Field. Give examples.

UNIT V - LATTICES AND BOOLEAN ALGEBRA

209. Define the following Set
    1. Subset
    2. Cardinality of a Set
    3. Union of Sets
    4. intersection of sets
    5. Symmetric difference of sets
    6. Power set
    7. finite Set
    8. infinite set
    9. complement of a Set
    10. Cartesian Product
210. Show that (a) A×B≠B×A by an example
    1. $A \subset B$ implies that $A \cup (B-A)=B$
    2. For Sets S, T and V, $(S \cap T) \times V=(S\times V) \cap (T\times V)$
211. $P(A) \cup P(B) \subseteq P(AUB)$, where $P(X)$ denotes the power set of X
212. State and Prove
    1. Distributive law
    2. Demorgan law
    3. Associative law
    4. Idempotent law
213. Show That for a set A of n elements $|P(A)|=2^n$.
214. Define binary relation and represent it graphically and in matrix notation.
215. Define the following properties of a relation on A
    1. Reflexive
    2. Symmetric
    3. Transitive
    4. Ir-reflexive
    5. Anti-symmetric
    6. Equivalence relation

7. Partial Ordering

216. Give examples

217. Let Z be the set of integers, define the relation R on Z by $R=\{(x, y)\in Z\times Z/x\text{-}y$ is divided by then show that R is an equivalence relation

218. Let Z^+ be the set of positive integers. Then define a relation R_1 on Z^+ by $R_1=\{(a, b) / a$ divides $b\}$ then show that R_1 is an partial ordering relation on Z^+.

219. Let R be a symmetric and transitive relation on a set A. Show that if for every $a\in A$ there exist $b\in A$ such that $(a, b)\in R$, then R is an equivalence relation

220. Let R be a transitive and reflexive relation on A. Let T be a relation on A such that $(a, b)\in T$ if both $(\underline{a}, b)$ and (b, a) are in R. Show that T is an equivalence relation.

221. Let R be a reflexive relation on a set A. Show that R is an equivalence relation if and only if $(a, b), (a, c)\in R \Rightarrow (b, c)\in R$

222. Define
    1. partition
    2. equivalence class

223. Let R be an equivalence relation on a set X. Then prove that
    1. For any $x\in X$, $x\in [x]$
    2. If $y\in [x]$ then $[x]=[y]$
    3. If $(x, y) \notin R$ then $[x] \cap [y]=\emptyset$.

224. Let X be any set. Let $A_1, A_2, \ldots A_n$ are sets such that $X=\bigcup_{i=1}^{n} A_i$ and $A_i \cap A_j=\emptyset$, $i\neq j$.

225. If relation R on X is given by $R= \{(x, y)\in (X\times X)/ x, y\in A_i$, for some $i\}$, then R is an equivalence relation on X.

226. Let X be any set. Prove that if a relation R on X is an equivalence relation then it induces unique partition on X. Conversely given a partition on X it induces an equivalence relation on X.

227. Give the partition on Z generated by the relation R is defined by 'congruence modulo3'

228. Define a relation R on Z by $R = \{(a, b) \in Z\times Z/ a^2=b^2\}$. Show that R is an equivalence relation. Also show the partition on Z induced by R.

229. Let A = {1, 2 , 3, 4, 5, 6, 7}; B={x, y, z}. Let F:A→B be a function defined by f(1)=x, f(2)=z, f(3)= x, f(4)=y, f(5)=x, f(6)=y, f(7)=x. Define a relation R on A by R=[(a, b) ∈ A×A/ f(a)= f(b)}. Prove that R is an equivalence relation on A.

230. Define

231. Prove that if the POSET $(A, \leq)$, has a greatest (least) element then that element is unique.

232. Let R be a transitive relation on A. Prove that R is a partial ordering on A if and only if $R\cap R^{-1}=\{(a,a)/ \ a\in A\}$ where $R^{-1}= \{(a, b) / (b, a) \in R\}$.

233. Define a relation R on Z by $R = \{(a, b) / a\text{-}b\}$is a non-negative even integers. Verify R is a partial ordering on Z.

234. Define lattice. Give example.
235. Define Hasse Diagram. Explain with an example.
236. In any lattice, $(L,\leq)$ prove that
    1. Idempotent
    2. Commutative
    3. Associative
    4. Absorption laws are satisfied.
237. Let $(L,\leq)$ be a lattice prove that for $a,b\in L$
    1. $a\leq b$
    2. $a*b=a$
    3. $a\oplus b=b$ are equivalent.

238. State and prove law of isotonicity on lattices.
239. State and prove distributive and modular inequality on lattices
240. Prove that every lattice $(L,\leq)$ is an algebraic system. Also prove the converse.
241. Define with example
    1. sublattice
    2. lattice homomorphism
    3. Complete lattice
    4. bounded lattice
    5. Complemented lattice
    6. distributive lattice
    7. Modular lattice.
242. Prove that every chain is a distributive lattice.
243. Show that the direct product of two lattices is a lattice.
244. Show that is a distributive lattice L, $a*b=a*c$ and $a\oplus b=a\oplus c\Rightarrow b=c$, for $a,b,c\in L$.
245. Show that in a lattice of $a\leq b$ and $c\leq d$ then $a*c\leq b*d$, for $a,b,c\in L$.
246. Prove that in a distributive lattice $(L,\leq)$ every element has an unique complement.
247. Show that every distributive lattice is modular.
248. Let L be a complemented distributive lattice for $a,b,c\in L$, Show that the following are
equivalent
    1. $a\leq b$
    2. $a\oplus b'=0$
    3. $a'*b=1$
    4. $b'\leq a'$
249. Prove that any chain is a modular lattice.
250. Give example

1. Not modular
2. Not modular but distributive
3. Not distributive

251. Show that every finite partially ordered set has a maximal and minimal element.
252. Show that if a lattice is distributive $(a*b)\oplus(b*c)\oplus(c*a)=(a\oplus b)*(b\oplus c)*(c\oplus a)$.
253. Show that every finite subset of a lattice has a glb and lub.
254. Show that every lattice is modular if none of its sublattice is isomorphic to the pentagon lattice
255. Show that every lattice is distributive if none of its sublattices is isomorphic to pentagan lattice or diamond lattice.
256. Define (i) Boolean algebra(ii) Atom(iii) Sub Boolean algebra (iv) Boolean isomorphism.
257. Show that $(P(X),\leq,U,\cap,\phi\ X)$ is a Boolean algebra.
258. State and prove Stone's Representation theorem.
259. Show that the cardinality of any Boolean algebra is 2^n for $n\in N$.
260. Show that $P(X)$ is isomorphic to B^n, where $X=\{x_1,x_2,......x_n\}$, B = {0, 1}.
