Signals and Systems

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Signal:

- An observed data representing a physical phenomenon is called a signal.

- A function of one or more independent variables which contain some information.

- Any physical quantity that varies with time is called signal.

- The signal is a continuously varying phenomenon which represented as a function of time.

  signal = function $x(t)$, $t \in R$

System: A set of elements or functional blocks that are connected together and produces an output in response to an input signal.

Examples:

Electrical signal – radio signal, TV signal, telephone signal, Computer signal

Acoustic Signal – Audio or Speech signal

Electrical system – filters, communication channels, amplifiers, attenuators etc.

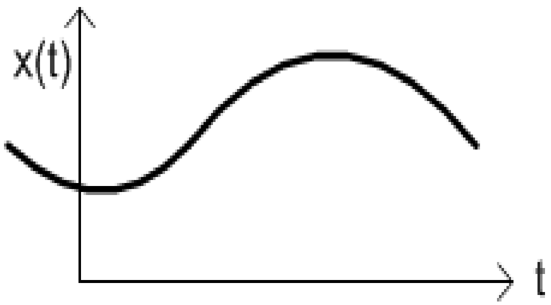
CLASSIFICATION OF SIGNALS

The signal are classified as

1. Continuous and Discrete time
2. Analog and Digital
3. Even and Odd
4. Periodic and Aperiodic
5. Deterministic and Random
6. Energy and power

1. Continuous time and discrete time signals

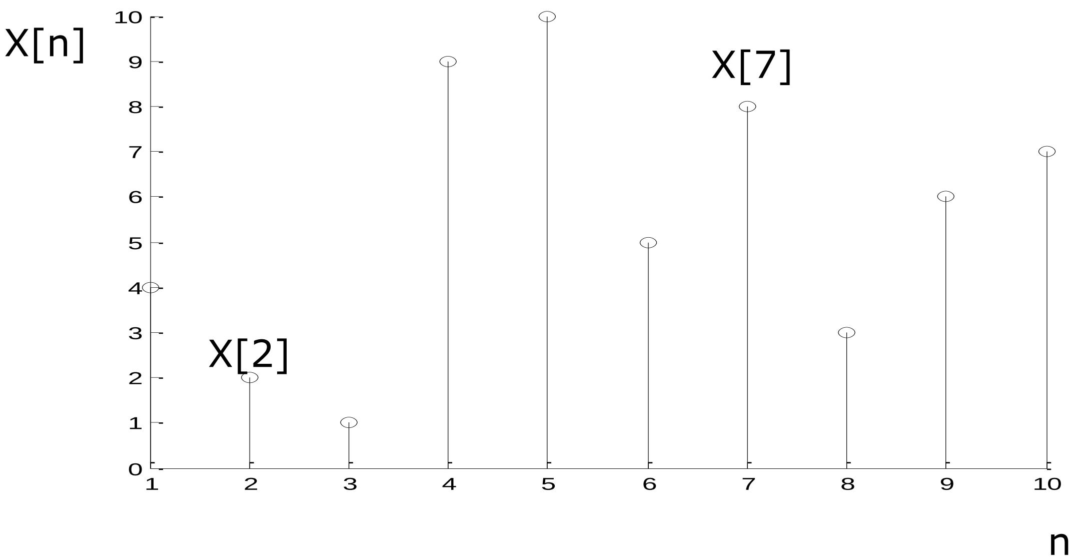
- A signal x(t) is said to be a continuous time signal if it is defined for all time t.
- Continuous time signal arises when a physical waveform is converted to electrical signal
- Example – Voltage, current, pressure, temperature, velocity etc.



- Discrete time signals – It is defined only at discrete instants of time.

- A discrete time signal is derived from a continuous time signal by sampling it at a uniform rate.

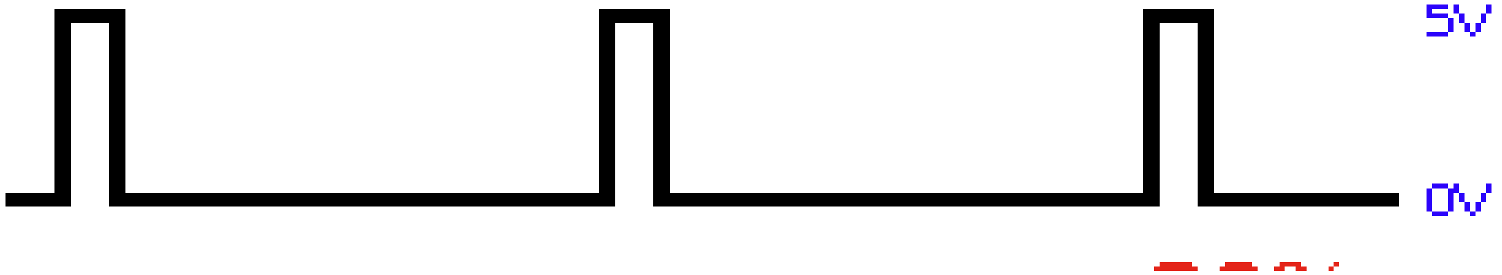
- A DT signal is mathematically represented as x[n], where n is time index, which always an integer (i.e.) n = 0,1,2,3,4,…….



2. Analog and Digital Signal

Analog Signal: The amplitude of the signal is defined for all the value of time. eg. All continuous time signals.

Digital Signal: The amplitude and time are quantized i.e., it takes only fixed values.

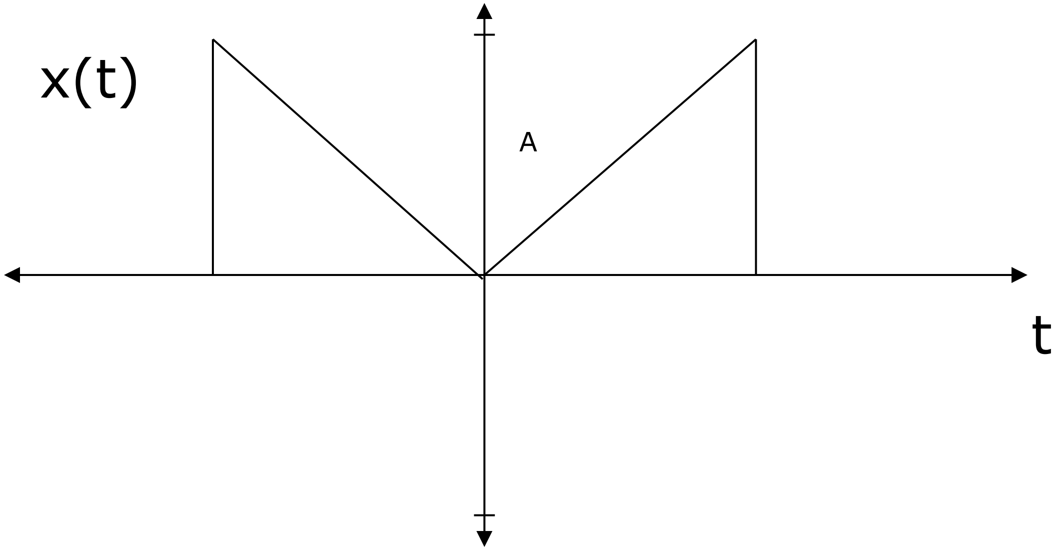


But for discrete time signal only time is quantized.

3. Even and Odd Signal

Even Signal: A CT signal x(t) is said to be an even signal if it satisfies the condition.

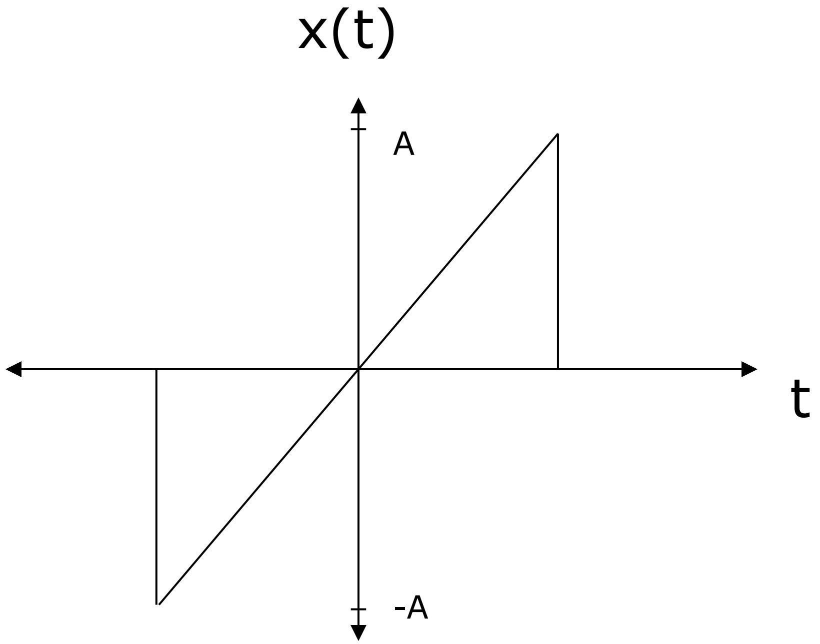
$$x(-t) = x(t) \text{ for all } t$$



Even signals are symmetric about time origin

Odd Signal: The signal x(t) is said to be an odd signal if it satifies the condition

$$x(-t) = -x(t) \text{ for all } t$$



Odd signals are anti-symmetric about time origin

- Any CT signal x(t) is expressed as

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(t)$ is the Even component and $x_o(t)$ is the Odd component of the signal x(t).

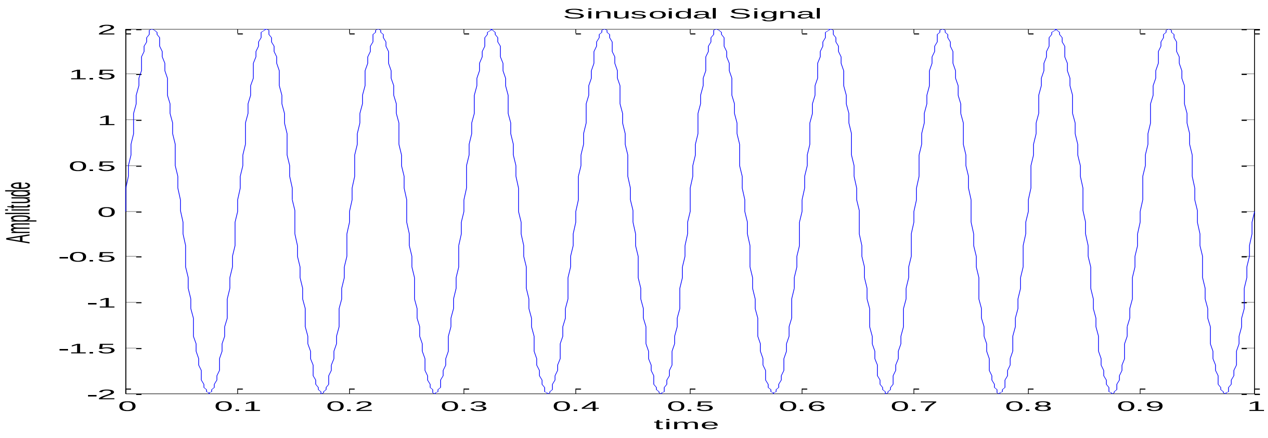
$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$

4. Periodic and nonperiodic signals

- A periodic signal x(t) is a function that satisfies the condition x(t) = x(t +T) for all t.

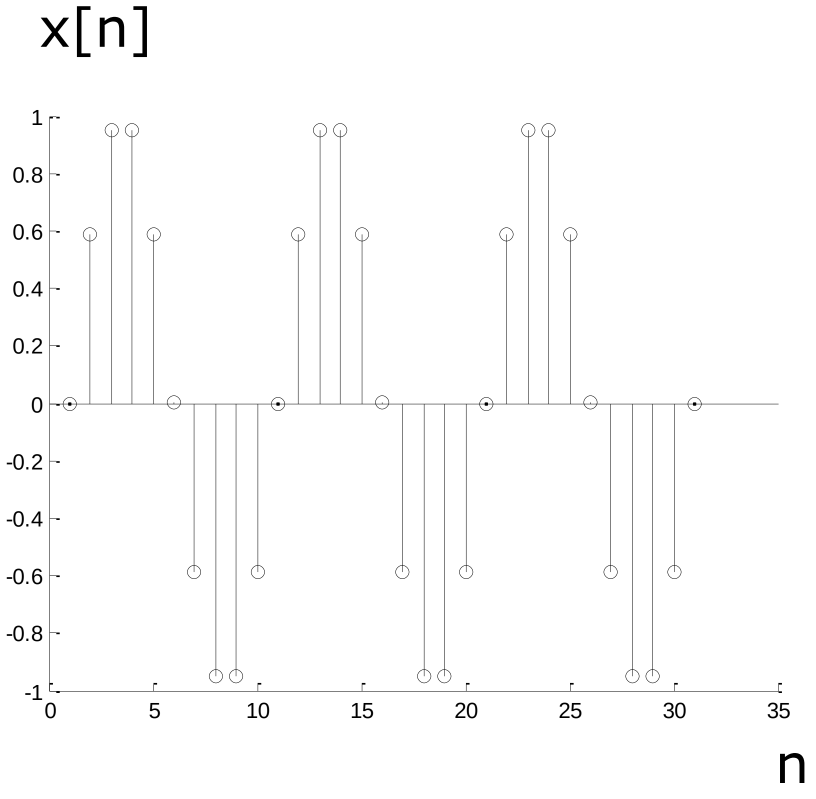
eg. Sinusoids, Square wave, sawtooth and etc.,



- The smallest value of T that satisfies the above condition is called fundamental period
- Fundamental frequency, f= 1/T
- Any signal for which there is no value of T to satisfy the condition is called aperiodic signal.

Discrete Periodic Signal:

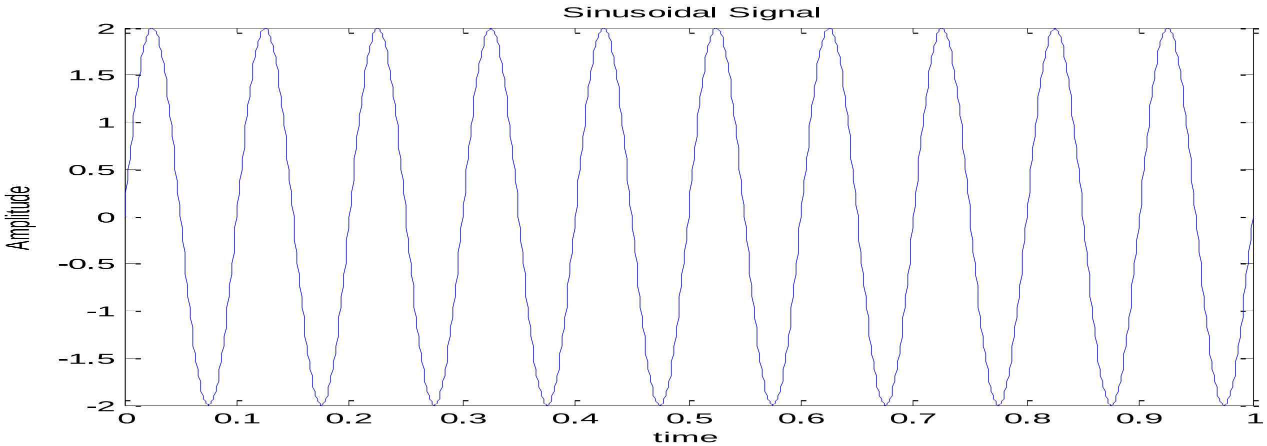
- A signal is discrete periodic if it satisfies the relation x[n] = x[n+N], where N is the period of the signal.



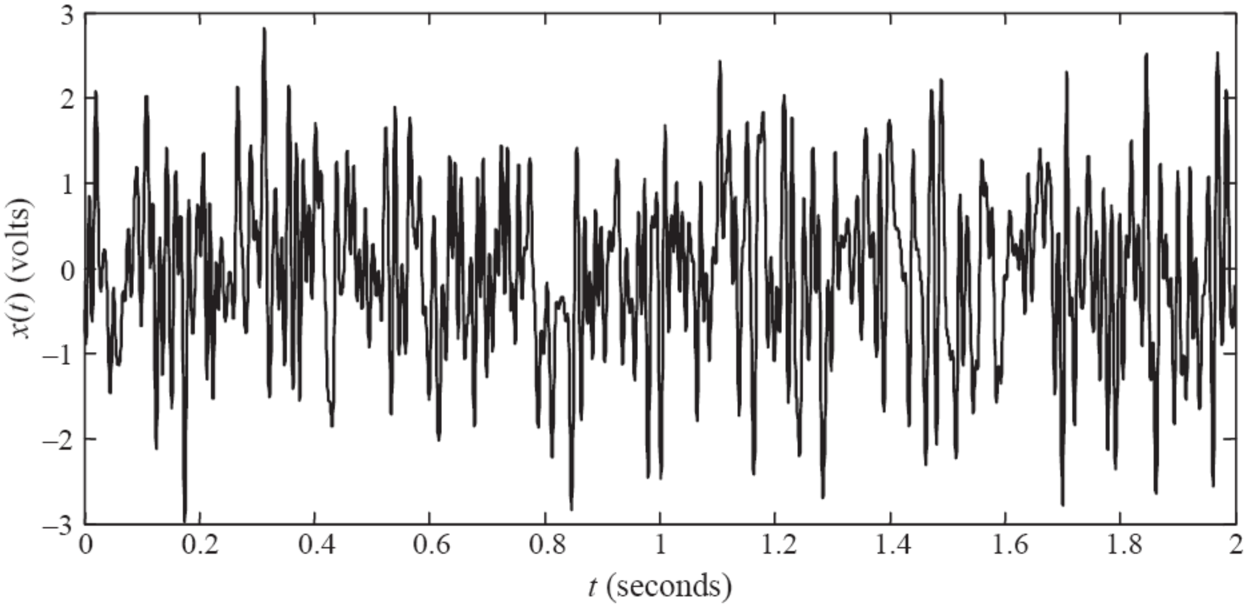
5. Deterministic and random signals:

- A deterministic signal is a signal about which there is no uncertainty with respect to its value at any time.

- Example of deterministic Signal



- Random signal is a signal about which there is uncertainty before its actual occurrence.



Eg. Noise generated in an amplifier

6. Energy signal and Power signal

- In electrical system, a signal can be represented as a voltage or current

- Consider a voltage $v(t)$ developed across R producing a current $i(t)$.

- The instantaneous power dissipated in the resistor is given by $p(t) = v^2(t)/R$

  or $p(t)= i^2(t)/R$
- For R=1 ohm , whether $x(t)$ is voltage or current , $p(t)=x^2(t)$.

- The total energy of a continuous time signal x(t) as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \mathrm{dt}$$

- For Discrete time signal x[n]

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- The average power is given by

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

- The average power of a periodic signal is given by

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

- The Average Power of a discrete time signal is

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

- For a Periodic Discrete time signal,

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

- The signal is referred to as an energy signal if and only if

$$0 < E < \infty$$

- The signal is referred to as an power signal if and only if

$$0 < P < \infty$$

- In particular an energy signal has zero average power whereas power signal has infinite energy.

Test your understanding

1. Consider the pair of signal shown . Which one of these two signals is even and which one is odd?

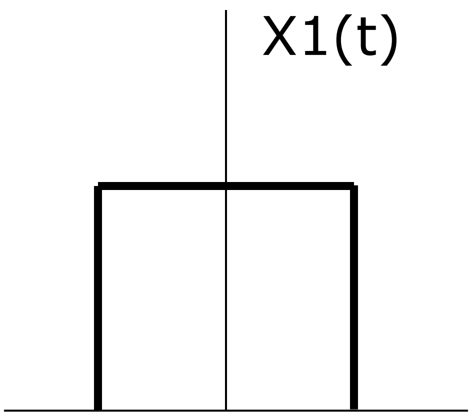


Fig a

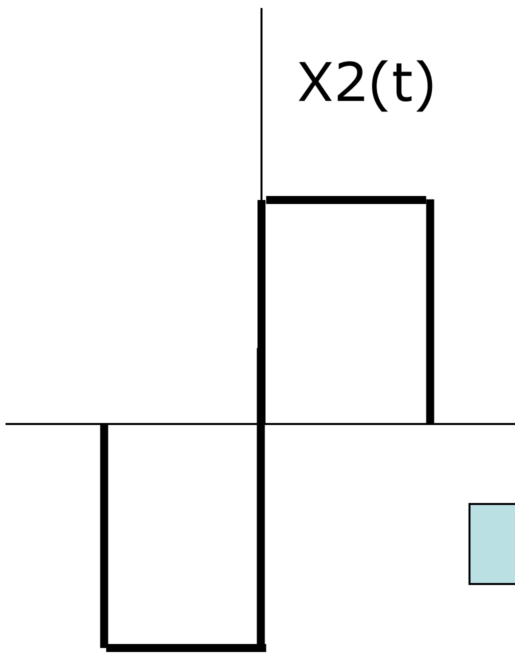


Fig b

Test your understanding

Ans : Fig a – even ,Fig b – odd

2. What is the fundamental frequency of the wave shown below?

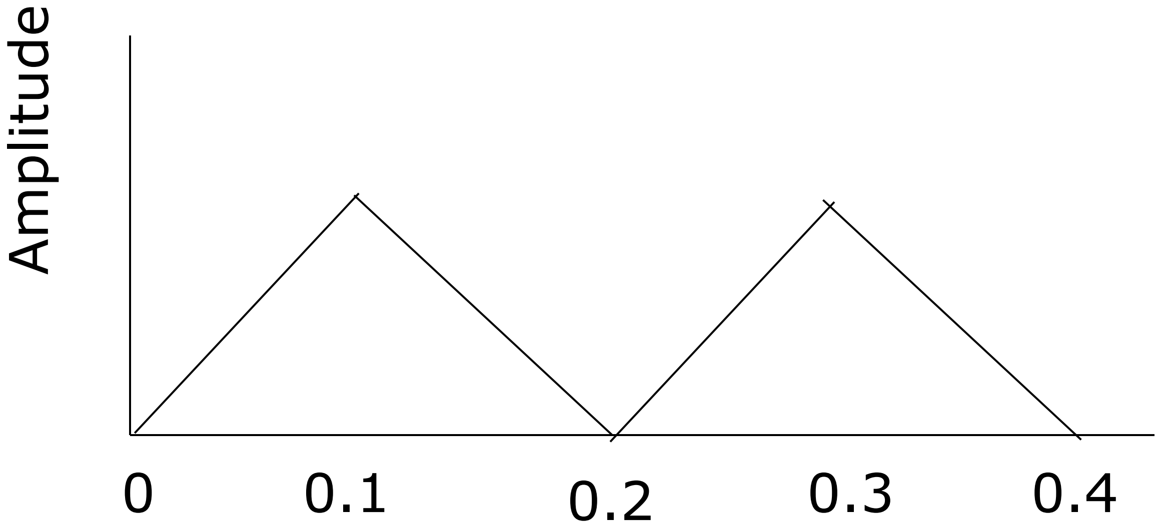


Fig : c

Test your understanding

2. Ans : Fundamental Period = 0.2

3. What is the average power of a triangular wave shown in fig: c?

   Ans : 1/3

Elementary Signals

1. Impulse Signal
2. Step Signal
3. Ramp Signal
4. Exponential Signal
5. Sinusoidal Signal

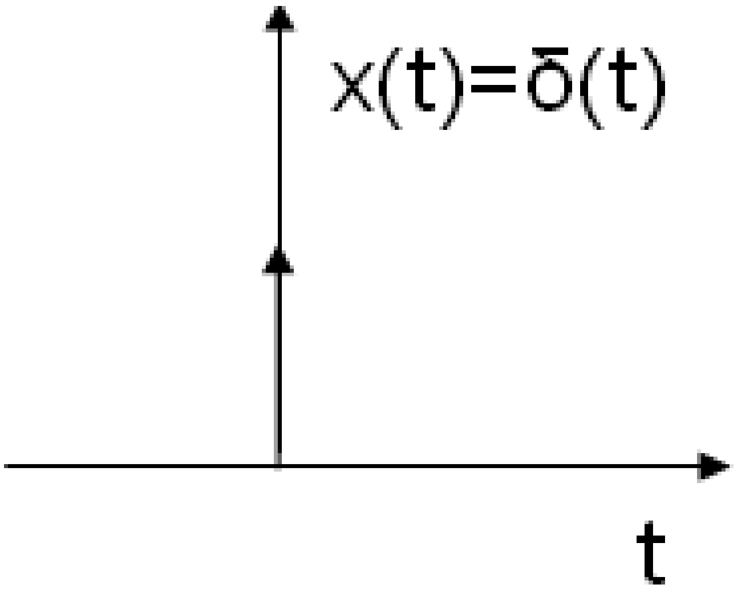
Types of signals (Continued)

Unit impulse function

$\delta(t) = 0$ for $t \neq 0$

$= 1$ for $t=0$

Area under unit impulse = 1



Types of signals (Continued)

Properties of impulse function

1. Shifting Property:

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$$

2. Replication Property:

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t)$$

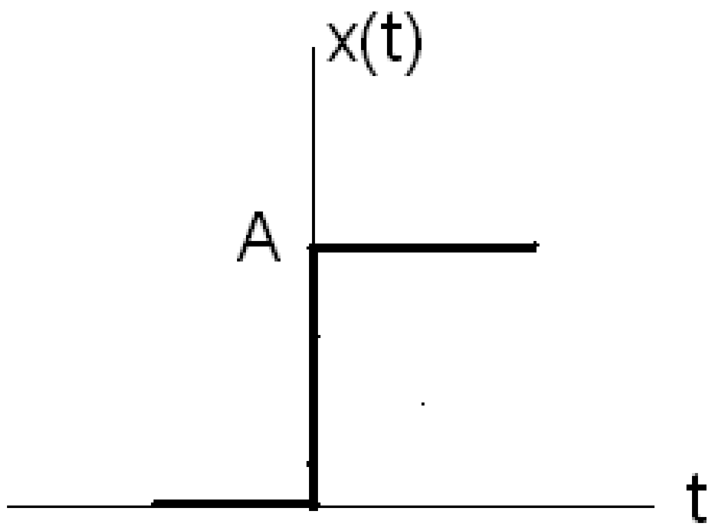
Elementary Signals (Continued)

Step Signal

It is a discontinuous function and is defined as

X(t)= 0 , t < 0

A, t >=0



Unit step function

U(t) = 0, t<0

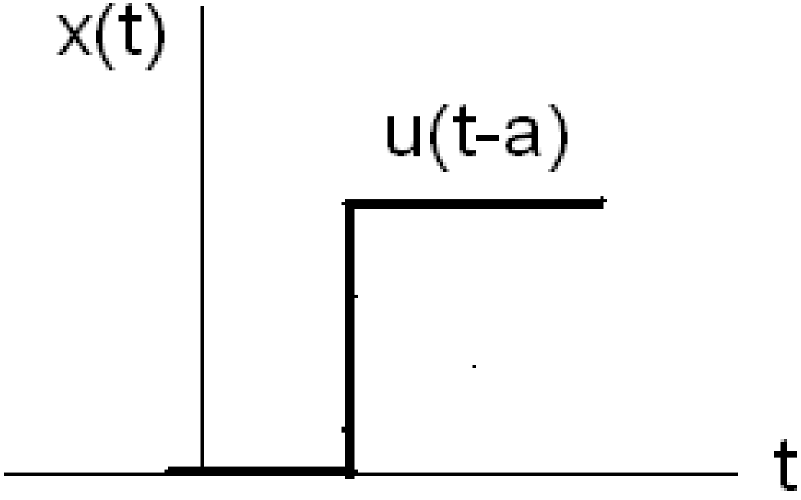
1, t>=0

Types of signals (Continued)

Time shifted Unit step function

U(t-a) =0 for t<a

1 for t>=a



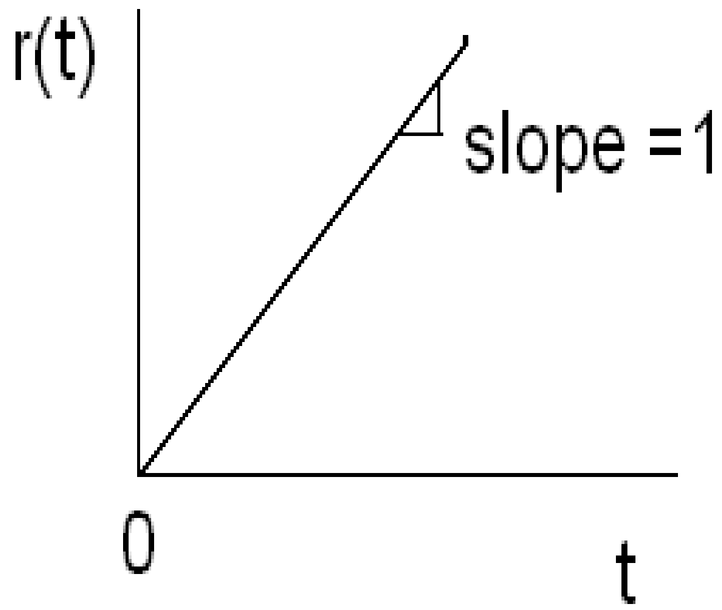
Types of signals (Continued)

Unit Ramp function

$r(t) = 0$ for $t<0$

$=t$ for $t>=0$

$r(t) = t\,u(t)$



Ramp function with any slope

$X(t) = 0$ for $t<0$

$=kr(t)$ for $t>=0$

$=kt\,u(t)$

Types of signals (Continued)

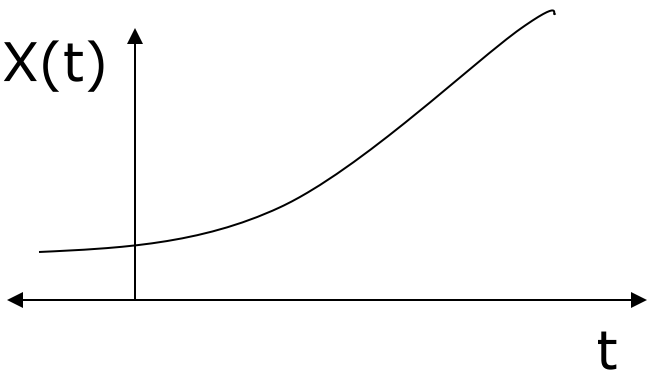
Exponential Signals:

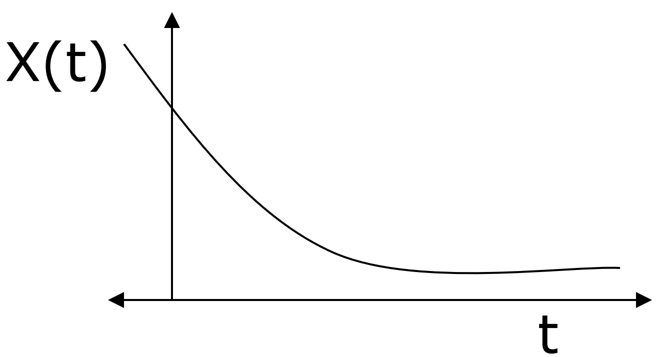
Complex exponential signals is of the form

$$x(t) = Ce^{at}$$

C,a complex numbers.

If C,a is real then x(t) is real exponential .





Relationship between step , ramp and Delta function

Unit ramp is defined as

r(t) = 0 for t<0

= t for t>=0

- Differentiating this equation on both sides with respect to t we get,

$$\frac{d}{dt}r(t) = U(t)$$

Relationship between step , ramp and Delta function

Consider a unit step function

U(t) = 0 for t<0

1 for t>=0

Integrate the above relation with respect to time we get,

$$\int u(t)dt = r(t)$$

Relationship between step , ramp and Delta function

U(t) = 0 for t<0

= 1 for t>=0

Differentiating with respect to t, we get

$$\frac{d}{dt}u(t) = \delta(t)$$

Similarly,

$$\int \delta(t)dt = u(t)$$

Relationship between step , ramp and Delta function

We Know,

$$\frac{d}{dt}r(t)=\delta(t)$$

Differentiating on both sides,

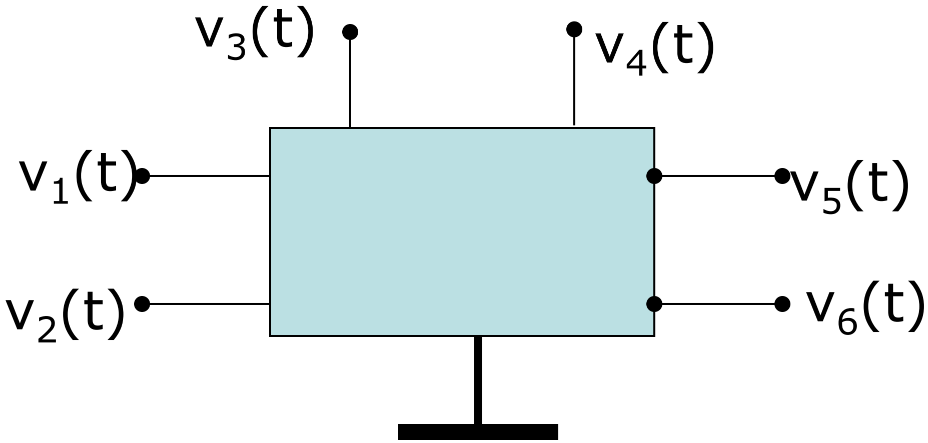
$$\frac{d}{dt}\left(\frac{d}{dt}r(t)\right)=\frac{d}{dt}u(t)$$

$$\frac{d^2}{dt^2}r(t)=\delta(t)$$

Multidimensional signal

An ordered set of one-dimensional signals

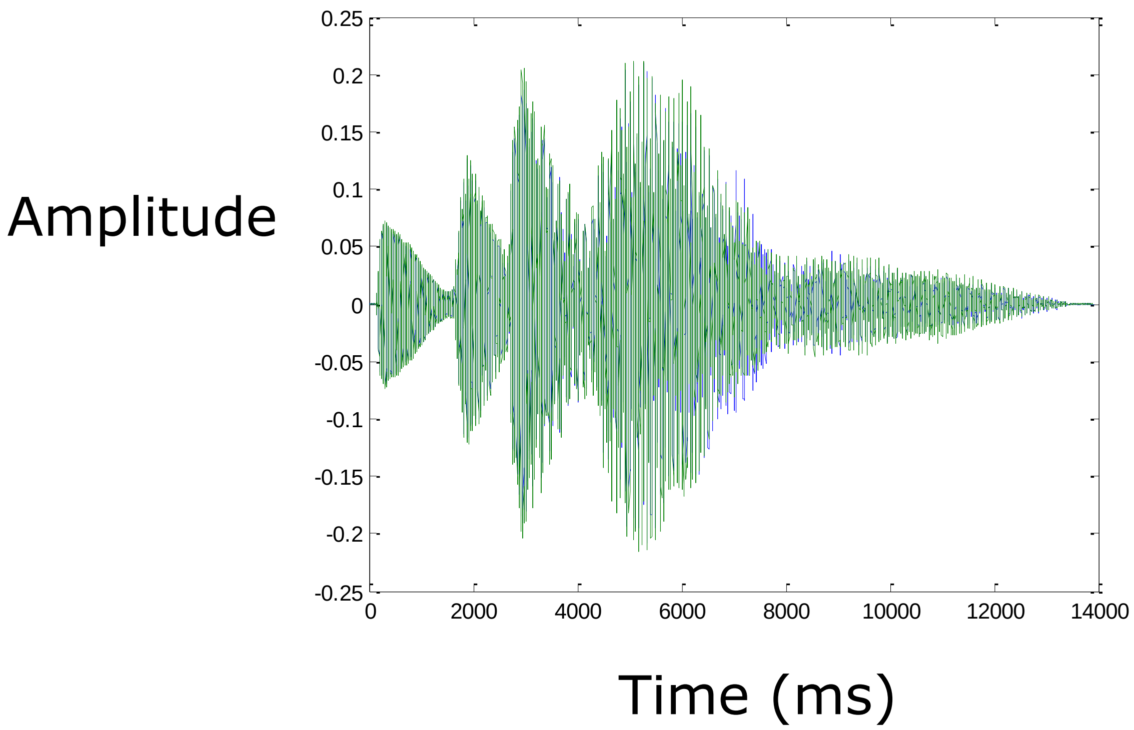
$V(t)=[v_1(t)\ ,v_2(t),\ v_3(t),\ \ldots,v_N(t)]$



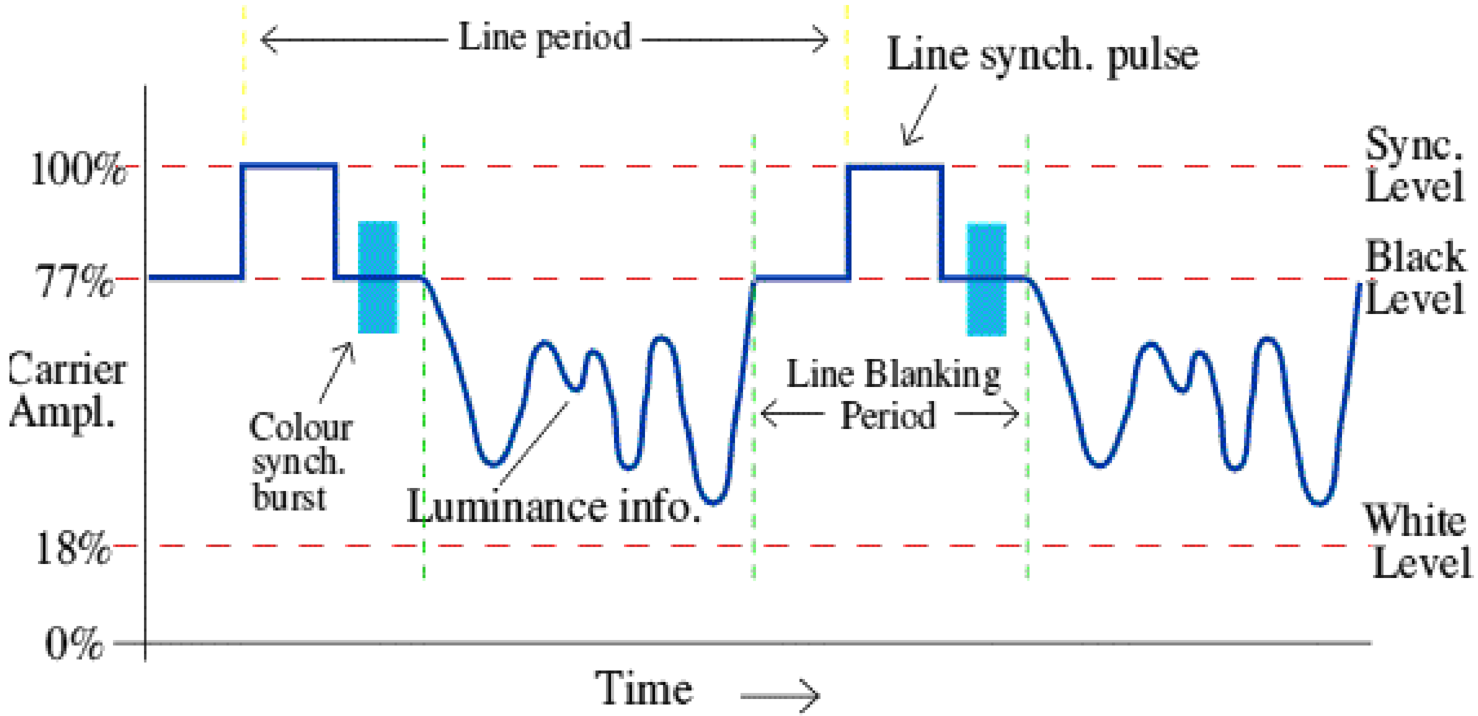
It is the set of voltages existing at the terminals of a multiport.

Examples

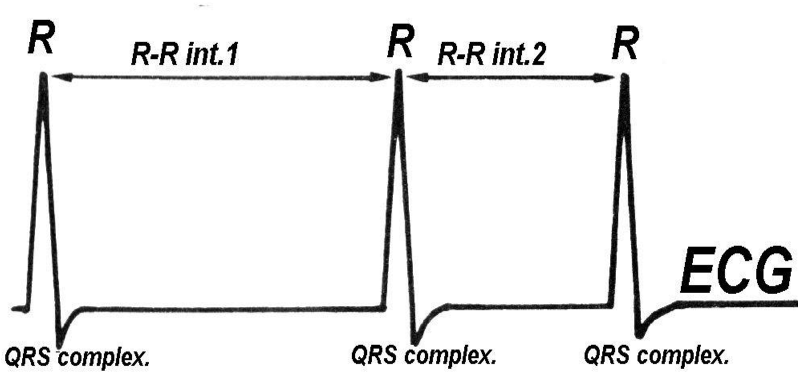
- Audio signal:



Video Signal



BIOMEDICAL SIGNALS



EEG Signal

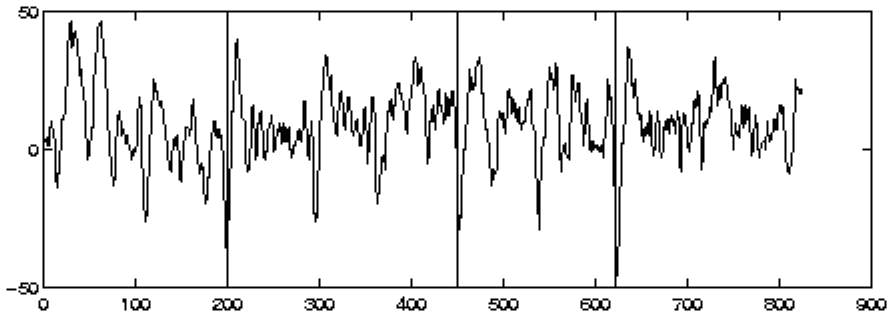
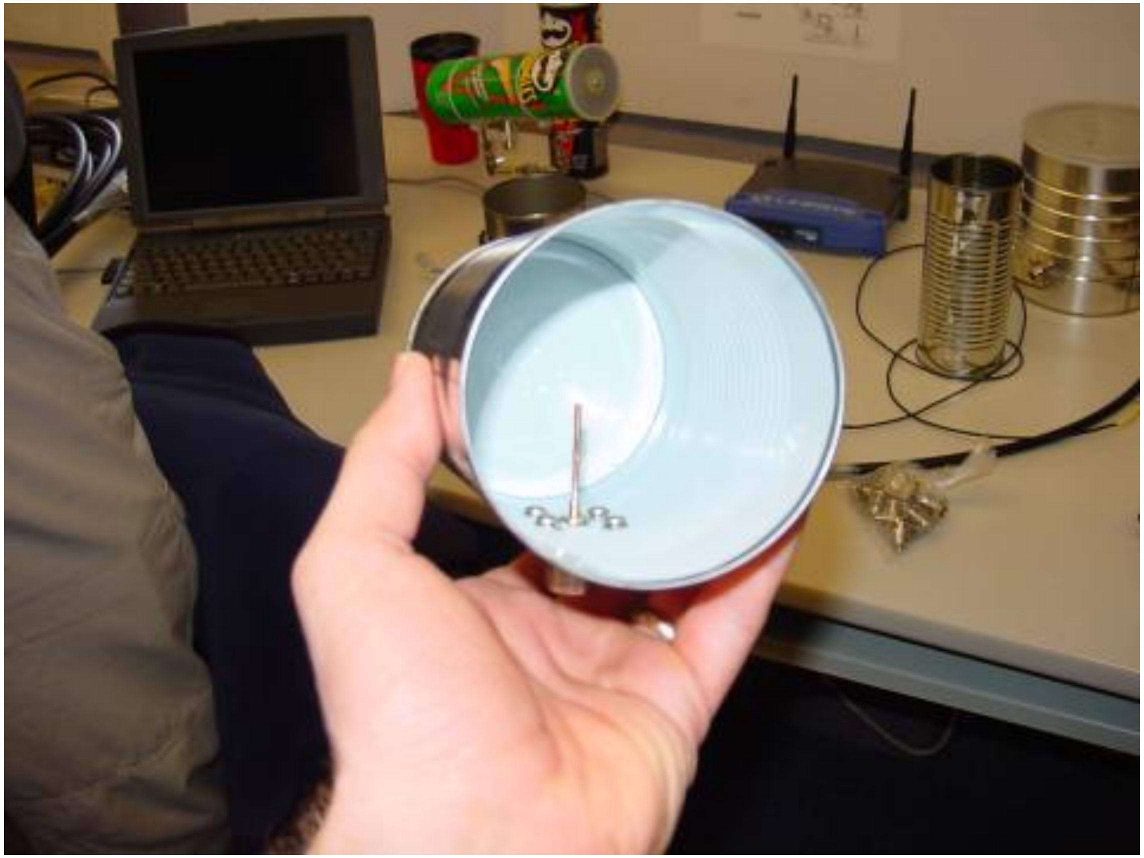


Image Signal



Discrete Time Signal - Types

(i) Unit sample Sequence

$$\delta(n)=\begin{cases}1, & n=0\\ 0, & n\neq 0\end{cases}=\left\{\cdots,0,0,\underset{\uparrow}{1},0,0,\cdots\right\}$$

(ii) Unit step sequence

$$u(n)=\begin{cases}1, & n\geq 0\\ 0, & n<0\end{cases}=\left\{\cdots,0,0,\underset{\uparrow}{1},1,1,\cdots\right\}$$

Discrete Time Signal – Types (Contd…)

(iii) Ramp Sequence

r(n) = n ; n>0

0 ; n<0

(iv) Exponential Sequence

$$x(n) = a^n, \forall n; a \in R$$

where x(n) = exp(n)