NFA WITH ε- MOVES

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AP/CSE

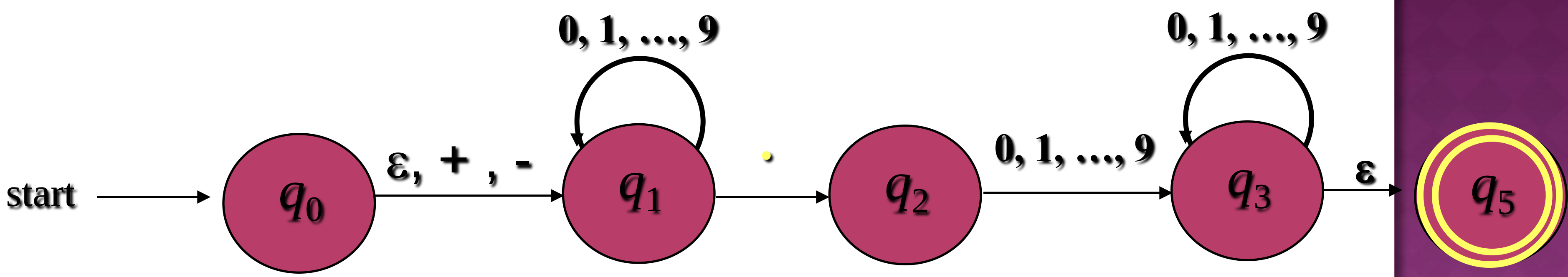
INTRODUCTION

- The NFA can be extended to include transitions on empty input ε
- The NFA with ε moves is defined by 5 tuple (Q, Σ,δ, q_0, F), with all components as in NFA except δ

  $$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow 2^Q$$

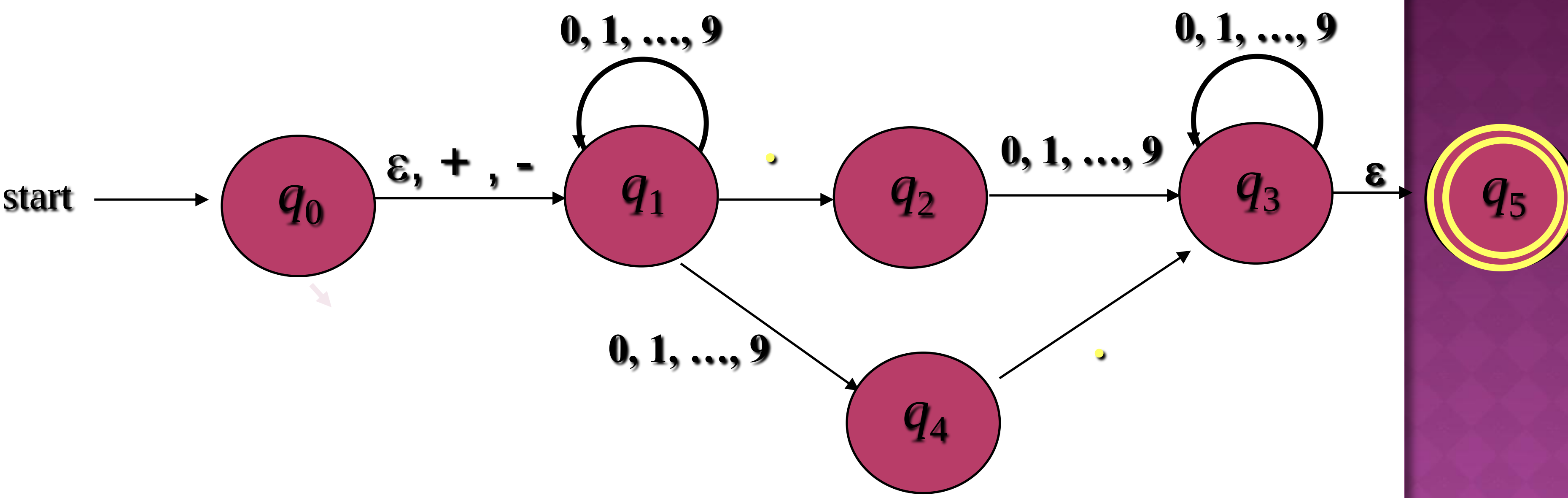
- The intention is that δ (q, a) will consists of all states p such that there is a transition labeled 'a' from q to p, where a is either ε or any symbol in Σ.

EXAMPLE



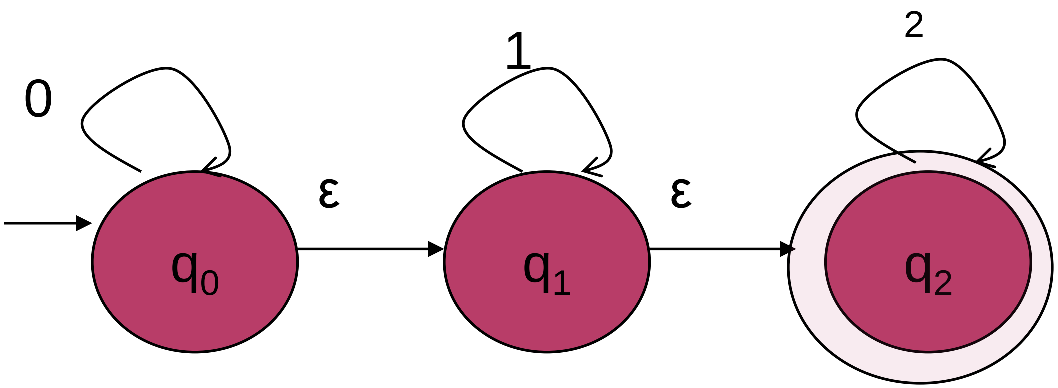
- An ε-NFA accepting decimal numbers like 2.15, .125, +1.4, -0.501…
- To accept a number like "+5." (nothing after the decimal point), add new state q_4.

EXAMPLE



- An ε-NFA accepting decimal numbers like 2.15, .125, +1.4, -0.501…
- To accept a number like "+5." (nothing after the decimal point), we have to add q_4.

EXAMPLE



- The transition diagram of the NFA accepts the language consisting of any number of 0's followed by any number of 1's followed by any number of 2's.
- For example, the string w = 002 is accepted by the NFA along the path – q_0, q_0, q_0, q_1, q_2, q_2, with arcs labeled 0, 0, ε, ε, 2.

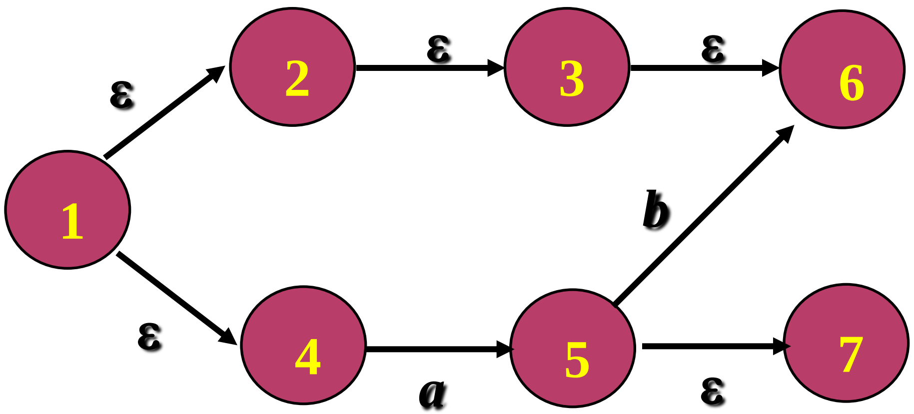
ε - CLOSURE

- We have to define the ε-closure to define the extended transition function for the ε-NFA.
- Formal recursive definition of **the set** ECLOSE(q) for q:
  - State q is in ECLOSE(q) (including the state itself);
  - If p is in ECLOSE(q), then all states accessible from p through paths of ε*'s* are also in ECLOSE(q).

EXAMPLE

- ε-closure for a set of states S:

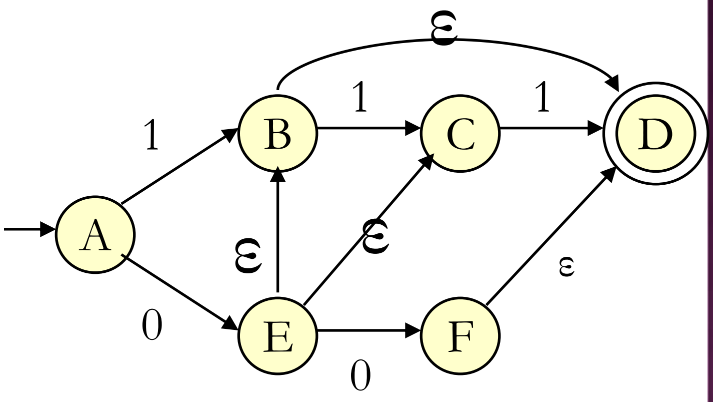
$$\text{ECLOSE}(S) = \bigcup_{q \in S} \text{ECLOSE}(q)$$



- ECLOSE(1) = {1, 2, 3, 4, 6}
- ECLOSE({3, 5}) = ECLOSE(3)∪ECLOSE(5) = {3, 6}∪{5, 7} = {3, 5, 6, 7}

EXAMPLE

- ECLOSE(A) = {A}
- ECLOSE(E) = {E,B,C,D}
- ECLOSE({C, D}) = {C, D}



EXTENDED TRANSITIONS OF ε-NFA

- Basis: $\hat{\delta}(q, \varepsilon) = \text{ECLOSE}(q)$.
- Induction:

$\hat{\delta}(q, xa)$ is computed as:

If $\hat{\delta}(q, x) = \{p_1, p_2, \ldots, p_k\}$ and

$$\bigcup_{i=1}^{k} \delta(p_i, a) = \{r_1, r_2, \ldots, r_m\},$$

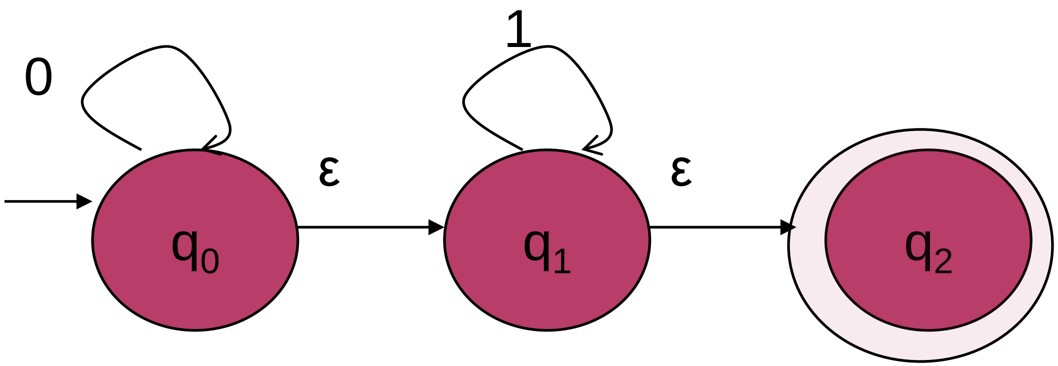
then $\hat{\delta}(q, xa) = \text{ECLOSE}(\{r_1, r_2, \ldots, {}_m\})$

$$= \text{ECLOSE}\left(\bigcup_{i=1}^{k} \delta(p_i, a)\right)$$

LANGUAGE OF ε-NFA

- The language accepted by NFA with ε - move is defined as:
- L(M) = $\{w \mid \hat{\delta}(q_0, w) \cap F \neq \varphi\}$

EXAMPLE



- Find $\delta(q_0, 01)$

EQUIVALENCE OF NFA & ε-NFA

- **Theorem**

  If L is accepted by NFA with ε -transitions, than L is accepted by an NFA without ε -transitions.

- **Proof**

- Let $M = (Q, \sum, \delta, q_0, F)$ be an NFA with ε - transitions. Construct M^1 which is NFA without ε - transition.

  $M^1 = (Q, \sum, \delta^1, q, F^1)$ where

  $$F^1 = \begin{cases} F \cup \{q\} & \text{if } \varepsilon\text{-CLOSURE } (q) \text{ contains a state of } F. \\ F & \text{otherwise} \end{cases}$$

PROOF

By induction :

δ^1 and $\hat{\delta}$ are same

δ and $\hat{\delta}$ are different

Let x be any string

$\delta^1(q_0, x) = \hat{\delta}(q_0, x)$

This statement is not true if

$x = \varepsilon$ because $\delta^1(q, \varepsilon) = \{q\}$ and

$\hat{\delta}(q_0, \varepsilon) = \varepsilon$ - CLOSURE (q_0)

PROOF

Basis step

$|x| = 1$

x is a symbol whose value is a

$\delta^1(q_0, a) = \hat{\delta}(q_0, a)$ (because by definition of $\hat{\delta}$)

PROOF

Induction step

let $x = wa$ where a is in Σ.

$$\delta^1(q_0, wa) = \delta^1(\delta^1(q_0, w), a)$$

$$= \delta^1(\hat{\delta}(q_0, w), a)$$

$$= \delta^1(p, a) \text{ [because by inductive hypothesis}$$

$$\delta(q_0, w) = \hat{\delta}(q_0, w) = p \text{(say)]}$$

Now we must show that

$$\delta^1(p, a) = \hat{\delta}(q_0, wa)$$

PROOF

But

$$\delta^1(p, a) = \bigcup_{q in P} \delta^1(q, a)$$

$$= \bigcup_{q in P} \hat{\delta}(q, a)$$

$$= \hat{\delta}(\hat{\delta}(q_0, w), a)$$

$$= \hat{\delta}(q_0, wa)$$

$$= \hat{\delta}(q_0, x)$$

Hence $\delta^1(q_0, x) = \hat{\delta}(q_0, x)$

SUMMARY

- Definition of ε-NFA
- Transition diagram, transition function and properties of transition function for ε-NFA.
- Equivalence of NFA & ε-NFA

TEST YOUR KNOWLEDGE

- State true or false?
  An NFA can be modified to allow transition without input alphabets, along with one or more transitions on input symbols.
- According to the given transitions, which among the following are the epsilon closures of q1 for the given NFA?
  $\delta\ (q1, \varepsilon) = \{q2, q3, q4\}$
  $\delta\ (q4, 1) = q1$
  $\delta\ (q1, \varepsilon) = q1$
  a) q4 b) q2 c) q1 d) q1, q2, q3, q4

REFERENCE

- Hopcroft J.E., Motwani R. and Ullman J.D, "Introduction to Automata Theory, Languages and Computations", Second Edition, Pearson Education, 2008