Introduction to Systems

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Overview

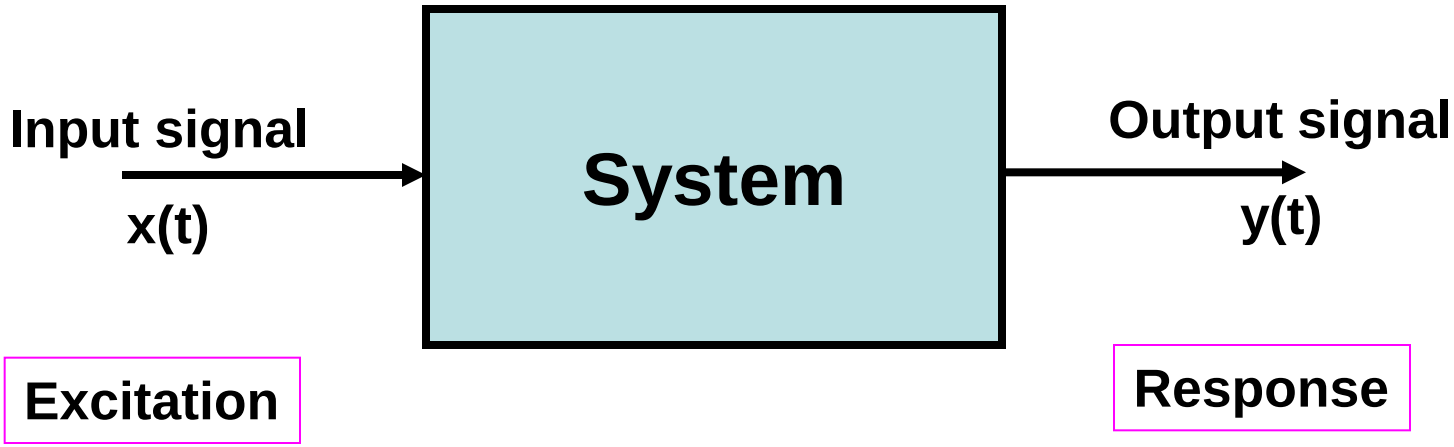
- Definition of a system
- Classification of Systems

What is a system?

- A Signal is a physical quantity, or quality, which conveys information

- A System is a device that take one or more signals as input, perform operations on the signals, and produce one or more signals as output

Basic Block Diagram of a system



Definition of a system

- **Implementation point-of-view**:

  A ***system*** is an arrangement of physical components connected or related in such a manner as to form and/or act as an entire unit.

- **Signal processing perspective**:

  A ***system*** can be viewed as any process that results in the transformation of signals, in which systems act on signals in prescribed ways.

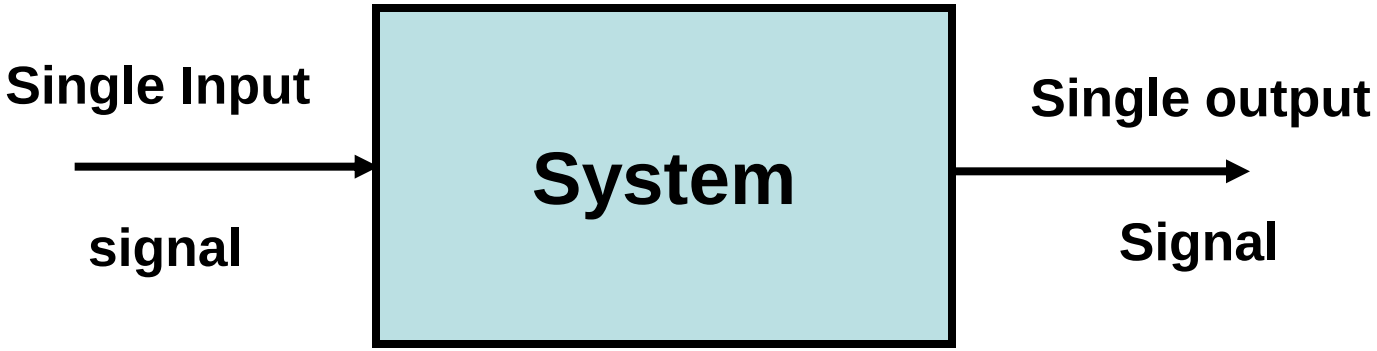
Definition of a system (continued)

- **Mathematical**:

  A ***system*** as a mapping of *N* input signals onto *M* output signals; the mapping carries out a transformation on the input signals according to a set of rules

Basic definitions

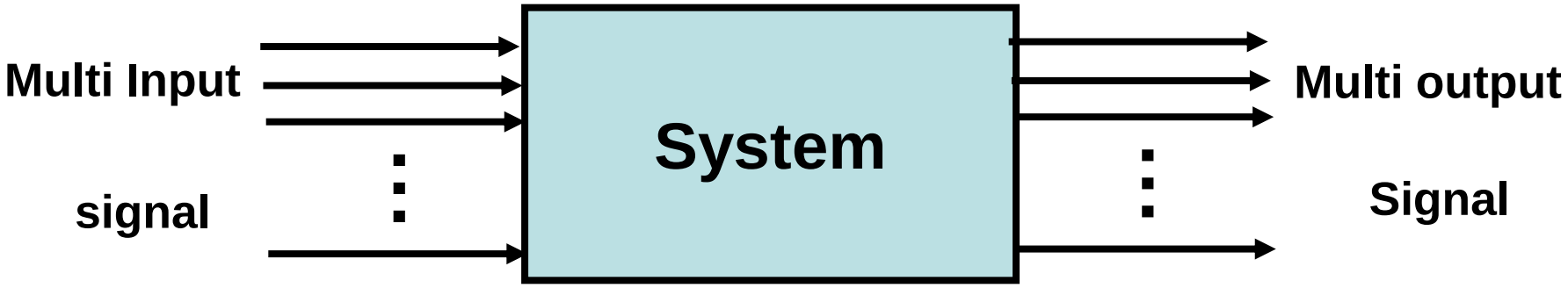
- ***Single-variable system*** (**SISO** system) has only one input and only one output



- Eg. **Audio Amplifier**

- ***Multivariable system*** (**MIMO** system)

  has more than one input or more than one output



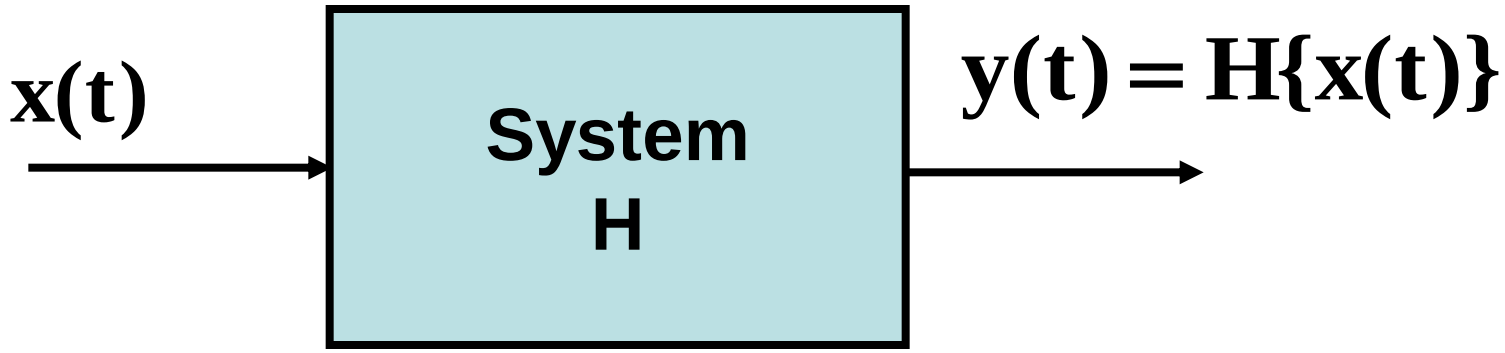
Eg. Mobile Phones

Input-output relationship

The general relationship between the input and the output of a system is expressed as

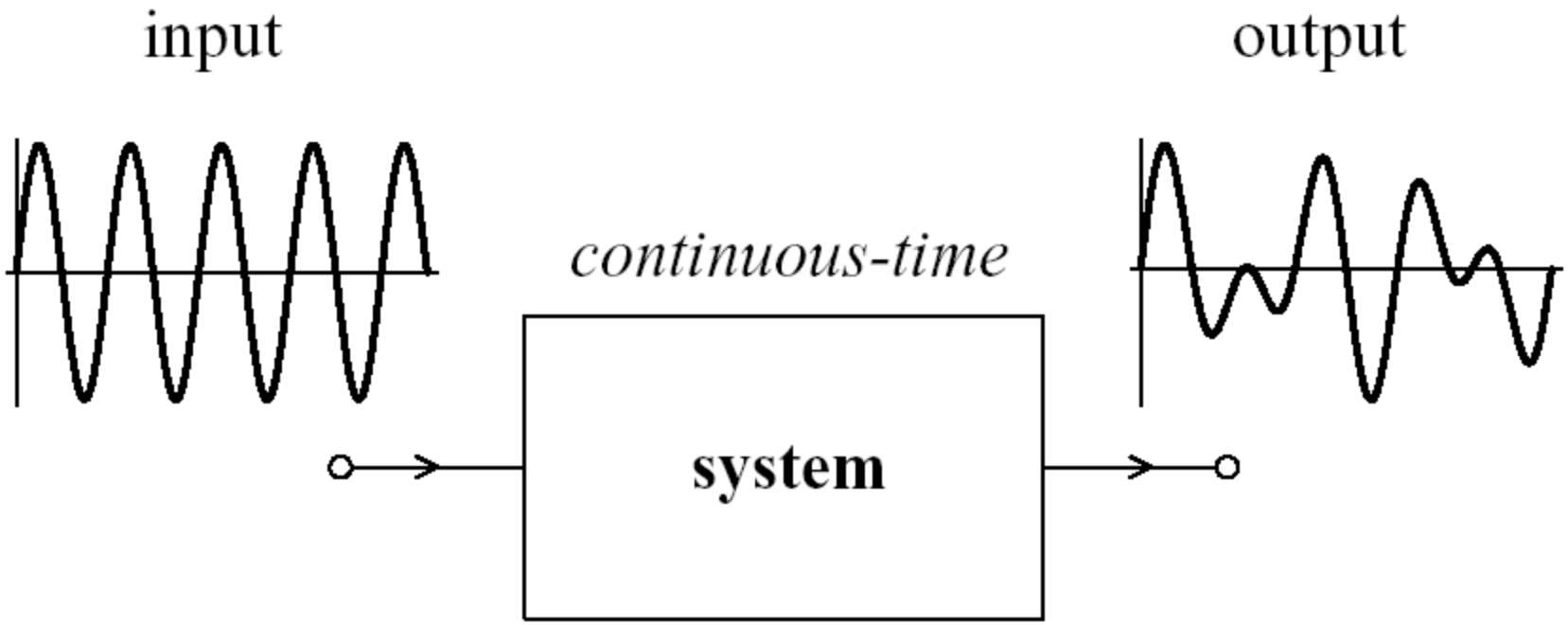
$$y(t) = H\{x(t)\}$$

Where H denotes transformation or operator.



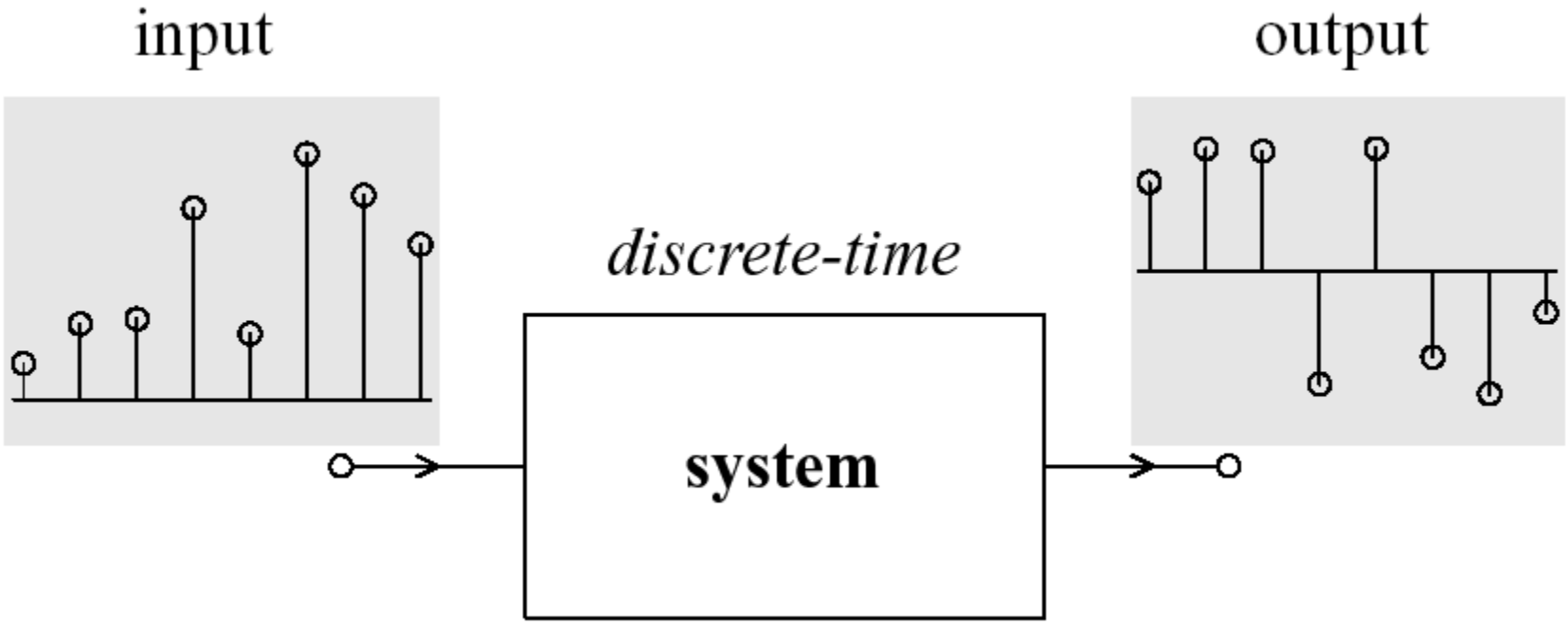
Black box concept:
the knowledge of the internal structure of a system is unavailable; the only access to the system is by means of the input ports and the output ports

Continuous-time system



Continuous-time system: the input and output signals are continuous time

Discrete-time system



Discrete-time system has discrete-time input and output signals

Classification of Systems

- Linear and Non-Linear
- Time-Invariant and Varying
- Causal
- Stable
- Memory and Memoryless
- Invertible and Non-Invertible

Linearity

- Linear Systems satisfy "Superposition principle"

Statement :

- The response of the system to a weighted sum of signals is equal to the corresponding weighted sum of the outputs of the system to each of the individual input signals.

Linear System

A system is linear if it satisfies the principle of superposition.

$$\mathbf{H\{a_1x_1(t)+a_2x_2(t)\}=a_1H\{x_1(t)\}+a_2H\{x_2(t)\}}$$

For any arbitrary inputs $x_1(t)$ and $x_2(t)$ and constants a_1 and a_2.

For example, $\mathbf{y(t)=ax(t)}$

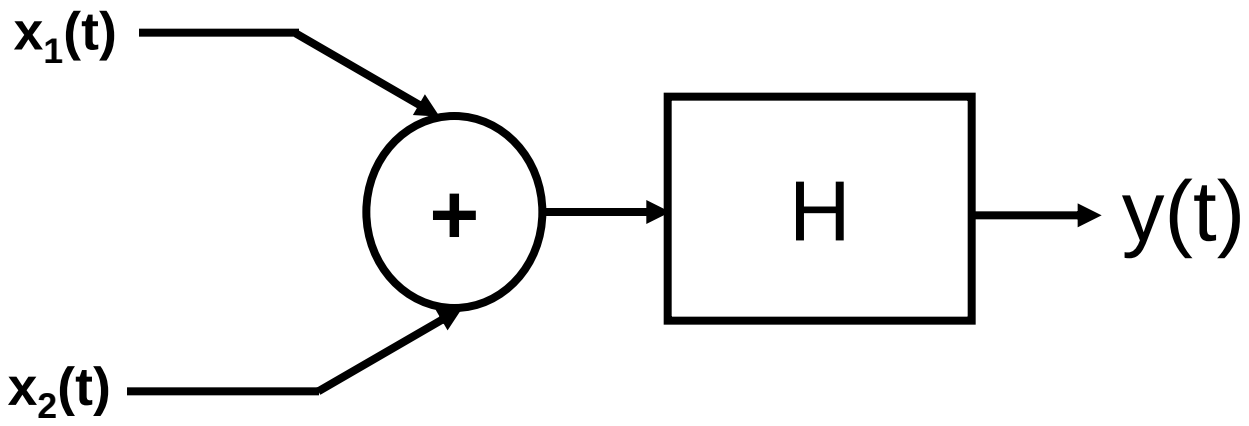
Principle of Superposition is described by

1. Additivity Property
2. Homogenity Property

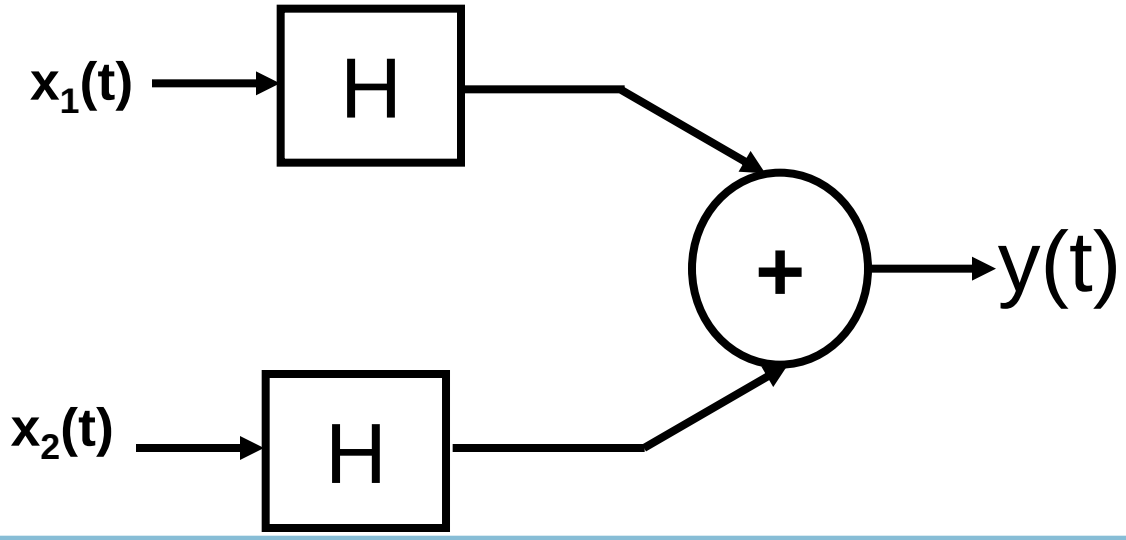
Additivity: $\mathbf{H\{x_1(t)+x_2(t)\}=H\{x_1(t)\}+H\{x_2(t)\}}$

Homogenity: $\mathbf{H\{Ax(t)\}=AH\{x(t)\}}$

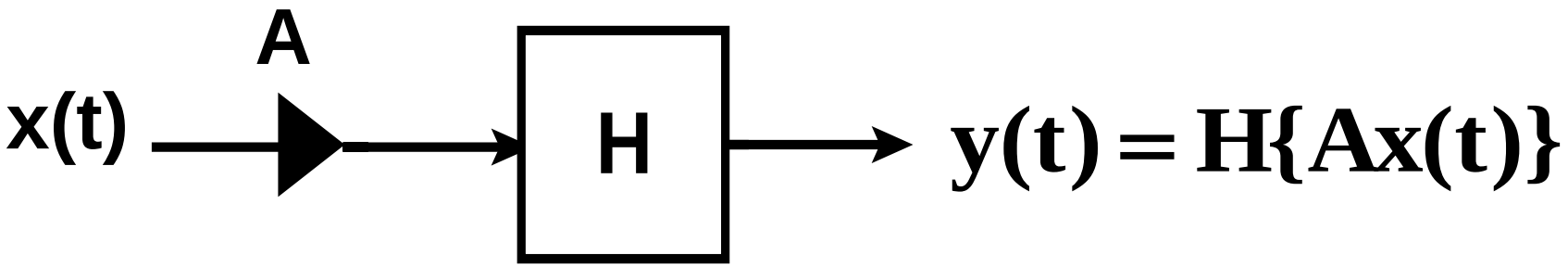
- Additivity



This is equivalent to



- Homogenity



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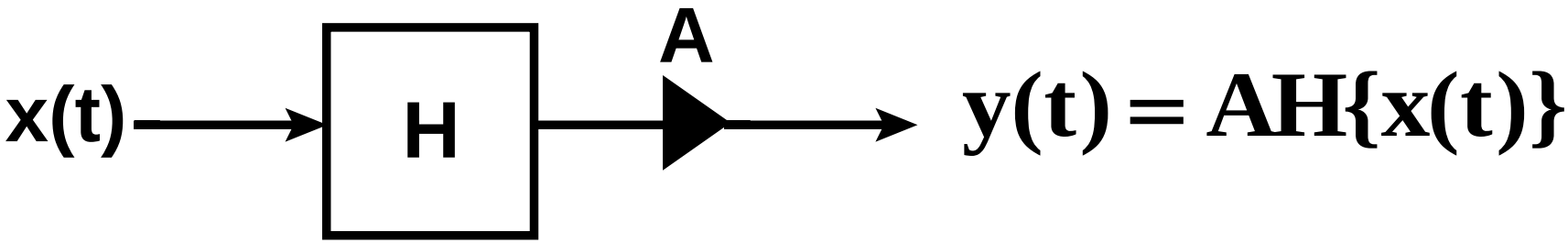


Non-Linear System

- The system is not satisfying principle of superposition.

The system is non-linear if it contains

a) Non-linear element.
b) Non-zero Initial conditions.
c) Internal Sources.

for example, $\mathbf{y(t) = ax(t) + b}$

Time (or) Shift Invariance

- A system is said to be "**time/shift invariant**" if it satisfies the condition

  $y(n,k) = y(n-k)$

  where,

  $y(n,k)$ = Delay in the input sequence by 'k' samples

  $y(n-k)$ = Delay if the output sequence by 'k' samples

Time Invariant or Shift Invariant system

- A system is time invariant, if a time shift in the input results in corresponding time shift in the output.

$$y(t) = H\{x(t)\}$$

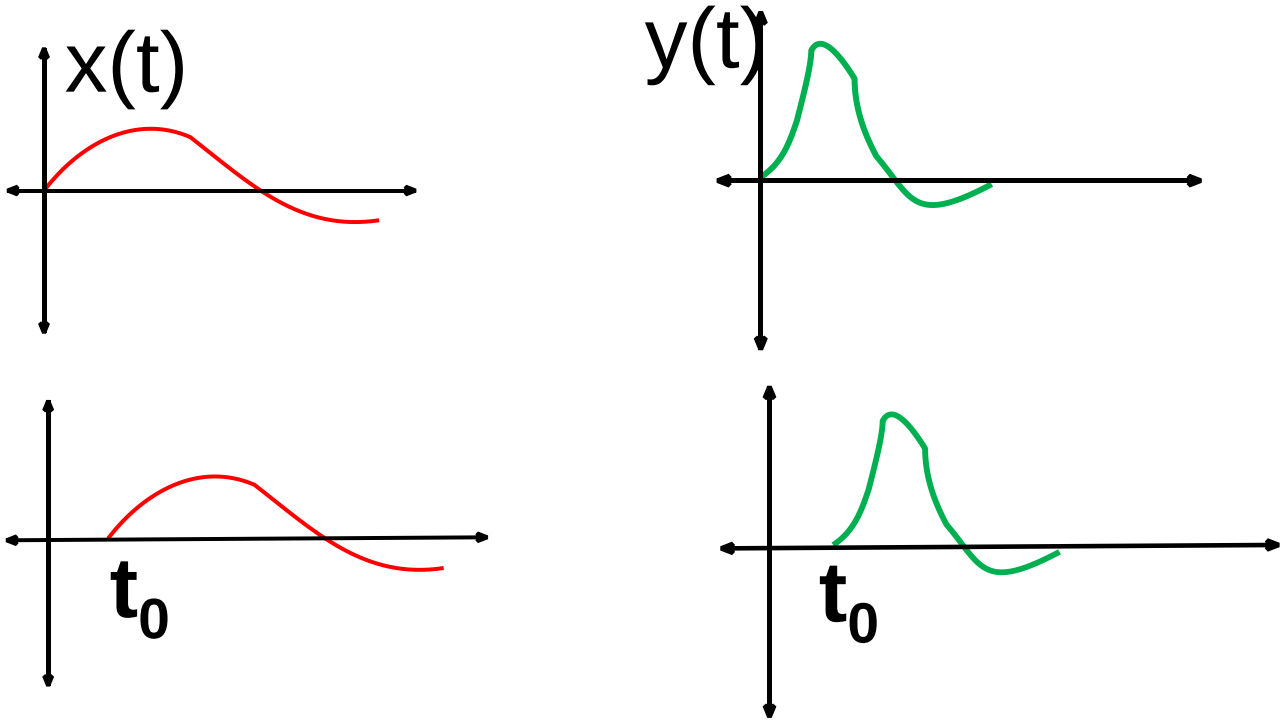
Let

$$y(t - t_0) = H\{x(t - t_0)$$

For example,

$$y(t) = x^2(t)$$

- If the input x(t) is delay by t_0 seconds, output is also delayed by same amount.



•Characteristics and parameters of a time-invariant system **do not change** with time

Time Varying System

- System that satisfying the equations

$$y(t) = H\{x(t)\}$$

$$y(t - t_0) \neq H\{x(t - t_0)\}$$

- For example, $y(t) = tx(t)$

Linear Time Invariant (LTI) Systems

A system which is linear and time invariant is called LTI systems.

eg. $$y(t) = x(t) + x(t-1)$$

The following is linear but time varying

$$y(t) = x(2t)$$

Causal System

- A system is called **causal** if the output depends only on the present and past values of the input

For example,

$$y(t) = x(t) + x(t-1)$$

(i.e.) The output of system does not depend on future values of the input

All real time systems are causal.

Non-Causal System

The Present output of the system depends on present, past and future values of the inputs.

For example,

$$y(t) = x(t) + x(t-1) + x(t+1)$$

Stable Systems

- A **stable system** is one that will remain at rest unless excited by an external source and will return to rest if all excitations are removed.

- An arbitrary system is said to be bounded input-bound output stable (BIBO), if every bounded input produces a bounded output.

Mathematically,

$$|\mathbf{x(t)}| \leq \mathbf{M_x} < \infty$$

$$|\mathbf{y(t)}| \leq \mathbf{M_y} < \infty \qquad \forall \mathbf{t}$$

For example, y(t)=x(t)

Memory and Memoryless Systems

- A system is memoryless (static) if the present output depends only on present input and it is independent of past and future inputs.

  For example, $y(t)= Ax(t)$

- A system with memory (dynamic) is one whose output depends on present and past values of the input

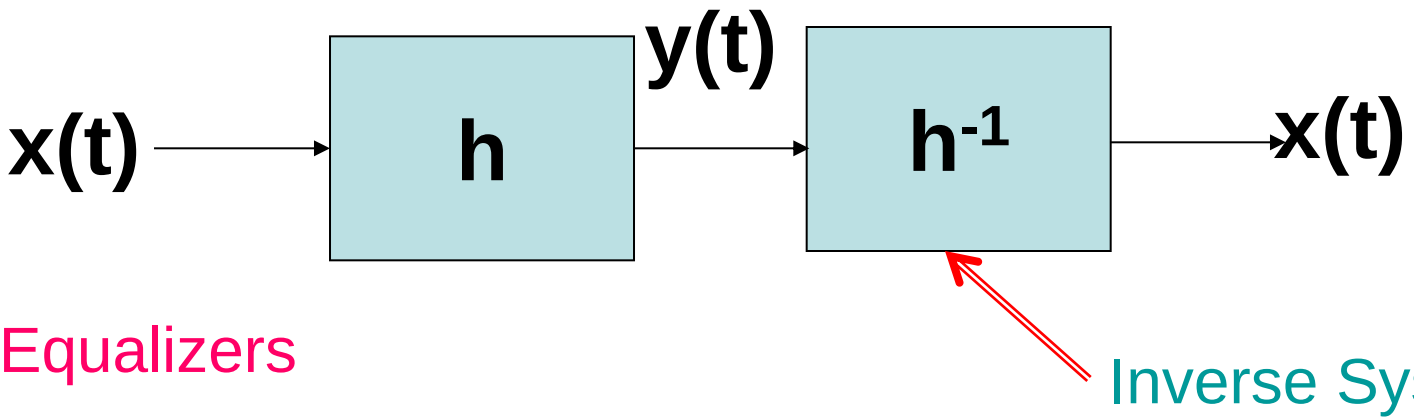
  For example, $y(t) = x(t) + x(t-1)$

*Physical systems with Inductors and Capacitors

Inverse System

- A system is said to be invertible if the input signal given to the system can be recovered from the output signal of the system.

- Inverse system is one when cascaded with the original system yields an output equal to the input.



eg. Equalizers

Problems

Check whether the following systems are linear, time invariant, causal, stable.

1. $y(t)=\log (x(t))$
2. $y(t)= C\, x(at)+1$
3. $Y(t)= A\, x(t+1)$