Computable Functions

Turing-Computable Functions

- A **total** function $f: \Sigma^* \rightarrow \Sigma^*$ is Turing-computable if there exists a DTM M such that for every x in Σ^*,

  $$(s, Bx\underline{B}) \vdash^* (h, Bf(x)\underline{B}).$$

- A **partial** $f: \Omega \rightarrow \Sigma^*$ is Turing-computable if there exists a DTM M such that $L(M)=\Omega$ and for every x in Ω,

  $$(s, Bx\underline{B}) \vdash^* (h, Bf(x)\underline{B}).$$

Construct a TM for successive function ?

$$f : N \rightarrow N, f(x) = x+1.$$

Assume that the input is encoded in UNARY form.

Let M = (Q, Σ, Γ, δ, *qo*, *B*, *F*)

Q = {*qo,q1*} *qo=start state q1=final state*

Σ = {o}

Γ = {o,B}

F = {*q1*}

States	Tape Symbols	
	0	B
q_0	$(q_0, 0, R)$	$(q_1, 0, R)$
q_1	–	–

Let us consider the input x=3, This is encoded as 000.

$$(q_0, \underline{0}00B) \vdash (q_0, 0\underline{0}0B) \vdash (q_0, 00\underline{0}B)$$

$$\vdash (q_0, 000\underline{B}) \vdash (q_1, 000\underline{0}B)$$

The machine halts in an accepting state q_1

by computing the successive of x

PROGRAMMING TECHNIQUES OF TURING MACHINES

- **Storage in the Finite Control**
- **Multiple Tracks**
- **Checking off Symbols**
- **Subroutines**

Storage in the Finite Control

- The finite control can be used to hold the finite amount of information.
- It is considered as a pair of elements, like *(q_0,a), where one exercising control* and second component stores a symbol in the finite control.
- Consider a turing machine M which accepts the language 01* + 10*
- Let M = (Q, {0,1}, {0,1,B}, δ, {q_0,B},B,F)
- Q = {q_0,q_1} x {0,1,B}

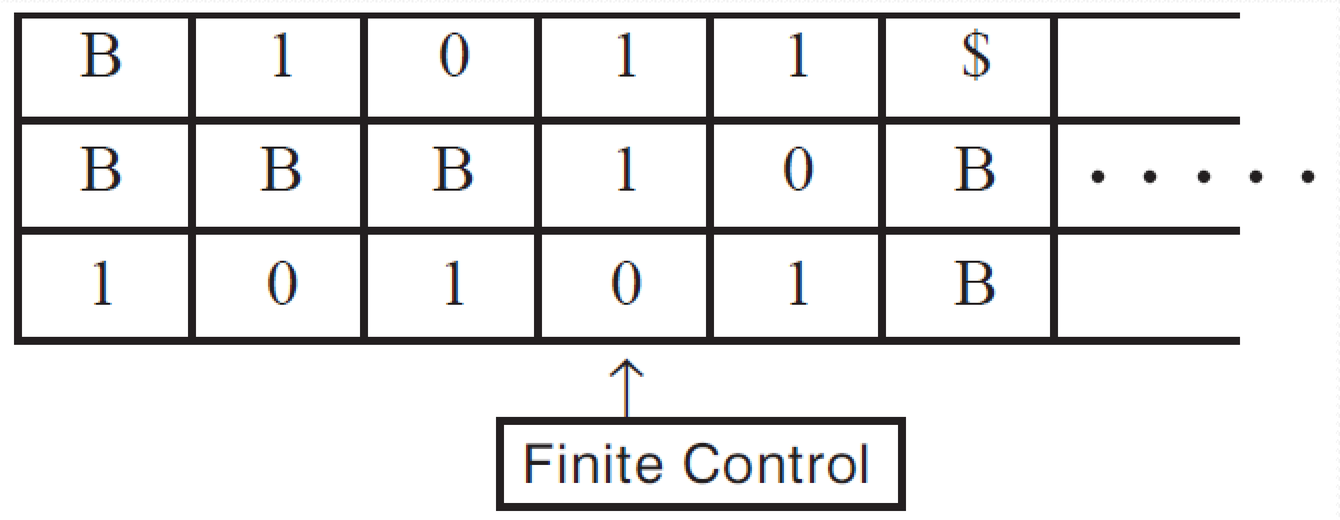
  = ([q_0,0], [q_0,1], [q_0,B], [q_1,0], [q_1,1], [q_1,B])
- F = {[q_1,B]}

$$\delta([q_0,B],0) = ([q_1,0], 0,R)$$
$$\delta([q_0,B],1) = ([q_1,1], 1, R)$$

$$\delta([q_1,0],1) = ([q_1,0], 1, R)$$
$$\delta([q_1,1],0) = ([q_1,1], 0, R)$$

$$\delta([q_1,0],B) = ([q_1,B],0,L)$$
$$\delta([q_1,1], B) = ([q_1,B], 1, L)$$

Multiple Tracks



Checking off Symbols

$\{ww^R | w \text{ in } \Sigma^*\}$

$\{a^i b^i : i \geq 1\}$

B	B	B	B	✓	✓	....	extra Track
a	*a*	*b*	*c*	*a*	*b*	....	

Consider a turing machine $M = (Q, \Sigma, \Gamma, \delta, q0, B, F)$ for the language $L = \{wcw \mid w \in \{a, b\}^+\}$

a) $Q = \{[q,d] \mid q = q_1\ q_2 \ldots\ q_9$ and $d = a, b$ or $B\}$

b) $\Sigma = \{[B, d] \mid d = a, b$ or $c\}$

c) $\Gamma = \{[x, d] \mid x = B$ or and $d = a, b, c$ or $B\}$

d) $q0 = [q_1, B]$

e) $F = \{[q_9, B]\}$

f) $B = [B, B]$

g) δ is defined for $d = a$ or b and $e = a$ or b.

$\delta([q_1, B], [B, d]) = ([q_2, d], [\checkmark, d], R)$

$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$

$\delta([q_2,d], [B, c]\} = ([q_3, d], [B, c], R)$

$\delta([q_3,d], [\checkmark, e]) = ([q_3,d], [\checkmark, e], R)$

$\delta([q_3,d], [B, d]) = ([q_4,B], [\checkmark, d], L)$

$\delta([q_4,B], [\checkmark, d]) = ([q_4,B], [\checkmark, d], L)$

$\delta([q_4,B], [B, c]) = ([q_5,B], [B, c], L)$

$\delta([q_5,B], [B, d]) = ([q_6,B], [B, d], L)$

$\delta([q_6,B], [B, d]) = ([q_6,B], [B, d], L)$

$\delta([q_6,B], [\checkmark, d]) = ([q_1,B], [\checkmark, d], R)$

$\delta([q_5,B], [\checkmark, d]) = ([q_7,B], [\checkmark, d], R)$

$\delta([q_7,B], [B, c]) = ([q_8,B], [B, c], R)$

$\delta([q_8,B], [\checkmark, d]) = ([q_8,B], [\checkmark, d], R)$

$\delta([q_8,B], [B, B]) = ([q_9,B], [, B], L)$

Subroutines

$\delta(q0,0) = (q6, B, R)$

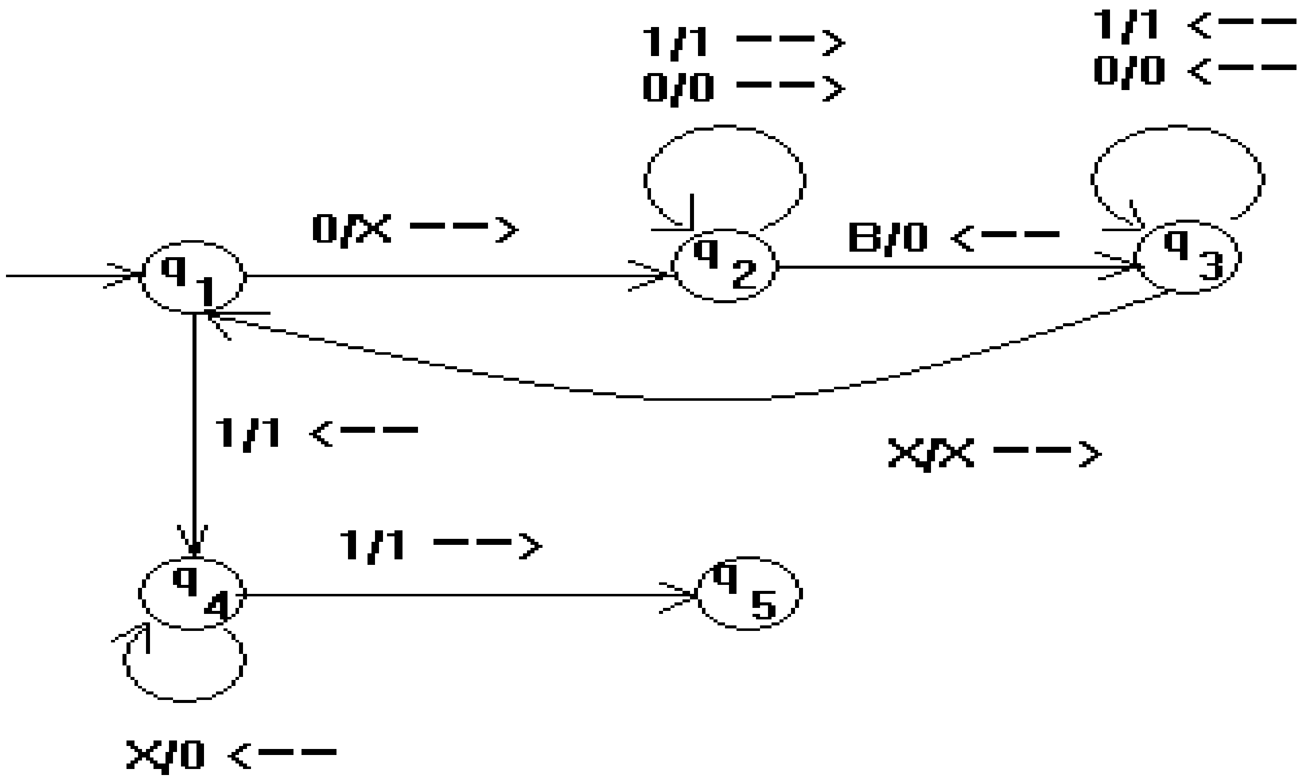
$\delta(q6, 0) = (q6, 0, R)$

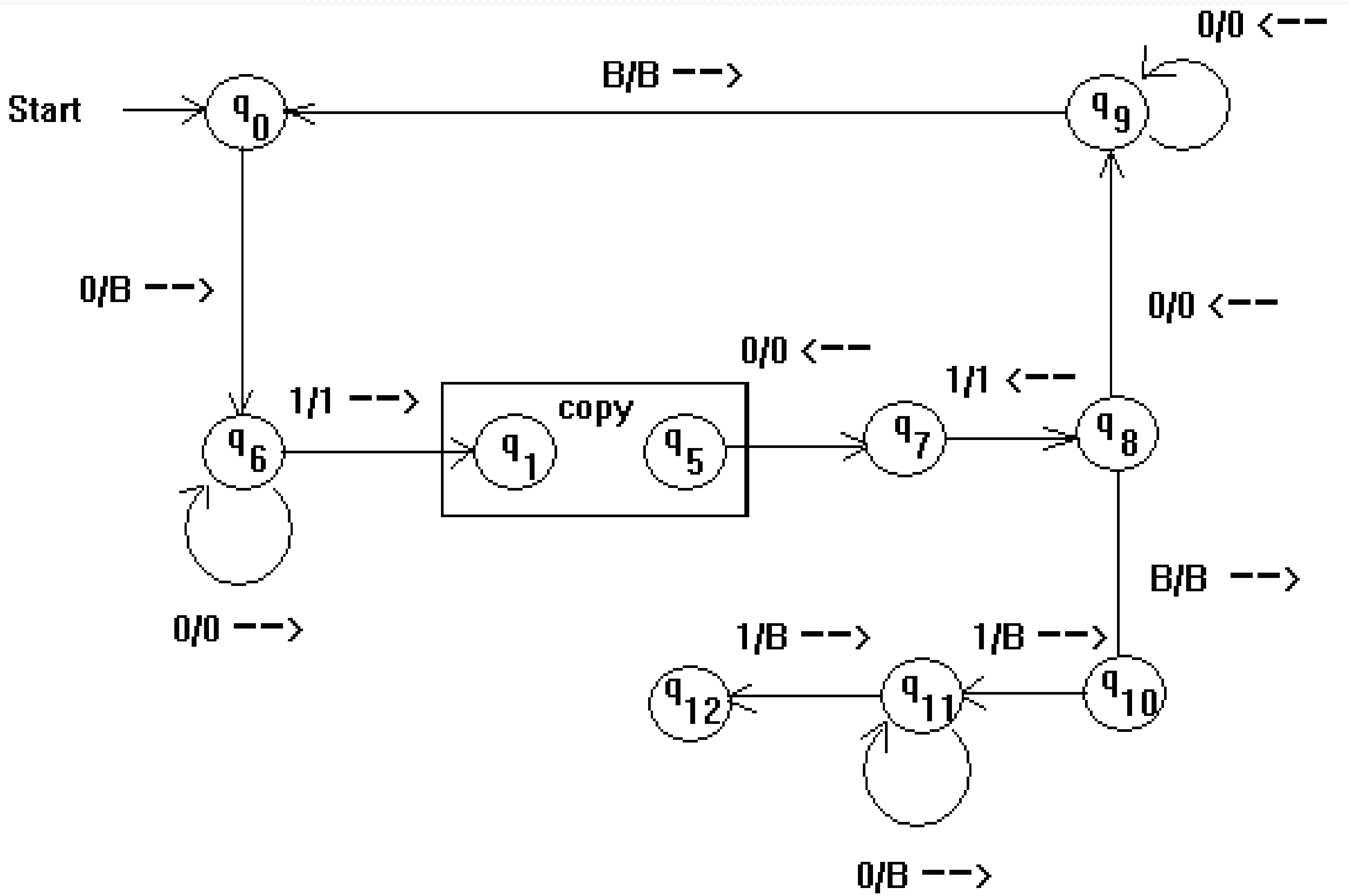
$\delta(q6, 1) = (q1, 1, R)$

δ for subroutine COPY.

States	Inputs			
	0	1	2	B
q_1	$(q_2, 2, R)$	$(q_4, 1, L)$		
q_2	$(q_2, 0, R)$	$(q_2, 1, R)$		$(q_3, 0, L)$
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_1, 2, R)$	
q_4		$(q_5, 1, R)$	$(q_4, 0, L)$	

States	Inputs			
	0	1	2	B
q_5	$(q_7,0,L)$			
q_7		$(q_8,1,L)$		
q_8	$(q_9,0,L)$			(q_{10},B,R)
q_9	$(q_9,0,L)$			$(q_0,B, R$
q_{10}		(q_{11},B,R)		
q_{11}	(q_{11},B,R)	(q_{12},B,R)		


1/1 -->
0/0 -->
1/1 <--
0/0 <--
0/X -->
B/0 <--
q1
q2
q3
1/1 <--
X/X -->
1/1 -->
q4
q5
X/0 <--



0/0 <--
B/B -->
Start
q_0
q_9
0/B -->
0/0 <--
0/0 <--
1/1 <--
1/1 -->
copy
q_6
q_1
q_5
q_7
q_8
B/B -->
0/0 -->
1/B -->
1/B -->
q_{12}
q_{11}
q_{10}
0/B -->

$\delta(q_0, 001001) \vdash Bq_6 01001B$

$\vdash B0q_6 1001B$

$\vdash B01q_1 001B$

$\vdash B01Xq_2 01B$

$\vdash B01X0q_2 1B$

$\vdash B01X01q_2 B$

$\vdash B01X0q_3 10$

$\vdash B01Xq_3 010$

$\vdash B01q_3 X010$

$\vdash B01Xq_1 010$

$\vdash B01XXq_2 10$

$\vdash B01XX1q_2 0$

$\vdash B01XX10q_2 B$

$\vdash B01XX1q_3 00$

$\vdash B01XXq_3 100$

$\vdash B01Xq_3 X100$

$\vdash B01XXq_1 100$

$\vdash B01Xq_4 X100$

$\vdash B01q_4 X0100$

$\vdash B0q_4 100100$

$\vdash B01q_5 00100$

$\vdash B0q_7 100100$

$\vdash Bq_8 0100100$

$\vdash q_9 B0100100$ (

$\vdash Bq_0 0100100$

$\vdash BBq_6 100100$

$\vdash BB1q_1 00100$

$\vdash BB1Xq_2 0100$

$\vdash BB1X0q_2 100$

$\vdash BB1X0q_2 100$

$\vdash BB1X01q_2 00$

$\vdash BB1X010q_2 0$

$\vdash BB1X0100q_2 B$

$\vdash BB1X010q_3 00$

$\vdash BB1X01q_3 000$

$\vdash BB1X0q_3 1000$

$\vdash BB1Xq_3 01000$

$\vdash BB1q_3 X01000$

$\vdash BB1Xq_1 01000$

$\vdash BB1XXq_2 1000$

$\vdash BB1XX1q_2 000$

$\vdash BB1XX10q_2 00$

$\vdash BB1XX100q_2 0$

$\vdash BB1XX1000q_2 B$

$\vdash BB1XX100q_3 00$

$\vdash BB1XX10q_3 000$

$\vdash BB1XX1q_3 0000$

$\vdash BB1XXq_3 10000$

$\vdash BB1Xq_3 X10000$

$\vdash BB1XXq_1 10000$

$|- BB1Xq_4X10000$
$|- BB1q_4X010000$
$|- BBq_410010000$
$|- BB1q_50010000$
$|- BBq_710010000$
$|- Bq_8B10010000$
$|- BBq_{10}10010000$
$|- BBBq_{11}0010000$
$|- BBBBq_{11}010000$
$|- BBBBBq_{11}10000$
$|- BBBBBBq_{12}0000$