Linear Convolution using Circular Convolution

I.Nelson
SSN College of Engineering

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- In signal processing applications, we are interested in the linear convolution of two finite duration sequences.

- In order to obtain the result of linear convolution from a circular convolution certain modifications are made.

- Let us consider two finite duration sequences x(n) and h(n). The duration of x(n) is L samples and that of h(n) is M samples. The linear convolution of x(n) and h(n) is given by the formula

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

- where y(n) is a finite duration sequence of L+M-1 samples. The circular convolution of x(n) and h(n) give N samples where N = max(L,M).

- The circular convolution then results in an L – point sequence that is M-1 points shorted than that given by linear convolution.

- In order to obtain the number of samples in circular convolution equal to L+M-1, both x(n) and h(n) mush be L+M-1 point sequences, which can be obtained by appending appropriate number of zero valued samples to both x(n) and h(n).

- Now we take L+M-1 point DFTs of x(n) and h(n) and multiply the DFTs to get Y(k). Then by finding inverse transform we obtain y(n).

- In other words, by increasing the length of the sequences x(n) and h(n) to L+M-1 points and then circularly convolving the resulting sequences we obtain the same result as would have been obtained with linear convolution.

Filtering Long Duration Sequences:

- Suppose that an input sequence x(n) is of long duration is to be processed with a system having impulse response of finite duration by convolving the two sequences.

- Because of the length of the input sequence, it would not be practical to store it all before performing linear convolution. Therefore, the input sequence must be divided into blocks. The successive blocks are processed separately one at a time and the results are combined later to yield the desired output sequence which is identical to the sequence obtained by linear convolution.

- Two methods are commonly used for filtering the sectioned data and combining the results are the overlap-save method and the overlap-add method.

Overlap – Save method:

- Let the length of the input sequence be Ls and the length of the impulse response be M.
- The input sequence is subdivided into blocks of data of size L+M-1.
- Each block consists of last M-1 data points of previous blocks followed by L new data points.
- For first block of data, first M-1 points are set to zero.
- L-1 zeros appended in impulse response.
- Perform Circular convolution between $x_i(n)$ and h(n).
- In $y_i(n)$, first M-1 points discarded.

Overlap – Add method:

- Let the length of the input sequence be Ls and the length of the impulse response be M.

- The input sequence divided into blocks of data size having length L and M-1 zeros are appended to it to make the data size of L+M-1.

- L-1 zeros are added to impulse sequence.

- Perform Circular convolution between $x_i(n)$ and h(n).

- Last M-1 points from each output block must be overlapped and added to the first M-1 points of the succeeding block.

Parameter selection to calculate DFT

- To find the DFT of a discrete-time signal the major parameters involved are

T – Sampling time (in secs)
F_s – Sampling rate (in Hz)
Δf – Frequency resolution (or) spacing between two successive frequency components (in Hz)
t_r – Record length (in secs)
F_N –Nyquist frequency (or) Folding frequency (in Hz)
F_m – Maximum frequency component present in the signal
N – Number of the samples

- According to Sampling theorem,

$$f_s \geq 2\, f_m$$

- This implies that the sampling time T must be selected according to $T \leq 1/2f_m$

- If the total number of samples in record is N. then the minimum record length is $t_r = NT$

- For the desired frequency resolution the minimum record length is $t_r = 1/\Delta f$

- To improve the frequency resolution we have to increase the record length. For a given N, this implies in increasing the sampling time, thus reducing the sampling rate.

- The reduction in sampling rate may cause aliasing.

- Therefore, the selection of N should be such that there is a trade off between the sampling rate and the frequency resolution.

- The only way in which the sampling rate of the frequency resolution can be improved while holding the other constant is to increase the number of points N.

- If f_m and Δf are both given, then $N \geq 2f_m/\Delta f$