Analog Chebyshev Filter

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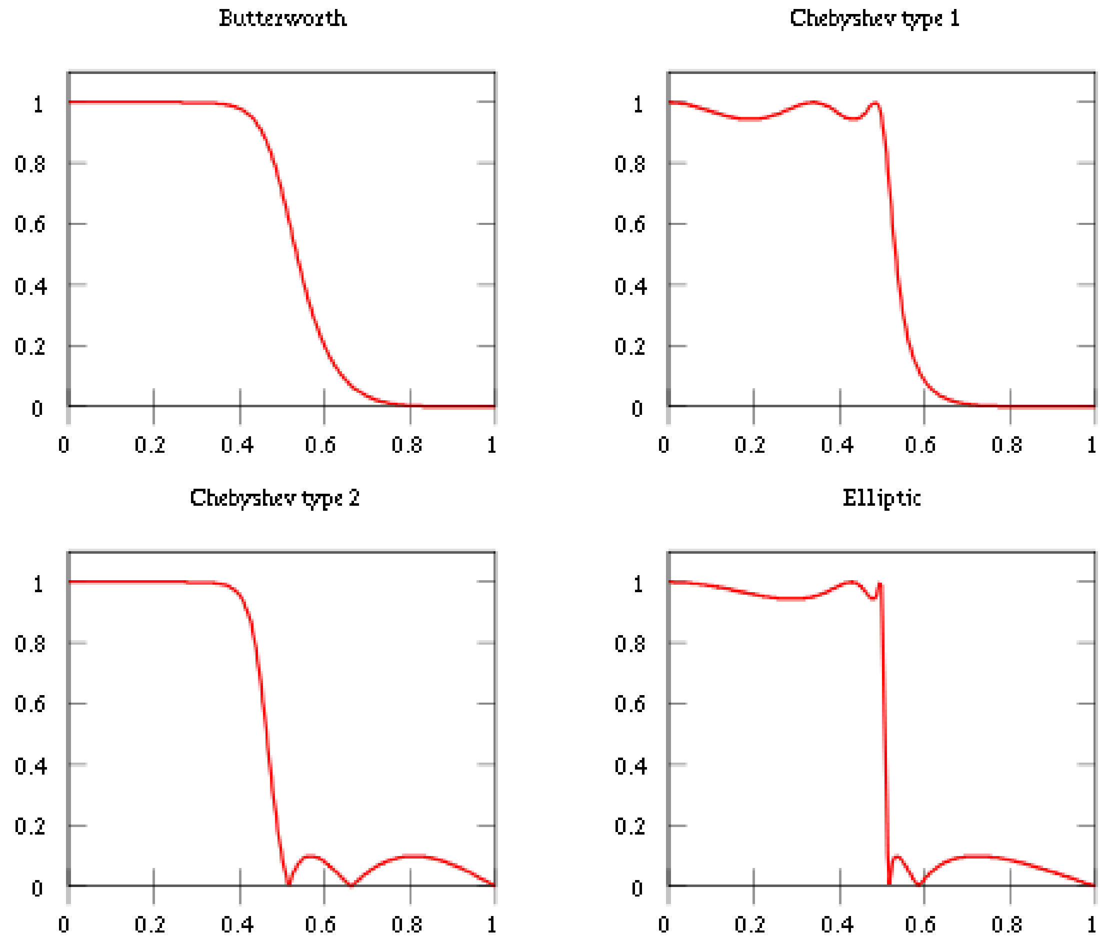
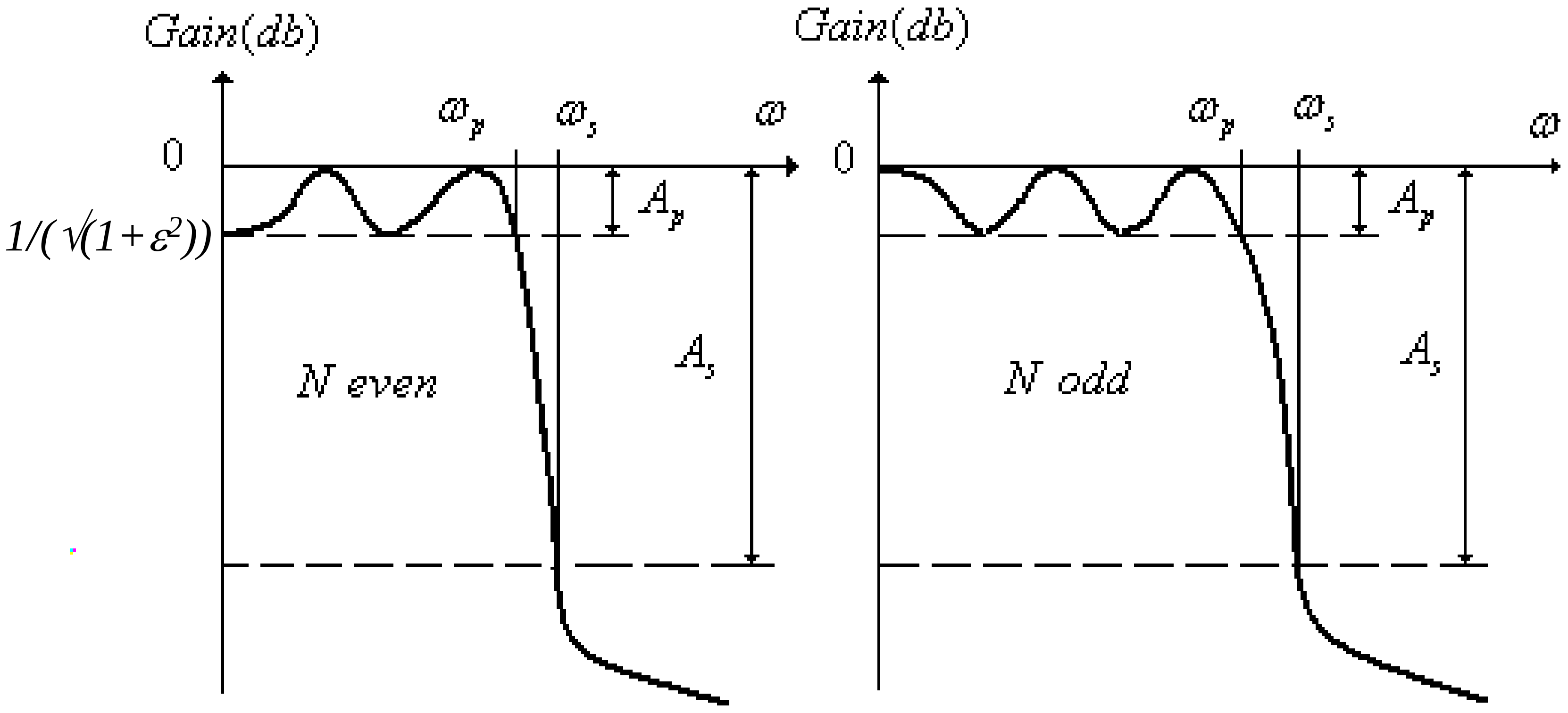
Low pass Chebyshev magnitude response:





Gain(db)
0
ω_p
ω_s
ω
$1/(\sqrt{(1+\varepsilon^2)})$
A_p
A_s
N even
Gain(db)
0
ω_p
ω_s
ω
A_p
A_s
N odd

Analog Lowpass Chebyshev filter:

- The magnitude function of the Chebyshev type I filter is given by,

$$|H(j\Omega)| = \frac{1}{\left[1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)\right]^{1/2}}; \qquad N = 1,2,\ldots\ldots..$$

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)}; \qquad N = 1,2,\ldots\ldots..$$

- The Nth order Chebyshev polynomial defined as,

$$C_N(x) = \cos(N \cos^{-1} x), \quad |x| \le 1 \text{ (Passband)}$$

$$C_N(x) = \cosh(N \cosh^{-1} x), \quad |x| > 1 \text{ (Stopband)}$$

- The Chebyshev polynomial is defined by the recursive formula,

$$C_N(x) = 2x\, C_{N-1}(x) - C_{N-2}(x), \quad N > 1$$

where $C_0(x) = 1$ and $C_1(x) = x$.

- The Chebyshev polynomials have the following properties:

1. (1) $C_N(x) = - C_N(-x)$ for N odd
   (2) $C_N(x) = C_N(-x)$ for N even
   (3) $C_N(0) = (-1)^{N/2}$ for N even
   (4) $C_N(0) = 0$ for N odd
   (5) $C_N(1) = 1$ for all N
   (6) $C_N(-1) = 1$ for N even
   (7) $C_N(-1) = -1$ for N odd

2. $C_N(x)$ oscillates with equal ripple between ± 1 for $|x| \leq 1$
3. For all N $0 \leq |C_N(x)| \leq 1$ for $0 \leq |x| \leq 1$
   $|C_N(x)| > 1$ $|x| > 1$
4. $C_N(x)$ is monotonically increasing for $|x| > 1$ for all N.

Taking logarithm for the magnitude response,

$$20\log|H(j\Omega)| = 10\log 1 - 10\log\left[1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)\right]$$

$$at\ \ \Omega = \Omega_p$$

$$20\log|H(j\Omega)| = -\alpha_p = -10\log\left[1+\varepsilon^2\right]$$

$$\varepsilon = \left(10^{0.1\alpha_p} - 1\right)^{1/2}$$

$$at\,\Omega = \Omega_s$$

$$20\log|H(j\Omega)| = -\alpha_s = -10\log\left[1+\varepsilon^2 C_N^2\left(\frac{\Omega_s}{\Omega_p}\right)\right]$$

$$\varepsilon^2 C_N^2\left(\frac{\Omega_s}{\Omega_p}\right) = 10^{0.1\alpha_s} - 1$$

$$N \geq \frac{\cosh^{-1}\left(\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}\right)}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

The poles of a Chebyshev type I filter can be determined by using,

$$s_k = a\cos\phi_k + jb\sin\phi_k$$

where

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \qquad k = 1,2,\ldots\ldots.N$$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}}$$

The poles of the Chebyshev transfer function are located on an ellipse in the s – plane.

Chebyshev Type II filter:

The magnitude square response is given by,

$$|H(j\Omega|^2 = \frac{1}{1+\varepsilon^2\left[\dfrac{C_N^2\left(\dfrac{\Omega_s}{\Omega_p}\right)}{C_N^2\left(\dfrac{\Omega_s}{\Omega}\right)}\right]}; \qquad N = 1,2,\ldots\ldots.$$

- The poles of a Chebyshev type II filter can be determined by using,

$$s_k = \frac{\Omega_s \sin\phi_k \sinh\theta}{(\sin\phi_k \sinh\theta)^2 + (\cos\phi_k \cosh\theta)^2} + j\frac{\Omega_s \cos\phi_k \cosh\theta}{(\sin\phi_k \sinh\theta)^2 + (\cos\phi_k \cosh\theta)^2}$$

- The order of the filter is,

$$N \geq \frac{\cosh^{-1}\left(\sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}\right)}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

Comparison between Butterworth filter and Chebyshev filter:

1. The magnitude response of Butterworth filter decreases monotonically as the frequency Ω increases from 0 to ∞, whereas the magnitude response of the Chebyshev filter exhibits ripples in the passband or stopband according to the type.

2. The transition band is more in Butterworth filter when compared to Chebyshev filter.

3. The poles of the Butterworth filter lie on a circle, whereas the poles of the Chebyshev filter lie on an ellipse.

4. For the same specifications, the number of poles in Butterworth are more when compared to Chebyshev filter, i.e., the order of the Chebyshev filter is less than that of Butterworth. Therefore less number of components are required to construct the Chebyshev filter.

Steps to design an analog Chebyshev low pass filter:

1. From the given specifications, find the order of the filter N.
2. Round off it to the next higher integer.
3. Calculate the poles of Chebyshev filter which lie on an ellipse.
4. Find the denominator polynomial of the transfer function using the above poles.
5. The numerator of the transfer function depends on the value of N.

   (a) For N odd, substitute s = 0 in the denominator polynomial and find the value, which is equal to the numerator polynomial.

   (b) For N even, substitute s = 0 in the denominator polynomial and divide the result by $(1+\varepsilon^2)^{1/2}$. This value is equal to the numerator.