Fast Fourier Transform
- Decimation in Frequency

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Steps of radix – 2 DIF – FFT algorithm:

1. The number of input samples $N=2^M$, where, M is an integer.
2. The input sequence is in natural order.
3. The number of stages in the flow graph is given by $M = \log_2 N$.
4. Each stage consists of N/2 butterflies.
5. Inputs / Outputs for each butterfly are separated by 2^{M-m} samples, where m represents the stage index, i.e., for first stage m=1 and for second stage m=2 and so on.
6. The number of complex multiplications is given by $(N/2)\log_2 N$.

7. The number of complex additions is given by $N\log_2 N$.
8. The twiddle factor exponents are a function of the stage index m and is given by $K = Nt/2^{M-m+1}$; $t= 0, 1, 2, 3,\ldots\ldots,2^{M-m}-1$
9. The number of sets or sections of butterflies in each stage is given by the formula 2^{m-1}.
10. The exponent repeat factor (ERF), which is the number of times the exponent sequence associated with m repeated is given by 2^{m-1}.

Differences between DIT & DIF algorithms

1. The input is bit-reversal while the output is in natural order in DIT. Whereas, for DIF the input is in natural order while the output is bit reversal order.
2. The DIF butterfly is slightly different from the DIT wherein DIF the complex multiplication takes place after the add – subtract operation.

Similarities between DIT & DIF algorithms

1. Both algorithms require same number of complex additions and multiplications to compute the DFT. Both algorithms uses bit – reversal at some place during the computation.

	DIT	DIF
Input sequence	Bit – reversal	Natural order
Output sequence	Natural order	Bit – reversal
Inputs / Outputs for each butterfly are separated by	2^{m-1} samples, where m represents the stage index, i.e., for first stage m=1 and for second stage m=2 and so on.	2^{M-m} samples
Twiddle factor exponents	$K = Nt/2^{m}$; t= 0, 1, 2, 3,……,2^{m-1}-1	$K = Nt/2^{M-m+1}$; t= 0, 1, 2, 3,……,2^{M-m}-1
Number of sets or sections of butterflies in each stage	2^{M-m}	2^{m-1}
Exponent repeat factor (ERF)	2^{M-m}	2^{m-1}

Problem2: Draw the Flow graph of 16 – point DIF – FFT.

Solution: Using the steps,

1. The Number of input samples, N = 16 = 2^8 (No. of stages, M = 8)
2. The output sequence is shuffled through bit – reversal shown below.
3. The number of stages M = $\log_2 16$ =4.
4. The number of butterflies per stage is N/2 = 8.
5. The inputs / outputs for each butterfly in stage m is separated by 2^{M-m} samples

   Stage 1 – Inputs / Outputs for each butterfly are separated by 8 sample.
   Stage 2 – Inputs / Outputs for each butterfly are separated by 4 samples.
   Stage 3 – Inputs / Outputs for each butterfly are separated by 2 samples.
   Stage 4 – Inputs / Outputs for each butterfly are separated by 1 samples.

6. The number of complex multiplications is given by
   $(N/2)\log_2 N = 8\log_2 16 = 32$
7. The number of complex additions is given by $16\log_2 16 = 64$.

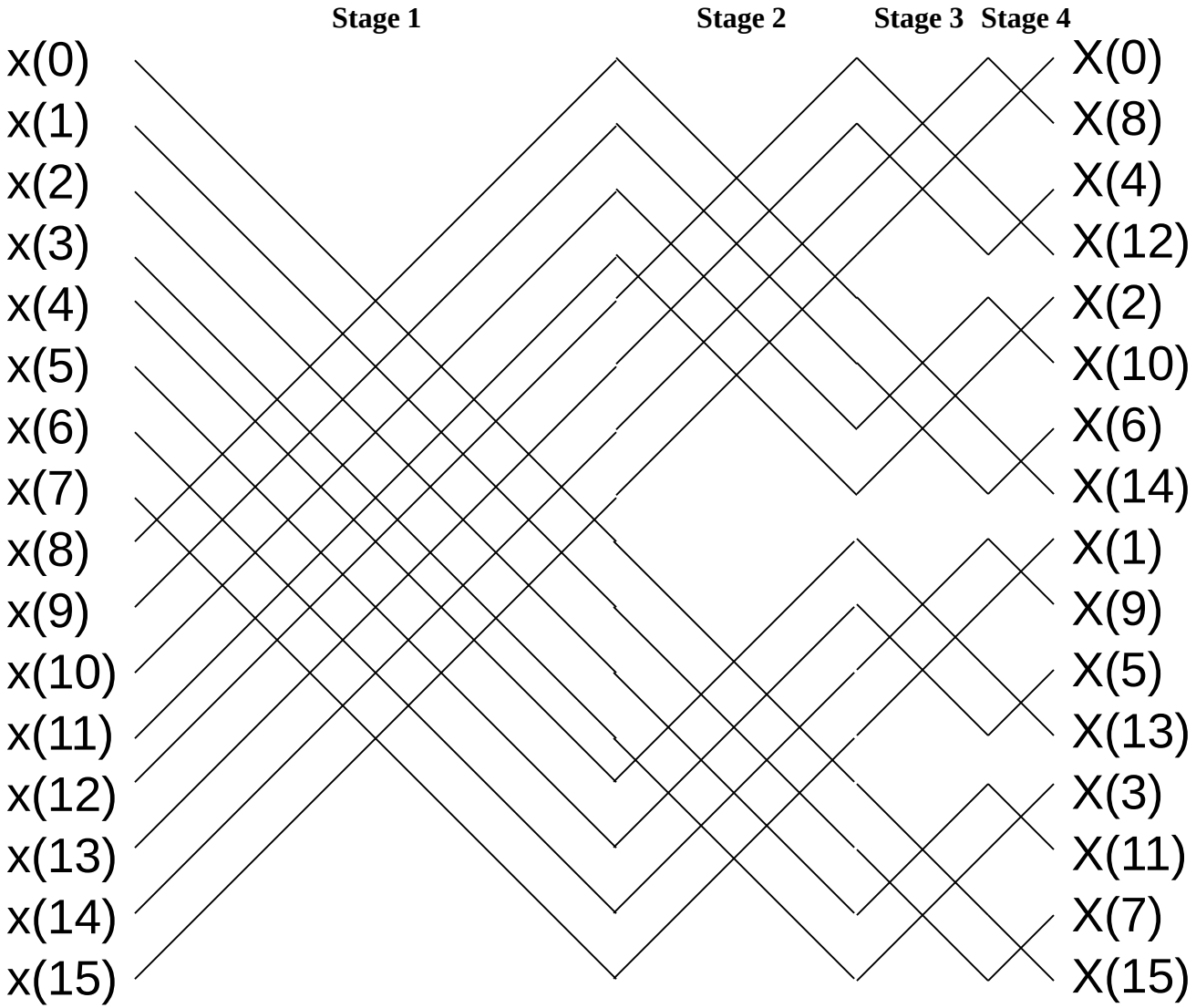
8. The twiddle factor exponents for each stage are given by
   $k = Nt/2^{M-m+1}$; $t = 0, 1, 2, 3, \ldots\ldots, 2^{M-m}-1$

   For Stage 1 the exponent is 0,1,2,3,4,5,6,7.
   For Stage 2 the exponent are 0,2,4,6.
   For Stage 3 the exponent are 0,4.
   For Stage 4 the exponent are 0.

9. The number of sets or sections of butterflies in each stage is given by 2^{m-1}.
   For Stage 1 the number of sets of butterflies are $2^{1-1} = 1$
   For Stage 2 the number of sets of butterflies are $2^{2-1} = 2$
   For Stage 3 the number of sets of butterflies are $2^{3-1} = 4$
   For Stage 4 the number of sets of butterflies are $2^{4-1} = 8$

10. The exponent repeat factor (ERF), which is the number of times the exponent sequence associated with m is repeat is given by 2^{m-1}.
   For Stage 1, ERF = 1
   For Stage 2, ERF = 2
   For Stage 3, ERF = 4
   For Stage 4, ERF = 8

Index	Binary Representation	Bit reversed Order	Bit reversed index
0	0000	0000	0
1	0001	1000	8
2	0010	0100	4
3	0011	1100	12
4	0100	0010	2
5	0101	1010	10
6	0110	0110	6
7	0111	1110	14
8	1000	0001	1
9	1001	1001	9
10	1010	0101	5
11	1011	1101	13
12	1100	0011	3
13	1101	1011	11
14	1110	0111	7
15	1111	1111	15


Stage 1
Stage 2
Stage 3
Stage 4
x(0)
x(1)
x(2)
x(3)
x(4)
x(5)
x(6)
x(7)
x(8)
x(9)
x(10)
x(11)
x(12)
x(13)
x(14)
x(15)
X(0)
X(8)
X(4)
X(12)
X(2)
X(10)
X(6)
X(14)
X(1)
X(9)
X(5)
X(13)
X(3)
X(11)
X(7)
X(15)

Problem3: Find the DFT of a sequence x(n) = {1,2,3,4,4,3,2,1} using DIF algorithm.

Solution:

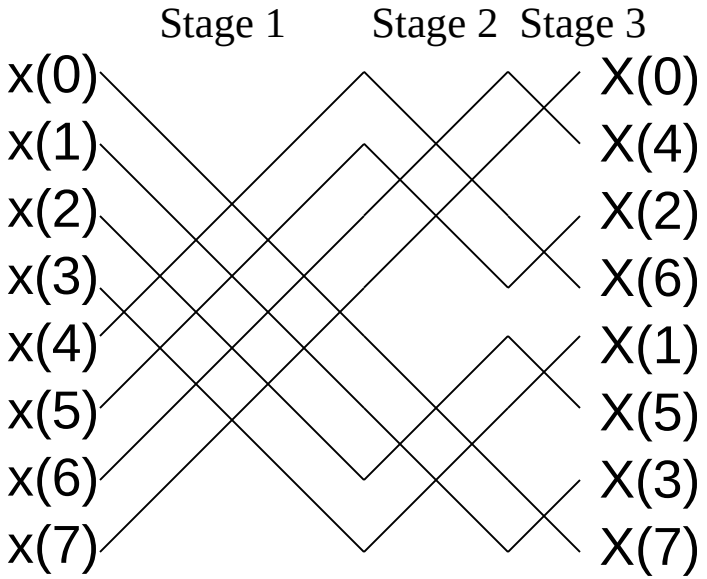
The twiddle factors associated with the flow graph are

$W_8^0 = 1$

$W_8^1 = e^{-j2\pi/8} = e^{-j\pi/4} = 0.707 - j0.707$

$W_8^2 = e^{-j4\pi/8} = e^{-j\pi/2} = -j$

$W_8^3 = e^{-j6\pi/8} = e^{-j3\pi/4} = -0.707 - j0.707$



Input	Output of Stage1	Output of Stage2	Output
x(0) = 1	1+4=5	5+5=10	10+10 = 20
x(1) = 2	2+3=5	5+5=10	(10-10)(1) = 0
x(2) = 3	3+2=5	(5-5)(1)=0	0+0 = 0
x(3) = 4	4+1=5	(5-5)(-j)=0	(0-0)(1) =0
x(4) = 4	(1-4)(1)=-3	(-3+-j)=-3-j	(-3-j) + (-2.828 –j1.414) = -5.828-j2.414
x(5) = 3	(2-3)(0.707 – j0.707) =- 0.707 + j0.707	(- 0.707 + j0.707) + (-2.121-j2.121) = -2.828-j1.414	((-3-j) - (-2.828 –j1.414))(1) = -0.172+j0.414
x(6) = 2	(3-2)(-j)=-j	(-3- -j)(1)=-3+j	(-3+j) + (2.828 –j1.414) = -0.172-j0.414
x(7) = 1	(4-1)(-0.707 – j0.707) =-2.121-j2.121	((- 0.707 + j0.707) - (-2.121-j2.121))(-j) = 2.828-j1.414	((-3+j) - (2.828 –j1.414))(1) = -5.828 +j2.414

X(k) = {20, -5.828-j2.414, 0, -0.172-j0.414, 0, -0.172-j0.414, 0, -5.828+j2.414}