Problems

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1. Design an eleventh order FIR lowpass filter for the following specification using Rectangular Window.

$$\mathbf{H(e^{j\omega})} = \begin{cases} \mathbf{1} & \mathbf{for}\ \dfrac{-\pi}{2} \le |\omega| \le \dfrac{\pi}{2} \\ \mathbf{0} & \mathbf{for}\ \dfrac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

Answer:

The transfer function of the filter is

$$H(z) = 0.0636 - 0.106\,z^{-1} + 0.3183\,z^{-2} + 0.5z^{-4} + 0.3183\,z^{-6} - 0.106\,z^{-8} + 0.0636\,z^{-10}$$

2. Design an eleventh order FIR lowpass filter for the following specification using Hamming Window.

$$\mathbf{H(e^{j\omega})} = \begin{cases} \mathbf{1} & \mathbf{for}\ \dfrac{-\pi}{2} \le |\omega| \le \dfrac{\pi}{2} \\ \mathbf{0} & \mathbf{for}\ \dfrac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

Answer: The transfer function of the filter is

$$H(z) = 0.0036 - 0.03z^{-2} - 0.1084z^{-3} - 0.2052z^{-4} + 0.75z^{-5} - 0.2052z^{-6} - 0.1084z^{-7} - 0.03z^{-8} + 0.0036z^{-10}$$

3. Design an ideal highpass filter with a frequency response

$$H_d(e^{j\omega}) = \begin{cases} 1 & ; \frac{\pi}{4} \le |\omega| \le \pi \\ 0 & ; \quad |\omega| < \frac{\pi}{4} \end{cases}$$

Determine the Transfer function of the filter for N=11. Use following window function: a) Rectangular Window b) Hanning Window c) Hamming Window

Answer:

a)

$$H(z) = 0.045 - 0.075z^{-2} - 0.159z^{-3} - 0.27z^{-4} + 0.75z^{-5} - 0.225z^{-6} - 0.159z^{-7} - 0.075z^{-8} + 0.045z^{-10}$$

b)

$$H(z) = -0.026z^{-2} - 0.104z^{-3} - 0.204z^{-4} + 0.75z^{-5} - 0.204z^{-6} - 0.104z^{-7} - 0.026z^{-8}$$

c)

$$H(z) = 0.0036 - 0.03z^{-2} - 0.1084z^{-3} - 0.2052z^{-4} + 0.75z^{-5} - 0.2052z^{-6} - 0.1084z^{-7} - 0.03z^{-8} + 0.0036z^{-10}$$

4. Design a 7-tap bandpass filter with a lower cutoff frequency of 2000 Hz, and upper cutoff frequency of 2400 Hz, and a sampling rate of 8000 Hz using a) Rectangular Window b) Hanning Window c) Hamming Window d) Blackman Window

Answer:

a)

$$H(z) = 0.1356 - 0.29z^{-1} - 0.0483z^{-2} + 0.31z^{-3} - 0.0483z^{-4} - 0.29z^{-5} + 0.1356z^{-6}$$

b) $$H(z) = -0.1579z^{-1} - 0.0789z^{-2} + 0.675z^{-3} - 0.0789z^{-4} - 0.1579z^{-5}$$

c)

$$H(z) = 0.0216 - 0.1789z^{-1} - 0.074z^{-2} + 0.6169z^{-3} - 0.074z^{-4} - 0.17892z^{-5} + 0.0216z^{-6}$$

d)

$$H(z) = -0.0964z^{-1} - 0.0778z^{-2} + 0.7924z^{-3} - 0.0778z^{-4} - 0.0964z^{-5}$$

5. Design a 7-tap bandstop filter with a lower cutoff frequency of 2000 Hz, and upper cutoff frequency of 2400 Hz, and a sampling rate of 8000 Hz using a) Rectangular Window b) Hanning Window c) Hamming Window d) Blackman Window

Answer:

a) $H(z) = -0.0424 + 0.091z^{-1} + 0.0151z^{-2} + 0.8731z^{-3} + 0.0151z^{-4} + 0.091z^{-5} - 0.0424z^{-6}$

b) $H(z) = 0.0241z^{-1} + 0.012z^{-2} + 0.9277z^{-3} + 0.012z^{-4} + 0.0241z^{-5}$

c)

$H(z) = -0.0036 + 0.0297z^{-1} + 0.0123z^{-2} + 0.9231z^{-3} + 0.0123z^{-4} + 0.0297z^{-5} - 0.0036z^{-6}$

d) $H(z) = 0.0129z^{-1} + 0.0104z^{-2} + 0.9534z^{-3} + 0.0104z^{-4} + 0.0129z^{-5}$

6. Design an eleventh order FIR lowpass filter for the following specification using Frequency sampling method.

$$H(e^{j\omega}) = \begin{cases} 1 & for \ \frac{-\pi}{2} \le |\omega| \le \frac{\pi}{2} \\ 0 & for \ \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

Answer:

$$H(z) = 0.0694 - 0.054\,z^{-1} - 0.1094\,z^{-2} + 0.0474\,z^{-3} + 0.3194\,z^{-4}$$
$$+ 0.4545\,z^{-5} + 0.3194\,z^{-6} + .0474\,z^{-7} - 0.1094\,z^{-8} - 0.054\,z^{-9} + 0.0694\,z^{-10}$$

7. Design a 11-tap FIR highpass filter for the following specification using Frequency Sampling method .

$$H(e^{j\omega}) = \begin{cases} 1 & for \ \frac{2\pi}{3} \le |\omega| \le \pi \\ 0 & elsewhere \end{cases}$$

Answer:

$$H(z) = 0.0497 - 0.0989\,z^{-1} - 0.0339\,z^{-2} - 0.1271\,z^{-3} - 0.2935\,z^{-4}$$
$$+ 0.3636\,z^{-5} - 0.2935\,z^{-6} - 0.1271\,z^{-7} - 0.0339\,z^{-8} - 0.0989\,z^{-9} + 0.0497\,z^{-10}$$

8. Design an eleventh order FIR lowpass filter for the following specification using Frequency sampling method.

$$H(e^{j\omega}) = \begin{cases} 1 & for \ \frac{-\pi}{2} \le |\omega| \le \frac{\pi}{2} \\ 0 & for \ \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

Answer:

$$H(z) = 0.0694 - 0.054\,z^{-1} - 0.1094\,z^{-2} + 0.0474\,z^{-3} + 0.3194\,z^{-4}$$
$$+ 0.4545\,z^{-5} + 0.3194\,z^{-6} + .0474\,z^{-7} - 0.1094\,z^{-8} - 0.054\,z^{-9} + 0.0694\,z^{-10}$$

9. Design a 13-tap FIR band pass filter for the following specification using Frequency Sampling method . Assume Sampling Frequency Fs = 8000Hz.

$$H(f) = \begin{cases} 1 & for\ 1000\ Hz \le |\omega| \le 2000\ Hz \\ 0 & elsewhere \end{cases}$$

Answer:

$$H(z) = 0.0211 - 0.1059\,z^{-1} + 0.0211\,z^{-2} - 0.2039\,z^{-3} - 0.2039\,z^{-4}$$
$$+ 0.1059\,z^{-5} + 0.3077\,z^{-6} + 0.1059\,z^{-7} - 0.2039\,z^{-8} - 0.2039\,z^{-9}$$
$$+ 0.0211\,z^{-10} + 0.1059\,z^{-11} + 0.0211\,z^{-12}$$