Programming Assignment 2 (RM Codes and Decoding)

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Instructions

- This assignment is created in a graded manner so as to aid you in writing the code for encoding and decoding of RM Codes. Hence proceed in the order as given in the assignment, solving each problem completely. Some questions (specifically the first) require you to find out the answers in a elaborate manner and check them with the inbuilt algebra functions or toolboxes of your favourite programming languages. This is to enable you to not depend on these high-level toolboxes and implement some basic algebra by yourself.
- Please use Python (whatever extension of Python is applicable) or Matlab.
- Your code should be clean and should carry helpful comments at every stage (please include comments about any special functions you use as it is useful for the reader).
- Please add helpful English statements when the code runs to enable the user to understand what (s)he is giving as input and how it should be given.
- Submit both the code and the outputs (make sure that the outputs are presented 'stage by stage' with suitable textual commentary to the reader, as though you are testing the code at each stage, and not just the final output is provided).
- Marks (3 questions (for now)): 2.5*4+5*4+(5+10)=45. This will be appropriately scaled. Additional question may be added later.

Assignment

1. Construct a program to encode any $RM(m, r)$ code. Using your code find the codewords corresponding to the message polynomials given in the respective codes.

(a) $X_1 + X_2 + X_3 + X_4$ in $RM(m = 5, r = 1)$.

(b) Verify the minimum distance for $RM(10, 4)$ code by computing the codeword whose weight is minimum distance.

(c) $X_4 + X_5 + X_8 + 1$ in $RM(15, 1)$

(d) $\sum_{A \in \mathcal{A}_9} X_A$ in $RM(20, 10)$, where $\mathcal{A}_9$ refers to all 9-sized subsets of $\{1, \ldots, 10\}$ and $X_A = \prod_{i \in A} X_i$.

2. For each of the codes in Q1, allow user to input message polynomial and also some value $\alpha \leq \frac{d_{min}-1}{2}$. Choose a random vector of weight α as the error vector. Add it to the specific codeword generated by the user specified message polynomial. Run Reed's Majority logic decoding algorithm to compute the message polynomial and the codeword correctly.

3.
   - Write a program to implement the Hadamard Transform and also the Fast Hadamard Transform for length 2^m for arbitrary length m. Show the comparison between the naive computation and the FHT in number of operations computed (use counter variables for every operation you do).
   - For the codes in Q1 (a) and (c) in Q1, decode (upto half minimum distance) using the Fast Hadamard Transform technique and compare your answer with the Majority logic decoding for same inputs of message and error vector as in Q2.