MACM 316 Asn 8 Report
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The double pendulum system, expressed as a first order ODE, is solved using Matlab's ode45 routine and Euler's method. When Euler's method is used to solve the ODE, we can see that as step size h increases $\varphi_1(t)$ and $\varphi_2(t)$ becomes more sinusoidal or periodic (see **figures 1** and **2**). These signs suggest that robustness improves with increasing h.

Figure 1: Approximated ϕ_1s to Double Pendulum Problem when 0 <= t <= 10

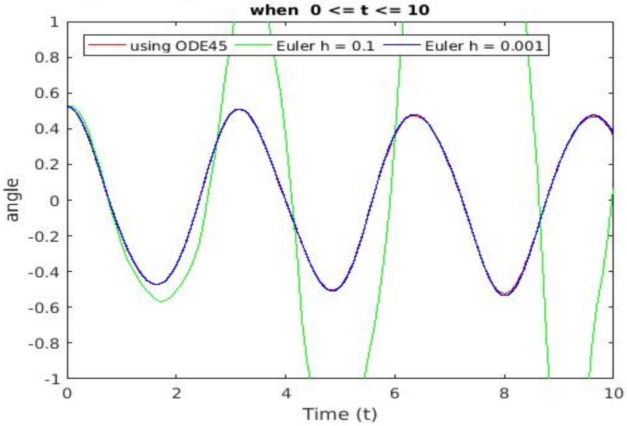
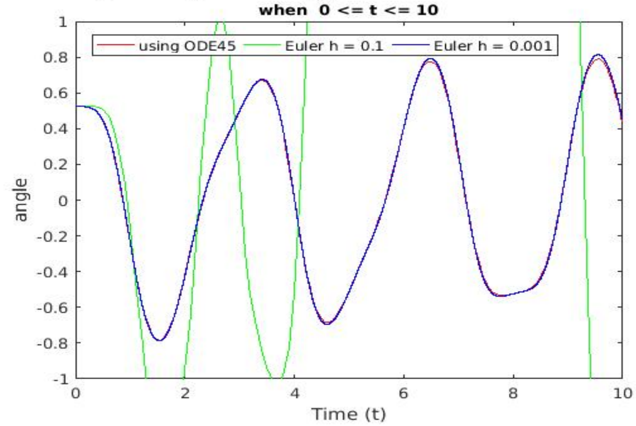


Figure 2: Approximated ϕ_2s to Double Pendulum Problem when 0 <= t <= 10



ode45 gives much better accuracy than the first degree Euler's method, for the former is an RK4 method with a global error of degree $O(n^4)$. Euler's method is also found to be less efficient (see **figure 3**) as it requires large number of time steps to produce more robust results . At low values of t – i.e. when $t_{end} = 10$ – with h = 0.001, $\varphi_1(t)_{euler}$ and $\varphi_2(t)_{euler}$ resembles $\varphi_1(t)_{ode45}$ and $\varphi_2(t)_{ode45}$ (see **figures 1** and **2**), but accumulation error will come into play as t increases. For instance, when using Euler's method to find the total energy of the system[1] (see **figure 5**), not only is the total energy not conserved, but the oscillations are getting worse with increasing values of t. Conversely, despite minor oscillations, total energy of the system calculated using ode45 is still preserved as the energy values across t are kept within the -59 ± 0.5 range (see **figure 4**). In other words, ode45 gives more trustworthy results.

Figure 3: Average computational time required to solve the Double Pendulum Problem (after 20 iterations)

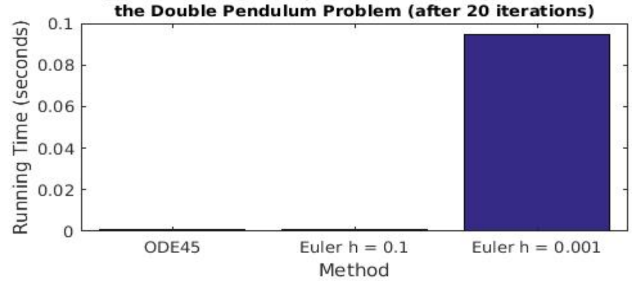


Figure 4: Estimated total energy of the system using ode45 when 0 <= t <= 200

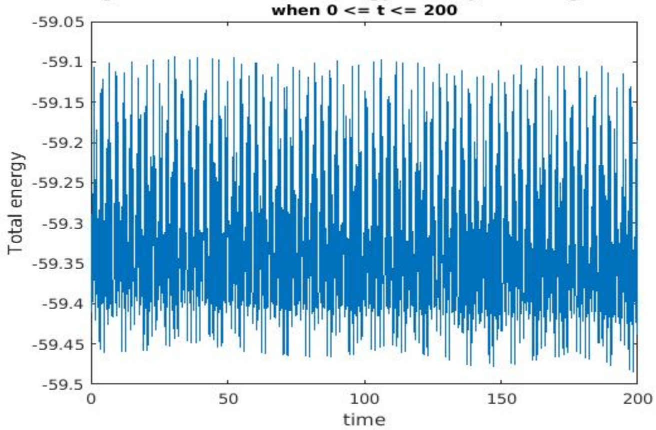
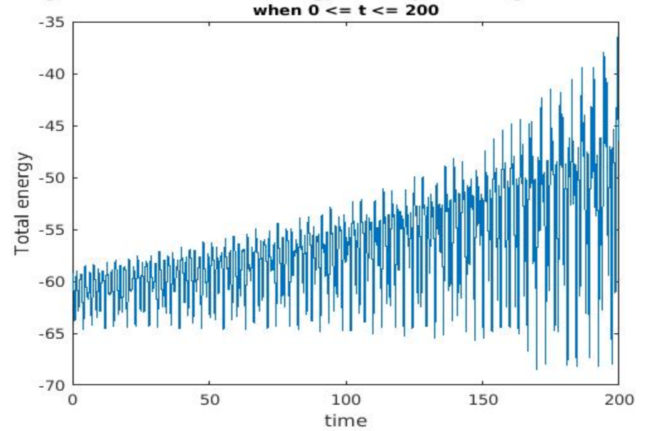


Figure 5: Estimated total energy of the system using Eulers method when 0 <= t <= 200



1 To improve computational efficiency and reduce cancellation and accumulation errors, P(t) and K(t) are solved as follows in the Matlab script:

$P(t) = -g\,[\,(m_1 + m_2)\,l_1 \cos(\varphi_1) + m_2\,l_2 \cos(\varphi_2)\,]$

$K(t) = \tfrac{1}{2}\,[\,(m_1 + m_2)\,\Omega_1^2 + m_2\,\Omega_2^2\,] + m_2\,\Omega_1\Omega_2\,[\,\cos(\varphi_1\varphi_2) + \sin(\varphi_1\varphi_2)\,],\ \Omega_1 = \varphi'_1\,l_1\ ;\ \Omega_2 = \varphi'_2\,l_2$