MACM 316 – Assignment # 8

Due Date: April 5th, 2017, at 11pm.

You must upload both your code (to Assignment #8 scripts/codes) and your report (to Assignment #8 computing report). The assignment is due at 11:00pm. I have set the due time in Canvas to 11:05pm and if Canvas indicates that you submitted late, you will be given 0 on the assignment. Your computing report must be exactly 1 page. There will be a penalty given if your report is longer than one page.

- Please read the **Guidelines for Assignments** first.
- Keep in mind that Canvas discussions are open forums.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

Computing Assignment – Simulating the double pendulum

This assignment requires you to do quite a bit of coding on your own so make sure that you start early. Also, you need to present a lot of results in your report so think about how you are going to organize the report. Do not use large figures but make sure that the figure fonts are large enough for the marker to be able to read them.

In this assignment you will simulate the motion of a double pendulum i.e. a pendulum with another pendulum attached to its end. The figure below shows a schematic depiction of the setup and introduces our notation for the different variables in this assignment. See

https://www.myphysicslab.com/pendulum/double-pendulum/double-pendulum-en.html

for an animation that you can play around with.

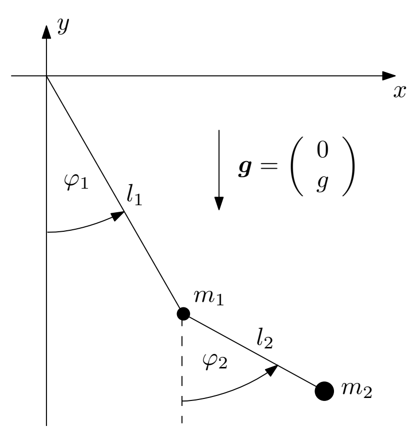

Figure 1: Schematic drawing of the double pendulum.

Assuming that the lengths l_1 and l_2 are constant and the connecting strings are massless one can show that the motion of the pendulums is governed by the following system of second order ODEs:

$$\begin{aligned} \phi_1''(t) + \frac{g}{l_1}\sin(\phi_1(t)) + \frac{m_2}{m_1+m_2}\frac{l_2}{l_1}\left[\cos(\phi_2(t)-\phi_1(t))\phi_2''(t) - \sin(\phi_2(t)-\phi_1(t)){\phi'}_2(t)^2\right] &= 0, \\ {\phi''}_2(t) + \frac{g}{l_2}\sin(\phi_2(t)) + \frac{l_1}{l_2}\left[\cos(\phi_2(t)-\phi_1(t)){\phi''}_1(t) + \sin(\phi_2(t)-\phi_1(t)){\phi'}_1(t)^2\right] &= 0. \end{aligned} \tag{$\star$}$$

In order to solve this system we introduce a new variable $\mathbf{x}(t) = (x_1(t), x_2(t), \cdots, x_9(t))^T$ (column vector) where

$$x_1(t) = \phi_1(t), \quad x_2(t) = \phi'{}_1(t), \quad x_3(t) = \phi_2(t), \quad x_4(t) = \phi'{}_2(t),$$
$$x_5(t) = g, \quad x_6(t) = m_1, \quad x_7(t) = m_2, \quad x_8(t) = l_1, \quad x_9(t) = l_2.$$

After some algebraic manipulation (which is not directly related to this assignment) we can rewrite $(\star)$ as a system of first order ODEs of the form

$$\mathbf{x}'(t) = \mathbf{f}(t, \mathbf{x}(t)). \tag{$\star\star$}$$

The script `double_pendulum_ODE.m` contains a function that evaluates the right hand side $\mathbf{f}$ at a given value of t and $\mathbf{x}$.

(a) Your first task in the assignment is to approximate the solution of this ODE system using the Euler's method for $0 \le t \le t_{\text{end}} = 10$. Use the initial condition

$$\mathbf{x}(0) = (\pi/6,\ 0,\ \pi/6,\ 0,\ 9.8,\ 2,\ 1,\ 2,\ 1)^T$$

and plot $\phi_1(t)$ and $\phi_2(t)$ (that is $x_1(t)$ and $x_3(t)$) as a function of time using time steps of size $h = 10^{-1}, 10^{-3}$. Make sure that you get the code runing with the smaller step size and a short time interval. Describe your obersvations and comment on robustness of your code with respect to the step size h. I suggest you write your code by completing the script `double_pendulum_skeleton.m` that contains the skeleton of the code that you need to write. Note that there are instructions in the skeleton code as well.

(b) Your next task is to repeat the above calculation using MATLAB's `ode45`. I suggest that you begin by reading the manual page for the `ode45` function (just google "MATLAB ode45" or search in MATLAB's help documentation). You should use the same initial conditions and time interval as in part (a). However, `ode45` does not require you to choose a time step. Plot $\phi_1(t)$ and $\phi_2(t)$ as before and compare your solution to those obtained using Euler's method. Which method is more accurate? which one is more efficient?

(c) The kinetic and potential energies of the double pendulum are given by the following expressions:

$$K(t) = \frac{1}{2}m_1\phi_1'(t)^2 l_1^2 + \frac{1}{2}m_2[\phi_1'(t)^2 l_1^2 + \phi_2'(t)^2 l_2^2 + 2\phi_1'(t) l_1 \phi_2'(t) l_2 \cos(\phi_1(t) - \phi_2(t))]$$
$$P(t) = -m_1 l_1 g \cos(\phi_1(t)) - m_2 g[l_1 \cos(\phi_1(t)) + l_2 \cos(\phi_2(t))]$$

Repeat the calculations in parts (a) and (b) for $t_{\text{end}} = 200$ and plot the total enery of this system $E(t) = K(t) + P(t)$ for the solution of the `ode45` solver and your Euler's solver with $h = 10^{-3}$. Describe your observations. Which method is more trustworthy?