homework-01

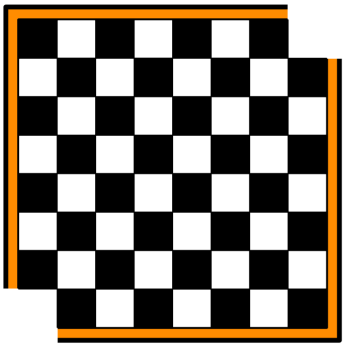
our group name here

February 25, 2017

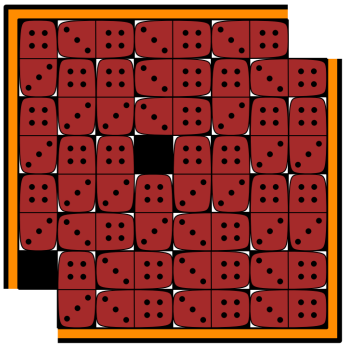
1 Broken Chessboard and Jumping With Coins

1.1 Tiling a Damaged Checkerboard

Exercise 1.1.
Firstly, we color the chessboard in black and white.



So now, we have 32 black squares and 30 white squares.
And if we put a tile in this chessboard, it will cover exactly a black square and a white squre, no matter how we put it.
So if we continue put tiles on it, we will have two black squares left in the end.



It means that whatever we do we will always get stuck because there are always two more black squares than white squares.
So it is obvious that we cannot tile this damaged chessboard.
Exercise 1.2.
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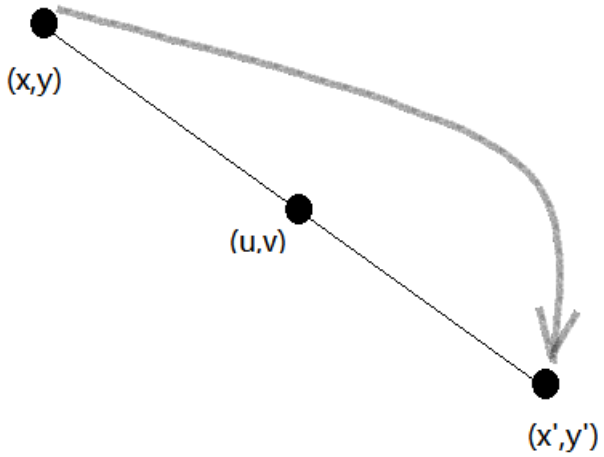
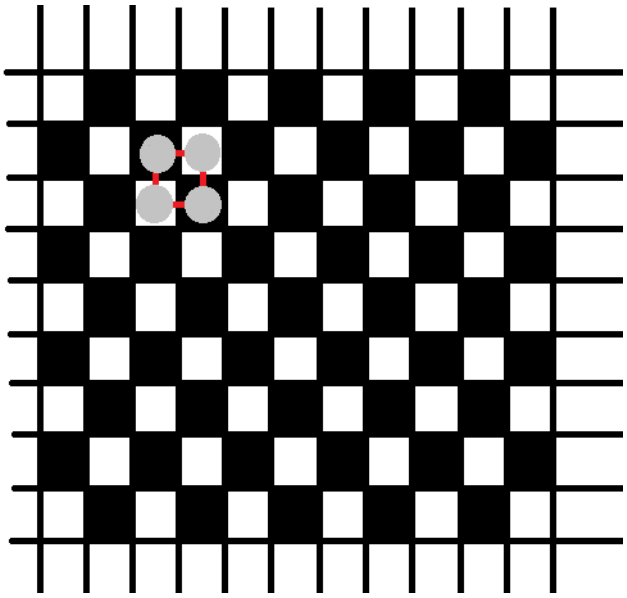
1.2 Jumping with Coins

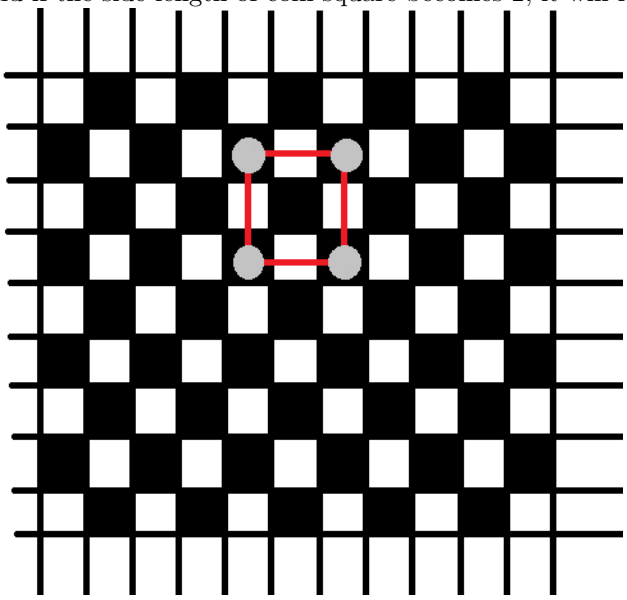
Exercise 1.3.
...
Exercise 1.4.
...
Exercise 1.5.
A coin is at a position $(x, y) \in \mathbf{R}^2$.
we assume that in the begning the pos are (0,0),(0,1),(1,0),(1,1).



$(u, v) + (u, v) - (x, y) = (2u - x, 2v - y)$
if $(u, v), (x, y) \in \mathbf{Z}^2$, then $(2u - x, 2v - y) \in \mathbf{Z}^2$.
Thus the coins will always be on an integer position. Now let (x,y) be the position of a coin,
if $(x, y) \in \mathbf{Z}^2$, we can say coin is $\begin{cases} white & \text{while x+y is odd} \\ black & \text{while x+y is even} \end{cases}$.
So, now the color of (x,y) is $x + y \mod 2$, the color of (2u-x,2u-y) is $(2u - x) + (2v - y) \mod 2 = (x + y) \mod 2$,they are on the same color square.
Considering that, we put four coins on a chessboard like this:



We can see there are two coins on white square and two on black square.
And if the side length of coin square becomes 2, it will look like this:



In this situation, all coins are on white (or black) square. But we have already proved that every coin's color will not change, so this pattern can never be achieved.

Exercise 1.6.

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Exercise 1.7.

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Exercise 1.8.
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2 Exclusion-Inclusion

2.1 Sets

Exercise 2.1.
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Exercise 2.2.
...
Exercise 2.3.
...

3 Feasible Intersection Patterns

Exercise 3.1.
...
Exercise 3.2.
...
Exercise 3.3.
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