CHAPTER 17

AN INTRODUCTION TO DECISION THEORY

LEARNING OBJECTIVES

When you have completed this chapter, you will be able to:

- **LO1** Identify and apply the three components of a decision.
- **LO2** Analyze a decision using expected monetary value.
- **LO3** Analyze a decision using opportunity loss.
- **LO4** Apply maximin, maximax, and minimax regret strategies to make a decision.
- **LO5** Compute and explain the expected value of perfect information.
- **LO6** Apply sensitivity analysis to evaluate a decision subject to uncertainty.
- **LO7** Use a decision tree to illustrate and analyze decision making under certainty.

Blackbeard's Phantom Fireworks is considering introducing two new bottle rockets. The company can add to the current line both, neither, or just one of the two. The success of these products depends on consumers' reaction. These reactions can be summarized as good, fair, or poor. The company's revenues are estimated in the payoff table in Exercise 11. Compute the expected monetary value for each decision. (Exercise 11a, LO2)

17.1 INTRODUCTION

A branch of statistics called **statistical decision theory** that uses probability has been developed since the early 1950s. As the name implies, the focus is on the process of making decisions and explicitly includes the payoffs that may result from selecting a particular decision alternative. In contrast, classical statistics focuses on estimating a parameter, such as the population mean, constructing a confidence interval, or conducting a hypothesis test. Classical statistics does not address the financial consequences.

Statistical decision theory is concerned with determining which decision, from a set of possible alternatives, is optimal for a particular set of conditions. Consider the following examples of decision-theory problems:

- Ford Motor Company must decide whether to purchase assembled door locks for the Ford F-150 truck or to manufacture and assemble the door locks at one of its plants. If sales of the F-150 truck continue to increase, it will be more profitable to manufacture and assemble the parts. If sales level off or decline, it will be more profitable to purchase the door locks assembled. Should Ford make or buy the door locks?
- Banana Republic developed a new line of winter jackets that are very popular in cold weather regions. It would like to purchase commercial television time during the upcoming Stanley Cup final. If both teams that play in the game are from the warm parts of Canada and the United States, it estimates that only a small proportion of the viewers will be interested in the jackets. However, a match-up between two teams that come from cold climates would reach a large proportion of viewers who wear jackets. Should Banana Republic purchase commercial television time?

- General Electric (GE) is considering three options with regard to the prices of refrigerators for next year. GE could (1) raise the prices by 5%, (2) raise the prices by 2.5%, or (3) leave the prices as they are. The final decision will be based on sales estimates and on GE's knowledge of what other refrigerator manufacturers might do.

In each of these cases, the decision is characterized by several alternative courses of action and several factors not under the control of the decision maker. For example, Banana Republic has no control over which teams reach the Stanley Cup final. These cases characterize the nature of decision making. Possible decision alternatives can be listed, possible future events determined, and even probabilities established, but *the decisions are made in the face of uncertainty*.

LO1 17.2 ELEMENTS OF A DECISION

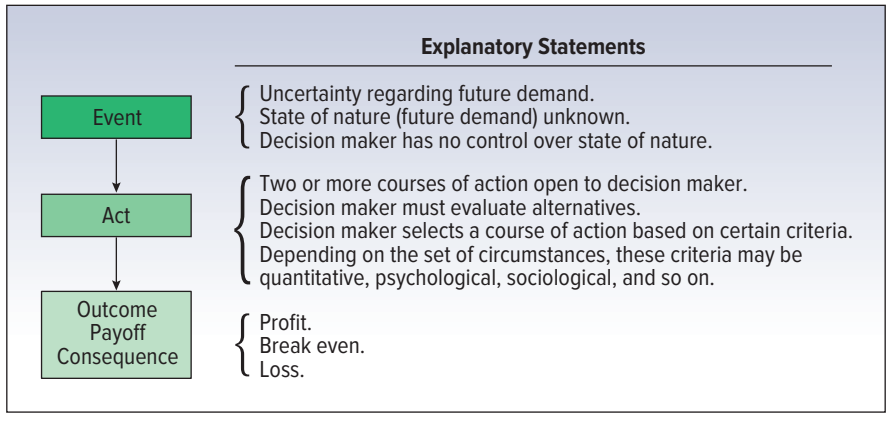
There are three components to any decision: (1) the choices available, or alternatives; (2) the states of nature, which are not under the control of the decision maker; and (3) the payoffs. These concepts will be explained in the following paragraphs.

The **alternatives**, or **acts**, are the choices available to the decision maker. Ford can decide to manufacture and assemble the door locks at one of its plants, or it can decide to purchase them. Banana Republic has two advertising alternatives: to purchase commercial television time or not. GE is considering three pricing alternatives regarding the marketing of refrigerators. To simplify our presentation, we assume the decision maker can select from a rather small number of outcomes. With the help of computers, however, the decision alternatives can be expanded to a large number of possibilities.

The **states of nature** are the uncontrollable future events. The state of nature that actually happens is outside the control of the decision maker. Ford does not know whether demand will remain high for the F-150 truck. Banana Republic cannot determine whether warm-weather teams or cold-weather teams will play in the Stanley Cup final. GE does not know how competitors will price refrigerators.

A **payoff** is needed to compare each combination of decision alternative and state of nature. Ford may estimate that if it assembles door locks at one of its plants and the demand for the new model trucks is low, the payoff will be $40 000. Conversely, if it purchases the door locks assembled and the demand is high, the payoff is estimated to be $22 000. Banana Republic needs to estimate the payoffs for purchasing commercial television time given estimates of the potential audience for the winter jacket. GE needs to estimate the payoffs for each pricing alternative given estimates of competitor's pricing.

The main elements of the decision under conditions of uncertainty are identified in the following diagram:



In many cases, we can make better decisions if we establish probabilities for the states of nature. These probabilities may be based on historical data or subjective estimates. Ford may estimate the probability of continued high demand as 0.70. Banana Republic may estimate a probability of 0.75 that the commercial will reach the targeted audience. GE may estimate the probability to be 0.25 that Amana and other manufacturers will raise the prices of their refrigerators.

17.3 DECISION MAKING UNDER CONDITIONS OF UNCERTAINTY

We begin this section with an example of decision making under uncertainty. The purpose of the example is to explain a logical process to evaluate a set of alternatives and select an alternative as the decision.

The first step is to set up a payoff table.

Payoff Table

Bob Hill, a small investor, has $1100 to invest. He has studied several common stocks and narrowed his choices to three, namely, Kayser Chemicals, Rim Homes, and Texas Electronics. He estimated that if his $1100 were invested in Kayser Chemicals and a strong bull market developed by the end of the year (i.e., stock prices increased drastically), the value of his Kayser stock would more than double, to $2400. However, if there were a bear market (i.e., stock prices declined), the value of his Kayser stock could conceivably drop to $1000 by the end of the year. His predictions regarding the value of his $1100 investment for the three stocks for a bull market and for a bear market are shown in Table 17–1. This table is a **payoff table**.

TABLE 17–1 Payoff Table for Three Common Stocks under Two Market Conditions

Purchase	Bull Market, S_1	Bear Market, S_2
Kayser Chemicals (A_1)	$2400	$1000
Rim Homes (A_2)	2200	1100
Texas Electronics (A_3)	1900	1150

The various choices are called the **decision alternatives** or the **acts**. There are three in this situation. Let A_1 be the purchase of Kayser Chemicals, A_2 the purchase of Rim Homes, and A_3 the purchase of Texas Electronics. Whether the market turns out to be bear or bull is not under the control of Bob Hill. These uncontrolled future events are the **states of nature**. Let the bull market be represented by S_1 and the bear market by S_2.

LO2

Expected Payoff

If the payoff table were the only information available, the investor might take a conservative action and buy Texas Electronics in order to be assured of at least $1150 at the end of the year (a slight profit). A speculative venture, however, might be to buy Kayser Chemicals, with the possibility of more than doubling the $1100 investment.

Any decision regarding the purchase of one of the three common stocks made solely on the information in the payoff table would ignore the valuable historical records kept by investment services relative to stock price movements over a long period. Suppose a study of these investment services revealed that during the past 10 years stock market prices increased six times and declined only four times. According to this information, the probability of a market rise is 0.60, and the probability of a market decline is 0.40.

Assuming these historical frequencies are reliable, we see that the payoff table and the probability estimates (0.60 and 0.40) can be combined to arrive at the **expected payoff** of buying each of the three stocks. Expected payoff is also called **expected monetary value**, shortened to EMV. It can also be described as the **mean payoff**. The calculations needed to arrive at the expected payoff for the act of purchasing Kayser Chemicals are shown in Table 17–2.

TABLE 17–2 Expected Payoff for the Act of Buying Kayser Chemicals, EMV (A_1)

State of Nature	Payoff ($)	Probability of State of Nature	Expected Value ($)
Market rise, S_1	$2400	0.60	$1440
Market decline, S_2	1000	0.40	400
			$1840

To explain one expected monetary value calculation, note that if the investor had purchased Kayser Chemicals and the market prices declined, the value of the stock would be only $1000 at the end of the year (from Table 17–1). Past experience, however, revealed that this event (a market decline) occurred only 40% of the time. In the long run, therefore, a market decline would contribute $400 to the total expected payoff from the stock, found by: $1000 × 0.40. Adding the $400 to the $1440 expected under rising market conditions gives $1840, the "expected" payoff in the long run.

These calculations are summarized as follows:

EXPECTED MONETARY VALUE

$$\text{EMV}(A_i) = \Sigma[P(S_j) \times V(A_i, S_j)] \qquad [17\text{–}1]$$

where:

- $\text{EMV}(A_i)$ refers to the expected monetary value of decision alternative, *i*. There may be many decisions possible. We will let *1* stand for the first decision, *2* for the second, and so on. The lower-case letter *i* represents the entire set of decisions.
- $P(S_j)$ refers to the probability of the states of nature. There can be an unlimited number, so we will let *j* represent this possible outcome.
- $V(A_i, S_j)$ refers to the value of the payoffs. Note that each payoff is the result of a combination of a decision alternative and a state of nature.

$\text{EMV}(A_1)$, the expected monetary value for the decision alternative of purchasing Kayser Chemicals stock, is computed by:

$$\begin{aligned}\text{EMV}(A_1) &= P(S_1) \times V(A_1, S_1) + P(S_2) \times V(A_1, S_2)\\ &= 0.60(\$2400) + 0.40(\$1000) = \$1840\end{aligned}$$

Purchasing Kayser Chemicals stock is only one possible choice. The expected payoffs for the acts of buying Kayser Chemicals, Rim Homes, and Texas Electronics are given in Table 17–3.

TABLE 17–3 Expected Payoffs for Three Stocks

Purchase	Expected Payoff ($)
Kayser Chemicals	$1840
Rim Homes	1760
Texas Electronics	1600

An analysis of the expected payoffs in Table 17–3 indicates that purchasing Kayser Chemicals would yield the greatest expected profit. This outcome is based on (1) the investor's estimated future value of the stocks and (2) historical experience with respect to the rise and decline of stock prices. It should be emphasized that although purchasing Kayser stock represents the best action under the expected-value criterion, the investor still might decide to buy Texas Electronics stock in order to minimize the risk of losing some of the $1100 investment.

self-review 17–1 Verify the conclusion, shown in Table 17–3, that the expected payoff for the act of purchasing Rim Homes stock is $1760.

EXERCISES

1. The following payoff table was developed. Let $P(S_1) = 0.30$, $P(S_2) = 0.50$, and $P(S_3) = 0.20$. Compute the expected monetary value for each of the alternatives. What decision would you recommend?

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$50	$70	$100
A_2	90	40	80
A_3	70	60	90

2. The Wilhelms Cola Company plans to market a new pineapple-flavoured cola this summer. The decision is whether to package the cola in returnable or in nonreturnable bottles. Currently, the provincial legislature is considering eliminating nonreturnable bottles. Tybo Wilhelms, president of Wilhelms Cola Company, has discussed the problem with his government representative and established the probability to be 0.70 that nonreturnable bottles will be eliminated. The table below shows the estimated monthly profits (in thousands of dollars) if the cola is bottled in returnable versus nonreturnable bottles. Of course, if the law is passed and the decision is to bottle the cola in nonreturnable bottles, all profits would be from out-of-province sales. Compute the expected profit for both bottling decisions. Which decision do you recommend?

Alternative	Law Is Passed ($ thousands), S_1	Law Is Not Passed ($ thousands), S_2
Returnable bottle	$80	$40
Nonreturnable bottle	25	60

LO3 Opportunity Loss

Another method to analyze a decision regarding which common stock to purchase is to determine the profit that might be lost because the state of nature (the market behaviour) was not known at the time the investor bought the stock. This potential loss is called **opportunity loss** or **regret**. To illustrate, suppose that the investor had purchased the common stock of Rim Homes and a bull market developed. Further, suppose the value of his Rim Homes stock increased from $1100 to $2200, as anticipated. But had the investor bought Kayser Chemicals stock and market values increased, the value of his Kayser stock would be $2400 (from Table 17–1). Thus, the investor missed making an extra profit of $200 by buying Rim Homes instead of Kayser Chemicals. To put it another way, the $200 represents the opportunity loss for not knowing the correct state of nature. If market prices did increase, the investor would have *regretted* buying Rim Homes. However, had the investor bought Kayser Chemicals and market prices increased, he would have had no regret, that is, no opportunity loss.

The opportunity losses corresponding to this example are given in Table 17–4. Each amount is the outcome (opportunity loss) of a particular combination of acts and a state of nature, that is, stock purchase and market reaction.

Note that the stock of Kayser Chemicals would be a good investment choice in a rising (bull) market, Texas Electronics would be the best buy in a declining (bear) market, and Rim Homes is somewhat of a compromise.

TABLE 17–4 Opportunity Losses for Various Combinations of Stock Purchase and Market Movement

	Opportunity Loss ($)	
Purchase	**Market Rise**	**Market Decline**
Kayser Chemicals	$0	$150
Rim Homes	200	50
Texas Electronics	500	0

self-review 17–2 Refer to Table 17–4. Verify that the opportunity loss for:

(a) Rim Homes, given a market decline, is $50.
(b) Texas Electronics, given a market rise, is $500.

EXERCISES

3. Refer to Exercise 1. Develop an opportunity loss table. Determine the opportunity loss for each decision.
4. Refer to Exercise 2, involving the Wilhelms Cola Company. Develop an opportunity loss table, and determine the opportunity loss for each decision.

Expected Opportunity Loss

The opportunity losses in Table 17–4 again ignore the historical experience of market movements. Recall that the probability of a market rise is 0.60 and that of a market decline 0.40. These probabilities and the opportunity losses can be combined to determine the *expected opportunity loss*. These calculations are shown in Table 17–5 for the decision to purchase Rim Homes. The expected opportunity loss is $140.

TABLE 17–5 Expected Opportunity Loss for the Act of Buying Rim Homes Stock

State of Nature	Opportunity Loss ($)	Probability of State of Nature	Expected Opportunity Loss ($)
Market rise, S_1	$200	0.60	$120
Market decline, S_2	50	0.40	20
			$140

Interpreting, the expected opportunity loss of $140 means that, in the long run, the investor would lose the opportunity to make an additional profit of $140 if he decided to buy Rim Homes stock. This expected loss would be incurred because the investor was unable to accurately predict the trend of the stock market. In a bull market, he could earn an additional $200 by purchasing the common stock of Kayser Chemicals, but in a bear market, an investor could earn an additional $50 by buying Texas Electronics stock. When weighted by the probability of the event, the expected opportunity loss is $140.

These calculations are summarized as follows:

EXPECTED OPPORTUNITY LOSS $$\text{EOL}(A_i) = \Sigma[P(S_j) \times R(A_i, S_j)] \quad [17\text{–}2]$$

where:

$\text{EOL}(A_i)$ refers to the expected opportunity loss for a particular decision alternative.
$P(S_j)$ refers to the probability associated with the states of nature, j.
$R(A_i, S_j)$ refers to the regret or loss for a particular combination of a state of nature and a decision alternative.

EOL(A_2), the regret, or expected opportunity loss, for selecting Rim Homes, is computed as follows:

$$\begin{aligned}\text{EOL}(A_2) &= P(S_1) \times R(A_2, S_1) + P(S_2) \times R(A_2, S_2)\\ &= 0.60(\$200) + 0.40(\$50) = \$140\end{aligned}$$

The expected opportunity losses for the three decision alternatives are given in Table 17– 6. The lowest expected opportunity loss is $60; that is, the investor would experience the least regret, on average, if he purchased Kayser Chemicals.

TABLE 17–6 Expected Opportunity Losses for the Three Stocks

Purchase	Expected Opportunity Loss ($)
Kayser Chemicals	$60
Rim Homes	140
Texas Electronics	300

Incidentally, note that the decision to purchase Kayser Chemicals stock, as it offers the lowest expected opportunity loss, reinforces the decision made previously–that Kayser stock would ultimately result in the highest expected payoff ($1840). These two approaches (lowest expected opportunity loss and highest expected payoff) will always lead to the same decision concerning which course of action to follow.

self-review 17–3 Referring to Table 17–6, verify that the expected opportunity loss for the act of purchasing Texas Electronics is $300.

EXERCISES

5. Refer to Exercises 1 and 3. Compute the expected opportunity losses.
6. Refer to Exercises 2 and 4. Compute the expected opportunity losses.

L04 17.4 MAXIMIN, MAXIMAX, AND MINIMAX REGRET STRATEGIES

Suppose that several financial advisors consider the purchase of Kayser Chemicals stock too risky. They note that the payoff might not be $1840, but only $1000 (from Table 17–1). Arguing that the stock market is too unpredictable, they urge the investor to take a more conservative position and buy Texas Electronics. This is called a **maximin strategy:** it maximizes the minimum gain. On the basis of the payoff table (see Table 17–1), they reason that the investor would be assured of at least a $1150 return, that is, a small profit. Those who subscribe to this somewhat pessimistic strategy are sometimes called *maximiners*.

At the other extreme are the optimistic *maximaxers*, who would select the stock that maximizes the maximum gain. If their *maximax strategy* were followed, the investor would purchase Kayser Chemicals stock. These optimists stress that there is a possibility of selling the stock in the future for $2400 instead of only $1150, as advocated by the maximiners.

Another strategy is the *minimax regret strategy*. Advisors advocating this approach would scan the opportunity losses in Table 17–4 and select the stock that minimizes the maximum regret. In this example, it would be Kayser Chemicals stock, with a maximum opportunity loss of $150. Recall that you wish to *avoid* opportunity losses! The maximum regrets were $200 for Rim Homes and $500 for Texas Electronics.

17.5 VALUE OF PERFECT INFORMATION

LO5

How much is "perfect" information worth?

Value of perfect information The difference between the maximum payoff under conditions of certainty and the maximum payoff under uncertainty.

Before deciding on a stock, the investor might want to consider ways of predicting the movement of the stock market. If he knew precisely what the market would do, he could maximize profit by always purchasing the correct stock. The question is: what is this advance information worth? The dollar value of this information is called the **expected value of perfect information**, written EVPI. In this example, if perfect information were available, Bob Hill would know what would happen in the future—whether the stock market would rise or decline. If Bob had this information, he would always make the best decision. The question is: what is the most that Bob should be willing to pay for perfect information?

An acquaintance, who is an analyst with a large brokerage firm, said that he would be willing to supply Bob with information that he might find valuable in predicting market rises and declines. Of course, there would be a fee, as yet undetermined, for this information, regardless of whether the investor used it. What is the maximum amount that Bob should pay for this special service? $10? $100? $500?

The value of the information from the analyst is, in essence, the expected value of perfect information because the investor would then be assured of buying the most profitable stock. In this example, it is the difference between the maximum value of the stock at the end of the year under conditions of certainty and the value associated with the optimal decision using the expected-value criterion.

To explain, the maximum expected value under conditions of certainty means that the investor would buy Kayser Chemicals if a market rise were predicted and Texas Electronics if a market decline were imminent. The expected payoff under conditions of certainty is $1900. (See Table 17–7.)

TABLE 17–7 Calculations for the Expected Payoff under Conditions of Certainty

State of Nature	Decision	Payoff ($)	Probability of State of Nature	Expected Payoff ($)
Market rise, S_1	Buy Kaiser	$2400	0.60	$1440
Market decline, S_2	Buy Texas Electronics	1150	0.40	460
				$1900

Recall that if the actual behaviour of the stock market were unknown (conditions of uncertainty), the stock to buy would be Kayser Chemicals; its expected value at the end of the period was computed to be $1840 (from Table 17–3). The value of perfect information is, therefore, $60, found by:

$1900	Expected value of stock purchased under conditions of certainty
−1840	Expected value of purchase (Kayser) under conditions of uncertainty
$ 60	Expected value of perfect information

In general, the expected value of perfect information is computed as follows:

EXPECTED VALUE OF PERFECT INFORMATION

$$\text{EVPI} = \text{Expected value under conditions of certainty} - \text{Expected value under conditions of uncertainty} \quad [17\text{–}3]$$

It would be worth up to $60 for the information the stock analyst might supply. In essence, the analyst would be "guaranteeing" a selling price on average of $1900, and if the analyst asked $40 for the information, the investor would be assured of a $1860 payoff, found by: ($1900 − $40). Thus, it would be worthwhile for the investor to agree to this fee ($40) because the expected outcome ($1860) would be greater than the expected value under conditions of uncertainty ($1840). However, if his acquaintance wanted a fee of $100 for the service, the investor would realize only $1800 on average, found by: $1900 − $100. Logically, the service

would not be worth $100 because the investor could expect $1840, on average, without agreeing to this financial arrangement. Note that the expected value of perfect information ($60) is the same as the minimum of the expected regrets (see Table 17–6). That is not an accident.

The output for the investment example using **Excel** is shown below. The expected payoff and the expected opportunity loss are the same as reported in Table 17–3 and Table 17–6. The calculations in the preceding investment example were kept at a minimum to emphasize the new terms and the decision-making procedures. When the number of decision alternatives and the number of states of nature become large, use of a software program or a spreadsheet is recommended.

	Payoff Table			Opportunity Loss Table		
	Bull Market	Bear Market	Expected Value	Bull Market	Bear Market	Expected Value
Kayser Chemicals	$2400	$1000	$1840	$0	$150	$60
Rim Homes	$2200	$1100	$1760	$200	$50	$140
Texas Electronics	$1900	$1150	$1600	$500	$1150	$300

LO6 17.6 SENSITIVITY ANALYSIS

In the foregoing stock selection situation, the set of probabilities applied to the payoff values was derived from historical experience with similar market conditions. Objections may be voiced, however, that future market behaviour may be different from past experiences. Despite these differences, *the rankings of the decision alternatives are frequently not highly sensitive to changes within a plausible range*. As an example, suppose the investor's brother believes that instead of a 60% chance of a market rise and a 40% chance of a decline, the reverse is true–that is, there is a 0.40 probability that the stock market will rise and a 0.60 probability of a decline. Further, the investor's cousin thinks the probability of a market rise is 0.50 and that of a decline is 0.50. A comparison of the original expected payoffs (left column), the expected payoffs for the set of probabilities suggested by the investor's brother (centre column), and those cited by the cousin (right column) is shown in Table 17–8. The decision is the same in all three cases–purchase Kayser Chemicals.

TABLE 17–8 Expected Payoffs for Three Sets of Probabilities

	Expected Payoffs		
Purchase	Historical Experience (probability of 0.60 rise, 0.40 decline) ($)	Brother's Estimate (probability of 0.40 rise, 0.60 decline) ($)	Cousin's Estimate (probability of 0.50 rise, 0.50 decline) ($)
Kayser Chemicals	$1840	$1560	$1700
Rim Homes	1760	1540	1650
Texas Electronics	1600	1450	1525

self-review 17–4 Referring to Table 17–8, verify that:

(a) The expected payoff for Texas Electronics for the brother's set of probabilities is $1450.
(b) The expected payoff for Kayser Chemicals for the cousin's set of probabilities is $1700.

A comparison of the three sets of expected payoffs in Table 17–8 reveals the best alternative would still be to purchase Kayser Chemicals. As might be expected, there are some differences in the expected future values for each of the three stocks.

If there are drastic changes in the assigned probabilities, the expected values and the optimal decision may change. As an example, suppose the prognostication for a market rise was

0.20 and for a market decline 0.80. The expected payoffs would be as shown in Table 17–9. In the long run, the best alternative would be to buy Rim Homes stock. Thus, sensitivity analysis lets you see how accurate the probability estimates need to be in order to feel comfortable with your choice.

TABLE 17–9 Expected Values for Purchasing the Three Stocks

Purchase	Expected Payoff
Kayser Chemicals	$1280
Rim Homes	$1320
Texas Electronics	$1300

self-review 17–5 Is there any choice of probabilities for which the best alternative would be to purchase Texas Electronics stock? (*Hint:* This can be arrived at algebraically or using a trial-and-error method. Try a somewhat extreme probability for a market rise.)

EXERCISES

7. Refer to Exercises 1, 3, and 5. Compute the expected value of perfect information.
8. Refer to Exercises 2, 4, and 6. Compute the expected value of perfect information.
9. Refer to Exercise 1. Revise the probabilities as follows: $P(S_1) = 0.50$, $P(S_2) = 0.20$, and $P(S_3) = 0.30$. Does this change the decision?
10. Refer to Exercise 2. Reverse the probabilities; that is, let $P(S_1) = 0.30$ and $P(S_2) = 0.70$. Does this alter your decision?

LO7

17.7 DECISION TREES

An analytic tool introduced in Chapter 4 that is also useful for studying a decision situation is a **decision tree**. It is a picture of all the possible courses of action and the consequent possible outcomes. A box is used to indicate the point at which a decision must be made, and the branches going out from the box indicate the alternatives under consideration. In Chart 17–1, on the left is a box with three branches radiating from it, representing the acts of purchasing Kayser Chemicals, Rim Homes, or Texas Electronics.

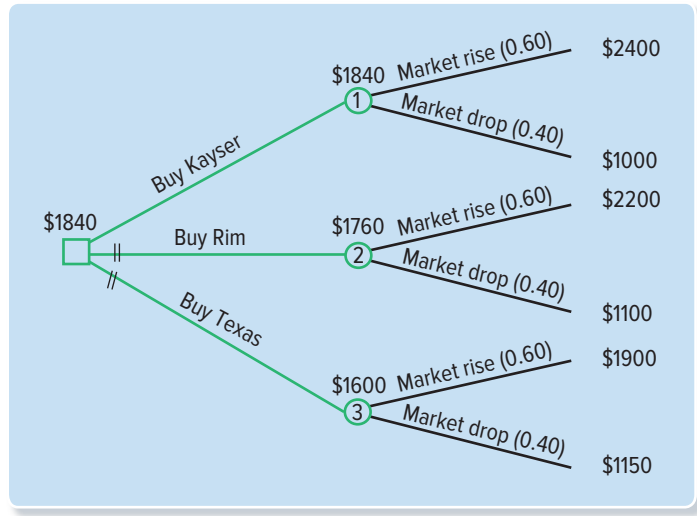


CHART 17–1 Decision Tree for the Investor's Decision

The three nodes, or circles, numbered 1, 2, and 3, represent the expected payoff of each of the three stocks. The branches going out to the right of the nodes show the chance events (market rise or decline) and their corresponding probabilities in parentheses. The numbers at the extreme ends of the branches are the estimated future values of ending the decision process at those points. This is sometimes called the *conditional payoff* to denote that the payoff depends on a particular choice of action and a particular chance outcome. Thus, if the investor purchased Rim Homes stock and the market rose, the conditional value of the stock would be $2200.

After the decision tree has been constructed, the best decision strategy can be found by what is termed *backward induction*. For example, suppose the investor is considering the act of purchasing Texas Electronics. Starting at the lower right in Chart 17–1 with the anticipated payoff given a market rise ($1900) versus a market decline ($1150) and going backward (moving left), the appropriate probabilities are applied to give the expected payoff of $1600 [found by: 0.60($1900) + 0.40($1150)]. The investor would mark the expected value of $1600 above circled node 3 as shown in Chart 17–1. Similarly, the investor would determine the expected values for Rim Homes and Kayser Chemicals.

Assuming the investor wants to maximize the expected value of his stock purchase, $1840 would be preferred over $1760 or $1600. Continuing to the left toward the box, the investor would draw a double bar across branches representing the two alternatives he rejected (numbers 2 and 3, representing Rim Homes and Texas Electronics). The unmarked branch that leads to the box is clearly the best action to follow, namely, buy Kayser Chemicals stock.

The expected value under *conditions of certainty* can also be portrayed via a decision tree analysis (see Chart 17–2). Recall that under conditions of certainty the investor would know *before the stock is purchased* whether the stock market will rise or decline. Hence, he would purchase Kayser Chemicals in a rising market and Texas Electronics in a falling market, and the expected payoff would be $1900. Again, backward induction would be used to arrive at the expected payoff of $1900.

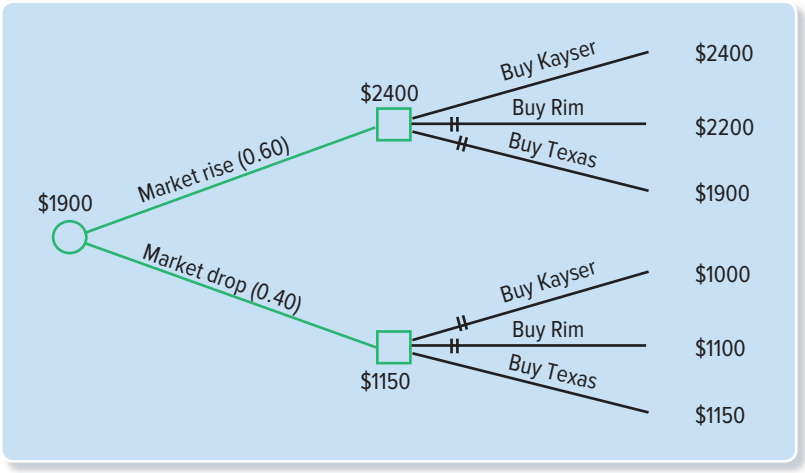


CHART 17–2 Decision Tree, Given Perfect Information

The monetary difference based on perfect information in Chart 17–2 and the decision based on imperfect information in Chart 17–1 is $60, found by: $1900 − $1840. Recall that the $60 is the value of perfect information.

Decision tree analysis provides an alternative way to perform the calculations presented earlier in the chapter. Some managers find that these graphic sketches help them follow the decision logic.

Chapter Summary

I. Statistical decision theory is concerned with making decisions from a set of alternatives.

- **A.** The various courses of action are called the *acts* or *alternatives*.
- **B.** The uncontrollable future events are called the *states of nature*. Probabilities are usually assigned to the states of nature.
- **C.** The consequence of a particular decision alternative and state of nature is called the *payoff*.
- **D.** All possible combinations of decision alternatives and states of nature result in a payoff table.

II. There are several criteria for selecting the best decision alternative.

- **A.** In the expected monetary value (EMV) criterion, the expected value for each decision alternative is computed, and the optimal one (largest—if profits; smallest—if costs) is selected.
- **B.** An opportunity loss table can be developed.
  1. An opportunity loss table is constructed by taking the difference between the optimal decision for each state of nature and the other decision alternatives.
  2. The difference between the optimal decision and any other decision is the opportunity loss or regret due to making a decision other than the optimal one.
  3. The expected opportunity loss (EOL) is similar to the expected monetary value. The opportunity loss is combined with the probabilities of the various states of nature for each decision alternative to determine the expected opportunity loss.
- **C.** The strategy of maximizing the minimum gain is referred to as *maximin*.
- **D.** The strategy of maximizing the maximum gain is called *maximax*.
- **E.** The strategy that minimizes the maximum regret is designated *minimax regret*.

III. The expected value of perfect information (EVPI) is the difference between the best expected payoff under certainty and the best expected payoff under uncertainty.

IV. Sensitivity analysis examines the effects of various probabilities for the states of nature on the expected values.

V. Decision trees are useful for structuring the various alternatives. They present a picture of the various courses of action and the possible states of nature.

Chapter Exercises

11. Blackbeard's Phantom Fireworks is considering two new bottle rockets. The company can add both to the current line, neither, or just one of the two. The success of these products depends on consumers' reactions to the products. These reactions can be summarized as "good," $P(S_1) = 0.30$; "fair," $P(S_2) = 0.50$; or "poor," $P(S_3) = 0.20$. The company's revenues, in thousands of dollars, are estimated in the following payoff table:

	State of Nature ($ thousands)		
Decision	S_1	S_2	S_3
Neither	$0	$0	$0
Product 1 only	125	65	30
Product 2 only	105	60	30
Both	220	110	40

- **a.** Compute the expected monetary value for each decision.
- **b.** What decision would you recommend?
- **c.** Develop an opportunity loss table.
- **d.** Compute the expected opportunity loss for each decision.
- **e.** Compute the expected value of perfect information.

12. A financial executive lives in Ottawa but must travel to Toronto frequently. She can go to Toronto by car, train, or plane. The cost for a plane ticket from Ottawa to Toronto is $285, and it is estimated that the trip takes 65 minutes in good weather and 70 minutes in bad weather. The cost for a train ticket is $190, and the trip takes four hours in good weather and 4 hours and 10 minutes in bad weather. The cost to drive her

own car from Ottawa to Toronto is $70, and this trip takes four hours in good weather and five in bad weather. The executive places a value of $75 per hour on her time. The weather forecast is for a 60% chance of bad weather tomorrow.

What decision would you recommend? (*Hint:* set up a payoff table, and remember that you want to minimize costs.) What is the expected value of perfect information?

13. The Thomas Manufacturing Company has $100 000 available to invest. Matthew Thomas, the president and CEO of the company, would like to either expand his production, invest the money in stocks, or purchase a certificate of deposit from the bank. Of course, the unknown is whether the economy will continue at a high level or there will be a recession. He estimates the likelihood of a recession at 0.20. Whether there is a recession or not, the certificate of deposit will result in a gain of 6%. If there is a recession, he predicts a 10% loss if he expands his production and a 5% loss if he invests in stocks. If there is not a recession, an expansion of production will result in a 15% gain, and stock investment will produce a 12% gain.

a. What decision should he make if he uses the maximin strategy?
b. What decision should Matthew Thomas make if the maximax strategy is used?
c. What decision would be made if he uses the expected-monetary-value criterion?
d. What is the expected value of perfect information?

14. The quality control department at Malcomb Products must either inspect each part in a lot or not inspect any of the parts. That is, there are two decision alternatives: inspect all the parts or inspect none of the parts. The proportion of parts defective in the lot, S_j, is known from historical data to assume the following probability distribution:

State of Nature, S_j	Probability, $P(S_j)$
0.02	0.70
0.04	0.20
0.06	0.10

For the decision not to inspect any parts, the cost of quality is $C = NS_j K$. For inspecting all the items in the lot, it is $C = Nk$, where:

$N =$ 20 (lot size)
$K =$ $18.00 (the cost of finding a defect)
$k =$ $0.50 (the cost of sampling one item)

a. Develop a payoff table.
b. What decision should be made if the expected-value criterion is used?
c. What is the expected value of perfect information?

15. Dude Ranches Incorporated was founded on the idea that many families do not have a sufficient amount of vacation time to drive to the dude ranches in the Rocky Mountain area for their vacations. Various surveys indicated, however, that there was a considerable interest in this type of family vacation, which includes horseback riding, cattle drives, swimming, fishing, and the like. Dude Ranches Incorporated bought a large farm near several eastern cities and constructed a lake, a swimming pool, and other facilities. However, to build a number of family cottages on the ranch would have required a considerable investment. Further, it reasoned that most of this investment would be lost should the ranch-farm complex be a financial failure.

Instead, it decided to enter into an agreement with the Mobile Homes Manufacturing Company to supply a very attractive, authentic, ranch-type mobile home. Mobile Homes agreed to deliver a mobile home on Saturday for $300 a week. Mobile Homes must know early Saturday morning how many mobile homes Dude Ranches Incorporated wants for the forthcoming week. It has other customers to supply and can only deliver the homes on Saturday. This presents a problem. Dude Ranches will have some reservations by Saturday, but indications are that many families do not make them. Instead, they prefer to examine the facilities before making a decision. An analysis of the various costs involved indicated that $350 a week should be charged for a ranch home, including all privileges. The basic problem is how many mobile ranch homes to order from Mobile Homes each week. Should Dude Ranches Incorporated order 10 (considered the minimum), 11, 12, 13, or 14 (considered the maximum)?

Any decision made solely on the information in the payoff table would ignore, however, the valuable experience that Dude Ranches Incorporated has acquired in the past four years (about 200 weeks) actually operating a dude ranch. Its records showed that it always had nine advance reservations. Also, it never had a demand for 15 or more cottages. The occupancy of 10, 11, 12, 13, or 14 ranch cottages, in part, represented families who drove in and inspected the facilities before renting. A frequency distribution showing the number of weeks in which 10, 11, . . . , 14 ranch cottages were rented during the 200-week period is found in the following table:

Number of Cottages Rented	Number of Weeks
10	26
11	50
12	60
13	44
14	20
	200

a. Construct a payoff table.
b. Determine the expected payoffs, and arrive at a decision.
c. Set up an opportunity loss table.
d. Compute the expected opportunity losses, and arrive at a decision.
e. Determine the value of perfect information.

16. The proprietor of the newly built Ski and Swim Lodge has been considering purchasing or leasing several snowmobiles for the use of guests. The owner found that other financial obligations made it impossible to purchase the machines. Snowmobiles Incorporated (SI) will lease a machine for $20 a week, including any needed maintenance. According to SI, the usual rental charge to the guests of the lodge is $25 a week. Gasoline and oil are extra. Snowmobiles Incorporated only leases a machine for the full season. The proprietor of Ski and Swim, knowing that leasing an excessive number of snowmobiles might cause a net loss for the lodge, investigated the records of other resort owners. The combined experience at several other lodges is shown below:

Number of Snow mobiles Demanded by Guests	Number of Weeks
7	10
8	25
9	45
10	20

a. Design a payoff table.
b. Compute the expected profits for leasing 7, 8, 9, and 10 snowmobiles based on the cost of leasing of $20, the rental charge of $25, and the experience of other lodges.
c. Which alternative is the most profitable?
d. Design an opportunity loss table.
e. Find the expected opportunity losses for leasing 7, 8, 9, and 10 snowmobiles.
f. Which act would give the least expected opportunity loss?
g. Determine the expected value of perfect information.
h. Suggest a course of action to the proprietor of the Ski and Swim Lodge. Include in your explanation the various figures, such as expected profit.

17. A furniture store has had numerous inquiries regarding the availability of furniture and equipment that could be rented for large outdoor summer parties. This includes such items as folding chairs and tables, a deluxe grill, propane gas, and lights. No rental equipment of this nature is available locally, and the management of the furniture store is considering forming a subsidiary to handle rentals.

An investigation revealed that most people interested in renting wanted a complete group of party essentials (about 12 chairs, four tables, a deluxe grill, a bottle of propane gas, tongs, etc.). Management decided not to buy a large number of complete sets because of the financial risk involved. That is, if the demand for the rental groups was not as large as anticipated, a large financial loss might be incurred. Further, outright purchase would mean that the equipment would have to be stored during the off-season.

It was then discovered that a firm in Collingwood leased a complete party set for $560 for the summer season. This amounts to about $5 a day. In the promotional literature from the Collingwood firm, a rental fee of $15 was suggested. For each set rented, a profit of $10 would thus be earned. It was then decided to lease from the Collingwood firm, at least for the first season.

The Collingwood firm suggested that on the basis of the combined experience of similar rental firms in other cities, either 41, 42, 43, 44, 45, or 46 complete sets be leased for the season. On the basis of this suggestion, management must now decide on the most profitable number of complete sets to lease for the season.

The leasing firm in Collingwood also made available some additional information gathered from several rental firms similar to the newly formed subsidiary. Note in the following table (which is based on the experience of the other rental firms) that for 360 days of the total of 6000 days' experience—or about 6% of the days—these rental firms rented out 41 complete party sets. On 10% of the days during a typical summer, they rented 42 complete sets, and so on.

Number of Sets Rented	Number of Days	Number of Sets Rented	Number of Days
40	0	44	2400
41	360	45	1500
42	600	46	300
43	840	47	0

a. Construct a payoff table. (As a check figure, for the act of having 41 complete sets available and the event of renting 41, the payoff is $410.)
b. The expected daily profit for leasing 43 complete sets from the Collingwood firm is $426.70; for 45 sets, $431.70; and for 46 sets, $427.45. Organize these expected daily profits into a table, and complete the table by finding the expected daily profit for leasing 41, 42, and 44 sets from the Collingwood firm.
c. On the basis of the expected daily profit, what is the most profitable action to take?
d. The expected opportunity loss for leasing 43 party sets from the Collingwood firm is $11.60; for 45 sets, $6.60; for 46 sets, $10.85. Organize these into an expected opportunity loss table, and complete the table by computing the expected opportunity loss for 41, 42, and 44.
e. On the basis of the expected opportunity loss table, what is the most profitable course of action to take? Does this agree with your decision for part (c)?
f. Determine the value of perfect information. Explain what it indicates in this problem.

18. Tim Waltzer owns and operates Waltzer's Wrecks, a discount car rental agency near the Cleveland Hopkins International Airport. He rents a wreck for $20 a day. He has an arrangement with Landrum Leasing to purchase used cars at $6000 each. His cars receive only needed maintenance and, as a result, are worth only $2000 at the end of the year of operation. Tim has decided to sell all his wrecks every year and purchase a complete set of used cars from Landrum Leasing.

	Numbers of Cars Rented per Day			
	20	21	22	23
Probability	0.10	0.20	0.50	0.20

His clerk-accountant provided him with a probability distribution with respect to the number of cars rented per day.

Tim is an avid golfer and tennis player. He is either on the golf course on weekends or playing tennis indoors. Thus, his car rental agency is only open weekdays. Also, he closes for two weeks during the summer and goes on a golfing tour.

The clerk-accountant estimated that it cost $1.50 per car rental for minimal maintenance and cleaning.

a. How many cars should he purchase to maximize profit?
b. What is the expected value of perfect information?

19. You sign up for a cell phone plan and are presented with this chart showing how your plan "automatically adjusts" to the minutes you use each month. For example: If you select Option 1 and you use 700 minutes the first month, you will only pay $79.99. If your usage then goes down to 200 minutes the second month, you will only pay $29.99. You guess your monthly usage will be 100, 300, 500, or 700 anytime minutes. Assume the probabilities for each event are the same.

Option 1 - Starting at $29.99 per Month	
Anytime Minutes	**Cost**
0–200	$29.99
201–700	$5 for each 50 minutes
Above 700	Additional anytime minutes only 10¢ each
Option 2 - Starting at $34.99 per Month	
Anytime Minutes	**Cost**
0–400	$34.99
401–900	$5 for each 50 minutes
Above 900	Additional anytime minutes only 10¢ each
Option 3 - Starting at $59.99 per Month	
Anytime Minutes	**Cost**
0–1000	$59.99
1001–1500	$5 for each 50 minutes
Above 1500	Additional anytime minutes only 10¢ each

a. Create a payoff (cost) table for this decision.
b. Using the expected monetary value principle, which decision would you suggest?
c. Using the optimistic (maximax cost) approach, which decision would you suggest?
d. Using the pessimistic (maximin cost) strategy, which decision would you suggest?
e. Work out an opportunity loss table for this decision.
f. Using the minimax regret strategy, which choice would you suggest?
g. What is the expected value of perfect information?

20. You are about to drive to your cottage. If your car's engine is out of tune, your gas cost will increase by $100. Having the engine tested will cost $20. If it is out of tune, repairs will cost $60. Before testing, the probability is 30% that the engine is out of tune. What should you do?

21. The following payoff table was developed. Let $P(S_1) = 0.35$, $P(S_2) = 0.45$, and $P(S_3) = 0.20$. Compute the expected monetary value for each of the alternatives. What decision would you recommend?

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$150	$170	$125
A_2	120	140	180
A_3	85	105	90

22. The following payoff table was developed. Let $P(S_1) = 0.15$, $P(S_2) = 0.45$, and $P(S_3) = 0.40$. Compute the expected monetary value for each of the alternatives. What decision would you recommend?

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$15	$25	$30
A_2	12	35	18
A_3	25	20	15

23. The following payoff table was developed. Let $P(S_1) = 0.35$, $P(S_2) = 0.45$, and $P(S_3) = 0.20$.

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$150	$170	$125
A_2	120	140	180
A_3	85	105	90

Develop an opportunity loss table, and determine the opportunity loss for each decision.

24. The following payoff table was developed. Let $P(S_1) = 0.15$, $P(S_2) = 0.45$, and $P(S_3) = 0.40$.

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$15	$25	$30
A_2	12	35	18
A_3	25	20	15

Develop an opportunity loss table, and determine the opportunity loss for each decision.

25. The following payoff table was developed. Let $P(S_1) = 0.35$, $P(S_2) = 0.45$, and $P(S_3) = 0.20$.

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$150	$170	$125
A_2	120	140	180
A_3	85	105	90

Compute the expected opportunity loss for each decision alternative.

26. The following payoff table was developed. Let $P(S_1) = 0.15$, $P(S_2) = 0.45$, and $P(S_3) = 0.40$.

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$15	$25	$30
A_2	12	35	18
A_3	25	20	15

Compute the expected opportunity loss for each decision alternative.

27. The following payoff table was developed. Let $P(S_1) = 0.35$, $P(S_2) = 0.45$, and $P(S_3) = 0.20$.

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$150	$170	$125
A_2	120	140	180
A_3	85	105	90

Compute the expected value of perfect information.

28. The following payoff table was developed. Let $P(S_1) = 0.15$, $P(S_2) = 0.45$, and $P(S_3) = 0.40$.

	State of Nature ($)		
Alternative	S_1	S_2	S_3
A_1	$15	$25	$30
A_2	12	35	18
A_3	25	20	15

Compute the expected value of perfect information.

29. Refer to Exercise 27. Revise the probabilities as follows: $P(S_1) = 0.50$, $P(S_2) = 0.20$, and $P(S_3) = 0.30$. Does this change the decision?

30. Refer to Exercise 28. Revise the probabilities as follows: $P(S_1) = 0.50$, $P(S_2) = 0.30$, and $P(S_3) = 0.20$. Does this change the decision?

Answers to Self-Reviews

17-1

Event	Payoff ($)	Probability of Event	Expected Value ($)
Market rise	$2200	0.60	$1320
Market decline	1100	0.40	440
			$1760

17-2 **(a)** Suppose the investor purchased Rim Homes stock, and the value of the stock in a bear market dropped to $1100 as anticipated (see Table 17–1). Instead, had the investor purchased Texas Electronics and the market declined, the value of the Texas Electronics stock would be $1150. The difference of $50, found by: $1150 − $1100, represents the investor's regret for buying Rim Homes stock.

(b) Suppose that the investor purchased Texas Electronics stock and then a bull market developed. The stock rose to $1900, as anticipated (see Table 17–1). However, had the investor bought Kayser Chemicals stock and the market value increased to $2400 as anticipated, the difference of $500 represents the extra profit the investor could have made by purchasing Kayser Chemicals stock.

17-3

Event	Payoff ($)	Probability of Event	Expected Opportunity Loss ($)
Market rise	$500	0.60	$300
Market decline	0	0.40	0
			$300

17-4 **(a)**

Event	Payoff ($)	Probability of Event	Expected Value ($)
Market rise	$1900	0.40	$ 760
Market decline	1150	0.60	690
			$1450

(b)

Event	Payoff ($)	Probability of Event	Expected Value ($)
Market rise	$2400	0.50	$1200
Market decline	1000	0.50	500
			$1700

17-5 For probabilities of a market rise (or decline) down to 0.333, Kayser Chemicals stock would provide the largest expected profit. For probabilities 0.333 to 0.037, Rim Homes would be the best buy. For 0.037 and below, Texas Electronics would give the largest expected profit. Algebraic solutions:

Kayser: $2400p + (1 - p)1000$

Rim: $2200p + (1 - p)1100$

$$1400p + 1000 = 1100p + 1100$$

$$p = 0.333$$

Rim: $2200p + (1 - p)\,1100$

Texas: $900p + (1 - p)\,1150$

$$1100p + 1100 = -250 + 1150$$

$$p = 0.037$$