

# Supplementary Information to surface second-harmonic generation enhanced by an ultrahigh- $Q$ microresonator

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## 1 Theoretical model

**Nonlinear coupling between the pump and the SH.** The nonlinear coupling is the result of the excitation from the nonlinear polarization  $\mathbf{P}$ , which contains the surface and the bulk response  $\mathbf{P} = \mathbf{P}_0^{\text{surf}} + \mathbf{P}^{\text{bulk}} = \mathbf{P}_0^{\text{surf}} + \mathbf{P}_\gamma^{\text{bulk}} + \mathbf{P}_\delta^{\text{bulk}}$ . The nonlinear coefficients  $\gamma$  and  $\delta$  represents the bulk multipole response with different forms  $\mathbf{P}_\gamma^{\text{bulk}} = \gamma \nabla(\mathbf{E} \cdot \mathbf{E})$  and  $\mathbf{P}_\delta^{\text{bulk}} = \delta(\mathbf{E} \cdot \nabla)\mathbf{E}$  [1], where  $\mathbf{E}$  is the electric field. The former one, as a longitudinal wave, only contributes at the surface, which can be written as an effective surface nonlinear susceptibility tensor with two non-zero components  $\chi_{\gamma\perp\perp\perp}$  and  $\chi_{\gamma\perp\parallel\parallel}$  [2].  $\mathbf{P}_0^{\text{surf}} + \mathbf{P}_\gamma^{\text{bulk}}$  can be written into an effective surface polarization  $\mathbf{P}^{\text{surf}}$  with the corresponding susceptibility  $\chi_s^{(2)} = \chi_{s0}^{(2)} + \chi_{s,\gamma}^{(2)}$ .

## References

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