Combustion Engine Energy Model

Rabie Ramadan

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1 Introduction

In this document, I summarize the procedure I used to construct an energy function that acts as a proxy for the fuel consumption of the Toyota Tacoma vehicle. The function I construct is $E(u, a)$, where u is the vehicle speed in m/s and a is the vehicle acceleration in m/s^2. E is measured in W or KW = 1000 W.

2 Procedure

I followed the following steps:

2.1 Step 1

I take the fuel consumption function for the Tacoma model as a function of u, and when $a = 0$, and convert it from grams/s to KW using the conversion rate 1 KW = 0.06666 gallons/hr = 0.05957 grams/s. This is the same conversion rate as 15 KW = 1 gallons/hr.

Then I fit a 3rd degree polynomial to this function (in least squares sense), to get

$$E_0(u) = C_0 + C_1 u + C_2 u^2 + C_3 u^3 \; .$$

This function is physics based, where the C_0 constant term represents power consumed by the vehicle when idle, the cubic term represents the power consumed to overcome air drag, and the linear and quadratic terms represent the power consumed to overcome the rolling friction.

The coefficients that I get are summarized below

$$\begin{aligned} C_0 &= 3405.5481762 \text{ W} \\ C_1 &= 83.123929917 \text{ kg} \cdot \text{m/s}^2 \\ C_2 &= 6.7650718327 \text{ kg/s} \\ C_3 &= 0.7041355229 \text{ kg/m} \end{aligned}$$

2.2 Step 2

Using Newton's law, I augment $E_0(u)$ with a term that accounts for accelerations as follows:

$$E_{\text{phys}}(u,a) = \max\left\{mau + E_0(u),\ 0\right\},$$

where $m = 2041$ kg is the vehicle mass. The max function is used here because we are dealing with a combustion engine that doesn't have negative fuel consumption.

2.3 Step 3

Using the actual fuel consumption function from the Tacoma model, and converting it to KW using the above conversion rate, I calculate the difference between the real fuel consumption and $E_{\text{phys}}(u,a)$. Then I propose that the difference resembles a bilinear polynomial function of the following form

$$\text{Crr}(u,a) = p_0 + p_1 a + p_2 u + p_3 au$$

in the domain $\{a \geq 0\}$. Since $E_{\text{phys}}(u,a)$ is already fit to the real Tacoma model when $a = 0$, the difference $\text{Crr}(u,0) \approx 0$. Therefore, I set $p_0 = 0$ and $p_2 = 0$, and then I find a least squares fit to the difference (in the domain $\{a \geq 0\}$) using the function with two parameters:

$$\text{Crr}(u,a) = p_1 a + p_3 au$$

The coefficients that I get are summarized below

$$\begin{aligned} p_1 &= 4598.7155 \text{ kg} \cdot \text{m/s} \\ p_3 &= 975.12719 \text{ kg} \end{aligned}$$

2.4 Step 4

Finally, I obtain the energy function E by adding the correction function Crr to E_{phys} as follows:

$$E(u,a) = E_{\text{phys}}(u,a) + \max\left\{\text{Crr}(u,a),\ 0\right\}.$$