Simulation and Modelling for Astrophysics Project

Group F

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Research question:

Can the Lidov-Kozai mechanism and tidal forces cause a system of moons (or exomoons) to align within the same orbital plane?

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1 Introduction

In recent decades, new and advanced space missions like CoRoT, Kepler, and TESS have discovered several thousand exoplanet-hosting stellar systems. Despite this newfound prevalence of exoplanets in the Universe, we have only just begun to scratch the surface of understanding exoplanet populations. One of the next upcoming frontiers in exoplanetology is the search for exomoon systems around exoplanets. As is common in astronomy, our search for exomoons starts in our host solar system, and the different moon systems found near home. One of the most breathtaking examples is the Jovian system and, in particular, Jupiter's four Galilean moons: Io, Europa, Ganymede and Callisto. This system can be easily observed through a small backyard telescope, and has been of significance to astronomy for several centuries. One notable feature of the Jovian system is the co-planar orbits of the Galilean moons. How common is it for a moon system to line up in such a way? Could captured moons, with random initial inclinations and eccentricities, given enough time, line up in much the same manner? And if so, what mechanisms would lead to such a configuration?

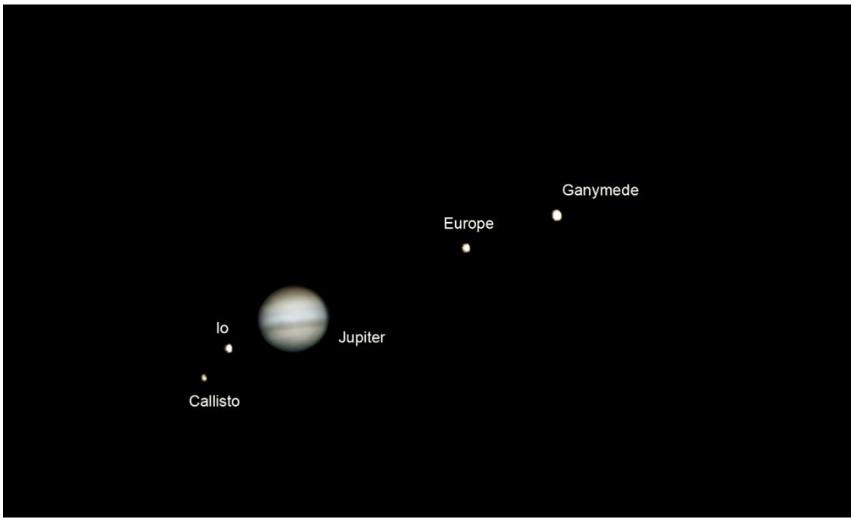

Figure 1: Jupiter and its Galilean Moons (Jean, Aug 2010). Note that the moons look larger than they are in reality due to over-exposure.

For our final project in Simulation and Modelling for Astrophysics, our group asks if Lidov-Kozai oscillations, in conjunction with tidal dissipation, can cause a system of exomoons to evolve into a co-planar alignment. We do this with a N-body gravitational code, influenced by tidal friction forces via an augmented bridge in AMUSE[1] (Portegies Zwart & McMillan, 2018). In doing so, we are able to create single and multiple moon systems that, given valid initial parameters (Section 2.1.2), simultaneously exhibit Lidov-Kozai oscillations and tidal dissipation. We obtain promising preliminary results, where the moon inclinations of a given system trend toward similar values.

[1] AMUSE code

2 Physical Background

2.1 The Lidov-Kozai Mechanism

The first mechanism we are addressing as a notable influence on exomoon systems are Lidov-Kozai oscillations (Kozai, 1962; Lidov, 1962). The Lidov-Kozai (sometimes called Kozai-Lidov, or just Kozai) mechanism is a dynamical effect that impacts hierarchical binary systems with at least 3 bodies. In astrophysics, systems that may experience Lidov-Kozai oscillations are fairly common, as the hierarchical binaries are a stable solution to the three body problem. These physics can be applied to studies of triple stars, black hole binaries, Hot Jupiter formation, and even the movement of artificial satellites around Earth (Naoz, 2016). The setup of Lidov-Kozai impacted systems is usually described in one of two ways:

1. A primary binary with a large, distant, outer perturbing body in orbit with the primary's centre of mass (COM).
2. A primary binary with a smaller body orbiting one of the binary's components.

These two interpretations ultimately describe the same system, the main difference being whether the inner or outer binary is the "primary." For our purposes, we stick to the language of the first.

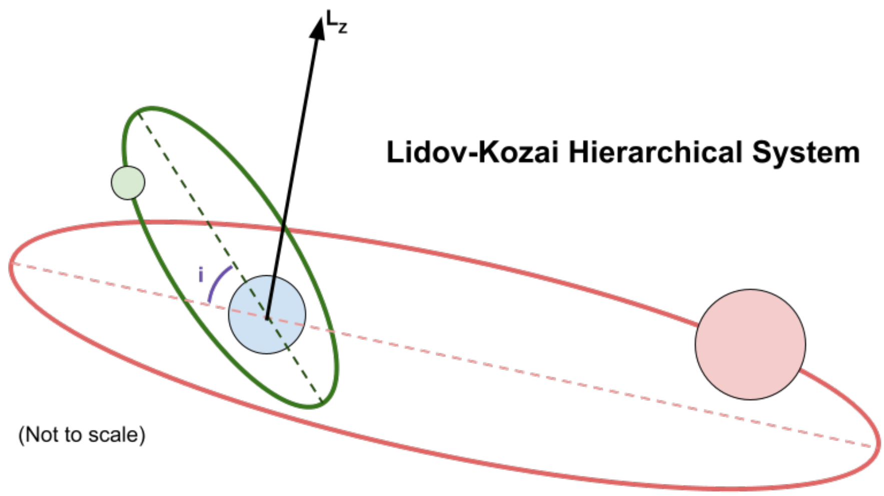

Figure 2: An example of a Lidov-Kozai hierarchical triple. In the context of this project, the inner or "primary" binary (green) represents an exoplanet and its moon(s), while the outer or "secondary" binary (red) consists of the COM of the inner binary and the host star. The inclination (purple) is measured as the angle between the inner and outer binaries' orbital planes, and the conserved component of system angular momentum (L_Z, black), is perpendicular to the plane of the outer binary. Figure is not to scale, eccentricities are exaggerated for illustration purposes.

In Lidov-Kozai oscillations, energy and angular momentum are exchanged between the secondary component of the inner binary and the outer perturber. In the context of our exomoon problem, the exomoon(s) is the perturbed secondary of the inner binary, and the system's host star is the outer perturber. The exchanges in energy and angular momentum cause the argument of periapsis of the inner binary to oscillate. This leads to a trade-off between inclination and eccentricity of the inner binary, the physics of which can be best understood in the test particle limit.

2.1.1 Test Particle Quadrupole Approximation

The test particle limit (TPL) is often used to simplify the mathematics of a Lidov-Kozai oscillating system, while retaining the key dynamical features that occur. In this approximation, the secondary element of the inner binary is assumed to be a massless test particle. The "Quadrupole" part of the approximation simply means the orbit of the outer binary is assumed to be near circular (as is the case for our example Jupiter-Sun outer binary). This is called "quadrupole" because the approximation retains the related system Hamiltonian to a quadrupole level (Naoz, 2016). In this approximation, the component of the total system angular momentum that is perpendicular to the outer binary's orbital plane (see Figure 2) is conserved:

$$L_z = \sqrt{1-e^2}\cos i = \sqrt{1-e_0^2}\cos i_0 = \text{constant}\,, \tag{1}$$

where i is the inclination between the two orbital planes and e is the eccentricity of the inner binary. This conservation of the z-component of the angular momentum vector is also known as the *Kozai-constant* (Kozai, 1962). As a consequence, when angular momentum is exchanged between the binaries, the changes in eccentricity and inclination are inverse. If the inclination is increased, eccentricity must decrease, and vice-versa.

2.1.2 Initial Parameters

Eq. 1 shows that the value of L_Z is dependent on the initial values of the system inclination and perturbed binary eccentricity. This limits the range of oscillations possible for given initial conditions. Combined with conservation of energy (Eqs. 17 and 18 in Naoz, 2016), one can find the maximum possible eccentricity for a given initial inclination:

$$e_{\text{max}} = \sqrt{1 - \frac{5}{3}\cos^2 i_0} \tag{2}$$

Solving the same system of equations for $\cos i_{\text{min}}$ yields:

$$\cos i_{\text{min}} = \pm\sqrt{\frac{3}{5}} \tag{3}$$

Which, in turn gives $i_{\text{min}} \approx 39.2°, 140.77°$. These values are understood to be the limits for initial system inclination at which significant Lidov-Kozai oscillations can occur.

Conveniently, the masses of the system bodies do not impact these values, and are only relevant to the timescale of oscillations. Therefore, for this exploration, we decide to set the values for the masses, radii, initial semi-major axes, and love parameters for the system bodies to be the same as those observed in the Sun-Earth-Galilean Moons system. This gives us confidence that the values we are using for these bodies are physically reasonable, while minimizing the free parameter space to be tested. This simplifies the problem to one more appropriate for the given course time constraint.

2.2 Tidal Friction

The second mechanism we are assessing in this project is tidal friction (sometimes called tidal dissipation). This process occurs between orbiting bodies that experience significant differential tidal forces from their binary partner. Generally speaking, a tidal force refers to the difference in gravitational force between the nearer and farther sides of a binary member, due to the change in distance from the other body (*see* Figure 3). Tidal forces are relevant in many areas of physics, from the Earth's ocean tides to 'spaghettification' around black holes. This differential force directly causes the nearer sides of the bodies to bulge out slightly, while inertial forces create a smaller, secondary bulge directly opposite the first.

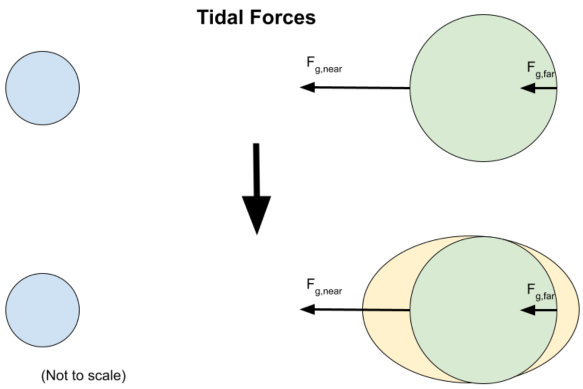

Figure 3: A quantitative illustration of tidal force effects. The side of the green body nearer its blue companion feels a stronger gravitational force than the far, causing the body to deform and bulge (shown in yellow). Figure and force vector arrows are not to scale.

As the system orbits, the resulting movement of these tidal bulges dissipates energy from the system. Generally, this dissipation has three primary possible effects (Rodríguez et al., 2011):

1. Orbital decay, where the periapsis of a nearly circular orbit decreases with time
2. Spin-orbit synchronisation, where one of the bodies becomes tidally locked (this is why we only see one side of the moon, for example).
3. Orbit circularization, where an eccentric orbit becomes generally more circular over time

The third of these, orbit circularization, being the particular effect of interest for this project. We believe the dissipation of energy due to tidal effects will minimise the strength of Lidov-Kozai oscillations over time by decreasing their eccentricities. During this process, we hope the moons will roughly align in some preferred orbital plane.

2.2.1 A Note On Orbit Circularization

In orbital mechanics and rocketry, it is known that the most efficient ways to circularize an orbit are to burn prograde at apoapsis, or retrograde at periapsis. In the case of tidal dissipation, however, the impulse on the orbiting system is not applied to a singular point. It is effectively a retrograde burn around the entire orbit, where the force is strongest at periapsis. As a result, instead of the clean, simple decrease in apoapsis that is often seen in orbital manoeuvres, the apoapsis can significantly decrease while a small increase in the periapsis may arise as the orbit circularizes.

3 Methodology

Having established our hypothesis and fleshed out our understanding of the problem at hand, it was time to translate our ideas to a working simulation. In order to answer our overarching question of whether or not a combination of the Lidov-Kozai mechanism and tidal forces could cause a system of moons (or exomoons) to become co-planar, we would have to model a particle system under the influence of gravity and tidal interactions and track its evolution through time. In doing so, the evolution of key orbital properties for each moon can be investigated when subjected to these external forces. In this section we outline our coding practices and how we leveraged AMUSE to our advantage for constructing our simulation. Furthermore, we describe how we chose to structure our code such that communication between our system could be easily achieved.

3.1 Code Framework

For this problem, it was important to ensure our coding procedures were well defined as the nature of our research question requires the fundamental effects of gravity and tidal friction to be modelled in a physically sound way. Therefore, we followed a sequence approach which involved testing the addition of each force one at a time before including them together. A flow chart illustrating this structure can be seen in Figure 4. Firstly we initialised our particle set of the Sun, Jupiter, and a single Galilean moon, giving each particle their respective bulk attributes such as their mass and radius. We evolved our system over time using a gravitational solver (as described in Section 3.2) in order to include the Lidov-Kozai effects exclusively, as seen in Figure 4a.
The next step was to introduce tidal interactions into our system. A script to compute the tidal forces between each binary was written and implemented into our code, as shown in Figure 4b. This procedure is described in more detail in Section 3.4. The Sun was removed from our particle set in order to eliminate the Lidov-Kozai mechanism since

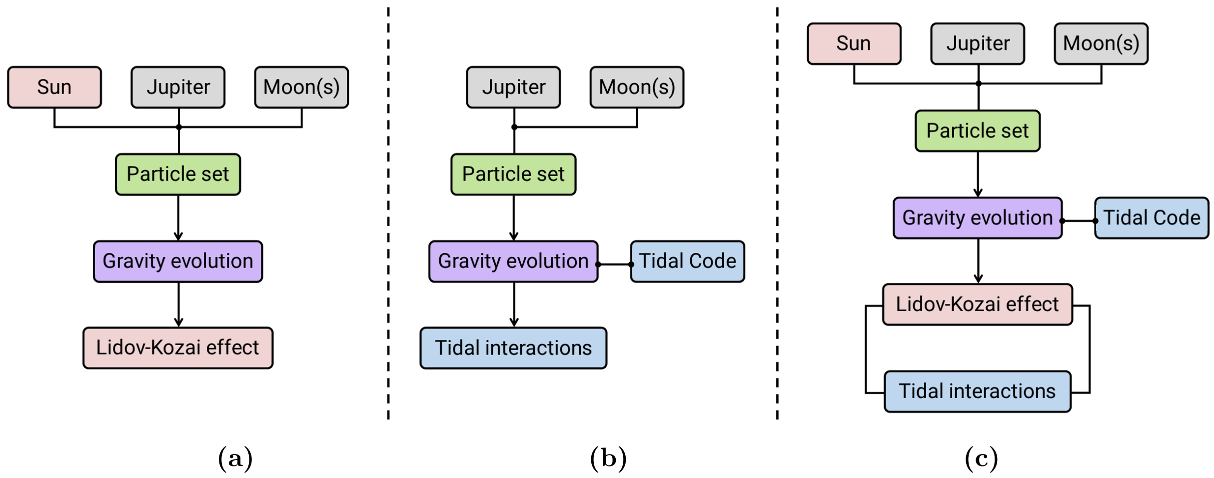

Figure 4: Flow charts of our coding procedures. Only the Lidov-Kozai effects are present in **(a)**, only the tidal interactions in **(b)**, and the combined Lidov-Kozai and tidal effects in **(c)**.

these effects result from the presence of the Sun which acts as the distant perturber in our system. This was done to observe the effects of the tidal interactions only so as to investigate the fidelity of our tidal code.

Finally, we added the Sun back into our system in Figure 4c to explore the combined Lidov-Kozai and tidal effects on our system. Following this, we added more moons into the system to begin investigating how the inclusion of multiple moon interactions impact the evolution of their orbital parameters.

3.2 Gravity Solver

Since our set-up consists of a dynamical system of particles, we would have to model the interactions between these bodies over time in order to investigate their evolution. This can be achieved through the use of a *N-body* code, which allows us to simulate the evolution of particle sets when subjected to external forces such as gravity. AMUSE is rich with community created N-body codes, with many designed to model the gravitational interactions between different particles. Three main choices exists which could be applied to solve for gravity: approximate methods, pure N-body codes, and direct N body codes. While the computational cost per step for gravitational N-body solvers generally scales as N^2 (where N is the number of particles), approximate methods relax this procedure when calculating the gravitational force. Tree codes are examples of approximate N-body solvers. These calculate the gravitational forces for nearby particles directly while grouping distant particles together and approximating their gravity by using the fact that neighbouring bodies have similar interactions. The result can be a reduction of the $O(N^2)$ complexity of a direct force calculation to $O(NlogN)$. While this speeds up computations, there is a cost to its complexity due to the fact that approximations are made to the more distant particle clumps. Since our system contains the Sun and a max of five separate particles all relatively close together, approximate methods was deemed unnecessary to use for our gravitational N-body solver since we want to model the interactions of our Galilean model to some detail.

Therefore, we would like to directly calculate the gravitational force for each particle. Both pure and direct N-body codes operate to solve for gravity in this way. The operation of these codes are quite similar, with some differences in their optimization. Pure N-body codes solve Newton's equations of motion with no free physical parameters, while direct methods employ additional parameters to help speed up computations. Although both solvers scale as N^2, it was found that pure N-body codes (such as ph4 and Hermite) were slower when tested against the Huayno direct N-body solver. This most likely arises from the order of the integrator used in each code. Hermite and ph4 are both 4th order integrators, while the Huayno code is symplectic. Generally, symplectic codes run quicker when integrating over large time-scales compared to higher order integrators, but can come at a cost of accuracy. When testing our code using these gravitational solvers, it was found that the results differed only slightly. For this reason and since our problem requires integration over large time-scales, we decided to use the Huayno direct code for our N-body solver.

3.3 Lidov-Kozai implementation

As discussed in Section 3.1, we firstly began by including the Lidov-Kozai mechanism. A simple particle set of the Sun, Jupiter, and a single Galilean moon was initialised using AMUSE. Most of the initial parameters for each celestial body were set from the present-day values (such as for the semimajor-axes, masses, and radii). However, the present-day orbital eccentricities and inclinations were not employed as these are the properties we are most interested in and their evolution over time is highly dependent upon their initial value. For our initial investigation, these parameters were arbitrarily chosen while ensuring the constraints described in Section 2.1.1 for the inclusion of the Lidov-Kozai mechanism were satisfied. The relative velocities for each body were obtained by creating binaries from the given orbital elements. A binary system was created for the Sun with Jupiter, while subsequent binaries were made for Jupiter and each moon (depending on how many moons were simulated). The system was then gravitationally integrated over time using the Huayno N-body solver.

Figure 5 demonstrates an example of the Lidov-Kozai cycles in effect for Jupiter's second moon, Europa. This plot shows the evolution of the orbital eccentricity and inclination over the course of 1000 years and illustrates the periodic exchange between both properties. Snap-shots of the system were taken every 10 years throughout its evolution, since the Lidov-Kozai mechanism operates on timescales much larger than that of the moon's orbital period (Antognini, 2015). The conservation law from Eq. 1 was also tested and the results were found to approximately agree with the theory that the z-component of the angular momentum vector should remain constant throughout time (*see* Figure A.10).

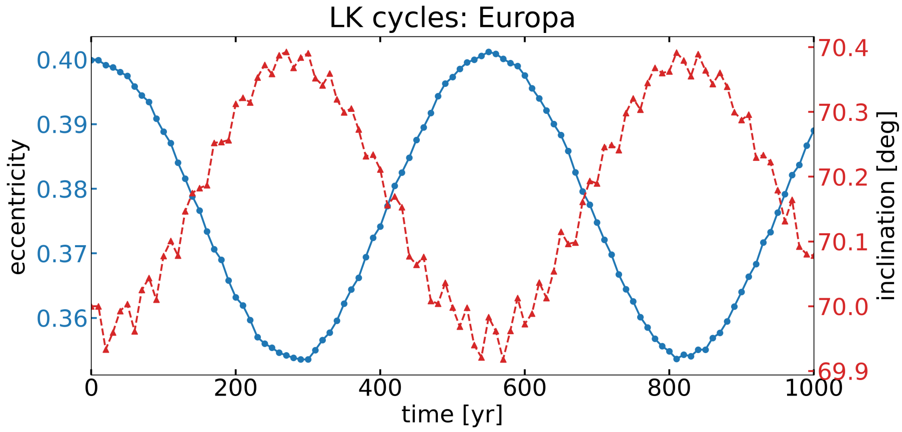

Figure 5: An example of the Lidov-Kozai (LK) cycles illustrated for Europa. Note the characteristic trade-off between the orbital eccentricity and inclination.

Following this, we began to add more moons into the system. This is illustrated in Figure 6, which shows an the Lidov-Kozai mechanism in effect for the full moon system. Once again, this plot conveys the evolution of the orbital eccentricities and inclinations over a period of 1000 years and demonstrates the periodic exchange between both properties. The evolutionary tracks here are more complex compared to those in Figure 5, when only a single moon was simulated. This is due to the moon-moon gravitational interactions, resulting in a more elaborate sequence of Lidov-Kozai cycles. Outer moons can begin to shield the inner bodies from the Lidov-Kozai oscillations in a process known as Kozai-shielding (Toonen, Hamers, & Portegies Zwart, 2016), whereby the Lidov-Kozai cycles can be suppressed. Although this effect results in more sporadic evolutionary tracks, the overall trade-offs between eccentricity and inclination can still be observed and the Lidov-Kozai mechanism can be discerned.

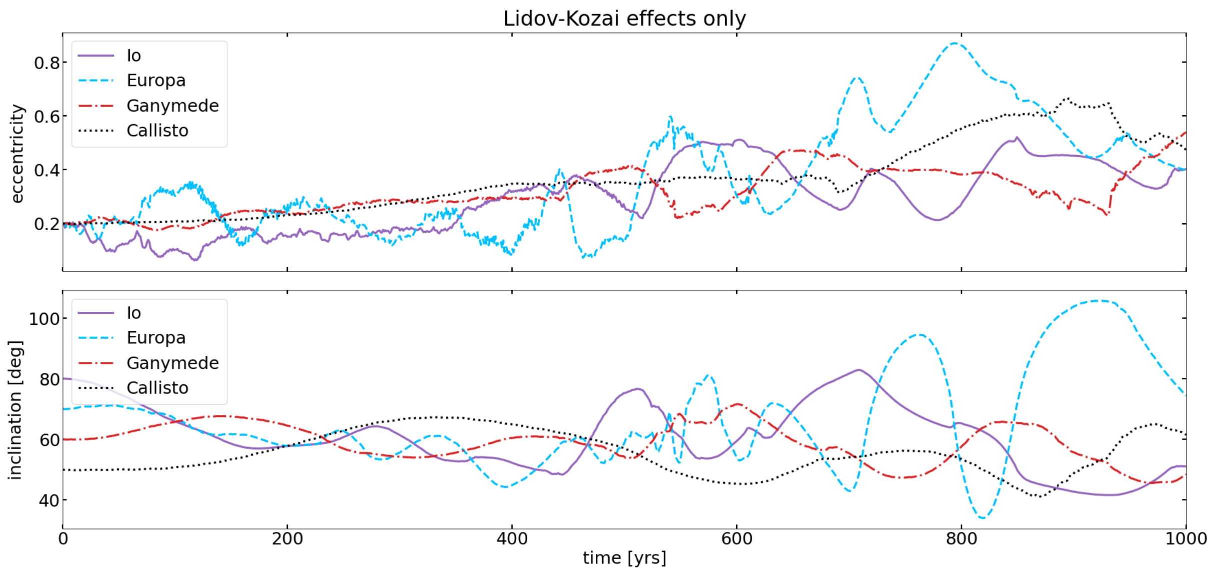

Figure 6: Lidov-Kozai oscillations for the full system of moons. More complicated evolutionary tracks are observed when multiple moons are simulated due to the gravitational interactions between each moon. The system was evolved with a time-step of 1 year.

3.4 Inclusion of Tidal Friction

Currently, a code within AMUSE to compute the tidal interactions between orbiting bodies does not exist. Therefore, a code was written during this project to add the tidal friction into our simulation. Initially, it was thought of to use an AMUSE community code for a full scale hydrodynamical simulation. However, this is unnecessary as the run time of the simulation will highly increase while the only interesting feature is the tidal friction. Therefore, it was decided to use an augmented bridge to simulate the friction. A bridge is a way to couple different codes together in AMUSE by using a "leapfrog" scheme to stagger the effects of each code on the overall system (Portegies Zwart & McMillan, 2018). An augmented bridge is a bridge that couples, in this case, an N-body solver to a class that gives kicks of an external force to the system. In our case, the augmented bridge couples our gravity code to a custom class for computing the tidal friction. When called upon, the class returns the accelerations due to the tidal forces which act as a kick to the gravitational code and updates the overall accelerations. The acceleration of a moon around Jupiter with the inclusion of tidal forces is given by:

$$\ddot{\vec{r}} = \ddot{\vec{r}}_{grav} + \frac{M+m}{Mm}(\vec{f} + \vec{f}_{rel}), \tag{4}$$

where $\ddot{\vec{r}}_{grav}$ is the acceleration due to the gravity which is included in the gravity code and M, m are the masses of Jupiter and the moon, respectively (Rodríguez et al., 2011). We use the expression for $\vec{f}$ and $\vec{f}_{rel}$ given by the Mignard model (Mignard, 1979) for tidal forces:

$$\vec{f} = -3k\Delta t \frac{Gm^2R^5}{r^{10}} \left[2\vec{r}(\vec{r}\cdot\vec{v}) + r^2(\vec{r}\times\vec{\Omega} + \vec{v})\right] \tag{5}$$

$$\vec{f}_{rel} = \frac{GMm}{c^2r^3}\left[\left(4\frac{GM}{r} - v^2\right)\vec{r} + 4(\vec{r}\cdot\vec{v})\vec{v}\right]. \tag{6}$$

Here $\vec{r}$, $\vec{v}$ and $\vec{\Omega}$ are the position, velocity and angular velocity of the moon relative to Jupiter. k is the Love number of Jupiter and Δt is related to the time lag in the deformation of the bodies due to tidal forces.

Because the gravitational part of the acceleration is already handled by the gravity code, only the second term of Eq. 4 needs to be implemented in the tidal force code. The value of $k\Delta t$ for the Galilean moons is approximately 0.02 s, but to increase the effect of the tidal friction a value of 360 s was taken (Rodríguez et al., 2011). This can be done because the tidal force is linear with $k\Delta t$. However, the consequence of this is that the timescales in the simulations do not represent the physical timescales.

The equations have been implemented in a tidal force class. This class contains a function `get_gravity_at_point` which returns the accelerations of the particles due to the tidal force at a specific moment. This function is used to kick the N-body code. The simulation was then first run for Jupiter and single moons, such that there is a two-body system without the Sun. This was done to check whether the tidal force code behaves in the way it is expected, as described in Section 2.2. The simulation was run for 5 Myr on ALICE, and the results can be seen in Figure 7.

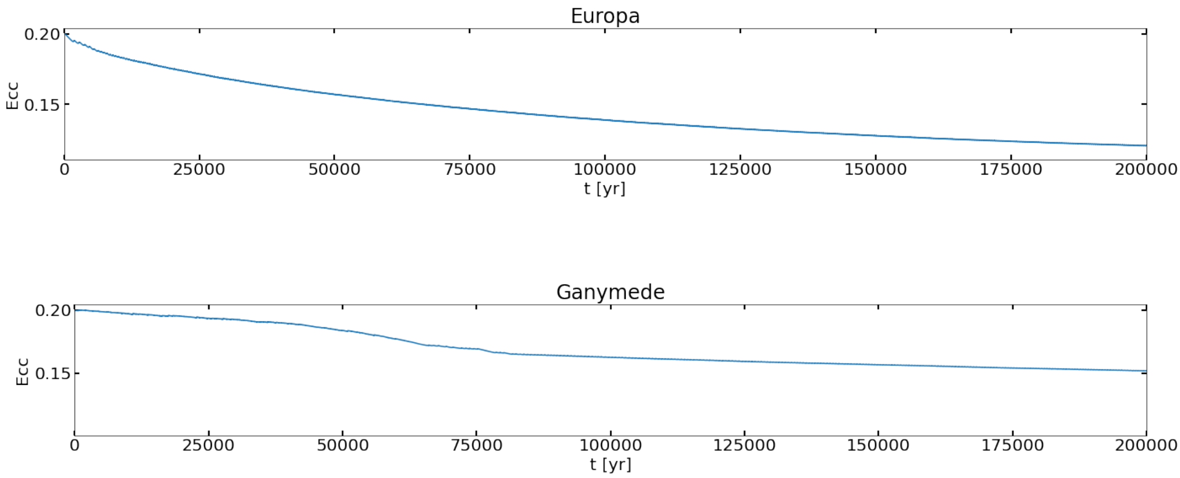

Figure 7: The evolution of the eccentricity of Europa and Ganymede, ran in individual simulations with only Jupiter. Only the first 200000 years are shown because this period sufficiently shows the tidal effects at work.

In these figures, the tidal forces are seen at work. For Europa and Ganymede the eccentricity drops slowly over time. This is the orbit circularization mentioned in Section 2.2.1. Though the eccentricities decrease, they do not drop to zero, even when the time range is increased. The eccentricities do decrease faster for Europa which is closer to Jupiter, which is in line with the expectations since the tidal force also scales with the inverse distance.

3.5 Combined Effects

The next step is to include the Sun in the previous simulations, such that we can see the Lidov-Kozai effect at work next to the tidal forces. For this effect to be observed, the initial inclination for each moon needs to satisfy the constraints described in Section 2.1.2, which shows the inclinations must be within the range $[40°, 140°]$. To be sure that the effect is seen, only initial inclinations larger than 50° and less than 130° were taken. Initial inclinations for Io, Europa, Ganymede and Callisto were arbitrarily chosen to be 80°, 70°, 60° and 50°, respectively. For simplicity, the initial eccentricity was taken to be 0.2 for all moons. Finally, all moons were added into the system and the full simulation was run using the same initial inclinations and eccentricities.

4 Results and Discussion

4.1 Single Moon Systems

The results from the simulations of Sun-Jupiter-moon systems are shown in Figure 8, for each of the Galilean moons. The combined effect of the Lidov-Kozai mechanism and tidal interactions on the evolution of the orbital eccentricity and inclination can be observed for each moon.

The first thing that is noticed are the clear Lidov-Kozai oscillations, which are present for all four moons. The eccentricity decreases for Europa, and for Ganymede it even drops to a near-zero value, indicating that the moon's orbit has become practically circularized. Compared to the results seen in Figure 7 without the presence of the Lidov-Kozai mechanism, the eccentricities for both Europa and Ganymede decrease at a much quicker pace. This is explained by the inclusion of the Lidov-Kozai cycles which enhances the tidal effects, resulting in the more rapid decrease in eccentricity and acts to circularize the orbits.

Although there are some jumps and drops in Io's eccentricity, this parameter seems to be reasonably constant over time. Interestingly for Callisto, the eccentricity increases until reaching a relatively constant value. A possible explanation comes from the fact that this moon is furthest away from Jupiter, meaning that the tidal forces felt by Callisto are much weaker and therefore they do not strongly effect its orbital parameters. On the other hand, the tidal forces felt by Io should be strongest as this is the closest moon to Jupiter. Although it's eccentricity remained relatively constant, it periodically fluctuated by $\sim 40\%$ from its initial value throughout its time range, and one could argue that a trend towards smaller eccentricities could be seen, something that could become more obvious on larger timescales.

Furthermore, Callisto was initialised with a relatively high inclination of $80°$ which could explain the slower decrease in its eccentricity over time. A further remark was observed for the orbit of Callisto - the moon crashed into Jupiter after the simulation had run for some time. Because this resulted in large erroneous parameters, this part of the simulation is not shown in Figure 8. A possible reason for this is that the Lidov-Kozai effect gets stronger than the tidal forces and force the orbits to become very elliptical. At a certain point the moon gets so close to Jupiter that it crashes into it. This increase in eccentricity was also seen in the when plotted for the full timescale.

Finally, it can be noted that the inclination for none of the moons decreased to zero, though Ganymede's inclination decreases steadily and continues to do so after the specified time range.

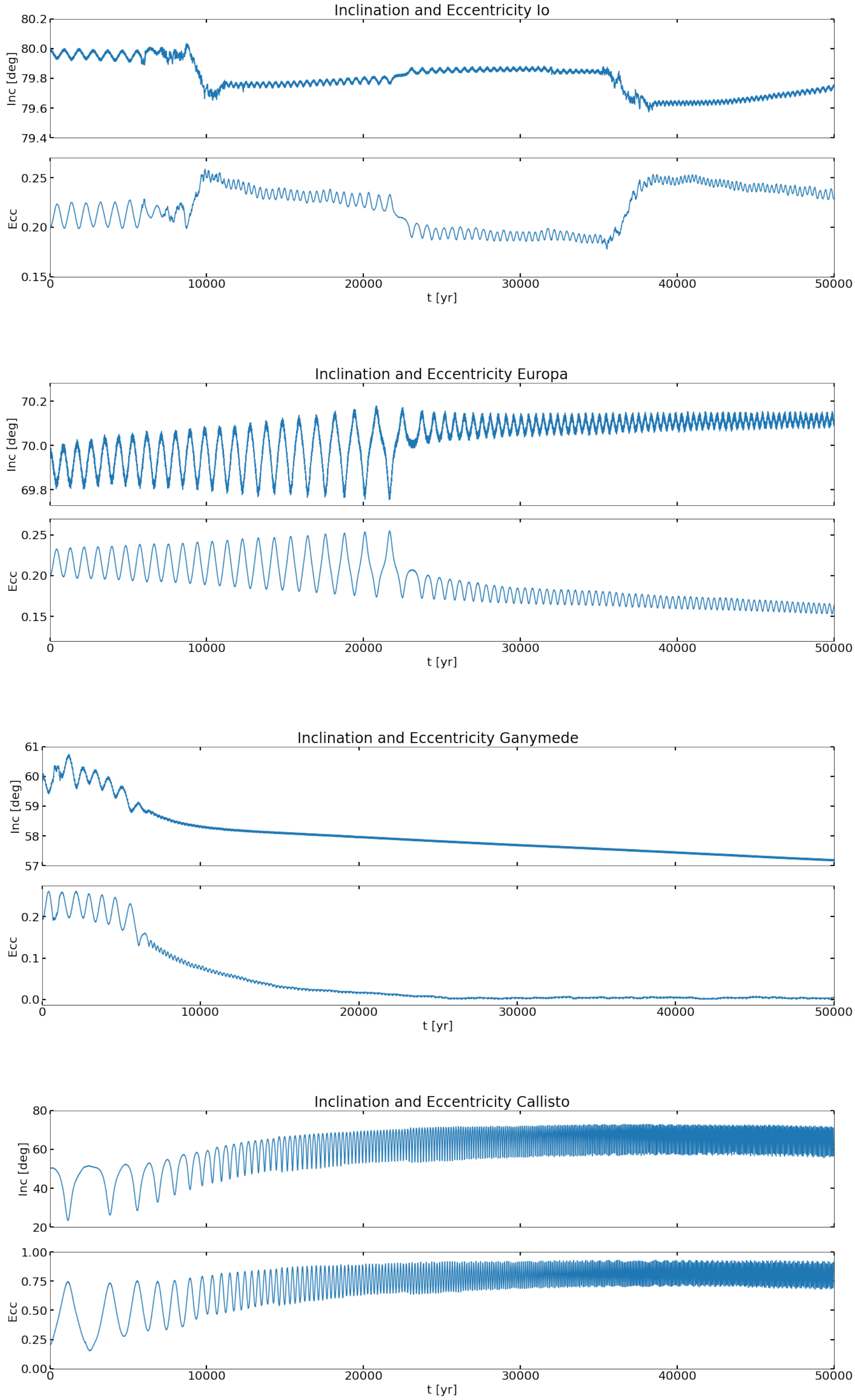

Figure 8: The evolution of the eccentricity and inclination of the single moon systems with the Sun included. For clarity purposes, the parameters are only plotted for the first 50000 years.

4.2 Multiple Moon Systems

The eccentricity and inclination evolutionary tracks under the influence of the combined Lidov-Kozai and tidal effects for the full system of moons can be observed in Figure 9. For this simulation, the system was evolved with a time-step of 0.2 days over 1000 years. Since tidal interactions develop on timescales similar to a tenth of an orbital period, this time-step was chosen to give a truer depiction of the tidal evolutions. Of course, this came with a cost to computational time, with this simulation taking ~ 20 hours to run on ALICE.

The same initial parameters as those employed in Figure 6 were used in this simulation, allowing us the ability to easily compare each plot. Although there is Kozai-shielding at play due to the moon-moon gravitational interactions, the characteristic periodic Lidov-Kozai cycles can still be observed. However, Kozai-shielding can act to prevent inner binaries from shrinking due to tidal dissipation (Toonen, Hamers, & Portegies Zwart, 2016), which could explain why there isn't an overall decrease amongst the eccentricities.

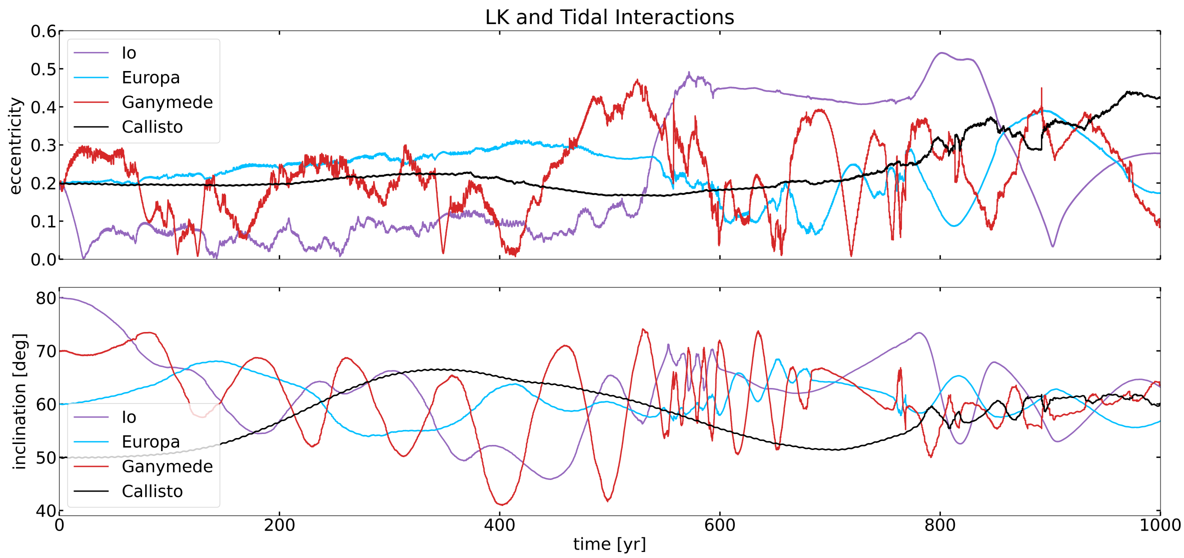

Figure 9: The combined Lidov-Kozai (LK) effects and tidal interactions at play for the full system of moons.

The evolution of the orbital inclinations is a highlight of Figure 9. While initialised with relatively large differences to each other, the final inclinations are observed to evolve towards a common value. Table 1 gives the values for these final inclinations, and it can even be seen that the final inclination for Io and Ganymede was found to be within $< 1°$ of each other. While the inclinations are not tending towards zero, they do seem to be approaching the same value, suggesting that the moons could become co-planar over time. Finally, the initial parameter space was briefly explored to help determine what initial conditions would be most ideal (if any) for evolving the system of moons to align within the same orbital plane. This was achieved by running a series of simulations in which the eccentricity and inclination for each moon was randomly assigned (while within the

Table 1: Initial and final inclinations (i) obtained from the full simulation shown in Figure 9.

Moon	**Initial i [deg]**	**Final i [deg]**
Io	80.00	~ 63.18
Europa	70.00	~ 56.82
Ganymede	60.00	~ 63.58
Callisto	50.00	~ 60.13

constraints of the Lidov-Kozai mechanism). While the results were helpful in eliminating a number of 'bad' configurations (i.e. those that caused a moon(s) to escape their orbit or crash into Jupiter), it was concluded that many more simulations would have to be run and at larger timescales to extract statistically significant information about the impact of these initial conditions. An interesting example of one of these simulations in which the orbit of Ganymede near-circularizes can be seen in Figure A.11.

5 Conclusions

Our project began with the question, 'Can a combination of the Lidov-Kozai mechanism and tidal forces cause a system of (exo)moons to align within the same orbital plane?'. An assortment of physical mechanisms were explored throughout this work and a deeper understanding of our research question was developed in the process. Elements of the AMUSE framework was probed in detail and our overall knowledge of the AMUSE environment significantly improved.

A system of particles modelled off of the Sun, Jupiter, and the Galilean moons was created and gravitationally evolved over time using the Huayno N-body solver to include the effects of the Lidov-Kozai mechanism. A custom class to compute the tidal friction was written using the equations based on the Mignard model for tidal forces. This was coupled to our system using an augmented bridge.

A variety of simulations were conducted to investigate the impact of the Lidov-Kozai mechanism, tidal interactions, and the combination of these influences for single moon and many moon systems. The combined influence from the Lidov-Kozai mechanism and tidal effects was then simulated for the entire 6-particle system of the Sun, Jupiter, and the Galilean moons.

A key result from this simulation was the gradual approach of the inclinations towards a common value, suggesting that the moons could be evolving towards a co-planar system. While this result is promising, one would have to further investigate the consistency of this behaviour in order to faithfully answer our research question. As the evolution of the orbital parameters of each moon are highly dependent upon their initial values, this could be achieved by running more simulations of varying initial conditions and timescales in order to get a broader spread of data.

Further work should involve the modification of our code to include separate time-step evolutions for the gravitational solver and the tidal code. As previously discussed, the Lidov-Kozai effects operate on much larger timescales than those for tidal interactions. If we set our time-step for evolving the system too large, we may not observe the full effects from the tidal interactions. On the other hand, a smaller time-step causes our simulations to become much more computationally expensive. Therefore, it would be beneficial to incorporate the functionality to evolve both forces at different time-steps to each other as the bridged system evolves. Optimising the tidal code in a faster language such as C^{++} or other low level languages could also help in this regard.

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A Appendix

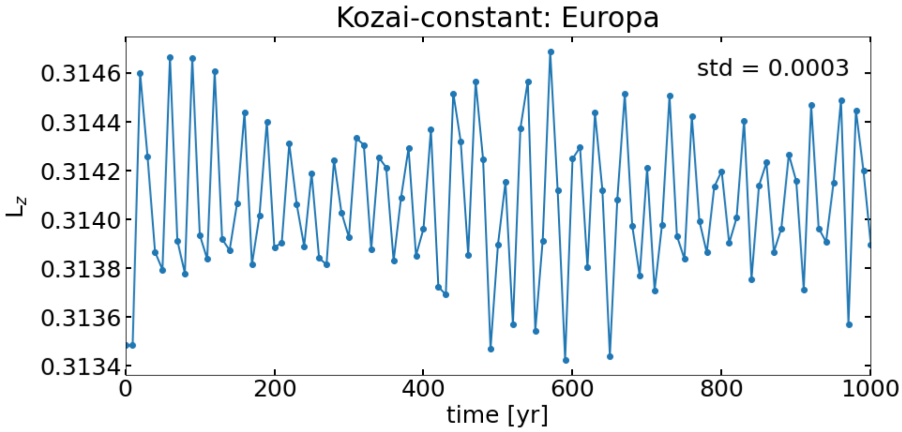

Figure A.10: The conservation law from Eq. 1, known as the *Kozai-constant*, illustrated for the case of Europa. The extremely low standard deviation of (std = 0.0003) highlights the practically constant relationship the z-component of the angular momentum vector (L_Z) has throughout time.

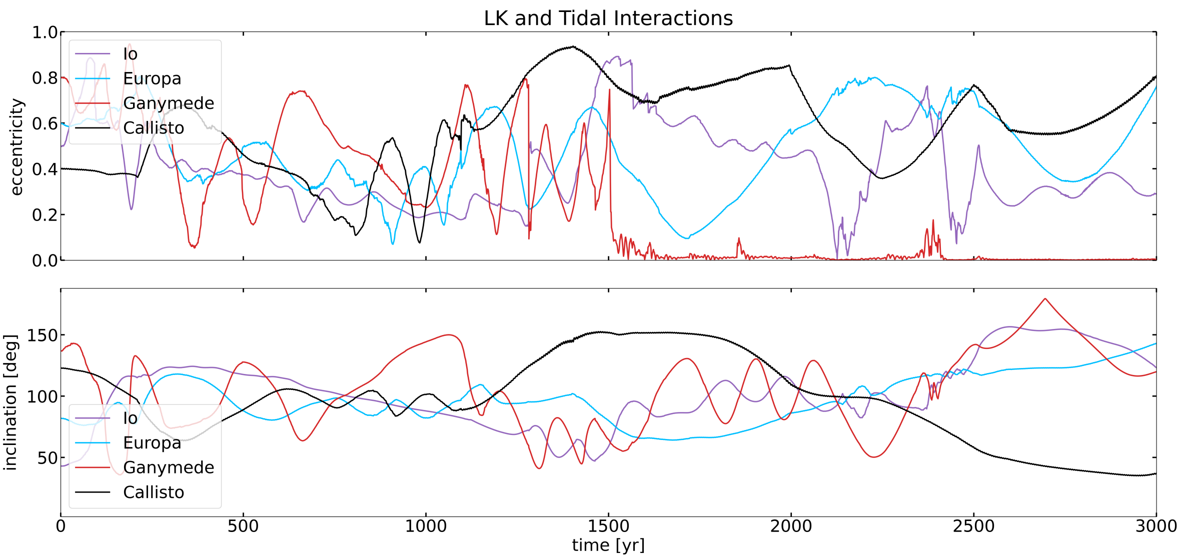

Figure A.11: An example of a simulation run from our investigation into the initial parameter space, whereby the inclination and eccentricity for each moon was randomly assigned. The Lidov-Kozai (LK) mechanism and tidal effects can be observed here for the full system of Galilean moons. Interestingly, Ganymede's orbit becomes near circular for these initial conditions, illustrating the strong tidal interactions between this moon and Jupiter.