Modeling Tidal Disruption Events in a Classical Regime

Rory Schadler and Catherine Slaughter

December 3, 2020

1 Introduction

A tidal disruption event occurs when a star falls within the innermost stable circular orbit (ISCO) of a very dense object such as a black hole. At this distance, the gravitational force from the dense object is so much stronger on the near side of the star than the far side, it causes the star to be torn apart. The resulting dust forms an accretion disk of material about the black hole. Our final project for CS89.18 aimed to recreate a simple version of the tidal disruption dynamics model in Clerici and Gomboc (2020). For this model, we used simple classical equations Newtonian gravitation and realistic, observationally and experimentally determined values for our model parameters. While it would be necessary to use equations from General Relativity to create a rigorous model of the gravitational potential around a black hole, we find that our classical model works more than adequately for the purposes of simulation and visualization. To first order, we are able to successfully recreate a tidal disruption event.

2 Implementation Details

2.1 Black Hole Model

In reality, a black hole exists as a point in space with infinite density, called a "singularity." In order to mimic this, we modeled our black hole as a point mass with $m_{BH} = 300 M_{\odot}$. This corresponds to the size of an intermediate-mass black hole, a type of black hole that is theorized to exist, but that astronomers have only observed on one occasion. In contrast, Clerici and Gomboc (2020) models a Massive Black Hole with $m_{BH} \approx 10^8 M_{\odot}$. We decided it would be easiest to place our simulation in the rest frame of the black hole, so we enforce that its position stays at the origin. The visualized sphere around our black hole represents the Schwarzschild Radius of the black hole, the point of no return for anything falling in.

2.2 Star Model

We model the star as a collection of discrete particles ($n = 211$), each with an equal amount of the star's mass, arranged in a spherical formation. The particles were spread in four concentric shells, and their relative positions were calculated in Mathematica and read in to our code from a file. In order to maintain that the mass of the star was set to $m_{star} = 1 M_{\odot}$, we set the mass of each particle to $m_{particle} = \frac{1}{n} M_{\odot}$. As is true in real life, the star is held together by the force of gravity. For each time step, the gravitational force is calculated between each pair of particles in the star, along with between each particle and the black hole. For comparison, Clerici and Gomboc (2020) uses $n = 10^5$ particles for their simulation.

2.3 Mathematical Model

For this project, we decided to keep our mathematical model for the interactions between particles within a classical (Newtonian) regime. This meant that interactions were governed by Newton's Law of Universal Gravitation,

$$F = G\frac{m_1 m_2}{r^2}\hat{n} \tag{1}$$

where G is the Gravitational constant, m_1 and m_2 are the masses of a given pair of interacting particles, r is the radius between the particles, and $\hat{n}$ is the unit vector pointing from particle 2 to particle 1. This was the primary physical equation used in this simulation. By comparison, the model in Clerici and Gomboc (2020) included a higher-order term to its gravitational force equation which accounted for the effects of General Relativity. The paper, however, was also modelling a system for a much more massive star (making the impact of GR greater), and was creating a model for rigorous scientific use. Our simplified classical model was more than sufficient for the purposes of creating a simulated video.

The secondary interaction we had to model was the collisions between individual particles in our star. We tried several methods for this including implicit spring forces and atomic potentials. Ultimately, we decided to use our intuition of Newton's second law ("every action has an equal and opposite reaction") to model the normal force of two particles that were near each other. In this case, we simply enforced that the net force between any given pair of neighboring particles be set to 0, as the normal force would balance the gravitational force. It was a somewhat nonphysical implementation of a physical phenomenon.

Although our mathematical model is approximated to a classical regime, we see some very good physical phenomena are properly simulated. Our final simulation looks similar to that of Clerici and Gomboc (2020) and other artist renditions. We are also able to see the effects of conservation of angular momentum on our system, as the resulting material clearly forms a flat accretion disk around the black hole, a known result of this conservation law. Additionally, we can quantitatively see Kepler's three laws of orbital motion in our simulation. As an early proof-of concept, we created a simulation of the sun and eight planets of the solar system in which these were especially apparent. The first law, which states that orbits are elliptical, can be seen by the lines traced out by the planets as they orbit the sun. The second law gives a mathematical intuition to how the orbital velocity of a particle should change as it goes around its orbit, we are able to observe in our simulation that the particles are moving more quickly near their periapsis (the point in the orbit nearest the black hole) than they are at their apoapsis (the furthest point). Lastly, in our solar system model, we are able to clearly see the kepler's third law relation that the larger the semimajor axis of an object's orbit, the longer it takes to make one complete cycle.

Ultimately, we used a mathematical model that, while relatively simple, provided some strong, physically-grounded simulations.

2.4 Numerical Algorithm

Implementing our mathematical model numerically was not too difficult. For our numerical algorithm we used a particle discretization, and an Explicit Euler solver. Newtonian gravitation is just an inverse-square law, so there are no differential operators that need to be discretized. In order to fit our simulation's scale, we calculated the Gravitational constant G to be 0.4.

We treated particles closer than 0.2 as being in contact with each other, and ignored their gravitational attraction. This had the effect of keeping the particles from overlapping, and implements an implicit normal force as noted in the previous section. We also ignored the gravitational interaction between particles more than 10 units apart, in order to keep the calculations manageable. At that distance the force is 2500 times smaller than the magnitude of the force at 0.2 units. Thus it is negligible, especially when compared to the force exerted by the black hole.

We also translated our solver so as to keep the star's center of mass constant. This was accomplished by maintaining a separate record of the star and black hole's visualized positions, while the calculations were carried out in the original frame where the black hole was kept stationary.

3 Solved Challenges

As mentioned previously, we ran into trouble keeping the star's component particles from overlapping with each other. We attempted to use implicit spring forces, but in order to match the stiffness necessary to keep the star's shape the spring constant k_s had to be larger than our Explicit Euler solver could handle, even when we shortened the time step. The solution we chose was to treat particles within 0.2 units of each other as in contact, and introduced a normal force that cancelled out the gravitational force between them. This is an abstraction of the forces that actually keep matter apart (van der Waals forces and the Pauli Exclusion principle), but it behaves well in our simulation so we believe it to be accurate, at least to first order.

4 Examples

The initial setup of the simulation can be seen in Figure 1. The star has an initial velocity down and to the left, and is in an unstable orbit. The first tidal disruption event can be seen in Figure 2, where the near side of the star is pulled towards the black hole, while the far side moves away in order to conserve angular momentum as the star spins. In Figure 3, the star is shown after its first full orbit. It has not settled back into a spherical shape, as it is still oscillating from the deformation that occurred in the first event. The second tidal disruption event can be seen in Figure 4, and in Figures 5 and 6 the star is shown reforming into a relatively localized body. The third tidal disruption event looks similar to the second, but the fourth (Figure 7) is more violent, with greater spread. Figure 8 is the fifth and final disruption event, and the full disintegration of the star is shown in Figure 9. The accretion disk formed after disintegration is shown in Figure 10, and is proof that angular momentum is conserved in our simulation despite never being enforced.

5 Conclusion and Contributions

While our model is not quite as complicated as that in Clerici and Gomboc (2020), we were ultimately able to create a reasonably physical simulation of a high-energy tidal disruption event. By enforcing only the Universal Law of Gravitation and a proxy for Newton's Third Law, we modeled a system in which we can observe Kepler's Laws of Planetary Motion (which are derived from Newton's Laws of Motion), conservation of angular momentum, and more. Additionally, we created some beautiful videos of tidal disruption that could have potential use in public education and outreach.

For this project, we were equal partners. Rory wrote the foundational code and put together the visualizations to create the black hole, star, sun, planets, and tracking lines. Catherine wrote the proof-of concept solar system driver. We both worked on bringing these things together for our final simulation, debugging, and problem solving issues that arose. Our code was based on the starter code given in class for our final toolkit. We worked together to present our work to the class (despite a small technical difficulty). For this report, Catherine wrote sections 1, 2.1, 2.2, 2.3, and 5. Rory wrote sections 2.4, 3, and 4, and was in charge of creating our visuals and final videos.

References

A. Clerici and A. Gomboc. A study on tidal disruption event dynamics around an sgr a*-like massive black hole. *A&A*, 642:A111, 2020. doi: 10.1051/0004-6361/202037641.

A Images

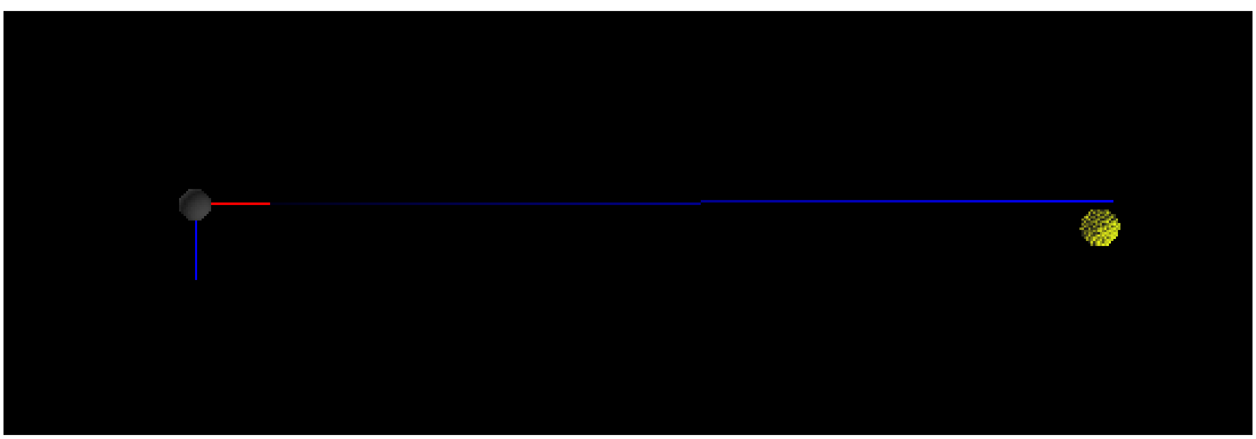

Figure 1: Starting arrangement of the simulation

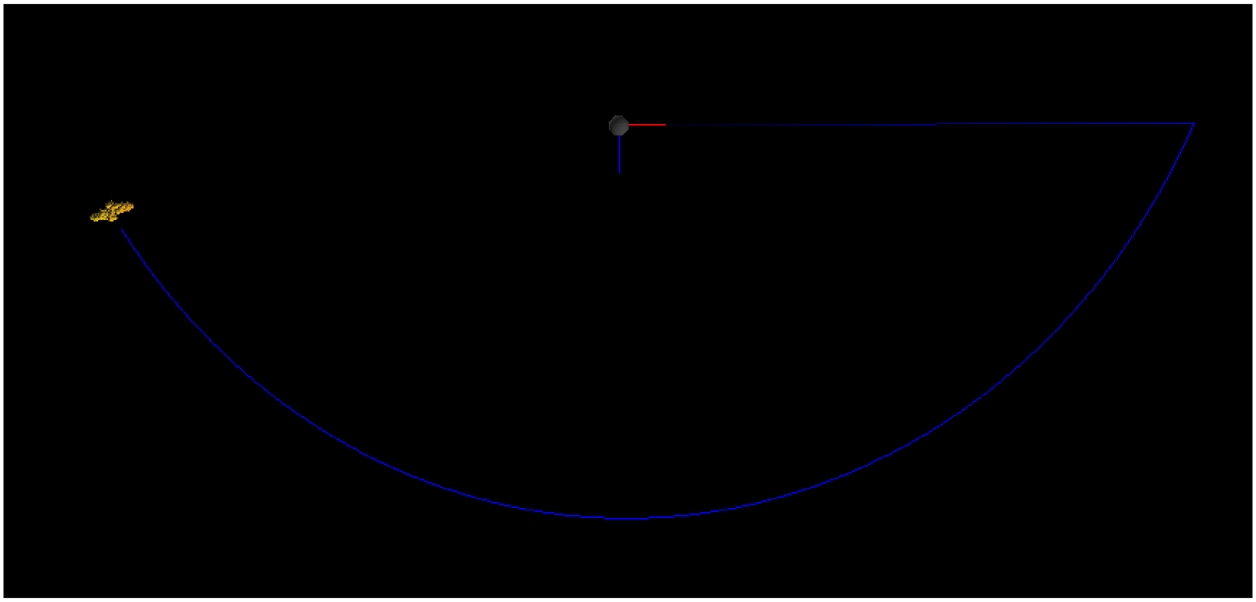

Figure 2: First tidal disruption event

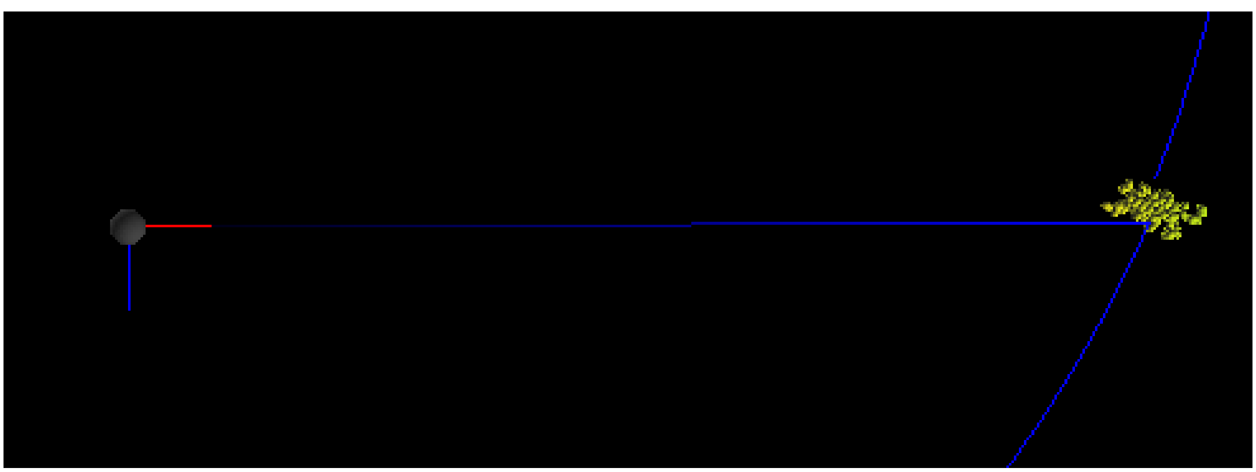

Figure 3: First full orbit

Figure 4: Second tidal disruption event

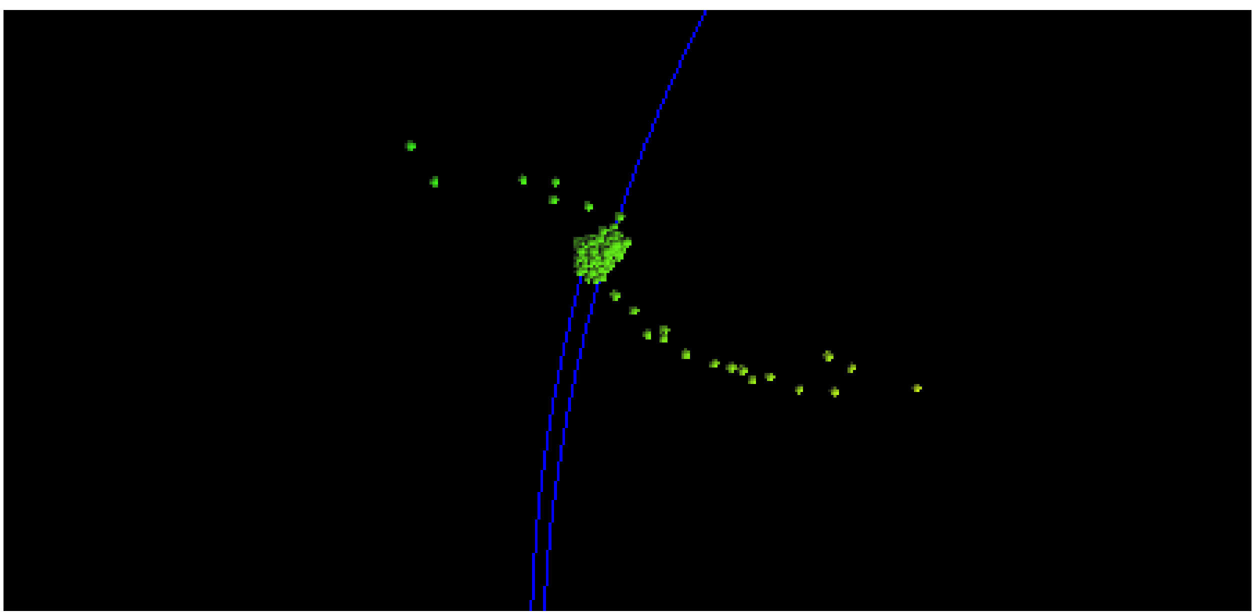

Figure 5: Star reforming after major tidal disruption event

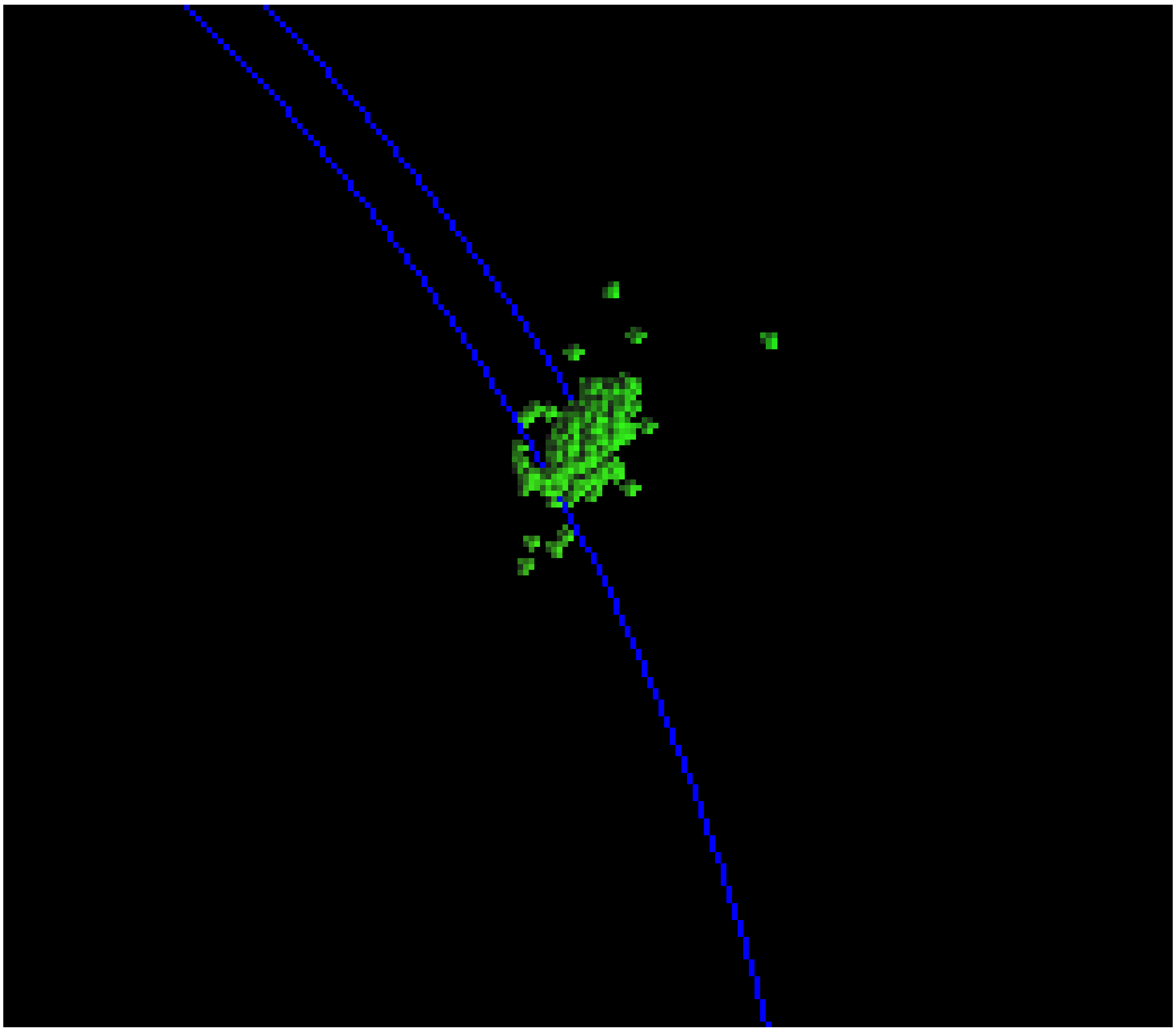

Figure 6: Star reformed after major tidal disruption event, second full orbit almost complete

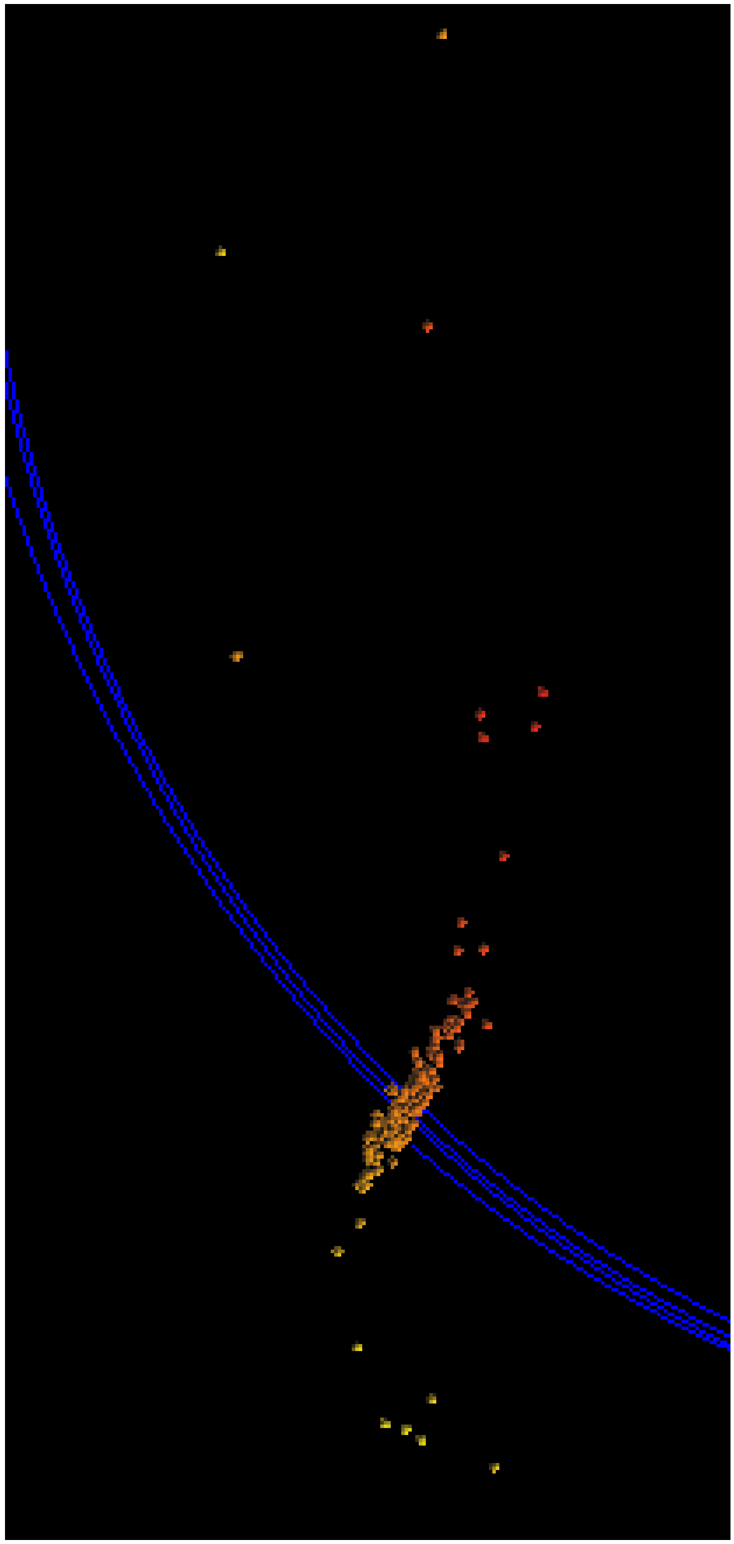

Figure 7: Fourth tidal disruption event

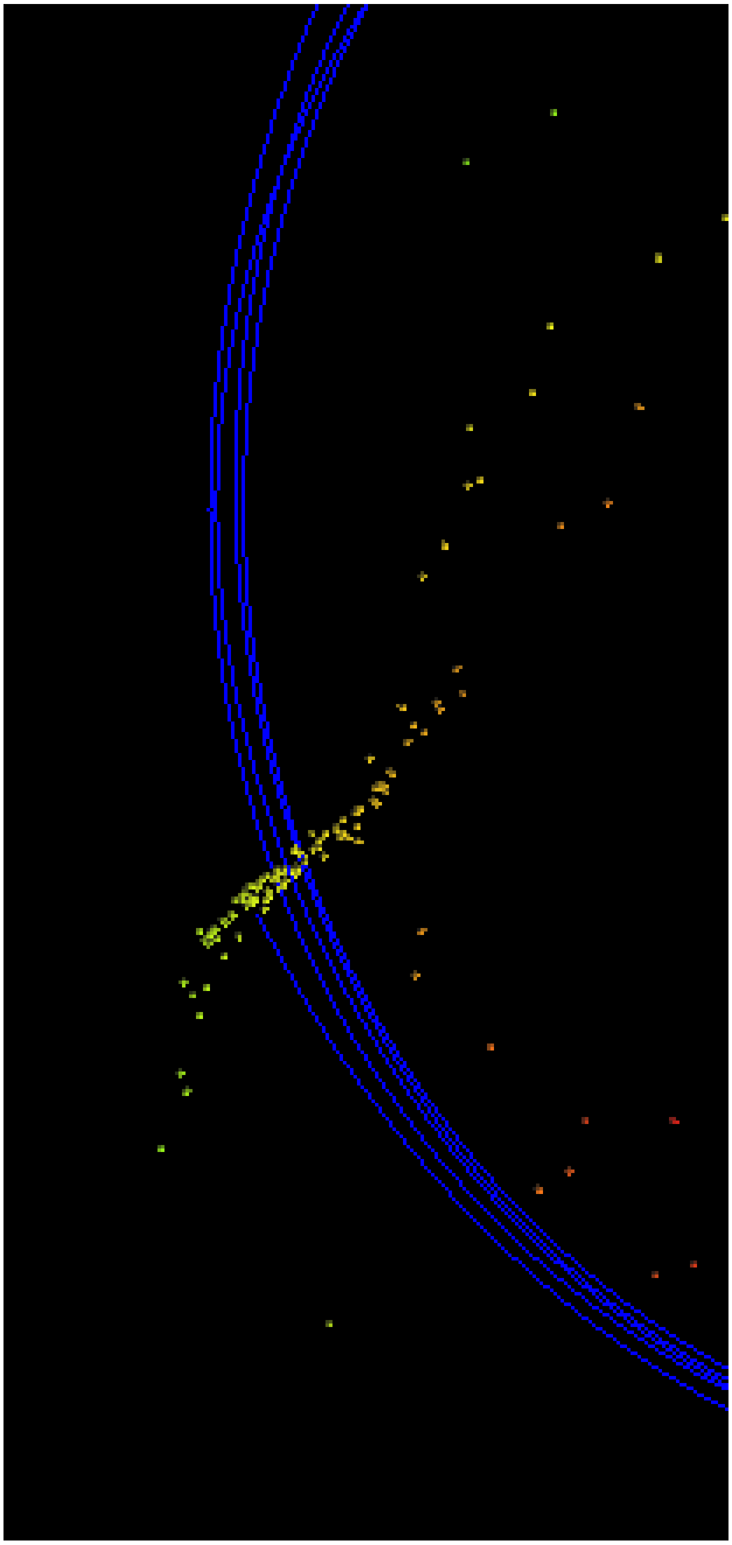

Figure 8: Fifth tidal disruption event

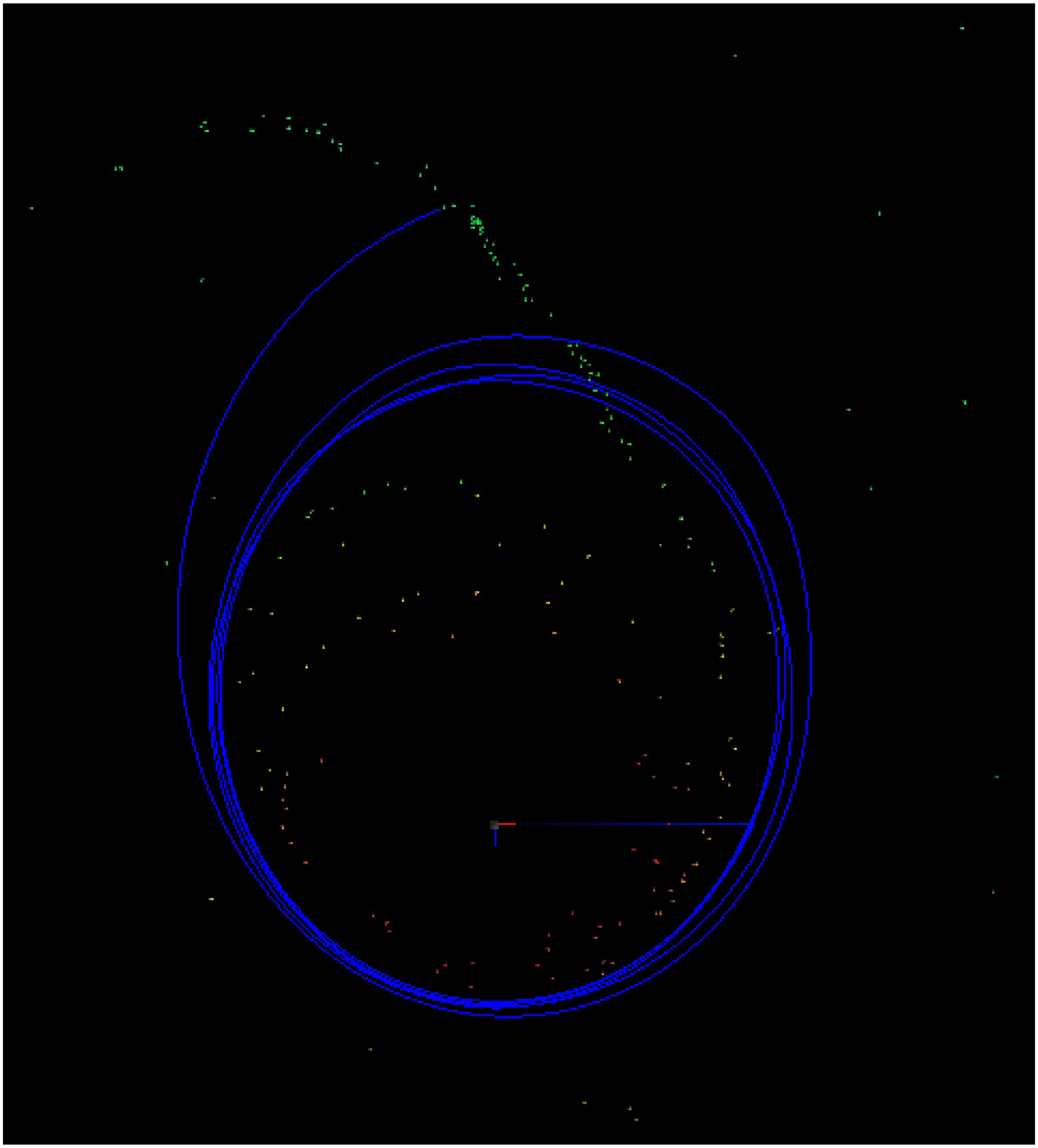

Figure 9: Final disintegration of the star

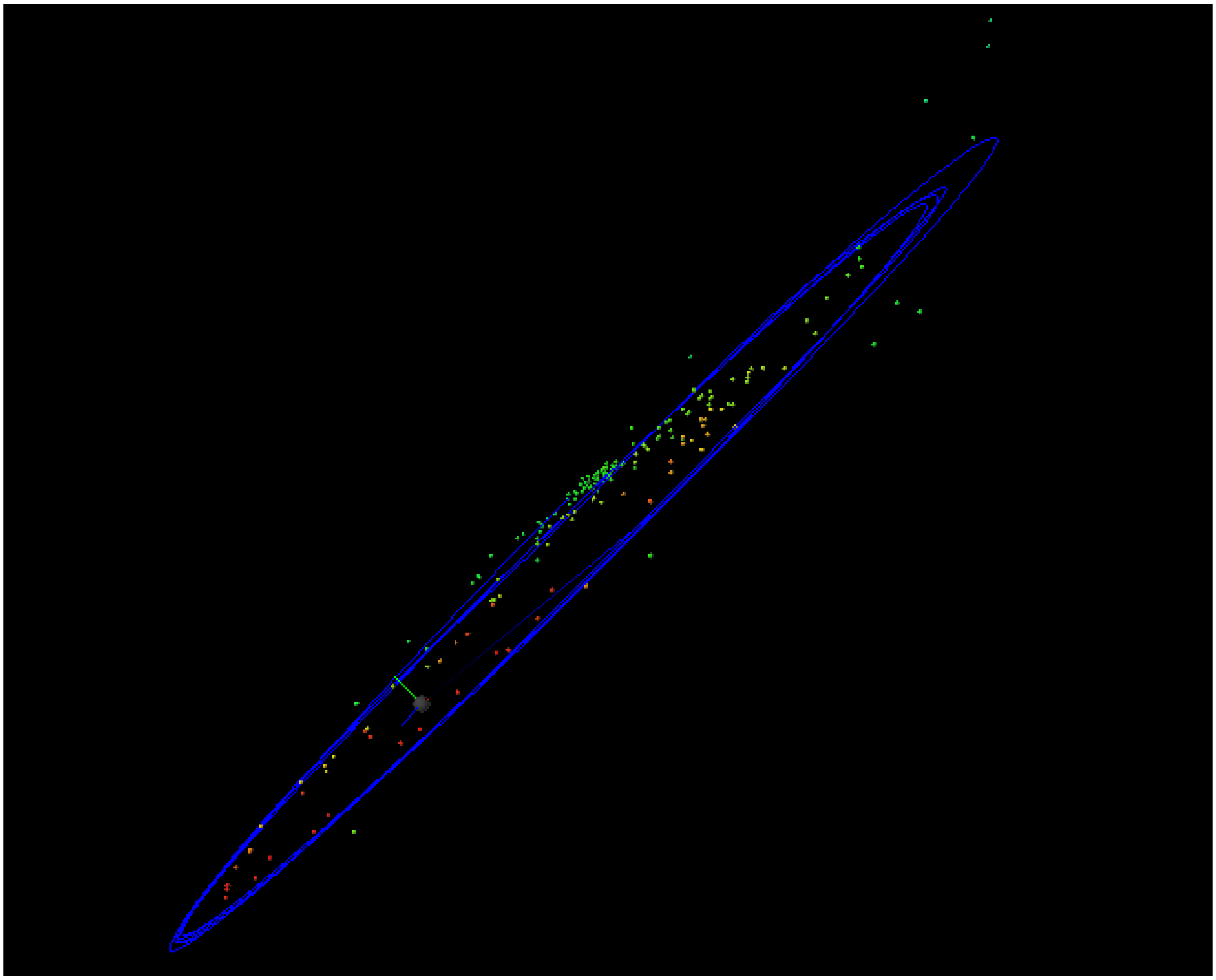

Figure 10: Accretion disk formed after final disintegration