

\ Step	Cost of each execution	Total # of times executed
1	1	1
2	1	$n + 1$
3	1	$\sum_{i=0}^{n-1} (n - i + 1) = n(n + 3) / 2$
4	1	$\sum_{i=0}^{n-1} (n - i) = n(n + 1) / 2$
5	1	(After solving nested summation) $n(n + 1)(n + 2) / 6$
6	5	(After solving nested summation) $n(n - 1)(n + 1) / 6$
7	7	$\sum_{i=0}^{n-1} (n - i) = n(n + 1) / 2$
8	2	1

$$T_1(n) = 4 + n + n(n + 3)/2 + n(n + 1)/2 + n(n + 1)(n + 2)/6 + n(n - 1)(n + 1)/6 + n(n + 1)/2$$

$$= 4 + n + n^2/2 + 3n/2 + n^2/2 + n/2 + (n^2 + n)(n + 2)/6 + (n^2 - n)(n + 1)/6 + n^2/2 + n/2$$

$$= 4 + n^2 + 3n + (n^3 + 2n^2 + n^2 + 2n)/6 + (n^3 + n^2 - n^2 - n)/6 + n^2/2 + n/2$$

$$= 4 + n^2 + 3n + n^3/6 + 3n^2/6 + 2n/6 + n^3/6 - n/6 + n^2/2 + n/2$$

$$= 4 + n^2 + 3n + 2n^3/6 + n^2 = 4 + 2n^2 + 3n + 2n^3/6 + 4n/6$$

$$= 24 + 12n^2 + 22n + 2n^3 = 2n^3 + 12n^2 + 22n + 24$$

Order of Complexity (c): $O(n^3)$

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	$n + 1$
3	1	n
4	1	$\sum_{i=0}^{n-1} (n - i + 1) = n(n + 3) / 2$
5	4	$\sum_{i=0}^{n-1} (n - i) = n(n + 1) / 2$
6	6	$\sum_{i=0}^{n-1} (n - i) = n(n + 1) / 2$
7	2	1

$$\begin{aligned}
T_2(n) &= 4 + 2n + n^2/2 + 3n/2 + 4(n^2/2 + n/2) + 6(n^2/2 + n/2) \\
&= 4 + 2n + n^2/2 + 3n/2 + 2n^2 + 2n + 3n^2 + 3n \\
&= 4 + 7n + 5n^2 + n^2/2 + 3n/2 = 8 + 14n + 10n^2 + n^2 + 3n \\
&= 11n^2 + 17n + 8
\end{aligned}$$

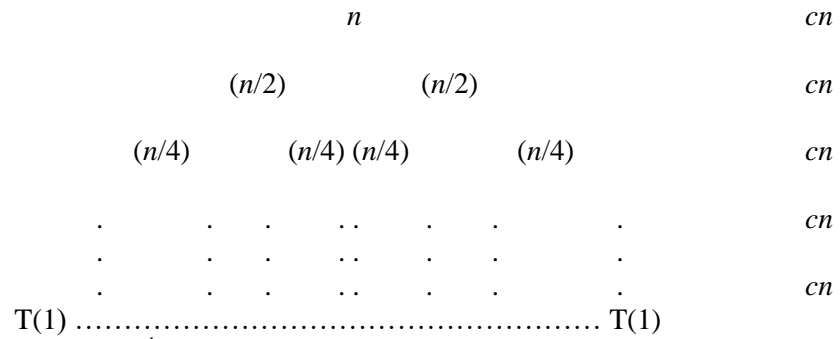
Order of Complexity (c): $O(n^2)$

Step	Cost of each execution	Total # of times executed
1	4	1
2	9	1
Steps executed when input is a base case: 1 and 2		
$T(0) = 4; T(1) = 9 \rightarrow$ Assume $T(1)$.		
3	5	1
4	2	1
5	1	$n / 2 + 1$
6	4	$n / 2$
7	6	$n / 2$
8	2	1
9	1	$n / 2 + 1$
10	5	$n / 2$
11	6	$n / 2$
12	4	1
13	1	1
14	1	1
15	14	1
Steps executed when input is NOT a base case: 3 - 15		

$$T(n > 1) = 2(T(n/2)) + 23n/2 + 31$$

$$\text{Simplified } T(n > 1) = 2(T(n/2)) + 23n/2 + c$$

Using recursion tree method:



$$2^h = 2^{\log_2 n}. \text{ Hence, height} = \log_2(n).$$

Based on the cost of each level and the number of levels, Order of Complexity (c): $O(n\log(n))$.

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	1
3	1	$n + 1$
4	9	n
5	6	n
6	2	1

$$T_4(n) = 16n + 5$$

Order of Complexity (c): $O(n)$