REBOIII Manual

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August 21, 2015

1 Formalism

The Reactive empirical bond order potential was fist developed by Brenner et. al. in 1990 [?]. It was developed to model CVD deposition of diamond. It was later modified into the REBOII potential [?].

1.1 Energy function

$$E_b = \sum_{i} \sum_{j>i} \left[V_R(r_{ij}) - \bar{b}_{ij} V_A(r_{ij}) \right]$$
 (1)

The pair terms are:

$$V^{R}(r_{ij}) = f^{C}(r_{ij}) (1 + Q/r_{ij}) A e^{-\alpha r_{ij}}$$
(2)

$$V^{A}(r_{ij}) = f^{C}(r_{ij}) \sum_{n=1,3} Be^{-\beta r_{ij}}$$
(3)

Where $f^{C}(r_{ij})$ is the cut off function

$$f^{c}(r_{ij}) = \begin{bmatrix} 1 & r_{ij} < D^{min} \\ [1 + cos[(r_{ij} - D^{min})/(D^{max} - D^{min})]] & D^{min} < r_{ij} < D^{max} \\ 0 & r_{ij} > D^{max} \end{bmatrix}$$
(4)

The bond order term is:

$$\bar{b}_{ij}(r_{ij}, r_{ik}, r_{jl}, r_{kl}) = \frac{1}{2} \left[b_{ij}^{\sigma - \pi} + b_{ji}^{\sigma - \pi} \right] + b_{ij}^{\pi}$$
(5)

where the σ and π bonding is accounted for by averaging the bond order terms of the i-j and j-i interactions. This term contians the three body terms dependant on bond angle θ_{ijk} and the coordination function P_{ij} .

$$b_{ij}^{\sigma-\pi} = \left[1 + \sum_{k \neq i,j} f_{ik}^{C}(r_{ik})G(\cos(\theta_{ijk}))e^{\lambda_{ijk}} + P_{ij}(N_i^C, N_i^H)\right]^{-\frac{1}{2}}$$
(6)

For the hydrocarbon centered angular function is a bicubic spline G_C and G_H . For the oxygen based interactions [?]

$$G(\cos(\theta_{ijk})) = a_0 + a_1 \left[a_2 - \cos(\theta) \right]^2 \tag{7}$$

where the cosine of θ can be calculated using the law of cosines:

$$\cos(\theta_{ijk}) = \frac{r_{ij}^2 + r_{ik}^2 - r_{kj}^2}{2r_{ij}r_{ik}} \tag{8}$$

and the function e^{λ} is used to remove spurious wells in the H angule function, otherwise $\lambda = 0$. Referring to the code it seems e^{λ} is complex that it seems.

$$e_{ijk}^{\lambda} = e^{\lambda_{ijk}(BL_{ik} - BL_{ij})} e^{\lambda_{ijk}(r_{ij} - r_{jk})} \tag{9}$$

where BL = 0 except for ijk = HCH and ijk = HHC when $BL_{HH} = 0.7415886997$ and $BL_{CH} = 1.09$. The coordination function is a bicubic spline dependant on the number of nieghbors of each elment type. Where the total number of elemental nieghbors is the sum of the cut off function f^c

$$N_i^{elm} = \sum_{k \neq i,j}^{atomsofelm} f_{ik}^c(r_{ik}) \tag{10}$$

 π bonding is taken into account with four body term:

$$b_{ij}^{\pi} = \Pi^{RC} + b_{ij}^{DH} \tag{11}$$

The radical term Π_{ij}^{RC} describes the radical nature of carbon bond as well as conjugation effects.

$$\Pi_{ij}^{RC} = F_{ij}(N_i^t, N_j^t, N_{ij}^{conj}) \tag{12}$$

where F_{ij} is a tricubic spline which depends on the total numbers of C and it conjucation N_{ij}^{conj} .

$$N_i^t = \sum_{elm}^{neighborsofi} N_i^{elm} \tag{13}$$

$$N_{ij}^{conj} = 1 + \left[\sum_{k \neq i,j}^{carbon} f_{ik}^{C}(r_{ik}) F(X_{ik})\right]^{2} + \left[\sum_{l \neq i,j}^{carbon} f_{jl}^{C}(r_{jl} F(X_{jl}))\right]^{2}$$
(14)

$$F(x_{ik}) = \begin{bmatrix} 1 & x_{ik} = 2 \\ 1 + \cos(\pi(x_{ik} - 2)) \end{bmatrix} / 2 \quad 2 < x_{ik} < 3 \\ 0 \quad x_{ik} > 3$$
 (15)

$$x_{ik} = N_k^t - f_{ik}^c(r_{ik}) \tag{16}$$

the dihedral term b_{ij}^{DH} is a four body torsional interaction

$$b_{ij}^{DH} = T_{ij} \left(N_i^t, N_j^t, N_{ij}^{conj} \right) \left[\sum_{(k \neq i, j)} \sum_{(l \neq i, j)} \left(1 - \cos^2(\Theta_{ijkl}) \right) f_{ik}^c(r_{ik}) f_{jl}^c(r_{jl}) \right]$$
(17)

where T_{ij} is a tricubic spline, and

$$cos(\Theta_{ijkl}) = e_{ijk}e_{ijl} \tag{18}$$

where

$$e_{ijk} = \hat{r}_{ji} \times \hat{r}_{ik} \tag{19}$$

$$e_{ijk} = \frac{r_{ji} \times r_{ik}}{|r_{ii}||r_{ik}|sin\theta_{jik}} \tag{20}$$

1.2 Derivatives

The energy function above is dependant on firts, second or third neighbors if screaning is implemented.

Therefore, the force $\frac{dE}{dr}$ on these atoms must be calculated.

$$E_b(r_{ij}, r_{ik}, r_{jl}) = \sum_{i} \sum_{j>i} \left[V_R(r_{ij}) - \bar{b}_{ij} V_A(r_{ij}) \right]$$
(21)

$$\frac{dE_b}{dr_{ij}} = \frac{dV_R(r_{ij})}{dr_{ij}} - \bar{b}_{ij}\frac{dV_A(r_{ij})}{dr_{ij}} - \frac{d\bar{b}_{ij}}{dr_{ij}}V_A(r_{ij})$$
(22)

$$\frac{dE_b}{dr_{ik}} = \frac{dV_R(r_{ij})}{dr_{ij}} - \bar{b}_{ij}\frac{dV_A(r_{ij})}{dr_{ij}} - \frac{d\bar{b}_{ij}}{dr_{ij}}V_A(r_{ij})$$
(23)

$$\frac{dV^{R}(r_{ij})}{dr_{ij}} = \frac{df^{c}(r_{ij})}{dr_{ij}} \left(1 + \frac{Q}{r_{ij}}\right) Ae^{-\alpha r_{ij}} + f^{c}(r_{ij}) \left[\frac{Q}{r_{ij}^{2}} Ae^{-\alpha r_{ij}} - \alpha r_{ij} \left(1 + \frac{Q}{r_{ij}}\right) Ae^{-\alpha r_{ij}}\right]$$
(24)

$$\frac{dV^A(r_{ij})}{dr_{ij}} = \frac{df^c(r_{ij})}{dr_{ij}} \sum_{n=1,3} Be^{-\beta_n r_{ij}} - f^C(r_{ij}) \sum_{n=1,3} \beta_n Be^{-\beta_n r_{ij}}$$
(25)

$$\frac{df^{c}(r_{ij})}{dr_{ij}} = -\frac{1}{2(D^{max} - D^{min})} sin[(r_{ij} - D^{min})/(D^{max} - D^{min})] \quad D^{min} < r_{ij} < D^{max} \quad (26)$$

$$r_{ij} > D^{max}$$

the mulitbody bond order term is dependant on r_{ij} , r_{ik} , r_{jl} and r_{kl} . Taking the derivative of b_{ij} with resplect to r_{ij}

$$\frac{d\bar{b}_{ij}}{dr_{ij}} = \frac{1}{2} \left[\frac{db_{ij}^{\sigma-\pi}}{dr_{ij}} + \frac{db_{ji}^{\sigma-\pi}}{dr_{ij}} \right] + \frac{db_{ij}^{\pi}}{dr_{ij}}$$

$$(27)$$

where

$$\frac{db_{ij}^{\pi}}{dr_{ij}} = \frac{d\Pi^{RC}}{dr_{ij}} + \frac{db_{ij}^{DH}}{dr_{ij}} \tag{28}$$

if

$$b_{ij}^{\sigma-\pi} = [d_{ij}]^{-\frac{1}{2}} \tag{29}$$

where

$$d_{ij} = 1 + \sum_{k \neq i} f_{ik}^{C}(r_{ik})G(\cos(\theta_{ijk}))e + P_{ij}(N_i^C, N_i^H)$$
(30)

then

$$\frac{db_{ij}}{dr_{ij}} = \frac{db_{ij}}{dd_{ij}} \frac{dd_{ij}}{dr_{ij}} \tag{31}$$

$$\frac{db_{ij}}{dd_{ij}} = -\frac{b_{ij}}{2d_{ij}} \tag{32}$$

$$\frac{dd_{ij}}{dr_{ij}} = \sum_{k \neq i,j} f_{ik}^{C}(r_{ik}) e^{\lambda_{ijk}} \frac{dG(\cos(\theta_{ijk}))}{dr_{ij}} + f_{ik}^{C}(r_{ik}) G(\cos(\theta_{ijk})) \frac{de^{\lambda_{ijk}}}{dr_{ij}} + \frac{dP_{ij}(N_{i}^{C}, N_{i}^{H})}{dr_{ij}}$$
(33)

$$\frac{dG(\cos(\theta_{ijk}))}{dr_{ij}} = \frac{dG(\cos(\theta_{ijk}))}{d\cos(\theta_{ijk})} \frac{d\cos(\theta_{ijk})}{dr_{ij}}$$
(34)

if $G(cos(\theta_{ijk}))$ is for oxygen or sulfur

$$\frac{dG(\cos(\theta_{ijk}))}{d\cos(\theta_{ijk})} = 2a_1 \left[\cos(\theta) - a_2\right] \tag{35}$$

 $cos(\theta_{ijk})$ is calculated with the law of cosines ??

$$\frac{d\cos(\theta_{ijk})}{dr_{ij}} = \frac{r_{ij}^2 - r_{ik}^2 + r_{kj}^2}{2r_{ij}r_{ik}r_{ij}}$$
(36)

$$\frac{de^{\lambda_{ijk}}}{dr_{ij}} = \lambda_{ijk}e^{\lambda_{ijk}} \tag{37}$$

$$\frac{dP_{ij}(N_i^C, N_i^H)}{dr_{ij}} = \sum_{elm} \frac{P_{ij}}{dN_{elm}} \frac{dN_{elm}}{dr_{ij}}$$
(38)

$$\frac{dN^{elm}}{dr_{ij}} = 0??? (39)$$

$$\frac{db_{ij}^{DH}}{dr_{ij}} = \frac{dT_{ij}}{dr_{ij}}DH_{sum} + T_{ij}\frac{dDH_{sum}}{dr_{ij}}$$

$$\tag{40}$$

where

$$DH_{sum} = \left[\sum_{(k \neq i,j)} \sum_{(l \neq i,j)} \left(1 - \cos^2(\Theta_{ijkl}) \right) f_{ik}^c(r_{ik}) f_{jl}^c(r_{jl}) \right]$$

$$(41)$$

$$\frac{dDH_{sum}}{dr_{ij}} = \left[\sum_{(k \neq i, j)} \sum_{(l \neq i, j)} \frac{d(1 - \cos^2(\Theta_{ijkl}))}{dr_{ij}} f_{ik}^c(r_{ik}) f_{jl}^c(r_{jl}) \right]$$
(42)

$$\frac{d\left(1 - \cos^2(\Theta_{ijkl})\right)}{dr_{ij}} = 2\frac{d\cos(\Theta_{ijkl})}{dr_{ij}} \tag{43}$$

$$\frac{d}{dr_{ij}} \left[\frac{(r_{ji} \times r_{ik})(r_{ij} \times r_{jl})}{|r_{ik}||r_{jl}||r_{ij}|^2 sin\theta_{ik} sin\theta_{jl}} \right] = \frac{d}{dr_{ij}} \left[\frac{rcross}{rmag} \right]$$
(44)

$$\frac{d}{dr_{ij}} \left[\frac{rcross}{rmag} \right] = \frac{\frac{drcross}{r_{ij}} rmag + rcross \frac{drmag}{dr_{ij}}}{rmag^2}$$
(45)

$$\frac{drmag}{dr_{ij}} = 2r_{ik}r_{jl}r_{ij}sin\theta_{ik}sin\theta_{jl} + r_{ik}r_{jl}r_{ij}^2sin\theta_{ik}\frac{dsin\theta_{jl}}{r_{ij}} + r_{ik}r_{jl}r_{ij}^2sin\theta_{jl}\frac{dsin\theta_{ik}}{r_{ij}}$$
(46)

$$\frac{dsin\theta_{ik}}{dr_{ij}} = \frac{d(1 - cos^2\theta_{ik})^{\frac{1}{2}}}{dr_{ij}} = -\frac{cos\theta_{ik}}{sin\theta_{ik}} \frac{dcos\theta_{ik}}{dr_{ij}}$$

$$(47)$$

$$\frac{dsin\theta_{jl}}{dr_{ij}} = \frac{d(1 - cos^2\theta_{jl})^{\frac{1}{2}}}{dr_{ij}} = -\frac{cos\theta_{jl}}{sin\theta_{jl}} \frac{dcos\theta_{jl}}{dr_{ij}}$$

$$(48)$$

Taking the derivative of b_{ij} with resplect to r_{ik}

$$\frac{d\cos(\theta_{ijk})}{dr_{ik}} = \frac{-(r_{ij}^2 - r_{ik}^2) + r_{kj}^2}{2r_{ij}r_{ik}r_{ik}}$$
(49)

$$\frac{de^{\lambda_{ijk}}}{dr_{ij}} = -\lambda_{ijk}e^{\lambda_{ijk}} \tag{50}$$

$$\frac{dN^{elm}}{dr_{ij}} = \sum_{k \neq i,j}^{atomsofelm} \frac{df_{ik}^c(r_{ik})}{r_{ik}}$$
(51)

$$\frac{drmag}{dr_{ik}} = |r_{jl}||r_{ij}|^2 sin\theta_{jl} sin\theta_{ik} + |r_{ik}||r_{jl}||r_{ij}|^2 sin\theta_{jl} \frac{dsin\theta_{ik}}{dr_{ik}}$$

$$(52)$$

$$\frac{dsin\theta_{ik}}{dr_{ik}} = -\frac{cos\theta_{ik}}{sin\theta_{ik}} \frac{dcos\theta_{ik}}{dr_{ik}} \tag{53}$$

$$\frac{drcross}{dr_{ik}} = \left[\frac{d}{dr_{ij}} \left(\vec{r}_{ji} \times \vec{r}_{ik}\right)\right] \cdot \left(\vec{r}_{ij} \times \vec{r}_{jl}\right)$$
(54)

$$\frac{drcross}{dr_{ik}} = \left[\frac{d}{dr_{ij}} \left(\begin{vmatrix} r_{ji}^{y} & r_{ji}^{z} \\ r_{ik}^{y} & r_{ji}^{z} \end{vmatrix} \hat{x} - \begin{vmatrix} r_{ji}^{x} & r_{ji}^{z} \\ r_{ik}^{x} & r_{ik}^{z} \end{vmatrix} \hat{y} + \begin{vmatrix} r_{ji}^{x} & r_{ji}^{y} \\ r_{ik}^{x} & r_{ik}^{y} \end{vmatrix} \hat{z} \right) \right] \cdot \left(\begin{vmatrix} r_{ij}^{y} & r_{ij}^{z} \\ r_{jl}^{y} & r_{jl}^{z} \end{vmatrix} \hat{x} - \begin{vmatrix} r_{ij}^{x} & r_{ij}^{z} \\ r_{jl}^{x} & r_{jl}^{z} \end{vmatrix} \hat{y} + \begin{vmatrix} r_{ij}^{x} & r_{ij}^{y} \\ r_{jl}^{x} & r_{jl}^{y} \end{vmatrix} \hat{z} \right) \right]$$

$$\frac{drcross}{dr_{ik}} = \left((r_{ji}^y - r_{ji}^z) \left| \begin{array}{cc} r_{ij}^y & r_{ij}^z \\ r_{jl}^y & r_{jl}^z \end{array} \right| \hat{x} + (r_{ji}^z - r_{ji}^x) \left| \begin{array}{cc} r_{jl}^x & r_{jl}^z \\ r_{ij}^x & r_{ij}^z \end{array} \right| \hat{y} + (r_{ji}^x - r_{ji}^y) \left| \begin{array}{cc} r_{ij}^x & r_{ij}^y \\ r_{jl}^x & r_{jl}^y \end{array} \right| \hat{z} \right) \ (55)$$

Taking the derivative of b_{ij} with resplect to r_{jl}

$$\frac{drmag}{dr_{il}} = |r_{ik}||r_{ij}|^2 sin\theta_{jl} sin\theta_{ik} + |r_{ik}||r_{jl}||r_{ij}|^2 sin\theta_{ik} \frac{dsin\theta_{jl}}{dr_{jl}}$$

$$(56)$$

$$\frac{dsin\theta_{jl}}{dr_{jl}} = -\frac{cos\theta_{jl}}{sin\theta_{jl}} \frac{dcos\theta_{jl}}{dr_{jl}} \tag{57}$$

Taking the derivative of b_{ij} with resplect to r_{il}

$$\frac{drmag}{dr_{il}} = |r_{ik}||r_{jl}||r_{ij}|^2 sin\theta_{ik} \frac{dsin\theta_{jl}}{dr_{il}}$$
(58)

$$\frac{dsin\theta_{jl}}{dr_i l} = -\frac{cos\theta_{jl}}{sin\theta_{il}} \frac{dcos\theta_{jl}}{dr_{il}}$$

$$(59)$$

Taking the derivative of b_{ij} with resplect to r_{kj}

$$\frac{d\cos(\theta_{ijk})}{dr_{kj}} = \frac{2r_{kj}}{2r_{ij}r_{ik}} \tag{60}$$

$$\frac{drmag}{dr_k j} = |r_{jl}||r_{ij}|^2 sin\theta_{jl} sin\theta_{ik} + |r_{ik}||r_{jl}||r_{ij}|^2 sin\theta_{jl} \frac{dsin\theta_{ik}}{dr_{ik}}$$

$$(61)$$

$$\frac{dsin\theta_{ik}}{dr_{ik}} = -\frac{cos\theta_{ik}}{sin\theta_{ik}} \frac{dcos\theta_{ik}}{dr_{ik}} \tag{62}$$

2 Parameters

2.1 sub.setpairs.f

Set pair parameters and angular function as well as potential cut off region.

- 1. A = DD(I, J)
- 2. $\alpha = DXL(I, J)$
- 3. Q = ED(I, J)
- 4. B = AD(I, J)
- 5. $\beta = AXL(I, J)$
- 6. $B_2 = BD(I, J)$
- 7. $\beta_2 = BXL(I,J)$
- 8. $B_3 = CD(I, J)$
- 9. $\beta_3 = CXL(I,J)$

where B_2 , B_3 and β_2 , β_3 are set to 1 and zero respectavely for all elements except carbon. RB1(I, J) and RB2 are the inner and outer points of the REBO cut off region used in f^C .

- 1. $RLIST(I, J) = (RB2(I, J) + RLL)^2$
- $2. RMAX = RB1^2$

Where RLL is the neighbor itemizecut off.

2.2 sub.setsplines.f

- 1. $XHO = P_O(N^H + N^C, N^O)$ knot values
- 2. $CLMOX = P_{HC,O}$ coefficients
- 3. $XHS = P_S(N^H + N^C, N^S)$ knot values
- 4. $CLMSX = P_{HC,S}$ coefficients
- 5. $SPGC = G_C$
- 6. $SPGH = G_H$
- 7. $XH(CC=1/CH=2,N^H,N^C),=P_{CC/CH}(N^H,N^C)$ knot values
- 8. $CLM() = P_{HC}$ coefficients

- 9. IN2 = bicubic spline exponents
- 10. IN3 = tricubic spline exponents
- 11. $CLMN() = F_{HC}$ coefficients
- 12. $TLMN() = F_{ij}$ torsional coefficients

2.3 mtable.f

Loop over all interactions

ntab the number of data points taken to define the pair potential between $0 - R_{max}$

ddtab(ki, kj) = dx the distance between two points when defining the potential table Loop over $\sum_{r=dx}^{(r_{cut}^{outer})}$

2.3.1 Calculate the cut off function and its derivative

 $f^c(r)$

С

If $r_{min}^C < r < r_{max}^C$

$$PID = frac\pi (D^{max} - D^{min}) \tag{63}$$

$$DTEMP = PID * (r - D^{min}) (64)$$

$$f^c(r) = \frac{1 + \cos(DTEMP)}{2}$$

which leads to the cut off function ?? and its deriviative.

$$\frac{df^c}{dr} = -\frac{1}{2(D^{MAX} - D^{min})} sin(\frac{r - D^{min}}{D^{Max} - D^{min}})$$

$$\tag{65}$$

Where if $f^c(D^{min}) = 1$ and $f^c(D^{max}) = 0$ fc and DFC are stored in tabe tabfc and tabdfc

2.3.2 Calculate pair terms

Attractive pair terms ??

$$\frac{dV^A}{dr} = \frac{df^c}{dr} \left[\sum_{n=1}^3 B_n e^{-\beta_n r} \right] + f^c \left[\sum_{n=1}^3 -\beta_n B_n e^{-\beta_n r} \right]$$

$$\tag{66}$$

Where

$$\sum_{n=1}^{3} B_n e^{-\beta_n r} = VA + VB + VC \tag{67}$$

and

$$\sum_{n=1}^{3} -\beta_n B_n e^{-\beta_n r} = DVA + DVB + DVC \tag{68}$$

Values at RC are stored in atable and datable

```
VV=(VA+VB+VC)/2.0D0
DVV=(DVA+DVB+DVC)/2.0D0
atable(ki,kj,i) = FC*VV
atable(kj,ki,i) = atable(ki,kj,i)
datable(ki,kj,i) = (FC*DVV+DFC*VV)/RC
datable(kj,ki,i) = datable(ki,kj,i)
```

not sure why there is a 1/2 in there ???

$$atable(KI, KJ, I) = \frac{1}{2} \sum_{n=1}^{3} B_n e^{-\beta_n r}$$
$$datable(KI, KJ, I) = -\frac{1}{2r} \frac{dV^A}{dr}$$

2.3.3 Repulsive pair terms

$$FF1 = Ae^{-\alpha r}$$

$$DF1 = -\alpha FF1$$

$$FF2 = \left(1 + \frac{Q}{r}\right)$$

$$DF2 = -\frac{Q}{r}$$

$$VV = (1 + \frac{Q}{r})Ae^{-\alpha r}$$

$$DVM = (-\frac{Q}{r}Ae^{-\alpha r}) + (-\alpha)(1 + \frac{Q}{r})Ae^{-\alpha r}$$

store replusive values in *rtable* and *drtable*

2.4 caguts.f

Check to see if neighbor list should be updated

```
IF(LCHK.EQ.1) THEN
```

where LCHK is set in the predictor corrector. If set to 1 then update nieghbor list.

```
K=0
           DO 302 I=1,NP
                NABORS(I)=K+1
                DO 299 L=1,3
                     RI(L)=RO(I,L)
299
                CONTINUE
                KI=KTYPE(I)
С
c cuts out all but C,H,Si, and Ge
                if(ki.ge.5) go to 302
С
                DO 301 J=1,NP
С
                     IF(I.EQ.J) GO TO 301
C
                     KJ=KTYPE(J)
c cuts out all but C,H,Si, and Ge
C
                     if(kj.ge.5) go to 301
                     RLIS=RLIST(KI,KJ)
С
                     RSQ=0.0D0
                     DO 298 L=1,3
                           RR(L)=RI(L)-RO(J,L)
                           RR(L)=RR(L) -
                                 CUBE(L)*ANINT(RR(L)/CUBE(L))
     &
                           RSQ=RSQ+RR(L)*RR(L)
                           IF(RSQ.GT.RLIS) GO TO 301
298
                     CONTINUE
С
405
                     CONTINUE
                     K=K+1
                     LIST(K)=J
                     IVCT2B(K)=I
                      JVCT2B(K)=J
C
```

```
301
                CONTINUE
302
           CONTINUE
C
           NABORS(NP+1)=K+1
           KEND=K
           if(kend.gt.nlmax) then
                 write(*,*) 'kend exceeds nlmax'
                 write(*,*) 'kend,nlmax = ',kend,nlmax
                 write(*,*) 'increase nlmax and recompile'
                 include 'close.inc'
                 stop
           endif
С
            write(*,*) 'kend= ',kend
      ENDIF
```

Loop over all neighbors and grab atom number and atom ID.

```
DO 320 K=1,KEND

I=IVCT2B(K)

J=JVCT2B(K)

KI=KTYPE(I)

KJ=KTYPE(J)
```

LCHECK determine if part of REBO parameter set Calculate inter nieghbor distance and store r^2 in COR

If distance is greater than cut off leave LCHECK = 0 and skip interaction.

```
IF(RSQ.GT.RMAX(KI,KJ)) GOTO 320
```

Otherwise set REBO atoms 1-10 to LCHECK = 1 and other atoms to LCHECK = 2 which was originally tight binding

```
if((kj.le.10).and.(ki.le.10)) LCHECK(K)=1
if((kj.ge.10).and.(ki.ge.10)) LCHECK(K)=2
```

Calculate table value of r and store r in RCOR

```
RC=SQRT(RSQ)
rt = rc/ddtab(ki,kj)
it = min(int(rt) + 1,ntab-1)
RCOR(K)=RC
```

$$rt = \frac{r_{ij}}{dr_{TAB}}$$

it = minimum(INT(rt))ortotalnumberofelementsinlookuptable)

Interpolate table values of pair potential by finding the table value and adding the difference of the current value and the next value times the difference between r and its truncated integer value it used for the look up table

```
WW(K)=TABFC(ki,kj,it)
             +(TABFC(ki,kj,it+1)-TABFC(ki,kj,it))*(rt-it+1)
&
      DWW(K)=TABDFC(ki,kj,it)
            +(TABDFC(ki,kj,it+1)-TABDFC(ki,kj,it))*(rt-it+1)
&
     EXX1(K) = atable(ki,kj,it)
               +(atable(ki,kj,it+1)-atable(ki,kj,it))*(rt-it+1)
&
     DEXX1(K) = datable(ki,kj,it) +
         (datable(ki,kj,it+1)-datable(ki,kj,it))*(rt - it +1)
&
     IF(I.GE.J) GO TO 320
     vv = rtable(ki,kj,it)
               +(rtable(ki,kj,it+1)-rtable(ki,kj,it))*(rt-it+1)
&
     rp = drtable(ki,kj,it)
               +(drtable(ki,kj,it+1)-drtable(ki,kj,it))*(rt-it+1)
&
```

The cut off function interpolation is writen explicitly the other values are calculated in the same manner.

$$WW = f^{C}(it) + (f^{C}(it + dr_{TAB}) - f^{C}(it))(r - it) = f^{C}(r_{TAB})$$

$$DWW = \frac{df^{c}(r_{TAB})}{dr}$$

$$EXX1 = V^{A}(r_{TAB})$$

$$DEXX1 = \frac{V^{A}(r_{TAB})}{dr}$$

$$vv = V^{R}(r_{TAB})$$

$$rp = \frac{V^{R}(r_{TAB})}{dr}$$

The values for f^C and V^A are stored to be later multiplied by b_{ij} , while V^R is added to the total energy.

tote = tote +
$$vv$$

eatom(i) = eatom(i) + $vv/2.0d0$
eatom(j) = eatom(j) + $vv/2.0d0$

$$E_{total} = \sum_{pair=1}^{K} E_{pair}$$
$$E_{atom} = \frac{1}{2} \sum_{pair} E_{pair}$$

Store repulsive force for pair

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$$F_{pair}(k, x/y/z) = \frac{dV^R}{dr}r$$

Add repulsive force to total force. Loop over all pairs and if $lcheck \neq 0$ ($r \not \in D^{MAX}$) and I < J

$$RNP(I,:) = \sum F_{pair}$$

2.5 pibond.f

Loop over all atoms and determine connectivity

```
DO 500 I=1,NP

JBEGIN=NABORS(I)

JEND=NABORS(I+1)-1

DO NN = 1,NTYPES

XHC(I,NN)=1.0d0

ENDDO

IF(JBEGIN.GT.JEND) GO TO 500

DO 490 J=JBEGIN,JEND

IF(LCHECK(J).ne.1) GO TO 490

JN=LIST(J)

XHC(I,KTYPE(JN))=XHC(I,KTYPE(JN))+WW(J)

490 CONTINUE

500 CONTINUE
```

If LCHECK is equal to 1 (REBO pair under D^{MAX} cut off distance) and atom number JN from LIST. Sum the cut off function of J ?? and store in XHC array.

$$XHC(I, KJ) = N_J^{KJ}$$

Loop over all atoms and its neighbors

Get \hat{r} values from COR array

```
DO 401 MM=1,3
CJ(MM)=COR(J,MM)
401 CONTINUE
```

Set SIJ = ||r|| [Var:??] and $RSQIJ = r \cdot r$ and KIKIJ = KI + KJ. Intial three body variables (NK,XSIJ,SSUMK,CONK) and local arrays.

```
NK=0
XSIJ=0.0d0
SSUMK=0.0d0
CONK=0.0D0
XNI(:)=XHC(I,:)
XNI(KJ) = XNI(KJ) - WW(J)
QI =-1.0d0* FLOAT(NTYPES)
DO NN = 1,NTYPES
QI = QI + XNI(NN)
ENDDO
SDALIK=0.0D0
```

Loop over all other neighbors other than J and keep track of how many there are with $\ref{eq:condition}$?

Calculate the cangle between K - I - J using the law of cosines.

$$cos(\theta) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$D0 \ 402 \ \text{MM=1,3}$$

$$\text{XK(NK,MM)=COR(K,MM)-CJ(MM)}$$

$$\text{RSQ2=RSQ2+XK(NK,MM)*XK(NK,MM)}$$

$$CONTINUE$$

$$SS=2.0d0*SIJ*S3$$

$$RR=RSQIJ-RSQ3$$

$$COSTH=(RSQIJ+RSQ3-RSQ2)/SS$$

$$(69)$$

$$XK(NK, D) = r_{KI}\hat{D} - r_{JI}\hat{D}$$

$$RSQ2 = (\hat{r}_K - \hat{r}_J)^2 = r_{KJ}^2 = c^2$$

$$SS = 2r_{KI}r_{JI} = 2ab$$

$$RR = r_{JI}^2 - r_{KI}^2 = a^2 - b^2$$

$$COSTH = \frac{r_{JI}^2 + r_{KI}^2 - r_{KJ}^2}{2r_{KI}r_{JI}}$$

Round in case ||cos|| > 1

IF(COSTH.GT.1.0D0) COSTH=1.0D0
IF(COSTH.LT.-1.0D0) COSTH=-1.0D0

Store $cos(\theta_{kij})$ in COSK and calculate sin

```
SINK(NK)=SQRT(1.0D0-COSTH*COSTH)
IF(ACOS(COSTH).GT.PI) SINK(NK)=-SINK(NK)
```

$$sin = \sqrt{1 - cos^2}$$

2.5.1 carbon anglur function

If KI is carbon Calculate spline index IG and spline values GANGLE and DGDTHET

```
IG=IGC(INT(-COSTH*12.0D0)+13)
                     IF(KI.EQ.1) THEN
                           IF(IG.NE.4) THEN
                                GANGLE=SPGC(1,IG)+SPGC(2,IG)*COSTH
                                DGDTHET=SPGC(2, IG)
                                DO 45 JJ=3,6
                                     GANGLE=GANGLE+SPGC(JJ,IG)
                                                   *(COSTH**(JJ-1))
     &
                                     DGDTHET=DGDTHET+SPGC(JJ,IG)*
     &
                                             (JJ-1)*(COSTH**(JJ-2))
45
                                CONTINUE
                           ELSE
                                ALI=0.0D0
                                DALI=0.0D0
                                IF(QI.LT.XQM) THEN
C####
                                      QQE=QI-2.0D0
С
                                      QQM=QQE/(QI-XQM)
                                      ALI=ATT*EXP(0.05D0*QQE*QQM)
С
                                      DALI=ALI*0.05D0*QQM*(2.0D0-QQM)
С
C####
                                     ALI=1.0D0
                                     IF(QI.GT.ATT) THEN
                                          DTEMP=PQ*(QI-ATT)
                                          ALI=(1.0D0+COS(DTEMP))/2.0D0
                                          DALI=-PQ/2.ODO*SIN(DTEMP)
                                     ENDIF
C####
                                ENDIF
                                GANGLE=SPGC(1,IG)+SPGC(2,IG)*COSTH
                                DGDTHET=SPGC(2,IG)
                                IG1=IG+1
                                GANGLE1=SPGC(1, IG1)+SPGC(2, IG1)*COSTH
```

$$IG = \begin{cases} 4 & 1 < \cos(\theta) < -\frac{1}{4} & 0 < \theta < 102.5 \\ 3 & -\frac{1}{4} < \cos(\theta) < -\frac{5}{12} & ? \\ 2 & -\frac{1}{2} < \cos(\theta) < -\frac{7}{12} & ? \\ 1 & -\frac{2}{3} < \cos(\theta) < -1 & ? \end{cases}$$

$$(70)$$

$$IG = 1, 2, 3GANGLE = a + bcos(\theta) + ccos(\theta)^{2} + dcos(\theta)^{3} + ecos(\theta)^{4} + fcos(\theta)^{5}$$
$$DGDTHET = b + 2ccos(\theta)^{1} + 3dcos(\theta)^{2} + 4ecos(\theta)^{3} + 5fcos(\theta)^{4}$$

2.5.2 Oxygen angular function

$$G(\theta) = a_s^0 + a_s^1 (a_s^0 - \cos(\theta))^2$$
$$\frac{dG(\theta)}{d\cos(\theta)} = 2a_s^1 (\cos(\theta) - a_s^0)$$

2.5.3 Hydrogen angular function

```
GANGLE=SPGH(1,IG)+SPGH(2,IG)*COSTH
DGDTHET=SPGH(2,IG)

DO 46 JJ=3,6
GANGLE=GANGLE+SPGH(JJ,IG)*(COSTH**(JJ-1))
DGDTHET=DGDTHET+SPGH(JJ,IG)*

&
(JJ-1)*(COSTH**(JJ-2))
CONTINUE
```

2.5.4 Conjugate Evaluation

From Equ. ?? where X_{ik} is the degree of saturation

```
FC=WW(K)
                DFC=DWW(K)
                CFUNI(NK)=0.0d0
                DCFUNI(NK)=0.0d0
                IF(KK.EQ.icarb.and.KI.NE.isulfur.and.
                   KJ.NE.isulfur ) THEN
&
                     FNTP = FLOAT(NTYPES)
                     XX = - FC -1.0d0*FNTP
                     DO NN = 1,NTYPES
                       XX=XX + XHC(KN,NN)
                     ENDDO
                     IF(XX.LT.3.0d0) THEN
                          IF(XX.LE.2.0d0) THEN
                               CFUNI(NK)=1.0d0
                          ELSE
                               PX=PI*(XX-2.0d0)
                               CFUNI(NK)=(1.0+COS(PX))/2.0d0
                               DCFUNI(NK)=-FC*SIN(PX)*PI/2.0d0
                          ENDIF
                          ENDIF
                ENDIF
                CONK=CONK+FC*CFUNI(NK)
```

$$FC = f_{ik}^{C}$$

$$DFC = \frac{df_{ik}^{C}}{d??}$$

$$XX(KJ) = X_{ik} = \sum_{k \neq i,j} f_{ik}^{KJ}$$

$$PX = \pi(X_{ik} - 2)$$

$$CFUNI = F(X_{ik})$$

$$DCFUNI = \frac{dF(X_{ik})}{dX_{ik}}$$

$$CONK = \sum_{k \neq i,j} f_{ik}^{C} * F(X_{ik})$$

Evaluate λ in b_{ij} if XDB not equal zero.

&

EXX=1.0

ENDIF

ELSE

$$XDB(2,2,2) = 4.0D0$$

$$XDB(2,1,2) = 4.0D0$$

$$XDB(2,2,1) = 4.0D0$$

$$XDB(2,2,1) = 4.0D0$$

$$XDB(2,1,1) = 4.0D0$$

$$XDB(1,2,1) = 0.0D0$$

$$XDB(1,2,2) = 0.0d0$$
(71)

$$XDB = \lambda(KI, KJ, KK)$$

2.5.5 Calculate dihedral angles

I think??

DCTDJK=-2.0d0/SS
DCTDIJ=(RR+RSQ2)/(SS*RSQIJ)
DCTDIK=(-RR+RSQ2)/(SS*RSQ3)
DCTJK(NK)=DCTDJK
DCTIJ(NK)=DCTDIJ
DCTIK(NK)=DCTDIK
GS=GANGLE*EXX

SSUMK=SSUMK + FC*GS

XTEMP=FC*EXX*DGDTHET

GFX=GS*FC*XDB(KI,KJ,KK)

XSIJ=XSIJ + XTEMP*DCTDIJ+GFX/SIJ

XSIK(NK)=(GS*DFC-GFX)/S3+XTEMP*DCTDIK

SDALIK=SDALIK+EXX*FC*DALDIK

XSJK(NK) = XTEMP*DCTDJK

DCTDJK =

Repeat for JI side of the bond

2.6 Corrdination function

Initialize spline values

EXNIJ=0.0d0 DEXNI(:)=0.0d0 EXNI2J = 0.0d0 DEXNI2(:) = 0.0d0

Set the thresh hold value of N^S to use sulfur spline.

```
pno_min = 1.01d0
pno_max = 1.21d0
delt_po = pno_max - pno_min
pie0 = dacos(-1.d0)
pij_min =-0.39d0
```

Round the number of nieghbors to find knots for spline evaluation. And remember all these values have a factor of +1 to allow for fortran 77 indexing, ie $N \neq 0$. Which leads to a lot of -1's

```
NH=INT(XNI(ihyd)+1.0D-12)
NC=INT(XNI(icarb)+1.0D-12)
NSX=INT(XNI(isulfur)+1.0D-12)
NTCH = NH + NC
```

IF C-C bond use P_{CC} and the number of sulfur atoms is less than the threshold value and set the derivative with respect to sulfur the same as the derivative with respect to carbon.

```
IF(KI.EQ.icarb.and.KJ.EQ.icarb) THEN
                  IF( (ABS(FLOAT(NH)-XNI(ihyd)).GT.1.0d-8).OR.
                      (ABS(FLOAT(NC+NSX)-XNI(icarb)-XNI(isulfur))
&
                      .GT.1.0d-8)) THEN
&
                       CALL BCUINT(KI, KJ, XNI(2), XNI(1)+XNI(3)-1.0d0
                                      ,NH,NC+NSX - 1,
&
                                     EXNIJ, DEXNI(2), DEXNI(1))
&
                 ELSE
                       EXNIJ=XH(KJ,NH,NC)
                       DEXNI(2)=XH1(KJ,NH,NC)
                       DEXNI(1)=XH2(KJ,NH,NC)
                 ENDIF
                       DEXNI(isulfur) = DEXNI(icarb)
```

Use if statement to determine if the value of N_J^{KJ} is close enough to an integer to evaluate the spline at the knots. Add N^S value to N^C value as a small correction.

$$N^{H} = \sum_{k \neq i,j}^{H} f_{IK}^{c}(r_{IK})N^{C} = \sum_{k \neq i,j}^{C} f_{IK}^{c}(r_{IK}) + \sum_{k \neq i,j}^{S} f_{IK}^{c}(r_{IK})P_{CC} = (N^{H}, N^{C}, 1)$$

If C-H bond calculate spline values from P_{CH} if the number of sulfur neighbors is less than the threshold

If the number of sulfur neighbors is greater than the threshold then use switching function. Interpolate values from each spline P^{CH} and P^{CHS} .

```
ELSEIF ( XNI(isulfur).GE.pno_max) THEN

IF((ABS(FLOAT(NTCH)-XNI(icarb)-XNI(ihyd)).GT.1.0d-8)

& .or.(ABS(FLOAT(NSX) - XNI(isulfur)).GT.1.0d-8)) THEN

CALL SULBCUINT (KI,KJ,XNI(icarb)+XNI(ihyd)-1.0d0,

XNI(isulfur),NTCH-1,NSX,EXNIJ

DEXNI(icarb),DEXNI(isulfur))
```

```
DEXNI(ihyd) = DEXNI(icarb)
                   IF ( EXNIJ.LT.pij_min) THEN
                      EXNIJ = pij_min
                   ENDIF
                 ELSE
                   EXNIJ = XHS(KI,KJ,NTCH-1,NSX)
                   DEXNI(icarb) = XHS1(KI,KJ,NTCH-1,NSX)
                   DEXNI(isulfur) = XHS2(KI,KJ,NTCH-1,NSX)
                   DEXNI(ihyd) = DEXNI(icarb)
                 ENDIF
               ELSE !switching region
                 IF ( (ABS(FLOAT(NH) - XNI(ihyd)).GT.1.0d-8).OR.
                       (ABS(FLOAT(NC) - XNI(icarb)).GT.1.0d-8)) THEN
  &
                   CALL BCUINT(KI, KJ, XNI(2), XNI(1), NH, NC,
                                   EXNIJ,DEXNI(2),DEXNI(1) )
  &
                 ELSE
                   EXNIJ=XH(KJ,NH,NC)
                   DEXNI(2)=XH1(KJ,NH,NC)
                   DEXNI(1)=XH2(KJ,NH,NC)
                 ENDIF
                 IF((ABS(FLOAT(NTCH)-XNI(icarb)-XNI(ihyd)).GT.1.0d-8)
                .or.(ABS(FLOAT(NSX) - XNI(isulfur)).GT.1.0d-8)) THEN
  &
                    CALL SULBCUINT (KI, KJ, XNI(icarb) + XNI(ihyd) - 1.0d0,
                                   XNI(isulfur),NTCH-1,NSX,EXNI2J
  &
                                  ,DEXNI2(icarb),DEXNI2(isulfur) )
  &.
                     DEXNI2(ihyd) = DEXNI2(icarb)
                   IF ( EXNIJ.LT.pij_min) THEN
                      EXNI2J = pij_min
                   ENDIF
                 ELSE
                   EXNI2J = XHS(KI,KJ,NTCH-1,NSX)
                   DEXNI2(icarb) = XHS1(KI,KJ,NTCH-1,NSX)
                   DEXNI2(isulfur) = XHS2(KI,KJ,NTCH-1,NSX)
                   DEXNI2(ihyd) = DEXNI2(icarb)
                 ENDIF
Then interpolate values
```

```
fijmid = 0.5d0*(1.d0+cos(pie0*(XNI(isulfur)-pno_min)
               /delt_po))
&
     dfijmid = - pie0 * 0.5d0 *sin(pie0*(XNI(isulfur)-pno_min)
               /delt_po)/delt_po
&
     pijtmp = EXNIJ*fijmid + EXNI2J*(1-fijmid)
     DEXNI(icarb) = DEXNI(icarb) * fijmid + DEXNI2(icarb) * (1 - fijmid)
     DEXNI(ihyd)=DEXNI(ihyd)*fijmid+DEXNI2(ihyd)*(1-fijmid)
```

```
DEXNI(isulfur) = DEXNI(isulfur)*fijmid +
                          DEXNI2(isulfur)*(1-fijmid) +
  &
  &
                          (EXNIJ-EXNI2J)*dfijmid
       EXNIJ = pijtmp
  fijmid = ??
If H- not to oxygen. set everything to zero
             ELSEIF (KI.EQ.ihyd.and.KJ.NE.isulfur) THEN
                EXNIJ = 0.0d0
                DEXNI(:) = 0.0d0
Otherwise H-O bond
             ELSE
                IF( (ABS(FLOAT(NTCH)-XNI(icarb)-XNI(ihyd)).GT.1.0d-8)
                  .or.(ABS(FLOAT(NSX)-XNI(isulfur)).GT.1.0d-8)) THEN
  &
                 CALL SULBCUINT(KI,KJ,XNI(icarb)+XNI(ihyd)-1.0d0,
                                    XNI(isulfur), NTCH-1, NSX, EXNIJ
  &
                                   ,DEXNI(icarb),DEXNI(isulfur) )
  &
                      DEXNI(ihyd) = DEXNI(icarb)
                    IF ( EXNIJ.LT.pij_min) THEN
                       EXNIJ = pij_min
                    ENDIF
                ELSE
                    EXNIJ = XHS(KI,KJ,NTCH-1,NSX)
                    DEXNI(icarb) = XHS1(KI,KJ,NTCH-1,NSX)
                    DEXNI(isulfur) = XHS2(KI,KJ,NTCH-1,NSX)
                    DEXNI(ihyd) = DEXNI(icarb)
                ENDIF
             ENDIF
Calculate b_{ij}, and retreave V^A from EXX1.
             DIJ=(1.0d0+EXNIJ+SSUMK)
             BIJ=DIJ**(-0.50D0)
             DJI=(1.0d0+EXNJI+SSUML)
             BJI=DJI**(-0.50D0)
             DBDZI=-0.50D0*BIJ/DIJ
             DBDZJ=-0.50D0*BJI/DJI
             VATT=EXX1(J)
```

$$DIJ = 1 + P_{IJ} + G(\theta)$$

$$BIJ = \frac{1}{(1 + P_{IJ} + G(\theta))^{\frac{1}{2}}}$$

$$DJI = 1 + P_{JI} + G(\theta)$$

$$BJI = \frac{1}{(1 + P_{JI} + G(\theta))^{\frac{1}{2}}}$$

$$DBDZI = \frac{1}{2} \frac{1}{(1 + P_{IJ} + G(\theta))^{\frac{3}{2}}}$$

$$DBDZJ = \frac{1}{2} \frac{1}{(1 + P_{JI} + G(\theta))^{\frac{3}{2}}}$$

$$VATT = V^{A}(J)$$

Calculate values for conjugate term.

```
DRADI=0.0d0
DRADJ=0.0d0
DRDC=0.0d0
CONJUG = 1.0D0 + (CONK**2) + (CONL**2)
XNT1 = 1.0d0 - 1.0d0*FLOAT(NTYPES)
XNT2 = 1.0d0 - 1.0d0*FLOAT(NTYPES)
DO NN = 1,NTYPES
    XNT1 = XNT1 + XNI(NN)
    XNT2 = XNT2 + XNJ(NN)
ENDDO
```

2.6.1 Energy

TOTE=TOTE-BTOT*VATT

```
eatom(i) = eatom(i) - btot*vatt/2.0d0
eatom(jn)= eatom(jn) - btot*vatt/2.0d0

VDBDI=VATT*DBDZI
VDBDJ=VATT*DBDZJ
VDRDC=VATT*DRDC
VDRDI=VATT*DRADI
VDRDJ=VATT*DRADJ
```

TOTE =

2.6.2 Forces

407

Start suming forces

3 Predictor corrector

4 Thermostats

4.1 Langevin Equation

```
DO 30 II=1,NTA
I=NLIST(II)
```

```
BM=BET*XMASS(KTYPE(I))

SM=SQRT(XMASS(KTYPE(I)))

DO 29 J=1,3

rrzp = RNP(I,J)

RNP(I,J)=RNP(I,J)-BM*R1(I,J)-SM*GL(II+(J-1)*NTA)

29 CONTINUE

30 CONTINUE
```

5 Terminology

- 1. atom number sequencial number of atom with in list of all atoms in the simulation
- 2. atom ID internal reference number 1-N total parameterized elements

6 Variables

- 1. I atom number [I]
- 2. KI atom ID [L]
- 3. J nieghbor atom number [L]
- 4. KJ niegghbor atom ID [L]
- 5. KTYPE(I) = KI atom number to atom ID [G]
- 6. SIJ = $||r_{ij}||$ [L]
- 7. NK = number of K nieghbors [L]
- 8. D = spacial dimension x,y,z [L]