

5. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If  $Q$  is the maximum charge on the capacitor, then the total energy is

$$U = \frac{Q^2}{2C} = \frac{(2.90 \times 10^{-6} \text{ C})^2}{2(3.60 \times 10^{-6} \text{ F})} = 1.17 \times 10^{-6} \text{ J}.$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If  $I$  is the maximum current, then  $U = LI^2/2$  leads to

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.168 \times 10^{-6} \text{ J})}{75 \times 10^{-3} \text{ H}}} = 5.58 \times 10^{-3} \text{ A}.$$

7. (a) The mass  $m$  corresponds to the inductance, so  $m = 1.25$  kg.

(b) The spring constant  $k$  corresponds to the reciprocal of the capacitance. Since the total energy is given by  $U = Q^2/2C$ , where  $Q$  is the maximum charge on the capacitor and  $C$  is the capacitance,

$$C = \frac{Q^2}{2U} = \frac{(175 \times 10^{-6} \text{ C})^2}{2(5.70 \times 10^{-6} \text{ J})} = 2.69 \times 10^{-3} \text{ F}$$

and

$$k = \frac{1}{2.69 \times 10^{-3} \text{ m / N}} = 372 \text{ N / m.}$$

(c) The maximum displacement corresponds to the maximum charge, so  $x_{\text{max}} = 1.75 \times 10^{-4} \text{ m}$ .

(d) The maximum speed  $v_{\text{max}}$  corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175 \times 10^{-6} \text{ C}}{\sqrt{(1.25 \text{ H})(2.69 \times 10^{-3} \text{ F})}} = 3.02 \times 10^{-3} \text{ A.}$$

Consequently,  $v_{\text{max}} = 3.02 \times 10^{-3} \text{ m/s}$ .

17. (a) We compare this expression for the current with  $i = I \sin(\omega t + \phi_0)$ . Setting  $(\omega t + \phi) = 2500t + 0.680 = \pi/2$ , we obtain  $t = 3.56 \times 10^{-4}$  s.

(b) Since  $\omega = 2500 \text{ rad/s} = (LC)^{-1/2}$ ,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500 \text{ rad/s})^2 (64.0 \times 10^{-6} \text{ F})} = 2.50 \times 10^{-3} \text{ H}.$$

(c) The energy is

$$U = \frac{1}{2} LI^2 = \frac{1}{2} (2.50 \times 10^{-3} \text{ H}) (1.60 \text{ A})^2 = 3.20 \times 10^{-3} \text{ J}.$$

18. (a) Since the percentage of energy stored in the electric field of the capacitor is  $(1 - 75.0\%) = 25.0\%$ , then

$$\frac{U_E}{U} = \frac{q^2 / 2C}{Q^2 / 2C} = 25.0\%$$

which leads to  $q / Q = \sqrt{0.250} = 0.500$ .

(b) From

$$\frac{U_B}{U} = \frac{Li^2 / 2}{LI^2 / 2} = 75.0\%,$$

we find  $i / I = \sqrt{0.750} = 0.866$ .

25. Since  $\omega \approx \omega'$ , we may write  $T = 2\pi/\omega$  as the period and  $\omega = 1/\sqrt{LC}$  as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$\begin{aligned} t = 50T &= 50 \left( \frac{2\pi}{\omega} \right) = 50 \left( 2\pi\sqrt{LC} \right) = 50 \left( 2\pi\sqrt{(220 \times 10^{-3} \text{ H})(12.0 \times 10^{-6} \text{ F})} \right) \\ &= 0.5104 \text{ s.} \end{aligned}$$

The maximum charge on the capacitor decays according to  $q_{\max} = Qe^{-Rt/2L}$  (this is called the *exponentially decaying amplitude* in §31-5), where  $Q$  is the charge at time  $t = 0$  (if we take  $\phi = 0$  in Eq. 31-25). Dividing by  $Q$  and taking the natural logarithm of both sides, we obtain

$$\ln \left( \frac{q_{\max}}{Q} \right) = -\frac{Rt}{2L}$$

which leads to

$$R = -\frac{2L}{t} \ln \left( \frac{q_{\max}}{Q} \right) = -\frac{2(220 \times 10^{-3} \text{ H})}{0.5104 \text{ s}} \ln(0.99) = 8.66 \times 10^{-3} \Omega.$$

26. The charge  $q$  after  $N$  cycles is obtained by substituting  $t = NT = 2\pi N/\omega'$  into Eq. 31-25:

$$\begin{aligned} q &= Qe^{-Rt/2L} \cos(\omega't + \phi) = Qe^{-RNT/2L} \cos[\omega'(2\pi N/\omega') + \phi] \\ &= Qe^{-RN(2\pi\sqrt{L/C})/2L} \cos(2\pi N + \phi) \\ &= Qe^{-N\pi R\sqrt{C/L}} \cos \phi. \end{aligned}$$

We note that the initial charge (setting  $N = 0$  in the above expression) is  $q_0 = Q \cos \phi$ , where  $q_0 = 6.2 \mu\text{C}$  is given (with 3 significant figures understood). Consequently, we write the above result as  $q_N = q_0 \exp(-N\pi R\sqrt{C/L})$ .

(a) For  $N = 5$ ,  $q_5 = (6.2 \mu\text{C}) \exp(-5\pi(7.2\Omega)\sqrt{0.0000032\text{F}/12\text{H}}) = 5.85 \mu\text{C}$ .

(b) For  $N = 10$ ,  $q_{10} = (6.2 \mu\text{C}) \exp(-10\pi(7.2\Omega)\sqrt{0.0000032\text{F}/12\text{H}}) = 5.52 \mu\text{C}$ .

(c) For  $N = 100$ ,  $q_{100} = (6.2 \mu\text{C}) \exp(-100\pi(7.2\Omega)\sqrt{0.0000032\text{F}/12\text{H}}) = 1.93 \mu\text{C}$ .